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Consensus Tracking via Iterative Learning Control for Singular Fractional-Order Multi-Agent Systems Under Iteration-Varying Topologies and Initial State Errors

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ABSTRACT This paper investigates the leader-following consensus tracking problems via iterative learning control for singular fraction-order multi-agent systems in the presence of iteration-varying switching topologies and initial state errors. First, in order to eliminate the impulsive effect of singular systems and handle iteration-varying topologies, the closed-loop $\mathcal{D}^{\alpha}$-type iterative learning control protocol is proposed. To deal with initial state errors, the initial state learning laws are introduced in light of the initial output errors of each follower agent. The developed $\mathcal{D}^{\alpha}$-type learning protocols based on initial state learning laws can guarantee each follower track perfectly the leader agent in the fixed time interval. Next, the sufficient convergent conditions of consensus tracking errors are provided. Moreover, the $\mathcal{D}^{\alpha}$-type learning protocols are extended to nonlinear singular fraction-order multi-agent systems with iteration-varying topologies and initial state errors. Finally, two numerical examples are presented to verify the validity of the proposed $\mathcal{D}^{\alpha}$-type learning scheme in this paper.

INDEX TERMS Iterative learning control, fractional-order, singular multi-agent systems, iteration-varying graphs, initial state errors.

I. INTRODUCTION

During the past decade, consensus analysis and cooperative control of multi-agent systems (MASs) have attracted extensive attention from scholars of different fields on account of their potential applications in several areas such as cooperative transportation by mobile robots [1], flocking [2], and formation control of vehicles [3], and so on. Consensus control has become a fundamental research topic for cooperative control, aiming to drive all follower agents to reach an agreement via the proper consensus protocol after a fixed time [4]. Actually, in some practical consensus scenes, such as satellite formation keeping [5], synchronisation of sensor networks [6], which needs to be achieved as perfectly as possible over a fixed time interval [7].

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Iterative learning control (ILC) has been widely utilized to cope with the repeated tracking control with high precision requirement in the fixed time interval due to its simplicity and effectiveness [8], [9]. Hence, ILC has been successfully implemented to many kinds of multi-agent systems in recent references, such as high-order nonlinear MASs [10], singular MASs [11], fractional-order MASs [12], and distributed parameter MASs [13]–[15], etc.. In [16], [17], the formation control problems of nonlinear MASs under switching interaction topologies were addressed by employing the ILC scheme. To handle the consensus tracking without a priori knowledge of the control direction, a new adaptive iterative learning control protocol was developed for uncertain nonlinear multi-agent systems under the fixed topology in [18]. Recently, the authors have investigated the problem of quantized iterative learning [19]–[21], and some practical factors including the finite-leveled quantizer with random packet losses was considered for the continuous-time MASs in [22]. Besides, in order to eliminate initial state errors and perform accurate consensus tracking, the initial state learning incorporated with ILC protocol was developed for MASs in [23]. However, it is worth noting that the whole aforesaid studies about ILC are focused on normal MASs.

Singular systems, also referred to as generalized state-space systems, semi-state systems and differential-algebraic systems, which can naturally represent a larger class of systems than the normal linear system model [24]. Applications of this class of systems can be extensively found in modeling and control of mechanical systems, interconnected systems, chemical processes, and other fields [25], [27]. Meanwhile, singular MASs, distinguishing from the norml MASs, possesses the characteristics of regularity and impulse behaviour [27]. And the consensus control of singular linear or nonlinear MASs has been reported in few literature, for instance, the non-fragile consensus control [4], the admissible consensus for homogenous descriptor MASs [29], the guaranteed-cost consensus for singular MASs under switching topologies [28]. It should be pointed out that all the aforementioned published works achieve consensus task after a finite time [28]. To accomplish the consensus task over a fixed time interval, the unified D-type iterative learning algorithm was firstly designed for a class of linear singular MASs in both continuous-time and discrete-time domain to ensure the outputs of followers converge to the leader's trajectory [11]. As is well known, fractional calculus has a long history which can be dated back to the 17th century, many researchers from physics, engineering and biology observe that a fruit number of systems can be modelled by fractional-order differential equations, such as, battery behavior, electromagnetic systems, etc. [30], [31]. Meanwhile, the consensus control of factional-order MASs have been widely concerned from different aspects [30]–[32]. Based on the memory property of fractional-order derivative, the $\mathcal{D}^{\alpha}$-type and PI^{β}-type iterative learning control protocols were applied to handle consensus tacking for nonlinear fractional-order MASs with fixed and iteration-varying communicating graphs, respectively [12], [32].

In the aforesaid references, the dynamic of MASs was governed by a normal system, a singular system or a fractional system. Recently, a novel system called singular fractional-order (SFO) system has been proposed [33], which can be considered as generalizations of singular MASs or fractional-order MASs and also has significant practical background, for instance, electrical networks with supercapacitors [34], swamp-floating plants [35], etc.. Till now, only a few meaningful results on above system have been achieved in [35]–[39]. As for the result of SFOMASs, the consensus problem of fractional-order singular MASs with uncertainties under fixed topology was firstly studied by virtue of robust admissible consensus protocols in [33]. However, there are still a great number of challenging and unsolved issues in the filed of SFOMASs, such as the switching topologies and different initial state errors, and so on. To the best of our knowledge, the consensus tracking of SFOMASs via iterative learning control in the presence of iteration-varying switching topologies and initial state errors has not been addressed in the literature yet.

In view of the above discussion, the main purpose of this paper is that the closed-loop $\mathcal{D}^{\alpha}$-type iterative learning update controllers with initial state learning laws are constructed for linear and nonlinear SFOMASs to achieve perfectly consensus tracking performances of the follower agents under iteration-varying switching topologies and initial state errors over a finite time interval. The distinctive features of this paper can be summarized as follows:

1) The consensus tracking of singular fractional-order MASs with iteration-varying switching topologies and initial state errors have been investigated accurately for the first time in this paper. Then the consensus tracking objective and the attenuating ability of impulse effect have been gradually achieved for SFOMASs.

2) The closed-loop $\mathcal{D}^{\alpha}$-type iterative learning control protocols based on initial state lerning laws via the outputs of each follower agent is proposed, which will be more practical than the distributed state protocol due to the fact that outputs are easier to measure. It is worth pointing out that we do not require that the singular fractional-order systems be impulse-free due to the effect of the developed closed-loop $\mathcal{D}^{\alpha}$-type learning algorithm.

3) The sufficient convergence conditions of consensus tracking errors of each follower agent under the proposed $\mathcal{D}^{\alpha}$-type ILC law are derived firstly for linear SFOMASs under fixed communication topology, and then extend the results to nonlinear SFOMASs under iteration-varying switching topologies case.

The layout of the paper is arranged as follows. The necessary preliminary about graph theory, fractional calculus, and useful lemmas are presented in Section 2. In Section 3, the developed ILC protocols are designed and main results on sufficient consensus tracking conditions are shown, respectively. Some numerical examples will be completed to verify that the achieved results are efficient in Section 4. Finally, some conclusions are drawn in Section 5.

Notations: $\mathbb{R}^n$ denotes n-dimensional Euclidean space. The superscript $'\mathrm{T}'$ represents the matrix transposition. The Kronecker product is $\otimes$ and $\mathbf{1}$ describes the column vector with each entry being 1. I is an identity matrix with appropriate dimensions. For A is matrix equipped with the matrix norm $\|A\| = \sqrt{\lambda_{\max}(A^{\mathrm{T}}A)}$, where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of A. For a vector function $\mathbf{Q}(\mathbf{t}) : [0, T] \to \mathbb{R}^n$ and a real constant $\lambda > 0$, the λ-norm is defined as $\|\mathbf{Q}\|_{\lambda} = \sup_{0\leqslant t\leqslant T} \{\|\mathbf{Q}(t)\|e^{-\lambda t}\}$.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. PRELIMINARIES

1) GRAPH THEORY

Let $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ be a weighted directed graph, where consists of the set of vertices $\mathscr{V} = \{1, 2, \ldots, N\}$, the set of edges $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ and the adjacency matrix $\mathscr{A}$. Here $\mathscr{V}$ also be the index set representing the agents in the interaction topology. A direct edge from i to j can be depicted by an ordered pair $(i, j) \in \mathscr{E}$, which means that the agent i can transmit information into the agent j. $\mathscr{A} = (a_{ij}) \in \mathbb{R}^{N \times N}$ denotes the weighted adjacency matrix of the graph $\mathscr{G}$, which is defined as $a_{ii} = 0$ and $a_{ij} > 0 \Leftrightarrow (j, i) \in \mathscr{E}$. Accordingly, denote $\mathscr{L} = \mathscr{D} - \mathscr{A}$ be the Laplacian matrix of the digraph $\mathscr{G}$, where $\mathscr{D} = diag\{d_1, d_2, \ldots, d_N\}$ with $d_i = \sum_{j=1}^{N} a_{ij}$ for $i \in \mathscr{V}$. A graph is said to contain a spanning tree, that there is a vertex called as the root such that exists a directed path from the root to all other vertex in the graph $\mathscr{G}$.

2) FRACTIONAL-ORDER INTEGRALS AND DERIVATIVES

Introducing a positive real number α, the Riemann-Liouville fractional-order integral is defined as

$$I^{\alpha} f(t) \triangleq \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad t > 0, \ \alpha \in \mathbb{R}^+,$$

where $\Gamma(\cdot)$ denotes the Gamma function.

An alternative definition for the fractional-order derivative is introduced by Caputo as follow:

$$\mathcal{D}^{\alpha} f(t) \triangleq I^{m-\alpha} \mathcal{D}^m f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau,$$

where $m - 1 < \alpha < m$, $m \in \mathbb{N}$.

