IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received September 17, 2020, accepted September 19, 2020, date of publication September 25, 2020, date of current version October 7, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3026748

Adaptive Control of a Piezo-Positioning Mechanism With Hysteresis and Input Saturation Using Time Delay Estimation

ZHIFU LI[1,2,3], (Member, IEEE), JUNHAI ZENG[1], YANGQUAN CHEN[3], (Senior Member, IEEE), GE MA[1,2], AND GUIYUN LIU[1,2]

[1]School of Mechanical and Electrical Engineering, Guangzhou University, Guangzhou 510006, China
[2]Center for Intelligent Equipment and Internet-connected Systems, Guangzhou University, Guangzhou 510006, China
[3]Mechatronics, Embedded Systems and Automation Laboratory, University of California, Merced, CA 95343, USA

Corresponding authors: Ge Ma (m_ge@gzhu.edu.cn) and Zhifu Li (lizhifu8@163.com)

This work was supported in part by the Science and Technology Program of Guangzhou under Grant 201804010085, in part by the National Natural Science Foundation of China under Grant 61603105, and in part by the China Scholarship Council (File No. 201908440074).

ABSTRACT In this paper, based on backstepping technique and time delay estimation (TDE) technique, an adaptive time delay compensation control scheme is developed for a class of piezoelectric positioning mechanical systems with Bouc-Wen hysteresis and input saturation constraint. The nonlinear part of the Bouc-Wen model is estimated online by TDE, and the TDE error introduced by TDE is compensated online by an adaptive law. Furthermore, an auxiliary variable system is used to deal with the input saturation constraint. Based on the Lyapunov method, the stability of the closed-loop system is analyzed and proved. Two simulation examples are given to demonstrate the effectiveness of the proposed control scheme.

INDEX TERMS Hysteresis, time delay control, adaptive control, backstepping.

I. INTRODUCTION

Hysteresis widely exists in modern electromechanical systems, especially in the systems that contain actuators and sensors made of intelligent materials, such as precision piezoelectric positioning mechanism [1], atomic force microscopy [2], hydraulic piezoelectric valves [3], fast cutting tool servo system [4] and so on. Without hysteresis compensation, it will seriously affect the performance of electromechanical systems, and even cause instability of the systems. For example, in an open-loop control, the error caused by hysteresis can reach up to 10%-15% [5]. Therefore, in recent years, modeling and compensation control of hysteresis have attracted significant attention [6]–[8]. There are four main methods for hysteresis modeling. The first is the physics-based modeling method, which mainly includes Jiles-Atherton model [9], Maxwell-slip model [10] and so on; The second method is based on differential equations, including Bouc-Wen model [11], Duhem model [12], and so on; The third one is based on operators, which includes Preisach model [13], Prandtl-Ishlinskii model [14], etc. The last one is intelligent modeling method based on computational intelligence, such as neural network model [15] and support vector machine model [16].

Hysteresis has the properties of multi-valued mapping, memory, rate-dependent, etc. Therefore, the control of systems with hysteresis is still an open question, which has attracted many scholars' interest [17]–[22]. Generally speaking, there are three methods for compensation control of hysteresis, the first is feed-forward compensation control, the second is feedback-feedforward compensation control, and the last is feedback compensation control.

Feedforward compensation method is a very effective and low-cost hysteresis compensation control method, which constructs a hysteresis inverse in the forward channel to eliminate the impact of hysteresis on systems. However, due to its open-loop mode, it completely depends on the accuracy of hysteresis modeling and parameter identification and is sensitive to external disturbances and model uncertainties. Therefore, this method has rarely been used in practical application.

Feedback-feedforward compensation control method is to construct an inverse model in the forward path to eliminate the hysteresis effects and design a controller in the feedback

The associate editor coordinating the review of this manuscript and approving it for publication was Feiqi Deng.

path to further improve the performance of the system and enhance the robustness of the system. A Bouc-Wen hysteresis inverse model is proposed in [17], which can eliminate the effect of the Bouc-Wen hysteresis, and an adaptive controller with hysteresis inverse is designed. The simulation results show that the control strategy is still effective in the presence of parameter perturbations of the hysteresis model. In [19], the continuous Prandtl-Ishlinskii model is decomposed into a finite number of discrete Prandtl-Ishlinskii operators, and the hysteresis is compensated by using the analytical inverse hysteresis model. For the error caused by the continuous model being converted into a discrete model, an adaptive control technique is used to compensate online. The feedback-feedforward compensation control method not only needs to construct hysteresis inverse models (constructing the hysteresis inverse model is a very difficult thing in itself), but also needs to know the exact parameters of the hysteresis model, and the number of these parameters is generally not small.

Feedback compensation control method does not need to construct the hysteresis inverse model or approximate inverse model but treats the nonlinear part of the hysteresis as a disturbance, and then uses sliding mode control, robust control and other methods to design feedback controllers. The Bouc-Wen hysteresis is decomposed into a linear term and a bounded nonlinear hysteresis term in [18]. Then, the nonlinear hysteresis term can be treated as a bounded disturbance. In [20], the hysteresis is directly regarded as a part of disturbance, and then the disturbance is eliminated by an adaptive sliding mode controller. The stability of the closed-loop system is proved theoretically. Although the feedback compensation control method does not need to construct a hysteresis inverse but treats the nonlinear term in the hysteresis as a bounded disturbance, it will affect the control accuracy to a certain extent.

Due to the limitations of physical conditions and sensors, not all system states are available in the practical systems. Therefore, state observer has become one of the main solutions. In [23], [24], state observers based on fuzzy logic system are designed for nonlinear systems. In [25], a nonlinear state observer is proposed for a stochastic nonlinear strict-feedback system. In [26], a high gain observer with updated gain and homogeneous correction terms is used to estimate the unknown states. Compared with the traditional observer, the high-gain observer not only has the performance of the state feedback controller when the gain is high enough, but also can suppress the disturbance to a certain extent as long as the gains design is reasonable [27]. Therefore, the high-gain observer has been widely used in many control problems.

In addition, due to physical constraints, in practical control systems, input saturation constraints are often encountered. Saturation often leads to excessive overshoot and large tracking error, which greatly limits the performance of the system. How to solve the problem of saturation nonlinearity is still a challenging task. Auxiliary variable method [28], [29], neural networks [30], [31], composite nonlinear feedback method [32]–[34] are used to deal with the input saturation constrains. Moreover, the problems of different control systems with input saturation have also attracted much attention [35], [36]. However, to the best of authors' knowledge, few results are available for adaptive time delay control of electromechanical systems with hysteresis.

The Bouc-Wen model is widely used in modelling hysteresis. And after the boundedness of the nonlinear hysteresis term was proved in [18], applications of Bouc-Wen hysteresis in control systems have received considerable attention. Based on time delay estimation(TDE) technique [37]–[39], a novel feedback control method is proposed for a class of electromechanical systems with Bouc-Wen hysteresis and input saturation constraint. In this control method, the TDE is used to estimate the nonlinear part of the Bouc-Wen model, and then an adaptive controller is designed by backstepping design method. The TDE error introduced by TDE is estimated and compensated by the adaptive law, so that the control performance of the system is further improved. The main contributions of this paper can be summarized as follows:

(i) A new method of hysteresis compensation control is proposed. Compared with [18], [20], this method uses the TDE technique to realize the on-line estimation of the nonlinear term of hysteresis, which can be directly used in the controller design, instead of treating the hysteresis nonlinearity as a bounded disturbance. Compared with the feedback-feedforward control in [17], [19], the proposed method does not need to construct the hysteresis inverse, and only one parameter of the hysteresis is used in the controller, which effectively reduces the calculation.

(ii) An adaptive method is proposed for the electromechanical system, which does not require knowledge of system dynamics.

(iii) Compared with [17]–[22], the input saturation constraint is considered in this paper, which makes the control method proposed in this paper can be better used in practical engineering systems.

The rest of this paper is organized as follows. The problem to be tackled is stated and the control objective is given in Section II. The proposed control scheme is given in Section III. In Section IV, simulation results are presented. Finally, conclusions are provided in Section V.

II. PROBLEM STATEMENT

Consider a piezoelectric positioning mechanic system [40], [41] preceded by an actuator with input hysteresis and input saturation:

$$\begin{cases} M\ddot{x} + D\dot{x} + Fx = w, \\ w = H[u](t) \end{cases} \tag{1}$$

where x, $\dot{x}$ and $\ddot{x}$ are the position, velocity and the acceleration, M, D and F denote the unknown mass, damping, and stiffness coefficients, u is the applied voltage to the piezoelectric positioning platform, $H[u](t)$ denotes the Bouc-Wen hysteresis nonlinearity, and the specific parameters are known.

The expression of $H[u](t)$ is given as follows [17], [18]:

$$w = H[u](t) = \mu\kappa u + (1-\mu)\kappa\vartheta = \mu_1 u + \mu_2\vartheta \quad (2)$$

where $0 < \mu < 1$ is a weighting parameter, κ is stiffness coefficient, μ_1 and μ_2 are constants with the same sign, and ϑ is given by the following nonlinear first-order differential equation:

$$\dot{\vartheta} = \dot{u} - \beta|\dot{u}||\vartheta|^{n-1}\vartheta - \chi\dot{u}|\vartheta|^{n} \quad (3)$$

where parameters β and χ describe the shape and amplitude of the hysteresis, respectively, n governs the smoothness of the transition from the initial slope to the slope of the asymptote, and $\beta > |\chi|, n \geq 1$.

