IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received July 31, 2020, accepted August 28, 2020, date of publication September 21, 2020, date of current version October 2, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3025635

Resource Allocation for Multicarrier Rate-Splitting Multiple Access System

LIHUA LI[1], (Member, IEEE), KEJIA CHAI[1], JILONG LI[2], AND XINGWANG LI[3], (Senior Member, IEEE)

[1]State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China
[2]The Academy of Broadcasting Science, SARFT, Beijing 100866, China
[3]School of Physics and Electronic Information Engineering, Henan Polytechnic University, Jiaozuo 454000, China

Corresponding author: Lihua Li (lilihua@bupt.edu.cn)

This work was supported in part by the National Key Research and Development Program of China under Grant 2018YFF0301201, and in part by the Scientific Research Project of Academy of Broadcasting Science, National Radio and Television Administration, under Grant JBKY2019027.

ABSTRACT In this article, we investigate the resource allocation problem for the multicarrier rate-splitting multiple access (RSMA) systems. On each subcarrier, messages are non-orthogonal superimposed on the power domain through the one-layer RSMA scheme. A novel three-step resource allocation algorithm is proposed to deal with the non-convex problem of sum rate maximization. In step 1, assuming average power allocation among subcarriers, we obtain the power distribution factors of the users in a single subcarrier by converting this problem into a difference of convex program (DCP), and approximate it by its first-order Taylor expansion. In step 2, we convert the user-subcarrier matching problem into an assignment problem and use the Hungarian algorithm to solve it. In step 3, the optimized power allocation algorithm is used to calculate the power allocation among the subcarriers, and then updates the power vector for each user. Numerical results show that our proposed three-step resource allocation algorithm could achieve comparable sum rate performance to the existing near-optimal solution with much lower computational complexity and outperforms orthogonal multiple access (OMA) scheme.

INDEX TERMS Multicarrier rate-splitting multiple access, resource allocation, difference of convex program, Hungarian algorithm, optimized power allocation algorithm.

I. INTRODUCTION

Multicarrier techniques have been widely adopted in broadband wireless communications over the last decade, due to their flexibility in resource allocation and multiuser diversity[1]. In conventional multicarrier systems, orthogonal frequency division multiple access (OFDMA) has been widely adopted, in which the whole system bandwidth is divided into multiple orthogonal subcarriers and each subcarrier is allocated to at most one user in order to avoid multiuser interference [2].

According to the statistics, the global mobile devices and connections will grow to 13.1 billion by 2023 [3]. To meet the demand for the future data traffic, many research works focus on non-orthogonal multicarrier systems, which can provide a 30% capacity gain over orthogonal multiple access (OMA) systems [4]. Considering the data rate and the error rate constraints, authors in [5] designed user grouping, subcarrier allocation and bit allocation schemes of multicarrier non-orthogonal multiple access (NOMA) systems to reduce the total transmit power. The authors of [6] developed an dynamic programming (DP) recursion framework to jointly optimize the subcarrier assignment, transmission power allocation and transmission rate adaptation of each user in the downlink multicarrier NOMA systems. And they further proposed a low-complexity algorithm based on the principles of block coordinate descent (BCD) and concave-convex procedure (CCCP) in [7].

On a parallel avenue, rate-splitting multiple access (RSMA) as a more general and efficacious transmission framework has been attract considerable interests, since it can achieve higher spectrum efficiency and energy efficiency than NOMA in any user deployments [8]. It uses linearly precoded rate-splitting (RS) at the transmitter to split the messages of users into multiple common and private messages, and transmits them simultaneously after superposition [9]. At the receiver, successive interference cancellation (SIC) is utilized to decode the common messages before decoding the private messages.

The associate editor coordinating the review of this manuscript and approving it for publication was Yanjiao Chen.

In [10]-[15], the RS-assisted single cell multi-antenna broadcast channels (BC) were considered, while the multiple cell interference (IC) were investigated in [16]-[21]. The recent work [22] investigated RS-assisted multi-cell BC by introducing RS into cloud-radio access networks (C-RAN) with partial cooperation among BSs. The work in [23] used RS transmission scheme to deal with a resource allocation problem for the multigroup multicast scenario. The multi-layer transmission structure of RSMA enables a superposition of multiple type of services, as the common message could be used to carry broadcast service, the sub-layer transmission tageted to partial users could be used for the transmission of multicast services, and the private lay is used to transmit unicast services. This is the main advantage of RSMA compared to other NOMA schemes, which could save the expenses for wireless resources for the transmission of broadcast and multicast services. NOMA and SDMA are just multiple schemes suitable for unicast service transmission,which is not suitable for broadcast and mulicast services. Therefore, in the paper, we consider the exploitation of RSMA for joint transmission of multi services.

Multicarrier transmission has become a primary scheme for wireless broadband communication systems as 4G, 5G and beyond. Multicarrier resources allocation is a key issue, which is more complicated for multicarrier non-orthoganal multiple access. However, the current research contributions on multicarrier resource allocation only focus on NOMA. To the best of our knowledge, a design of power allocation and user subcarrier matching for multicarrier RSMA (MC-RSMA) systems is missing. To this end, we propose a resource allocation algorithm to maximize the sum rate of the RSMA-based system. The main contributions of this article are summarized as follows:

- We propose a three-step low-complexity resource allocation framework. One dimensional problem such as power allocation on a single subcarrier, matching between user and subcarrier and power allocation among different subcarriers are solved in steps. Each sub-problem is analyzed and solved by the optimal algorithm.
- In step 1, assuming that the power of different subcarriers is evenly distributed, the non-convex problem of power distribution of a single subcarrier is transformed into a convex problem with the aid of difference of convex program (DCP) and approximate it by its first-order Taylor expansion around a feasible point.
- In step 2, we transform the user-subcarrier 0-1 matching problem into an assignment problem. To solve the assignment problem, an Hungarian algorithm based user-subcarrier matching algorithm is proposed.
- In step 3, the power allocation problem between different subcarriers problem is proved to be a concave function. We transform it into a Lagrange function and solved it by an optimized power allocation algorithm, which provide better sum-rate performance than equal power allocation.
- For transmit beamforming, multicarrer resource allocation and power allocation, channel state information (CSI) is required at the transmitter. Howerver perfect CSI is impossible to be obtained due to channel estimation error, channel quantization error and CSI feedback delay. CSI errors bring distinct performance degradation for beamformning, power and mulicarrier resource allocation. To be more practical, we evaluate the proposed sheme with imperfect CSI considerations.

In summary, we propose a power and muticarrier resource allocation scheme for muli-user multi-carrier RSMA for joint transmission of broadcast and unicast services, where random beamforming and zero-forcing beamforming are used for common and private messages considering complexity issue. The joint optimization of beamforming vectors is also an important and complicated problem, which is out of the scope of this article. The rest of this article is organized as follows. In Section II, the system model for the RSMA scheme and the sum rate maximization problem mathematically are presented. Section III proposes the three-step resource allocation framework. The power allocation algorithm for one subcarrier, the Hungarian algorithm for user-subcarrier matching problem and the power allocation algorithm between subcarriers are analyzed in Section IV, Section V and Section VI respectively. Simulation results are provided in Section VII, and finally, conclusions are drawn in Section VIII.

Notation: Bold upper and lower letters denote matrices and vectors respectively. A symbol not in bold font denotes a scalar. $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^{\perp}$respectively denote the Hermitian, transpose and the null space of a matrix or vector. $\|\cdot\|$ refers to the 2-norm of a vector. $\mathbb{E}[\cdot]$ refers to the expectation of a random variable. $(\cdot)^+$ denotes the value is non-negative.

