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Topology Control Algorithm of Underwater Sensor Network Based on Potential-Game and Optimal Rigid Sub-Graph

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ABSTRACT Aiming at the problems of unbalanced energy consumption, redundant links, short life cycle in underwater sensor networks, a topology control algorithm for underwater wireless sensor networks based on potential-game and optimal rigid sub-graph is proposed. Firstly, based on the potential game theory, the topology control model of underwater sensor network is constructed, which considers network coverage, connectivity, transmission energy consumption, end-to-end delay, transmission success rate, node residual energy and so on. It is proved that the model is an ordinal potential game and has a Nash equilibrium solution. Then, the link weight function of node load and node residual energy is introduced, and the redundant links in the network are eliminated by using the principle of optimal rigid sub-graph. Simulation experiments and contrast analysis show that compared with other network models, the network topology model constructed in this paper has lowed node load and stronger energy balance, prolongs the life cycle of the network and more conforms to the underwater environment.

INDEX TERMS Redundant links, topology control, potential-game, optimal rigid sub-graph, life cycle.

I. INTRODUCTION

Underwater wireless sensor network is a kind of physical network including sound, magnetic field and electrostatic field. It has been widely used in ocean data collection, pollution prediction, ocean mining and ocean monitoring, and it will play an important role in future naval operations.

Network topology control is one of the key technologies in UWSN research field. Due to the complicated underwater environment, the transmission of underwater acoustic signals are subject to such uncertain factors as high bit error rate, extended propagation time, intermittent interruption of links, which result in frequent changes of network topology and poor energy efficiency of UWSN [1]–[3]. Therefore, considering the effects of energy consumption balance and energy efficiency on UWSN with High Performance, further study of the optimized network topology control algorithm can not only improve the communication efficiency of UWSN sensor

nodes, but also prolong the network life cycle. It is also the basis of key technologies such as location of underwater sensor nodes, clock synchronization, Mac protocol and so on. It also provides theoretical support for the application of UWSN [4]–[6]. Through reviewing the relevant literature, this paper mainly constructs the UWSN topology control model from the following aspects.

In the aspect of the evaluation index of network topology, all kinds of topology control algorithms have totally different hypotheses and design goals because of the different demands of network topology in different application scenarios. Connectivity and coverage are the most basic requirements for network topology, compared with the land environments, the underwater environment is more complex and the channel delay of transmission is usually large. Therefore, the communication conflict is likely to occur, and the multi-path effect of transmission and the channel are easy to be disturbed, which also leads to the poor reliability of data transmission. To ensure that the data can be sent successfully, the nodes need to resend the data many times, which makes the nodes

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consume energy quickly. Therefore, the network topology constructed in this paper should take the effects of underwater complex environmental factors on the network topology into full consideration.

In the aspect of energy balance of network topology, with the operation of the network, the energy distribution of nodes in the network may become more and more unbalanced. Some nodes die because of the imbalance of energy distribution, which leads to the gradual paralysis of the network. Therefore, the network topology dynamically adjusts the network load of nodes to balance the energy consumption of nodes considering the remaining energy of nodes

In the aspect of network robustness, because of the frequent changes of the underwater environment, the network topology should have a certain anti-destruction ability, namely network robustness, when there are ocean currents or impacts by large marine organisms. Meanwhile, in UWSN, "node failure" is one of the reasons for the frequent failures of network topology. Therefore, how to ensure that the network cannot affect the information transmission of other nodes in case of the failure of some nodes is the most important part of the network robustness. Node degree is one of the methods to measure the network robustness. If the node degree is too high, the information transmitted between nodes will cause serious interference and conflict, and there are many redundant links and the load of nodes is large. However, the low degree of nodes will lead to a longer link between the nodes and poorer network balance, meanwhile, the "bottleneck nodes" are more, and the nodes are more likely to fail. Hence, the effects of robustness on network topology should be considered when constructing underwater network topology.

To sum up, the network topology model should meet the above conditions, that is, there are many underwater factors involved in topology, energy consumption balance and strong robustness. In view of the above problems, this paper constructs a network topology control model with multiple underwater factors. In the aspect of topology energy balance, the paper introduces the residual energy and transmitting power of nodes, uses the definition of potential-game Nash equilibrium solution, constructs the topology control model of potential-game network, proves the Nash equilibrium solution of network topology, and makes the topology have the ability of periodic adjustment, so as achieving the purpose of energy consumption equilibrium. In the aspect of robustness, the Optimal Rigid Sub-graph is used to reduce the node load and eliminate redundant links.

The text structure of this paper is as follows: section 1 introduces the research significance of the UWSN topology control algorithm. Section 2 introduces the related work in detail. Section 3 presents the process of constructing the model. Section 4 gives the details of how this paper executes the model. Section 5 verifies the validity of the model through simulation experiments. The last section summarizes the whole works and looks forward to the future research.

II. RELATED WORK

At present, scholars at home and abroad have made some achievements on the topology control algorithm of terrestrial wireless sensor networks. A clustering control algorithm was proposed in References [7], which reduced the average energy consumption of the network, but the topology robustness was not strong. In order to effectively improve the robustness of the network topology. Anderson *et al.* used the characteristics of the rigid matrix in the rigid graph to locate the network, and added the eigenvalues of the rigid matrix in the selection of the location points, which improved the robustness of the network [8]. However, the influence of residual energy of sensor nodes on the life cycle of the network was not considered. With the aim of effectively prolonging the network life cycle, a topology control algorithm EBTCa based on the proximity graph was proposed in References [9], which took the residual energy of sensor nodes and the link length of nodes as the weight of link to reduce and balance network energy consumption and prolong the network life cycle. In order to give better consideration to prolonging network life cycles and improving network robustness. In reference [10], aiming at the problem of uneven energy consumption of wireless sensor networks, a topology control algorithm OREE with energy balances is proposed based on the optimal rigid graph. By introducing a link weight function which considered energy consumption and residual energy synthetically, the network topology can be reconstructed periodically according to the residual energy of the current node through the algorithm. A network topology optimization algorithm based on the optimal rigid graph, LQETCORg, was proposed in References [11] to solve the problem of uneven energy and poor communication quality of the network. The algorithm introduces a link weight function which considers the link quality and residual energy synthetically to establish a topology with high-quality link communication and balanced energy consumption. Furthermore, Luo *et al.* combined the rigid graph and potential-game into a two-stage constraint solving problem, which greatly enhanced the network life cycle [12].

However, because of the complex underwater environment, the topology control algorithm for terrestrial wireless sensor networks cannot be directly applied in the underwater wireless sensor networks. At present, much research on the topology control algorithm of UWSN mainly considers network evaluation indexes such as connectivity and coverage [13]. On the condition of maintaining network connectivity and maximized communication coverage, Yang G *et al.* Proposed a deployment scheme based on underwater clustering [14]. In order to improve the coverage and connectivity of the network effectively, References [15] used a mobile node Mules to optimize the topology of UWSN. A topology control algorithm PCMP was proposed in References [16] to optimize the connectivity and coverage of underwater wireless sensor networks, and the data packet transmission rate between nodes was used to calculate and

determine the connectivity probability of the nodes. However, the underwater sensor node is influenced by uncertain factors such as water flow and fish swarm, making the original network failed locally and the network paralyzed, thus the robustness of the network topology is poor. Tan Y J *et al.* Introduced the concept of network structure entropy and node connectivity, and took the network structure entropy, connectivity and coverage as the evaluation indexes of network performance, which can truly reflect the robustness of UWSN topology [17].

However, the above network topology control algorithms are carried out in the ideal situations, that is, the node is selfless, but actually, the node will selfishly reduce the energy consumption to save its own energy.

An energy control algorithm EFPC for underwater wireless sensor networks was proposed in References [18], which introduced game theory to avoid node selfishness, balanced network energy consumption and had a Nash equilibrium. By limiting power level to avoid disturbing underwater organisms, the algorithm realized good network topology control, improved network performance, and adopted game theory to equalize node energy consumption, but the underwater environment is complex and some important factors affecting network energy consumption are not considered. References [19] mapped the coverage, connectivity, network energy consumption, communication link delay and transmission success rate optimization into potential-game problems, constructing the UWSN topology control model of multiple target quality of service optimization and designing the corresponding distributed node regulation algorithm. However, it can't guarantee the robustness of the network. Therefore, Liu L *et al.* Constructed a scale-free topology by using complex networks, then analyzed the state of the nodes, classified the nodes according to the coverage probability and communication probability between nodes, and reduced the energy consumption of nodes by the states of sleep and awake so that the topology had higher coverage, less energy consumption and the robustness of the network structure [20]. Wang Y *et al.* Designed a three-dimensional underwater fault-tolerant topology to enhance the robustness of the network by maintaining K-connectivity [21], but the algorithm did not take into account the node degree, link redundancy, propagation delay and so on in the network topology, which will cause the nodes with more energy to run out of energy and die.

