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# Structured Multi-Agent-Based Model for Bankruptcy Contagion With Cash Flow

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**ABSTRACT** Numerous multi-agent models have been proposed for various economic phenomena, especially for the bankruptcy contagion phenomenon, which is a seriously destructive occurrence that begins with the bankruptcy of a small group of individuals and spreads to a large scale like an infectious disease. However, almost none of these bankruptcy models can be applied in plural environments or consider the difficulty of implementation. Furthermore, cash flow between firms, which is a highly influential factor in bankruptcy contagion situations, is considered in few such models. To address these shortcomings, in this article, a new relationship, called the cash flow relationship, is first presented based on several relationships related to cash flows. A graphical structure called a cash flow graph is then presented to record cash flow relations between financial institutions and provide a discussion of the nature of the cash flow is then presented. Next, a multi-agent bankruptcy contagion model based on the cash flow graph is introduced. Finally, inferences are drawn from the proposed model and experiments conducted to explore the bankruptcy contagion phenomenon and confirm these inferences. This proposed model can be applied in multiple environments related to cash flows to successfully address the limitations of the existing multi-agent models for bankruptcy contagion.

**INDEX TERMS** Agent decision-making, bankruptcy diffusion, structured decision-making, social network.

## I. INTRODUCTION

According to the opinion put forward by Farmer and Foley [1] in *Nature* in 2009, agent-based models are more promising than empirical statistical models and conventional equilibrium models for modeling complex economic phenomena. Since then, numerous researchers have given attention to modeling various economic phenomena using multi-agent systems. As an important and complex economic phenomenon, the bankruptcy contagion phenomenon has obviously attracted considerable attention from researchers.

A multi-agent system is a computerized system for representing multiple interactions among intelligent agents within an environment. These agents have autonomy and sociality [2]. Because of their autonomy, the agents can behave autonomously, according to their own beliefs, desires, and intentions. As a result, autonomy can make agents in the same system exhibit different behavior patterns. Because of their sociality, the agents can communicate, coordinate, and cooperate with each other to achieve a common goal. Thus, a simple definition of a multi-agent system is a system that contains multiple agents communicating, coordinating, and cooperating with each other to achieve one goal. Based on this definition, a complete multi-agent system can be considered to consist of three components: agents, relations, and interaction protocols. The multi-agent-based model proposed for bankruptcy contagions in this article is presented in terms of aspects of each of these three components.

The bankruptcy contagion phenomenon is similar to the concept of dominoes, in which the bankruptcy of a few financial institutions brings about a large number of bankruptcies in a financial network. As a complicated economic phenomenon, bankruptcy contagion scenarios have been explored by researchers for many years, but no great progress in modeling the bankruptcy contagion phenomenon has yet been achieved. In previous studies, actual bankruptcy

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propagation has always been found to be so complicated that some simplifications had to be made to attempt to model the bankruptcy contagion phenomenon. For example, researchers have usually only considered one type of relation in such studies or have constructed simplified models that have only addressed some specific factors in bankruptcy contagion. The main reason for these simplifications is that it is very difficult to consider numerous factors involved in bankruptcy contagions using conventional equilibrium models.

Multi-agent models offer a very different approach. From a multi-agent perspective, a bankruptcy contagion can be viewed as a complex interactive process that occurs within a group of agents who have particular relations to each other. We can design agents to model financial institutions involved in bankruptcy propagation and then simulate an actual bankruptcy contagion process using these modeled agents. This method provides a simpler, more accurate, and more adaptive means of constructing a bankruptcy contagion model than is possible using conventional equilibrium modeling. Therefore, numerous researchers have investigated modeling of the bankruptcy contagion phenomenon via multi-agent models in recent years.

Many researchers have explored bankruptcy contagions via multi-agent models, but their studies have possessed some shortcomings. First, in most previous studies on this subject, researchers have not considered the difficulty of implementing such a model, which is not an easy task, especially for domain experts who are not familiar with coding. Second, to the best of our knowledge, each of the previous research efforts on multi-agent modeling of bankruptcy contagion has been focused on one particular relationship, such as the supply relationship, debt relationship, or parent–subsidiary relationship, for simplicity. Third, the previously developed models have placed little if any emphasis on the cash flows between financial institutions, which play an important role in bankruptcy contagions.

To address the shortcomings of previous modeling efforts, we studied the common features among various relationships that should be considered in modeling the bankruptcy contagion phenomenon within a multi-agent system. The main contributions of this study are as follows:

1) We modeled the bankruptcy contagion phenomenon within the framework of a multi-agent system. In this way, the model could be divided into several simpler components, reducing the difficulty of coding.
2) We derived a more general relationship, referred to as the cash flow relationship, from several common relationships between firms, such as the supply relationship and debt relationship. The cash flow relationship can represent all the relationships related to the cash flows between financial institutions. Based on this relationship, we propose a bankruptcy contagion model that can be applied to multiple relationships.
3) The liquidity of financial institutions and the cash flows between them are significant factors in the bankruptcy of any one institution or a bankruptcy contagion. We therefore considered the effects of the amounts of cash flows and liquidity in our model and explored how they affect the bankruptcy contagion process.

The remainder of this article is organized as follows. An overview of the related literature is presented in Section II. Section III proposes the methodological process, which consists in a succession of models, to normalize the development procedure. In Section IV, we define the cash flow relationship that forms the basis of our model and show how cash flow relations between agents can be represented graphically. Section V presents our multi-agent bankruptcy contagion model, based on the cash flow graph. In Section VI, we present the results of a series of simulation experiments conducted to explore the processes of bankruptcy contagions. Some advantages and limitations of the model developed in this study are discussed in Section VII.

## II. RELATED WORK

### A. DIFFUSION IN FINANCE

Diffusion in finance has received considerable attention in the field of economics [3]–[5]. Kiyotaki and Moore [6] first proposed in theory that a trade credit chain will form a channel of liquidity shock and, based on the ''lost decade'' of Japan in the 1990s, noted that financial contagion can spread through the effect of balance sheets. Researchers have therefore been concerned with verifying the trade credit chain as a potential and important transmission mechanism of enterprise bankruptcy infection and, more broadly, examining how a financial contagion phenomenon in trade credit chain increases ''bankruptcy''—a remarkable effect. For example, Raddatz [7] verified that a trade credit chain increases the relevance of the output between two enterprises using data describing the relationship between the input and output for 378 manufacturing enterprises in 43 countries. Acemoglu *et al.* [8] proposed that the severe effects produced by bankruptcy would not only spread through direct trade relationships, but also increase downstream through indirect trade relationships. This type of phenomenon is widespread in various industries. Jacobson and von Schedvin [9] analyzed a set of Swedish enterprise data collected during 2007–2011 that included information on bankruptcies and related trade credit chains, verifying the transmission mechanism of bankruptcy in trade credit chains.

Other researchers have focused on examining concrete cases in financial markets and on larger scales. For example, Morales and Andreosso-O'Callaghan [10] used several types of econometric models to analyze the effects of contagions in the global economy. Ahmad *et al.* [11] investigated the effects of financial and marketing diffusion in several countries in southern Europe and the United States, Britain, and Japan to the stock markets of the BRIICKS countries (Brazil, Russia, India, Indonesia, China, South Korea, and South Africa). They also analyzed the degree of influence of the effects of contagions on different countries.

There have been many similar studies and published papers in the areas of economics and finance. Much of this research has been empirical and has been focused on specific issues or specific phenomena through analyses of real data and attempts to identify the underlying reasons for the observed phenomena to provide guidance for economic policymakers [12]–[17]. The main purpose of this type of research is to predict future financial situations by analyzing existing financial data. The results of this type of research mainly depend on the authenticity and validity of the data used and the rationality of the analysis employed.

### B. APPLICATION OF MULTI-AGENT TECHNOLOGY IN FINANCIAL AND ECONOMIC RESEARCH

Because of the advantages of multi-agent modeling in analyzing complex, large-scale social systems, researchers have begun to apply it to financial and economic research in recent years [1], [18]–[20]. For example, Hernes *et al.* [21] proposed a multi-agent model for an early warning system for economic crises. This model can assess the possibility of an economic crisis in advance and remind users to take preventive action. Admittedly, it is difficult to grasp the dynamics of the financial market, which has led to inefficiency in many financial market rules proposed in the past. However, agent-based modeling offers a more convenient way of understanding and analyzing the dynamics of financial markets. For example, Bae *et al.* [12] developed a financial market model based on agents that includes consideration of debt and can improve the ability to analyze the capital structure of an agent dynamically, which makes this model well suited to analyzing the actual conditions of capital markets. Zhang *et al.* [23] designed a multi-agent-based system that was integrated with an agent-oriented approach and ontology to achieve a common understanding of a problem domain by focusing on the valuation effects of bankruptcy filings through inter-firm linkage.

Grilli *et al.* [24] studied macroeconomic stability and the interconnection among banks by constructing an agent-based model. Their study considered the complex characteristics of the credit market in detail and described the interaction and evolution of the credit market in the form of a network. Georg [25] proposed a dynamic multi-agent model of a banking system with a central bank and studied the effects of the interaction network structure between bank agents on financial contagions and common shocks.