Consider the voltage actuator input constraint, and u is given by:

$$u = sat(v) = \begin{cases} sign(v)u_{max}, & |v| \geq u_{max} \\ v, & |v| < u_{max} \end{cases} \quad (4)$$

where v is the control signal to be designed, and u_{max} is the known saturation limit.

Let $x_1 = x$, $x_2 = \dot{x}$, and $\Gamma = \mu_2\vartheta$, using (2), system (1) can be rewritten as:

$$\begin{cases} \dot{x}_1 = x_2, \\ M\dot{x}_2 = \mu_1 u + \Gamma - Dx_2 - Fx_1. \end{cases} \quad (5)$$

The control objective is to design a control scheme for the system (1)-(4) such that the displacement x can track the desired trajectory x_d.

Lemma 1 [18]: For any piecewise continuous signals u and $\dot{u}$ (bounded or not), the solution $\vartheta(t)$ of (3) is bounded by $|\vartheta(t)| \leq max\{|\vartheta(0)|, \sqrt[n]{\frac{dx_1}{\beta+\chi}}\}$, where $\vartheta(0)$ is the initial condition of (3).

Remark 1: The boundedness of the $\vartheta(t)$ means that it can be considered as a bounded disturbance in the system. And the initial value is usually set to $\vartheta(0) = 0$.

Remark 2: Although the piezoelectric positioning mechanic system in [40], [41] is discussed herein, the control method can also be used for the systems represented as Equation (1), such as the piezo-actuated stage described in [42] and the mechanical system in [43].

Assumption 1: The desire trajectory x_d and its first and second derivatives are known and bounded.

Lemma 2 [44]: Assume the function $y(t)$ and its first n derivatives are bounded, thus $|y^{(k)}| < Y_k$ for $k = 0, \ldots, n$, where Y_k are positive constants. Consider the following linear system:

$$\begin{aligned} &\epsilon\dot{\omega}_i = \omega_{i+1}, \ for\ i = 1, \ldots, n-1, \\ &\epsilon\dot{\omega}_n = -\lambda_1\omega_n - \lambda_2\omega_{n-1} - \ldots - \omega_1 + x_1 \end{aligned} \quad (6)$$

where ϵ is any small positive constant and the parameters $\lambda_1, \ldots, \lambda_{n-1}$ are chosen so that the polynomial $s^{n-1} + \lambda_1 s^{n-2,\ldots,} + 1$ is Hurwitz. Then there exist positive constants l_k for $k = 2, \ldots, n$ and t^*, such that for all $t > t^*$, we have

$$|\frac{\omega_{k+1}}{\epsilon^k} - y^{(k)}| \leq \epsilon l_{k+1}, \ for\ k = 1, \ldots, n-1. \quad (7)$$

III. CONTROL DESIGN

In this paper, two cases are investigated for the piezoelectric positioning mechanic system (5): (i) full state feedback control design, that is, x_1 and x_2 are known; and (ii) output feedback control design, that is, only x_1 is known. For the second case where x_2 cannot be directly measured, a high-gain observer is proposed to estimate x_2.

A. FULL STATE FEEDBACK CONTROL

Before using the backstepping technique to design the control law, the following change of coordinates is made:

$$\begin{cases} z_1 = x_1 - x_d, \\ z_2 = x_2 - \dot{x}_d - \alpha_1 \end{cases} \quad (8)$$

where α_1 is a virtual control law to be designed.

Step 1: From (5) and (8), we have

$$\dot{z}_1 = z_2 + \alpha_1 \quad (9)$$

then the virtual control law α_1 is designed as:

$$\alpha_1 = -c_1 z_1 \quad (10)$$

where c_1 is a positive parameter to be designed. From (8)-(10), we obtain

$$z_1\dot{z}_1 = -c_1 z_1^2 + z_1 z_2 \quad (11)$$

Step 2: From (5) and (8), we get

$$\begin{aligned} M\dot{z}_2 &= M\dot{x}_2 - M\ddot{x}_d - M\dot{\alpha}_1 \\ &= \mu_1 u + \Gamma - Dx_2 - Fx_1 - M\ddot{x}_d - M\dot{\alpha}_1. \end{aligned} \quad (12)$$

The following auxiliary design system is used to deal with the input saturation:

$$\dot{\xi} = \begin{cases} -K\xi - \dfrac{|z_2\mu_1\Delta u| + (\frac{1}{2}\mu_1)^2(\Delta u)^2}{\xi} \\ +\mu_1(v-u), & |\xi| \geq \sigma \\ 0, & |\xi| < \sigma \end{cases} \quad (13)$$

where $\Delta u = u - v$, ξ is the state of the auxiliary design system, K is a positive parameter to be designed, and parameter σ is a small positive constant.

From (13), we have

$$\xi\dot{\xi} = -K\xi^2 - (\frac{1}{2}\mu_1)^2(\Delta u)^2 - |z_2\mu_1\Delta u| - \mu_1\Delta u\xi. \quad (14)$$

Then, considering the input saturation effect, we design the following control law:

$$\begin{cases} v = \mu_1^{-1}v_0 \\ v_0 = -z_1 - c_2 z_2 - K_v(z_2+\xi) - \hat{\Gamma} + \hat{D}x_2 \\ \quad + \hat{F}x_1 + \hat{M}(\ddot{x}_d + \dot{\alpha}_1) - \hat{B}sign(z_2) \end{cases} \quad (15)$$

where parameters c_2 and K_v are positive constants to be designed, $\hat{D}$, $\hat{F}$ and $\hat{M}$ are the estimates of the parameters D, F and M, $\hat{B}$ is the estimate the bound B of the TDE error which will be introduced in the following.

Unlike traditional hysteresis feedback compensation control design methods, instead of treating Γ as a bounded disturbance, we estimate it using the TDE technique. Let $\hat{\Gamma}$ denote the estimation of Γ, then, with the help of (1) and (5), we can obtain $\hat{\Gamma}$ using the TDE scheme

$$\begin{aligned}\hat{\Gamma} = \Gamma(t-h) &= M\dot{x}_2(t-h) + Dx_2(t-h) \\ &\quad + Fx_1(t-h) - \mu_1 u(t-h) \\ &= w(t-h) - \mu_1 u(t-h) \end{aligned} \tag{16}$$

where h is an adequate small delay time. In practice, h is set to one unit of sampling time. According to the principle of TDE technique, as long as h is small enough, then $\Gamma(t) \cong \Gamma(t-h) = \hat{\Gamma}(t)$. However, there still be a TDE error

$$e_d(t) = \Gamma - \hat{\Gamma}.$$

Using B to denote the bound of the TDE error, then we have $|e_d(t)| \leq B$.

Remark 3: From Lemma 1, it is reasonable to assume that $e_d(t)$ is bounded. Furthermore, in practice, $\hat{\Gamma}$ is computed according to (16). Since u is given by the controller, $u(t-h)$ can be assumed to be known. If signal w is available, then we can obtain $\hat{\Gamma}$ from $w(t-h) - \mu_1 u(t-h)$. Otherwise, we can use $\hat{D}(t-h)$, $\hat{F}(t-h)$ and $\hat{M}(t-h)$ to estimate the parameters D, F and M, then get the estimation $\hat{\Gamma}$.

The parameters updating laws are designed as

$$\dot{\hat{D}} = -\gamma_D z_2 x_2 \tag{17}$$

$$\dot{\hat{F}} = -\gamma_F z_2 x_1 \tag{18}$$

$$\dot{\hat{M}} = -\gamma_M z_2(\ddot{x}_d + \dot{\alpha}_1) \tag{19}$$

$$\dot{\hat{B}} = \gamma_B z_2 sign(z_2) \tag{20}$$

where $\tilde{D} = D - \hat{D}$, $\tilde{F} = F - \hat{F}$, $\tilde{M} = M - \hat{M}$, $\tilde{B} = B - \hat{B}$, and γ_D, γ_F, γ_M and γ_B are positive constants to be designed.

Choose a Lyapunov function candidate as

$$\begin{aligned} V = \frac{1}{2}z_1^2 + \frac{1}{2}Mz_2^2 + \frac{1}{2}\xi^2 + \frac{1}{2\gamma_D}\tilde{D}^2 \\ + \frac{1}{2\gamma_F}\tilde{F}^2 + \frac{1}{2\gamma_M}\tilde{M}^2 + \frac{1}{2\gamma_B}\tilde{B}^2. \end{aligned} \tag{21}$$

Using (11) and (12), take the time derivative of V, we obtain

$$\begin{aligned} \dot{V} &= -c_1 z_1^2 + z_1 z_2 + z_2 M\dot{z}_2 + \xi\dot{\xi} - \frac{1}{\gamma_D}\tilde{D}\dot{\hat{D}} \\ &\quad - \frac{1}{\gamma_F}\tilde{F}\dot{\hat{F}} - \frac{1}{\gamma_M}\tilde{M}\dot{\hat{M}} - \frac{1}{\gamma_B}\tilde{B}\dot{\hat{B}} \\ &= -c_1 z_1^2 + z_1 z_2 + z_2(\mu_1 u + \hat{\Gamma} + e_d - Dx_2 \\ &\quad - Fx_1 - M\ddot{x}_d - M\dot{\alpha}_1) + \xi\dot{\xi} - \frac{1}{\gamma_D}\tilde{D}\dot{\hat{D}} \\ &\quad - \frac{1}{\gamma_F}\tilde{F}\dot{\hat{F}} - \frac{1}{\gamma_M}\tilde{M}\dot{\hat{M}} - \frac{1}{\gamma_B}\tilde{B}\dot{\hat{B}}. \end{aligned} \tag{22}$$