II. SYSTEM MODEL

We consider a downlink system, which comprises a base station (BS) with N_t transmit antennas and K users each with a single-antenna receiver, denoted by a set $\mathcal{K} = \{1, 2, \ldots, K\}$. The overall system bandwidth W is divided into G orthogonal subcarriers, indexed by $\mathcal{G} = \{1, 2, \ldots, G\}$. Unlike OFDMA system, a single subcarrier can be shared by multiple users. According to the channel state information (CSI) feedback from the users, the BS can obtain the channel of user k on the subcarrier g, denoted as $\mathbf{h}_{k,g}$.

The generalized RS framework is very general and can be used to identify the best possible performance, but its implementation can be complex due to the large number of SIC layers and common messages involved [8]. To overcome this problem, we use one-layer RS in this work. Each message is split into a common part and a private part. That is, a new data stream is added to the original data streams on each subcarrier. In each receiver, Each user firstly decodes the common message and proceeds to decode its own private message via SIC. The two-user RSMA transmission model is shown in Fig. 1.

Denote the messages superimposed on subcarrier g is $\mathcal{K}_g$, $|\mathcal{K}_g| = K_g$. Let $\mathbf{s}_{g,c}$ represent the super common message

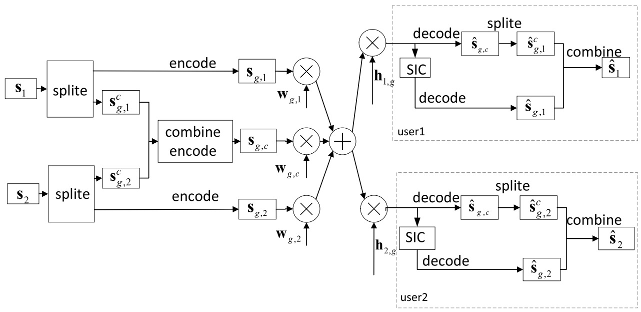


FIGURE 1. **Two-user RSMA transmission model.**

that consists of common message split from K_g pieces of messages. For user k, the rest of the private part is denoted by $\mathbf{s}_{g,k}$. The power domain multiplexing technique is used to encode the signals. And the signal sent on the subcarrier g can be expressed as

$$\mathbf{x}_g = P_{g,c}\mathbf{w}_{g,c}\mathbf{s}_{g,c} + \sum_{k\in\mathcal{K}_g} P_{g,k}\mathbf{w}_{g,k}\mathbf{s}_{g,k} \tag{1}$$

The private message is trasmitted along a ZF-precoder as $\mathbf{w}_{g,k} \subseteq \text{span}\left(\left\{\hat{\mathbf{h}}^{\perp}_{k',g}\right\}_{k'\in\mathcal{K}_g\setminus\{k\}}\right)$ and $\mathbf{w}_{g,c}$ is a random precoder. The received signal at user k is given by

$$\mathbf{y}_k = \mathbf{h}^H_{k,g}\left(P_{g,c}\mathbf{w}_{g,c}\mathbf{s}_{g,c} + \sum_{k\in\mathcal{K}_g} P_{g,k}\mathbf{w}_{g,k}\mathbf{s}_{g,k}\right) + \mathbf{n}_k \tag{2}$$

We assume P_{tot} is the total transmitting power of BS, and the allocated transmit power to subcarrier g is P_g, subject to $\sum_{g=1}^{G} P_g \leq P_{tot}$. $P_{g,c}$ represents the allocated transmit power to the common message of subcarrier g and $P_{g,k}$ represents the allocated transmit power to user k on subcarrier g, with $P_{g,c} + \sum_{k\in\mathcal{K}_g} P_{g,k}=P_g$.

At the user k, the super common message is decoded first, and the rest of the messages are taken as interference. Thus, the SINR of the super common message is

$$\text{SINR}^c_{g,k} = \frac{P_{g,c}\left|\mathbf{h}^H_{k,g}\mathbf{w}_{g,c}\right|^2}{\sum_{k\in\mathcal{K}_g} P_{g,k}\left|\mathbf{h}^H_{k,g}\mathbf{w}_{g,k}\right|^2 + N_0} \tag{3}$$

The common part is subtracted from the received signal after been decoded, and then the private messages received by the user is decoded. The SINR of the private message of the user k is

$$\text{SINR}_{g,k} = \frac{P_{g,k}\left|\mathbf{h}^H_{k,g}\mathbf{w}_{g,k}\right|^2}{\sum_{k\in\mathcal{K}_g, k'\neq k} P_{g,k'}\left|\mathbf{h}^H_{k,g}\mathbf{w}_{g,k'}\right|^2 + N_0} \tag{4}$$

where N_0 denotes the power of noise. Based on the Shannon's law, the achievable rate of super common message on subcarrier g can be expressed as

$$R_{g,c} = \log_2\left(1 + \min\left\{\text{SINR}^c_{g,k}\right\}\right), k \in \mathcal{K}_g \tag{5}$$

The achievable rate of the private message of the user k on subcarrier g is

$$R_{g,k} = \log_2\left(1 + \text{SINR}_{g,k}\right) \tag{6}$$

A. OPTIMIZATION PROBLEM FORMULATION

For the wireless communication system, a higher transmission rate over limited resources is an important criterion for measuring the performance of system. So we aim at maximizing the sum of data rates subject to power constraints in this work. The user-subcarrier matching factor and power allocation factor are taken as variables. Let $\mathbf{P}=\{P_1, \ldots, P_G\}$ denote the power allocation factor set between different subcarriers. We set $\boldsymbol{\rho} = \left[\rho_{k,g}\right]_{\forall k\in\mathcal{K}_g, \forall g\in G}$ indicate the user-subcarrier matching matrix. And we assume the set of power allocation factor between messages on each subcarrier is $\mathbf{P}_g = \left[P_{g,c}, P_{g,1}, \ldots, P_{g,K}\right]$.

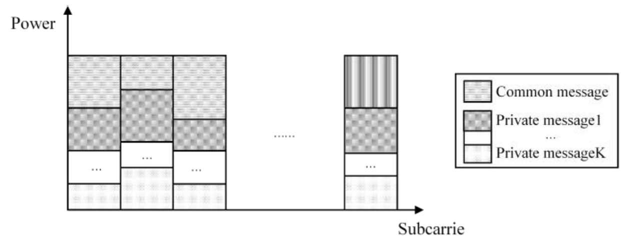
The problem can be formulated mathematically as follows:

$$\begin{aligned}
\max_{\mathbf{P},\boldsymbol{\rho},\mathbf{P}_g} \; & R = \sum_{g=1}^{G}\left(R_{g,c} + \sum_{k\in\mathcal{K}_g} R_{g,k}\right) \\
s.t.\; C1 : & \sum_{g=1}^{G} P_g \leq P_{tot} \\
C2 : & P_g \geq 0, \forall g \\
C3 : & P_{g,c} + \sum_{k\in\mathcal{K}_g} P_{g,k}=P_g \\
C4 : & \sum_{g=1}^{G} \rho_{k,g} = 1, \forall k \in \mathcal{K}_g \\
C5 : & \sum_{k=1}^{K} \rho_{k,g} \geq 1, \forall g \in G \\
C6 : & \rho_{k,g} \in \{0, 1\}, \forall k \in \mathcal{K}_g, \forall g \in G
\end{aligned} \tag{7}$$

where $C1$ indicates that the sum of power allocated on each subcarrier is not greater than the maximum power that the BS can transmit. $C2$ is the non-negative transmit power constraint on each subcarrier. $C3$ represents that the sum of the power of each message on subcarrier g is equal to the power allocated to subcarrier g by the BS. $C4$ stands for that each message is superimposed on only one subcarrier. $C5$ specifies that more than one data message can be superimposed on each subcarrier. In constraint $C6$, $\rho_{k,g}$ is a boolean variable indicating the allocation of subcarriers. $\rho_{k,g} = 1$ means that user k is assigned to subcarrier g for superimposed transmission.