Another common research direction has involved using methods based on agent mechanisms to study and analyze the behavior and mechanisms of economic markets, such as auction mechanisms, negotiation mechanisms, and so on. The greatest challenge in this research approach is the determination of the optimal mechanism design [26]: when a seller agent has one or several items to sell and presumably knows little about valuation of the goods by the buyer agent, how should the goods be sold and distributed to maximize the revenue of the seller agent [27], [28]? Studies in this vein have recently been focused on crowd-sourcing markets. For instance, Karger *et al.* [29] used agents to simulate the behavior of individuals in a crowd-sourcing market and proposed an approximate algorithm based on the low-rank matrix to determine task assignments and extract solutions. This method can achieve satisfactory reliability at low cost and outweighs the formerly proposed ‘‘majority voting’’ rule. Taking this approach a step further, Tran-Thanh *et al.* [30] proposed an agent-based budget allocation algorithm that can allocate funds among different tasks to obtain a relatively lower error rate and better performance than the methods proposed by Karger *et al.* [29].

In summary, applications of agent technology in financial and economic research have mainly fallen into two categories: 1) modeling an economic or financial body in terms of agents and setting up an agent system to simulate and analyze the economic or financial behavior of a large-scale social systems [1], [31], [32]; or 2) using an agent mechanism or game theory to analyze and design an economic market mechanism [33]–[35]. Overall, the research in this area has mainly been based on using agents to represent specific economic and financial behavior or create examples to model and analyze.

### C. AGENT-BASED INFORMATION DIFFUSION

There is a close relationship between multi-agent interaction and the diffusion of information [36], [37]. The main body of propagation can be modeled using interactive agents, the medium can be modeled as an interactive environment in the multi-agent system, the contents of the propagation can be modeled as a negotiation object in the multi-agent system, and the propagation model can be modeled as the interaction protocol and decision-making mechanism in the multi-agent system. In recent years, many research studies on agent-based information diffusion have been presented [38]–[41], particularly at the International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS).

Korkmaz *et al.* [42] used multi-agent game theory model to analyze the type of information diffusion that occurs on social communication platforms such as Facebook, which exploits the unique classical threshold propagation model and general knowledge mechanism to study the dynamic characteristics of cluster behavior. Small and Mason [43] used the Iterated Local Transitivity (ILT) model to study decision-making in competitive information diffusion and explored how two competing agents achieve Nash equilibrium in information diffusion.

The disease infection model is a very common model in information diffusion. Mao [44] used agent simulation to study three types of transmission process: the spread of disease, spread of information about a disease, and prevention of the spread of disease in an urban area of one million people. The model employs methods based on individuals, network models, behavioral theory, and stochastic processes and can effectively simulate real propagation processes and provide advice for use in disease control.

Gong and Xu [45] combined a complex network model and a multi-agent model to study multi-agent information propagation in a scale-free network environment and identified delayed propagation characteristics of different propagation processes in scale-free networks. López-Pintado [46] used an agent threshold propagation model of random network connections in different connection distributions to determine its propagation thresholds.

In summary, the related research has mainly involved modeling the main body of propagation as an agent and then, based on classical communication mechanisms (e.g., a threshold model or infectious disease model), modeling and analyzing the features and rules of the specific information diffusion process [47], [48]. In addition, agent game theory has been used to analyze the main body in the propagation process of interaction and to study the effects of different networks on the propagation process [49]–[51].

## III. METHODOLOGICAL PROCESS

In this section, we describe in detail the process of our model framework based on structured multi-agent. Through this process, we aim to develop the structured multi-agent-based model for analyzing the bankruptcy contagion phenomenon, by utilizing domain knowledge and construct technology. The whole development is composed of the following steps, as depicted in Fig. 1.

1) Identification of conceptual model: Analysis of the knowledge involved in various relations between companies to model into a cash flow graph, and definition of the concepts of structured analysis of the graph.
2) Definition of operational model: Identification of multi-agent systems development theories to define the appropriate model base on the cash flow graph, and determination of the interaction rules.
3) Design of simulation experiments: Development of some experiments to explore the bankruptcy contagion phenomenon and confirm the inferences drawn before.

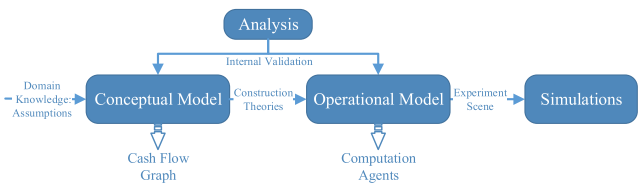


**FIGURE 1.** The methodology of the modeling process.

## IV. DEFINITION AND NATURE OF CASH FLOW GRAPH

It is well known that the relationships between companies are important routes for bankruptcy contagions. It is therefore logical to consider the relations between companies in studies of bankruptcy contagions. In previous studies, researchers have presumed that one type of relation, e.g., supply relations or debt relations, exists between companies. Considering only one type of relation offers the advantage of simplicity, but we think that a more general relationship exists between companies. Given the fact that supply relations, debt relations, and other type of financial relations all involve cash flows, we chose to use cash flow relations between companies as a proxy for all of the relationships of other types that involve cash flows. Cash flow relationships can be represented graphically in what we refer to as a cash flow graph.

### A. CASH FLOW GRAPH

A cash flow describes the transmission of cash from one company to another. All of the behaviors related to cash between companies can be expressed by cash flows. For example, a situation in which company $A$ supplies products to company $B$ can be viewed as a cash flow from company $B$ to company $A$. We think that cash flow relationships are more essential than supply relationships or debt relationships to describe financial relations between companies. Thus, it is vital to consider cash flow relations in modeling the bankruptcy contagion phenomenon within a group of companies. To express the notion of cash flows explicitly, a supply chain can be taken as an example, as described below.

If we use nodes to represent companies and directed edges to represent supply relations between companies, then a supply chain network can be described by a directed graph $G =< A, R >$. $A$ indicates a group of companies, which can be modeled as agents later, and $R$ indicates the supply relations within the group of companies.

Figure 2 is a directed graph that shows a simple supply chain network. In the figure, nodes $a$, $b$, $c$, $d$, and $e$ represent companies, and the value of a directed edge represents a supply relation indicating the amount of periodic trading of cash between two companies. For example, in Fig. 2, the value of edge $< a, b >$ is 15, denoted as $V_{ab} = 15$, which can be interpreted as a periodic process of company $a$ suppling 15 units of goods to company $b$ in each cycle. However, if we think about it inversely, it also can be interpreted as company $b$ providing 15 units of cash to company $a$ in each cycle. Based on this example, we can see that a cash flow graph can be obtained by simply reversing the edges in the supply chain graph. A definition of a cash flow graph is provided below.

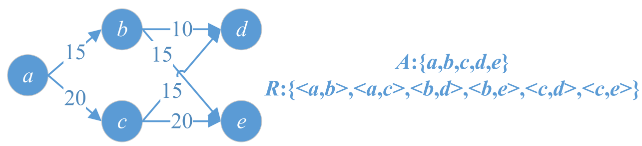


**FIGURE 2.** Simple supply chain network graph that illustrates the structure of a supply chain network.

*Definition 1 (Cash Flow Graph):* For graph $G =< A, R >$, $\forall < i, j > \in R$, the value of edge $< i, j >$ is $V_{ij} = k$, which indicates that there are $k$ units of cash transmitted from $i$ to $j$. Graph $G$ is a cash flow graph.

Figure 3 is a cash flow graph corresponding to Fig. 2. The directed edges in the graph indicate cash flows between

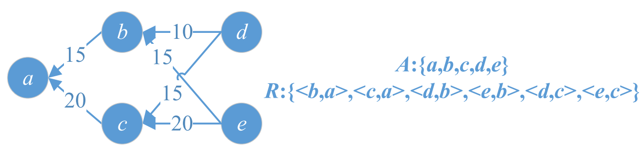


**FIGURE 3.** Simple cash flow graph corresponding to the supply chain network graph in Fig. 2.

companies, and the value on each edge represents the amount of cash flow during a given period between two companies. As an example, edge $< d, b >$ in Fig. 3 indicates there are 10 units of cash flow that occur periodically from $d$ to $b$. Given the notion of the cash flow graph, we describe some aspects of such a graph in the next section.

### *B. NATURE OF A CASH FLOW*

Jiang *et al.* [52] presented an analysis of interaction structures via graph theory. Similarly, in this section, we discuss the nature of cash flow graphs via graph theory. First, it is necessary to define some concepts related to cash flow graphs.

*Definition 2 (Cash Dependence Structure):* For cash flow graph $G =< A, R >, \forall a \in A$, the cash dependence structure of $a$ is the set of all of the incoming edges of $a$ in $G$, which is denoted as $IR_a$. The set $IR_a$ can be formulated as follows:

$$IR_a = \{< k, a > |k, a \in A\wedge < k, a >\in R\} \tag{1}$$

In Fig. 3, nodes $a, b, c, d,$ and $e$ represent companies, and their respective cash dependence structures are $IR_a = \{< b, a >, < c, a >\}$, $IR_b = \{< d, b >, < e, b >\}$, $IR_c = \{< d, c >, < e, c >\}$, $IR_d = \emptyset$, and $IR_e = \emptyset$.

*Definition 3 (Cash Deciding Structure):* For cash flow graph $G =< A, R >, \forall a \in A$, the cash deciding structure of $a$ is the set of all of the outgoing edges of $a$ in $G$, which is denoted as $OR_a$. Similarly, the set $OR_a$ can be formulated as follows:

$$OR_a = \{< a, k > |k, a \in A\wedge < a, k >\in R\} \tag{2}$$

In Fig. 3, we have the following cash deciding structures: $OR_a = \emptyset$, $OR_b = \{< b, a >\}$, $OR_c = \{< c, a >\}$, $OR_d = \{< d, b >, < d, c >\}$, and $OR_e = \{< e, b >, < e, c >\}$.