B. PROBLEM FORMULATION

Consider the singular fractional-order multi-agent systems (SFOMASs) consisting of N agents. At the kth iteration, the dynamics of the ith agent is described by:

$$\begin{cases} E\mathcal{D}_t^{\alpha} z_{k,i}(t) = Az_{k,i}(t) + Bu_{k,i}(t), \\ y_{k,i}(t) = Cz_{k,i}(t), \end{cases} \tag{1}$$

where k denotes the iteration index; $i \in \mathscr{V}$ represents the ith follower agent; $t \in [0, T]$ is the time variable; $\mathcal{D}_t^{\alpha}(\cdot)$ denotes the Caputo fractional derivative; $z_{k,i}(t) \in \mathbb{R}^n$, $u_{k,i}(t) \in \mathbb{R}^m$ and $y_{k,i}(t) \in \mathbb{R}^m$ represent the state, control input and output, respectively; $E \in \mathbb{R}^{n \times n}$ is a singular matrix and $0 < rank(E) = r < n$; $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{m \times n}$ are the constant matrices. Throughout this study, the SFOMASs (1) is assumed to satisfy the regularity condition, i.e. $\det(s^{\alpha}E - A)$ is not identically zero. Regarding above N agents, the interaction topology among them is described by the directed digraph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$.

The desired trajectory for consensus tracking $y_d(t)$ is defined on a finite-time interval $[0, T]$, which is generated by the following dynamics:

$$\begin{cases} E\mathcal{D}_t^{\alpha} z_d(t) = Az_d(t) + Bu_d(t), \\ y_d(t) = Cz_d(t), \end{cases} \tag{2}$$

where $z_d(t)$ and $u_d(t)$ are the desired state and control input, respectively. In particular, we assume that only a subset of followers know the desired trajectory, which can be regarded as a virtual leader and indexed by vertex 0. Together with $\mathscr{G}$, the extended topology including both leader and all followers can be depicted by $\tilde{\mathscr{G}} = (\mathscr{V} \cup \{0\}, \tilde{\mathscr{E}}, \tilde{\mathscr{A}})$, where $\tilde{\mathscr{E}}$ is the corresponding edge set and $\tilde{\mathscr{A}}$ is the adjacency matrix of $\tilde{\mathscr{G}}$.

Before addressing the consensus tracking problem of SFOMASs, the basic Assumptions are given firstly.

Assumption 1: The graph $\tilde{\mathscr{G}}$ contains a spanning tree with the leader as its root.

The control objective of this paper is to design an appropriate iterative algorithm to generate a control input sequence $u_{k,i}(t)$ such that each follower agent can track the leader's trajectory perfectly for all $t \in [0, T]$ as $k \to \infty$, i.e.

$$\lim_{k \to \infty} \left\| y_d(t) - y_{k,i}(t) \right\| = 0, \quad i \in \mathscr{V}. \tag{3}$$

III. MAIN RESULTS

This section contains two subsections. The $\mathcal{D}^{\alpha}$-type ILC updating law and its convergence properties are adequately revealed for linear SFOMASs with initial state errors and the fixed graph in Section III-A. Then, the iteration-varying topology condition is considered. In Section III-B, the theoretical results of consensus tracking are extended to the nonlinear SFOMASs.

A. CONVERGENCE ANALYSIS OF LINEAR SFOMAS

Based on the fixed topology, denote the available information $\gamma_{k,i}(t)$ at the $(k+1)$th iteration for the agent i as

$$\gamma_{k,i}(t) = \sum_{j=1}^{N} a_{ij}(y_{k,i}(t) - y_{k,j}(t)) + s_i(y_d(t) - y_{k,i}(t)), \tag{4}$$

where s_i is the weight between agent i and the leader. If agent i can access the desired trajectory, then $s_i = 1$; otherwise, $s_i = 0$. Let $e_{k,i}(t) = y_d(t) - y_{k,i}(t)$ be the tracking error, then Eq.(4) can be rewritten as

$$\gamma_{k,i}(t) = \sum_{j=1}^{N} a_{ij}(e_{k,j}(t) - e_{k,i}(t)) + s_i e_{k,i}(t). \tag{5}$$

In order to handle the consensus tracking problem of SFOMASs (1), the closed-loop $\mathcal{D}^{\alpha}$-type iterative learning control algorithms for $i \in \mathscr{V}$ are constructed as follow:

$$u_{k+1,i}(t) = u_{k,i}(t) + \Psi_1 \mathcal{D}_t^{\alpha} \gamma_{k+1,i}(t), \tag{6}$$

and the initial state learning mechanism is designed by

$$z_{k+1,i}(0) = z_{k,i}(0) + \Psi_2 \gamma_{k,i}(0), \tag{7}$$

where $\Psi_1 \in \mathbb{R}^{m \times m}$ and $\Psi_2 \in \mathbb{R}^{m \times m}$ denote two learning gain matrices to be designed.

Remark 1: It has been known from Zhang [24,26,39] that impulse terms exist in the response of the singular fractional-order system. The impulse terms may result in control saturation or even deteriorate the system performance, and thereby it is expected to eliminate. Furthermore, with the developed closed-loop $\mathcal{D}^{\alpha}$-type ILC law, the singular fractional-order system can be transformed into a normal system, where the impulsive effects can be removed. For more details, please refer to [26,39].

For simplicity, define the following column stack vectors:

$$\gamma_k(t) = \left[\gamma_{k,1}^{\mathrm{T}}(t), \gamma_{k,2}^{\mathrm{T}}(t), \ldots, \gamma_{k,N}^{\mathrm{T}}(t)\right]^{\mathrm{T}},$$
$$z_k(t) = \left[z_{k,1}^{\mathrm{T}}(t), z_{k,2}^{\mathrm{T}}(t), \ldots, z_{k,N}^{\mathrm{T}}(t)\right]^{\mathrm{T}},$$
$$u_k(t) = \left[u_{k,1}^{\mathrm{T}}(t), u_{k,2}^{\mathrm{T}}(t), \ldots, u_{k,N}^{\mathrm{T}}(t)\right]^{\mathrm{T}},$$
$$e_k(t) = \left[e_{k,1}^{\mathrm{T}}(t), e_{k,2}^{\mathrm{T}}(t), \ldots, e_{k,N}^{\mathrm{T}}(t)\right]^{\mathrm{T}}.$$

Then, Eqs.(5)-(7) via using Kronecker product, one obtains

$$\gamma_k(t) = [(\mathscr{L} + \mathscr{S}) \otimes I_m]\, e_k(t), \tag{8}$$
$$u_{k+1}(t) = u_k(t) + [(\mathscr{L} + \mathscr{S}) \otimes \Psi_1]\, \mathcal{D}_t^{\alpha} e_{k+1}(t), \tag{9}$$
$$z_{k+1}(0) = z_k(0) + [(\mathscr{L} + \mathscr{S}) \otimes \Psi_2]\, e_k(0), \tag{10}$$

where $\mathscr{L}$ is the Laplacian matrix of $\mathscr{G}$ in kth iteration, $\mathscr{S} = diag\{s_1, s_2, \cdots s_N\}$ is associated with $\tilde{\mathscr{G}}$.

Introducing $\delta z_{k,i}(t) = z_d(t) - z_{k,i}(t)$ and $\delta u_{k,i}(t) = u_d(t) - u_{k,i}(t)$, and it can be obtained form Eq.(1) and Eq.(2) that

$$E\mathcal{D}_t^{\alpha} \delta z_{k,i}(t) = A\delta z_{k,i}(t) + B\delta u_{k,i}(t). \tag{11}$$

Define the following vectors:

$$\delta z_k(t) = \left[\delta z_{k,1}^{\mathrm{T}}(t), \delta z_{k,2}^{\mathrm{T}}(t), \ldots, \delta z_{k,N}^{\mathrm{T}}(t)\right]^{\mathrm{T}},$$
$$\delta u_k(t) = \left[\delta u_{k,1}^{\mathrm{T}}(t), \delta u_{k,2}^{\mathrm{T}}(t), \ldots, \delta u_{k,N}^{\mathrm{T}}(t)\right]^{\mathrm{T}}.$$

Then, Eq.(11) can be rewritten in the compact form

$$(I_N \otimes E)\mathcal{D}_t^{\alpha} \delta z_k(t) = (I_N \otimes A)\delta z_k(t) + (I_N \otimes B)\delta u_k(t). \tag{12}$$

To investigate the variation of $\delta u_k(t)$ between two consecutive iterations from the Eq.(9), one gets

$$\begin{aligned}\delta u_k(t) &= \delta u_{k-1}(t) - [(\mathscr{L} + \mathscr{S}) \otimes \Psi_1]\, \mathcal{D}_t^{\alpha} e_k(t)\\ &= \delta u_{k-1}(t) - [(\mathscr{L} + \mathscr{S}) \otimes \Psi_1]\, (I_N \otimes C)\mathcal{D}_t^{\alpha} \delta z_k(t)\\ &= \delta u_{k-1}(t) - [(\mathscr{L} + \mathscr{S}) \otimes \Psi_1 C]\, \mathcal{D}_t^{\alpha} \delta z_k(t).\end{aligned} \tag{13}$$

At the same time, it can be obtained from Eq.(10) that

$$\delta z_{k+1}(0) = \delta z_k(0) - [(\mathscr{L} + \mathscr{S}) \otimes \Psi_2]\, e_k(0). \tag{14}$$

Substituting Eq.(13) into Eq.(12) results in

$$\begin{aligned}(I_N \otimes E)\mathcal{D}_t^{\alpha} \delta z_k(t) &= (I_N \otimes A)\delta z_k(t) + (I_N \otimes B)(\delta u_{k-1}(t)\\ &\quad -((\mathscr{L} + \mathscr{S}) \otimes \Psi_1 C)\mathcal{D}_t^{\alpha} \delta z_k(t))\\ &= (I_N \otimes A)\delta z_k(t) + (I_N \otimes B)\delta u_{k-1}(t)\\ &\quad - [(\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C]\, \mathcal{D}_t^{\alpha} \delta z_k(t),\end{aligned}$$

which implies

$$\begin{aligned}&[(I_N \otimes E) + ((\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C)]\, \mathcal{D}_t^{\alpha} \delta z_k(t)\\ &\qquad = (I_N \otimes A)\delta z_k(t) + (I_N \otimes B)\delta u_{k-1}(t).\end{aligned}$$

There exists a learning gain matrix Ψ_1 such that $[(I_N \otimes E) + ((\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C)]$ is non-singular, one has

$$\mathcal{D}_t^{\alpha} \delta z_k(t) = \tilde{A}\delta z_k(t) + \tilde{B}\delta u_{k-1}(t), \tag{15}$$

with $\tilde{A} = [(I_N \otimes E) + ((\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C)]^{-1}(I_N \otimes A)$, $\tilde{B} = [(I_N \otimes E) + ((\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C)]^{-1}(I_N \otimes B)$.

