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# Residual Flux Density Measurement Method of Single-Phase Transformer Core Based on Time Constant

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**ABSTRACT** Residual flux density in the single-phase transformer core can dramatically increase inrush current when the transformer is energized. To reduce inrush current, it is necessary to study residual flux density measurement. This paper proposes a new residual flux density measurement method based on time constant. Firstly, the generation principle of residual flux density is analyzed under different magnetization states, and it is found that the positive relative differential permeability is smaller than the negative at different residual flux density. To obtain the relative differential permeability, when an appropriate DC excitation is applied, the measurement circuit is equivalent to a first-order RL circuit. Then, combining magnetic circuit and transient circuit analysis, the relationship between time constant and relative differential permeability is obtained. It is conclusion that the positive time constant is less than the negative. Residual flux density direction is determined by comparing the positive and negative time constant, and the magnitude of residual flux density is calculated by the relationship between residual flux density and the difference of the positive and negative time constant. Finally, the empirical formula between residual flux density and time constant difference of the square core is obtained in finite element method, and then verified on the experimental platform. Compared with other measurement methods, the relative error of proposed empirical formula is within 4.58 %, and it has higher accuracy in this paper. The proposed method in this paper can provide a reference for selecting the demagnetization voltage, which improves the effectiveness of demagnetization.

**INDEX TERMS** Residual flux density, relative differential permeability, time constant difference, empirical formula.

## I. INTRODUCTION

When the power transformer is cut-out operation and in various test operations [1]–[3], the internal magnetic flux density in the iron core does not fall to zero and thus an unknown residual flux density ( $B_r$ ) is produced, which is resulted by the hysteresis characteristics of iron core materials [4]–[6]. When the transformer is switched to connect with power grid again, a magnetizing inrush current will be produced as the existence of  $B_r$  [7], [8]. Sometimes this current may be 6 to 8 times higher than the rated state current [9], which will result the transformer failing to connect with the power grid. To suppress inrush current, synchronous closing

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technology and phase selection closing technology are proposed in [10]–[12], on the premise of  $B_r$  magnitude and direction being obtained [13]. Moreover, demagnetization without knowing the magnitude of  $B_r$  in the transformer core is not only time consuming and laborious, but also has a poor demagnetization effect [14]. Thus, it is of great significance to accurately detect  $B_r$  in the transformer core for the safe operation of large power transformers and even power grids.

At present, there are five main methods for measuring the  $B_r$  in the transformer core:

1) Estimation method. When large power transformers leave the factory, empirical estimation method is generally used to estimate  $B_r$  in the iron core. And, it is considered that  $B_r$  is 20 % - 80 % of the saturated flux density [15]. In [16],  $B_r$  is found not to higher than 0.7 times of the saturated flux

density by analyzing more than 500 transformers. However, this method can only roughly estimate the range of  $B_r$  and cannot accurately predict the magnitude of  $B_r$  in the iron core.

2) Indirect measurement method. In [17], the  $B_r$  before energization is calculated based on the obtained peak value of the magnetizing inrush current after the transformer is energized. However, it is not sure that this method can be applied to  $B_r$  measurement before the transformer is closed.

3) Pre-magnetization method. By applying larger current to the core, the internal flux density of the core is changed from original  $B_r$  to saturated magnetic flux density. After that the phase selection closing technology or demagnetization method is used to suppress the magnetizing inrush current [18]–[20]. In [21], the DC voltage excitation with varying polarity to measure  $B_r$  is proposed, which is essentially the same as the pre-magnetization method. Although the magnitude of  $B_r$  can be measured, the  $B_r$  has been changed to saturated flux density, and a large current to saturate the iron core is required. Finally, for demagnetizing  $B_r$ , the extra time and expensive equipment are required.

4) Voltage integration method. This method is based on the electromagnetic induction law [22], [23]. By recording the voltage waveform at the transformer opening time, the  $B_r$  is obtained by the voltage integration method. However, the upper limit of the integration (the time of opening) is not easily determined. Due to hysteresis characteristics of the ferromagnetic material, measured  $B_r$  at the moment of opening is different from  $B_r$  after stabilization and thus the feasibility of this method is limited.

5) Transient excitation measurement method. In [24], the  $B_r$  is obtained by establishing the relationship between  $B_r$  and magnetizing current. However, this method uses the magnetization curve in the simulation model, thus this accuracy is greatly reduced and the  $B_r$  direction is not judged. To address the limitations of this approach, in [25], author analyzes the relationship between  $B_r$  and transient current difference. However, the measurement time of transient current difference is not easily determined, which may result in a lower accuracy of calculated  $B_r$ . In [26], the  $B_r$  is calculated based on the positive time constant. However, the  $B_r$  is only measured in known  $B_r$  directions, and not in unknown directions. Moreover, the influence of external excitation on  $B_r$  is not analyzed. Therefore, there is no effective method for measurement  $B_r$  in the iron core.

To accurately measure  $B_r$  in the iron core, this paper proposes a new  $B_r$  measurement method. This method has several improvements over the above methods as follows:

1) The relationship between  $B_r$ , the difference of the positive and negative time constant, and external DC voltage is theoretically analyzed. Then, the square iron core is taken as the research object, and the empirical formula for measuring  $B_r$  is obtained by finite element method (FEM).

2) Through analysis the relationship between the relative differential permeability ( $\mu_{rd}$ ) and the corresponding time constant at  $B_r$ , the  $B_r$  direction is determined.

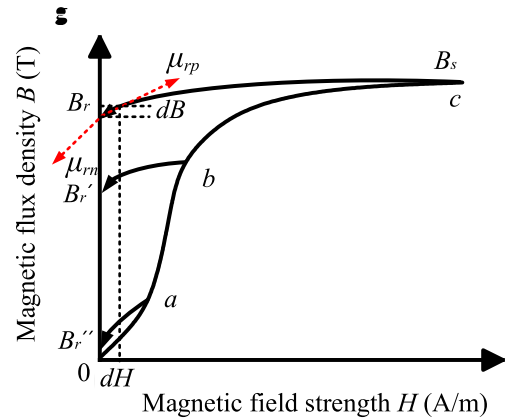


FIGURE 1. The generation principle of  $B_r$  in the iron core under different magnetization states.