Note that $u = v + \Delta u$ and $z_2 e_d \leq |z_2|B$, and substituting (15) into (22), we obtain

$$\begin{aligned} \dot{V} &= -c_1 z_1^2 - c_2 z_2^2 + z_2[-\tilde{D}x_2 - \tilde{F}x_1 \\ &\quad - \tilde{M}(\ddot{x}_d + \dot{\alpha}_1)] + |z_2|\tilde{B} + z_2\mu_1\Delta u \\ &\quad - K_v z_2^2 - K_v z_2\xi + \xi\dot{\xi} - \frac{1}{\gamma_D}\tilde{D}\dot{\hat{D}} \\ &\quad - \frac{1}{\gamma_F}\tilde{F}\dot{\hat{F}} - \frac{1}{\gamma_M}\tilde{M}\dot{\hat{M}} - \frac{1}{\gamma_B}\tilde{B}\dot{\hat{B}}. \end{aligned} \tag{23}$$

Substituting (14) and (17)-(20) into (23), we have

$$\begin{aligned} \dot{V} \leq -c_1 z_1^2 - c_2 z_2^2 - K\xi^2 - K_v z_2^2 - K_v z_2\xi \\ - (\frac{1}{2}\mu_1)^2(\Delta u)^2 - \mu_1\Delta u\xi. \end{aligned} \tag{24}$$

Considering the facts:

$$0 \leq \xi^2 + (\frac{1}{2}\mu_1)^2(\Delta u)^2 + \mu_1\Delta u\xi - K_v z_2\xi \leq \frac{1}{2}z_2^2 + \frac{1}{2}K_v^2\xi^2$$

From (24), we have

$$\begin{aligned} \dot{V} &\leq -c_1 z_1^2 - c_2 z_2^2 - K_v z_2^2 - K_v z_2\xi \\ &\quad - (K-1)\xi^2 \\ &\leq -c_1 z_1^2 - c_2 z_2^2 - K_v z_2^2 - (K-1)\xi^2 \\ &\quad + \frac{1}{2}z_2^2 + \frac{1}{2}K_v^2\xi^2 \\ &\leq -c_1 z_1^2 - c_2 z_2^2 - (K_v - \frac{1}{2})z_2^2 \\ &\quad - (K - 1 - \frac{1}{2}K_v^2)\xi^2. \end{aligned} \tag{25}$$

Remark 4: To ensure the stability of the close-loop system, the control parameters should satisfy the following condition: $K_v - \frac{1}{2} > 0$ and $K - 1 - \frac{1}{2}K_v^2 > 0$. Thus, $\dot{V}$ will be negative definite.

Theorem 1: Considering the plant (1)-(4) under Assumption 1, the adaptive backstepping time delay control scheme, consisting of the control laws (10) and (15), auxiliary system (13), the updated laws (17)-(20), and the TDE (16), guarantees that all signals in the closed-loop system are globally uniformly bounded and the asymptotic tracking is achieved, i.e.

$$\lim_{t\to\infty}[x(t) - x_d(t)] = 0.$$

Proof: From (25) and Remark 4, it can be concluded that V is globally uniformly bounded, i.e., the signals $z_1, z_2, \xi, \tilde{D}, \tilde{F}, \tilde{M}, \tilde{B}$ are bounded. Thus, $\hat{D}$, $\hat{F}$, $\hat{M}$ and $\hat{B}$ are bounded. According to Assumption 1, (8) and (10), the states x_1, x_2 are also bounded. Therefore, v and v_0 are bounded from (15). By applying the LaSalle-Yoshizawa Theorem to (25), we have that $z_1(t) \to 0$ as $t \to \infty$, which ensures that $\lim_{t\to\infty}[x(t) - x_d(t)] = 0$.

B. OUTPUT FEEDBACK CONTROL

The proposed control law (15) is designed under the assumption that all outputs are measurable. However, some output information may not be measurable due to practical issues such as cost and dimensions. In this section, a high gain observer is used to estimate the unmeasurable term x_2. According to Lemma 2, the high gain observer for the system (5) is considered with $n = 2$, and the unmeasurable state $x_2 = \dot{x}$ can be approximated by

$$\hat{x}_2 = \frac{\omega_2}{\epsilon}. \tag{26}$$

Then, the unmeasurable backstepping state signal z_2 can be estimated by:

$$\hat{z}_2 = \frac{\omega_2}{\epsilon} - \dot{x}_d - \alpha_1 \tag{27}$$

where the dynamics of ω_2 are given as follows:

$$\epsilon\dot{\omega}_1 = \omega_2 \tag{28}$$

$$\epsilon\dot{\omega}_2 = -\lambda_1\omega_2 - \omega_1 + x_1. \tag{29}$$

According to Lemma 2, there exist constants t^* and l_2 such that $\forall t > t^*$, we have

$$|\frac{\omega_2}{\epsilon} - \dot{x}_1| \le \epsilon l_2. \tag{30}$$

Note that

$$\begin{aligned}\tilde{x}_2 &= x_2 - \hat{x}_2\\ &= \dot{x}_1 - \frac{\omega_2}{\epsilon}\\ \tilde{z}_2 &= z_2 - \hat{z}_2\\ &= \dot{x}_1 - \dot{x}_d - \alpha_1 - \frac{\omega_2}{\epsilon} + \dot{x}_d + \alpha_1\\ &= \dot{x}_1 - \frac{\omega_2}{\epsilon}.\end{aligned}$$

From (30), we have

$$\tilde{z}_2^2 \le (\epsilon l_2)^2 = \beta \tag{31}$$

$$\tilde{x}_2^2 \le \beta \tag{32}$$

Now the control law and parameter updating laws are designed as follows:

$$\begin{cases} v = \mu_1^{-1} v_0\\ v_0 = -z_1 - c_{21}\hat{z}_2 - d_1\hat{z}_2 - K_{v1}(\hat{z}_2 + \xi_1) - \hat{\Gamma}\\ \qquad + \hat{D}\hat{x}_2 + \hat{F}x_1 + \hat{M}(\ddot{x}_d + \dot{\alpha}_1) - \hat{B}sign(\hat{z}_2) \end{cases} \tag{33}$$

$$\dot{\hat{D}} = -\gamma_{D1}(\hat{z}_2\hat{x}_2 + \sigma_D\hat{D}) \tag{34}$$

$$\dot{\hat{F}} = -\gamma_{F1}(\hat{z}_2 x_1 + \sigma_F\hat{F}) \tag{35}$$

$$\dot{\hat{M}} = -\gamma_{M1}(\hat{z}_2(\ddot{x}_d + \dot{\alpha}_1) + \sigma_M\hat{M}) \tag{36}$$

$$\dot{\hat{B}} = \gamma_{B1}(\hat{z}_2 sign(\hat{z}_2) - \sigma_B\hat{B}) \tag{37}$$

where $c_{21}, d_1, K_{v1}, \gamma_{D1}, \gamma_{F1}, \gamma_{M1}, \gamma_{B1}, \sigma_D, \sigma_F, \sigma_M$ and σ_B are positive constants to be designed.

And the auxiliary design system is designed as follows:

$$\dot{\xi}_1 = \begin{cases} -\dfrac{|\hat{z}_2\mu_1\Delta u| + ((\frac{1}{2}\mu_1)^2 + \frac{1}{2})(\Delta u)^2}{\xi_1} \\ -K_1\xi_1 + \mu_1(v-u), & |\xi_1| \ge \sigma_1\\ 0, & |\xi_1| < \sigma_1. \end{cases} \tag{38}$$

where ξ_1 is the state of the auxiliary design system, K_1 is a positive constant to be designed, and parameter σ_1 is a small positive constant.

Considering a Lyapunov function candidate as

$$\begin{aligned}V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}Mz_2^2 + \frac{1}{2}\xi_1^2 + \frac{1}{2\gamma_{D1}}\tilde{D}^2 + \frac{1}{2\gamma_{F1}}\tilde{F}^2\\ + \frac{1}{2\gamma_{M1}}\tilde{M}^2 + \frac{1}{2\gamma_{B1}}\tilde{B}^2 + \frac{1}{2}\tilde{z}_2^2.\end{aligned} \tag{39}$$

Note that $\frac{1}{2}\tilde{z}_2^2 \le \frac{1}{2}\beta$, therefore, we only need to determine the stability of $V_2 = V_1 - \frac{1}{2}\tilde{z}_2^2$.