Now, the following useful Lemmas are given, which will be utilized in the proof of main theorems.

Lemma 1: (see [31]) If the function $f(x, t)$ is continuous, then the initial value problem

$$\begin{aligned}{}_{t_0}^{C}\mathcal{D}_t^{\alpha} x(t) &= f(x_t, t), \quad 0 < \alpha < 1,\\ x(t_0) &= \varphi.\end{aligned}$$

is equivalent to the following nonlinear Volterra integral equation

$$x(t) = x(t_0) + \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t - \tau)^{\alpha-1} f(x_\tau, \tau) d\tau.$$

and its solutions are continuous.

Lemma 2: (see [40]) Suppose that two non-negative real series $\{a_k\}_{k=0}^{\infty}$ and $\{b_k\}_{k=0}^{\infty}$, where a_k and b_k are bounded for any given integer $k \geqslant 0$, and satisfying

$$0 \leqslant a_{k+1} \leqslant ra_k + b_k,$$

with $0 \leqslant r < 1$, $\lim_{k\to\infty} b_k = 0$, then one has

$$\lim_{k\to\infty} a_k = 0.$$

Lemma 3: Consider the linear SFOMASs (1) under the fixed graph and Assumption 1 holds, the following estimation is obtained

$$\|\Delta\delta z_k\|_{\lambda} \leqslant 2a_2\|e_k\|_{\lambda} + 2a_1\, \|e_k(0)\|\, e^{-\lambda T},$$

where $\Delta\delta z_k(t)$ is defined as $\delta z_{k+1}(t) - \delta z_k(t)$, and

$$a_1 = \|\tilde{B}\, [(\mathscr{L} + \mathscr{S}) \otimes \Psi_1] - (\mathscr{L} + \mathscr{S}) \otimes \Psi_2\|,$$
$$\text{and} \quad a_2 = \|\tilde{B}\|\, \|(\mathscr{L} + \mathscr{S}) \otimes \Psi_1\|.$$

Proof: With Lemma 1, integrating both sides of Eq.(15) from 0 to t and combining with Eq.(14) yields,

$$\delta z_k(t) = \delta z_k(0) + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{\tilde{A}\delta z_k(\tau) + \tilde{B}\delta u_{k-1}(\tau)}{(t - \tau)^{1-\alpha}} d\tau.$$

Accordingly, one has

$$\begin{aligned}\Delta\delta z_k(t) &= \delta z_{k+1}(0) - \delta z_k(0)\\ &\quad + \frac{\tilde{A}}{\Gamma(\alpha)} \int_0^t \frac{\delta z_{k+1}(\tau) - \delta z_k(\tau)}{(t - \tau)^{1-\alpha}} d\tau\\ &\quad + \frac{\tilde{B}}{\Gamma(\alpha)} \int_0^t \frac{\delta u_k(\tau) - \delta u_{k-1}(\tau)}{(t - \tau)^{1-\alpha}} d\tau.\end{aligned} \tag{16}$$

Substituting Eq.(13) and Eq.(14) into Eq.(16) results in

$$\begin{aligned}&\Delta\delta z_k(t)\\ &= -\, [(\mathscr{L} + \mathscr{S}) \otimes \Psi_2]\, e_k(0)\\ &\quad + \frac{\tilde{A}}{\Gamma(\alpha)} \int_0^t \frac{\Delta\delta z_k(\tau)}{(t - \tau)^{1-\alpha}} d\tau\end{aligned}$$

$$\begin{aligned}
&-\frac{\tilde{B}[(\mathscr{L}+\mathscr{S})\otimes\Psi_1]}{\Gamma(\alpha)}\int_0^t \frac{\mathcal{D}_t^{\alpha}e_k(t)}{(t-\tau)^{1-\alpha}}d\tau\\
&=-[(\mathscr{L}+\mathscr{S})\otimes\Psi_2]\,e_k(0)\\
&\quad-\tilde{B}[(\mathscr{L}+\mathscr{S})\otimes\Psi_1](e_k(t)-e_k(0))\\
&\quad+\frac{\tilde{A}}{\Gamma(\alpha)}\int_0^t\frac{\Delta\delta z_k(\tau)}{(t-\tau)^{1-\alpha}}d\tau\\
&=\Big[\tilde{B}((\mathscr{L}+\mathscr{S})\otimes\Psi_1)-(\mathscr{L}+\mathscr{S})\otimes\Psi_2\Big]e_k(0)\\
&\quad-\tilde{B}[(\mathscr{L}+\mathscr{S})\otimes\Psi_1]\,e_k(t)+\frac{\tilde{A}}{\Gamma(\alpha)}\int_0^t\frac{\Delta\delta z_k(\tau)}{(t-\tau)^{1-\alpha}}d\tau.
\end{aligned}\tag{17}$$

Introducing $a_1=\|\tilde{B}[(\mathscr{L}+\mathscr{S})\otimes\Psi_1]-(\mathscr{L}+\mathscr{S})\otimes\Psi_2\|$, a_2=$\|\tilde{B}\|\,\|(\mathscr{L}+\mathscr{S})\otimes\Psi_1\|$ and $a_3=\|\tilde{A}\|$, and taking norm operations on both sides of Eq.(17), one has

$$\begin{aligned}
\|\Delta\delta z_k(t)\|&\leqslant a_1\|e_k(0)\|+a_2\|e_k(t)\|\\
&\quad+\frac{a_3}{\Gamma(\alpha)}\int_0^t\frac{\|\Delta\delta z_k(\tau)\|}{(t-\tau)^{1-\alpha}}d\tau.
\end{aligned}\tag{18}$$

Furthermore, multiplying $e^{-\lambda t}$ on both sides of Ineq.(18), one can obtain

$$\begin{aligned}
&\|\Delta\delta z_k(t)\|\,e^{-\lambda t}\\
&\quad\leqslant a_1\|e_k(0)\|\,e^{-\lambda t}+a_2\|e_k(t)\|\,e^{-\lambda t}\\
&\qquad+\frac{a_3}{\Gamma(\alpha)}e^{-\lambda t}\int_0^t\frac{e^{\lambda\tau}e^{-\lambda\tau}\|\Delta\delta z_k(\tau)\|}{(t-\tau)^{1-\alpha}}d\tau\\
&\quad\leqslant a_1\|e_k(0)\|\,e^{-\lambda t}+a_2\|e_k\|_\lambda\\
&\qquad+\frac{a_3}{\Gamma(\alpha)}e^{-\lambda t}\int_0^t\frac{e^{\lambda\tau}}{(t-\tau)^{1-\alpha}}d\tau\,\|\Delta\delta z_k\|_\lambda.
\end{aligned}\tag{19}$$

According to Hölder inequality, select an appropriate $p\in(1,\frac{1}{1-\alpha})$ such that

$$\int_0^t(t-\tau)^{\alpha-1}e^{\lambda\tau}d\tau\leqslant\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,t^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}e^{\lambda t},\tag{20}$$

where $\frac{1}{p}+\frac{1}{q}=1$ and $p,q>0$.

Combining Ineq.(19) and Ineq.(20), one gives

$$\begin{aligned}
&\|\Delta\delta z_k(t)\|\,e^{-\lambda t}\\
&\quad\leqslant a_1\|e_k(0)\|\,e^{-\lambda t}+a_2\|e_k\|_\lambda\\
&\qquad+\frac{a_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,t^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}\|\Delta\delta z_k\|_\lambda.
\end{aligned}\tag{21}$$

Next, taking supremum for Ineq.(21) w.r.t. t, one has

$$\begin{aligned}
&\|\Delta\delta z_k\|_\lambda\\
&\quad\leqslant a_1\|e_k(0)\|\,e^{-\lambda T}+a_2\|e_k\|_\lambda\\
&\qquad+\frac{a_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}\|\Delta\delta z_k\|_\lambda.
\end{aligned}\tag{22}$$

Obviously, for selecting λ sufficient large enough, i.e.,

$$\lambda\geqslant\Theta_1:=\frac{1}{q}\left(\frac{1}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,t^{\frac{1}{p}-(1-\alpha)}2a_3\right)^q.\tag{23}$$

Then, one has

$$\|\Delta\delta z_k\|_\lambda\leqslant 2a_2\|e_k\|_\lambda+2a_1\|e_k(0)\|\,e^{-\lambda T}.\tag{24}$$

□

Based on the Assumptions and Lemmas given above, the main results of this paper are presented as follow.