III. THE THREE-STEP OPTIMIZATION FRAMEWORK FOR MC-RSMA

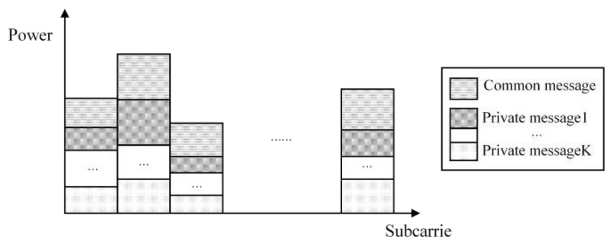
The maximization problem (7) is a mixed non-linear programming (MNLP) problem, which is in general difficult to solve directly. In order to solve this problem, we propose a low-complexity three-step framework that reduces the dimension of multidimensional variables. Specifically, one-dimensional problems such as power allocation among different users on a single subcarrier, matching between users



Step 1: RSMA messages layers power allocation

subcarrier / user	1	2	3	···	G
1	$\rho_{1,1}$	$\rho_{1,2}$	$\rho_{1,3}$	···	$\rho_{1,G}$
2	$\rho_{2,1}$	$\rho_{2,2}$	$\rho_{2,3}$	···	$\rho_{2,G}$
···	···	···	···	···	···
K	$\rho_{K,1}$	$\rho_{K,2}$	$\rho_{K,3}$	···	$\rho_{K,G}$

Step 2: user-subcarrier matching



Step 3: subcarriers power allocation

FIGURE 2. Three-step optimization framework for MC-RSMA.

and subcarriers, and power allocation among different subcarriers are solved step by step. Each sub-problem is analyzed and solved by the optimal algorithm.

The steps of the three-step resource allocation algorithm can be given as follows, while the schetch diagram is shown in Fig. 2.

step 1: We first assume that the transmit power of the system is evenly distributed on different subcarriers and determines the power assigned scheme to each message on a single subcarrier.

step 2: We assign subcarriers to users based on the power allocation solution obtained in Step 1. The user-subcarrier matching problem can be transformed into an assignment problem. To solve the assignment problem, the Hungarian algorithm can be used under a fixed power limit.

step 3: We calculate the power distribution coefficients between different subcarriers to optimize the sum-rate maximization problem. And then update the final power allocation of each user.

In a word, we first determine which subcarriers the user is assigned can maximize the rate, and then adjust the power allocation coefficient between the subcarriers to maximize the sum rate of the considered system. For RSMA we exploit ZF precoding for private messages, therefore the channel state information from BS to users are needed at the transitter(CSIT). At the same time, CSIT are still necessary for the proposed resource allocation. For FDD mode, CSI need to be feedback to the BS transmitter. While in TDD mode, CSIT could be get by channel reciprocity. This is signalling overhead of our proposed algorithm.

IV. POWER ALLOCATION SCHEME IN ONE SUBCARRIER

We denote the user messages set superimposed on the subcarrier g is $\mathcal{K}_g$. According to (5) and (6), the sum rate on the subcarrier g is $R_g = R_{g,c} + \sum_{k\in\mathcal{K}_g} R_{g,k}$. Based on the RSMA scheme, the common messages have the decoding priority. In order to ensure decode correctly, the received power of the common messages must be no less than the private messages. With the maximum sum-rate on the subcarrier as the optimization objective, the optimization problem can be expressed as follows:

$$\begin{aligned}
&\max_{\mathbf{P}_g} \; R_{g,c} + \sum_{k\in\mathcal{K}_g} R_{g,k} \\
&s.t.\; C1: P_{g,c}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,c}\right|^2 > P_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2 \\
&\quad C2: P_{g,c} + \sum_{k\in\mathcal{K}_g} P_{g,k} = P_g \\
&\quad C3: P_{g,k} \geq 0 \\
&\quad C4: P_{g,c} \geq 0
\end{aligned} \tag{8}$$

where $C1$ means the received power of the common messages is no less than the private messages. $C2$ represents that the sum of the power of each message on the subcarrier is equal to the power allocated to the subcarrier by the BS. $C3$ and $C4$ indicate that the power allocated to the common and private messages are non-negative.

The formula (8) can be rewritten as

$$\begin{aligned}
&\max_{\mathbf{P}_g} \sum_{k\in\mathcal{K}_g} \log_2\left(1 + \frac{P_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2}{\sum\limits_{\substack{k\in\mathcal{K}_g\\ k'\neq k}} P_{g,k'}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2 + N_0}\right) + R_{g,c} \\
&s.t.\; R_{g,c} \leq \frac{P_{g,c}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,c}\right|^2}{\sum\limits_{k\in\mathcal{K}_g} P_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2 + N_0}, \forall k\in\mathcal{K}_g \\
&\quad P_{g,c}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,c}\right|^2 > P_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2, \forall k\in\mathcal{K}_g \\
&\quad P_{g,c} + \sum_{k\in\mathcal{K}_g} P_{g,k} = P_g \\
&\quad P_{g,k} \geq 0 \\
&\quad P_{g,c} \geq 0
\end{aligned} \tag{9}$$

The problem (9) is a non-convex problem and is known as NP-hard. We can transfer the original problem into a DCP [24]. Note that the difference between concave and convex functions is formally equivalent. Thus, we reformulate the problem as

$$\begin{aligned}
\min\ & \sum_{k\in\mathcal{K}_g}\log_2\left(\sum_{k\in\mathcal{K}_g,k'\neq k}P_{g,k'}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2+N_0\right)\\
&-\sum_{k\in\mathcal{K}_g}\log_2\left(\sum_{k\in\mathcal{K}_g}P_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2+N_0\right)-R_{g,c}\\
s.t.\ & P_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2-P_{g,c}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,c}\right|^2<0,\forall k\in\mathcal{K}_g\\
& P_{g,c}+\sum_{k\in\mathcal{K}_g}P_{g,k}=P_g\\
& -P_{g,k}\leq 0\\
& -P_{g,c}\leq 0
\end{aligned}\tag{10}$$

Then we can approximate it by its first-order Taylor expansion around a feasible point. First we let

$$\begin{aligned}
&f\left(R_{g,c},P_{g,c},P_{g,1},\ldots,P_{g,K_g}\right)\\
&=\sum_{k\in\mathcal{K}_g}\log_2\left(\sum_{k\in\mathcal{K}_g,k'\neq k}P_{g,k'}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2+N_0\right)
\end{aligned}\tag{11}$$

and do first-order Taylor expansion of this function at differentiable points $\left(R_{g,c}^0,P_{g,c}^0,P_{g,1}^0,\ldots,P_{g,K_g}^0\right)$. Then we have

$$\frac{\partial f}{\partial R_{g,c}}=0\tag{12}$$

$$\frac{\partial f}{\partial P_{g,c}}=0\tag{13}$$

$$\frac{\partial f}{\partial P_{g,k}}=\sum_{\substack{k'\in\mathcal{K}_g\\k'\neq k}}\frac{P_{g,k}\left|\mathbf{h}_{k',g}^H\mathbf{w}_{g,k}\right|^2}{\ln 2\left(\sum_{\substack{k''\in\mathcal{K}_g\\k''\neq k'}}P_{g,k''}\left|\mathbf{h}_{k',g}^H\mathbf{w}_{g,k''}\right|^2+N_0\right)}\tag{14}$$

$$\begin{aligned}
f=&\sum_{k\in\mathcal{K}_g}\log_2\left(\sum_{k\in\mathcal{K}_g,k'\neq k}P_{g,k'}^0\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2+N_0\right)\\
&+\sum_{\substack{k'\in\mathcal{K}_g\\k'\neq k}}\frac{\left(P_{g,k}-P_{g,k'}^0\right)\left|\mathbf{h}_{k',g}^H\mathbf{w}_{g,k}\right|^2}{\ln 2\left(\sum_{k''\in\mathcal{K}_g,k''\neq k'}P_{g,k''}\left|\mathbf{h}_{k',g}^H\mathbf{w}_{g,k''}\right|^2+N_0\right)}
\end{aligned}\tag{15}$$

