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The Generalized Median Tour Problem: Modeling, Solving and an Application

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ABSTRACT We introduce, formulate, and solve the Generalized Median Tour Problem, which is motivated in the health supplies distribution for urban and rural areas. A region comprises districts that must be served by a specialized vehicle visiting its health facilities. We propose a distribution strategy to serve these health facilities efficiently. A single tour is determined that visits a set of health facilities (nodes) composed of disjoint clusters. The tour must visit at least one facility within each cluster, and the unvisited facilities are assigned to the closest facility on the tour. We minimize the sum of the total tour distance and the access distance traveled by the unvisited facilities. Efficient formulations are proposed and several solution strategies are developed to avoid subtours based on branch & cut. We solve a set of test instances and a real-world instance to show the efficiency of our solution approaches.

INDEX TERMS Combinatorial optimization, health supply chain, generalized median tour.

I. INTRODUCTION

Humanity is frequently exposed to different types of natural disasters or sanitary crises worldwide, such as earthquakes, tsunamis (e.g., Chile 1960 and 2010, Japan 2011 and Indonesia 2004), tornadoes (e.g., Katrina 2005 and Irma 2017), and pandemics (e.g., Covid-19, H1N1, SARS, MERS, etc.). Therefore, an efficient supply chain management is crucial to assure the distribution of essential goods (food and water for people and animals) and health supplies (medical samples, tests, vaccines, blood, and face masks). In the case of natural disasters, the main challenge, after it occurs, is to redesign, repair, or completely restore the distribution networks. On the other hand, in a sanitary crisis, the need for adapting and supplying the existing health facilities is required to cover and protect affected inhabitants and regions in a short time. Thus, the aim is to avoid large transportation distances and times, while yielding an effective, efficient, and fast distribution of the required health supplies.

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In the case of critical health supplies (e.g., blood or vaccines), distribution traceability and items conservation are relevant to ensure their quality and effectiveness. Hence, the number of stops or transfer stations should be reduced (cross-docking or intermediate facilities). Simultaneously, special and expensive transportation modes are usually required (e.g., airplanes, ambulances, and special trucks), which provide proper conditions inside the vehicles, including cooling, security, and stability [1]–[3]. This issue is particularly relevant and challenging for geographically isolated rural areas in undeveloped and developing countries, given the limited, or even rudimentary nature of their local transportation systems.

Accordingly, this research is motivated by the challenge of designing a distribution network for essential health supplies for a set of mixed urban/rural districts or regions under a sanitary crisis of a pandemic or epidemic. Under these scenarios, political authorities are forced to establish lock-downs and cordons sanitaires in specific regions, usually coinciding with districts or municipalities. Furthermore, the distribution network must allow a fast distribution of the health supplies to a set of health facilities that can receive and deliver these supplies to the involved inhabitants.

An inherent constraint that arises in these scenarios, associated with the political and administrative organization matters, states that each non-selected facility must be assigned only to facilities at the same district to which it belongs. This constraint ensures the duty for the distribution process to specific related sanitary and political authorities at the districts. Besides, it facilitates the distribution process, avoiding travels between different districts, and thus helping to observe lockdowns and cordons sanitaires.

Summarizing, at least one of the existing health facilities at each district has to be selected for a first-stage distribution, and every non-selected facility must be assigned to a single selected facility in the same district. The first-stage distribution consists of a visit sequence considering only the selected facilities at all involved districts from a known specific depot (e.g., an airport or a regional hospital). At the same time, the second stage-distribution process is defined as direct trips between the selected health facilities and the assigned (non-selected) facilities. Hence, the distribution system must be optimally designed to minimize the total time of the first-stage distribution process plus the total time associated with the second-stage distribution process.

A relevant additional practical concern is observed, which requires that a truck that visits the selected facilities at all districts may enter and exit each district only once, ensuring proper and safe handling of the supplies inside the truck and proper traceability of the distribution process.

Fig. 1 presents a representation of a southern area of Chile (generated in Google My Maps ©), containing 12 districts (framed areas with red lines), where the circles represent the health facilities, and the grey circles represent the selected facilities for the first distribution stage. Thus, the health supplies arrive at the main airport (yellow circle) and subsequently are distributed by a refrigerated truck (black lines) to the selected points on the tour. Next, all remaining facilities are assigned to a selected facility in its district (brown lines). Note that it is possible to select more than one health facility at each district to be part of the truck's route. Finally, the objective is to minimize the total travel time of the vehicle and the travel time between the selected facilities and the other non-selected health facilities.

In order to address the previously discussed distribution network design problem, this paper aims at introducing, modeling, and solving the Generalized Median Tour Problem (GMTP). In this novel problem, the nodes (health facilities) are grouped into clusters (districts), and a single tour or route must visit all the clusters, such that the nodes within each cluster that are not in the tour must be assigned to the closest node within the tour. Also, the tour must visit each cluster exactly one time, and the tour visits at least one node per cluster, like the Generalized Traveling Salesman Problem [4], GTSP, and Insular Traveling Salesman Problem [5], InTSP.

Similar applications of this problem arise when: both construction and access costs (or distances) assumed by clients

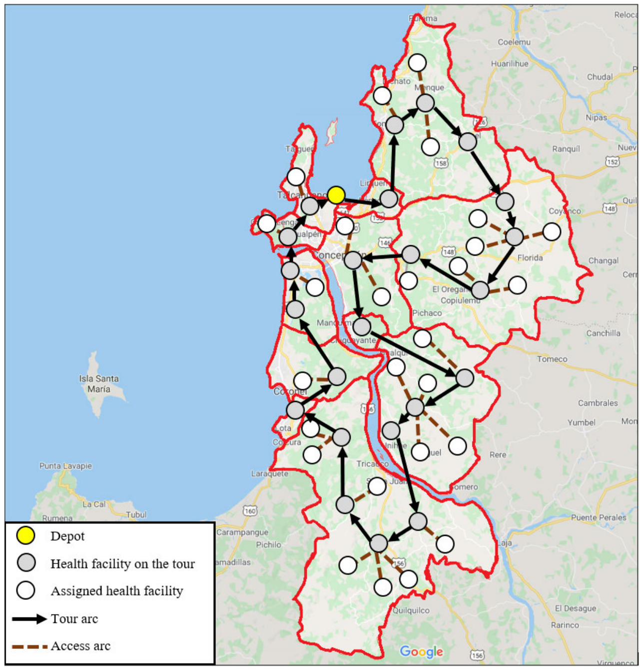


FIGURE 1. Representation of the health supplies distribution network in a geographical area with 12 zones.

are significant; there is no capacity constraint for the vehicle, and the clients are grouped into disjoints clusters. The studied problem may be applied in food delivery in rural areas, water delivery systems in humanitarian logistics, parcel delivery, school bus routing, telecommunication network design, etc.