Theorem 1: For the linear SFOMASs (1) under the iterative learning algorithm Eq.(7), if the learn gain Ψ_2 for initial state learning process satisfies

$$\|I_{mN}-(\mathscr{L}+\mathscr{S})\otimes C\Psi_2\|<1,$$

then one obtains

$$\lim_{k\to\infty}\|e_k(0)\|=0.$$

Proof: According to the definition of $e_k(t)$, one gives

$$\begin{aligned}
e_{k+1}(0)&=e_k(0)+y_k(0)-y_{k+1}(0)\\
&=e_k(0)+(I_N\otimes C)(z_k(0)-z_{k+1}(0))\\
&=e_k(0)-[(\mathscr{L}+\mathscr{S})\otimes C\Psi_2]\,e_k(0)\\
&=[I_{mN}-(\mathscr{L}+\mathscr{S})\otimes C\Psi_2]\,e_k(0).
\end{aligned}\tag{25}$$

Tanking norm on both sides of Eq.(25), one yields

$$\|e_{k+1}(0)\|\leqslant\|I_{mN}-(L+S)\otimes C\Psi_2\|\,\|e_k(0)\|.$$

Thus, with the convergence condition of Theorem 1, one obtains

$$\lim_{k\to\infty}\|e_k(0)\|=0.$$

The proof of Theorem 1 is completed. □

Theorem 2: Considering linear SFOMASs (1) with the iterative learning algorithm Eq.(9) and the learning gain Ψ_2 meets the requirement in Theorem 1, and Assumption 1 is satisfied. Then, if there exists the gain matrix Ψ_1 satisfying

$$\|I_{mN}-\tilde{B}[(\mathscr{L}+\mathscr{S})\otimes C\Psi_1]\|<0.5,$$

with $\tilde{B}=[(I_N\otimes E)+((\mathscr{L}+\mathscr{S})\otimes B\Psi_1C)]^{-1}(I_N\otimes B)$, *the control goal can achieve, i.e.,*

$$\lim_{k\to\infty}\|e_k(t)\|=0.$$

Proof: Note that the tracking error of the ith agent is

$$\begin{aligned}
&e_{k+1,i}(t)\\
&=e_{k,i}(t)-(y_{k+1,i}(t)-y_{k,i}(t))\\
&=e_{k,i}(t)+(y_d(t)-y_{k+1,i}(t))-(y_d(t)-y_{k,i}(t))\\
&=e_{k,i}(t)+(Cz_d(t)-Cz_{k+1,i}(t))-(Cz_d(t)-Cz_{k,i}(t))\\
&=e_{k,i}(t)+C\delta z_{k+1,i}(t)-C\delta z_{k,i}(t).
\end{aligned}\tag{26}$$

Then, it can be rewritten as

$$e_{k+1}(t)=e_k(t)+(I_N\otimes C)(\delta z_{k+1}(t)-\delta z_k(t)).\tag{27}$$

Substituting Eq.(17) into Eq.(27), then one has

$$\begin{aligned}
&e_{k+1}(t)\\
&= e_k(t) - (I_N \otimes C)\tilde{B}\left[(\mathscr{L}+\mathscr{S})\otimes \Psi_1\right] e_k(t)\\
&\quad +(I_N \otimes C)\left[\tilde{B}((\mathscr{L}+\mathscr{S})\otimes \Psi_1)-(\mathscr{L}+\mathscr{S})\otimes\Psi_2\right] e_k(0)\\
&\quad +\frac{(I_N\otimes C)\tilde{A}}{\Gamma(\alpha)}\int_0^t \frac{\Delta\delta z_k(\tau)}{(t-\tau)^{1-\alpha}}d\tau)\\
&= \left[\tilde{B}((\mathscr{L}+\mathscr{S})\otimes C\Psi_1) - (\mathscr{L}+\mathscr{S})\otimes C\Psi_2\right] e_k(0)\\
&\quad +\left[I_{mN} - \tilde{B}((\mathscr{L}+\mathscr{S})\otimes C\Psi_1)\right] e_k(t)\\
&\quad +\frac{(I_N\otimes C)\tilde{A}}{\Gamma(\alpha)}\int_0^t \frac{\Delta\delta z_k(\tau)}{(t-\tau)^{1-\alpha}}d\tau.
\end{aligned} \tag{28}$$

Introducing the following notations

$$\begin{aligned}
b_1 &= \|\tilde{B}\left[(\mathscr{L}+\mathscr{S})\otimes C\Psi_1\right] - (\mathscr{L}+\mathscr{S})\otimes C\Psi_2\|,\\
b_2 &= \|I_{mN} - \tilde{B}\left[(\mathscr{L}+\mathscr{S})\otimes C\Psi_1\right]\|,\\
b_3 &= \|I_N\otimes C\|\|\tilde{A}\|,
\end{aligned}$$

and taking norm on Eq.(28) derives

$$\|e_{k+1}(t)\| \leqslant b_1\|e_k(0)\| + b_2\|e_k(t)\| + \frac{b_3}{\Gamma(\alpha)}\int_0^t \frac{\|\Delta\delta z_k(\tau)\|}{(t-\tau)^{1-\alpha}}d\tau. \tag{29}$$

Then, multiplying $e^{-\lambda t}$ on both sides of Ineq.(29) yields,

$$\begin{aligned}
&\|e_{k+1}(t)\|\,e^{-\lambda t}\\
&\leqslant b_1\|e_k(0)\|\,e^{-\lambda t} + b_2\|e_k(t)\|\,e^{-\lambda t}\\
&\quad +\frac{b_3}{\Gamma(\alpha)}e^{-\lambda t}\int_0^t \frac{e^{\lambda\tau}e^{-\lambda\tau}\,\|\Delta\delta z_k(\tau)\|}{(t-\tau)^{1-\alpha}}d\tau\\
&\leqslant b_1\|e_k(0)\|\,e^{-\lambda t} + b_2\|e_k\|_\lambda\\
&\quad +\frac{b_3}{\Gamma(\alpha)}e^{-\lambda t}\int_0^t \frac{e^{\lambda\tau}}{(t-\tau)^{1-\alpha}}d\tau\,\|\Delta\delta z_k\|_\lambda\\
&\leqslant b_1\|e_k(0)\|\,e^{-\lambda t} + b_2\|e_k\|_\lambda\\
&\quad +\frac{b_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,t^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}\|\Delta\delta z_k\|_\lambda,
\end{aligned}$$

which implies that

$$\begin{aligned}
&\|e_{k+1}\|_\lambda\\
&\leqslant b_1\|e_k(0)\|\,e^{-\lambda T} + b_2\|e_k\|_\lambda\\
&\quad +\frac{b_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}\|\Delta\delta z_k\|_\lambda.
\end{aligned} \tag{30}$$

By using Lemma 3, it indicates that

$$\begin{aligned}
&\|e_{k+1}\|_\lambda\\
&\leqslant b_1\|e_k(0)\|\,e^{-\lambda T} + b_2\|e_k\|_\lambda\\
&\quad +\frac{b_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}2a_1\|e_k(0)\|\,e^{-\lambda T}\\
&\quad +\frac{b_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}2a_2\|e_k\|_\lambda\\
&= (b_1 + \frac{2a_1b_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}})\\
&\quad\times\|e_k(0)\|\,e^{-\lambda T}\\
&\quad +(b_2 + \frac{2a_2b_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}})\|e_k\|_\lambda.
\end{aligned} \tag{31}$$

Finally, selecting some sufficient large λ such that both Ineq.(23) and the following one meets,

$$\lambda \geqslant \Theta_2 \coloneqq \frac{1}{q}\left(\frac{1}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}\,t^{\frac{1}{p}-(1-\alpha)}\frac{2a_2b_3}{b_2}\right)^q. \tag{32}$$

Therefore,

$$\|e_{k+1}\|_\lambda \leqslant 2b_2\|e_k\|_\lambda + (b_1 + \frac{a_1b_2}{a_2})\,\|e_k(0)\|\,e^{-\lambda T}. \tag{33}$$

Since the initial state learning mechanism Eq.(14),

$$\lim_{k\to\infty}\|e_k(0)\|=0. \tag{34}$$

According to Lemma 2, if $b_2 < 0.5$, one has

$$\lim_{k\to\infty}\|e_k\|_\lambda=0. \tag{35}$$

The proof of Theorem 2 is completed. □

Actually, the fixed topology is difficult and restrictive to achieve for multi-agent systems. The fixed graph extended to the iteration-varying graph, which means that the controller is more robust to topology variations. The iteration-varying graphs are defined as follows:

$$\mathscr{H}_k = \mathscr{L}_k + \mathscr{S}_k, \tag{36}$$

where $\mathscr{L}_k$ is the Laplacian matrix of $\mathscr{G}$ in kth iteration, $\mathscr{S}_k = diag\{s_{k,1}, s_{k,2}, \cdots s_{k,N}\}$. If agent i can access the desired trajectory, then $s_{k,i}=1$; otherwise, $s_{k,i}=0$, $i=1,2,\ldots,N$ is associated with $\bar{\mathscr{G}}$. Now Eqs.(8)-(10) are written in an iteration-varying form as follows:

$$\gamma_k(t) = (\mathscr{H}_k\otimes I_m)e_k(t), \tag{37}$$

$$u_{k+1}(t) = u_k(t) + (\mathscr{H}_k\otimes\Psi_1)\mathcal{D}_t^\alpha e_{k+1}(t), \tag{38}$$

$$z_{k+1}(0) = z_k(0) + (\mathscr{H}_k\otimes\Psi_2)e_k(0). \tag{39}$$

Corollary 1: For the system (1) with the iterative learning algorithm Eq.(9) and the initial state learning mechanism Eq.(10), Assumption 1 is satisfied. Then the consensus tracking objective (3) holds if there exists the gain matrix satisfying

$$\|I_{mN} - \tilde{B}(\mathscr{H}_k\otimes C\Psi_1)\| < 0.5, \tag{40}$$

with $\tilde{B} = [(I_N\otimes E) + (\mathscr{H}_k\otimes B\Psi_1C)]^{-1}(I_N\otimes B)$.