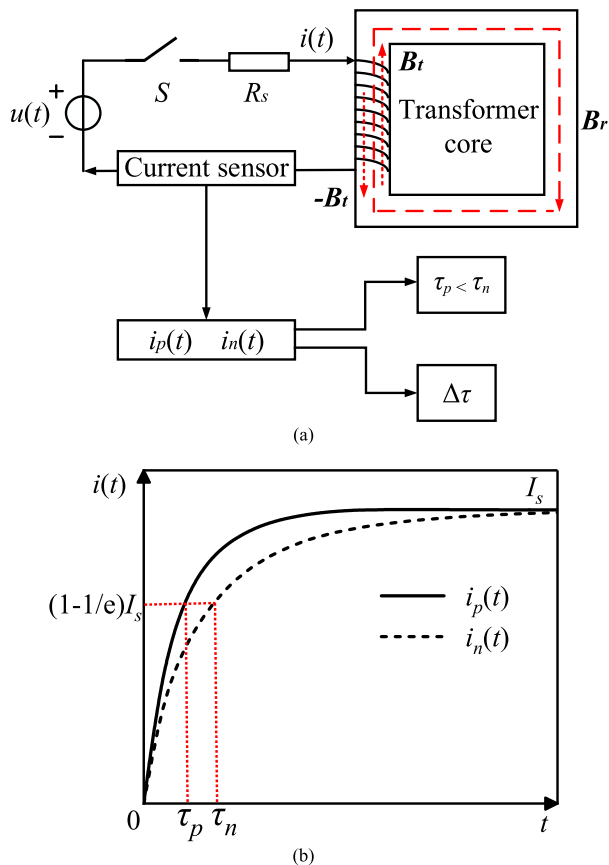
3) Through numerical simulation analysis, the range of applied DC excitation is analyzed and determined. Then the  $B_r$  test platform of the square core is established to verify the accuracy of proposed method. Experimental results prove that the  $B_r$  measurement method proposed in this paper has higher accuracy compared with existing methods.

The rest of the paper is organized as follows. The basic principle of proposed  $B_r$  measurement method is analyzed in Section II. Section III obtains the proposed empirical formula of the square iron core in FEM. In Section IV, the accuracy of proposed empirical formula and the feasibility of proposed  $B_r$  measurement method are verified in the square iron core test platform. Section V concludes the paper.

## II. PRINCIPLE OF PROPOSED RESIDUAL FLUX DENSITY MEASUREMENT METHOD

### A. GENERATION PRINCIPLE OF RESIDUAL FLUX DENSITY

When the transformer core is externally excited, due to the magnetic domain structure changes inside the core material, the ferromagnet exhibits certain magnetic properties to the outside [27]. Fig. 1 shows  $B_r$  generation principle under different magnetization states. As shown, the magnetization process is divided into three states: 1) Reversible magnetization state. When the external magnetic field strength ( $H$ ) increases from 0 to point a, the magnetic domains change will occur in reversible magnetization stage. And then when  $H$  disappears,  $B_r''$  is almost not generated. 2) Irreversible magnetization state. As  $H$  increases to point b, the magnetic domains movement begins to change from reversible magnetic domain wall displacement to irreversible magnetic domain wall displacement, thereby exhibiting stronger magnetism to the outside world. When  $H$  suddenly disappears, the magnetic domain state cannot be restored to the original state, but formed a new state, which results in a phase difference between  $B$  and  $H$ , which is called hysteresis. At this time,  $B_r'$  in the iron core cannot be ignored. 3) Saturated magnetization state. When  $H$  continues to increase to point c, the magnetic domain movement enters the saturated state. After  $H$  disappears, a large  $B_r$  is generated and is not ignored.



**FIGURE 2.** The basic principle of proposed  $B_r$  measurement method in this paper, (a) the measurement circuit of the proposed method and (b) the positive and negative transient current waveforms.

**B. PROPOSED RESIDUAL FLUX DENSITY MEASUREMENT METHOD**

The square iron core is selected as the research object in this paper. Fig. 2 shows the basic principle of proposed  $B_r$  measurement method. As shown in Fig. 2(a), the measurement circuit of proposed method is presented.  $B_r$  is the initial residual flux density in the iron core determined by a large current. The DC voltage  $u(t)$  is applied to the measuring winding to exacton a test flux density ( $B_t$ ) on initial  $B_r$ . When a positive DC voltage with the same polarity is applied,  $B_t$  is generated on the basis of the initial  $B_r$  so that the actual  $B_r$  in the iron core will increase. In addition, when using a negative DC voltage with the opposite polarity to  $B_r$ ,  $-B_t$  is generated on the initial  $B_r$  and the magnitude of  $B_r$  will decrease. At the same time, corresponding transient current  $i(t)$  is generated, as shown in Fig. 2(b).

When a positive DC voltage is applied in the measuring winding, the positive transient current  $i_p(t)$  increases. While a negative DC voltage with the opposite polarity to  $B_r$  is applied, the negative transient current  $i_n(t)$  will be decreased. However, compared to the first process, due to the hysteresis phenomenon of the iron material, the change rate of  $i_p(t)$  is slower than  $i_n(t)$  (see Fig. 2 (b)). The change rate of  $i_p(t)$  or  $i_n(t)$  is reflected by the positive or negative time constant ( $\tau_p$  or  $\tau_n$ ) in the transient process. Time constant

is defined as this moment when the steady-state value  $I_s$  of the transient current drops  $1/e$ . Thus, the direction of  $B_r$  is determined by comparing  $\tau_p$  and  $\tau_n$ , and the relationship between  $B_r$  and the difference  $\Delta\tau$  of  $\tau_p$  and  $\tau_n$  is found to determine the magnitude of  $B_r$  in the iron core.  $\Delta\tau$  at  $B_r$  is expressed as

$$\Delta\tau = |\tau_p - \tau_n| \tag{1}$$

According to above analysis, the proposed relationship for calculating  $B_r$  in the iron core is expressed as

$$B_r = f(\Delta\tau) \tag{2}$$

This method also considers the influence of different external DC voltage (It is also the applied  $B_t$  value) on  $B_r$  in the iron core. To ensure higher accuracy of proposed measurement method, when the DC voltage is applied, it is acceptable to keep the change rate of  $B_r$  is within 10 %. In the following sections, this relationship will be obtained by FEM and verified by experiments.