To solve these problems, in this paper proposes a topology control algorithm of underwater wireless sensor network based on potential-game and optimal rigid sub-graph. This algorithm considers underwater factors fully, designs UWSN topology control algorithm which includes network connectivity, coverage, energy consumption, transmission delay, data transmission success rate, signal to interference plus noise ratio and other optimization targets, and uses the principle of optimal rigid sub-graph to eliminate redundant links in network topology and reduce the load of nodes; then it can improve the network robustness,

balance the network load and extend the network life cycle.

III. FRAMEWORK DESIGN

A. NETWORK MODEL AND RELEVANT HYPOPAPER

In the three-dimensional space, the wireless sensor network can be mapped into an undirected graph $G(V, E, P)$, in which $V = \{v_1, v_2, \dots, v_N\}$ is the set of underwater sensor nodes; $E = \{e_{ij}^j, i \in V, j \in V, i \neq j\}$ represents the link set of inter-node communication; $P = \{(p_1, p_2, \dots, p_n) : p_i \in [p_{\min}, p_i^{\max}]\}$ is the power set of the node, in which p_{\min} is the receiving threshold and p_i^{\max} is the maximum power. In underwater networks, for any two nodes $i, j \in V$, if the Euclidean distance d_{ij} between them accords with $d_{ij} \leq r_c$, we call the nodes i and j as neighbor nodes, and r_c as the radius of communication. If nodes i and j can communicate with each other, then $\exists e_{ij}^j \in E$.

To facilitate later research, the following constraints are applied to UWSN:

- (1) The underwater sensor nodes can adjust the suspension in any depth according to its own pressure sensor; the sensing range of the node is spherical, which can be accurately perceived inside the ball but not outside the ball.
- (2) The working mode between the sensor nodes is half-duplex mode, and the communication range of the node i refers to the sphere in the center of the node v_i and the radius R_i , while the sensing range of nodes i is the sphere with the radius $R_s (R_s \leq R_i)$.
- (3) In underwater three-dimensional networks, N nodes are randomly deployed, and each node is rational and selfish.
- (4) The initial energy of each node is isomeric, whose value range is as follows: Poisson distribution for a specific value λ .
- (5) In UWSN, each sensor node has a unique D_i identification.
- (6) When all nodes in the network choose the maximized communication radius, the connectivity and coverage of the network can be guaranteed.
- (7) The death cycle of a single node is the network lifetime.

B. ORDINAL POTENTIAL-GAME MODEL

The potential-game is a kind of strategy game. In the strategy game $T = \langle N, A, U(A) \rangle$, there are three main factors: participants, strategy set and utility function. The detailed description is as follows:

- 1) Participants $N = \{1, 2, \dots, n\}$, in which n is the number of participants in the game.
- 2) Strategy set A : in which A_i represents the set of behaviors of the participant i , and if i has k behaviors, there is $A_i = \{a_1, a_2, \dots, a_k\}$, $A = \times_{i=1}^N A_i$ makes a_i as a specific behaviour of participant i , thus $a_{-i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$ presents the behaviour set of other participants except i , usually we use $a = (a_i, a_{-i})$ to represents a particular set of behaviors.

3) Utility function $U(A)$: in which, U can be expressed as $U = \{u_1, u_2, \dots, u_n\}$. $U_i(a_i, a_{-i}) : A \rightarrow R$ can be used to show the utility of participant i under the strategic mix (a_i, a_{-i}) .

Definition 1 (Nash Equilibrium [22]): Given a game model $T = \langle N, A, U(A) \rangle$ with n participants, for $\forall i \in N$ and $a_i \in A_i$, provided other participants $a_{-i}^* = (a_1^*, \dots, a_{i-1}^*, a_{i+1}^*, \dots, a_n^*)$, the behavior a_i^* is the optimal behaviour of participant i . There are:

$$U_i(a_i^*, a_{-i}^*) \geq U_i(a_i, a_{-i}^*) \quad (1)$$

so $a^* = (a_1^*, a_2^*, \dots, a_n^*)$ is a Nash equilibrium of the game model $T = \langle N, A, U(A) \rangle$.

Definition 2 (Ordinal Potential-Game and Ordinal Potential Function [23]): In a game model $T = \langle N, A, U(A) \rangle$, if there are two different strategies $a_i^1, a_i^2 \in A_i$ for $\forall i \in N$, if there is a function $\varphi : A \rightarrow R$ that makes

$$\begin{aligned} \text{sgn}(U_i(a_i^1, a_{-i}^1) - U_i(a_i^2, a_{-i}^2)) \\ = \text{sgn}(\varphi(a_i^1, a_{-i}^1) - \varphi(a_i^2, a_{-i}^2)) \end{aligned} \quad (2)$$

Definition 3 (Pareto Optimality [24]): If there isn't a strategy set $(a_1, a_2, \dots, a_n) \in A$ that makes $U_i(a_i^*, a_{-i}^*) \leq U_i(a_i, a_{-i}) \forall i \in N$ and there is at least one $\forall j \in N$ to make $U_j(a_j^*, a_{-j}^*) \leq U_j(a_j, a_{-j})$ hold, then the strategy $(a_1, a_2, \dots, a_n) \in A$ is a Pareto optimality.

C. RIGID-GRAPH MODEL

In this section, we give the concept of rigid-graph and the extension of rigid-graph. The rigid-graph is a kind of undirected connected graph, so it satisfies the properties of graph. In the underwater three-dimensional topology, the topology of underwater wireless sensor network can be described by a graph. The details are as follows:

1) Rigid-graph: In the graph $G(V, E)$, for any two vertices $(i, j) \in E$, the motion trajectory $f(t)$ of the node corresponds with $\|f_i(t) - f_j(t)\| = d$, and d is a constant, that is, the motion trajectory of the vertex of the undirected-graph is invariable, we call the undirected-graph as a rigid-graph, otherwise it is a deformation-graph. In the underwater three-dimensional environment, the rigid-graph can be combined with the network topology, the vertex of the rigid-graph can be represented as the node of the network topology, and the edge of the rigid-graph can be represented by the communication link between the nodes. As shown in Figure 1, (a) and (b) represent rigid topology and variable topology composed of five nodes in a three-dimensional environment.

Property 1: From the Definition of the rigid-graph, it can be seen that a rigid-graph is a non-deformable graph, that is, the topology graph constructed by a rigid-graph has strong stability.

2) Smallest rigid-graph: The smallest rigid-graph is a special kind of rigid-graph, which is a kind of graph which keeps rigidity and can't be deleted any edge. If a graph

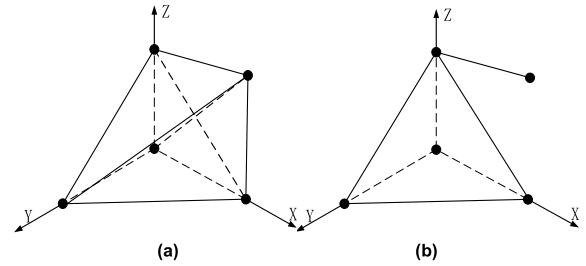


FIGURE 1. (a) Rigid-graph. (b) Deformation-graph.

is rigid and has no edges without losing rigidity can be removed, then it is the least rigid.

Definition 4: If any edge removed from a graph will cause the graph to be rigid while maintaining the rigidity of the graph, this graph is called the smallest rigid-graph.

Lemma 1: The maximum number of links of a graph constructed by n nodes is $n(n - 1)/2$, in a rigid-graph of r -dimensional space, the graph with the number of links as $n \times r - r(r + 1)/2$ is a smallest rigid-graph [25].

3) Optimal rigid-graph: If a topological graph is minimally rigid and the weighted sum of edges is the smallest among all rigid-graphs composed of the same vertices, then the topological graph is an optimal rigid-graph, and the optimal rigid-graph is a rigid-graph with the minimum number of edges and the weighted sum of edges.