So, the optimization problem can be rewriten as

$$\begin{aligned}
\min\ & \sum_{k\in\mathcal{K}_g}\log_2\left(\sum_{k\in\mathcal{K}_g,k'\neq k}P_{g,k'}^0\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2+N_0\right)\\
&+\sum_{\substack{k'\in\mathcal{K}_g\\k'\neq k}}\frac{\left(P_{g,k}-P_{g,k'}^0\right)\left|\mathbf{h}_{k',g}^H\mathbf{w}_{g,k}\right|^2}{\ln 2\left(\sum_{k''\in\mathcal{K}_g,k''\neq k'}P_{g,k''}\left|\mathbf{h}_{k',g}^H\mathbf{w}_{g,k''}\right|^2+N_0\right)}\\
&-\sum_{k\in\mathcal{K}_g}\log_2\left(\sum_{k\in\mathcal{K}_g}P_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2+N_0\right)-R_{g,c}
\end{aligned}\tag{16}$$

By extracting the constant term, the optimization problem can be sorted out into (17).

$$\begin{aligned}
\min\ & \sum_{\substack{k'\in\mathcal{K}_g\\k'\neq k}}\frac{P_{g,k}\left|\mathbf{h}_{k',g}^H\mathbf{w}_{g,k}\right|^2}{\ln 2\left(\sum_{k''\in\mathcal{K}_g,k''\neq k'}P_{g,k''}\left|\mathbf{h}_{k',g}^H\mathbf{w}_{g,k''}\right|^2+N_0\right)}\\
&-\sum_{k\in\mathcal{K}_g}\log_2\left(\sum_{k\in\mathcal{K}_g}P_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2+N_0\right)\\
&-R_{g,c}+\text{const}\\
s.t.\ & P_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2-P_{g,c}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,c}\right|^2<0,\forall k\in\mathcal{K}_g\\
& P_{g,c}+\sum_{k\in\mathcal{K}_g}P_{g,k}=P_g\\
& -P_{g,k}\leq 0\\
& -P_{g,c}\leq 0
\end{aligned}\tag{17}$$

We note that the optimization objective is convex, while the constraint condition is non-convex. So it cannot be directly solved, and further processing is needed. In this section, the one-dimensional iterative search method is adopted to solve the problem. First, we initialize $R_{g,c}$ randomly, and the problem can be transformed into a convex problem by substituting $R_{g,c}$ into (17). The convex problem can be solved efficiently by standard convex program solvers such as CVX [25]. Thus, the optimal solution $P_{g,c}^*,P_{g,1}^*,\ldots,P_{g,K_g}^*$ corresponding to the current $R_{g,c}$ can be obtained. Then substitute $P_{g,c}^*,P_{g,1}^*,\ldots,P_{g,K_g}^*$ into the (17) to get the optimal $R_{g,c}^*$. Repeat the above process until $P_{g,c}^*,P_{g,1}^*,\ldots,P_{g,K_g}^*$ converges. Finally, we can find the solution $P_{g,c}^*,P_{g,1}^*,\ldots,P_{g,K_g}^*$ that maximizes the sum rate over a single subcarrier.

V. USER-SUBCARRIER MATCHING SCHEME

Due to the channel response on each subcarrier is different, the user-subcarrier matching scheme will affect the throughput of the system. Therefore, reasonable and effective user matching scheme plays an important role in system performance gain. If the full search method is used, a large number of redundant calculations will lead to low efficiency. And if the fixed matching scheme is adopted, it cannot be flexibly changed according to the current demand and channel state, resulting in bad performance. We find that this user-subcarrier matching problem is similar to the assignment problem where the number of users is less than the number of tasks, so the deformed Hungarian algorithm can be used, which can achieve the tradeoff between algorithm complexity and performance.

In this section, the assignment problem and the Hungarian algorithm are firstly described respectively, then the user-subcarrier matching problem is discussed and solved.

A. ASSIGNMENT ISSUES

Suppose there are n tasks that need n people to complete, and each person's work efficiency is different. Each task can only be assigned to one person, and one person can only do one task. The assignment problem is how to assign tasks that can minimize the total working time.

Let C be the efficiency matrix, its element $c_{i,j} > 0, i, j = 1, 2, \ldots n$ represents the time for the person i to complete task j. The efficiency matrix is described in Table 1.

We denote the binary variable $x_{i,j}$ represents whether to assign the person i to complete task j, i.e.

$$x_{i,j} = \begin{cases} 1, \text{the person } i \text{ is assigned to task } j \\ 0, \text{the person } i \text{ is not assigned to task } j \end{cases} \tag{18}$$

The model of the assignment problem can be abstracted into the following formula:

$$\begin{aligned} \min\ z &= \sum_{i=1}^{n}\sum_{j=1}^{n} c_{i,j}x_{i,j} \\ s.t.\ &\sum_{i=1}^{n} x_{i,j} = 1, j = 1, 2, \ldots, n \\ &\sum_{j=1}^{n} x_{i,j} = 1, i = 1, 2, \ldots, n \\ &x_{i,j} \in \{0, 1\} \end{aligned} \tag{19}$$

The first constraint restricts the task j can only be assigned to one person, and the second constraint requires the person i can only complete one task. The assignment problem is a special case of 0-1 integer programming.

B. HUNGARIAN ALGORITHM

Hungarian algorithm is a combination optimization algorithm for solving the task assignment problem in polynomial time. The basic idea is to subtract a constant from the rows and columns of the efficiency matrix, and turn it into zero elements in different rows and columns. Let the variables corresponding to these zero elements become 1 and the remaining variables become 0 to obtain the optimal solution of the assignment problem.

Theorem 1: If the element with a value of 1 in the feasible solution matrix $\mathbf{x}^* = (x_{i,j})$, which the corresponding element in the efficiency matrix $C = (c_{i,j})$ is 0, the optimal solution is $\mathbf{x}^*$.

Theorem 2: Given the assignment problem G with efficiency matrix $C = (c_{i,j})$, the new matrix C' obtained by adding or subtracting the same number from the row or column of C has the same efficiency with C.

Proof: for $\forall s \in \mathbb{R}$,

$$\begin{aligned} z' &= \sum_{i=1, i\neq k}^{n}\sum_{j=1}^{n} c_{i,j}x_{i,j} + \sum_{j=1}^{n}\left(c_{k,j} \pm s\right)x_{k,j} \\ &= \sum_{i=1, i\neq k}^{n}\sum_{j=1}^{n} c_{i,j}x_{i,j} + \sum_{j=1}^{n} c_{k,j}x_{k,j} + (\pm s)\sum_{j=1}^{n} x_{k,j} \\ &= \sum_{i=1, i\neq k}^{n}\sum_{j=1}^{n} c_{i,j}x_{i,j} + \sum_{j=1}^{n} c_{k,j}x_{k,j} + (\pm s) \\ &= z + (\pm s) \end{aligned} \tag{20}$$

TABLE 1. Efficiency matrix.

Task \ Worker	A_1	A_2	$\ldots$	A_n
B_1	$c_{1,1}$	$c_{1,2}$	$\ldots$	$c_{1,n}$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
B_n	$c_{n,1}$	$c_{n,2}$	$\ldots$	$c_{n,n}$

TABLE 2. Rate matrix.

message \ subcarrier	1	$\ldots$	2	$\ldots$	G
$\mathbf{s}_1$	$r_{1,1}$	$\ldots$	$r_{1,g}$	$\ldots$	$r_{1,G}$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\mathbf{s}_k$	$r_{k,1}$	$\ldots$	$r_{k,g}$	$\ldots$	$r_{k,G}$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\mathbf{s}_K$	$r_{K,1}$	$\ldots$	$r_{K,g}$	$\ldots$	$r_{K,G}$

C. SOLUTIONS TO USER-SUBCARRIER MATCHING PROBLEMS

The matching problem can be likened to the assignment problem, where the subcarrier is equivalent to the worker, the user is equivalent to the task, and the user's transmission rate on the subcarrier is equivalent to the work efficiency. However, this targeted problem is different from the traditional assignment problem. In traditional assignment problem, users and tasks have the same number, and the goal is to obtain the minimum total value. While in this user-subcarrier matching problem, the number of users is greater than the subcarriers, and the target number is the maximum total value.