*Definition 4 (Cash Source Set):* For cash flow graph $G =< A, R >, \forall a \in A$, the cash source set of $a$ is the set of all of the nodes connected by the incoming edges of $a$ in $G$, i.e., the set of all of the nodes linked by the edges in $IR_a$ excluding $a$. The cash source set of $a$ is denoted as $I_a$ and can be formulated as follows:

$$\begin{aligned} I_a &= \{k|k, a \in A\wedge < k, a >\in IR_a\} \\ &= \{k|k, a \in A\wedge < k, a >\in R\} \end{aligned} \tag{3}$$

There are five cash source sets in Fig. 3: $I_a = \{b, c\}$, $I_b = \{d, e\}$, $I_c = \{d, e\}$, $I_d = \emptyset$, and $I_e = \emptyset$.

*Definition 5 (Cash Sink Set):* For cash flow graph $G =< A, R >, \forall a \in A$, the cash sink set of $a$ is the set of all of the nodes connected by the outgoing edges of $a$ in $G$, i.e., the set of all of the nodes linked by the edges in $OR_a$ excluding $a$ itself. The cash sink set of $a$ is denoted as $O_a$ and can be formulated as follows:

$$\begin{aligned} O_a &= \{k|k, a \in A\wedge < a, k >\in OR_a\} \\ &= \{k|k, a \in A\wedge < a, k >\in R\} \end{aligned} \tag{4}$$

In Fig. 3, there are five cash sink sets: $O_a = \emptyset$, $O_b = \{a\}$, $O_c = \{a\}$, $O_d = \{b, c\}$, and $O_e = \{b, c\}$.

*Definition 6 (Interaction Circumstance):* For cash flow graph $G =< A, R >, \forall a \in A$, the interaction circumstance of $a$ is the set of all of the directed edges of $a$ in $G$, namely, the union of $IR_a$ and $OR_a$. $S_a$ is used to denote the interaction circumstance of $a$, and $S_a$ can be formulated as follows:

$$\begin{aligned} S_a = IR_a \cup OR_a = &\{< k, a > |k, a \in A\wedge < k, a >\in R\} \\ &\cup\{< a, k > |k, a \in A\wedge < a, k >\in R\} \end{aligned} \tag{5}$$

According to the definition stated above, the interaction circumstances of nodes $a$, $b$, $c$, $d$, and $e$ in Fig. 3 can be listed as follows: $S_a = \{< b, a >, < c, a >\}$, $S_b = \{< d, b >, < e, b >, < b, a >\}$, $S_c = \{< d, c >, < e, c >, < c, a >\}$, $S_d = \{< d, b >, < d, c >\}$, and $S_e = \{< e, b >, < e, c >\}$.

*Definition 7 (Isomorphic):* For cash flow graph $G =< A, R >, \forall a \in A$, if $\exists b \in A$, which meets the following conditions: $I_a = I_b$, and $O_a = O_b$. We can then say that $a$ is isomorphic with $b$ or that $b$ is isomorphic with $a$.

In Fig. 3, nodes $b$ and $c$ are isomorphic with each other.

From the above seven definitions, presuming that the cash flow graph is a directed acyclic graph (DAG), we can draw the following inferences. Again, based on **Definition 7,** if $a_i$ and $a_j$ are isomorphic with each other, then $a_i$ and $a_j$ must share very similar interaction circumstances.

*Inference 1:* Cash flow graph $G =< A, R >$ is a DAG. $\forall a \in A$, there exists such an inference: $I_a \cap O_a = \emptyset$. Namely, $b \in I_a \Rightarrow b \notin O_a$ and $b \in O_a \Rightarrow b \notin I_a$.

*Proof:*
Assume that $\exists b \in A$, and $b \in I_a \cap O_a$.
Then, according to the following deductive process:
$b \in I_a \Rightarrow< b, a >\in R$, and $b \in O_a \Rightarrow< a, b >\in R$,
the edges $< b, a >$, and $< a, b >$ form a cycle, which is contradictory with the fact that the graph $G$ is a DAG.

So $I_a \cap O_a = \emptyset$; **Inference 1** has been proven.

In addition, considering the definition of isomorphism, we find that if $a$ is isomorphic with $b$, then they have similar significances in the graph. Specifically, we can draw another inference as follows.

*Inference 2:* There is a cash flow graph $G =< A, R >$. $\forall a, b \in A$, if $a$ is isomorphic with $b$, then $< k, a >\in S_a \Leftrightarrow < k, b >\in S_b$, and $< a, k >\in S_a \Leftrightarrow< b, k >\in S_b$.

*Proof:*
Prove $< k, a >\in S_a \Leftrightarrow< k, b >\in S_b$.
$a$ is isomorphic with $b$, so we have $I_a = I_b$.

1. To prove $< k, a >\in S_a \Leftrightarrow< k, b >\in S_b$, that is,
$< k, a >\in S_a \Rightarrow k \in I_a \Rightarrow k \in I_b \Rightarrow< k, b >\in S_b$;
2. To prove $< k, b >\in S_b \Leftrightarrow< k, a >\in S_a$.
$< k, b >\in S_b \Rightarrow k \in I_b \Rightarrow k \in I_a \Rightarrow< k, a >\in S_a$.
Then, we have $< k, a >\in S_a \Leftrightarrow< k, b >\in S_b$.

The process of the other proof ($< a, k >\in S_a \Leftrightarrow< b, k >\in S_b$) is the same as above.

**Inference 2** has been proven.

## V. MULTI-AGENT BANKRUPTCY CONTAGION MODEL BASED ON A CASH FLOW GRAPH

In this section, we propose a multi-agent model for the bankruptcy contagion phenomenon based on cash flow graphs. A multi-agent model consists of agent models and interactive rules between agents. First, we need an agent model to simulate the behavior of a company in a bankruptcy contagion situation. We can use various agent models to simulate different behaviors of companies. Second, we should predefine the interactive rules between agents in a simulation case. These rules let agents know with whom and what they can talk. Finally, we can construct a simulation case by combining agents according to the cash flow graph and presetting the parameters of the agents.

In the following sections, we first explore the influences of a single bankruptcy event. Then, we introduce the agent modeling for the company. Third, we predefine the interactive rules among agents. Finally, we present some inferences drawn from the results obtained with our model.

### A. INFLUENCES OF A SINGLE BANKRUPTCY EVENT

Before trying to construct an agent model for companies in bankruptcy propagation, we must establish how a company will react to bankruptcy events. We first discuss what effects the surrounding companies will experiences when a company is bankrupt.

For company $b$ in a cash flow graph, there are two types of relative locations of the surrounding companies. In Fig. 4, companies $a$ and $c$ illustrate the two types of locations relative to company $b$. According to the definitions proposed in Section IV, company $a$ is a cash sink set of company $b$ ($O_b$) and company $c$ is a cash source set of company $b$ ($I_b$). The union of $O_b$ and $I_b$ is the set of all companies surrounding company $b$. The question we want to answer in this section can be expressed as follows. For any company $b$, a company $a$ in $O_b$ and company $c$ in $I_b$ exist, as Fig. 4 shows. We seek to determine what effects company $a$ and $c$ will experience when company $b$ is bankrupt.

- With respect to company $c$: company $c$ is one element of the cash source set of company $b$, namely, $c \in I_b$, and $V_{cb} = Y$. When company $b$ is bankrupt, company $c$ does not need to provide cash to $b$ any more. This situation sounds advantageous for company $c$, but if we consider it from a realistic perspective, we find that it means that the normal trading activities of company $c$ will be affected by the bankruptcy of company $b$. The cash situation of company $c$ will then experience a negative impact.
  Suppose that there is a reverse influence factor $\alpha$ and that $0 < \alpha < 1$. When company $b$ is bankrupt at time $t$, company $c$ will suffer a reduction in cash at time $t+1$. We use $\Delta C_c$ to denote the amplitude of the reduction and $C_c^t$ to denote the cash of company $c$ at time $t$. Their formulas are as follows:
  $$\Delta C_c = V_{cb} \cdot \alpha \tag{6}$$
  $$C_c^{t+1} = C_c^t - \Delta C_c = C_c^t - V_{cb} \cdot \alpha \tag{7}$$
  Equation (6) describes how much company $c$ suffers when company $b$ is bankrupt, and Eq. (7) shows the process of the cash of company $c$ decreasing.
- With respect to company $a$: company $a$ is one element of the cash sink set of company $b$, namely, $a \in O_b$. Once company $b$ is bankrupt, company $a$ will face a lack of $X$ units of cash per period. Assuming that this lack will be fully reflected in the cash reduction of the company, the cash of company $a$ will suffer a reduction because of the bankruptcy of company $b$.
  As a member of the cash sink set of company $b$, the consequence that company $a$ suffers can be computed using the following formulas:
  $$\Delta C_a = V_{ba} \tag{8}$$
  $$C_a^{t+1} = C_a^t - \Delta C_a = C_a^t - V_{ba} \tag{9}$$
  Equation (9) is similar to Eq. (7); they express the cash-decreasing behaviors of companies $a$ and $c$, respectively. We have thus analyzed the effects that the surrounding companies will experience when company $b$ is bankrupt.

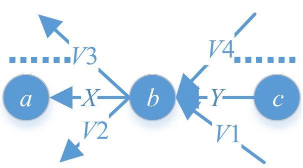


**FIGURE 4. Two types of surrounding companies ($a$, $c$) with different locations relative to $b$.**

In summary, company $b$ will have negative effects on its surrounding companies when it goes bankrupt. These negative effects are manifested as cash reductions for the other companies. We also need to clarify the bankruptcy conditions of the companies.

It is common that a company will become bankrupt when its cash is reduced to a particular extent. Here, we refer to this particular extent as the bankruptcy threshold, denoted by an upper letter, $U$. The following hypothesis applies to the bankruptcy threshold $U$.