Time derivative V_2 is

$$\begin{aligned}\dot{V}_2 \le& -c_1z_1^2 + z_1z_2 + z_2(\mu_1 u + \hat{\Gamma} + e_d - Dx_2\\ &- Fx_1 - M\ddot{x}_d - M\dot{\alpha}_1) + \xi_1\dot{\xi}_1 - \frac{1}{\gamma_{D1}}\tilde{D}\dot{\hat{D}}\\ &- \frac{1}{\gamma_{F1}}\tilde{F}\dot{\hat{F}} - \frac{1}{\gamma_{M1}}\tilde{M}\dot{\hat{M}}.\end{aligned} \tag{40}$$

Substituting (33) into (40), we obtain

$$\begin{aligned}\dot{V}_2 \le& -c_1z_1^2 - c_{21}z_2\hat{z}_2 + z_2(-\tilde{F}x_1 - \tilde{M}(\ddot{x}_d + \dot{\alpha}_1)\\ &+ \hat{D}\hat{x}_2 - Dx_2) + z_2\mu_1\Delta u - K_{v1}z_2(\hat{z}_2 + \xi_1)\\ &+ z_2e_d - z_2\hat{B}sign(\hat{z}_2) - d_1z_2\hat{z}_2 + \xi_1\dot{\xi}_1 - \frac{1}{\gamma_{D1}}\tilde{D}\dot{\hat{D}}\\ &- \frac{1}{\gamma_{F1}}\tilde{F}\dot{\hat{F}} - \frac{1}{\gamma_{M1}}\tilde{M}\dot{\hat{M}} - \frac{1}{\gamma_{B1}}\tilde{B}\dot{\hat{B}}.\end{aligned} \tag{41}$$

According to (34)-(38), from (41), we have

$$\begin{aligned}\dot{V}_2 \le& -c_1z_1^2 - c_{21}z_2\hat{z}_2 - z_2\tilde{F}x_1 + \tilde{F}\hat{z}_2x_1 + \sigma_F\tilde{F}\hat{F}\\ &- z_2\tilde{M}(\ddot{x}_d + \dot{\alpha}_1) + \tilde{M}\hat{z}_2(\ddot{x}_d + \dot{\alpha}_1) + \sigma_M\tilde{M}\hat{M}\\ &+ z_2(\hat{D}\hat{x}_2 - Dx_2) + \tilde{D}\hat{z}_2\hat{x}_2 + \sigma_D\tilde{D}\hat{D} - d_1z_2\hat{z}_2\\ &+ z_2\mu_1\Delta u - K_{v1}z_2\hat{z}_2 - K_{v1}z_2\xi_1 - K_1\xi_1^2\\ &- [(\frac{1}{2}\mu_1)^2 + \frac{1}{2}](\Delta u)^2 - |\hat{z}_2\mu_1\Delta u| - \mu_1\Delta u\xi_1\\ &+ z_2e_d - z_2\hat{B}sign(\hat{z}_2) - \tilde{B}\hat{z}_2 sign(\hat{z}_2) + \sigma_B\tilde{B}\hat{B}.\end{aligned} \tag{42}$$

Note that

$$\begin{aligned}\tilde{F}\hat{F} &= \tilde{F}(F - \tilde{F}) = \tilde{F}F - \tilde{F}^2\\ \tilde{F}F &\le \frac{1}{2}\tilde{F}^2 + \frac{1}{2}F^2.\end{aligned}$$

Thus, the following inequality holds

$$\tilde{F}\hat{F} \le -\frac{1}{2}\tilde{F}^2 + \frac{1}{2}F^2. \tag{43}$$

Similarly, we have

$$\tilde{D}\hat{D} \le -\frac{1}{2}\tilde{D}^2 + \frac{1}{2}D^2 \tag{44}$$

$$\tilde{M}\hat{M} \le -\frac{1}{2}\tilde{M}^2 + \frac{1}{2}M^2. \tag{45}$$

Considering the (3-5)th terms of (42), we have

$$\begin{aligned}
&-z_2\tilde{F}x_1+\tilde{F}\tilde{z}_2x_1+\sigma_F\tilde{F}\hat{F}\\
&\quad=-z_2\tilde{F}x_1+\tilde{F}(z_2-\tilde{z}_2)x_1+\sigma_F\tilde{F}\hat{F}\\
&\quad=-\tilde{F}\tilde{z}_2x_1-\sigma_F\tilde{F}\hat{F}\\
&\quad=-\tilde{F}\tilde{z}_2(z_1+x_d)+\sigma_F\tilde{F}\hat{F}\\
&\quad=-\tilde{F}\tilde{z}_2z_1-\tilde{F}\tilde{z}_2x_d+\sigma_F\tilde{F}\hat{F}\\
&\quad\le\frac{1}{2}\tilde{F}^2+\frac{1}{2}\tilde{z}_2^2z_1^2+\frac{1}{2}\tilde{F}^2+\frac{1}{2}\tilde{z}_2^2x_d^2-\frac{\sigma_F}{2}\tilde{F}^2+\frac{\sigma_F}{2}F^2\\
&\quad\le-(\frac{\sigma_F}{2}-1)\tilde{F}^2+\frac{\beta}{2}z_1^2+\frac{\beta}{2}x_d^2+\frac{\sigma_F}{2}F^2. \qquad (46)
\end{aligned}$$

From (8)-(10), we have

$$\begin{aligned}
\dot{\alpha}_1&=-c_1z_1=-c_1(\dot{x}_1-\dot{x}_d)=-c_1(x_2-\dot{x}_d)\\
&=-c_1(z_2+\alpha_1)=-c_1(z_2-c_1z_1). \qquad (47)
\end{aligned}$$

Considering the terms of the second line of (42), we have

$$\begin{aligned}
&-z_2\tilde{M}(\ddot{x}_d+\dot{\alpha}_1)+\tilde{M}\tilde{z}_2(\ddot{x}_d+\dot{\alpha}_1)+\sigma_M\tilde{M}\hat{M}\\
&\quad=-\tilde{M}\tilde{z}_2(\ddot{x}_d+\dot{\alpha}_1)+\sigma_M\tilde{M}\hat{M}\\
&\quad=-\tilde{M}\tilde{z}_2\ddot{x}_d-\tilde{M}\tilde{z}_2\dot{\alpha}_1+\sigma_M\tilde{M}\hat{M}\\
&\quad=-\tilde{M}\tilde{z}_2\ddot{x}_d-\tilde{M}\tilde{z}_2(-c_1z_2+c_1^2z_1)+\sigma_M\tilde{M}\hat{M}\\
&\quad\le\frac{1}{2}\tilde{M}^2+\frac{1}{2}\tilde{z}_2^2\ddot{x}_d^2+\frac{c_1}{2}\tilde{M}^2+\frac{c_1}{2}\tilde{z}_2^2z_2^2+\frac{c_1^2}{2}\tilde{M}^2\\
&\qquad+\frac{c_1^2}{2}\tilde{z}_2^2z_1^2-\frac{\sigma_M}{2}\tilde{M}^2+\frac{\sigma_M}{2}M^2\\
&\quad\le-(\frac{\sigma_M}{2}-\frac{1}{2}-\frac{c_1}{2}-\frac{c_1^2}{2})\tilde{M}^2+\frac{\beta}{2}\ddot{x}_d^2\\
&\qquad+\frac{c_1\beta}{2}z_2^2+\frac{c_1^2\beta}{2}z_1^2+\frac{\sigma_M}{2}M^2. \qquad (48)
\end{aligned}$$

Considering the terms of the third line of (42), we obtain

$$\begin{aligned}
&z_2(\hat{D}\hat{x}_2-Dx_2)+\tilde{D}\hat{z}_2\hat{x}_2+\sigma_D\tilde{D}\hat{D}-d_1z_2\hat{z}_2\\
&\quad=z_2((D-\tilde{D})\hat{x}_2-Dx_2)+\tilde{D}\hat{z}_2\hat{x}_2+\sigma_D\tilde{D}\hat{D}-d_1z_2\hat{z}_2\\
&\quad=(\tilde{z}_2+\hat{z}_2)(-\tilde{D}\hat{x}_2-D\tilde{x}_2)+\tilde{D}\hat{z}_2\hat{x}_2+\sigma_D\tilde{D}\hat{D}-d_1z_2\hat{z}_2\\
&\quad=-\tilde{D}\tilde{z}_2\hat{x}_2-Dz_2\tilde{x}_2+\sigma_D\tilde{D}\hat{D}-d_1z_2\hat{z}_2\\
&\quad=-\tilde{D}\tilde{z}_2(\hat{z}_2+x_d+\alpha_1)-Dz_2\tilde{x}_2+\sigma_D\tilde{D}\hat{D}-d_1z_2\hat{z}_2\\
&\quad=-\tilde{D}\tilde{z}_2z_2+\tilde{D}\tilde{z}_2^2-\tilde{D}\tilde{z}_2x_d+c_1\tilde{D}\tilde{z}_2z_1-Dz_2\tilde{x}_2\\
&\qquad-\sigma_D\tilde{D}\hat{D}-d_1z_2(z_2-\tilde{z}_2)\\
&\quad\le\frac{1}{2}\tilde{D}^2+\frac{1}{2}\tilde{z}^2z_2^2+\tilde{D}\beta+\frac{1}{2}\tilde{D}^2+\frac{1}{2}\tilde{z}^2x_d^2+\frac{c_1}{2}\tilde{D}^2\\
&\qquad+\frac{c_1}{2}\tilde{z}^2z_1^2+\frac{1}{2}z_2^2+\frac{1}{2}D^2\tilde{x}^2-\frac{\sigma_D}{2}\tilde{D}^2+\frac{\sigma_D}{2}D^2\\
&\qquad+\frac{d_1}{2}\tilde{z}^2-\frac{d_1}{2}z_2^2\\
&\quad\le-(\frac{d_1}{2}-\frac{1}{2}-\frac{\beta}{2})z_2^2-(\frac{\sigma_D}{2}-1-\frac{c_1}{2}-\beta)\tilde{D}^2+\frac{c_1}{2}\beta z_1^2\\
&\qquad+(\frac{\sigma_{D+\beta}}{2})D^2+\frac{\beta}{2}x_d^2+\frac{d_1\beta}{2}. \qquad (49)
\end{aligned}$$