Since the current number of users is greater than the number of subcarriers, the problem solved can be considered as the case where the number of workers is less than the number of tasks in the assignment problem.

Table 2 represents the rate matrix $\mathcal{R}$ between user messages and subcarriers, and each element represents the rate of this message transmitted on this subcarrier. $r_{k,g} = \left\|\mathbf{h}_{k,g}\right\|^2$. That is because we assume that the power allocated to each subcarrier is equal and $R_{k,g} = \log_2\left(1 + P_g\left\|\mathbf{h}_{k,g}\right\|^2/N_0\right)$, so $R_{k,g}$ is only related to $\left\|\mathbf{h}_{k,g}\right\|^2$.

For the case where the number of subcarriers G is less than the number of users K, we add $K - G$ virtual subcarriers with the rate of 0 to the rate matrix C. An assignment problem with the number of subcarriers equals to the number of users is defined as follows:

$$\begin{aligned} \max_{\boldsymbol{\rho}}\ &\sum_{k=1}^{K}\sum_{g=1}^{G} r_{k,g}\rho_{k,g} \\ s.t.\ &\sum_{g=1}^{G}\rho_{k,g} = 1, \forall k \in \mathcal{K}_g \\ &\sum_{k=1}^{K}\rho_{k,g} \geq 1, \forall g \in G \\ &\rho_{k,g} \in \{0, 1\}, \forall k \in \mathcal{K}_g, \forall g \in G \end{aligned} \tag{21}$$

Algorithm 1 Hungarian Algorithm Based User-Subcarrier Matching Algorithm

Input: channel matrix $\mathbf{h}_{kg}$, user messages $\mathbf{s}_1, \ldots, \mathbf{s}_K$

Output: user-subcarrier matching matrix $\boldsymbol{\rho}$

1: initialize $K' \leftarrow K$, $G' \leftarrow G$, $\mathcal{R} = \left\{r_{k,g} | r_{k,g} = \left\|\mathbf{h}_{k,g}\right\|^2\right\}$
2: $r_{max} \leftarrow \max\left\{r_{k,g} | \forall k \in \mathcal{K}, g \in \mathcal{G}\right\}$
3: $r_{k,g} \leftarrow r_{max} - r_{k,g}, \forall k \in \mathcal{K}, g \in \mathcal{G}$
4: **if** $K' > G'$ **then**
5:     add $K' - G'$ dummy columns with all values to 0 in R
6: **else if** $K' < G'$ **then**
7:     add $G' - K'$ dummy rows with all values to 0 in R
8: **end if**
9: $N \leftarrow \max\left\{K', G'\right\}$
10: **for** $k = 1$ to N **do**
11:     $r_{k,g}^{min} \leftarrow \min\left\{r_{k,g} | \forall g \in \{1, \ldots, N\}\right\}$
12:     $r_{k,g} \leftarrow r_{k,g} - r_{k,g}^{min}$
13: **end for**
14: **for** $g = 1$ to n **do**
15:     $r_{k,g}^{min} \leftarrow \min\left\{r_{k,g} | \forall k \in \{1, \ldots, N\}\right\}$
16:     $r_{k,g} \leftarrow r_{k,g} - r_{k,g}^{min}$
17: **end for**
18: Find the minimum number l_{num} of lines through the row and columns to cover all zero elements.
19: **if** $l_{num} = N$ **then**
20:     Select a set of 0 so that each row or column has only one selected, i.e. $R_{temp} = \left\{\tilde{r}_{k,g}\right\}$
21:     **for** $\tilde{r}_{k,g} \in R_{temp}$ **do**
22:         **if** $k < K'$ and $g < G'$ **then**
23:             $\rho_{k,g} \leftarrow 1$
24:         **end if**
25:         $K' \leftarrow K' - \{k\}$
26:     **end for**
27:     **if** $K' == \varnothing$ **then**
28:         break
29:     **end if**
30:     go back to Step 4
31: **else if** $l_{num} < N$ **then**
32:     Find the smallest entry r_{min} not covered by any line.
33:     Subtract r_{min} from each row that is not crossed out.
34:     Add r_{min} to each column that is crossed out.
35:     go back to Step 18
36: **end if**

The object of this problem is to find the best matching matrix $\boldsymbol{\rho}$, where the element $\rho_{k,g}$ indicates whether the message k is allocated for superposition transmission on the subcarrier g. We derive the user-subcarrier matching algorithm based on the Hungarian algorithm as described in Algorithm 1. In Algorithm 1, the operations from step 1 to 3 make a transformation of $r_{k,g}$, so that the maximization problem could be transformed into a minization problem. The rest steps carry a Hungarian task assignment algorithm to find a minization taget value.

VI. POWER ALLOCATION BETWEEN SUBCARRIERS

The previous section is based on the assumption that the power allocated on each subcarrier is equal, but in practice, due to the different messages superimposed on each subcarrier and the differences between channels, the power requirements of each subcarrier are different. Therefore, the purpose of this section is to calculate the power distribution factors among different subcarriers under the total power limit of BS transmission, so as to make more reasonable use of power resources and improve the performance of the system. The optimal power allocation factors between subcarriers can be obtained by solving the following optimization problem:

$$\begin{aligned} \max_{\mathbf{P}} \ & R = \sum_{g=1}^{G}\left(R_{g,c} + \sum_{k\in\mathcal{K}_g} R_{g,k}\right) \\ s.t. \ & \sum_{g=1}^{G} P_g \le P_{tot} \\ & P_g \ge 0 \end{aligned} \tag{22}$$

where P_g represents the power allocated on each subcarrier g. Let $t_{g,c}$ be the proportion of the power of common messages on the subcarrier, and $t_{g,k}$ be the proportion of private messages $\mathbf{s}_k$, with $t_{g,c} + \sum_{k\in\mathcal{K}_g} t_{g,k}=1$.

According to the definition of $R_{g,k}$, we rewrite (22) as follows:

$$R = \sum_{g=1}^{G}\left(R_{g,c} + \sum_{k\in\mathcal{K}_g} \log_2\left(1 + \frac{t_{g,k}P_g\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2}{\sum_{k\in\mathcal{K}_g,k'\neq k} t_{g,k'}P_g\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2+N_0}\right)\right) \tag{23}$$

It can be shown that the rate on each subcarrier depends on the channel state of the subchannel and the power allocated to this subcarrier. It remains to show that R is made up of a series of *log* functions, i.e. $R=\log_2 m_1\left(P_g\right)+\log_2 m_2\left(P_g\right)+\ldots$, and

$$m_k\left(P_g\right)=1 + \frac{t_{g,k}P_g\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2}{\sum_{k\in\mathcal{K}_g,k'\neq k} t_{g,k'}P_g\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2 + N_0} \tag{24}$$

Let $\alpha=t_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2$, $\beta=\sum_{k\in\mathcal{K}_g,k'\neq k} t_{g,k'}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2$, $m_k\left(P_g\right)$ can be rewriten as

$$m_k\left(P_g\right)=1 + \frac{\alpha P_g}{\beta P_g + N_0} \tag{25}$$

Taking the second derivative of $m_k\left(P_g\right)$, then we have

$$\frac{\partial^2 m_k\left(P_g\right)}{\partial {P_g}^2} = -\frac{2\alpha\beta N_0}{\left(\beta P_g + N_0\right)^3} < 0 \tag{26}$$