*Hypothesis 1:* Company $x$ is bankrupt when its amount of cash is less than $U$, namely $C_x < U$; otherwise, company $x$ is in a healthy state.

When a company experiences some losses in cash but is still in a healthy state, the company has the ability to recover from the effects of bankruptcy. Its recovery ability is represented by emergent behaviors of the company, such as sales, staff reduction and borrowing of money. We need to determine the extent to which recovery is possible, which can be formalized as described below.

*Hypothesis 2:* For any company $x$, at each cycle, if the cash of company $x$ is not less than $U$ but is less than the original cash of company $x$ ($C_{x\_orin}$), namely,

$(C_x \geq U) \wedge (C_x < C_{x\_orin})$, then the cash of $x$ will recover autonomously. Its increment formula is as follows:

$$C_x^{t+1} = \begin{cases} C_x^t + \left(C_{x\_orin} - C_x^t\right)/\theta, & \left(C_{x\_orin} - C_x^t\right) > T \\ C_x^t + T, & \left(C_{x\_orin} - C_x^t\right) \leq T \end{cases} \tag{10}$$

In this equation, parameter $\theta$ will influence the recovery speed of $C_x$. The smaller the value of $\theta$, the faster the recovery. The parameter $T$ is a constant for company $x$, referring to the minimal value of recovery in each cycle.

### B. AGENT MODELING OF COMPANIES

Given the great similarity between the diffusion process and a multi-agent system, we can view the bankruptcy contagion phenomenon as the complex interactive behaviors of a group of agents, if we use agents to simulate real companies. In this section, we focus on how to model an agent for a company in a bankruptcy contagion.

Basole and Bellamy [53] proposed a bankruptcy contagion model that considers the network visibility, health state, and surrounding states. Learning form this model, we put forward a simplified agent model here. Given the hypotheses proposed in Section V.A, we can use a triplet $< C, R, G >$ to represent the agent model:

- $C$, which represents the current cash of the agent, is an inherent state value. When $C < U$, the agent is bankrupt; otherwise, the agent is in a healthy state. Every agent has an initial amount of cash, which can be the same amount that another agent has or a different amount. As long the agent does not encounter any bankruptcy event, the cash of the agent stays the same all the time.
- $R$, which represents the reactive rules to bankruptcy events of an agent, is a series of reactive behaviors corresponding to bankruptcy events in the outside world. Agents will perform the following actions sequentially.
  - *For every bankruptcy event, the agent will diminish its cash according to its location relative to the source of the bankruptcy event. For the two types of relative locations, Eqs. (6)-(9) are used to compute the influences.*
  - *The agent detects its own current cash and checks whether the current cash is smaller than U. If so, then the state of the agent is changed to a bankrupt state; otherwise, the agent is kept in a healthy state.*
- $G$, which represents a set of handling rules when an agent is bankrupt, is a series of actions. In our model, when the state of an agent is changed to a bankrupt state, it will take the following actions in sequence.
  - *The agent will send a bankruptcy message to every agent in its cash source set and cash sink set.*
  - *The agent will remove the corresponding node and its interaction circumstance from the cash flow graph.*

The above triplet is a literal description of our agent model of companies in bankruptcy propagations. We next discuss agent structures from the perspective of software engineering. Referring to the idea of componentization in agent modeling [54], we have the structure design shown in Fig. 5.

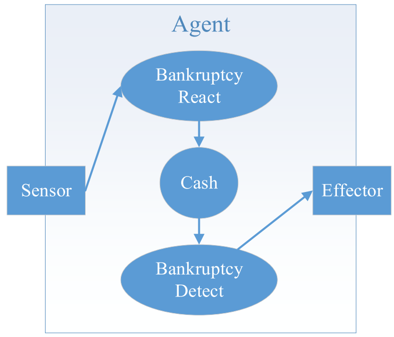


**FIGURE 5. Structure of our agent model, based on the idea of componentization. An agent has three components in total, in addition to the sensor and effector.**

Figure 5 shows the complete working process of an agent. When a bankruptcy event occurs around the agent, the sensor of the agent receives the bankruptcy event and delivers the message to the "Bankruptcy React" component. The "Bankruptcy React" component computes the exact influence value according to the bankruptcy message and delivers the result to the "Cash" component. Next, the "Cash" component diminishes the result from the current cash of the agent. Afterward, the "Bankruptcy Detect" component runs automatically. It checks whether the current cash of the agent is less than $U$ to determine whether the agent is bankrupt. Then, the "Bankruptcy Detect" component delivers the result to the effector. Finally, the effector takes actions according to the result received from the "Bankruptcy Detect" component, namely, the current health state of the agent. If the agent is in a bankrupt state, the effector will perform corresponding bankruptcy actions; otherwise, the effector will not take any action. The concrete responsibilities of each component are described below.

The "Bankruptcy React" component is responsible for reacting to the bankruptcy event and handling it. In our model, the component computes the exact value of influence of a bankruptcy event. The exact procedures are shown in Fig. 6.

The "Cash" component has an attribute that refers to the current cash of the agent. The "Cash" component has the following two responsibilities directly related to cash: (1) the actual operation of the current cash of the agent, such as addition, subtraction, reading, or writing of the current cash and (2) the automatic recovery of the cash at each cycle, which conforms to the particular rule of ***Hypothesis 2***. Equation (10) is a formula expressing this rule. Figure 7 is a flowchart of the automatic recovery function, demonstrating the process flow of the "Cash" component in the automatic recovery function.

The "Bankruptcy Detect" component is responsible for judging whether the agent is bankrupt at the end of each cycle, so the "Bankruptcy Detect" component will run at each

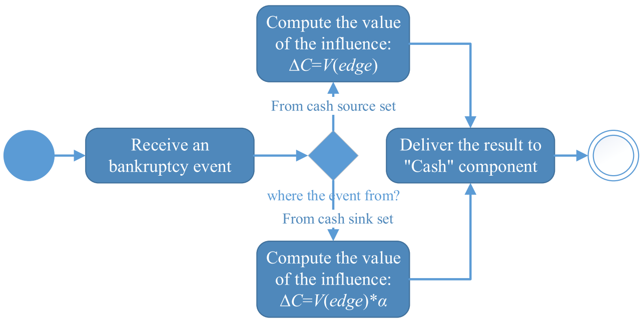


**FIGURE 6.** Flowchart of the "Bankruptcy React" component, describing the process flow of the component.

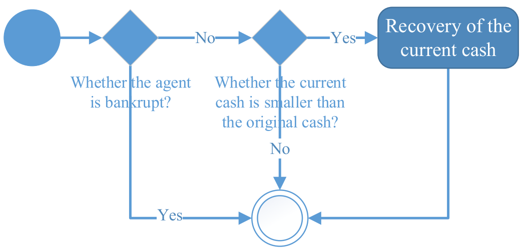


**FIGURE 7.** Flowchart of the automatic recovery function in the "Cash" component.

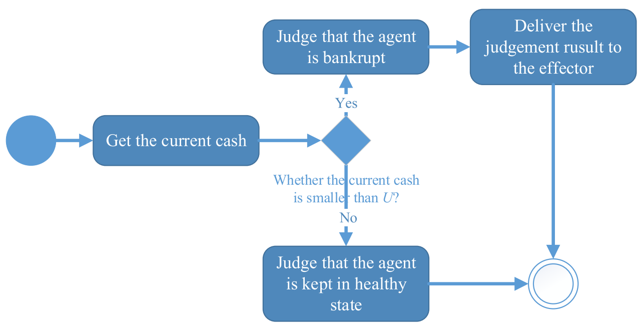


**FIGURE 8.** Flowchart of the bankruptcy detecting function in the "Bankruptcy Detect" component.

cycle and may modify the state of the agent according to the current cash situation. Referring to ***Hypothesis 1*** mentioned in Section V.A, the component needs to compare the current cash with the bankruptcy threshold $U$ to determine whether the agent is bankrupt. Figure 8 is a flowchart of the process flow of the bankruptcy detection function in the component.

### C. INTERACTION RULES OF AGENTS

After elaborating an agent model, the only step remaining is to predefine a set of interactive rules for agents. In this section, we describe a complete multi-agent system based on the cash flow graph. At the end of this section, we present a simple example of bankruptcy propagation using our model.

From a multi-agent perspective, the bankruptcy propagation phenomenon can be understood as a consequence of the interactions within a group of agents. The interaction process determines the final result of bankruptcy propagation, i.e., the interactive rules affect how bankruptcy propagation occurs.

In our model, we suppose that a bankruptcy message can be diffused only by means of particular relations between agents. The bankruptcy message must be transferred according the cash flow relation, regardless of what order it is in. If we use a cash flow graph to record the cash flow relations between agents, as mentioned before, then the rules of the interactions between agents in our multi-agent model can be described as follows: when an agent is bankrupt, the agent can deliver its bankruptcy messages to agents in its cash source set or cash sink set only by way of directed edges connected to itself, regardless of the directions of the edges. In another formalized expression, the rules of the interactions between agents can be predefined as described below:

*Use a cash flow graph $G =< A, R >$ to represent the cash flow relations between agents. Then, $\forall$ agent $x \in A$, if there is a bankruptcy event occurring in agent x, agent x can only transfer its bankruptcy messages along directed edge $< x, d >$ in forward order or directed edge $< u, x >$ in reverse order. Among them, d and u are an agents in set A, whereas the directed edges $< x, d >$ and $< u, x >$ are in set R.*

The agent model mentioned in Section V.B can tell us what an agent should do in bankruptcy propagation. The interactive rules discussed in this section tell us to whom an agent should deliver its message. We are thus able to obtain a complete multi-agent model for bankruptcy propagations. To clarify our bankruptcy propagation model further, a simple example of bankruptcy propagation is given as follows.