Considering the terms of the fourth and fifth lines of (42), we obtain

$$\begin{aligned}
&z_2\mu_1\Delta u-K_{v1}z_2\hat{z}_2-K_{v1}z_2\xi_1-K_1\xi_1^2\\
&\quad-[(\frac{1}{2}\mu_1)^2+\frac{1}{2}](\Delta u)^2-|\hat{z}_2\mu_1\Delta u|-\mu_1\Delta u\xi_1\\
&\quad\le-(K_1-1-\frac{1}{2}K_{v1}^2)\xi_1^2-(K_{v1}-\frac{1}{2})z_2^2+K_{v1}\tilde{z}_2z_2\\
&\qquad+z_2\mu_1\Delta u-|(z_2-\tilde{z}_2)\mu_1\Delta u|-\frac{1}{2}(\Delta u)^2\\
&\quad\le-(K_1-1-\frac{1}{2}K_{v1}^2)\xi_1^2-(K_{v1}-\frac{1}{2})z_2^2+\frac{K_{v1}}{2}z_2^2\\
&\qquad+\frac{K_{v1}\beta}{2}+|\tilde{z}_2\mu_1\Delta u|-\frac{1}{2}(\Delta u)^2\\
&\quad\le-(K_1-1-\frac{1}{2}K_{v1}^2)\xi_1^2-(\frac{K_{v1}}{2}-\frac{1}{2})z_2^2\\
&\qquad+\frac{K_{v1}\beta}{2}+\frac{1}{2}\mu_1^2\tilde{z}_2^2\\
&\quad\le-(K_1-1-\frac{1}{2}K_{v1}^2)\xi_1^2-(\frac{K_{v1}}{2}-\frac{1}{2})z_2^2\\
&\qquad+\frac{K_{v1}\beta}{2}+\frac{\mu_1^2\beta}{2}. \qquad (50)
\end{aligned}$$

Considering the terms of the last line of (42), we get

$$\begin{aligned}
&z_2e_d-z_2\hat{B}sign(\hat{z}_2)-\tilde{B}\hat{z}_2sign(\hat{z}_2)+\sigma_B\tilde{B}\hat{B}\\
&\quad\le|z_2|B-(\tilde{z}_2+\hat{z}_2)\hat{B}sign(\hat{z}_2)-\tilde{B}\hat{z}_2sign(\hat{z}_2)+\sigma_B\tilde{B}\hat{B}\\
&\quad\le|\tilde{z}_2|B+|\hat{z}_2|B-\tilde{z}\hat{B}sign(\hat{z}_2)-\hat{B}|\hat{z}_2|-\tilde{B}|\hat{z}_2|+\sigma_B\tilde{B}\hat{B}\\
&\quad\le|\tilde{z}_2|B-\tilde{z}\hat{B}sign(\hat{z}_2)+\sigma_B(B-\hat{B})\hat{B}\\
&\quad\le\sqrt{\beta}B+\frac{1}{2}\tilde{z}_2^2sign(\hat{z}_2)^2+\frac{1}{2}\hat{B}^2+\sigma_BB\hat{B}-\sigma_B\hat{B}^2\\
&\quad\le\sqrt{\beta}B+\frac{1}{2}\beta+\frac{1}{2}\hat{B}^2+\frac{\sigma_B}{2}B^2+\frac{\sigma_B}{2}\hat{B}^2-\sigma_B\hat{B}^2\\
&\quad\le-(\frac{\sigma_B}{2}-\frac{1}{2})\hat{B}^2+\sqrt{\beta}B+\frac{1}{2}\beta+\frac{\sigma_B}{2}B^2\\
&\quad\le-(\frac{\sigma_B}{2}-\frac{1}{2})(B-\tilde{B})^2+\sqrt{\beta}B+\frac{1}{2}\beta+\frac{\sigma_B}{2}B^2\\
&\quad\le-(\frac{\sigma_B}{2}-\frac{1}{2})(2B^2+2\tilde{B}^2)+\sqrt{\beta}B+\frac{1}{2}\beta+\frac{\sigma_B}{2}B^2\\
&\quad\le-(\sigma_B-1)\tilde{B}^2+\sqrt{\beta}B+\frac{1}{2}\beta+(1-\frac{\sigma_B}{2})B^2. \qquad (51)
\end{aligned}$$

According to (46) and (48)-(51), (42) can be rewritten as

$$\begin{aligned}
\dot{V}_2\le&-c_1z_1^2-\frac{c_{21}}{2}z_2^2+\frac{c_{21}}{2}\beta-(\frac{\sigma_F}{2}-1)\tilde{F}^2+\frac{\beta}{2}z_1^2\\
&+\frac{\beta}{2}x_d^2+\frac{\sigma_F}{2}F^2-(\frac{\sigma_M}{2}-\frac{1}{2}-\frac{c_1}{2}-\frac{c_1^2}{2})\tilde{M}^2\\
&+\frac{\beta}{2}\ddot{x}_d^2+\frac{c_1\beta}{2}z_2^2+\frac{c_1^2\beta}{2}z_1^2+\frac{\sigma_M}{2}M^2\\
&-(\frac{d_1}{2}-\frac{1}{2}-\frac{\beta}{2})z_2^2-(\frac{\sigma_D}{2}-1-\frac{c_1}{2}-\beta)\tilde{D}^2\\
&+\frac{c_1}{2}\beta z_1^2+(\frac{\sigma_{D+\beta}}{2})D^2+\frac{\beta}{2}x_d^2+\frac{d_1\beta}{2}\\
&-(K_1-1-\frac{1}{2}K_{v1}^2)\xi_1^2-(\frac{K_{v1}}{2}-\frac{1}{2})z_2^2\\
&+\frac{K_{v1}\beta}{2}+\frac{\mu_1^2\beta}{2}-(\sigma_B-1)\tilde{B}^2+\sqrt{\beta}B\\
&+\frac{1}{2}\beta+(1-\frac{\sigma_B}{2})B^2
\end{aligned}$$

$$
\begin{aligned}
\leq\ & -(c_1 - \frac{\beta}{2} - \frac{c_1^2\beta}{2} - \frac{c_1\beta}{2})z_1^2 - (\frac{c_{21}}{2} - \frac{c_1\beta}{2})z_2^2 \\
& - (\frac{d_1}{2} - \frac{1}{2} - \frac{\beta}{2})z_2^2 - (\frac{K_{v1}}{2} - \frac{1}{2})z_2^2 - (K_1 - 1 \\
& - \frac{1}{2}K_{v1}^2)\xi_1^2 - (\frac{\sigma_D}{2} - 1 - \frac{c_1}{2} - \beta)\tilde{D}^2 - (\frac{\sigma_F}{2} - 1)\tilde{F}^2 \\
& - (\frac{\sigma_M}{2} - \frac{1}{2} - \frac{c_1}{2} - \frac{c_1^2}{2})\tilde{M}^2 - (\sigma_B - 1)\tilde{B}^2 + \frac{c_{21}}{2}\beta \\
& + \frac{\beta}{2}x_d^2 + \frac{\sigma_F}{2}F^2 + \frac{\beta}{2}\ddot{x}_d^2 + \frac{\sigma_M}{2}M^2 + (\frac{\sigma_{D+\beta}}{2})D^2 \\
& + \frac{\beta}{2}x_d^2 + \frac{d_1\beta}{2} + \frac{K_{v1}\beta}{2} + \frac{\mu_1^2\beta}{2} + \sqrt{\beta}B \\
& + \frac{1}{2}\beta + (1 - \frac{\sigma_B}{2})B^2 \\
\leq\ & -\kappa_1 V_2 + C_1
\end{aligned} \tag{52}
$$

where

$$
\begin{aligned}
\kappa_1 = min\Big(& 2(c_1 - \frac{\beta}{2} - \frac{c_1^2\beta}{2} - \frac{c_1\beta}{2}), \frac{2}{M}(\frac{c_{21}}{2} - \frac{c_1\beta}{2} \\
& + \frac{d_1}{2} - 1 - \frac{\beta}{2} + \frac{K_{v1}}{2}), 2(K_1 - 1 - \frac{1}{2}K_{v1}^2), \\
& 2\gamma_{D1}(\frac{\sigma_D}{2} - 1 - \frac{c_1}{2} - \beta), 2\gamma_{F1}(\frac{\sigma_F}{2} - 1), \\
& 2\gamma_{M1}(\frac{\sigma_M}{2} - \frac{1}{2} - \frac{c_1}{2} - \frac{c_1^2}{2}), 2\gamma_{M1}(\sigma_B - 1)\Big)
\end{aligned} \tag{53}
$$

$$
\begin{aligned}
C_1 =\ & \frac{c_{21}}{2}\beta + \frac{\beta}{2}x_d^2 + \frac{\sigma_F}{2}F^2 + \frac{\beta}{2}\ddot{x}_d^2 + \frac{\sigma_M}{2}M^2 \\
& + (\frac{\sigma_{D+\beta}}{2})D^2 + \frac{\beta}{2}x_d^2 + \frac{d_1\beta}{2} + \frac{K_{v1}\beta}{2} \\
& + \frac{\mu_1^2\beta}{2} + \sqrt{\beta}B + \frac{1}{2}\beta + (1 - \frac{\sigma_B}{2})B^2.
\end{aligned} \tag{54}
$$

Remark 5: In order to guarantee that $\dot{V}_2$ is negative definite, the parameters of the controller need to fulfil the following criteria: $c_1 - \frac{\beta}{2} - \frac{c_1^2\beta}{2} - \frac{c_1\beta}{2} > 0$, $\frac{c_{21}}{2} - \frac{c_1\beta}{2} + \frac{d_1}{2} - 1 - \frac{\beta}{2} + \frac{K_{v1}}{2} > 0$, $K_1 - 1 - \frac{1}{2}K_{v1}^2 > 0$, $\frac{\sigma_D}{2} - 1 - \frac{c_1}{2} - \beta > 0$, $\frac{\sigma_F}{2} - 1 > 0$, $\frac{\sigma_M}{2} - \frac{1}{2} - \frac{c_1}{2} - \frac{c_1^2}{2} > 0$, $\sigma_B - 1 > 0$.