Definition 5: If a topology graph is the smallest rigid-graph and has the minimum weighted sum of links under the condition of the same vertex, then the graph is called optimal rigid graph.

Definition 6: For two arbitrary topology sub-graphs $G(V, E)$ and $G' = (V', E')$, if $V' \subseteq V$ and $E' \subseteq E$, we call G' as the sub-graph of G only if the graph G' is an optimal rigid graph, we call G' is an optimal rigid sub-graph of G .

Property 2: From Lemma 1 and Definition 5 in r -dimensional space, the optimal rigid-graph is a topology constructed with fewer links, the minimum total link weight, and at least one vertex connected to r links, i. e. the optimal rigid graph is r -connected, which has strong robustness.

D. THE UTILITY FUNCTION

Due to the complexity of underwater environments, it is difficult to quantify the benefits of nodes. In order to truly reflect the network situation, in this paper considers the utility function U from the following aspects:

1) **Network connectivity:** Network connectivity is a necessary condition for the normal operation of the network. By adding this parameter, we can ensure that the network can remain connected after many game iterations when the node reduces its transmitting power. So, the connection function is set as follows:

$$F(a_i, a_j) = \begin{cases} 1 & \text{connected} \\ 0 & \text{else} \end{cases} \quad (3)$$

- 2) **Network coverage:** the network coverage function is set as follows:

$$C(a_i, a_j) = \begin{cases} 1 & \text{covered} \\ 0 & \text{else} \end{cases} \quad (4)$$

- 3) **Network energy consumption:** The underwater acoustic communication, energy consumption model is different from the land radio energy consumption model, and its influence factors are more, so there are many Definitions in the node energy consumption model. In this paper, the energy consumption model in References [26] is cited. Therefore, the energy consumption of the data sent by the node can be expressed as:

$$E_t(l, r) = l \times E_{elec} + C \times H \times r \times e^{\alpha(f) \times r} \times T \quad (5)$$

Of which: $C = 2\pi \times 0.67 \times 10^{-9.5}$, f is the transmitting frequency; l is the size of the data packet; E_{elec} is the energy consumed by receiving unit data; T is the data transmission time; H is the average water depth of the node, $\alpha(f) = \frac{0.11f^2}{1+f^2} + \frac{44f^2}{4100+f^2} + 2.75 \times 10^{-4}f^2 + 0.003$, r is the distance of communication between nodes. The energy consumption of the data packet with a node receiving length $E_r(l)$ is:

$$E_r(l) = l \times E_{elec} \quad (6)$$

- 4) **End-to-end delay:** The effect on the physical properties of water and the network transmission characteristics on the transmission delay is analyzed synthetically. The end-to-end delay of the receiving node to the sending node is as follows:

$$D(i, j) = T_{i \rightarrow j}^D = \begin{cases} k_s \times \left\{ \frac{l}{R_{ij}} + \frac{D_{ij}(t)}{c} + \Delta\tau_k \right\}, & \text{if } p_{ij} \geq \kappa \\ \infty, & \text{else } p_{ij} < \kappa \end{cases} \quad (7)$$

Of which: k_s is the number of data packet re-transmission, l is the length of the data packet, R_{ij} is the transmission rate; $D_{ij}(t)$ is the distance from node i to node j at the moment t , c is the propagation speed of sound wave. $\Delta\tau_k$ is the maximum multi-path transmission delay difference caused by multi-path propagation in the k_s re-transmission of the data packet, p_{ij} is the power of inter-node communication, and κ is the signal threshold value that the receiver can process.

- 5) **Signal to interference plus noise ratio (SINR):** In underwater wireless networks, SINR is an important index to evaluate signal quality. It is defined as the ratio of signal size received in the sum of the noise power of the interference power. References [27] in this paper defines it as:

$$\gamma_{ij} = \frac{2B_n}{R_{ij}} \cdot \frac{p_{ij} \cdot r_{ij}^{-a} \cdot 10^{-(\alpha(f)r_{ij}/1000)/10}}{\sum_{k=1}^m \mu_{ik} p_{ik} r_{ik}^{-a} 10^{-(\alpha(f)r_{ik}/1000)/10} + \sigma^2} \quad (8)$$

Of which: B_n is the system bandwidth, $\alpha(f)$ is the medium absorption coefficient and r_{ij} is the transmission distance. $\sum_{k=1}^m \mu_{ik} p_{ik} r_{ik}^{-a} 10^{-(\alpha(f)r_{ik}/1000)/10}$ divides μ_{ik} which are same as the same i into a group, totally m groups, σ^2 is noise variance.

- 6) **Success rate of transmission:** The success rate of transmission is established by the References [28] as:

$$S(i, j) = f(\gamma_{ij}) = \left(1 - \frac{e^{-\frac{\gamma_{ij}}{2}}}{\sqrt{2\pi\gamma_{ij}}}\right)^l \quad (9)$$

The relationship between the number of re-transmissions k_s and the success rate of transmission is:

$$P(X = k_s) = S(i, j)[1 - S(i, j)]^{k_s - 1} \quad (10)$$

- 7) **Residual energy of node:** The residual energy of a node is the main consideration of topology control of underwater network, which can reflect the life cycle of networks and the equilibrium of energy consumption. In order to realize the equilibrium of energy consumption in the network, it is necessary to purposefully adjust the nodes with more residual energy to participate in the forwarding task. In this paper, the factor $\frac{E_r(i)}{E_o(i) - E_r(i)}$ is added to the utility function, of which $E_o(i)$ and $E_r(i)$ are the initial energy and the residual energy of the node i , respectively. At the same time, the factor $\frac{E_r(j)}{E_o(j) - E_r(j)}$ is added to the average residual energy of the neighbor node. k is the number of neighbor nodes of node i when its transmitting power is p_i .

In a comprehensive analysis, network connectivity, coverage, transmission energy consumption, transmission end-to-end delay, SINR, transmission success rate, node residual energy are the main optimization goals of UWSN. Because of the multiplicity of these goals and the contradiction between targets, it is difficult to achieve the optimal performance. Therefore, the distributed multi-objective optimization is transformed into the game to solve, and the dynamic solution to the distributed multi-objective optimization problem is realized by the repeated game process. Therefore, when satisfied with the properties of the utility function described in the References [29], for $\forall i \in N$ the utility function of the game model $T = \langle N, A, U(A) \rangle$ is defined as follows:

$$U_i(a_i, a_{-i}) = F_i(a_i, a_{-i}) C_i(a_i, a_{-i}) (\alpha E_i^{TX} \max \frac{E_r(i)}{E_o(i) - E_r(i)} + \beta \overline{E_i(a_i)}) + \lambda S_i(a_i, a_{-i}) - \alpha E_i^{TX} D_i(a_i, a_{-i}) \frac{E_r(i)}{E_o(i) - E_r(i)} \quad (11)$$

Of which: α , β , λ are the weight adjustment factors and are positive.

E. MATRIX OF OPTIMAL RIGID SUB-GRAPH

(1) The construction of the link weight function, the link weight function $\lambda_{ij}(t)$ is represented as:

$$\lambda_{ij}(t) = \chi_1 d(i, j)^\zeta + \chi_2 \frac{1}{E_r(t)} \quad (12)$$

where $\chi_1 + \chi_2 = 1$, of which, ζ denotes the link quality adjustment factor, $d(i, j)$ denotes the node communication distance, $E_r(t)$ denotes the node residual energy.