**C. DETERMINING DIRECTION OF RESIDUAL FLUX DENSITY**

Due to the hysteresis phenomenon of the ferromagnetic material, its magnetization state cannot be restored to the magnetic neutral state, but maintains the  $B_r$  state. Therefore, the  $B_r$  depends mainly on the magnetic domain structure change from maximum flux density state to  $B_r$  state, which is reflected by the change of the relative differential permeability ( $\mu_{rd}$ ) at  $B_r$  [26]. As shown in Fig. 1,  $\mu_{rd}$  at  $B_r$  is defined as the ratio of magnetic flux density increment  $dB$  and magnetic field strength increment  $dH$ .

$$\mu_{rd} = \frac{1}{\mu_0} \frac{dB}{dH} \tag{3}$$

When  $H$  changes from positive to negative in the vicinity of  $B_r$ , the magnetization ability of magnetic domain increases from small to large and thus  $\mu_{rd}$  at  $B_r$  also increases from small to large, as shown in Fig. 1. The relationship between positive and negative relative differential permeability ( $\mu_{rp}$  and  $\mu_{rn}$ ) at  $B_r$  is as follows:

$$\mu_{rp} < \mu_{rn} \tag{4}$$

Based on the magnetic circuit analysis [25], the relationship between the equivalent inductance  $L_{eq}$  and  $\mu_{rd}$  at  $B_r$  is expressed as

$$L_{eq} = \frac{N^2 S \mu_0}{l} \mu_{rd} \tag{5}$$

where  $N$  expresses the ratio of measuring winding,  $S$  and  $l$  represent the cross-sectional area and the average magnetic path length of the iron core, respectively.  $\mu_0$  expresses the vacuum permeability of air, and its value is  $4\pi \times 10^{-7}$  H/m.

When the DC voltage is applied, the measurement circuit is modeled as a  $RL$  parallel equivalent circuit, as shown in Fig. 3. As shown,  $R_s$  includes the internal source resistance,

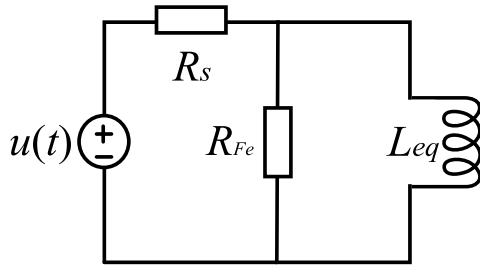


FIGURE 3. The equivalent circuit when the DC excitation is applied.

connecting wires resistance, and winding resistance.  $L_{eq}$  represents equivalent magnetized inductance of measuring winding, which is effect by the magnetic flux in the iron core.  $R_{Fe}$  expresses the iron loss resistance. When the appropriate DC voltage is applied, the change rate of  $B_r$  in the iron core is within 10 %. At this time, resistance  $R_{Fe}$  can be ignored in the transient progress.

When the external DC voltage is applied, the time constant  $\tau$  in the circuit is described as:

$$\tau = \frac{L_{eq}}{R} \tag{6}$$

where  $R$  expresses the total resistance in the circuit. According to (5) and (6), the relationship between  $\tau$  and  $\mu_{rd}$  at  $B_r$  is expressed as

$$\tau = \frac{N^2 S \mu_0}{R l} \mu_{rd} \tag{7}$$

It can be seen that the changes of  $\mu_{rp}$  and  $\mu_{rn}$  at  $B_r$  can be directly reflected by  $\tau_p$  and  $\tau_n$  in the transient circuit. Based on (4) and (7), the relationship between  $\tau_p$  and  $\tau_n$  at  $B_r$  is expressed as

$$\tau_p < \tau_n \tag{8}$$

Thus, the direction of  $B_r$  is determined by comparing  $\tau_p$  and  $\tau_n$  at  $B_r$  when the appropriate DC voltage is applied.

### III. PARAMETERS DETERMINATION AND SIMULATION ANALYSIS

In this paper, an experiment platform which adopts square stacked steel sample is built, as shown in Fig. 12. The main dimension of the iron core is determined as the  $S = 0.0016 \text{ m}^2$ ,  $l = 1.92 \text{ m}$ . Fig. 4 shows the square iron core model in FEM. As shown, the turns of preset  $B_r$  winding are 50, and the turns of measuring winding are 10. Before the numerical simulation analysis, it is necessary to discuss the range of preset  $B_r$  and the selection of external DC excitation.

#### A. RANGE OF RESIDUAL FLUX DENSITY

When the measurement frequency is 5 Hz, the magnetization curve of stacked steel sample is obtained, as shown in Fig. 5. As shown, the saturated flux density of the iron core material can reach 1.8 T. According to the empirical estimation method [15], [16], the  $B_r$  is 20 % -70 % of the saturated flux density in the iron core. Thus, the range of preset  $B_r$  is

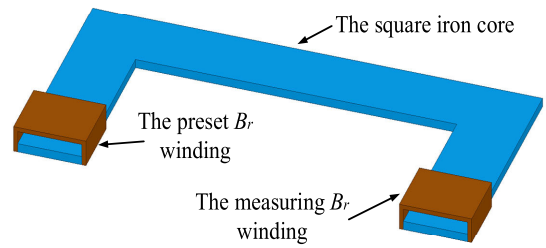


FIGURE 4. The square iron core model set in FEM.

TABLE 1. Preset residual flux density values.