Proof: The proof is similar to Theorem 2. Firstly, one can obtain the following agents' state trajectories via Eqs.(38) and (39):

$$\begin{aligned}
&\Delta\delta z_k(t)\\
&= \left[\tilde{B}(\mathscr{H}_k\otimes\Psi_1) - (\mathscr{H}_k\otimes\Psi_2)\right] e_k(0)\\
&\quad -\tilde{B}(\mathscr{H}_k\otimes\Psi_1)e_k(t) + \frac{\tilde{A}}{\Gamma(\alpha)}\int_0^t \frac{\Delta\delta z_k(\tau)}{(t-\tau)^{1-\alpha}}d\tau.
\end{aligned} \tag{41}$$

From Eq.(41), one obtains

$$\|\Delta\delta z_k\|_\lambda \leqslant 2\tilde{a}_2\|e_k\|_\lambda + 2\tilde{a}_1 \|e_k(0)\| e^{-\lambda T}, \quad (42)$$

with $\tilde{a}_1 = \|\tilde{B}(\mathscr{H}_k \otimes \Psi_1) - (\mathscr{H}_k \otimes \Psi_2)\|$ and $\tilde{a}_2 = \|\tilde{B}\|\|\mathscr{H}_k \otimes \Psi_1\|$.

Substituting Eq.(41) into Eq.(27), one has

$$\begin{aligned} e_{k+1}(t) &= \left[\tilde{B}(\mathscr{H}_k \otimes C\Psi_1) - (\mathscr{H}_k \otimes C\Psi_2)\right] e_k(0) \\ &\quad + \left[I_{mN} - \tilde{B}(\mathscr{H}_k \otimes C\Psi_1)\right] e_k(t) \\ &\quad + \frac{(I_N \otimes C)\tilde{A}}{\Gamma(\alpha)} \int_0^t \frac{\Delta\delta z_k(\tau)}{(t-\tau)^{1-\alpha}} d\tau. \end{aligned} \quad (43)$$

Taking λ norm on both sides on Eq.(43) yields,

$$\|e_{k+1}\|_\lambda \leqslant 2\tilde{b}_2\|e_k\|_\lambda + (\tilde{b}_1 + \frac{\tilde{a}_1\tilde{b}_2}{\tilde{a}_2}) \|e_k(0)\| e^{-\lambda T}. \quad (44)$$

where $\tilde{b}_1 = \|\tilde{B}(\mathscr{H}_k \otimes C\Psi_1) - (\mathscr{H}_k \otimes C\Psi_2)\|$ and $\tilde{b}_2 = \|I_{mN} - \tilde{B}(\mathscr{H}_k \otimes C\Psi_1)\|$.

Finally, according to Lemma 2 and Eq.(40), we deduce $\lim_{k\to\infty} \|e_k\|_\lambda = 0$.

The proof is completed. □

B. EXTENSION TO NONLINEAR SFOMAS

As an extension of the previous subsection, the $\mathcal{D}^\alpha$-type updating law for nonlinear SFOMASs with the fixed/iteration-varying graph and initial state error is formulated in this section.

The dynamics of the ith agent at kth iteration as follow

$$\begin{cases} E\mathcal{D}_t^\alpha z_{k,i}(t) = Az_{k,i}(t) + f(z_{k,i}(t), t) + Bu_{k,i}(t), \\ y_{k,i}(t) = Cz_{k,i}(t), \end{cases} \quad (45)$$

where $f(z_{k,i}(t), t) \in \mathbb{R}^n$ represents a continuous nonlinear function about $z_{k,i}(t)$, the following assumption is satisfied:

Assumption 2: $f(z, t)$ satisfies Lipschitz conditions, which means that for any $u, v \in \mathbb{R}^n$, there exists a constant $l_f > 0$ such that

$$\|f(u, t) - f(v, t)\| < l_f \|u - v\|.$$

The desired consensus tracking trajectory is generated by the following dynamics:

$$\begin{cases} E\mathcal{D}_t^\alpha z_d(t) = Az_d(t) + f(z_d(t), t) + Bu_d(t), \\ y_d(t) = Cz_d(t). \end{cases} \quad (46)$$

Introducing $f(\delta z_{k,i}(t), t) = f(z_d(t), t) - f(z_{k,i}(t), t)$ and defining the following vectors:

$$\begin{aligned} &f(\delta z_k(t), t) \\ &= \left[f^{\mathrm{T}}(\delta z_{k,1}(t), t), f^{\mathrm{T}}(\delta z_{k,2}(t), t), \ldots, f^{\mathrm{T}}(\delta z_{k,N}(t), t)\right]^{\mathrm{T}}. \end{aligned}$$

It follows from Eq.(45) and Eq.(46) that

$$E\mathcal{D}_t^\alpha \delta z_{k,i}(t) = A\delta z_{k,i}(t) + f(\delta z_{k,i}(t), t) + B\delta u_{k,i}(t), \quad (47)$$

which implies

$$\begin{aligned} &[(I_N \otimes E) + ((\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C)]\, \mathcal{D}_t^\alpha \delta z_k(t) \\ &= (I_N \otimes A)\delta z_k(t) + f(\delta z_k(t), t) \\ &\quad + (I_N \otimes B)\delta u_{k-1}(t). \end{aligned} \quad (48)$$

There exists a learning gain matrix Ψ_1 such that $[(I_N \otimes E) + ((\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C)]$is non-singular, one has

$$\mathcal{D}_t^\alpha \delta z_k(t) = \tilde{A}\delta z_k(t) + \tilde{F}f(\delta z_k(t), t) + \tilde{B}\delta u_{k-1}(t), \quad (49)$$

with $\tilde{F} = [(I_N \otimes E) + ((\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C)]^{-1}$,

$$\begin{aligned} \tilde{A} &= [(I_N \otimes E) + ((\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C)]^{-1}(I_N \otimes A), \\ \tilde{B} &= [(I_N \otimes E) + ((\mathscr{L} + \mathscr{S}) \otimes B\Psi_1 C)]^{-1}(I_N \otimes B). \end{aligned}$$

Lemma 4: Consider the nonlinear SFOMASs (46) under the fixed graph and Assumptions 1 and 2 are satisfied, then the following estimation meets

$$\|\Delta\delta z_k\|_\lambda \leqslant 2a_1 \|e_k(0)\| e^{-\lambda T} + 2a_2\|e_k\|_\lambda,$$

with $a_1 = \|\tilde{B}[(\mathscr{L} + \mathscr{S}) \otimes \Psi_1] - (\mathscr{L} + \mathscr{S}) \otimes \Psi_2\|$ and $a_2 = \|\tilde{B}\| \|(\mathscr{L} + \mathscr{S}) \otimes \Psi_1\|$.

Proof: This proof is similar to Lemma 3. With Lemma 1, integrating both sides of Eq.(49) from 0 to t and combining with Eq.(13), one can obtain the following results

$$\begin{aligned} &\delta z_k(t) \\ &= \delta z_k(0) \\ &\quad + \frac{1}{\Gamma(\alpha)} \int_0^t \frac{\tilde{A}\delta z_k(\tau) + \tilde{F}f(\delta z_k(\tau), \tau) + \tilde{B}\delta u_{k-1}(\tau)}{(t-\tau)^{1-\alpha}} d\tau. \end{aligned} \quad (50)$$

Then, one has

$$\begin{aligned} &\Delta\delta z_k(t) \\ &= \delta z_{k+1}(t) - \delta z_k(t) \\ &= \delta z_{k+1}(0) - \delta z_k(0) + \frac{\tilde{A}}{\Gamma(\alpha)} \int_0^t \frac{\delta z_{k+1}(\tau) - \delta z_k(\tau)}{(t-\tau)^{1-\alpha}} d\tau \\ &\quad + \frac{\tilde{F}}{\Gamma(\alpha)} \int_0^t \frac{f(\delta z_{k+1}(\tau), \tau) - f(\delta z_k(\tau), \tau)}{(t-\tau)^{1-\alpha}} d\tau \\ &\quad + \frac{\tilde{B}}{\Gamma(\alpha)} \int_0^t \frac{\delta u_k(\tau) - \delta u_{k-1}(\tau)}{(t-\tau)^{1-\alpha}} d\tau. \end{aligned} \quad (51)$$

Substituting Eq.(18) and Eq.(19) into Eq.(51) results in

$$\begin{aligned} \Delta\delta z_k(t) &= \left[\tilde{B}((\mathscr{L} + \mathscr{S}) \otimes \Psi_1) - (\mathscr{L} + \mathscr{S}) \otimes \Psi_2\right] e_k(0) \\ &\quad - \tilde{B}[(\mathscr{L} + \mathscr{S}) \otimes \Psi_1] e_k(t) \\ &\quad + \frac{\tilde{A}}{\Gamma(\alpha)} \int_0^t \frac{\Delta\delta z_k(\tau)}{(t-\tau)^{1-\alpha}} d\tau \\ &\quad + \frac{\tilde{F}}{\Gamma(\alpha)} \int_0^t \frac{\Delta f(\delta z_k(\tau), \tau)}{(t-\tau)^{1-\alpha}} d\tau. \end{aligned} \quad (52)$$