A distinctive feature of the GMTP in comparison to the InTSP (where the ship only can visit the docks of the islands) is that the route can access all the demand nodes in the GMTP. Additionally, the InTSP considers that each cluster may be visited (enter and exit) more than once. Notice that if the access distances between clusters in the GMTP are arbitrarily high, then the problem is reduced to the Median Tour Problem, MTP [6].

Note that GMTP is NP-Hard. If the access distance is negligible, the problem is also reduced to the Generalized Traveling Salesman Problem, which is NP-Hard. Furthermore, if the access distances are extremely high, the solution implies to visit all the nodes for each cluster, obtaining the Clustered Traveling Salesman Problem (CTSP).

The paper is organized as follows. Section II presents an updated literature review of related problems. Section III introduces a MIP formulation for the problem assuming at least one node visited per cluster. Section IV exposes the numerical results to solve the GMTP and a real application about health supplies in a region in southern Chile. Section V presents conclusions and future work.

II. LITERATURE REVIEW

The GMTP is motivated by the InTSP, and it is an extension of the GTSP and the MTP. The InTSP, recently introduced by [5], involves maritime and ground transportation costs with a bi-objective perspective. This problem consists of defining

TABLE 1. Main differences of the GMTP in relation with closest previous problems.

Related Problems	Visited nodes per cluster			Entrances/exits per cluster		Secondary cost in the		Bi-objective
	One	At least one	All	One	Al least one	Objective	Constraint	
RSP						✓		
MCP							✓	
MTP						✓		✓
GTSP	✓			✓				
Insular-TSP		✓			✓	✓		✓
Cluster-TSP			✓	✓				
GMTP		✓		✓		✓		

optimal sequences for collecting the waste generated inside a set of islands. Each island has a known set of docks or ports that potentially may be used to collect the generated waste. Besides, there is a ship departing from a fixed, known port. The ship must collect the waste from all the islands and returns to the starting port, having enough capacity to transport all waste on a single trip. Moreover, the waste generated on each island is transported inside the island to one or more docks where the ship will arrive, allowing an island to be visited more than once. Consequently, if each island (cluster) is visited once (in one or more docks), then this problem is similar to the GMTP. On the other hand, in the GTSP (introduced by [7]), the nodes form a set of clusters, and the objective is to build a minimum cost tour that starts and ends in a depot, visiting a single node of each cluster. If the distance or access cost between nodes inside the same cluster is 0, then the GMTP becomes the GTSP.

Some related vehicle routing problems that address the demand allocation to the visited nodes along the route is found in the extensive facility location problem literature [8]–[10]. Recently, [11], provide an updated review afterward commented and referenced by three specialized researchers [12]–[14], who provide remarkable future research directions. This family of the network design problems is characterized as connected structures aimed at serving a set of clients. Such structures may have the shape of a path, tree, tour, or sub-graph, and the demand nodes that are embraced by these topologies must be assigned to a node that belongs to the structure. There are vehicle routing applications where it is not possible to visit all clients directly, by economic, practical, or geographical considerations. In this case, the clients that are not visited must travel to the closest visited customer. For example, the Median Tour Problem (MTP) [6], determines a tour that must visit just p of the n possible nodes of the network. Two objectives are minimized: the total distance of the tour and the total travel distance for the nodes, reaching their closest stop on the tour. A closest problem to the MTP are the Ring Star Problem (RSP) and the Median Cycle Problem (MCP), where a single cycle through a set of nodes, minimizing the cost of the cycle, and the assignment costs of the nodes that are not in the cycle [9], [15]. An extensive review of transportation problems where the objective is to design a single cycle in a non-directed network is presented in [16].

As previously mentioned, another related problem is the GTSP, introduced by [7]. In this problem, the nodes form a set of clusters, and the objective is to build a minimum cost tour that starts and ends in a depot, visiting a single node of each cluster [17]. This problem has several real applications. [18] suggest that a wide variety of optimization problems can be modeled as a GTSP, e.g., the design of postbox collection routes, goods distribution by sea, etc. [19], propose an integer formulation for the GTSP. [20] introduce facets that define valid inequalities for the GTSP, and [4] develop a branch & cut algorithm to the symmetric case of the GTSP. Since the GTSP is a NP-Hard problem, there have been a lot of articles that develop heuristics, providing good feasible solutions to the problem in a short time [21]–[23].

A significantly related variant is the Clustered Traveling Salesman Problem (CTSP), where the nodes are separated in clusters, and all the nodes of each cluster must be visited consecutively before departing to another cluster or the depot. If the salesman visits a cluster, it cannot leave the cluster until all clients have been served [24]. Applications of this problem cover a wide range of areas. Several algorithms have been developed for solving the CTSP: approximation algorithms [25], [26], tabu search [27], Lagrangian relaxation [28]; genetic algorithms, [29], Grasp, [30]; hybrid heuristics [31]; memetic algorithms, [32]; and other heuristics [33]. A special case of the CTSP is the Ordered Clustered Traveling Salesman Problem (OCTSP), where a visit sequence to the clusters is defined a priori, [34], [35].

According to the previous review, the GMTP is a new combinatorial problem. Table 1 details the differences with closest problems presented in literature. The clients are separated into clusters that must be visited by a tour. The problem consists of determining a tour or a simple cycle that visits each cluster once, and the nodes within a cluster could be visited at least once. Some nodes may be out of the tour, which must be assigned to their closest node on the selected tour. Two types of distances are considered: the total distance of the tour, and the total access distance (sum of the access distances to reach the nearest node on the tour). The GMTP is an extension of the cycle location problems, using the mini-sum access criteria, such as the RSP, MCP, MTP, among others, where the nodes are grouped in disjoint clusters.