Sub-graph

(2) matrix. For topology graph $G = (V, E)$, the sub-graph matrix G^i composed of the node i and its neighbor node is as follows:

$$G_{xy}^i = \begin{cases} 1 & \text{if } \exists e_j^i \in E \\ 0 & \text{else} \end{cases} \quad (13)$$

(3) Rigid matrix. In r -dimensional space, the coordinate of the node i is usually expressed as $(s_i^1, s_i^2, \dots, s_i^r)$, and then in three-dimensional space, the coordinate can be described as $(s_i^1 = x_i, s_i^2 = y_i, s_i^3 = z_i)$. When n nodes are deployed randomly in r -dimensional space, their coordinates based on their serial number are as follows:

$$\{(s_1^1, s_1^2, \dots, s_1^r), (s_2^1, s_2^2, \dots, s_2^r), \dots, (s_i^1, s_i^2, \dots, s_i^r), \dots, (s_j^1, s_j^2, \dots, s_j^r), \dots, (s_n^1, s_n^2, \dots, s_n^r)\} \quad (14)$$

In the link set $E = \{e_j^i, i \in V, j \in V, i \neq j\}$ of undirected-graph $G = (V, E)$, each link can be converted to a row vector of a rigid matrix. When a rigid matrix $M_{|e| \times 3n}$ ($|e|$ denotes the total number of links in a topology graph) is constructed in three-dimensional space, the row vector m_k made up of elements in the k row corresponding to the k link e_j^i in the matrix is as follows:

$$m_k = [0, \dots, s_i^1 - s_j^1, s_i^2 - s_j^2, s_i^3 - s_j^3, \dots, s_j^1 - s_i^1, s_j^2 - s_i^2, s_j^3 - s_i^3, \dots, 0] \quad (15)$$

Then, a rigid matrix $M_{|e| \times 3n}$ consisting of $|e|$ row vectors deployed in three-dimensional space is represented as below:

$$M = \begin{bmatrix} m_1 \\ m_2 \\ \dots \\ m_{|e|} \end{bmatrix} = (m_1^T, m_2^T, \dots, m_{|e|}^T) \quad (16)$$

Lemma 2: If a matrix M is constructed by an undirected-graph with n vertices in r -dimensional space, the graph is the smallest rigid graph, if and only if the rank of its rigid matrix satisfies the following [30]:

$$\text{rank}(M) = n * r - r(r + 1)/2 \quad (17)$$

Hence, from Formula (16), it is known that the rank of the smallest rigid graph with n vertices in three-dimensional space can be represented as: $\text{rank}(M) = 3n - 6$.

Property 3: From Definitions 6, 7 and Lemma 2 above, we can see that the link weight set W of the rigid matrix

is constructed from the Formula (11) after the construction of rigid matrix M of all links in r -dimensional space. The weight set W is arranged in ascending order, and the rigid matrix is initially represented as $M_c = M(1)$. Then the rows in the rigid matrix M are added to the rigid matrix M_c according to the link weight order, if matrix M_c is full rank, adding continuously until the whole set W is covered. Finally matrix M_c generated is the optimal rigid matrix.

Theorem 1: The game model $T = \langle N, A, U(A) \rangle$ constructed in this paper is an ordinal potential-game. The function $\varphi : A \rightarrow R$ is an ordinal potential function of the game model $T = \langle N, A, U(A) \rangle$, and there is a Nash equilibrium solution. The ordinal potential function is defined as:

$$\begin{aligned} \varphi(a_i, a_{-i}) &= \sum_{i \in N} [F_i(a_i, a_{-i})C_i(a_i, a_{-i})(\alpha E_i^{TX \max} \frac{E_r(i)}{E_o(i) - E_r(i)} + \beta \overline{E_i(a_i)} + \lambda S_i(a_i, a_{-i})) - \alpha E_i^{TX} D_i(a_i, a_{-i}) \frac{E_r(i)}{E_o(i) - E_r(i)}] \quad (18) \end{aligned}$$

According to Definition 2, if the strategy of any player in the game changes, the utility function and potential function are all satisfied with Formula (2). Theorem 1 is proved as below.

Proof: According to the utility function in Formula (11), if a_i^1 and a_i^2 are two different strategies of node i , and $a_i^1, a_i^2 \in A_i$, The difference between the utility functions of the two strategies of the node can be calculated as shown in the formula (19), as shown at the bottom of the next page. Then the utility difference of potential function can be expressed as a formula (20), as shown at the bottom of the next page. At the same time, because $F_i(a_i, a_{-i})$ and $C_i(a_i, a_{-i})$ are monotone non-decreasing functions, so formula (21), as shown at the bottom of the next page, is correct.

Since $F_i(a_i^1, a_{-i}^1)C_i(a_i^1, a_{-i}^1), F_i(a_i^2, a_{-i}^2)C_i(a_i^2, a_{-i}^2)$ in Formula (19) and strategies a_i^1 and a_i^2 of node i have the same changing trend (here $\Delta\varphi_{-i} \geq 0$), the utility function $U_i(a_i, a_{-i})$ and the potential function $\varphi(a_i, a_{-i})$ have the same symbolic change and always have the same symbol, that is: $\text{sgn}(U_i) = \text{sgn}(\varphi_i)$, no matter how strategies of node i change. According to Definition 1 and Definition 2, we know that the game model $T = \langle N, A, U(A) \rangle$ is an ordinal potential-game, the function φ_i is an ordinal potential function of $T = \langle N, A, U(A) \rangle$, and there must be a Nash equilibrium solution.

Theorem 2: The Nash equilibrium solution of the game model $T = \langle N, A, U(A) \rangle$ constructed in this paper is Pareto optimally.

Proof: Due to the limited number of nodes in the network, strategies of nodes selected are limited. According to References [31], the finite ordinal potential-game will surely converge to the Nash equilibrium. Based on the establishment of the network model, the easy-to-know node maximizes its utility benefit by adjusting its own strategy. That is, the nodes in the network will continuously reduce their emissions and prolong their survival time until the power of all nodes can not

change (Nash equilibrium state). When Nash equilibrium is reached, if one node reduces its transmitting power, the connectivity and coverage of the network will be destroyed, resulting in the reduction of the utility benefit of other nodes. Therefore, the Nash equilibrium solution of the game model $T = \langle N, A, U(A) \rangle$ constructed in this paper is Pareto optimal solution according to the Definition 3.

IV. PG-OSTCG TOPOLOGY CONTROL ALGORITHM

A. NETWORK TOPOLOGY CONSTRUCTION

Based on the analysis of inter-node communication distance and link quality, the maximum communication radius of the initial sensor node i is R_C , and the sensing radius is R_S . As shown in Table 1, the node i broadcasts the information packet NCK to the surrounding nodes, where $NCK = \{ID_i, S_i, E_r^i\}$, ID denotes i identification code, S_i is the position coordinate of i , E_r^i is the residual energy of i . When the sensor node j receives the information package NCK of node i , node j sends the information package ACK to node i , in which $ACK = \{ID_j, P_j, E_r^j, E_i^{ij}, \Delta, r_{ij}, U_i\}$, and Δ denotes the dynamic topology response capability, r_{ij} denotes the success rate of the node i -to-node j path transmission, U_i is the value of utility function of the node i in the network. When node i receives the acknowledgement of information packet ACK from the surrounding node j , it adds node j to its neighboring information table. In this way, the maximum

TABLE 1. Information package.

		NCK information package			Residual energy	
		location				
		ACK information package				
ID	Position	Residual energy	Energy	delay	Success rate	Function value

overall network topology view G_{max} can be obtained, which provides the strategy choice for the subsequent topology game stage.

B. IMPLEMENTATION STAGE OF NETWORK TOPOLOGY GAME

The main task of the topology game stage is to dynamically adjust the network topology to prolong the life cycle and anti-destruction ability of the underwater network based on the energy consumption and the robustness of the network in the underwater environment. The method of network topology adjustment stage adopted in this paper is to adjust the transmission power of the node, and set it to be the optimal transmission power in different underwater environment, so as to obtain the network topology which conforms to the underwater environment. Firstly, First, as shown in Table 2, when the topology is established, node i calculates the current power p_i through the Formula (5), and obtain the residual energy E_r^i and the optional power set of the node: $P_i = \{(p_1, p_2, \dots, p_m) : p_i \in [p_i^{min}, p_i^{max}]\}$, where p_i^{min} and p_i^{max}