For simplicity, denote $\bar{a}_3 = \|\tilde{A}\| + l_f\|\tilde{F}\|$. And taking norm and multiplying the factor $e^{-\lambda t}$ on both sides of above expression, one has

$$\begin{aligned} \|\Delta\delta z_k(t)\| e^{-\lambda t} &\leqslant a_1 \|e_k(0)\| e^{-\lambda t} + a_2\|e_k\|_\lambda \\ &\quad + \frac{\bar{a}_3}{\Gamma(\alpha)} e^{-\lambda t} \int_0^t \frac{e^{\lambda\tau}}{(t-\tau)^{1-\alpha}} d\tau \|\Delta\delta z_k\|_\lambda. \end{aligned} \quad (53)$$

Combining with Ineq.(20) and take supremum on Ineq.(53) on $[0,T]$, it indicates that

$$\begin{aligned}&\|\Delta\delta z_k\|_\lambda\\ &\leqslant a_1\,\|e_k(0)\|\,e^{-\lambda T}+a_2\|e_k\|_\lambda\\ &\quad+\frac{\tilde{a}_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}\|\Delta\delta z_k\|_\lambda. \qquad (54)\end{aligned}$$

Obviously, choosing some λ large enough, i.e.,

$$\lambda\geqslant\Theta_1:=\frac{1}{q}\left(\frac{1}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}t^{\frac{1}{p}-(1-\alpha)}2a_3\right)^q. \qquad (55)$$

Furthermore, one has

$$\|\Delta\delta z_k\|_\lambda\leqslant 2a_2\|e_k\|_\lambda+2a_1\,\|e_k(0)\|\,e^{-\lambda T}. \qquad (56)$$

□

For the nonlinear SFOMASs, the following convergence result is given:

Theorem 3: For the nonlinear SFOMASs (45) with the $\mathcal{D}^\alpha$-type iterative learning algorithm Eq.(9) and the initial state learning mechanism applied with learning gain Ψ_2 in Eq.(10) meets Theorem 1, and Assumptions 1-2 are satisfied. If there exists the gain matrix Ψ_1 satisfying

$$\|I_{mN}-\tilde{B}[(\mathscr{L}+\mathscr{S})\otimes C\Psi_1]\|<0.5,$$

where $\tilde{B}=[(I_N\otimes E)+((\mathscr{L}+\mathscr{S})\otimes B\Psi_1C)]^{-1}(I_N\otimes B)$, the consensus tracking error $e_k(t)$ converges to zero as iteration number $k\to\infty$ for all $t\in[0,T]$.

Proof: Substituting Eq.(52) into Eq.(27) results in

$$\begin{aligned}&e_{k+1}(t)\\ &=\Big[\tilde{B}((\mathscr{L}+\mathscr{S})\otimes C\Psi_1)-((\mathscr{L}+\mathscr{S})\otimes C\Psi_2)\Big]e_k(0)\\ &\quad+\Big[I_{mN}-\tilde{B}((\mathscr{L}+\mathscr{S})\otimes C\Psi_1)\Big]e_k(t)\\ &\quad+\frac{(I_N\otimes C)\tilde{A}}{\Gamma(\alpha)}\int_0^t\frac{\Delta\delta z_k(\tau)}{(t-\tau)^{1-\alpha}}d\tau\\ &\quad+\frac{(I_N\otimes C)\tilde{F}}{\Gamma(\alpha)}\int_0^t\frac{\Delta f(\delta z_k(\tau),\tau)}{(t-\tau)^{1-\alpha}}d\tau. \qquad (57)\end{aligned}$$

Taking norm on Eq.(57) derives

$$\begin{aligned}&\|e_{k+1}(t)\|\\ &\leqslant b_1\,\|e_k(0)\|+b_2\,\|e_k(t)\|\\ &\quad+\frac{\|I_N\otimes C\|\left\|\tilde{A}\right\|+l_f\,\|I_N\otimes C\|\left\|\tilde{F}\right\|}{\Gamma(\alpha)}\int_0^t\frac{\|\Delta\delta z_k(\tau)\|}{(t-\tau)^{1-\alpha}}d\tau. \qquad (58)\end{aligned}$$

Denote $\bar{b}_3=\|I_N\otimes C\|\|\tilde{A}\|+l_f\|I_N\otimes C\|\|\tilde{F}\|$. Multiplying by the factor $e^{-\lambda t}$ on both sides of Ineq.(58), one has

$$\begin{aligned}&\|e_{k+1}(t)\|\,e^{-\lambda t}\\ &\leqslant b_1\,\|e_k(0)\|\,e^{-\lambda t}+b_2\|e_k\|_\lambda\\ &\quad+\frac{\bar{b}_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}t^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}\|\Delta\delta z_k\|_\lambda, \qquad (59)\end{aligned}$$

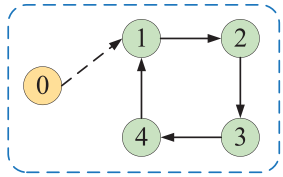


FIGURE 1. Directed fixed topology among agents in the network.

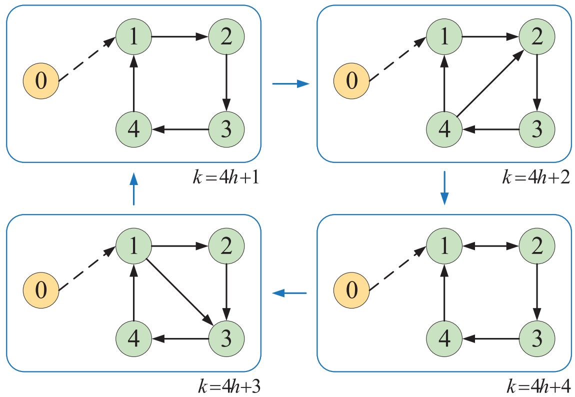


FIGURE 2. Directed iteration-varying topology among agents in the network.

which implies the following results

$$\begin{aligned}&\|e_{k+1}\|_\lambda\\ &\leqslant b_1\,\|e_k(0)\|\,e^{-\lambda T}+b_2\|e_k\|_\lambda\\ &\quad+\frac{\bar{b}_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}}\|\Delta\delta z_k\|_\lambda. \qquad (60)\end{aligned}$$

According to Lemma 4, it indicates that

$$\begin{aligned}&\|e_{k+1}\|_\lambda\\ &\leqslant(b_1+\frac{2a_1\bar{b}_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}})\,\|e_k(0)\|\,e^{-\lambda T}\\ &\quad+(b_2+\frac{2a_2\bar{b}_3}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}T^{\frac{1}{p}-(1-\alpha)}\frac{1}{\sqrt[q]{q\lambda}})\|e_k\|_\lambda. \qquad (61)\end{aligned}$$

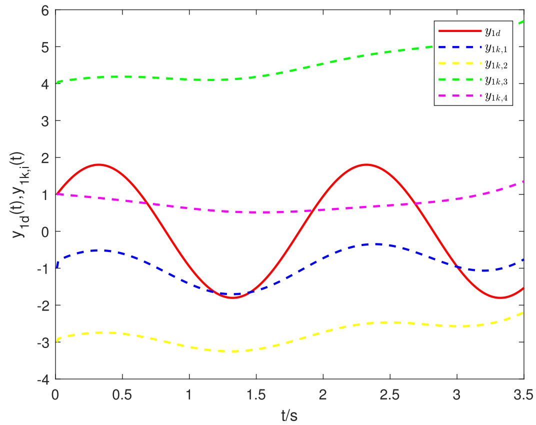
Finally, for some sufficient large λ, satisfying Ineq.(55) and the following one, i.e.,

$$\lambda\geqslant\bar{\Theta}_2:=\frac{1}{q}\left(\frac{1}{\Gamma(\alpha)}\sqrt[p]{\frac{1}{1-p(1-\alpha)}}t^{\frac{1}{p}-(1-\alpha)}\frac{2a_2\bar{b}_3}{b_2}\right)^q, \qquad (62)$$

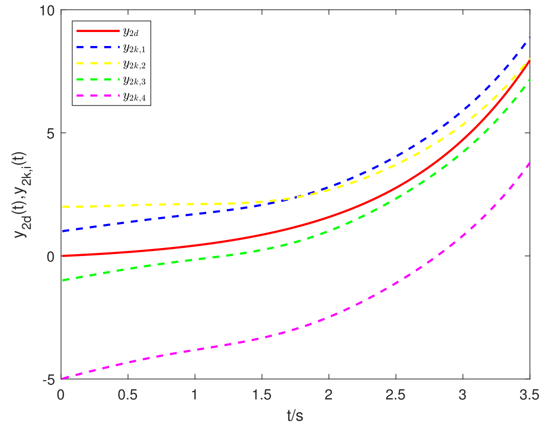
Furthermore, one yields

$$\|e_{k+1}\|_\lambda\leqslant 2b_2\|e_k\|_\lambda+(b_1+\frac{a_1b_2}{a_2})\,\|e_k(0)\|\,e^{-\lambda T}. \qquad (63)$$

The rest of proof is similar to Theorem 2. □

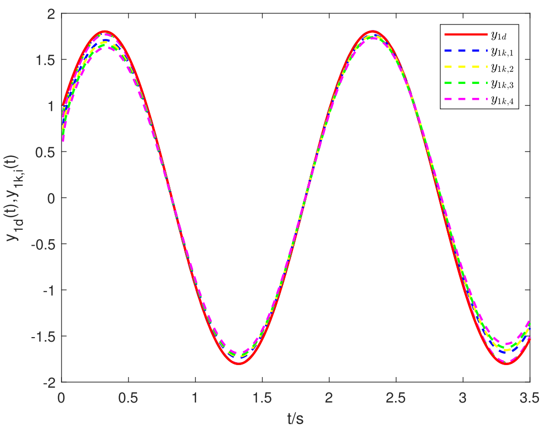


(a) Output trajectory 1 $y_{1k,i}(t), (k = 1)$

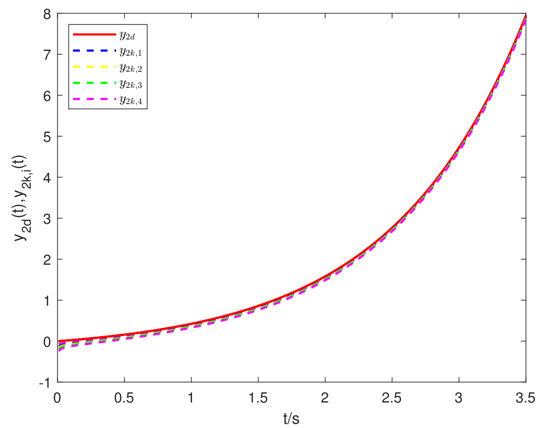


(b) Output trajectory 2 $y_{2k,i}(t), (k = 1)$

FIGURE 3. Output trajectories of all agents at 1st iteration in Example 1.