III. THE INTEGER PROGRAMMING MODEL FOR THE GENERALIZED MEDIAN TOUR PROBLEM

This Section aims at formulating de GMTP and also at proposing different valid cuts and strategies for solving the proposed GMTP formulation. Subsections III-A and III-B present the notation and the proposed IP formulation, respectively. Subsections III-C and III-D propose alternative valid inequalities for the GMTP, while Subsection III-E, establishes a combined strategy employing the two types of valid inequalities. Finally, Subsection III-F discusses some computational issues, and Subsection III-G describes the separation algorithm employed.

A. PROBLEM DESCRIPTION AND NOTATION

The GMTP consists in defining a single tour to serve a set of nodes, where the nodes belong to several disjoint clusters. Additionally, there is a single vehicle with enough capacity for serving all demands on a single tour. Thus, the tour must visit at least one node per cluster. Each non-visited node must be connected to one of the visited nodes at the same cluster.

Let $G = (N, A)$ be an asymmetric complete graph, where N is the set of nodes, and A is the set of arcs. Let K be the set of disjoint clusters, where N_p, with $p \in K$, is a subset of N. For the sake of simplicity, the cluster $N_1 = \{1\}$ contains only the depot node. Let c_{ij} be the travel distance over the arc $(i, j) \in A$. Also, every arc inside each cluster p, $(i, j) \in A_p$ with $i, j \in N_p$, has an access distance d_{ij}.

Let A' be the set of access arcs, i.e., $A' = \bigcup_{p \in K} A_p$. Notice that, if the vehicle travels from node i to node j, it travels in a distance c_{ij}. On the other hand, if a node i is not visited by the vehicle, it must be assigned to the visited node inside the cluster, traveling an access distance d_{ij}.

The decision variables are defined as follows:

$$x_{ij} = \begin{cases} 1 & \text{if the arc } (i, j) \in A \text{ is on the tour} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if node } i \text{ is assigned to the node } j\text{: } (i, j) \in A' \\ 0 & \text{otherwise} \end{cases}$$

B. PROBLEM FORMULATION

Accordingly, the problem is formulated as an integer programming (IP) model as follows:

GMTP

$$Min \sum_{i \in N} \sum_{j \in N \setminus \{i\}} c_{ij} x_{ij} + \sum_{p \in K} \sum_{i \in N_p} \sum_{j \in N_p \setminus \{i\}} d_{ij} y_{ij} \quad (1)$$

Subject to:

$$\sum_{i \in N_p} \sum_{j \in N \setminus N_p} x_{ij} = 1 \quad \forall p \in K \quad (2)$$

$$\sum_{i \in N \setminus N_p} \sum_{j \in N_p} x_{ij} = 1 \quad \forall p \in K \quad (3)$$

$$\sum_{i \in N \setminus \{j\}} x_{ij} = \sum_{i \in N \setminus \{j\}} x_{ji} \quad \forall j \in N \quad (4)$$

$$\sum_{j \in N \setminus \{i\}} x_{ij} \leq 1 \quad \forall i \in N \quad (5)$$

$$\sum_{i \in N} \sum_{j \in N \setminus \{i\}} x_{ij} \geq |K| \quad (6)$$

$$\sum_{j \in N_p} y_{ij} = 1 \quad \forall p \in K, i \in N_p \quad (7)$$

$$\sum_{i \in N \setminus \{j\}} x_{ij} = y_{jj} \quad \forall p \in K, j \in N_p \quad (8)$$

$$y_{ij} \leq y_{jj} \quad \forall p \in K, i, j \in N_p : i \neq j \quad (9)$$

$$\sum_{i \in S} \sum_{j \in S \setminus \{i\}} x_{ij} \leq |S| - 1 \quad \forall S \subset N \setminus \{1\} : |S| \geq 2 \quad (10)$$

$$x_{ij}, y_{ab} \in \{0, 1\} \quad \forall i, j \in N, p \in K, a, b \in N_p : (a, b) \in A' \quad (11)$$

The objective function (1) jointly minimizes the distance of the tour plus the access distance to the tour. Constraints (2) and (3) force the route to a single visit per cluster. Constraints (4) state the flow balance to each node. Constraints (5) assure a maximum of one visit per node. Constraint (6) assures that each cluster is visited at least once. Constraints (7) force the assignment of each node in a cluster to its closest node visited by the tour. Constraint set (8) indicates the assignment of node j to itself if the tour visits that node. Constraints (9) force the assignment of a node i to node j only if the node j is on the route. Constraints (10) preclude disconnected tours using flow variables. Constraint (11) states the binary nature of the variables. Note that the variables y_{ij} can be relaxed as continuous.

The number of constraints in set 10 is $O(2^{|N|})$, which makes this formulation intractable as it is. Thus, the next subsections present different strategies to deal with constraints (10) to solve the GMTP efficiently. First, a set of valid and tighter inequalities called *Packing cuts* are presented. Second, the Gavish and Graves [36] constraints are displayed. Third, a combination of the Gavish and Graves constraints and Packing cuts is used. Finally, we present a branch & cut scheme and separation algorithms for the required strategies.

C. PACKING CUTS

The constraints (12), also known as *Packing cuts*, are valid cuts for the GMTP:

$$\sum_{i \in S} \sum_{j \in S \setminus \{i\}} x_{ij} + \sum_{i \in S} \sum_{j \in S \setminus \{i\} : (i,j) \in A'} y_{ij} \leq |S| - 1 \quad \forall S \subset N \setminus \{1\}, |S| \geq 2 \quad (12)$$

Proof [37]: Constraints (12) replace constraints (10). Thus, constraints (12) are added iteratively to break subtours between clusters and within clusters. Fig. 2 shows an example of a subtour $S = \{2, 4, 5, 7, 8, 9, 12, 15, 17, 21\}$ observed in the scheme. This subtour is removed with constraint (12):

Indeed, in this example, $\sum_{i \in S} \sum_{j \in S \setminus \{i\}} x_{ij} = 5$; $\sum_{i \in S} \sum_{j \in S \setminus \{i\} : (i,j) \in A'} y_{ij} = 5$, and $|S| = 10$. Thus, the constraint $\sum_{i \in S} \sum_{j \in S \setminus \{i\}} x_{ij} + \sum_{i \in S} \sum_{j \in S \setminus \{i\} : (i,j) \in A'} y_{ij} = 10 > |S| - 1 = 9$ precludes the subtour.