$B_{r1}/\text{T}$	$B_{r2}/\text{T}$	$B_{r3}/\text{T}$	$B_{r4}/\text{T}$	$B_{r5}/\text{T}$	$B_{r6}/\text{T}$	$B_{r7}/\text{T}$	$B_{r8}/\text{T}$
0.44	0.68	0.76	0.83	0.97	1.02	1.08	1.18

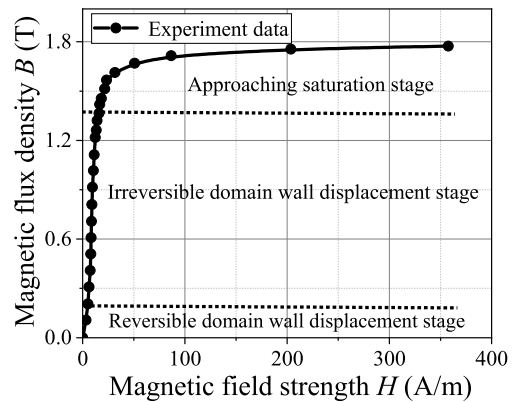


FIGURE 5. The magnetization curve of steel sample at measurement frequency is 5 Hz.

0.36 T-1.26 T in the simulation and experiments. Preset  $B_r$  values selected in this paper are shown in TABLE 1.

#### B. SELECTION OF EXTERNAL DC EXCITATION

When  $B_r$  is zero,  $B_t$  value is determined by applying DC voltage. Then, when  $B_r$  is not zero, DC voltages of the same magnitude and different directions are loaded. Due to the hysteresis effect of the iron core, the change of  $B_r$  is smaller than the actual applied  $B_t$  value. Moreover, when applied  $B_t$  value is larger, the change of initial  $B_r$  becomes larger. To improve the measurement accuracy, the applied DC voltage cannot make the change rate of initial  $B_r$  exceed 10 %.

As shown in Fig. 5, the overall magnetization curve is divided into three stages: 1) In the stage of reversible magnetic domain wall displacement. The maximum flux density in the iron core can approximately reach 0.2 T. When  $H$  disappears, the magnetization can return to the original state along the original path. At this time,  $B_r$  is close to zero. 2) In the stage of irreversible magnetic domain wall displacement. When  $H$  disappears, the magnetization cannot be restored to the original magnetization state, but there is a phase difference between  $B$  and  $H$ , resulting in hysteresis.

At this time,  $B_r$  cannot be ignored. 3) In the approaching the saturation stage. Hysteresis phenomenon is more obvious than the previous two stages. When  $H$  disappears, generated  $B_r$  is too large to be ignored. When the applied  $B_t$  is greater than the flux density in the reversible magnetic domain wall displacement stage, the change of initial  $B_r$  is larger affected. Therefore, the applied  $B_t$  value is not larger than 0.2 T. At this time, the relationship between  $B$  and  $H$  is almost linear, as shown in Fig. 5. The applied  $B_t$  is expressed as

$$B_t = \mu H \quad (9)$$

where the permeability  $\mu$  is obtained by the data fitting and its value is 0.05 H/m.

According to the ampere loop law, the above equation is obtained:

$$\frac{u(t)}{R} \cdot N = H \cdot l \quad (10)$$

where  $N$  expresses the turns of the measuring winding and its value is 10.  $R$  expresses the total resistance in the measurement circuit and its value is 5.04  $\Omega$ .

According to (9) and (10), the external DC voltage  $u(t)$  can be obtained.

$$u(t) = \frac{Rl}{\mu N} B_t \quad (11)$$

When a positive or negative DC voltage is applied, due to the hysteresis characteristics of core material,  $B_r$  in the iron core is changed from initial  $B_r$  to new  $B_{r1}$  or  $B_{r2}$ . The change  $\Delta B$  of flux density is different in the positive and negative directions. To describe the effect of external excitation on  $B_r$ , the change rate  $B_r \%$  of  $B_r$  is defined as

$$B_r \% = \left| \frac{\Delta B}{B_r} \right| \times 100\% \quad (12)$$

where the positive  $\Delta B$  is expressed by  $B_{r1} - B_r$ , and the negative  $\Delta B$  is expressed by  $B_{r2} - B_r$ .

Fig. 6 shows the change of  $B_r \%$  under different  $B_t$  when  $B_r$  is 0.83 T. As shown, the positive  $B_{r1} \%$  and negative  $B_{r2} \%$  increase as applied  $B_t$  value increases. Since the change of  $\mu_{rp}$  at  $B_r$  is smaller than  $\mu_{rn}$ , the magnetization ability in the positive direction is weaker than that of the negative, and thus  $B_{r1} \%$  is less than  $B_{r2} \%$ . When  $-B_t$  is applied, the variation of  $B_r \%$  is greater than that with the positive. Therefore, when  $B_{r2} \%$  is less than 10 %, the maximum value of  $B_t$  is determined.

Fig. 7 shows the change of  $B_r \%$  under different  $B_r$  and different  $B_t$ . As shown, as  $B_r$  increases, corresponding  $B_{r1} \%$  and  $B_{r2} \%$  gradually become smaller. And,  $B_{r2} \%$  is less than 10 % under the situations: 1) when  $B_t$  is less than 0.10 T, corresponding  $B_{r2} \%$  is less than 10 % in the range of preset  $B_r$  and 2) when  $B_t$  is larger than 0.10 T and less than 0.15 T, corresponding  $B_{r2} \%$  is less than 10 % at  $B_r > 1.0$  T. The above two situations can meet the conditions of  $B_t$  selection. Thus,  $B_t$  value is selected in the range of 0.15 T.