Since the second derivative is less than 0, we can infer that $m_k\left(P_g\right)$ is a concave function with respect to the variable P_g. So the $\log_2 m_k\left(P_g\right)$ is a concave function of P_g, and we can

Algorithm 2 Optimized Power Allocation Algorithm

Input: subcarrier set $\mathcal{G}=\{1, 2, \ldots, G\}$

Output: power allocation set $\mathbf{P}=[P_1, P_2, \ldots, P_G]$

1: **for** all $g \in \mathcal{G}$ **do**
2: compute $P_g(\lambda)$ by solving (29)
3: **end for**
4: $\lambda \leftarrow \sum_{g=1}^{G} P_g(\lambda) = P_{tot}$
5: **for** all $g \in \mathcal{G}$ **do**
6: update P_g by (30) for a given λ
7: **if** $P_g == 0$ **then**
8: $\mathcal{G} \leftarrow \mathcal{G} - \{g\}$
9: go back to step 1
10: **end if**
11: **end for**

conclude that R is the concave function of P_g. We rewrite the problem (22) in an equivalent convex optimization problem as follows:

$$\begin{aligned} \min_{P_g} & -\sum_{g=1}^{G}\left(R_{g,c}+\sum_{k\in\mathcal{K}_g} R_{g,k}\right) \\ s.t. & \sum_{g=1}^{G} P_g - P_{tot} \leq 0 \\ & -P_g \leq 0 \end{aligned} \tag{27}$$

Note that the optimal power allocation problem between subcarriers meets the KKT condition. It can be solved by using the Lagrangian multiplier method. We construct the Lagrange function first.

$$\begin{aligned} L(\lambda, P_g) &= -\sum_{g=1}^{G}\left(R_{g,c}+\sum_{k\in\mathcal{K}_g} R_{g,k}\right)+\lambda\left(\sum_{g=1}^{G} P_g - P_{tot}\right) \\ &= -\sum_{g=1}^{G}\left(R_{g,c}+\sum_{k\in\mathcal{K}_g}\log_2\right. \\ &\quad \left.\times\left(1+\frac{t_{g,k}P_g\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2}{\sum_{k\in\mathcal{K}_g,k'\neq k} t_{g,k'}P_g\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2+N_0}\right)\right) \\ &\quad +\lambda\left(\sum_{g=1}^{G} P_g - P_{tot}\right) \end{aligned} \tag{28}$$

where $\lambda > 0$ is the Lagrange multiplier. Let $\alpha_k=t_{g,k}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k}\right|^2$, $\beta_k=\sum_{k\in\mathcal{K}_g,k'\neq k} t_{g,k'}\left|\mathbf{h}_{k,g}^H\mathbf{w}_{g,k'}\right|^2$, the optimal power allocation between subcarriers should satisfy:

$$\frac{\partial L}{\partial P_g} = -\frac{1}{\ln 2}\cdot\sum_{k\in\mathcal{K}_g}\frac{\alpha_k N_0}{(\alpha_k+\beta_k)P_g+N_0}+\lambda=0 \tag{29}$$

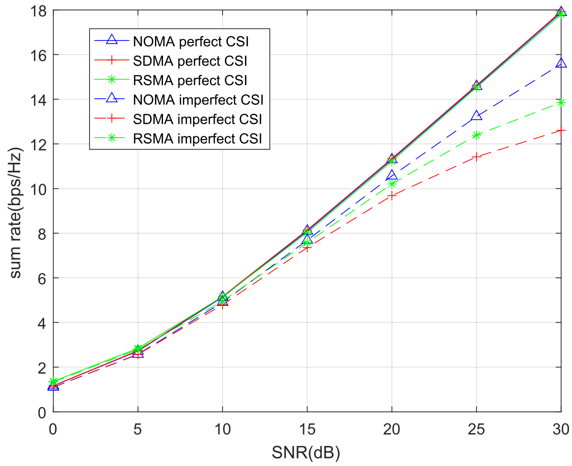


FIGURE 3. Performance comparison of RSMA, NOMA and SDMA with perfect CSI and imperfect CSI ($\sigma^2 = 0.5$).

Solving the fractional equation, the optimal power allocation can be obtained as

$$P_g^{*}=P_g(\lambda)^{+} \tag{30}$$

Formula (30) cannot be solved directly by numerical methods. The proposed optimized power allocation algorithm is summarized in Algorithm 2. Since the Algorithm 2 is a dynamic process, whose complexity is related to Pg condition. So the complexity is between $O(\sum_{g=1}^{G} K_g N_t) \sim O(G\sum_{g=1}^{G} K_g N_t)$.

VII. SIMULATION RESULT AND DISCUSSION

In this section, we evaluate the performance of the proposed resource allocation scheme in MC-RSMA based system. We consider a downlink MISO transmission scenario, where the transmitter has 4 antennas and each user has one antenna. We only consider small scale fading as Rayleigh fading in the simulation. CSI is needed for SIC receiver and precoder respectively, however it is hard to obtain perfect CSI. Therefore in our simulation in section VII, schemes are evaluated in the case of imperfect CSI. The imperfect channel is modeled as $\mathbf{h} = \hat{\mathbf{h}} + \mathbf{e}$, where $\hat{\mathbf{h}}$ is the estimated channel of $\mathbf{h}$ and $\mathbf{e}$ denotes the CSI error, $\mathbf{e} \sim \mathcal{CN}(0, \sigma^2)$. Meanwhile, we normalize the noise, i.e. the SNR is equal to the power constraint P. The performance of the proposed scheme is evaluated by Monte Carlo simulations and the number of Monte Carlo cycles is set to 5000.

A. SIMULATION OF POWER ALLOCATION ON A SINGLE SUBCARRIER

To begin with, we investigate the performance of RSMA, NOMA and SDMA schemes with perfect CSI and imperfect CSI ($\sigma^2 = 0.5$) in a downlink senario, which is shown in the Fig. 3. It is assumed that only two user messages are superimposed on this subcarrier, and an average power distribution between messages is adopted for SDMA and RSMA. For RSMA, half of the total power is used to transmit

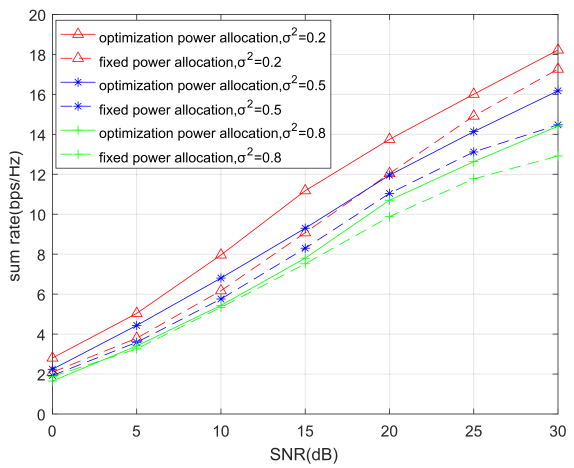


FIGURE 4. **The sum rate of MC-RSMA versus SNR for the optimized power allocation scheme and the fixed power allocation scheme among different message layers on each subcarrier under different CSI errors.**