Theorem 2: Considering the plant (1)-(4) under Assumption 1, the adaptive output time delay control scheme, consisting of the control laws (10) and (33), auxiliary system (38), the updated laws (34)-(37), and the TDE (16), guarantees that the closed-loop system is semi-globally stable in the sense that all the closed-loop signals are bounded. Furthermore, the tracking error signal z_1 converges asymptotically to the compact set Ω defined by

$$\Omega := \{z_1 \in R \,|\, |z_1| \leq \sqrt{Z}\} \tag{55}$$

where $Z = 2(V_2(0) + \frac{C_1}{\kappa_1})$ with C_1, κ_1 are given in (53) and (54).

Proof: From (25), and following steps in [44], we can can be concluded that z_1 converges to the compact set defined by (55). By using the method of the proof for Theorem 1, it is easily proved that all the signals in the closed-loop system are bounded, and hence is omitted.

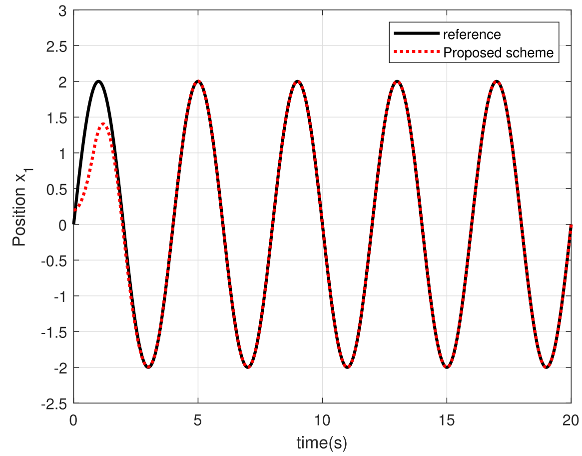


FIGURE 1. **Tracking performance of the proposed scheme under state feedback control.**

Remark 6: To avoid the possible chattering problem caused by the sign function, the sign function in (15) and (33) can be replaced by a hyperbolic tangent function as in [19]:

$$tanh(\frac{z_2}{\varsigma}) = \frac{sinh(\frac{z_2}{\varsigma})}{conh(\frac{z_2}{\varsigma})} = \frac{e^{\frac{z_2}{\varsigma}} - e^{-\frac{z_2}{\varsigma}}}{e^{\frac{z_2}{\varsigma}} + e^{-\frac{z_2}{\varsigma}}} \tag{56}$$

where ς is a small positive constant. Using the inequality $0 \leq |z| - ztanh(\frac{z_2}{\varsigma}) \leq 0.2785\varsigma$, it is easy to conclude that Theorem 2 is still true and that the asymptotic tracking performance in Theorem 1 would change to asymptotically converge to a compact set as in Theorem 2.

Remark 7: The transient performance of the system can be improved under the condition of ensuring the stability of the system by the following tuning methods: (i) Large c_1, c_2, c_{21} can improve the tracking error of the system, but it may lead to system oscillation and large control energy. Therefore, c_1, c_2 and c_{21} should not be chosen too large when the tracking error is guaranteed. (ii) The adaptive scaling factors $\gamma_D, \gamma_F, \gamma_M, \gamma_B, \gamma_{D1}, \gamma_{F1}, \gamma_{M1}, \gamma_{B1}$ are usually designed to be small, and the other constants (such as K, σ_B and so on) are designed to meet the requirements in Remark 4 and Remark 5, mainly to ensure the stability of the system. Then, the fine-tuning of these parameters is carried out to improve the transient performance of the system.

Remark 8: Both the control laws and the adaptive update laws have the same complexity as the classical adaptive backstepping controller in [45]. The auxiliary variable system consists of only a first order differential equation, and the state of the auxiliary variable system remains unchanged when the system enters a steady state (i.e. $\xi < \sigma_1$). The TDE is calculated from the system state and parameters at time $(t - h)$. Because the system state and parameters at time $(t - h)$ are known, the implementation is also convenient and simple. Therefore, in general, the control scheme proposed in this paper has clear objectives of each component and is easy to implement.

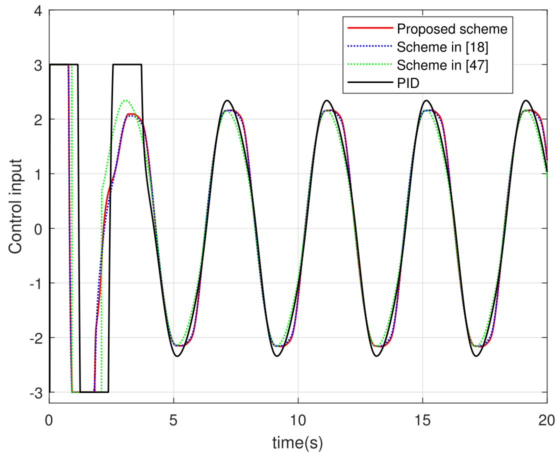


FIGURE 2. Control inputs of the four comparing control schemes under state feedback control.

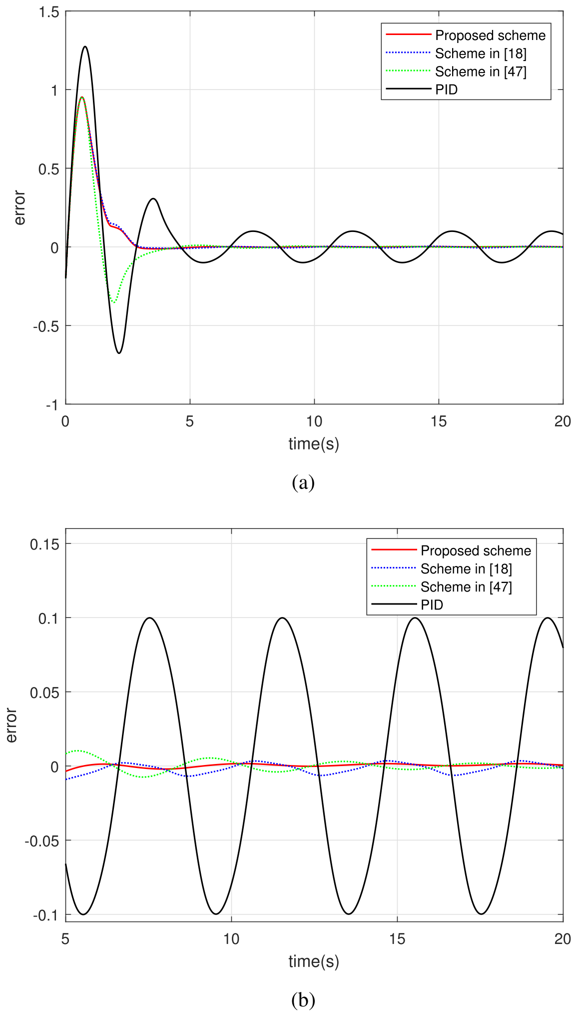


FIGURE 3. Tracking errors of the four comparing control schemes under state feedback control: (a) Tracking errors for $t = [0\ 20]$; (b) Local enlargement of tracking errors for $t = [5\ 20]$.

IV. SIMULATION STUDY

In this section, we will provide two cases to demonstrate the effectiveness of our proposed controller (15) and (33) for system (1)-(4), respectively, i.e. (i) full state feedback control simulation; and (ii) output feedback control simulation. The actual values of the system parameters are selected as follows: $D = 0.15\ Ns/m, M = 1\ Kg, F = 1\ M/m$. The Bouc-Wen hysteresis parameters are chosen as $\beta = 1, n = 2, \chi = 0.5, \mu_1 = 1$, and $\mu_2 = 1$. The input saturation limit is given as $u_{max} = 3$. The control objective is to drive the system displacement x to track the desire trajectory $x_d = 2sin(0.5\pi t)$. The initial state is chosen as x(0)=0.2, and the delay time of TDE is selected as $h = 0.001s$.

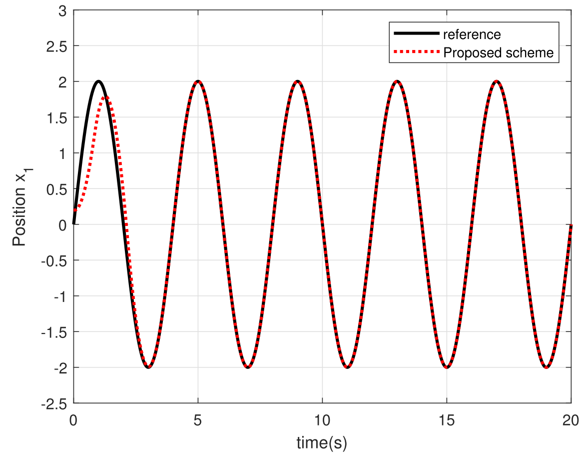


FIGURE 4. Tracking performance of the proposed control scheme under output feedback control.