At the same time, when the positive and negative  $B_t$  are applied,  $\tau_p$  and  $\tau_n$  are obtained by corresponding transient

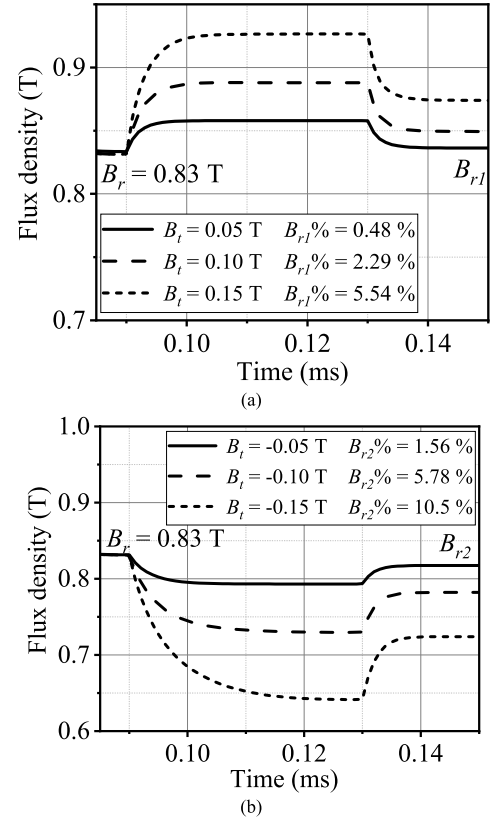


FIGURE 6. The change of  $B_r \%$  under different  $B_t$  when  $B_r$  is 0.83 T, (a) the change of  $B_{r1} \%$  and (b) the change of  $B_{r2} \%$ .

TABLE 2. Applied test flux density and DC voltage value.

$B_t/T$	0.05	0.10	0.15
$u(t)/V$	1.00	2.00	3.00

current. When applied  $B_t$  value is 0.05 T, 0.1 T and 0.15 T, the changes of  $\tau_p$  and  $\tau_n$  are shown in Fig. 8. It can be seen that corresponding  $\Delta \tau$  is more and more obvious as  $B_r$  increases. When  $B_t = 0.05$  T and preset  $B_r < 0.44$  T,  $\tau_p$  is close to  $\tau_n$  and thus corresponding  $\Delta \tau$  is almost to zero. At this time, the direction of  $B_r$  in the iron core is not easily determined. To make the direction of  $B_r$  can be accurately judged, the applied  $B_t$  value is not less than 0.05 T. Based on the above analysis, applied  $B_t$  value varies within the range of 0.05 T - 0.15 T, and then corresponding  $u(t)$  is obtained by equation (11), as shown in TABLE 2.

### C. EMPIRICAL FORMULA FOR CALCULATING RESIDUAL FLUX DENSITY

Fig. 9 shows the  $i_p(t)$  and  $i_n(t)$  waveforms when  $B_t$  is 0.1 T and  $B_r$  is 0.83 T. As shown,  $\tau_p$  and  $\tau_n$  can be obtained by  $i_p(t)$  and  $i_n(t)$  waveforms. The time constant is mainly affected by the equivalent inductance value in the equivalent circuit, which is related to  $\mu_{rd}$  at  $B_r$ . When the positive and negative  $B_t$  is applied, as  $B_r$  increases, the change of  $\mu_{rp}$  is smaller than  $\mu_{rn}$ . Thus,  $\tau_p$  is less than  $\tau_n$  at different  $B_r$ . By comparing  $\tau_p$  and  $\tau_n$ , the direction of  $B_r$  in the iron core is determined.

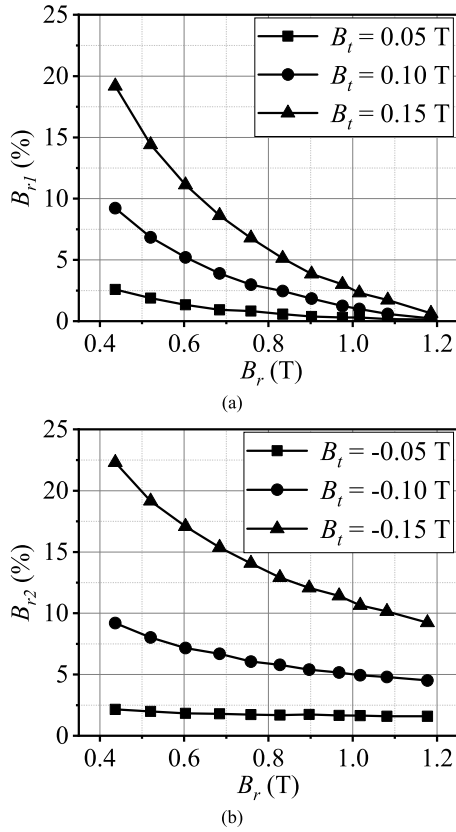


FIGURE 7. The change of  $B_r$  % under different  $B_r$  and different  $B_t$ , (a) the change of  $B_{r1}$  % and (b) the change of  $B_{r2}$  %.

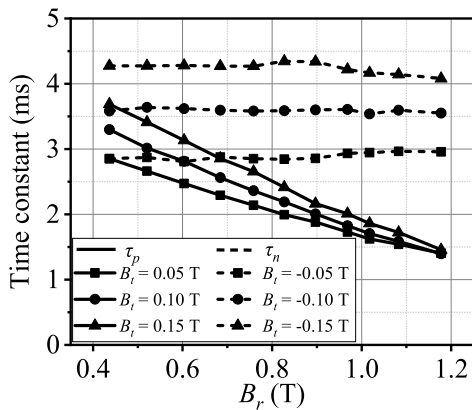


FIGURE 8. The change of the  $\tau_p$  and  $\tau_n$  at different  $B_r$  when the  $B_t$  is 0.05 T, 0.10 T and 0.15 T.

After determining  $B_r$  direction in the iron core, to obtain the empirical formula of square iron core, the relationship between  $B_r$  and  $\Delta\tau$  at different  $B_r$  and different  $B_t$  is analyzed. When  $B_t$  is 0.1 T and -0.1 T, the  $i_p(t)$  and  $i_n(t)$  waveforms and corresponding time constant changes are shown in Fig. 10(a) and Fig. 10(b). It can be seen that  $\tau_p$  or  $\tau_n$  is obtained by corresponding transient current waveforms (see Fig. 10(c)). As shown in Fig. 10(c), the change of  $\tau_p$  linearly decreases as  $B_r$  increases, which is greatly affected by  $B_r$  in the iron core. While the change of  $\tau_n$  is slow with the increase of  $B_r$  and is small under the influence of  $B_r$ .