$$\begin{aligned} \Delta U_i &= U_i(a_i^1, a_{-i}^1) - U_i(a_i^2, a_{-i}^2) \\ &= F_i(a_i^1, a_{-i}^1)C_i(a_i^1, a_{-i}^1)(\alpha E_i^{TX \max} \frac{E_r(i)}{E_o(i) - E_r(i)} + \beta \overline{E_i(a_i^1)} + \lambda S_i(a_i^1, a_{-i}^1)) - \alpha E_i^{TX} D_i(a_i^1, a_{-i}^1) \frac{E_r(i)}{E_o(i) - E_r(i)} \\ &\quad - F_i(a_i^2, a_{-i}^2)C_i(a_i^2, a_{-i}^2)(\alpha E_i^{TX \max} \frac{E_r(i)}{E_o(i) - E_r(i)} + \beta \overline{E_i(a_i^2)} + \lambda S_i(a_i^2, a_{-i}^2)) + \alpha E_i^{TX} D_i(a_i^2, a_{-i}^2) \frac{E_r(i)}{E_o(i) - E_r(i)} \\ &= \alpha E_i^{TX \max} \frac{E_r(i)}{E_o(i) - E_r(i)} [F_i(a_i^1, a_{-i}^1)C_i(a_i^1, a_{-i}^1) - F_i(a_i^2, a_{-i}^2)C_i(a_i^2, a_{-i}^2)] + \beta [F_i(a_i^1, a_{-i}^1)C_i(a_i^1, a_{-i}^1) \overline{E_i(a_i^1)} \\ &\quad - F_i(a_i^2, a_{-i}^2)C_i(a_i^2, a_{-i}^2) \overline{E_i(a_i^2)}] \\ &\quad + \lambda [F_i(a_i^1, a_{-i}^1)C_i(a_i^1, a_{-i}^1)S_i(a_i^1, a_{-i}^1) - F_i(a_i^2, a_{-i}^2)C_i(a_i^2, a_{-i}^2)S_i(a_i^2, a_{-i}^2)] \\ &\quad - \alpha E_i^{TX} \frac{E_r(i)}{E_o(i) - E_r(i)} [D_i(a_i^1, a_{-i}^1) - D_i(a_i^2, a_{-i}^2)] \end{aligned} \tag{19}$$

$$\begin{aligned} \varphi &= \varphi(a_i^1, a_{-i}^1) - \varphi(a_i^2, a_{-i}^2) \\ &= \sum_{i \in N} \left(F_i(a_i^1, a_{-i}^1)C_i(a_i^1, a_{-i}^1)(\alpha E_i^{TX \max} \frac{E_r(i)}{E_o(i) - E_r(i)} + \beta \overline{E_i(a_i^1)} + \lambda S_i(a_i^1, a_{-i}^1)) - \alpha E_i^{TX} D_i(a_i^1, a_{-i}^1) \frac{E_r(i)}{E_o(i) - E_r(i)} \right) \\ &\quad - \sum_{i \in N} \left(F_i(a_i^2, a_{-i}^2)C_i(a_i^2, a_{-i}^2)(\alpha E_i^{TX \max} \frac{E_r(i)}{E_o(i) - E_r(i)} + \beta \overline{E_i(a_i^2)} + \lambda S_i(a_i^2, a_{-i}^2)) - \alpha E_i^{TX} D_i(a_i^2, a_{-i}^2) \frac{E_r(i)}{E_o(i) - E_r(i)} \right) \\ &= \Delta U_i + \sum_{k \in N, k \neq i} \left\{ [F_k(a_i^1, a_{-i}^1)C_k(a_i^1, a_{-i}^1) - F_k(a_i^2, a_{-i}^2)C_k(a_i^2, a_{-i}^2)] \cdot \left(\alpha E_k^{TX \max} \frac{E_r(i)}{E_o(i) - E_r(i)} + \beta \overline{E_k(a_k)} + \lambda S_k(a_k, a_{-k}) \right) \right. \\ &\quad \left. - \alpha E_k^{TX} D_i(a_k, a_{-k}) \frac{E_r(i)}{E_o(i) - E_r(i)} \right\} = \Delta U_i + \Delta \varphi_{-i} \end{aligned} \tag{20}$$

$$F_i(a_i, a_{-i})C_i(a_i, a_{-i})(\alpha E_i^{TX \max} \frac{E_r(i)}{E_o(i) - E_r(i)} + \beta \overline{E_i(a_i)} + \lambda S_i(a_i, a_{-i})) \geq \alpha E_i^{TX} D_i(a_i, a_{-i}) \frac{E_r(i)}{E_o(i) - E_r(i)} \tag{21}$$

TABLE 2. The stage of topological game.

Initialization
01: Node i broadcasts "Hello" message at p_i^{\max}
02: Determine the neighbor set $M_{\max}(i)$
03: Determine the optional power set P_i for node i
Game execution stage
01: $P_i = \{p_1, p_2, \dots, p_m\}$, descending sort
02: while p_i is not NE
03: for $i = 1, i \leq N, i++$
04: choose power according to $p_i^* = \arg \max_{p_i \in P_i} U_i(p_i, p_{-i})$
05: if $U_i(p_i^*, p_{-i}) \geq U_i(p_i, p_{-i})$
06: if p_i^* is NE
07: $p_i = p_i^*$, update $p_i, M_{\max}(i)$
08: end if
09: end if
10: end for
11: Broadcast a "Hello" message including the new power p_i
12: end while

denote the minimum power and the maximum power of the node.

Secondly, each node selects the power in turn according to the power set, and in each round, only one node adjusts the power, while the power of other nodes remains the same. In order to ensure convergence to Nash equilibrium, this paper adopts the optimal reflection strategy proposed in References [32] to update scheme, which will surely converge to Nash equilibrium in the finite ordinal potential-game. Therefore, in this paper, the optimal response strategy of node i can be represented as: $p_i^* = \arg \max_{p_i \in P_i} U_i(p_i, p_{-i})$,

when the power of other participants is deployed as p_{-i} . In the implementation of the game, when the node chooses the power to communicate which is lower than the current power, observe whether the gain is becoming larger or not. If so, the lower power is more suitable for the communication power of the node; otherwise, the node should keep the current power unchanged. When the power of each node i is better than that of other nodes, which is to say, the power of all nodes reaches the optimal state, and the power set P of each node does not change the power of any node to make the network gain larger, the network reaches a kind of equilibrium state, that is, Nash equilibrium. At this point, the set R of the node's neighboring nodes is constructed from the information table of each node, and the network topology graph is generated.

C. REDUNDANT LINK DELETION STAGE

After the last stage, although the network reaches the Nash equilibrium solution, the energy consumption equilibrium and the network, anti-destruction ability are overemphasized in the network model, so there are some problems with the nodes in the network such as link redundancy and large link weight. In this paper, the principle of optimal rigid graph is introduced to eliminate redundant links in the net-

work. As shown in Table 2, Each node i calculates its own number k of neighbors according to the set R of neighbor nodes, builds its sub-graph matrix G^i of weight link set W_i from Formula (12) and (13), and determines whether the link set it builds satisfies the construction of a rigid matrix based on Definition 4-6 and Lemma 2. If so, a rigid matrix M can be constructed from Formula (14) - (16), and an optimal sub-rigid matrix M_c will be constructed according to Property 3. Finally, the link set D to be deleted can be obtained from the optimal rigid sub-graph matrix, using which, the neighbor node set R of each node can be updated.

V. PERFORMANCE EVALUATION OF PG-OSTCG ALGORITHM

In this part, we use Python to design four groups of comparison simulation, a group of algorithm weight factor selection experiments, compared with three groups of algorithms to verify the effectiveness of the algorithm. The three algorithms are described as follows:

In order to better reflect the influence of underwater factors on network topology, DEBA [31] algorithm is selected for comparison. This algorithm is a kind of terrestrial wireless sensor network, but it only considers the impact of node residual energy on the network topology life cycle, and uses the game theory to balance the network energy consumption. It can also be used in underwater network topology under certain conditions, so it can better reflect the influence of underwater factors on network topology.

In order to verify the robustness of the network topology, this paper selects the classic three-dimensional fault-tolerant topology 3Dk-RNG [21]. This algorithm uses graph theory to construct a kind of network topology which can keep three-connected all the time and has strong robustness.

Similarly, EFPC optimizes the transmission power by applying a utility function based on Nash equilibrium. The proposed power control algorithm can be used for parallel transmission to improve the quality of network communication and avoid interference to marine mammals. This algorithm has strong network connectivity quality.

To sum up, this paper compares the network feasibility from four aspects: network topology robustness, network link communication quality, network energy consumption balance and network life cycle.