Figure 9 shows a bankruptcy event occurring for agent $x$ at time 0. We initially set the state of agent $x$ to bankrupt to initiate the bankruptcy propagation.

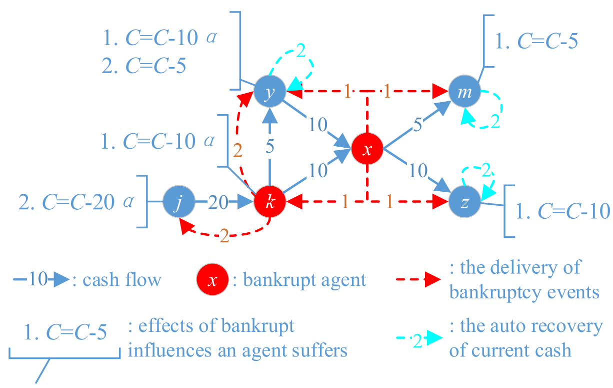


**FIGURE 9.** Simple example of bankruptcy propagation using our model.

The first diffusion process occurs during period 1. Agents $m$, $z$, $y$, and $k$ will receive a bankruptcy message from agent $x$, and the current cash of each of them will be decreased simultaneously. On the one hand, as elements in the cash sink set of agent $x$, the reduction values of the current cash of agents $m$ and $z$ are $V_{xm} = 5$ and $V_{xz} = 10$, respectively. At this point, the current cash of agents $m$ and $z$ is still greater than bankruptcy threshold $U$, so neither of them will be bankrupt. On the other hand, as elements of the cash source set of agent $x$, the current cash of agents $y$ and $k$ is reduced by $V_{yx} \cdot \alpha = 10\alpha$ and $V_{kx} \cdot \alpha = 10\alpha$, respectively. At this point, the current cash of agent $y$ is greater than $U$

as well, so agent $y$ is still in a healthy state, whereas the current cash of agent $k$ is less than $U$, which results in the occurrence of a bankruptcy event for agent $k$ at time 1. This event is the second propagation of bankruptcy. Because of the bankruptcy of agent $k$, agent $k$ will execute its bankruptcy actions next. Because agent $x$ was already bankrupt, agent $k$ can only diffuse its bankruptcy message along directed edges $< k, y >$ and $< j, k >$.

At time 2, agents $y$ and $j$ receive the bankruptcy message from agent $k$, resulting in reductions in their current cash. Simultaneously, all the agents in a healthy state recover automatically. The exact value of the recovery can be computed by Eq. (10), given the values of $\theta$ and $T$. As one element in the cash sink set of agent $k$, the current cash of agent $y$ is reduced by $V_{ky} = 5$. At this point, the current cash of agent $y$ is still greater than $U$, so agent $y$ is not bankrupt. On the other hand, as one element in the cash source set of agent $k$, the current cash of agent $j$ is reduced by $V_{jk} \cdot \alpha = 20\alpha$. At the same moment, the current cash of agent $j$ is greater than bankruptcy threshold $U$ as well. Consequently, agent $j$ remains in a healthy state. Because no other bankruptcy event occurs, the total process of bankruptcy propagation is terminated.

### D. INFERENCES BASED ON OUR MULTI-AGENT BANKRUPTCY CONTAGION MODEL

Now we have a complete multi-agent model for bankruptcy propagation. Furthermore, we can simulate bankruptcy propagation by implementing this model. Even without implementing the model, we can draw the following inferences based on the proposed model and the nature of the cash flow graph.

- In the bankruptcy propagation process, the more sources of bankruptcy propagation, the larger the influence scale of bankruptcy propagation. In our proposed model, for any agent $a$, the scope that agent $a$ can influence contains all the agents involved in the interaction circumstance of agent $a$, i.e., all the nodes connected with agent $a$ in the cash flow graph. So, the more diffusion sources of bankruptcy, the larger the sphere of influence of the first bankruptcy propagation process. Obviously, this characteristic will increase the scale of bankruptcy propagation influences. On the other hand, the bankruptcy probability of an agent, as well the number of diffusion sources, will become greater because an agent may encounter multiple bankruptcy events at one time. This behavior can also increase the scale of bankruptcy propagation effects.
- The influence scale of bankruptcy propagations becomes larger when the value of parameter $\alpha$ increases. In our model, for any agent $x$, $\exists$ agent $i \in O_x$, and the value of edge $< x, i >$ is denoted as $V_{xi}$. Then, if agent $i$ is bankruptcy, the value describing the influence agent $x$ suffers is $\Delta C_x = V_{xi} \cdot \alpha$. From this formula, we can see that the bankruptcy possibility of agent $x$ increases as the value of the parameter $\alpha$ increases. Consequently, the final influence scale of bankruptcy propagation will become larger.
- For any agent $a$, the greater the initial cash of agent $a$, the lower the probability of bankruptcy of agent $a$. In our model, the effect of influence when agent $a$ receives a bankruptcy message is a reduction in the current cash of agent $a$. It is more difficult to make the current cash reach a level lower than the bankruptcy threshold $U$ when the initial cash of agent $a$ is higher. This is the same as the normal phenomenon of a more solid company with more assets being less likely to go bankrupt.
- The greater the value of parameter $t$ is, the larger the final influence scale of the bankruptcy propagation is. From Eq. (9), in which the parameter $t$ appears, we find that the recovery speed of cash increases when the value of the parameter $t$ increases. A faster recovery speed will contribute to reducing the bankruptcy probability of an agent. As a consequence, the final influence scale of the bankruptcy propagation will become smaller.
- The final influence scale of bankruptcy propagation becomes larger as the value of the parameter $k$ increases. From Eq. (9), in which the parameter $k$ appears, we find that the parameter $k$ can affect the recovery speed of an agent. The smaller the value of the parameter $k$ is, the greater the recovery speed is. As with the parameter $t$, when the value of the parameter $k$ decreases, the recovery speed of the current cash increases, which leads to a smaller influence scale of the bankruptcy propagation. In contrast, the final influence scale of a bankruptcy propagation will get larger when the value of the parameter $k$ increases.

The above five inferences were all deduced from our model and hypotheses mentioned before through some simple reasoning processes, but without the verifications of facts or experiments. Such verifications were beyond the scope of this study, but we will conduct some experiments to verify these inferences in our future research efforts.

## VI. CASE STUDY AND EXPERIMENTS

In this section, we describe the implementation of the multi-agent-based bankruptcy contagion model proposed above in the Java language, using JDK1.6. A series of simulation experiments considering 500 agents were conducted to analyze the bankruptcy contagion process using the model implemented as described.

Because of the difficulty of obtaining statistics on financial interactions and supply relationships between firms, we had to make some simplifications to perform the simulation experiments. First, by considering the rules resulting from the financial statistics in the 2014 Annual Reports of the Beiqi Foton Motor Co., Ltd.,[1] the Gujing Group Co., Ltd.,[2] and

[1] http://static.sse.com.cn/disclosure/listedinfo/announcement/c/2015-08-31/600166_2015_z.pdf

[2] http://disclosure.szse.cn/finalpage/2015-04-29/1200934004.PDF

the Quanchai Engine Co., Ltd,[3] we constructed the cash flow graph. The model parameters were set as follows. For any Agent $a$:

- The bankruptcy threshold $U_a = 0.3\ C_{orin}$, where $C_{orin}$ is the original cash of agent $a$;
- The reverse influence factor $\alpha_a = 0.5$;
- The minimal recovery value at each cycle $t_a = 2$;
- The recovery speed of cash $k_a = 10$.

In addition, the following three indicators were used to measure different contagions in our simulation experiments:

- $BAR^{end}$, the final bankruptcy ratio in a simulation. In a bankruptcy contagion simulation, the bankruptcy ratio at time $t$ can be calculated using the following formula:
$$BAR^t = NB^t/NA \tag{11}$$
where $NB^t$ indicates the number of bankruptcy agents at time $t$ and $NA$ is the total number of agents, which was 500 in the simulation described here. Thus, $BAR^{end}$ can be calculated as $BAR^{end} = NB^{end}/500$, where "$end$" is used to represent the end time of the simulation.
- $\Delta BAR$, the bankruptcy ratio increment from the start time of the simulation. This quantity is a supplementary indicator related to the indicator $BAR^{end}$, which is useful in situations with high initial bankruptcy ratios. $\Delta BAR$ can be calculated via the formula below:
$$\Delta BAR = BAR^{end} - BAR^{start} \tag{12}$$
where "$start$" represents the start time of a simulation, namely, time 0, and $BAR^{start}$ is the initial bankruptcy ratio preset for the simulation.
- $FOB_{target}$, the bankruptcy frequency of the target agent, is a statistical indicator whose values are determined from the results of $m$ simulations. To determine this value, we first select an agent as the observation target. Then, the simulation is performed $m$ times in total. If in these simulations, the target agent becomes bankrupt $n$ times in total, then we obtain $FOB_{target} = n/m$.

### A. EFFECTS OF DIFFERENT INITIAL BANKRUPTCY RATIOS

The initial bankruptcy ratio is the ratio of the agents we preset to bankruptcy for the purpose of simulating bankruptcy contagions. Figure 10 shows the simulation results for the indicator $\Delta BAR$ with increasing initial bankruptcy ratio from 0 to 1 in increments of 0.01. Four groups of simulation results with different bankruptcy threshold conditions are presented together in Fig. 10:

- $u = 0.1c$. In these simulations, for any agent $a$, its bankruptcy threshold $U_a$ is set to ($0.1\ C_{orin}$), that is, $U_a = 0.1\ C_{orin}$;
- $u = 0.3c$. In these simulations, for any agent $a$, its bankruptcy threshold $U_a = 0.3\ C_{orin}$;
- $u = 0.5c$. In these simulations, for any agent $a$, namely, $U_a = 0.5\ C_{orin}$;
- $u = 0.7c$. In these simulations, for any agent $a$, $U_a = 0.7\ C_{orin}$.