D. GAVISH-GRAVES CONSTRAINTS

We adapted the Gavish-Graves constraints (GGC) for all nodes [36]. This set of constraints guarantee a connected tour. Therefore, we define f_{ij} as a continuous variable indicating

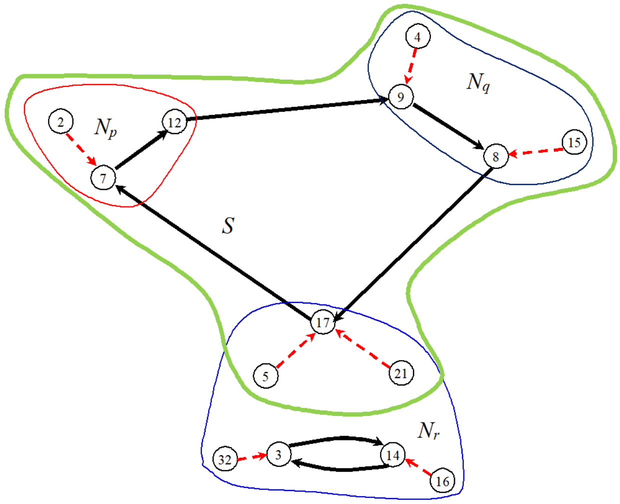


FIGURE 2. Example of a subtour S.

flow between nodes i and j, and the Gavish-Graves constraints are as follows:

$$\sum_{j \in N \setminus \{1\}} f_{1j} = \sum_{i \in N} y_{ii} - 1 \quad (13)$$

$$\sum_{i \in N \setminus \{j\}} f_{ij} = y_{jj} + \sum_{i \in N \setminus \{j\}} f_{ji} \quad \forall j \in N \setminus \{1\} \quad (14)$$

$$f_{ij} \leq (|N| - 1) x_{ij} \quad \forall i, j \in N : i \neq j \quad (15)$$

$$f_{ij} \geq 0 \quad \forall i, j \in N : i \neq j \quad (16)$$

Constraints (13) – (16) replace constraints (10). Constraint (13) states the maximum flow that leaves the depot. Constraints (14) force the flow continuity for each node j. Constraints (15) assure if the arc (i, j) is not on the tour (i.e. $x_{ij} = 0$), the flow variable $f_{ij} = 0$. Constraints (16) assure the domain of variables f_{ij}.

E. GAVISH-GRAVES CONSTRAINTS COMBINED WITH PACKING CUTS

We propose a combination of the Gavish-Graves constraints and packing cuts (GGC + Packing) for solving the GMTP. Packing cuts are used to break subtours S_p strictly generated within clusters N_p. Besides, the GGCs avoid subtours between clusters. Thus, we replace constraints (12) by constraints (17) as follows:

$$\sum_{i \in S_p} \sum_{j \in S_p \setminus \{i\}} x_{ij} + \sum_{i \in S_p} \sum_{j \in S_p \setminus \{i\}} y_{ij} \leq |S_p| - 1$$
$$\forall p \in K \setminus \{1\}, S_p \subset N_p, |S_p| \geq 2 \quad (17)$$

Note that (17) is a particular case of (12). Fig. 3 shows an example of a cluster composed of nodes $N_p = \{2, 3, 4, 7, 8, 9\}$. A subtour $S = \{2, 7, 8, 9\}$ is observed in the scheme. This subtour is removed with constraint (17):

Indeed, in this example, $\sum_{i \in S_p} \sum_{j \in S_p \setminus \{i\}} x_{ij} = 3$; $\sum_{i \in S_p} \sum_{j \in S_p \setminus \{i\}} y_{ij} = 1$, and $|S_p| = 4$. Thus, the constraint $\sum_{i \in S_p} \sum_{j \in S_p \setminus \{i\}} x_{ij} + \sum_{i \in S_p} \sum_{j \in S_p \setminus \{i\}} y_{ij} = 4 > |S_p| - 1 = 3$ precludes the subtour.

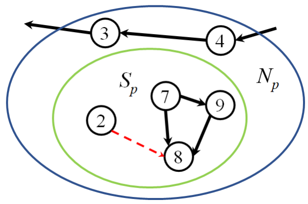


FIGURE 3. Example of a subtour S_p within a cluster N_p.

Alternatively, for this case, we now redefine variables f_{pq} as the flow when cluster p precedes cluster q in the tour, and the flow balance constraints are as follows:

$$\sum_{q \in K \setminus \{1\}} f_{1q} = |K| - 1 \quad (18)$$

$$\sum_{p \in K \setminus \{q\}} f_{pq} = 1 + \sum_{p \in K \setminus \{q\}} f_{qp} \quad \forall q \in K \setminus \{1\} \quad (19)$$

$$f_{pq} \leq (|K| - 1) \sum_{i \in N_p} \sum_{j \in N_q} x_{ij} \quad \forall p \in K,$$
$$q \in K \setminus \{1\} : p \neq q \quad (20)$$

$$f_{pq} \geq 0 \quad \forall p \in K, q \in K \setminus \{1\} : p \neq q \quad (21)$$

Thus, constraints (17) – (21) replace constraints (10). Constraint (17) deletes the subtours generated within each cluster. Constraint (18) states the flow that leaves the depot. Constraints (19) force the flow balance for each cluster q. Constraints (20) establish that if there is a flow between two clusters p and q, there is travel between both clusters. Constraints (21) assure the domain of variables f_{pq}.

F. SOLUTION PROCEDURES

All mathematical models presented are solved by the commercial solver CPLEX 12.8. Nonetheless, The number of constraints in sets (12) and (17) are $O(2^{|N|})$, which makes the formulations using packing constraints intractable as they are. As a consequence, we propose two approaches to deal with them.