To make it convenient to understand the simulation process, performance evaluation indexes given for experimental simulations are:

- 1 Robustness of network topology:** With network connectivity as the main objective, when there is an interrupt in a network link, the network can select other links to transmit data at a high speed, that is, the better the connectivity of the network is, the stronger its robustness will be.
- 2. Average degree of nodes:** In underwater sensor networks, the ratio of the sum of node degrees of each sensor to the total number of network nodes is called

the average degree of nodes D_{av} , that is:

$$D_{av} = \frac{1}{N} \sum_{i=1}^N d_i^\alpha \quad (22)$$

3. **Average length of communication links:** In underwater sensor networks, the ratio of the sum of length of each communication link between nodes to the total number of network links is called the average length of links l_{av} , that is:

$$l_{av} = \frac{1}{L} \sum_{i=1}^N \sum_{j=1}^{N_1} l_{ij} \quad (23)$$

Of which, l_{ij} is length of communication link between Node i and j, and L is the total number of network links.

4. **Network life cycle:** In underwater sensor networks, the difference of the time when the first dead node appears and that when the network starts working is called network life cycle T_L , that is:

$$T_L = T_D - T_B \quad (24)$$

Of which, T_D is the time when the first node is dead, and T_B is the time when the network starts working.

A. ALGORITHM COMPLEXITY ANALYSIS

Algorithm complexity is the standard to measure the amount of computation. In this paper, we mainly analyze the time complexity of the algorithm. The algorithm in this paper mainly includes two stages, namely, the stage of topology game and the stage of eliminating redundant links. The details are as follows:

Firstly, in the stage of topology game, as shown in Table 2, all nodes traverse their own strategies, and calculate the revenue by the formula (11), and determine the set of neighbors by the revenue. In this game stage, assume that the maximum number of neighbor nodes is M (the maximum number of strategies of each node), where $1 \leq M < n$, that is, the time complexity of this stage is $o(M^2n)$.

In the redundant link elimination stage, as shown in Table 3, all nodes need to construct the rigid graph matrix from the neighbor set, and perform the rigid transformation algorithm, the complexity of which is $o(Mn)$. To sum up, the complexity of this algorithm is $o(M^2n)$.

B. ANALYSIS ON INFLUENCE OF WEIGHT FACTORS ON NETWORK TOPOLOGY

In this section, 80 nodes are randomly placed in a three-dimensional monitoring region ($400 \times 400 \times 400$), adjust the parameters in α, β, λ in proportion, and then select the appropriate parameters. The specific experimental parameters are shown in Table 4.

From the utility function in equation (11), it can be seen that the formula mainly includes three parts:energy consumption balance, transmission success rate and transmission delay. When the proportion of the parameter α is large, the energy consumption of the network is mainly balanced, but the success rate will be reduced. When the proportion of β

TABLE 3. Pseudo code of PG-OSTCG algorithm.

```

Redundant link deletion
01 :for  $i = 1, i \leq N, i++$ 
02: using  $M_{\max}(i)$  Determine the neighbor set  $R$ 
03: using Formulate (12),(13) calculate  $W_i, G^i$ 
04: using  $G^i$  set up  $M_c$  and Initialization  $M_c=M(1)$ 
05: for  $k = 1$  to  $|W_i| \times (|W_i|-1) / 2$  do
06:   if  $Rank(M_c) \leq 3|W_i| - 6$  then
07:     if  $Rank(\begin{bmatrix} M_c \\ M(k+1) \end{bmatrix}) = Rank(M_c) + 1$  then
 $M_c = \begin{bmatrix} M_c \\ M(k+1) \end{bmatrix}$ ;
08:   end if
09: end if
10: end for
11: using  $M_c$  update  $R$ 
12: end for
    
```

TABLE 4. Parameter setting of simulation environment.

Parameter	Description	Initial value
N	Number of nodes	80
V	Node arrangement region	$400m \times 400m \times 400m$
E_o	Node initial energy	300J
E_r	Node residual energy	Poisson distribution $\lambda=250$
p_{min}	Receiving threshold	$3.5 \times 10^{-5}W$
L	Size of data package	2000 B
R_{ij}	Data transmission rate	2000bit / s
BE(i,j)	Link bandwidth	(100,200)kbps
v	sound velocity	1500 m / s
τ_E	Energy threshold value	0.2
f	Transmitting frequency	700HZ
K	Receiving sensitivity	-80dB

and λ is large, the transmission delay and energy consumption balance will be poor. It can be seen that the parameter α, β, λ have a certain proportional relationship. Therefore, to facilitate the selection of parameters, we will under the condition when the value of any two weight factors is 1 within limited, the other weight factor is regulated to analyze influences of weight factors on network topology performances in the algorithm.

It can be seen from Figure 2-4, after numerous number of data experiments, we intercepted 20 groups of experiments (experimental serial number), and analyzed the influence of parameters α, β, λ on network topology from three aspects of average transmission success rate, average node degree and average residual energy of neighbor nodes.

It can be seen from equation (8) that when the transmission power is large, it will lead to a large signal interference to

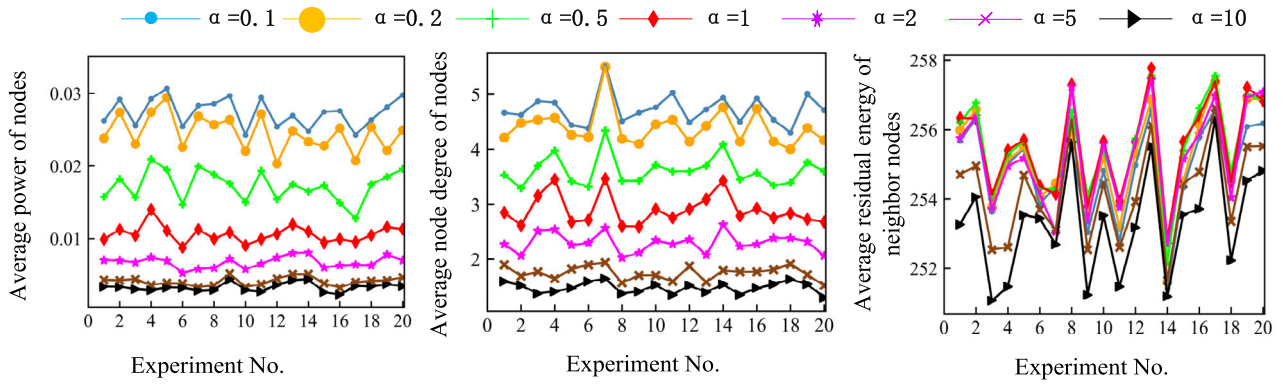


FIGURE 2. When $\beta, \lambda = 1$, the impact of value α on network performance.

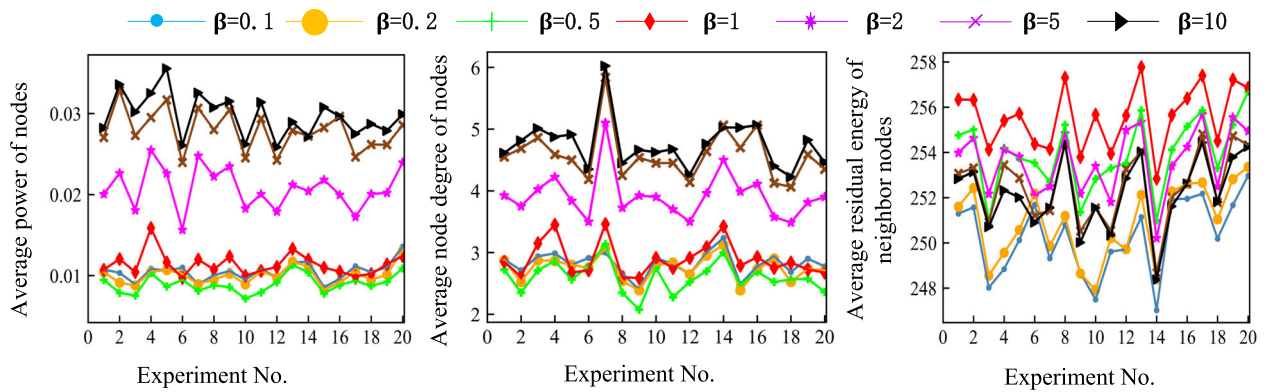


FIGURE 3. When $\alpha, \lambda = 1$, the impact of value β on network performance.