(a) Output trajectory 1 $y_{1k,i}(t), (k = 20)$



(b) Output trajectory 2 $y_{2k,i}(t), (k = 20)$

FIGURE 4. Output trajectories of all agents at 20th iteration in Example 1.

Corollary 2: For the nonlinear SFOMASs (1) with the iterative learning algorithm Eq.(9) and the initial state learning mechanism Eq.(10), Assumptions 1 and 2 are satisfied. Then the consensus tracking objective (3) holds if there exists the gain matrix satisfying

$$\|I_{mN} - \tilde{B}(\mathscr{H}_k \otimes C\Psi_1)\| < 0.5, \tag{64}$$

where $\tilde{B} = [(I_N \otimes E) + (\mathscr{H}_k \otimes B\Psi_1 C)]^{-1}(I_N \otimes B)$.

Proof: The proof is similar to Corollary 1, we omit it here. □

IV. SIMULATION EXAMPLES

In this section, two simulation results on tracking examples are given to demonstrate the effectiveness of the developed iterative learning algorithm (9) and (10).

In the following examples, we set $\alpha = 0.85$ and the initial state at first iteration is chosen as $z_{1,1}(0) = \begin{bmatrix}-1 & 1\end{bmatrix}^{\mathrm{T}}, z_{1,2}(0) = \begin{bmatrix}-3 & 2\end{bmatrix}^{\mathrm{T}}, z_{1,3}(0) = \begin{bmatrix}4 & -1\end{bmatrix}^{\mathrm{T}}, z_{1,4}(0) = \begin{bmatrix}1 & -5\end{bmatrix}^{\mathrm{T}}$. The initial control signal $u_{1,i}(0) = 0, i = 1, 2, 3, 4$ for all agents.

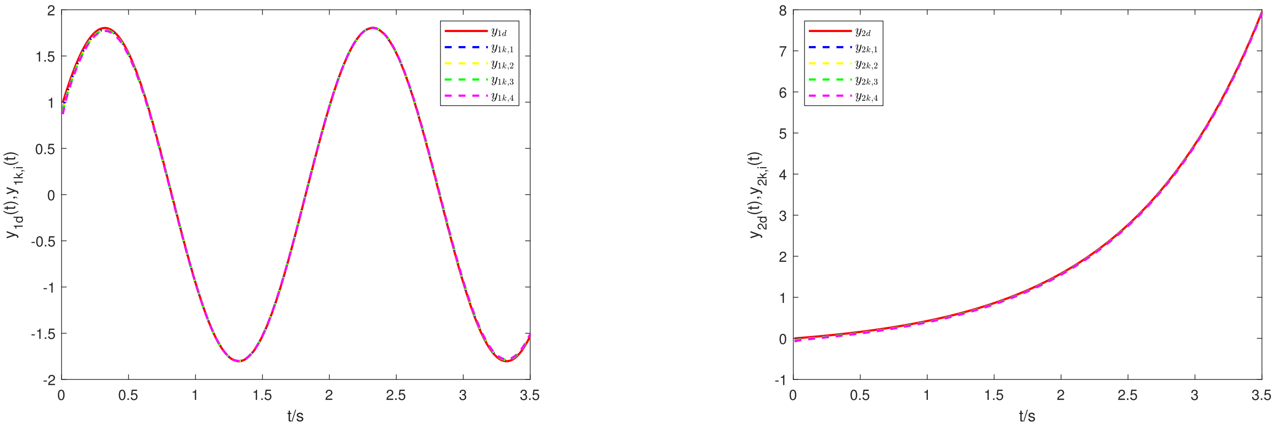
Example 1: Consider the linear SFOMASs with fixed graphs and initial state errors. The dynamics of the ith agent is described by

$$\begin{cases} E\mathcal{D}_t^{\alpha} z_{k,i}(t) = Az_{k,i}(t) + Bu_{k,i}(t), \\ y_{k,i}(t) = Cz_{k,i}(t), \end{cases} \tag{65}$$

where $i = 1, 2, 3, 4$, $t \in [0, 3.5]$ and

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & -0.1 \end{bmatrix},$$
$$B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$z_{k,i}(t) = \begin{bmatrix} z_{1k,i}(t) \\ z_{2k,i}(t) \end{bmatrix}, \quad y_{k,i}(t) = \begin{bmatrix} y_{1k,i}(t) \\ y_{2k,i}(t) \end{bmatrix}.$$

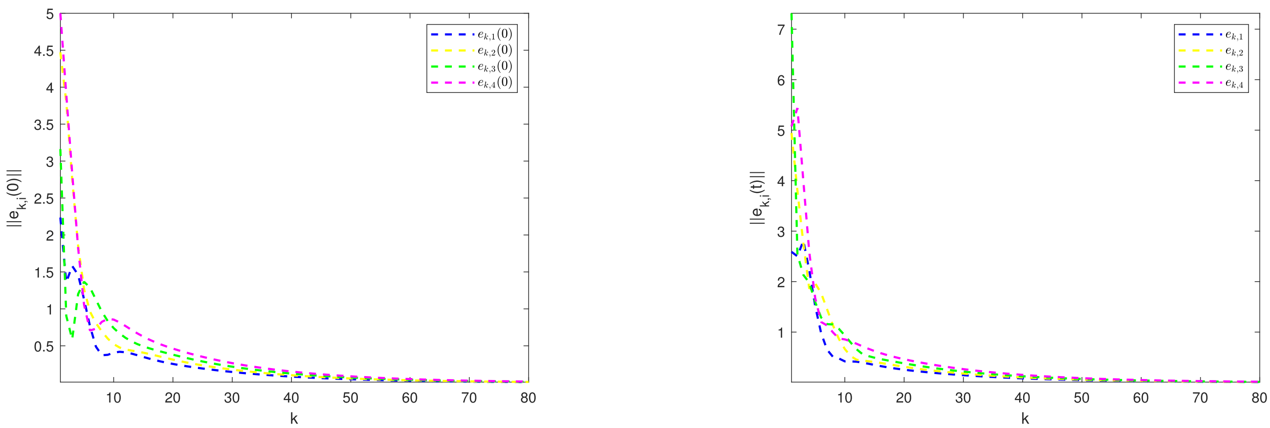
The fixed graph $\mathscr{G}$ in the network is shown in Fig. 1. Vertex 0 represents the leader agent, and only agent 1 can receive the leader's information. The Laplacian matrix for followers



(a) Output trajectory 1 $y_{1k,i}(t)$, $(k = 40)$

(b) Output trajectory 2 $y_{2k,i}(t)$, $(k = 40)$

FIGURE 5. Output trajectories of all agents at 40th iteration in Example 1.



(a) Initial state errors $e_{k,i}(0)$

(b) Tracking errors $e_{k,i}(t)$

FIGURE 6. Initial state errors and tracking errors for all agents in each iteration in Example 1.

is given by

$$\mathscr{L}=\begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

and $\mathscr{S} = diag\{1, 0, 0, 0\}$. It can be seen that the graph $\tilde{\mathscr{G}}$ has a spanning tree with the leader as its root, which satisfies Assumption 1.

According to the $\mathcal{D}^\alpha$-type iterative learning algorithm (9) and (10), the gain matrix is taken as

$$\Psi_1 = \begin{bmatrix} 0.9 & 0 \\ 0 & 1.2 \end{bmatrix} \quad \text{and} \quad \Psi_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

The leader's trajectory is given by

$$y_d = \begin{bmatrix} 1.5\sin(\pi t) + \cos(\pi t) \\ 0.25(e^t - 1) \end{bmatrix}, \quad \forall t \in [0, 3.5].$$

Fig. 3-5 show the output trajectories of all agents at 1st, 10th and 40th iterations, respectively. As the iteration number increases, all output of each follower agent approach consistently to the leader's trajectory. Moreover, Fig. 6 (a) depicts the initial errors along the iteration axis. It is observed that the initial states converge to the desired initial state. It can be seen from Fig. 6 (b) that the consensus tracking is achieved by the proposed learning learning algorithm (9) and (10).