[3]http://static.sse.com.cn/disclosure/listedinfo/announcement/c/2015-03-19/600218_2014_n.pdf

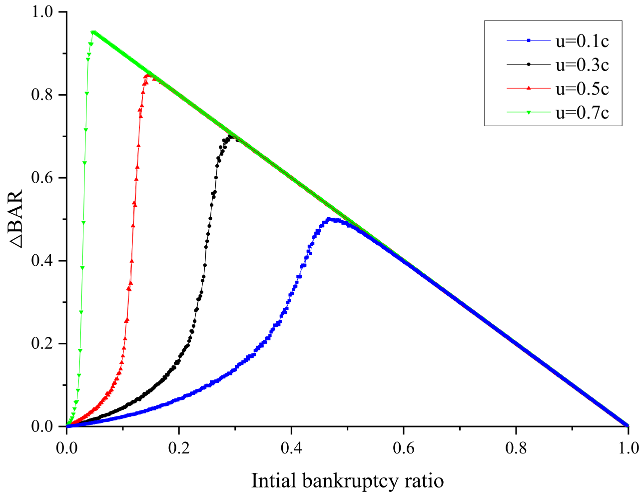


**FIGURE 10. Effects of the initial bankruptcy ratio under different bankruptcy threshold conditions.**

The four curves in Fig. 10 share a common variation trend. Based on this trend, the curves can each be divided into two parts.

- In the first part, as the initial bankruptcy ratio increases, $\Delta BAR$ increases first and then decreases after the peak;
- In the second part, $\Delta BAR$ decreases linearly with decreasing initial bankruptcy ratio, which can be fitted to the function $y = 1 - x$. Because $\Delta BAR = BAR^{end} - BAR^{start} = BAR^{end} - x$, in the second part, $BAR^{end}$ is always 1, which means that all the agents in the simulation are bankrupt. This phenomenon can only be observed in the experimental environment because of the limit on the number of agents considered, so we can ignore the second part of the curves.

The above common trend of the curves indicates that in a real bankruptcy contagion environment, which would have no limit on the number of individuals involved, the influence scale of a bankruptcy contagion will always increase as the initial bankruptcy ratio increases.

On the other hand, in comparing the four curves and concentrating on their differences, we find that, under the same conditions, the lower the bankruptcy threshold $U$, the smaller the corresponding value of the indicator $\Delta BAR$, indicating a smaller influence scale of the bankruptcy contagion.

Moreover, the trend of the four curves in Fig. 10 is very similar to the classical SIR (Susceptible-Infected-Removed) propagation mechanism. In SIR model, we use $s, i, r$ to represent the amount of three groups of persons, that is, $S$ (Susceptible), $I$ (Infected), and R (Removed). Then the derivative of $s$ (i.e., $s$') is shown as $s' = -s \cdot i$. Assume the total number of persons is $N$, then we have $s = N - i - r$. Take it to the above formula, got: $s' = i^2 - N \cdot i - r \cdot i$. From the above formula, we can see that, as $i$ increases, $s$' will first decrease from 0 and then increase to 0. Therefore, as $i$ increases, the decrease in the number of susceptible people ($s$) per unit time will first increase and then decrease, just like the curves' changing trend shown in Fig. 10.

### B. EFFECTS OF THE REVERSE INFLUENCE FACTOR

The reverse influence factor $\alpha$ indicates the decay factor of the bankruptcy influence in a reverse direction. Figure 11 shows the simulation results obtained by increasing the value of $\alpha$ step by step. There are four groups of results with different bankruptcy threshold parameters in the figure.

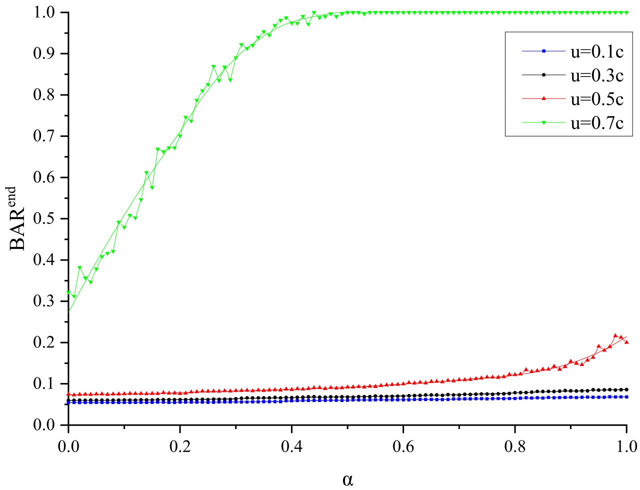


**FIGURE 11. Effects of the reverse influence factor $\alpha$ under different bankruptcy threshold conditions.**

The four curves all share a common variation trend: as the reverse influence factor $\alpha$ increases, the indicator $BAR^{end}$ increases, which indicates a larger influence scale of the bankruptcy contagion. This result is consistent with our expectation.

As Fig. 11 shows, with an increase in the bankruptcy threshold, which means that it is easier to bankrupt an agent, the effects of the influence factor $\alpha$ become more remarkable. This characteristic indicates that only when the bankruptcy threshold is relatively large can the reverse influence factor $\alpha$ have a significant effect on a bankruptcy contagion.

### C. EFFECTS OF THE ORIGINAL CASH OF AGENTS

It is common sense that a firm with more liquid cash will be more difficult to bankrupt in a bankruptcy contagion situation, i.e., it will be more able to resist the effects of the bankruptcy contagion. In the next experiment, we selected a target agent and set various different values for its original cash. The indicator $FOB_{target}$ was used to measure the degree of difficulty of bankrupting the target agent.

Figure 12 shows the simulation results. We can see from the figure that the trends of the curves are consistent with common sense, except for the results when the bankruptcy threshold is $U = 0.7\ C_{orin}$.

The common trend shown is that as the original cash increases, the indicator $FOB_{target}$ decreases continuously until it reaches 0. Thus, an increase in liquid cash will enhance the ability of an agent to resist the impacts of bankruptcy.

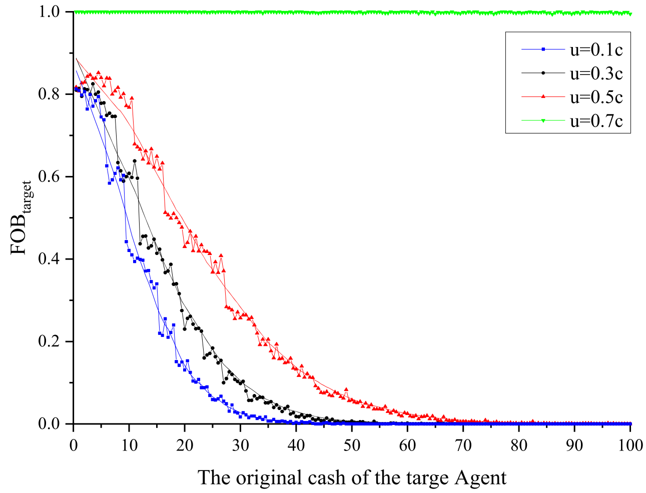


**FIGURE 12. Effects of the original cash under different bankruptcy threshold conditions.**

Second, we focus on the curve for $u = 0.7c$, which means that the bankruptcy threshold parameter is $U = 0.7C_{orin}$. This is a horizontal straight line that can be modeled as $y = 1$, which means that the target agent will definitely go bankrupt under the condition of such a high bankruptcy threshold. Thus, in a situation with high bankruptcy threshold, having more liquid cash does little to assist an agent in resisting the impacts of bankruptcy contagions.

### D. EFFECTS OF MINIMAL RECOVERY OF THE AGENT

Usually, a firm that suffers from the influence of a bankruptcy event but does not go fully bankrupt will recover from this disaster by itself. That is, a firm always has a self-recovery ability. In our model, we use two parameters to control the strength of the self-recovery abilities of agents: the minimal recovery value $t$ and recovery speed $k$ of an agent. The minimal recovery value of an agent is the minimal value of cash that an agent can recover in a time step. Fig. 13 shows the effects of different minimal recovery values on the

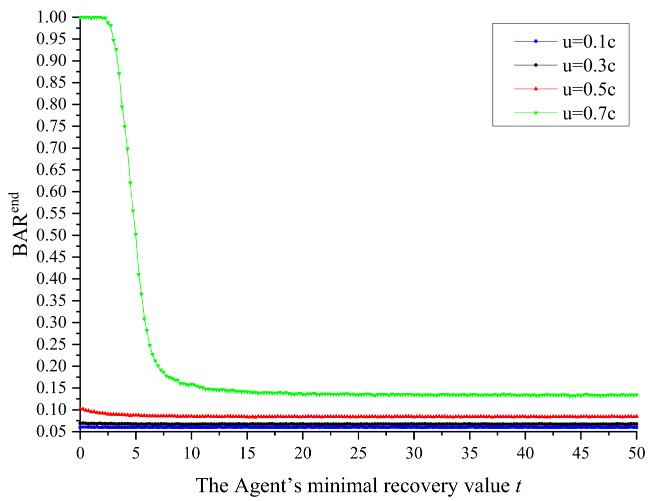


**FIGURE 13. Effects of $t$ under different bankruptcy threshold conditions.**

indicator $BAR^{end}$, which indicate the influence scale of a bankruptcy contagion simulation.