The separation procedure follows the steps of the algorithm described in [38]. We apply this procedure to check the integer *candidate* solutions along the tree of the branch & bound algorithm, and to delete subtours within the branch & cut algorithm. This procedure is performed by employing CPLEX 12.8, which allows us to monitor the *candidate* solutions during the optimization process using its generic callbacks.

G. SEPARATION ALGORITHM

This procedure is applied independently over two mathematical models. The first one, is stated by constraints (1) – (9), and (11), which we will refer as model A. The second model, is represented by constraints (1) – (9), (11), and (18) - (21), which we will refer as model B. This procedure deletes subtours by iteratively adding constraints (12) to A, and (17) to B.

When solving model A or B, once a *candidate* solution is found by a generic callback, we build a support graph

TABLE 2. Results for test instances.

	Packing Cuts (PC)				GGC				Packing Cuts + GGC			
Instance	UB	GAP (%)	CPU (s)	Nodes	UB	GAP (%)	CPU (s)	Nodes	UB	GAP (%)	CPU (s)	Nodes
10att48	35,170	0.00	6	2,120	35,170	0.00	44	2,964	35,170	0.00	11	2,964
24Araque51	5,459	0.00	4	0	5459	0.00	10	1,266	5459	0.00	10	1,768
11eil51	418	0.00	3	0	418	0.00	10	163	418	0.00	4	163
14st70	680	0.00	10	27	680	0.00	79	633	680	0.00	15	633
16eil76	542	0.00	13	25	542	0.00	50	205	542	0.00	33	205
16pr76	109,769	0.00	25	2,966	109,769	0.00	335	4,911	109,769	0.00	58	4,911
20gr96	498	0.00	31	0	498	0.00	610	3,333	498	0.00	79	3,333
20rat99	1,245	0.00	29	853	1,245	0.00	907	2,485	1245	0.00	43	2,485
20kroA100	20,403	0.00	23	1,478	20,403	0.00	573	2,506	20,403	0.00	59	2,506
20kroB100	22,047	0.00	46	2,507	22,047	0.00	1,110	2,981	22,047	0.00	98	2,981
20kroC100	20,235	0.00	41	1,112	20,235	0.00	901	2,490	20,235	0.00	72	2,490
20kroD100	21,748	0.00	52	3,163	21,748	0.00	834	3,395	21,748	0.00	180	3,395
20kroE100	22,089	0.00	66	2,086	22,089	0.00	2,153	16,205	22,089	0.00	126	16,205
20rd100	7,951	0.00	27	1,169	7,951	0.00	512	1,834	7,951	0.00	84	1,834
21eil101	628	0.00	17	71	628	0.00	125	696	628	0.00	86	696
21lin105	14,020	0.00	37	2,425	14,020	0.00	999	2,431	14,020	0.00	80	2,431
22pr107	42,114	0.00	49	763	42,114	20.45	3,600	6,255	42,114	0.00	202	6,255
25pr124	58,636	0.00	127	1,578	87,635	38.86	3,600	4,794	58,636	0.00	1,044	4,794
26bier127	119,140	0.00	86	2,598	119,140	0.00	2,434	2,562	119,140	0.00	177	2,562
28pr136	100,107	0.00	127	1,262	100,263	2.27	3,600	6,785	100,107	0.00	257	6,785
28gr137	713	0.00	130	3,752	713	0.00	2,947	2,460	713	0.00	158	2,460
29pr144	58,671	0.00	291	22,541	85,807	35.84	3,600	1,426	58,671	0.00	923	1,426
30kroA150	26,314	0.00	248	4,545	29,986	13.29	3,600	2,451	26,314	0.00	653	2,451
30kroB150	25,799	0.00	146	3,304	27,856	10.72	3,600	2,388	25,799	0.00	404	2,388
31pr152	73,921	0.00	830	25,778	94,164	33.45	3,600	1,686	73,921	0.00	1,485	1,686
32u159	42,519	0.00	325	1,225	44,651	6.26	3,600	2,427	42,519	0.00	397	2,427
39rat195	2,416	0.00	688	15,457	2,729	15.38	3,600	944	2,416	0.00	1,666	944
40d198	15,842	0.00	668	1,365	28,582	55.78	3,600	623	15,842	0.00	874	623
40kroA200	29,112	0.00	528	7,785	48,462	45.13	3,600	896	29,112	0.00	2,425	319
40kroB200	29,603	0.00	575	2,584	54,918	50.06	3,600	698	30,292	4.90	3,600	18,345
41gr202	485	0.00	874	1,627	530	11.79	3,600	393	485	0.00	2,667	393
45ts225	130,272	1.12	3,600	84,438	176,082	30.34	3,600	265	138,250	8.36	3,600	29,725
46pr226	79,731	0.00	1,159	6,771	148,844	53.29	3,600	538	359,749	78.73	3,600	18,469
46gr229	1,828	0.00	1,214	5,789	3,319	50.28	3,600	144	2,200	20.15	3,600	16,280
53gil262	2,390	0.00	1,994	6,882	7,706	71.31	3,600	0	8,503	74.49	3,600	2,413
53pr264	51,979	0.00	2,390	26,784	163,488	74.33	3,600	51	52,943	3.19	3,600	10,668
60pr299	51,114	7.42	3,600	16,376	80,861	43.92	3,600	0	198,262	76.16	3,600	7,014

$G_s(N, A_s)$ considering all variables x_{ij} and y_{ij}, whose associated values in the candidate solution are different from zero. In order to identify subtours, a super-node S is built to implement the shrinking technique [39]. Let $\bar{N} = N$. The searching process begins in the depot (node 1), which is added in S. We include in S all nodes of the graph such that $x_{ij} = 1 : i \in S, j \in \bar{N} \setminus S$ or $y_{ij} = 1 : j \in S, i \in \bar{N} \setminus S : (i, j) \in A'$. Once there are no more nodes to add to S and $|S| < |\bar{N}|$, a subtour is identified. Then, to break the subtour S, the corresponding subtour elimination constraints are added into models A or B. The identified nodes in S are removed from $\bar{N}$, i.e. $\bar{N} \setminus S$. Next, the set S is cleaned and the search starts again selecting a node $i \in \bar{N}$, which is added to S. This process is repeated until $\bar{N} = \emptyset$. The order of this algorithm is $O(|N|^4)$.