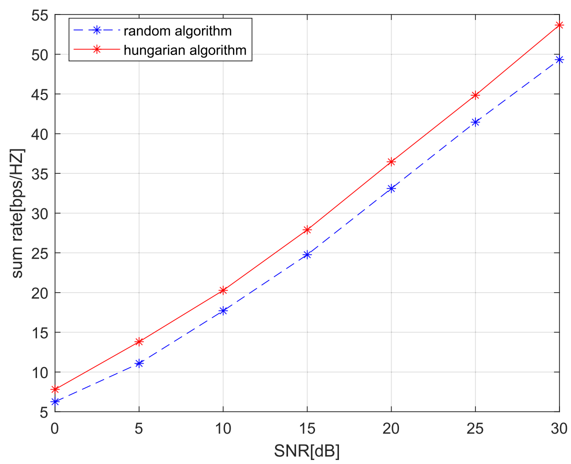


FIGURE 5. **The sum rate of MC-RSMA versus SNR for the Hungarian matching algorithm and the random matching scheme with imperfect CSI ($\sigma^2 = 0.5$).**

a common message to both users. Different from RSMA and SDMA, we exploit a proportional power allocation for NOMA. ZF precoding is used for RSMA, NOMA and SDMA downlink multiuser beamforming. It can be seen that the three schemes have same performance under perfect CSI. This is because the ZF precoding algorithm can completely eliminate the interference between users and achieve the maximum system sum rate. NOMA performs better than RSMA and SDMA under imperfect CSI. This is because NOMA is assumed to be able to decode the interference signal completely in SIC, while RSMA could not cancel interference due to imperfect CSI, and SDMA is to treat the interference as completely as noise. Due to the limitations of the NOMA deployment scenario and the complexity of the transmitter and receiver, it is not suitable for superimposing more messages on the same subcarrier, so RSMA has more advantages. On the other aspect, RSMA can jointly transmit broadcast services and unicast services without additional radio resources, which make RSMA to be a good structure for multiple type services transmission with high spectrum efficiency.

Fig. 4 simulates the system sum rate of the optimized power allocation among different messages with different CSI error levels ($\sigma^2 = 0.2$, $\sigma^2 = 0.5$, $\sigma^2 = 0.8$). In this step, equal power are assigned to each subcarriers while the optimized power allocation are carried to find the power for common and private messages. It can be seen from the figure that the optimized power allocation scheme algorithm can effectively improve the performance of the system under different SNRs compared to the fixed power allocation scheme, and can improve more performance under high SNR.

B. SIMULATION ANALYSIS OF USER AND SUBCARRIER MATCHING

In this subsection, we evaluate the matching between user and subcarrier with the aim of maximizing the system sum rate. Suppose that there are 5 user messages to be allocated to 3 subcarriers. The superimposed users on each subcarrier adopt the non-orthogonal transmission mode of one-layer rate splitting, and the power allocated to each subcarrier are same.

According to the suboptimal power distribution algorithm, the power distribution factor of superimposed user messages on the same subcarrier can be obtained. And the problem of selecting user combinations in the rate matrix can be transformed into an assignment problem and the Hungarian algorithm based user-subcarrier matching algorithm can deal with it. Compared with the performance of the random assignment algorithm in this transmission scenario, the simulation results can be obtained as shown in the Fig. 5.

In Fig. 5, we compare the system sum rate changes for the Hungarian algorithm based user and multicarrier mapping and the random subcarrier scheme under the condition of CSI error $\sigma^2 = 0.5$. As SNR increases, the system sum rate increases. Because the Hungarian algorithm is to find the user and subcarrier matching that can achieve the maximum rate among the existing users, it can be seen from the figure that the system performance of the Hungarian algorithm based scheme is better than that of the random matching scheme.

We compare the complexity of random matching scheme, Hungarian matching algorithm and full search matching scheme. K represents the total number of users and G represents the total number of subcarriers in Table 3. It can be seen from the table that the optimal full-search matching scheme is of high complexity and is not suitable for use in practical projects. The Hungarian algorithm can reduce the computational complexity and improve the performance compared with the random matching scheme, thus achieving a balance between complexity and performance.

C. SIMULATION ANALYSIS OF POWER ALLOCATION BETWEEN DIFFERENT SUBCARRIERS

The previous user and subcarrier matching scheme is based on equal power distribution among subcarriers. However, users have different channel conditions on different

TABLE 3. Comparison of computational complexity of different matching schemes

scheme	random matching	Hungarian matching	full search matching
computation complexity	$O(KG)$	$O(K^2G)$	$O(K^G)$

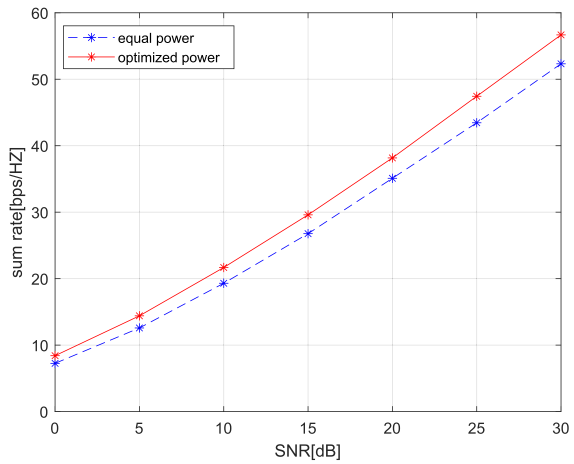


FIGURE 6. The sum rate of MC-RSMA versus SNR for the optimized power distribution algorithm and the equal power distribution scheme among subcarriers with imperfect CSI ($\sigma^2 = 0.5$).

subcarriers, and different users have different QoS requirements. Equal power distribution between subcarriers cannot guarantee the optimal performance of the system. Based on the above analysis, this section simulates and analyzes the power allocation algorithm.

After determining the power matching factor between the superimposed messages on the same subcarrier, the optimized algorithm is used to allocation power among subcarriers. Performance comparison with the equal power distribution scheme is shown in Fig. 6. The sum rate of these two power distribution schemes increases with the increase of SNR, and the performance of the system using the optimized power distribution algorithm is better than that of the equal power distribution scheme.

VIII. CONCLUSION

In this work, we investigate the MC-RSMA network resource allocation scheme. To maximize the system sum rate, a resource allocation model of joint power distribution and user-subcarrier matching is formulated. To solve this non-convex problem, we propose a three-step low complexity algorithm to disassemble the original problem and solve the variables in turn. Firstly, by converting this problem into a DCP and approximating it by its first-order Taylor expansion, the power distribution factor between the superposed messages on the same subcarrier can be solved. Then the matching factor between the user and the subcarrier is solved based on the Hungarian algorithm. Finally the problem is transformed into the Lagrange function to solve the power distribution factor between different subcarriers through the optimized power distribution algorithm. The simulation results prove the feasibility and superiority of the proposed scheme of resource allocation in the RSMA network under imperfect CSI.

REFERENCES

[1] Y. Sun, D. W. K. Ng, Z. Ding, and R. Schober, "Optimal joint power and subcarrier allocation for MC-NOMA systems," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Washington, DC, USA, Dec. 2016, pp. 1–6.

[2] Y. Fu, L. Salaun, C. W. Sung, and C. S. Chen, "Subcarrier and power allocation for the downlink of multicarrier NOMA systems," *IEEE Trans. Veh. Technol.*, vol. 67, no. 12, pp. 11833–11847, Dec. 2018.

[3] Cisco, Cisco Annual Internet Report. *Cisco Annual Internet Report (2018–2023) White Paper*. [Online]. Available: https://www.cisco.com/c/en/us/solutions/collateral/executive-perspectives/ annual-internet-report/white-paper-c11-741490.html

[4] C.-L. Wang, T.-Y. Chen, Y.-F. Chen, and D.-S. Wu, "Low-complexity resource allocation for downlink multicarrier NOMA systems," in *Proc. IEEE 29th Annu. Int. Symp. Pers., Indoor Mobile Radio Commun. (PIMRC)*, Bologna, Italy, Sep. 2018, pp. 1–6.

[5] J.-H. Tseng, Y.-F. Chen, and C.-L. Wang, "User selection and resource allocation algorithms for multicarrier NOMA systems on downlink beamforming," *IEEE Access*, vol. 8, pp. 59211–59224, 2020.

[6] J. Chen, L. Zhang, Y.-C. Liang, and S. Ma, "Effective-throughput maximization for multicarrier NOMA in short-packet communications," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Waikoloa, HI, USA, Dec. 2019, pp. 1–6.