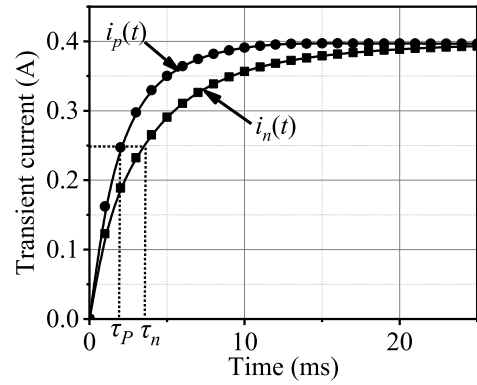


FIGURE 9. The  $i_p(t)$  and  $i_n(t)$  waveforms when  $B_t$  is 0.1 T and  $B_r$  is 0.83 T.

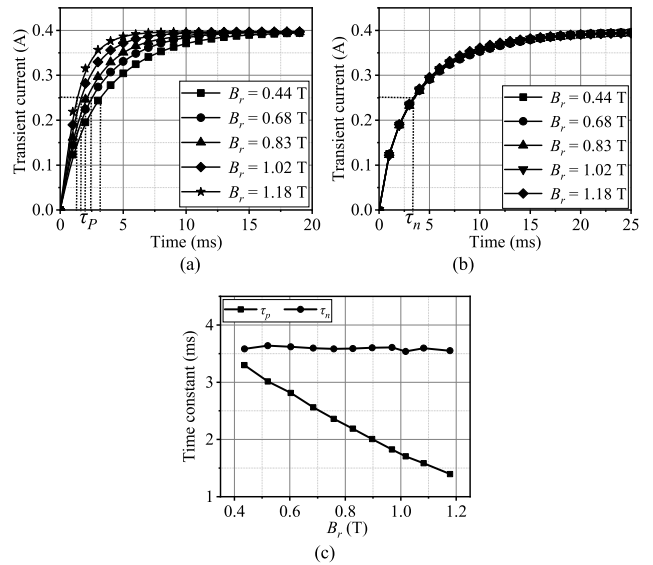


FIGURE 10. The  $i_p(t)$  and  $i_n(t)$  waveforms and corresponding  $\tau_p$  and  $\tau_n$  at different  $B_r$  when  $B_t$  is 0.10 T and -0.10 T, (a) the  $i_p(t)$  waveforms, (b) the  $i_n(t)$  waveforms, and (c) the changes of the  $\tau_p$  and  $\tau_n$ .

TABLE 3. Time constant value at different residual flux density.

$B_r/\text{T}$	$\tau_p/\text{ms}$	$\tau_n/\text{ms}$	$\Delta\tau/\text{ms}$
0.44	3.30	3.58	0.28
0.52	3.02	3.64	0.62
0.60	2.81	3.62	0.81
0.68	2.56	3.60	1.03
0.76	2.36	3.58	1.22
0.83	2.19	3.59	1.40
0.90	2.01	3.60	1.59
0.97	1.83	3.61	1.78
1.02	1.71	3.54	1.83
1.08	1.59	3.60	2.01
1.18	1.40	3.55	2.16

As shown in Fig. 10,  $\tau_p$  and  $\tau_n$  are obtained by the  $i_p(t)$  and  $i_n(t)$  waveforms at different  $B_r$  when  $B_t$  is 0.1 T and -0.1 T. And then  $\Delta\tau$  is obtained by (1). TABLE 3 shows the value of  $\tau_p$ ,  $\tau_n$  and  $\Delta\tau$  at different  $B_r$  when  $B_t$  is 0.1 T and -0.1 T.

Through data fitting method, the proposed empirical formula between  $B_r$  and  $\Delta\tau$  is expressed as

$$B_r = f(\Delta\tau) = a\Delta\tau + b \quad (13)$$

TABLE 4. Fitted parameters valve.

$B_t/T$	$a$	$b$
0.05	0.46	0.44
0.10	0.39	0.29
0.15	0.36	0.20

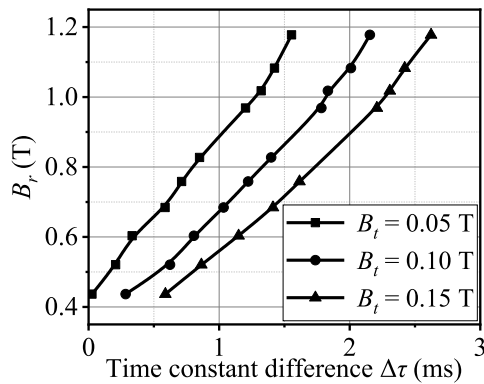


FIGURE 11. The fitting relationship between  $B_r$  and  $\Delta\tau$  at different  $B_t$ .

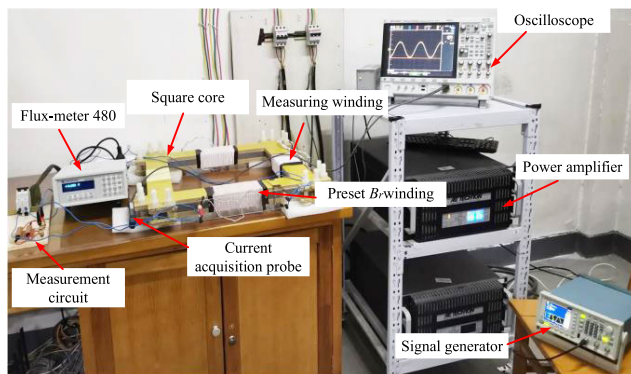


FIGURE 12. The experimental platform of proposed  $B_r$  measurement method.

where the parameters  $a$  and  $b$  are the fitting coefficient, respectively. TABLE 4 shows the parameters  $a$  and  $b$  at different  $B_t$ , and corresponding the relationship between  $B_r$  and  $\Delta\tau$  is shown in Fig. 11. When  $B_r$  in the iron core increases,  $\Delta\tau$  increases linearly at different  $B_t$ . After that the accuracy of proposed empirical formula will be verified in the experiments in Section IV.