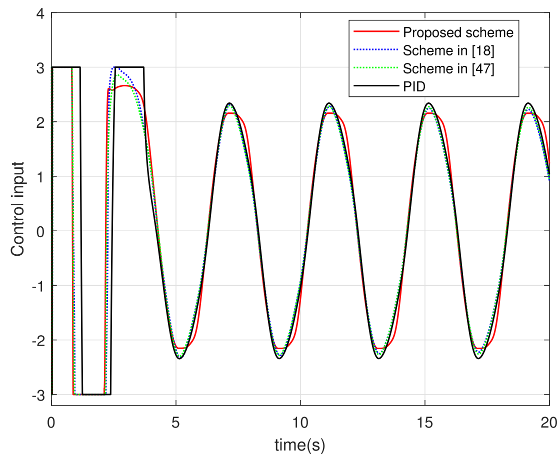
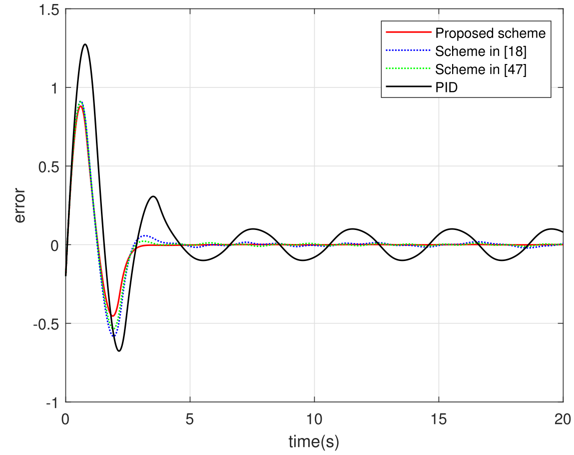


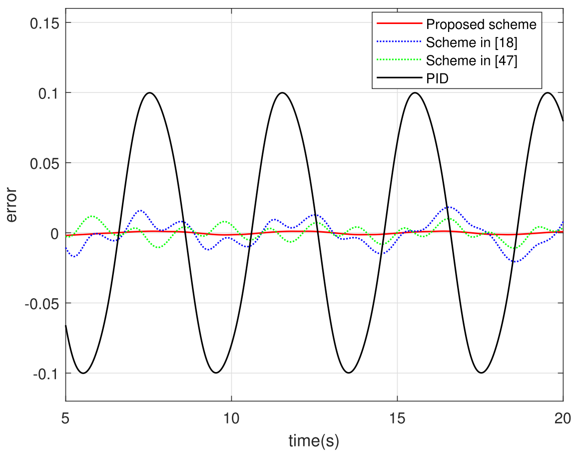
FIGURE 5. Control inputs of the four comparing control schemes under output feedback control.

The simulation parameters satisfying Theorem 1 for the full state feedback control scheme are chosen as: $c_1 = 0.4, c_2 = 17.2, \hat{D}(0) = 0.2, \hat{F}(0) = 1.2, \hat{M}(0) = 0.6, \xi(0) = 0.1, \hat{B}(0) = 1,, \gamma_D = 0.005, \gamma_F = 20, \gamma_M = 0.2, \gamma_B = 0.2, K = 10, \sigma = 0.1, K_v = 1$. To avoid the chattering problem caused by the sign function, the parameter in (56) is selected as $\varsigma = 0.01$.

The simulation parameters satisfying Theorem 2 for the output feedback control scheme are chosen as:$c_1 = 10, c_{21} = 0.4, d_1 = 1, \hat{D}(0) = 0.2, \hat{F}(0) = 1.3, \hat{M}(0) = 0.6, \xi(0) = 0.02, \hat{B}(0) = 0.8,, \gamma_{D1} = 0.001, \gamma_{F1} = 0.00001,$



(a)



(b)

FIGURE 6. Tracking errors of the four comparing control schemes under output feedback control: (a) Tracking errors for $t = [0\ 20]$; (b) Local enlargement of tracking errors for $t = [5\ 20]$.

$\gamma_{M1} = 0.001, \gamma_{B1} = 0.01, \sigma_D = 35, \sigma_F = 2.1, \sigma_M = 145, \sigma_B = 2, \epsilon = 0.0197, K_1 = 10, \sigma_1 = 0.01, K_{v1} = 1, \varsigma = 0.1$.

To illustrate the effectiveness of the proposed control scheme, the schemes in [18], [46] and the classic PID controller are also applied to the system (1)-(4). In [18], the nonlinear part of the hysteresis is considered as a bounded disturbance. In [46], the sliding mode control with perturbation estimation is used to deal with the hysteresis. Fig. 1 shows the tracking performance using the proposed method under state feedback control. It can be clearly seen that the output of the system can track the desired trajectory well. The control input under state feedback control is shown in Fig.2. And Fig.3 shows the tracking error under the state feedback control. Obviously, the proposed scheme is better than the other three schemes under state feedback control. As for output feedback control simulation, Fig.4-Fig.6 show the tracking performance, control input and tracking error, respectively. It can be concluded that the proposed scheme is effective and gives better performance than the other three control methods in comparison.

It should be mentioned that simulations for several different desired trajectories with different initial conditions have also been conducted. Results show that they all have similar behaviours as the one shown in this paper. The simulations for $h = 0.01$, which means larger delay time for TDE, give almost identical results. This further demonstrates the effectiveness of the proposed control scheme.

V. CONCLUSION

In this paper, a new time delay feedback control strategy with hysteresis compensation is proposed for a class of second-order electromechanical systems with Bouc-Wen hysteresis and input saturation constraint. The controller is designed by backstepping design method. In the controller design process, the TDE is used to estimate the nonlinear part of the Bouc-Wen model, the adaptive law is applied to eliminate the TDE error, and the auxiliary variable system is used to deal with the input saturation problem. Based on Lyapunov direct method, the corresponding controllers are designed for the full state feedback control and the output feedback control, respectively, and the stability of the closed-loop system is analyzed. Finally, the effectiveness of the control scheme is further demonstrated by simulations.

In the proposed control scheme, only a classic backstepping feedback controller is designed with the help of the time delay control. In the future, fractional order controller, sliding mode controller, reinforcement learning controller can be combined with the time delay control to compensate the hysteresis. In addition, the hysteresis compensation control scheme based on TDE can be applied to the controller design of other systems with hysteresis, such as piezoelectric positioning stages, atomic force microscope, etc.

REFERENCES

[1] Y. Jian, D. Huang, J. Liu, and D. Min, "High-precision tracking of piezoelectric actuator using iterative learning control and direct inverse compensation of hysteresis," *IEEE Trans. Ind. Electron.*, vol. 66, no. 1, pp. 368–377, Jan. 2019.

[2] D. Croft, G. Shed, and S. Devasia, "Creep, hysteresis, and vibration compensation for piezoactuators: Atomic force microscopy application," *J. Dyn. Syst., Meas., Control*, vol. 123, no. 1, pp. 35–43, Mar. 2001.

[3] F. Stefanski, B. Minorowicz, J. Persson, A. Plummer, and C. Bowen, "Non-linear control of a hydraulic piezo-valve using a generalised Prandtl–Ishlinskii hysteresis model," *Mech. Syst. Signal Process.*, vol. 82, no. 1, pp. 412–431, Jan. 2017.

[4] J. Li, H. Tang, Z. Wu, H. Li, G. Zhang, X. Chen, J. Gao, Y. Xu, and Y. He, "A stable autoregressive moving average hysteresis model in flexure fast tool servo control," *IEEE Trans. Autom. Sci. Eng.*, vol. 16, no. 3, pp. 1699–1708, Jul. 2019.

[5] L. Zhang, J. Huang, Y. Tang, R. Li, Q. Huang, R. Cheng, and C. Wang, "Independent driving method for significant hysteresis reduction of piezoelectric stack actuators," *Rev. Sci. Instrum.*, vol. 90, no. 11, Nov. 2019, Art. no. 115006.

[6] Z. Li, N. Huang, Y. Zhong, J. Zeng, and G. Ma, "Fractional order modeling and experimental verification of hysteresis nonlinearities in piezoelectric actuators," *Opt. Precis. Eng.*, vol. 28, no. 5, pp. 1124–1131, May 2020.

[7] J. Wang, Z. Liu, Y. Zhang, and C. L. P. Chen, "Neural adaptive event-triggered control for nonlinear uncertain stochastic systems with unknown hysteresis," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 11, pp. 3300–3312, Nov. 2019.

[8] X. Zhang, Y. Wang, C. Wang, C.-Y. Su, Z. Li, and X. Chen, "Adaptive estimated inverse output-feedback quantized control for piezoelectric positioning stage," *IEEE Trans. Cybern.*, vol. 49, no. 6, pp. 2106–2118, Jun. 2019.

[9] X. Gao and Y. Liu, "Research on control strategy in giant magnetostrictive actuator based on Lyapunov stability," *IEEE Access*, vol. 7, pp. 77254–77260, 2019.

[10] M. Goldfarb and N. Celanovic, "Modeling piezoelectric stack actuators for control of micromanipulation," *IEEE Control Syst.*, vol. 17, no. 3, pp. 69–79, Jun. 1997.

[11] M. H. M. Ramli, T. V. Minh, and X. Chen, "Pseudoextended Bouc–Wen model and adaptive control design with applications to smart actuators," *IEEE Trans. Control Syst. Technol.*, vol. 27, no. 5, pp. 2100–2109, Sep. 2019.

[12] B. D. Coleman and M. L. Hodgdon, "A constitutive relation for rate-independent hysteresis in ferromagnetically soft materials," *Int. J. Eng. Sci.*, vol. 24, no. 6, pp. 897–919, Jan. 1986.

[13] X. Li, D. Kim, S. M. Neumayer, M. Ahmadi, and S. V. Kalinin, "Estimating preisach density via subset selection," *IEEE Access*, vol. 8, pp. 61767–61774, 2020.

[14] P. Krejci and K. Kuhnen, "Inverse control of systems with hysteresis and creep," *IEE Proc. Control Theory Appl.*, vol. 148, no. 3, pp. 185–192, May 2001.

[15] X. Zhao and Y. Tan, "Modeling hysteresis and its inverse model using neural networks based on expanded input space method," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 3, pp. 484–490, May 2008.