Example 2: In Example 2, the nonlinear SFOMASs with iteration-varying graph and initial state error are considered. Considering the ith agent model as follows:

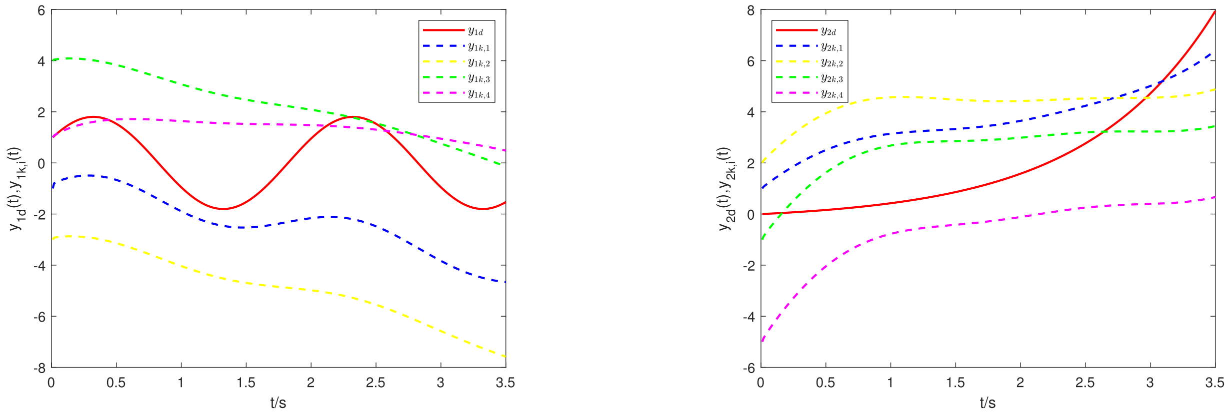
$$\begin{cases} E\mathcal{D}_t^\alpha z_{k,i}(t) = Az_{k,i}(t) + f(z_{k,i}(t), t) + Bu_{k,i}(t), \\ y_{k,i}(t) = Cz_{k,i}(t). \end{cases} \quad (66)$$

where the remain parameter settings are the same as Example 1. Nevertheless, the nonlinear term is expressed as

$$f(z_{k,i}(t), t) = \begin{bmatrix} 0.5\sin(z_{1k,i}(t)) - 0.8z_{2k,i}(t) \\ 0.3z_{1k,i}(t) + 0.2\sin(z_{2k,i}(t)) \end{bmatrix}. \quad (67)$$

Accordingly, it satisfies Assumption 2.

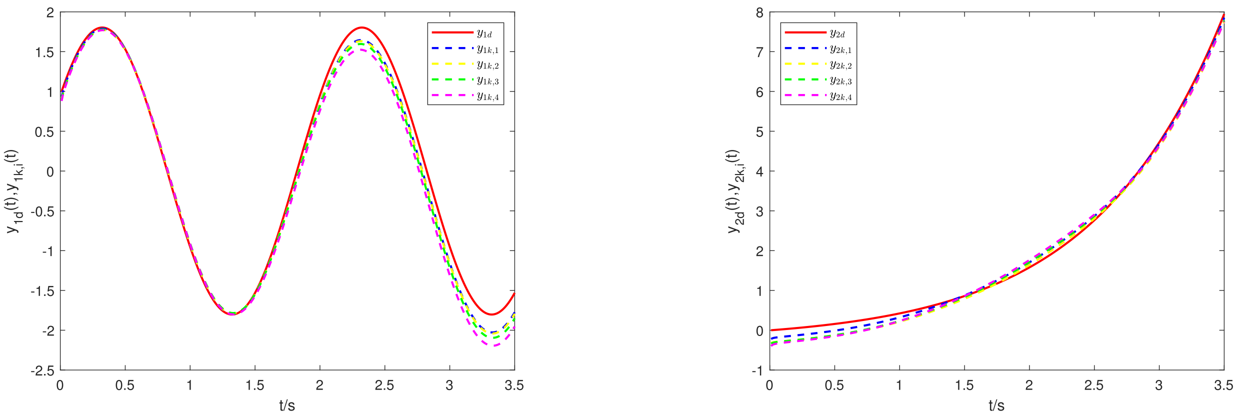
The iteration-varying topologies are depicted in Fig. 2, in which there are four graphs. In each iteration, the graph



(a) Output trajectory 1 $y_{1k,i}(t), (k = 1)$

(b) Output trajectory 2 $y_{2k,i}(t), (k = 1)$

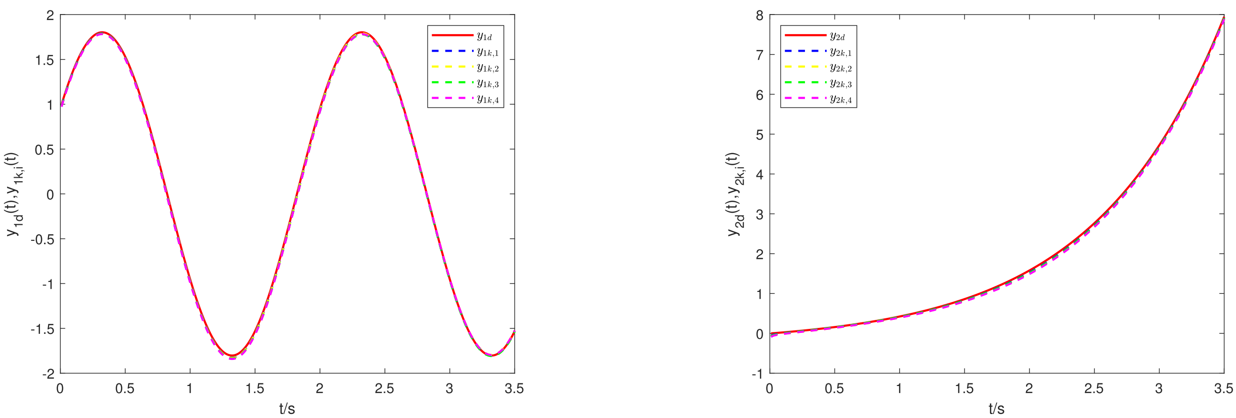
FIGURE 7. **Output trajectories of all agents at 1st iteration in Example 2.**



(a) Output trajectory 1 $y_{1k,i}(t), (k = 30)$

(b) Output trajectory 2 $y_{2k,i}(t), (k = 30)$

FIGURE 8. **Output trajectories of all agents at 30th iteration in Example 2.**



(a) Output trajectory 1 $y_{1k,i}(t), (k = 60)$

(b) Output trajectory 2 $y_{2k,i}(t), (k = 60)$

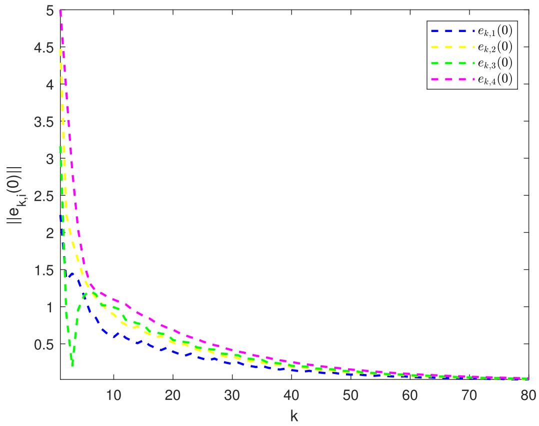
FIGURE 9. **Output trajectories of all agents at the 60th iteration in Example 2.**

is chosen by the selection function $k = 4h + j, j = 1, 2, 3, 4$, and h is a non-negative integer.

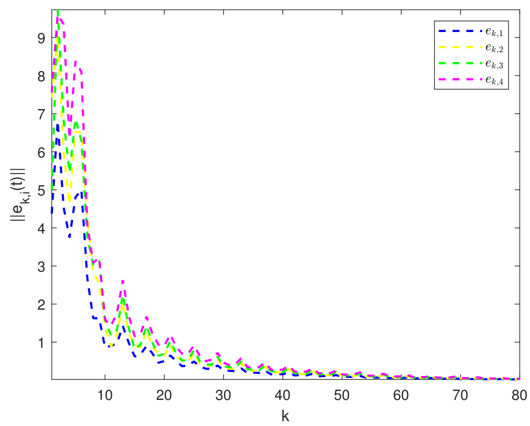
$$\mathscr{H}_1 = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \mathscr{H}_2 = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 2 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

$$\mathscr{H}_3 = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \mathscr{H}_4 = \begin{bmatrix} 2 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

Fig. 7-9 show the output trajectories of all agents in 1st, 10th and 60th iterations, respectively. As the iteration number



(a) Initial state errors $e_{k,i}(0)$



(b) Tracking errors $e_{k,i}(t)$

FIGURE 10. Initial state errors and tracking errors for all agents in each iteration in Example 2.

increases, all output profiles can track the desired trajectory. In view of Fig. 10 (a), all initial states can converge to the desired initial state during the initial state learning process. Fig. 10 (b) depicts the tracking errors in each iteration, which demonstrates the precise tracking performance over a finite time interval.

V. CONCLUSION

This paper has investigated the consensus tracking for both linear and nonlinear singular fractional-order MASs under iteration-varying topologies and initial state errors over a finite time interval. The closed-loop $\mathcal{D}^{\alpha}$-type ILC protocols based on initial state learning laws have been proposed for the follower agents and the update laws depend on the information available from the neighbour agents. Then, with the developed $\mathcal{D}^{\alpha}$-type ILC protocols, the sufficient convergence conditions have been presented and the perfect tracking can be achieved for both linear and nonlinear SFOMASs asymptotically. Two simulation examples demonstrate that the designed $\mathcal{D}^{\alpha}$-type ILC protocols with initial state learning laws in this paper are effective. Future efforts will now turn to add from the respect of robustness against uncertainties which include not only external disturbances but also find the the feasibility condition of Theorem 3 cases.

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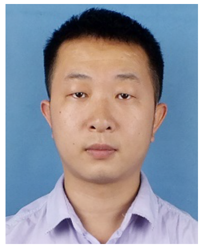

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