IV. RESULTS

In this section, we detail the results of a computational results for the GMTP. The first subsection presents results for 37 test instances up to 299 nodes and 60 clusters. All instances were taken from [4], which are symmetric and Euclidean (i.e., $c_{ij} = c_{ji}$; $d_{ij} = d_{ji}$). For each test instance, the first node (node 1 or 0) is considered as a cluster, such that $N_1 = \{1\}$ or $N_1 = \{0\}$.

In the second subsection, we present results for a real-world instance. The models were coded and solved using Visual C++ and CPLEX 12.8. All experiments were run on a PC with 32GB of RAM and Processor Intel Core i7-7700.

A. TEST INSTANCES

Table 2 shows the results using three subtour elimination strategies for the GMTP. The CPU time limit was set in 1 hour. UB is the best feasible integer solution found in the time limit. GAP is the integrality gap for each instance, and the CPU is the time reported for each instance. If the optimal solution is found, the GAP = 0, and the CPU time is reported. The instance label *ANameB* indicates that the instance has A clusters and B nodes. The column "Nodes" denotes the number of nodes explored within the branch & bound algorithm.

As observed in Table 2, the optimal solution (GAP $=$ 0.00%) is reported for most of the instances. Table 3 presents summarized results of Table 2. The strategy of using packing cuts shows better behavior in all instances, which is reflected in the integrality GAP and CPU time when it is compared with the use of GGC constraints between clusters.

Besides, the branch & cut strategy outperforms the other strategies in terms of the Integrality gap and computing time. However, the GGC + Packing Cuts strategy allows us to find a feasible solution in most cases, especially in the larger instances. It is important to note that not all instances reach the optimal solution or even a feasible solution within one hour, which can be explained by the combinatorial nature of this variant of the problem (GMTP).

TABLE 3. Summary of results using the three procedures.

Procedure	# Optimal solutions of 37 instances	Average		
		GAP (%)	CPU Time (s)	#B&B
PC	35	0.2	542.7	7,114
GGC	18	17.9	2,244.1	2,332
PC + GGC	29	7.2	1,069.5	5,066

B. REAL-WORLD INSTANCE

We solved a real-world instance in a southern region in Chile. The region comprises 23,890 km^2 and 1,557,414 inhabitants. The region contains three provinces: Arauco, Biobío, Concepción. Each province has a set of districts. The regional health services use the same provinces for budget and administrative purposes. Subsequently, each provincial health service is independent, and it is responsible for providing supplies to their districts. Consequently, each province is addressed and solved separately.

Furthermore, each province has a set of health facilities, such as hospitals, primary health centers, and rural health centers. All distances between each pair of nodes were determined using Google Maps ©. Notice that, the network is asymmetric, i.e. $c_{ij} \neq c_{ji}$ and $d_{ij} \neq d_{ij}$.

Fig. 4 shows a map (by Google My Maps ©), where the three provinces are framed in red, and Table 4 shows the area (km^2), number of districts, and number of health facilities per province.

TABLE 4. Information about each province.

Provinces	Area km^2	# Districts (clusters)	# Health Facilities
Biobío	14,987.9	16	105
Concepción	3,439.0	12	85
Arauco	5,457.2	7	46

Each district corresponds to a cluster, and each node is a health facility. Thus, a distribution route for health supplies is presented for each province. The depot is located near to the main airport in the region (node "A" in Fig. 4).

The objective function (1) minimizes the total distance of the tour plus the total distance traveled by nodes outside the tour. In some instances, the objectives might be in different units or magnitudes. In our problem, we add this feature to normalize the magnitude for both terms of the objective an make them comparable [40]. θ is the set of feasible solutions by (2)-(11), and $Y \in \theta$ is a feasible solution. $\alpha \in [0, 1]$ is a weight of 22. Notice that for a fixed α, the problem is still mono-objective, as shown in (22), where $TP(Y)$ is the total route distance, $AS(Y)$ is the total travel access distance for each solution Y, (TP_{min}, AS_{min}) is the ideal point, and (TP_{max}, AS_{max}) is the anti-ideal point. TP_{min} is the minimum value for the shortest tour, and it is associated with the worst access distance AS_{max}. Besides, AS_{min} is the minimum value for the access distance, and it is associated with the worst tour distance TP_{max}.

$$Min\ \alpha \times \left[\frac{TP(Y) - TP_{min}}{TP_{max} - TP_{min}}\right] + (1-\alpha) \times \left[\frac{AS(Y) - AS_{min}}{AS_{max} - AS_{min}}\right] \quad (22)$$