[7] J. Chen, L. Zhang, Y.-C. Liang, and S. Ma, "Optimal resource allocation for multicarrier NOMA in short packet communications," *IEEE Trans. Veh. Technol.*, vol. 69, no. 2, pp. 2141–2156, Feb. 2020.

[8] Y. Mao, B. Clerckx, and V. O. K. Li, "Rate-splitting multiple access for downlink communication systems: Bridging, generalizing, and outperforming SDMA and NOMA," *EURASIP J. Wireless Commun. Netw.*, vol. 2018, no. 1, p. 133, May 2018.

[9] Y. Mao, B. Clerckx, and V. O. K. Li, "Rate-splitting multiple access for coordinated multi-point joint transmission," in *Proc. IEEE Int. Conf. Commun. Workshops (ICC Workshops)*, May 2019, pp. 1–6.

[10] S. Yang, M. Kobayashi, D. Gesbert, and X. Yi, "Degrees of freedom of time correlated MISO broadcast channel with delayed CSIT," *IEEE Trans. Inf. Theory*, vol. 59, no. 1, pp. 315–328, Jan. 2013.

[11] C. Hao, Y. Wu, and B. Clerckx, "Rate analysis of two-receiver MISO broadcast channel with finite rate feedback: A rate-splitting approach," *IEEE Trans. Commun.*, vol. 63, no. 9, pp. 3232–3246, Sep. 2015.

[12] M. Dai, B. Clerckx, D. Gesbert, and G. Caire, "A rate splitting strategy for massive MIMO with imperfect CSIT," *IEEE Trans. Wireless Commun.*, vol. 15, no. 7, pp. 4611–4624, Jul. 2016.

[13] H. Joudeh and B. Clerckx, "Sum-rate maximization for linearly precoded downlink multiuser MISO systems with partial CSIT: A rate-splitting approach," *IEEE Trans. Commun.*, vol. 64, no. 11, pp. 4847–4861, Nov. 2016.

[14] Y. Mao, B. Clerckx, and V. O. K. Li, "Rate-splitting for multi-antenna non-orthogonal unicast and multicast transmission," in *Proc. IEEE 19th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jun. 2018, pp. 1–5.

[15] Y. Mao, B. Clerckx, and V. O. K. Li, "Energy efficiency of rate-splitting multiple access, and performance benefits over SDMA and NOMA," in *Proc. 15th Int. Symp. Wireless Commun. Syst. (ISWCS)*, Aug. 2018, pp. 1–5.

[16] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[17] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

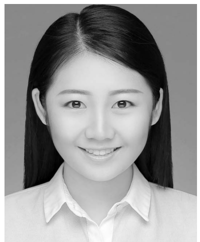
[18] H. Dahrouj and W. Yu, "Multicell interference mitigation with joint beamforming and common message decoding," *IEEE Trans. Commun.*, vol. 59, no. 8, pp. 2264–2273, Aug. 2011.

[19] C. Hao and B. Clerckx, "MISO networks with imperfect CSIT: A topological rate-splitting approach," *IEEE Trans. Commun.*, vol. 65, no. 5, pp. 2164–2179, May 2017.

[20] C. Hao, B. Rassouli, and B. Clerckx, "Achievable DoF regions of MIMO networks with imperfect CSIT," *IEEE Trans. Inf. Theory*, vol. 63, no. 10, pp. 6587–6606, Oct. 2017.

[21] X. Du, L. Li, Z. Li, and G. Lu, "Topological rate-splitting based power allocation scheme in K-cell MISO interference channel with imperfect CSIT," in *Proc. Int. Conf. Comput., Netw. Commun. (ICNC)*, Honolulu, HI, USA, Feb. 2019, pp. 960–964.
[22] A. A. Ahmad, H. Dahrouj, A. Chaaban, A. Sezgin, and M.-S. Alouini, "Interference mitigation via rate-splitting in cloud radio access networks," in *Proc. IEEE 19th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jun. 2018, pp. 1–5.
[23] H. Chen, D. Mi, B. Clerckx, Z. Chu, J. Shi, and P. Xiao, "Joint power and subcarrier allocation optimization for multigroup multicast systems with rate splitting," *IEEE Trans. Veh. Technol.*, vol. 69, no. 2, pp. 2306–2310, Feb. 2020.
[24] Z. Li, L. Li, and X. Du, "Rate-splitting based transmission strategy design in FDD-based system," in *Proc. IEEE/CIC Int. Conf. Commun. China (ICCC)*, Aug. 2018, pp. 773–777.
[25] M. Grant and S. Boyd. (Jan. 2020). *CVX: MATLAB Software for Disciplined Convex Programming, Version 2.2*. [Online]. Available: http://cvxr.com/cvx

LIHUA LI (Member, IEEE) received the B.E. and Ph.D. degrees from the Beijing University of Posts and Telecommunications (BUPT), Beijing, China, in 1999 and 2004, respectively. She had been a Visiting Scholar with the University of Oulu, Finland, in 2010, and Stanford University, USA, in 2015. She is currently a Professor with BUPT. Her research interests include MIMO and massive MIMO, cooperative transmission technologies, link adaptation etc., relating to new generation mobile communication systems such as 5G and beyond. She has published 95 articles and five books. She has applied 23 national invention patents and one international patent. She was selected and funded as one of the New Century Excellent Talents by the Chinese Ministry of Education, in 2008. She won the Second Prize of the State Technological Invention Award (top-3 China national awards), in 2008, and the First Prize of the China Institute of Communications Science and Technology Award, in 2006, for research achievements of "Wideband Wireless Mobile TDD-OFDM-MIMO Technologies."



KEJIA CHAI received the B.E. degree in telecommunication engineering from the Beijing University of Posts and Telecommunications (BUPT), China, in 2017, where she is currently pursuing the M.E. degree with the State Key Laboratory of Networking and Switching Technology, School of Information and Communication Engineering. Her research interests include wireless communications and networks, coordinated multi-point transmission, and rate-splitting technology.

JILONG LI received the B.S. degree in electronic engineering from the University of Electronic Science and Technology of China, Chengdu, China, the M.S. degree in remote sensing from the Chinese Academy of Sciences, Beijing, China, in 2002, and the Ph.D. degree in electronic engineering from the Beijing University of Posts and Telecommunications, Beijing, in 2005. He is currently working with The Academy of Broadcasting Science as a Senior Engineer. His research interests include 5G broadcast, radio and wireless convergence networks, radio digital broadcasting, channel coding, and modulation technology.

XINGWANG LI (Senior Member, IEEE) received the B.Sc. degree from Henan Polytechnic University, in 2007, the M.Sc. degree from the University of Electronic Science and Technology of China, in 2010, and the Ph.D. degree from the Beijing University of Posts and Telecommunications, in 2015.

From 2010 to 2012, he was working with Comba Telecom Ltd., Guangzhou, China, as an Engineer. He spent one year, from 2017 to 2018, as a Visiting Scholar with Queen's University Belfast, Belfast, U.K. He is currently an Associate Professor with the School of Physics and Electronic Information Engineering, Henan Polytechnic University, Jiaozuo, China. His research interests include MIMO communication, cooperative communication, hardware constrained communication, non-orthogonal multiple access, physical layer security, unmanned aerial vehicles, and the Internet of Things. He has served as the TPC/Co-Chair, such as IEEE Globecom, IEEE/CIC ICCC, IEEE WCNC, IEEE VTC, and IEEE/IET CSNDSP. He is currently an Editor on the Editorial Board of IEEE Access, *Computer Communications*, *Physical Communication*, and the *KSII Transactions on Internet and Information Systems*. He is also a Lead Guest Editor of the Special Issue on "Recent Advances in Physical Layer Technologies for 5G-Enabled Internet of Things" of *Wireless Communications and Mobile Computing*.