[16] X. Mao, Y. Wang, X. Liu, and Y. Guo, "A hybrid feedforward-feedback hysteresis compensator in piezoelectric actuators based on least-squares support vector machine," *IEEE Trans. Ind. Electron.*, vol. 65, no. 7, pp. 5704–5711, Jul. 2018.

[17] J. Zhou, C. Wen, and T. Li, "Adaptive output feedback control of uncertain nonlinear systems with hysteresis nonlinearity," *IEEE Trans. Autom. Control*, vol. 57, no. 10, pp. 2627–2633, Oct. 2012.

[18] F. Ikhouane, V. Mañosa, and J. Rodellar, "Adaptive control of a hysteretic structural system," *Automatica*, vol. 41, no. 2, pp. 225–231, Feb. 2005.

[19] Z. Li, Y. Hu, Y. Liu, T. Chen, and P. Yuan, "Adaptive inverse control of non-linear systems with unknown complex hysteretic non-linearities," *IET Control Theory Appl.*, vol. 6, pp. 1–7, Jan. 2012.

[20] Q. Xu, "Precision motion control of piezoelectric nanopositioning stage with chattering-free adaptive sliding mode control," *IEEE Trans. Autom. Sci. Eng.*, vol. 14, no. 1, pp. 238–248, Jan. 2017.

[21] Z. Li, N. Huang, Y. Zhong, G. Ma, G. Liu, and D. Zhu, "Feedback feed-forward control of piezoelectric positioning stages and experimental evaluation," in *Proc. Chin. Autom. Congr. (CAC)*, Xian, China, Nov. 2018, pp. 3758–3762.

[22] X. Chen, "Discrete-time adaptive control design for ionic polymer-metal composite actuators," *IEEE Access*, vol. 6, pp. 28114–28121, 2018.

[23] S. Tong, K. Sun, and S. Sui, "Observer-based adaptive fuzzy decentralized optimal control design for strict-feedback nonlinear large-scale systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 569–584, Apr. 2018.

[24] Y. Li, K. Sun, and S. Tong, "Observer-based adaptive fuzzy fault-tolerant optimal control for SISO nonlinear systems," *IEEE Trans. Cybern.*, vol. 49, no. 2, pp. 649–661, Feb. 2019.

[25] W. Chen, L. Jiao, J. Li, and R. Li, "Adaptive NN backstepping output-feedback control for stochastic nonlinear strict-feedback systems with time-varying delays," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 40, no. 3, pp. 939–950, Jun. 2010.

[26] V. Andrieu, L. Praly, and A. Astolfi, "High gain observers with updated gain and homogeneous correction terms," *Automatica*, vol. 45, no. 2, pp. 422–428, Feb. 2009.

[27] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

[28] Z. Zhao, Y. Liu, and F. Luo, "Output feedback boundary control of an axially moving system with input saturation constraint," *ISA Trans.*, vol. 68, pp. 22–32, May 2017.

[29] M. Chen, S. S. Ge, and B. Ren, "Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints," *Automatica*, vol. 47, no. 3, pp. 452–465, Mar. 2011.

[30] Q. Hu and B. Xiao, "Intelligent proportional-derivative control for flexible spacecraft attitude stabilization with unknown input saturation," *Aerosp. Sci. Technol.*, vol. 23, no. 1, pp. 63–74, Dec. 2012.

[31] M. Li, M. Hou, and C. Yin, "Adaptive attitude stabilization control design for spacecraft under physical limitations," *J. Guid. Control. Dyn.*, vol. 39, no. 9, pp. 2176–2180, Sep. 2016.

[32] B. M. Chen, T. H. Lee, K. Peng, and V. Venkataramanan, "Composite nonlinear feedback control for linear systems with input saturation: Theory and an application," *IEEE Trans. Autom. Control*, vol. 48, no. 3, pp. 427–439, Mar. 2003.

[33] S. Rasoolinasab, S. Mobayen, A. Fekih, P. Narayan, and Y. Yao, "A composite feedback approach to stabilize nonholonomic systems with time varying time delays and nonlinear disturbances," *ISA Trans.*, vol. 101, pp. 177–188, Jun. 2020.

[34] S. Mobayen and J. Ma, "Robust finite-time composite nonlinear feedback control for synchronization of uncertain chaotic systems with nonlinearity and time-delay," *Chaos, Solitons Fractals*, vol. 114, pp. 46–54, Sep. 2018.

[35] L. Zhao, J. Yu, and C. Lin, "Command filter based adaptive fuzzy bipartite output consensus tracking of nonlinear coopetition multi-agent systems with input saturation," *ISA Trans.*, vol. 80, pp. 187–194, Sep. 2018.

[36] Y. Yu, H.-K. Lam, and K. Y. Chan, "T–S fuzzy-model-based output feedback tracking control with control input saturation," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3514–3523, Dec. 2018.

[37] K. Youcef-Toumi and O. Ito, "A time delay controller for systems with unknown dynamics," *J. Dyn. Syst., Meas., Control*, vol. 112, no. 1, pp. 133–142, Mar. 1990.

[38] S. Roy and I. N. Kar, *Adaptive-Robust Control With Limited Knowledge on Systems Dynamics: An Artificial Input Delay Approach and Beyond*. Singapore: Springer, 2011.

[39] Y. Wang, F. Yan, J. Chen, F. Ju, and B. Chen, "A new adaptive time-delay control scheme for cable-driven manipulators," *IEEE Trans. Ind. Informat.*, vol. 15, no. 6, pp. 3469–3481, Jun. 2019.

[40] J. Zhou, C. Wen, and C. Zhang, "Adaptive backstepping control of A piezo-positioning mechanism with hysteresis," *Trans. Can. Soc. Mech. Eng.*, vol. 31, no. 1, pp. 97–110, Mar. 2007.

[41] F.-J. Lin, H.-J. Shieh, and P.-K. Huang, "Adaptive wavelet neural network control with hysteresis estimation for piezo-positioning mechanism," *IEEE Trans. Neural Netw.*, vol. 17, no. 2, pp. 432–444, Mar. 2006.

[42] C. Lin and S. Yang, "Independent driving method for significant hysteresis reduction of piezoelectric stack actuators," *Mechatronics*, vol. 16, no. 7, pp. 417–426, Sep. 2006.

[43] R. Ouyang and B. Jayawardhana, "Absolute stability analysis of linear systems with Duhem hysteresis operator," *Automatica*, vol. 50, no. 7, pp. 1860–1866, Jul. 2014.

[44] W. He, S. S. Ge, Y. Li, E. Chew, and Y. S. Ng, "Neural network control of a rehabilitation robot by state and output feedback," *J. Intell. Robotic Syst.*, vol. 80, no. 1, pp. 15–31, Oct. 2015.

[45] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design*. New York, NY, USA: Wiley, 1995.

[46] R. Xu, X. Zhang, H. Guo, and M. Zhou, "Sliding mode tracking control with perturbation estimation for hysteresis nonlinearity of piezo-actuated stages," *IEEE Access*, vol. 6, pp. 30617–30629, 2018.

ZHIFU LI (Member, IEEE) received the B.Sc. and M.Sc. degrees in control theory and control engineering from Central South University, Changsha, Hunan, China, in 2003 and 2006, respectively, and the Ph.D. degree in control theory and control engineering from the South China University of Technology, Guangzhou, Guangdong, China, in 2012.

From 2012 to 2015, he was a Postdoctoral Fellow in mechatronics with the South China University of Technology. He has been with the School of Mechanical and Electrical Engineering, Guangzhou University, since 2015, where he is currently an Associate Professor. He has published over 30 research articles. His research interests include nonlinear control and its application, swarm intelligence optimization, applied fractional calculus, robust adaptive control, and machine vision.

JUNHAI ZENG received the B.Sc. degree in automation from the Zhongkai University of Agriculture and Engineering, Guangzhou, Guangdong, China, in 2018. He is currently pursuing the master's degree in mechanical engineering with the School of Mechanical and Electrical Engineering, Guangzhou University, Guangzhou. His research interests include swarm intelligence optimization, intelligent control, and system identification theory.

YANGQUAN CHEN (Senior Member, IEEE) received the Ph.D. degree in advanced control and instrumentation from Nanyang Technological University, Singapore, in 1998.

He was on the Faculty of Electrical and Computer Engineering, Utah State University, before he joined the School of Engineering, University of California, Merced, CA, USA, in 2012, where he teaches "Mechatronics" for juniors and "Fractional Order Mechanics" for graduates. His research interests include mechatronics for sustainability, cognitive process control and hybrid lighting control, multi-UAV based cooperative multispectral "personal remote sensing" and applications, applied fractional calculus in controls, signal processing and energy informatics, as well as distributed measurement and distributed control of distributed parameter systems using mobile actuator and sensor networks.

GE MA received the B.Sc. degree in mathematics from Shandong Jianzhu University, Jinan, Shandong, China, in 2010, and the Ph.D. degree in control theory and control engineering from the South China University of Technology, Guangzhou, Guangdong, China, in 2016.

She has been with the School of Mechanical and Electrical Engineering, Guangzhou University, since 2016, where she is currently a Lecturer. Her research interests include image processing, machine vision, control theory, and intelligence optimization.

GUIYUN LIU received the B.Sc. degree from Qinghai University, Xining, China, in 2006, and the Ph.D. degree from the South China University of Technology, Guangzhou, Guangdong, China, in 2012.

He has been with the School of Mechanical and Electrical Engineering, Guangzhou University, since 2012, where he is currently an Associate Professor. His research interests include energy-efficient wireless communications, intelligence, and optimization.