$$Y \in \theta \quad (23)$$

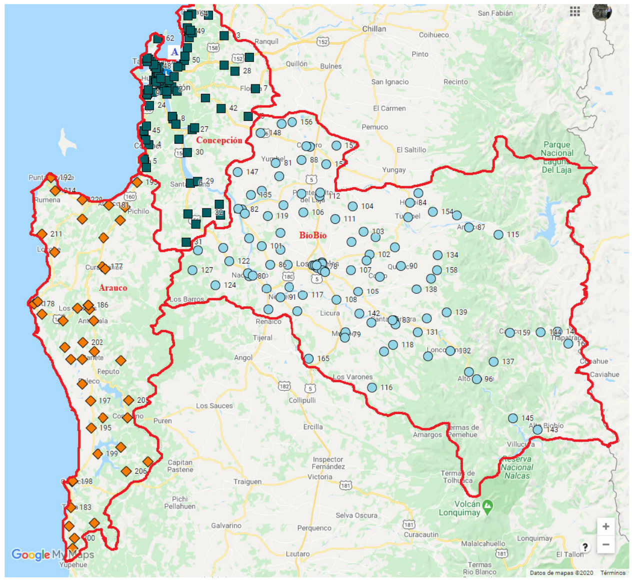
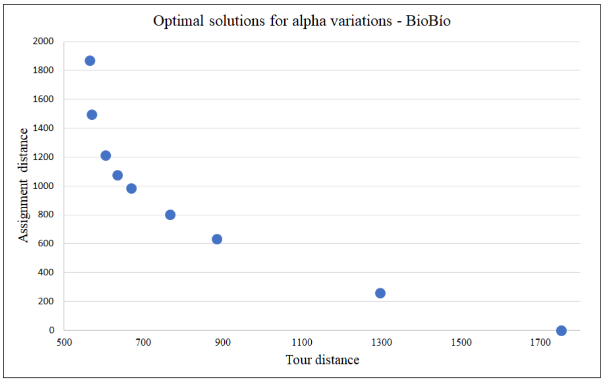
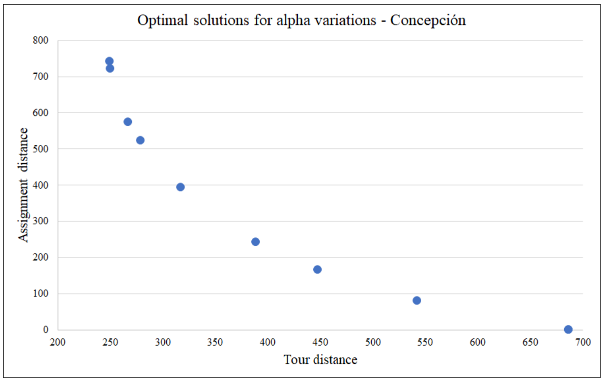


FIGURE 4. Representation of the health supplies distribution network in a geographical area.

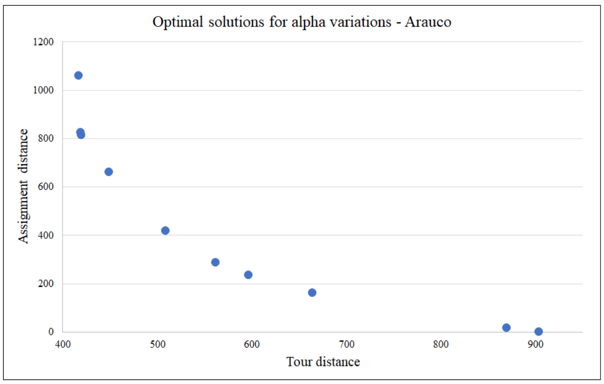
The results of the application of the model in the three provinces are presented in Table 5, where a set of optimal solutions is found by varying α from 0.1 to 1.0, using increments of 0.1 units. We used the Packing Cuts strategy to solve each instance. The notation is as follows: α is the weight for each objective; TP represents the total distance, in kilometers, of the tour; AS represents the total distance (in kilometers) traveled by the facilities not belonging to the tour; #St indicates the number of stops in the route (including the depot); #Cuts indicates the number of cuts found by the branch & cut algorithm; Time is the reported CPU time (in seconds) for each run, and GAP is the integrality Gap. Table 6 presents the average for cuts (#Cuts), CPU time (Time), and



(a) Biobío



(b) Concepción



(c) Arauco

FIGURE 5. A set of optimal solutions for α variations for the three provinces.

GAP (%). Notice that the instances of Concepción province take more time to solve due to the nodes are closer to each other.

Table 7 indicates that the tour distance is longer in BioBío province because of the starting node is the airport and the facilities are more scattered. It should be noticed that the Biobío and Arauco areas are composed mainly of rural areas. It reflects the average distance from each health facility to the tour. As opposed, the tour distance is the shortest in Concepción, because this is a smaller and dense are. This fact implies that health facilities are closer to each other.

TABLE 5. Detailed results for the three provinces.

Biobío						
α	TP (km)	AS (Km)	#St	#Cuts	Time (s)	GAP (%)
0.1	1,752.9	0.0	105	314	28	0
0.2	1,752.9	0.0	105	303	32	0
0.3	1,297.4	259.5	85	340	40	0
0.4	884.8	630.2	62	302	56	0
0.5	767.5	800.1	54	589	244	0
0.6	670.0	982.7	45	627	270	0
0.7	636.0	1,073.0	39	999	697	0
0.8	605.9	1,211.6	35	1,092	12,555	0
0.9	570.4	1,495.4	27	986	13,605	0
1	565.2	1,868.4	18	896	1,166	0
Concepción						
α	TP (km)	AS (Km)	#St	#Cuts	Time	GAP
0.1	686.8	0.0	85	189	11	0
0.2	686.8	0.0	85	236	12	0
0.3	542.6	803.0	76	287	20	0
0.4	447.8	166.2	59	524	125	0
0.5	389.2	242.7	48	738	1,674	0
0.6	317.4	392.8	38	1,284	13,316	0
0.7	279.2	522.4	24	1,644	14,403	4
0.8	267.5	574.1	19	1,210	792	0
0.9	250.5	720.6	14	1,202	211	0
1	249.9	741.9	13	720	93	0
Arauco						
α	TP (km)	AS (Km)	#St	#Cuts	Time	GAP
0.1	903.9	0.0	46	170	1	0
0.2	870.2	172.0	43	167	2	0
0.3	664.7	161.4	35	196	7	0
0.4	596.8	233.4	29	251	13	0
0.5	561.9	287.4	26	360	22	0
0.6	508.9	416.3	22	383	40	0
0.7	449.0	661.6	17	317	37	0
0.8	420.0	812.2	14	342	27	0
0.9	419.3	823.6	13	426	136	0
1	416.9	1,058.3	11	287	16	0

TABLE 6. Summary of results for each province.