#### IV. EXPERIMENTAL VALIDATION

##### A. EXPERIMENTAL PLATFORM

To verify the accuracy of proposed empirical formula, this paper establishes an experimental platform of proposed  $B_r$  measurement method, as shown in Fig. 12. The current signal  $I_m$  in preset  $B_r$  winding and the applied DC voltage  $u(t)$  in the measuring winding are respectively obtained by the signal generator. Firstly, a large current signal  $I_m$  is applied in the preset  $B_r$  winding to generate an initial  $B_r$  in the iron core. Then,  $u(t)$  is sent through the signal generator, and amplified by the power amplifier, applied in the measuring

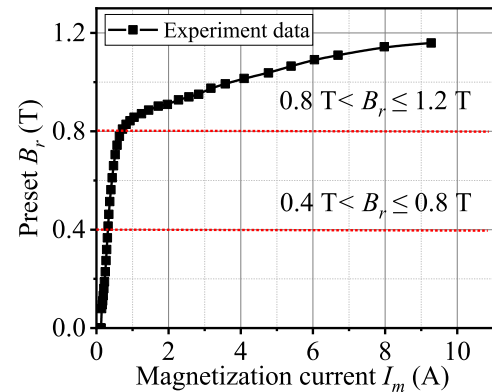


FIGURE 13. The relationship between preset  $B_r$  and magnetization current  $I_m$ .

winding to result  $B_t$ . When  $B_t$  is applied, the change of average flux density in the iron core is observed by the Flux-meter 480. Furthermore, the positive and negative transient current waveforms are observed and stored by the oscilloscope. Finally, through the positive and negative transient current waveforms, corresponding  $\tau_p$  and  $\tau_n$  are obtained. From this the direction of  $B_r$  in the iron core is determined by comparing  $\tau_p$  and  $\tau_n$ , and the magnitude of  $B_r$  is calculated by proposed empirical formula between  $B_r$  and  $\Delta\tau$ .

##### B. PRESET RESIDUAL FLUX DENSITY

Fig. 13 shows the relationship between preset  $B_{rp}$  and magnetization current  $I_m$ . As shown, there is a piecewise linear relationship between preset  $B_{rp}$  and  $I_m$ . When  $B_r$  changes from 0.4 T to 0.8 T, the relationship between preset  $B_{rp}$  and corresponding  $I_m$  is expressed as:

$$B_{rp} = 5.54I_m^2 + 3.62I_m - 0.48 \quad (14)$$

When  $B_r$  changes from 0.8 T to 1.26 T, the relationship between preset  $B_{rp}$  and corresponding  $I_m$  is expressed as:

$$B_{rp} = 0.05I_m + 0.81 \quad (15)$$

Through (14) and (15),  $B_r$  value in the iron core can be predicted more accurately.

##### C. RESIDUAL FLUX DENSITY MEASUREMENT

Fig. 14 shows the  $i_p(t)$  and  $i_n(t)$  waveforms when  $B_r$  is 0.83 T and  $B_t$  is 0.1 T. It can be seen that  $\tau_p$  and  $\tau_n$  are obtained by corresponding transient current waveforms. Since the change rate of  $i_p(t)$  is greater than that of  $i_n(t)$ ,  $\tau_p$  is less than  $\tau_n$  in the transient progress. By comparing  $\tau_p$  and  $\tau_n$ , the smaller  $\tau_p$  is determined as the positive time constant, and the positive direction of  $B_r$  is determined. And, the larger  $\tau_n$  is determined as the negative time constant, and the negative direction of  $B_r$  in the iron core is determined. In addition, the simulation results are in good agreement with the measured transient current waveforms. Experimental results prove the accuracy and feasibility of proposed measurement method in FEM.

Fig. 15 shows the  $i_p(t)$  and  $i_n(t)$  waveforms under different  $B_r$  when  $B_t$  is 0.1 T and -0.1 T. When preset  $B_r$  is 1.18 T,

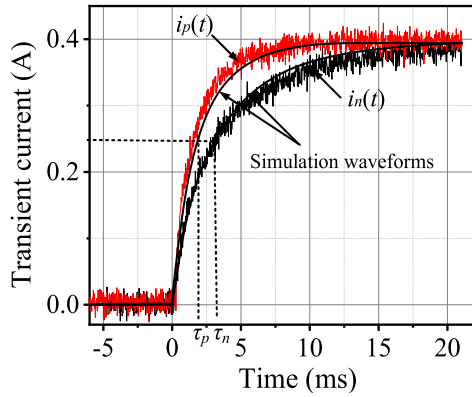


FIGURE 14. The  $i_p(t)$  and  $i_n(t)$  waveforms when  $B_r$  is 0.83T and  $B_t$  is 0.1T.

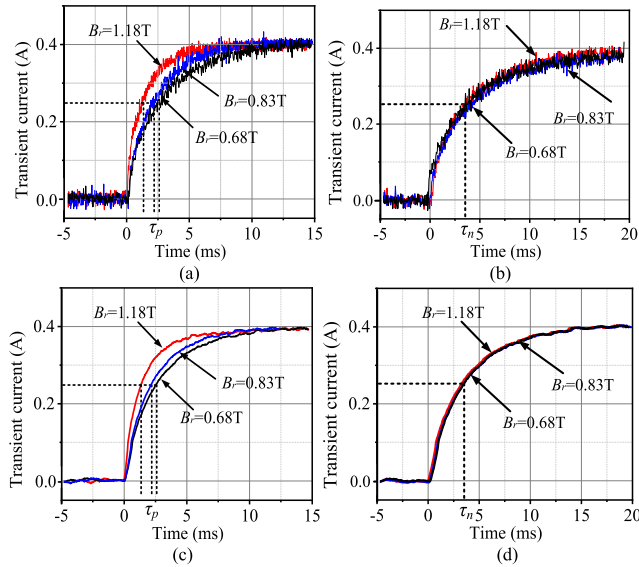


FIGURE 15. The  $i_p(t)$  and  $i_n(t)$  waveforms at different  $B_r$  when  $B_t$  is 0.1 T and -0.1 T, (a) measured  $i_p(t)$  waveforms, (b) measured  $i_n(t)$  waveforms, (c) processed  $i_p(t)$  waveforms, and (d) processed  $i_n(t)$  waveforms.