From the figure, we can see that, in general, the greater $t$, the smaller $BAR^{end}$. Thus, a larger $t$ can help prevent bankruptcy. On the other hand, in Fig. 13, the curves all exhibit little change, except the green curve for $u = 0.7c$, which corresponds to a bankruptcy threshold of $U = 0.7\ C_{orin}$. These results show that changes in $t$ do not significantly affect the progress of bankruptcy contagions, unless the bankruptcy threshold is very high.

### E. EFFECTS OF THE CASH RECOVERY SPEED OF THE AGENT

In our model, $k$ is a positive parameter that describes the recovery speed of an agent. The smaller $k$, the faster the recovery, and the greater power of self-recovery of an agent. Fig. 14 demonstrates the effects of the agent recovery speed on the indicator $BAR^{end}$. We can see effects similar to those for the minimal recovery value of the agent.

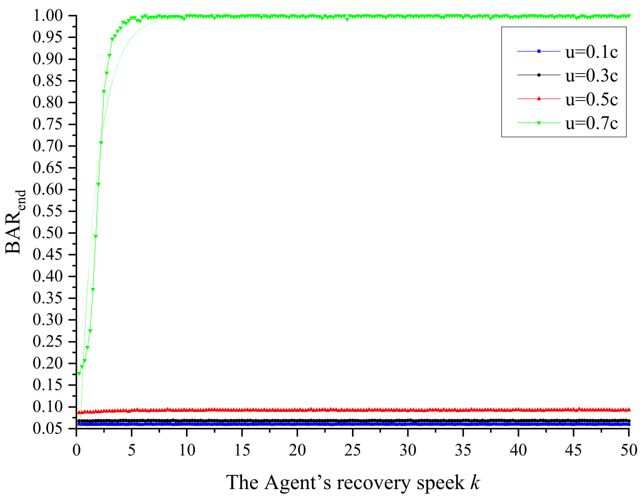


**FIGURE 14. Effects of $k$ under different bankruptcy threshold conditions.**

All of the curves are almost straight lines except the green curve for $u = 0.7c$, which corresponds to a bankruptcy threshold of $U = 0.7\ C_{orin}$. Thus, $k$ only has a slight effect on the bankruptcy contagions unless the bankruptcy threshold is very high. In other words, the self-recovery ability of an agent has little effect on preventing bankruptcy in general.

We observe the common trend of the curves and conclude that with increasing $k$, $BAR^{end}$ increases as well, indicating a greater scale of influence of the bankruptcy contagion.

## VII. CONCLUSION AND FUTURE WORK

In this article, we presented a new common relationship called the cash flow relationship, which is more essential than other relationships, such as the supply relationship, that are usually considered in multi-agent models of bankruptcy contagion. We also presented a graphical structure called a cash flow graph to record cash flow relations between financial institutions and provide a discussion of the nature of the cash flow graph. We concerned ourselves mainly with the effects of cash flow and considered relations between agents in a structural way, then presented our model from a software engineering perspective to illustrate the considerations involved in implementing the model in a software application. Furthermore, the model was constructed using componentization to simplifying the subsequent implementation process.

Compared with previous models, our model can be applied in more circumstances, because of its underlying relationship, cash flow relationship, which is a more essential relationship than considered before, such as supply relationship. Meanwhile, there are some defects in our model as well. First, although the formulas proposed are considered the linear effects of cash flows, the model is only a rough measure. More quantitative data from experiments results or a real-life case are needed to improve the scalability of the model. Second, although referring to some real financial statistics in the experiments, we don't offer a method to model a cash flow graph from a real scene, and how to calculate an accurate value for cash flow edge is still a question now.

In the future, we will endeavor to expand this research effort in two directions. On the one hand, we will consider more factors in our agent model to complicate our formulas. On the other hand, we will work to find out a practicable method to model a cash flow graph from real companies' scene accurately, in order to apply our model in the real market circumstances.

## REFERENCES

[1] J. D. Farmer and D. Foley, "The economy needs agent-based modelling," *Nature*, vol. 460, no. 7256, pp. 685–686, Aug. 2009, doi: 10.1038/460685a.

[2] M. Wooldridge and N. R. Jennings, "Intelligent agents: Theory and practice," *Knowl. Eng. Rev.*, vol. 10, no. 2, pp. 115–152, Jun. 1995, doi: 10.1017/S0269888900008122.

[3] W. S. Frame and L. J. White, "Technological change, financial innovation, and diffusion in banking," Federal Reserve Bank Atlanta, Atlanta, GA, USA, Working Paper 2451/33549, Mar. 2009. [Online]. Available: http://ssrn.com/abstract=2380060

[4] M. M. Croce, P. Farroni, and I. Wolfskeil, "When the markets get COVID: Contagion, viruses, and information diffusion," CEPR, Washington, DC, USA, CEPR Discuss. Paper DP14674, Apr. 2020. [Online]. Available: https://ssrn.com/abstract=3594307

[5] L. Yarovaya, J. Brzeszczynski, J. W. Goodell, B. M. Lucey, and C. K. Lau. (May 16, 2020). *Rethinking Financial Contagion: Information Transmission Mechanism During the COVID-19 Pandemic*. [Online]. Available: https://ssrn.com/abstract=3602973, doi: 10.2139/ssrn.3602973.

[6] N. Kiyotaki and J. Moore, "Balance-sheet contagion," *Amer. Econ. Rev.*, vol. 92, no. 2, pp. 46–50, Apr. 2002, doi: 10.1257/000282802320188989.

[7] C. Raddatz, "Credit chains and sectoral comovement: Does the use of trade credit amplify sectoral shocks?" *Rev. Econ. Statist.*, vol. 92, no. 4, pp. 985–1003, Nov. 2010, doi: 10.1162/rest_a_00042.

[8] D. Acemoglu, V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi, "The network origins of aggregate fluctuations," *Econometrica*, vol. 80, no. 5, pp. 1977–2016, 2012, doi: 10.3982/ECTA9623.

[9] T. Jacobson and E. von Schedvin, "Trade credit and the propagation of corporate failure: An empirical analysis," *Econometrica*, vol. 83, no. 4, pp. 1315–1371, 2015, doi: 10.3982/ECTA12148.

[10] L. Morales and B. Andreosso-O'Callaghan, "The global financial crisis: World market or regional contagion effects?" *Int. Rev. Econ. Finance*, vol. 29, pp. 108–131, Jan. 2014, doi: 10.1016/j.iref.2013.05.010.

[11] W. Ahmad, S. Sehgal, and N. R. Bhanumurthy, "Eurozone crisis and BRIICKS stock markets: Contagion or market interdependence?" *Econ. Model.*, vol. 33, pp. 209–225, Jul. 2013, doi: 10.1016/j.econmod.2013.04.009.

[12] K.-H. Bae, G. A. Karolyi, and R. M. Stulz, "A new approach to measuring financial contagion," *Rev. Financial Stud.*, vol. 16, no. 3, pp. 717–763, Jul. 2003, doi: 10.1093/rfs/hhg012.

[13] R. Iyer and J.-L. Peydró, "Interbank contagion at work: Evidence from a natural experiment," *Rev. Financial Stud.*, vol. 24, no. 4, pp. 1337–1377, Apr. 2011, doi: 10.1093/rfs/hhp105.

[14] W. Mohti, A. Dionísio, I. Vieira, and P. Ferreira, "Financial contagion analysis in frontier markets: Evidence from the US subprime and the eurozone debt crises," *Phys. A, Stat. Mech. Appl.*, vol. 525, pp. 1388–1398, Jul. 2019, doi: 10.1016/j.physa.2019.03.094.

[15] R. Handika, G. Soepriyanto, and S. A. H. Havidz, "Are cryptocurrencies contagious to asian financial markets?" *Res. Int. Bus. Finance*, vol. 50, pp. 416–429, Dec. 2019, doi: 10.1016/j.ribaf.2019.06.007.

[16] M. Dungey, T. J. Flavin, and D. Lagoa-Varela, "Are banking shocks contagious? Evidence from the eurozone," *J. Banking Finance*, vol. 112, Mar. 2020, Art. no. 105386, doi: 10.1016/j.jbankfin.2018.07.010.

[17] Z. Zhao, H. Wen, and K. Li, "Identifying bubbles and the contagion effect between oil and stock markets: New evidence from China," *Econ. Model.*, to published, doi: 10.1016/j.econmod.2020.02.018.

[18] E. Dweck, M. T. Vianna, and A. da Cruz Barbosa, "Discussing the role of fiscal policy in a demand-led agent-based growth model," *Economia*, vol. 21, no. 2, pp. 185–208, May 2020, doi: 10.1016/j.econ.2019.03.004.

[19] J. Lussange, S. Bourgeois-Gironde, S. Palminteri, and B. Gutkin, "Stock price formation: Useful insights from a multi-agent reinforcement learning model," 2019, *arXiv:1910.05137*. [Online]. Available: http://arxiv.org/abs/1910.05137

[20] J. Liu and J. Dong, "A multi-agent simulation of investment choice in the P2P lending market with bankruptcy risk," *J. Simul.*, to be published, doi: 10.1080/17477778.2020.1759386.

[21] M. Hernes, M. Maleszka, N. T. Nguyen, and A. Bytniewski, "A model of a multiagent early warning system for crisis situations in economy," in *Proc. Int. Conf. Comput. Collective Intell. Technol. Appl. (ICCCI)*, 2015, pp. 47–56, doi: 10.1007/978-3-319-24069-5_5.