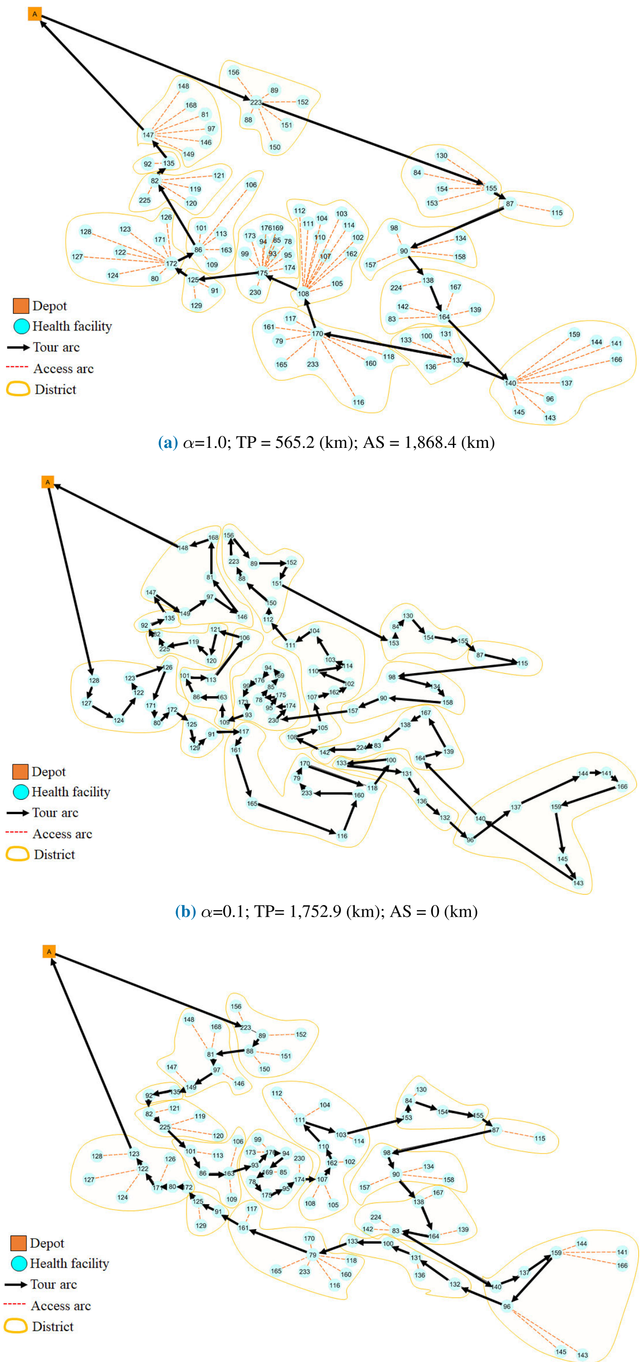
Provinces	Average		
	#Cuts	Time (s)	GAP (%)
Biobío	644.8	2,869.3	0.0
Concepción	803.4	3,065.7	0.4
Arauco	289.9	30.1	0.0

Fig. 5 presents a set of optimal solutions for each province. The longer the tour, the shorter the total access distances. On the other hand, the shorter the tour, the longer the total access distances. Naturally, extreme solutions might not be practical. The tour distance would be the longest in the right extreme because all nodes are visited. In the left extreme, the total tour distance would be the shortest, but the number of nodes assigned is too high.

Fig. 6 presents three solutions taken from Table 5. The first one (Fig. 6a represents the shortest route (like a GTSP solution). The second picture (Fig. 6b indicates the largest route, where all health facilities are visited. Finally, Fig. 6c presents an intermediate solution, with $\alpha = 0.5$.

TABLE 7. Analyses of tour and access distances.

Provinces	Tour distance			Access distance			Average access distance		
	Max	Min	Average	Max	Min	Average	Max	Min	Average
Biobío	1,752.9	565.2	950.3	1,868.4	0.0	832.1	17.8	0.0	7.9
Concepción	686.8	249.9	411.8	803.0	0.0	416.4	9.4	0.0	4.9
Arauco	903.9	416.9	581.2	1,058.3	0.0	462.6	23.0	0.0	10.1



(a) α=1.0; TP = 565.2 (km); AS = 1,868.4 (km)

(b) α=0.1; TP= 1,752.9 (km); AS = 0 (km)

(c) α=0.5; TP = 767.5 (km); AS= 800.1 (km)

FIGURE 6. Three optimal solutions for the Biobío province.

V. CONCLUSION AND FUTURE WORK

In this article, we propose, model, and solve a novel problem called Generalized Median Tour Problem (GMTP). In this problem, the nodes naturally conform clusters, and a single tour must visit each one of them, such that the nodes within each cluster that are not included in the tour must be assigned to the closest node on the tour. The solution minimizes the weighted sum of the tour distances (associated with the length of the tour), and the access distances (associated with the unvisited nodes to the selected nodes in the tour within each cluster).

Three solution strategies are proposed. The first one is based on employing a set of packing constraints within a branch & cut algorithm. The second one is based on using the Gavish and Graves constraints. The third method is based on the combination of these two first methods. The results show the suitability of the proposed and implemented procedures, solving benchmark instances up to 299 nodes.

We present and solve a real-world application for the health supplies distribution in the Biobío Region, Chile, from the main regional airport to the existing health facilities. Particularly, the analyzed cases are motivated by the relevant need for a fast distribution system and strategy for critical health supplies, such as vaccines for COVID-19 (or under other pandemic scenarios), and demanded blood under natural disasters (e.g., earthquakes). Also, we present a sensitivity analysis yielding different configurations for the distribution system, evidencing the suitability and convenience of the proposed methodology to support public authorities and decision-makers in the task of planning and designing such strategic distribution systems.

The tour could visit one or more nodes within each cluster according to the weights of the objective function. If the tour arcs are more significant than the access arcs, the tour will visit the smallest possible number of nodes per cluster. Oppositely, if the tour arcs are less relevant in comparison with the access arcs, the tour will visit more than one node within each cluster.

Future works may include the development of heuristic procedures to solve the different variants of the GMTP in a shorter CPU times. Other research directions may comprise the column generation approach and a branch & price strategy. Several GMTP extensions and variants can be explored, including multi-period and multi-vehicle formulations, capacitated vehicles, time-windows, and advanced transportation processes inside the clusters, such as consolidation strategies, sequencing with secondary vehicles, tree structures, among others.

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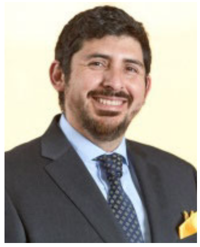

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