0.83 T and 0.68 T,  $\tau_p$  and  $\tau_n$  are measured by corresponding transient current waveforms. As shown in Fig. 15(a) and Fig. 15(b), measured  $i_p(t)$  and  $i_n(t)$  waveforms noise are relatively large. To reduce the measurement error caused by this noise, the moving average method is used to suppress this noise [29]. The processed transient current waveforms are shown in Fig. 15(c) and Fig. 15(d). Through the processed transient current waveforms,  $\tau_p$  and  $\tau_n$  are calculated and then corresponding  $\Delta\tau$  is substituted into (13) to calculate  $B_r$ . The measurement data are shown in TABLE 5. As shown, the preset  $B_{rp}$  is obtained by (14) and (15), and the  $B_{rv}$  measured in the voltage integration method [21] is obtained by Flux-meter 480. The relative error  $\varepsilon_1\%$  between  $B_{rp}$  and calculated  $B_r$  and the relative error  $\varepsilon_2\%$  between  $B_{rp}$  and  $B_{rv}$  are expressed as

$$\varepsilon_1\% = \frac{|B_r - B_{rp}|}{B_{rp}} \times 100\% \quad (16)$$

$$\varepsilon_2\% = \frac{|B_{rv} - B_{rp}|}{B_{rp}} \times 100\% \quad (17)$$

TABLE 5. Measurement data.

$B_{rp}/T$	$B_{rv}/T$	$\Delta\tau/ms$	$B_r/T$	$\varepsilon_1/\%$	$\varepsilon_2/\%$
0.44	0.49	0.44	0.46	4.58	10.85
0.68	0.75	1.08	0.71	4.40	10.22
0.76	0.84	1.12	0.73	4.36	9.98
0.83	0.91	1.29	0.79	4.24	9.85
0.90	0.99	1.47	0.86	4.12	9.52
1.02	1.11	1.77	0.98	4.05	8.89
1.08	1.17	1.91	1.04	4.01	8.43
1.18	1.27	2.16	1.13	3.90	7.75

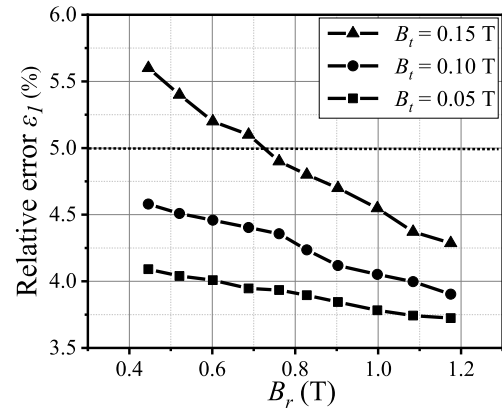


FIGURE 16. The change of  $\varepsilon_1\%$  at different  $B_r$  when  $B_t$  is 0.05 T, 0.1 T and 0.15 T.

As shown in TABLE 5, when  $B_{rp}$  changes from 0.44 T to 1.18 T,  $\varepsilon_1\%$  is within 4.58%, which is less than  $\varepsilon_2\%$ . Compared with voltage integration method, proposed empirical formula has higher accuracy. Fig. 16 shows the change of  $\varepsilon_1\%$  when  $B_r$  value is 0.05 T, 0.1 T and 0.15 T. In engineering, it is best to control the relative error range within 5%. When  $B_t$  is 0.05 T, the measured transient current signal is weaker so that there is a higher accuracy necessary to the experimental device. And the external electromagnetic interference is large, which is not conducive to signal detection. When  $B_t$  is 0.1 T,  $\varepsilon_1\%$  is within 4.58%, which meets the requirements of engineering error. When  $B_t$  is 0.15 T, the smaller the  $B_r$  is, the greater the  $\varepsilon_1\%$  is. And when  $B_r$  is less than about 0.7 T,  $\varepsilon_1\%$  is greater than 5%, indicating that the range of measurement  $B_r$  is narrow. In summary, when  $B_t$  is 0.1 T, the measurement is best in a wide range of  $B_r$  in the iron core.

## V. CONCLUSION

This paper presents a residual flux density measurement method of the single-phase transformer core based on time constant. Compared with existing residual flux density measurement methods, the main advantages of proposed method include: 1) when the appropriate DC excitation that is the positive or negative direction with residual flux density is applied, the direction of residual flux density is accurately determined by comparing the positive and negative time constant. 2) The magnitude of residual flux density in different directions is calculated by proposed relationship between



residual flux density and time constant difference, and its accuracy is verified for a square iron core. When the applied DC voltage is same, the relative error of proposed method is within 4.58 %, which has higher accuracy than voltage integration method. At the same time, it also meets the requirements of engineering error. 3) When an appropriate DC excitation is applied, the magnitude of residual flux density in different ranges is accurately calculated by proposed empirical formula. But the measurement results have higher requirements for the accuracy of measurement instruments. In addition, the proposed measurement method is applicable to transformer cores with different structures which correspond to different empirical formulas. In the future, we hope to analyze different empirical formulas and find the connection between different empirical formulas to solve this problem.

This research provides a reference for selecting the direction and amplitude of demagnetization voltage in the transformer core. In the next step study, the proposed measurement method will be applied to the actual single-phase and three-phase power transformer cores to carry out residual flux density measurement research.

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