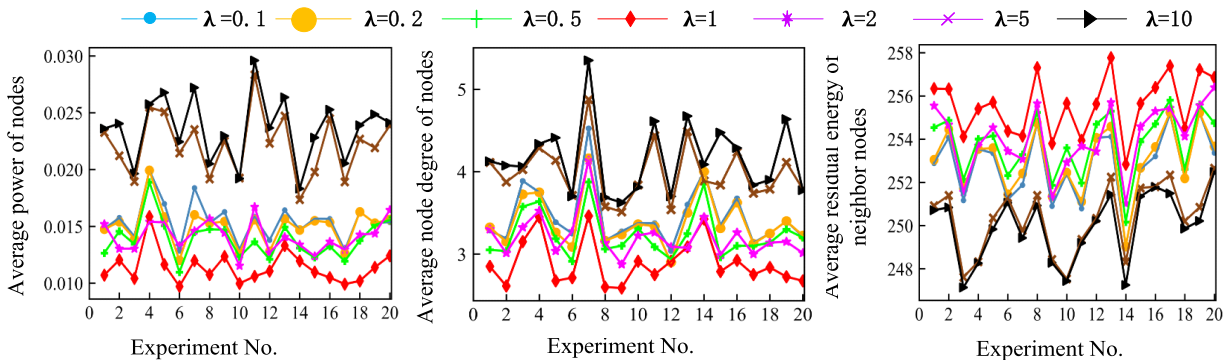


FIGURE 4. When $\alpha, \beta = 1$, the impact of value λ on network performance.

noise ratio; from equation (9), it can be seen that a higher signal interference to noise ratio will reduce the transmission success rate; at the same time, a lower transmission success rate will lead to link interruption, which will also reduce the transmission success rate, thus affecting energy consumption balance. Therefore, from the utility function, we can see that the ratio of parameters α, β, λ should not be too high or too low. At the same time, it can be seen from the figure, average transmitting power and average node degree of nodes as well as average residual energy of neighbor

nodes increase with the increase of α , meanwhile the above 3 indexes have a relatively stable variations after $\alpha \geq 2$. In the same way, it can be known through generalized theory of network topological structure that, when transmitting power of nodes in the network is relatively low, meanwhile there is a moderate node degree, the topological structure then is relatively perfect. Thus, based on introduction of operational capacity of network nodes and network performance indexes in References [30], α is set as 1, β is set as 2 and λ is set as 2 in this paper.

C. ANALYSIS ON ROBUSTNESS OF PG-OSTCG

In this section, we choose DEBA, EFPC and 3DK-RNG, which are two game theories based topology control algorithms of underwater network, and compare them with a PG-OSTCG. First of all, we randomly generate 80 nodes in a three-dimensional monitoring area ($400 \times 400 \times 400$), and compare the corresponding network topology. At the same time, the maximum node degree and the average node degree of the four algorithms are compared by pointing out the number of nodes from 70 to 150 to verify the topology robustness.

Figure 5 shows the network topology structure constructed by DEBA algorithm based on the method of balanced node energy consumption of game theory. It can be seen in the figure. That there are many “bottleneck nodes” with a little residual energy, which lead to failure to complete guarantee of network full coverage and connectivity; Figure 6 and 7 respectively show EFPC, a network topological structure constructed through pure strategy game and power control and 3DK-RNG, a fault-tolerant topological control used in underwater environments (that is, it can bear a certain node/link failures), although the above 2 algorithms effectively reduce “bottleneck nodes”, due to the extra-high node degrees and

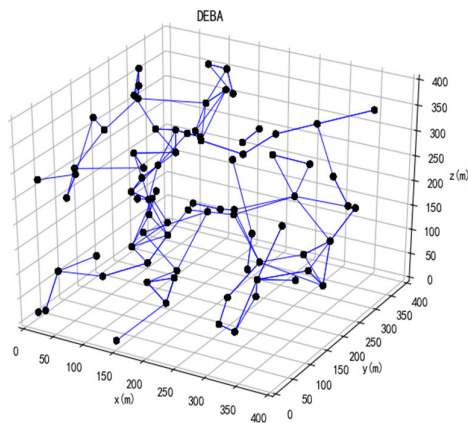


FIGURE 5. The DEBA algorithm network topology.

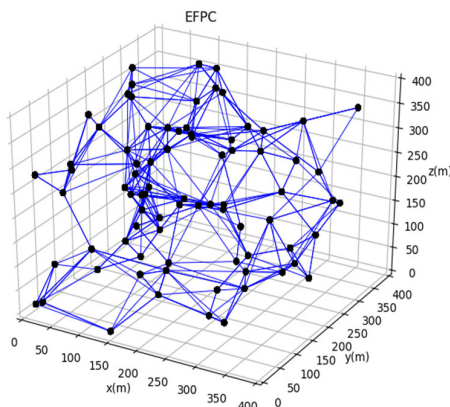


FIGURE 6. The EFPC algorithm network topology.

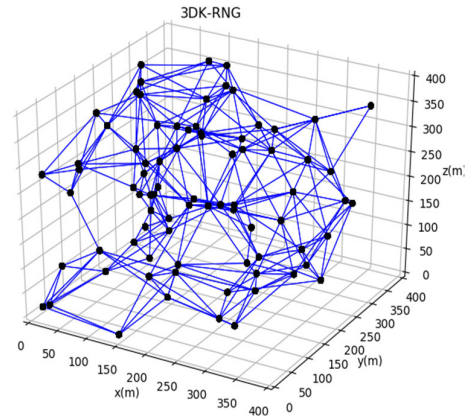


FIGURE 7. The 3DK-RNG algorithm network topology.

large number of redundant links, conflicts are caused by information transmission among sensor nodes, leading to unnecessary energy consumption.

Figure 8 shows an optimized network topological structure constructed by PG-OSTCG algorithm through integration of potential-game and optimal rigid sub-graph model, which takes both node loads and energy consumption as well as balance of network energy consumption into consideration. With nodes containing higher amount of redundant energy as relay forwards node of data, it relieves early deaths of key and fringe nodes caused by quick consumption of energy, meanwhile it also reduces redundant links to the network, thus effectively extending life cycles of the network.

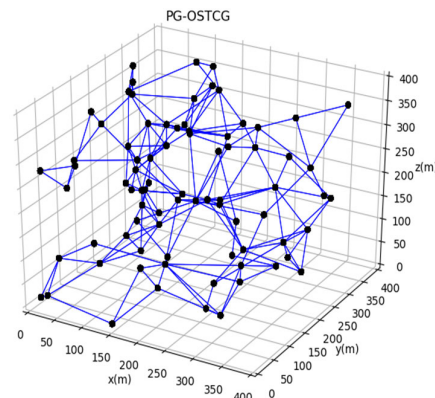


FIGURE 8. The PG-OSTCG algorithm network topology.

Figure 9 and 10 show comparisons between average and maximum degrees of nodes of DEBA, EFPC and 3DK-RNG as well as PG-OSTCG algorithm put forward in this paper. It is generally shown that in all the above 3 algorithms, average and maximum degrees of nodes increase as the number of nodes increases, when the number of nodes reaches a certain value, the average degree of nodes reaches a relatively steady state. While it can be seen from Figure 10 that, maximum degrees of nodes by DEBA and EFPC algorithms are relatively high, which is because the two algorithms are mainly

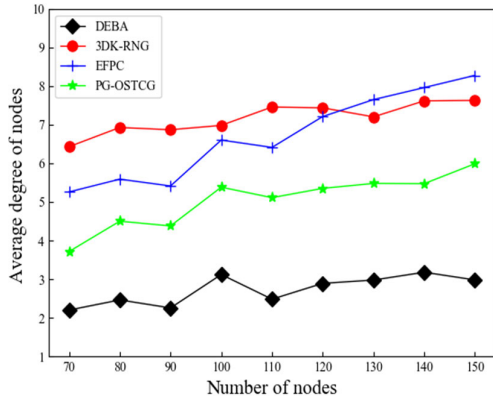


FIGURE 9. Contrast diagram of average node degree.

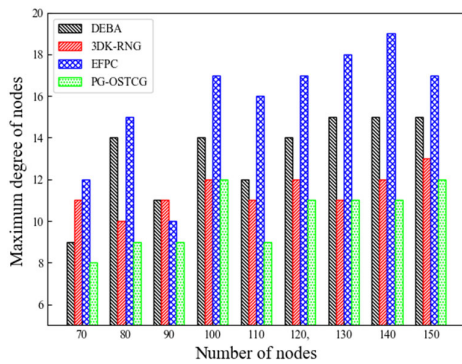


FIGURE 10. Contrast diagram of maximum node degree.

based on balanced node energy consumption and powers, thus leading to a large relative deviation from average and maximum degrees of those nodes.