[22] T. Fischer and J. Riedler, "Prices, debt and market structure in an agent-based model of the financial market," *J. Econ. Dyn. Control*, vol. 48, pp. 95–120, Nov. 2014, doi: 10.1016/j.jedc.2014.08.013.

[23] J. Zhang, L. Cheng, and H. Wang, "A multi-agent-based decision support system for bankruptcy contagion effects," *Expert Syst. Appl.*, vol. 39, no. 5, pp. 5920–5934, Apr. 2012, doi: 10.1016/j.eswa.2011.11.112.

[24] R. Grilli, G. Tedeschi, and M. Gallegati, "Bank interlinkages and macroeconomic stability," *Int. Rev. Econ. Finance*, vol. 34, pp. 72–88, Nov. 2014, doi: 10.1016/j.iref.2014.07.002.

[25] C.-P. Georg, "The effect of the interbank network structure on contagion and common shocks," *J. Banking Finance*, vol. 37, no. 7, pp. 2216–2228, Jul. 2013, doi: 10.1016/j.jbankfin.2013.02.032.

[26] R. B. Myerson, "Optimal auction design," *Math. Oper. Res.*, vol. 6, no. 1, pp. 58–73, Feb. 1981, doi: 10.1287/moor.6.1.58.

[27] A. Pavan, I. Segal, and J. Toikka, "Dynamic mechanism design: A myersonian approach," *Econometrica*, vol. 82, no. 2, pp. 601–653, 2014, doi: 10.3982/ECTA10269.

[28] S. Liu, Q. Hu, and Y. Xu, "Optimal inventory control with fixed ordering cost for selling by Internet auctions," *J. Ind. Manage. Optim.*, vol. 8, no. 1, pp. 19–40, 2012, doi: 10.3934/jimo.2012.8.19.

[29] D. R. Karger, S. Oh, and D. Shah, "Budget-optimal crowdsourcing using low-rank matrix approximations," in *Proc. 49th Annu. Allerton Conf. Commun., Control, Comput. (Allerton)*, Sep. 2011, pp. 284–291, doi: 10.1109/Allerton.2011.6120180.

[30] L. Tran-Thanh, M. Venanzi, A. Rogers, and N. R. Jennings, "Efficient budget allocation with accuracy guarantees for crowdsourcing classification tasks," in *Proc. 12th Int. Conf. Auton. Agents Multiagent Syst. (AAMAS)*, St. Paul, MN, USA, May 2013, pp. 901–908. [Online]. Available: https://dl.acm.org/doi/10.5555/2484920.2485063

[31] B. LeBaron, "Agent-based computational finance," in *Handbook of Computational Economics: Agent-Based Computational Economics*, vol. 2, K. L. Judd and L. Tesfatsion, Eds. Amsterdam, The Netherlands: North-Holland, 2006, ch. 24, pp. 1187–1233.

[32] N. Oriol and I. Veryzhenko, "Market structure or traders' behavior? A multi agent model to assess flash crash phenomena and their regulation," *Quant. Finance*, vol. 19, no. 7, pp. 1075–1092, Jul. 2019, doi: 10.1080/14697688.2018.1548771.

[33] M. P. Wellman, W. E. Walsh, P. R. Wurman, and J. K. MacKie-Mason, "Auction protocols for decentralized scheduling," *Games Econ. Behav.*, vol. 35, nos. 1–2, pp. 271–303, Apr. 2001, doi: 10.1006/game.2000.0822.

[34] S. Dughmi and J. Vondrák, "Limitations of randomized mechanisms for combinatorial auctions," *Games Econ. Behav.*, vol. 92, pp. 370–400, Jul. 2015, doi: 10.1016/j.geb.2014.01.007.

[35] Q. Luo, Y. Shi, X. Zhou, and H. Li, "Research on the effects of institutional liquidation strategies on the market based on multi-agent model," *Comput. Econ.*, to be published, doi: 10.1007/s10614-020-09987-z.

[36] Y. Jiang and J. C. Jiang, "Understanding social networks from a multi-agent perspective," *IEEE Trans. Parallel Distrib. Syst.*, vol. 25, no. 10, pp. 2743–2759, Oct. 2014, doi: 10.1109/TPDS.2013.254.

[37] Y. Jiang and J. C. Jiang, "Diffusion in social networks: A multiagent perspective," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 45, no. 2, pp. 198–213, Feb. 2015, doi: 10.1109/TSMC.2014.2339198.

[38] A. Fetta, P. Harper, V. Knight, and J. Williams, "Predicting adolescent social networks to stop smoking in secondary schools," *Eur. J. Oper. Res.*, vol. 265, no. 1, pp. 263–276, Feb. 2018, doi: 10.1016/j.ejor.2017.07.039.

[39] W. Li, Q. Bai, M. Zhang, and T. D. Nguyen, "Modelling multiple influences diffusion in on-line social networks," in *Proc. 17th Int. Conf. Auton. Agents Multiagent Syst. (AAMAS)*, Richland, SC, USA, Jul. 2018, pp. 1053–1061, doi: 10.5555/3237383.3237854.

[40] P. Bhattacharya, S. Ekanayake, C. J. Kuhlman, C. Lebiere, D. Morrison, S. Swarup, M. L. Wilson, and M. G. Orr, "The matrix: An agent-based modeling framework for data intensive simulations," in *Proc. 18th Int. Conf. Auton. Agents MultiAgent Syst. (AAMAS)*, Richland, SC, USA, May 2019, pp. 1635–1643, doi: 10.5555/3237383.3237854.

[41] A. Casals, A. Belbachir, and A. E. Fallah-Seghrouchni, "Adaptive and collaborative agent-based traffic regulation using behavior trees," in *Proc. 19th Int. Conf. Auton. Agents MultiAgent Syst. (AAMAS)*, Auckland, New Zealand, May 2020, pp. 1789–1791. [Online]. Available: https://dl.acm.org/doi/10.5555/3398761.3398983.

[42] G. Korkmaz, C. J. Kuhlman, A. Marathe, M. V. Marathe, and F. Vega-Redondo, "Collective action through common knowledge using a Facebook model," in *Proc. 13th Int. Conf. Auton. Agents Multiagent Syst. (AAMAS)*, Paris, France, May 2014, pp. 253–260. [Online]. Available: https://dl.acm.org/doi/10.5555/2615731.2615774

[43] L. Small and O. Mason, "Information diffusion on the iterated local transitivity model of online social networks," *Discrete Appl. Math.*, vol. 161, nos. 10–11, pp. 1338–1344, Jul. 2013, doi: 10.1016/j.dam.2012.10.029.

[44] L. Mao, "Modeling triple-diffusions of infectious diseases, information, and preventive behaviors through a metropolitan social network—An agent-based simulation," *Appl. Geogr.*, vol. 50, pp. 31–39, Jun. 2014, doi: 10.1016/j.apgeog.2014.02.005.

[45] X. Gong and J. Xu, "Research on delay characteristics of information in scale-free networks based on multi-agent simulation," *Procedia Comput. Sci.*, vol. 17, pp. 989–1002, Jan. 2013, doi: 10.1016/j.procs.2013.05.126.

[46] D. López-Pintado, "Contagion and coordination in random networks," *Int. J. Game Theory*, vol. 34, no. 3, pp. 371–381, Oct. 2006, doi: 10.1007/s00182-006-0026-5.

[47] E. Kiesling, M. Günther, C. Stummer, and L. M. Wakolbinger, "Agent-based simulation of innovation diffusion: A review," *Central Eur. J. Oper. Res.*, vol. 20, no. 2, pp. 183–230, Jun. 2012, doi: 10.1007/s10100-011-0210-y.

[48] G. Jiang, X. Feng, W. Liu, and X. Liu, "Clicking position and user posting behavior in online review systems: A data-driven agent-based modeling approach," *Inf. Sci.*, vol. 512, pp. 161–174, Feb. 2020, doi: 10.1016/j.ins.2019.09.053.

[49] M. Rodríguez-Achach, L. Casillas, and F. J. Espinosa, "Diffusion of innovations in a social network under mixed Pareto–Nash strategies," *Phys. A, Stat. Mech. Appl.*, vol. 335, nos. 3–4, pp. 671–676, Apr. 2004, doi: 10.1016/j.physa.2003.12.025.

[50] G. Fu, F. Chen, J. Liu, and J. Han, "Analysis of competitive information diffusion in a group-based population over social networks," *Phys. A, Stat. Mech. Appl.*, vol. 525, pp. 409–419, Jul. 2019, doi: 10.1016/j.physa.2019.03.035.

[51] H. Cui, R. Wang, and H. Wang, "An evolutionary analysis of green finance sustainability based on multi-agent game," *J. Cleaner Prod.*, vol. 269, Oct. 2020, Art. no. 121799, doi: 10.1016/j.jclepro.2020.121799.

[52] Y. Jiang, J. Hu, and D. Lin, "Decision making of networked multiagent systems for interaction structures," *IEEE Trans. Syst., Man, Cybern. A, Syst., Humans*, vol. 41, no. 6, pp. 1107–1121, Nov. 2011, doi: 10.1109/TSMCA.2011.2114343.

[53] R. C. Basole and M. A. Bellamy, "Supply network structure, visibility, and risk diffusion: A computational approach," *Decis. Sci.*, vol. 45, no. 4, pp. 753–789, Aug. 2014, doi: 10.1111/deci.12099.

[54] J. Zhang, C. Xia, and W. Zhang, "Accessory-based multi-agent simulating platform on the Web," in *Proc. 10th Int. Workshop Agents Data Mining Interact. (ADMI)*, Paris, France, May 2014, pp. 52–63, doi: 10.1007/978-3-319-20230-3_5.

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