D. ANALYSIS ON LINK QUALITY OF PG-OSTCG

In this section, average links lengths of EFPC, 3DK-RNG and PG-OSTCG algorithms are compared and analyzed through changing the number of nodes under the above experimental simulation environment, and then link quality of PG-OSTCG algorithm is analyzed under different values of coefficient α . As there are many “bottleneck nodes” and “fringe nodes” in DEBA algorithm, we will not compare them in the following sections.

Figure 11 shows the change of the average link length of the 3DK-RNG, EFPC and PG-OSTCG algorithm as the number of nodes increases from 70 to 150. As can be seen from the figure, average link length of topological structure constructed by 3DK-RNG is the largest, that is, when there are equal nodes, the link quality of network topology generated through 3DK-RNG is poor with a high energy consumption. In addition, the average link length of EFPC algorithm is smaller than that of the 3DK-RNG algorithm while being larger than that of a PG-OSTCG algorithm, which indicates that communication quality of network topology generated through PG-OSTCG algorithm is higher than that of 3DK-RNG and EFPC algorithms.

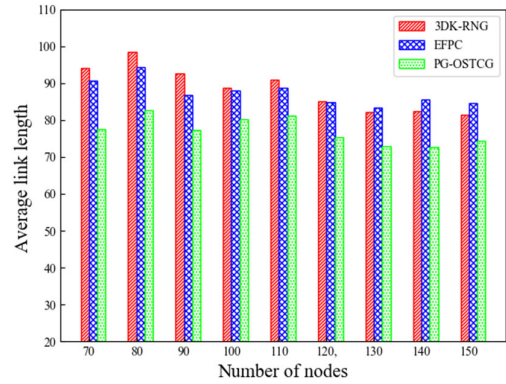


FIGURE 11. Contrast diagram of average link length with different numbers of nodes.

It can be acquired from Figure 12 that, when there is a few numbers of the same nodes, average link length will decrease to the continual increase in α , and the link quality is slightly increased, while the decrease of the average link length is small, and balanced energy consumption of PG-OSTCG algorithm increases; This shows that it feasible to regulate network life cycle by algorithm put forward in this paper through regulating weight factors (that is, sacrificing or increasing a small amount of communication quality).

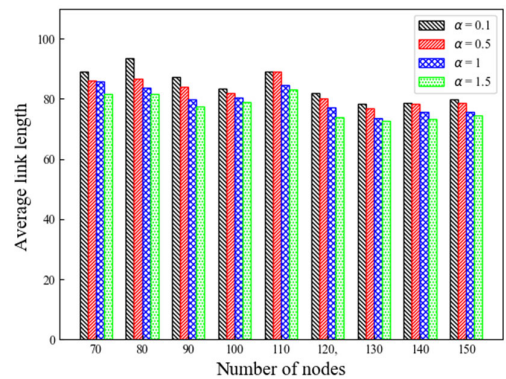


FIGURE 12. The Contrast Diagram of Average Link Length with Variation of Adjustment Coefficient α .

E. ANALYSIS ON BALANCED ENERGY CONSUMPTION OF PG-OSTCG

Energy consumption of balanced nodes should take both residual energies of nodes and loads with nodes into consideration. Balanced energy consumption of 3DK-RNG, EFPC and PG-OSTCG algorithms during network operations processes under the same experimental simulation environment as mentioned is compared and analyzed.

Figure 13 shows comparison between standard deviations from node residual energy through the 3 algorithms, namely 3DK-RNG, EFPC and PG-OSTCG as network operational time goes on. As load energy consumption of the nodes is not taken into consideration by 3DK-RNG algorithm, rising speed of residual energy standard deviation of this algorithm

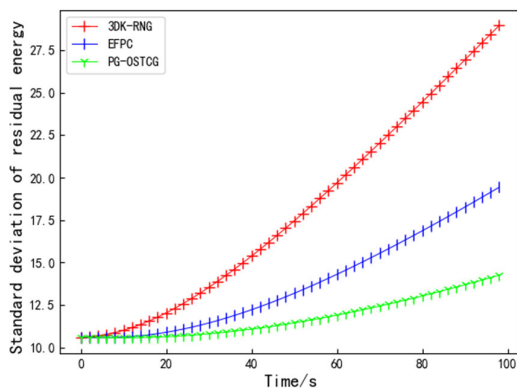


FIGURE 13. Contrast diagram of standard deviation of node residual energy.

is fast, energy consumption of some nodes are large and its imbalance in energy consumption is high. The EFPC algorithm balances the node energy consumption through the node game strategy, and the growth rate of the standard deviation of the node residual energy is relatively slow, and energy consumption balancing capacity of the network is relatively good, while as conditions of energy consumption of sensor nodes are not taken into consideration, standard deviation of residual energy of sensor nodes is larger than that of PG-OSTCG algorithm. When considering balanced energy consumption of node residual energy through game theory, PG-OSTCG algorithm makes use of rigid graph theory and link weight functions with node loads and residual energy to reject redundant links in the network, and conducts a periodical reconstruction on the network to further to avoid the overload of nodes, so variations of standard deviation of residual energy in PG-OSTCG algorithm are small as operational time goes on.

F. ANALYSIS OF NETWORK LIFE CYCLE EXTENSION EFFECT THROUGH PG-OSTCG

It is verified in this section that to extend network life cycle through PG-OSTCG algorithm is the main objective of balanced energy consumption. Thus, under the same experimental simulation environment as mentioned above, network life cycles of 3DK-RNG, EFPC and PG-OSTCG algorithms under different number of nodes are compared.

Figure 14 shows comparison of network life cycles of the 3 algorithms under different node numbers, namely 3DK-RNG, EFPC and PG-OSTCG. It can be seen from the Figure, that, the PG-OSTCG algorithm is superior to 3DK-RNG and EFPC algorithms as the number of nodes increases, which is because 3DK-RNG and EFPC algorithms simply a concern about network topological robustness, but not problems about loads and residual energy of nodes. Although EFPC algorithm reduces node energy consumption through game theory, it fails to balance network energy consumption. The PG-OSTCG algorithm constructed in this paper has an overall consideration of loads and residual energy of nodes as well as problems about redundant links

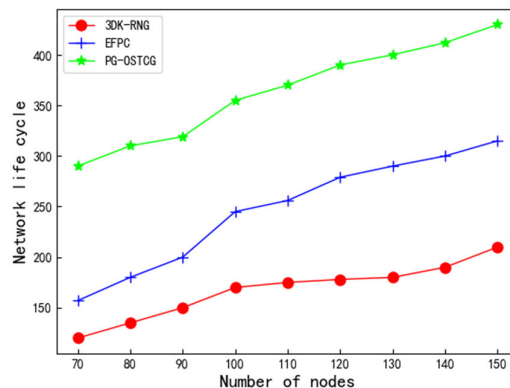


FIGURE 14. Contrast diagram of network life cycle with different number of nodes.

in the network, meanwhile the network has adaptive and periodical reconstructing capacities, so the network life cycle of PG-OSTCG algorithm is superior to that of 3DK-RNG and EFPC algorithms.

VI. CONCLUSION

In this paper, we propose an underwater sensor topology control algorithm based on potential game and optimal rigid sub-graph, with specific contributions as follows:

First of all, on the aspect of network evaluation indexes, factors including connectivity, coverage, transmission energy consumption, end-to-end transmission delay, SINR, transmission success rate and node residual energy, etc. of underwater networks are comprehensively considered in algorithm of this thesis, which are converted to a multi-target game solution process that is more suitable for underwater environments.

Secondly, on the aspect of robustness, optimal rigid sub-graph model is put forward in this thesis, redundant links in the network are rejected, node loads are reduced and the network life cycle is extended through constructing a weighted link containing node loads and residual energy, etc. which gives the network a strong robustness. It is indicated by the simulation experiment that, compared with existing network topology models, the network topology model constructed in this thesis conforms more with underwater environments, has fewer redundant links in its network, a lower node load, a stronger network robustness and balance as well as a longer network life cycle.

Although the network topology model constructed in this thesis is superior to existing network topology models on all aspects, there is no detailed method to regulate weight factors in the model (artificial experience settings are adopted to regulate the weight factors in this thesis). Thus, emphasis of follow-up works of this thesis should be laid on regulating method of weight factors of this model.

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