IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received August 18, 2020, accepted August 28, 2020, date of publication September 14, 2020, date of current version September 28, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3024150

Hybrid Sliding Mode/H-Infinity Control Approach for Uncertain Flexible Manipulators

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ABSTRACT In this article, a hybrid control approach combining sliding mode and H-infinity is proposed for an uncertain single-link flexible manipulator. The sliding mode controller stabilizes the nonlinear manipulator system, while the H-infinity controller enhances the noise rejection capability of the system by reducing the total system nonlinearity. The proposed hybrid controller is designed with the goal of rejecting external noises, hence providing a higher system performance, compared to a pure sliding mode controller. To avoid unintentional consequences of switching between the sliding mode and H-infinity controller, a fuzzy neural network weighting method is designed providing a smooth synthesis of both controller outputs. The neuro-fuzzy method applies a weighted combination of the two controller outputs to the manipulator system. In addition, a novel fuzzy estimation method is used to characterize the unstructured nonlinear disturbances in manipulator systems. The proposed hybrid control approach along with the fuzzy estimator is capable of providing a versatile means to stabilize flexible manipulator systems while maintaining a precise reference trajectory tracking in presence of unstructured uncertainty and nonlinear dynamics, as demonstrated by simulation results.

INDEX TERMS Flexible manipulator, sliding mode control, H-infinity, fuzzy neural network, fuzzy estimator.

I. INTRODUCTION

Flexible manipulators have received great attention in recent years, especially in high speed, high precision applications [1]–[6]. For instance, they have been extensively used in space exploration technologies [7], where weight reduction and lightness are in demand and flexibility becomes an inherent feature of such technologies. Also, several applications, including construction automation and mining operations [1], incorporate flexible manipulators in their fast, precise tasks. While providing higher performance in damping and precision and being substantially lighter than their rigid counterparts, flexible manipulator systems possess more uncertain behavior and nonlinearity, making their operation and control more challenging. Therefore, efficient control approaches are required to deal with the uncertainty and nonlinearity present in such systems. Various approaches, including impedance control [8], fuzzy-PID [9], hybrid linear control [10], sliding mode [11]–[16], and iterative learning control [17], [18], have been proposed to date to stabilize flexible manipulators and provide an acceptable performance with respect to the uncertainties. For instance, a hybrid linear control of input shaping and feedback control was proposed in [19] for a flexible manipulator. In [20], a sliding mode approach was used to drive the joint to the desired position, and an adaptive neural network was applied to approximate the unknown disturbances. In another work [21], the authors used an iterative learning identification method to construct a Fourier basis function space model for a flexible manipulator. Then, a pseudo inverse type iterative learning law was used to estimate the inverted stable non-minimum phase system. Most of the controllers designed in the literature for the flexible manipulators have been nonlinear approaches and incorporation of such controllers increases the total nonlinearity of the entire system resulting in adverse consequences such as higher sensitivity to the noise appearance. In addition, the implementation cost of such nonlinear controllers is relatively high. Therefore, a significant gap exists in the literature with respect to a cost-efficient controller that can effectively reject disturbances/noises in a flexible manipulator system and this work intends to bridge the gap by proposing a novel approach to reduce the nonlinearity in such systems and

The associate editor coordinating the review of this manuscript and approving it for publication was Jun Shen.

provide a precise reference trajectory tracking in presence of unstructured uncertainty.

In this work, the sliding mode and H-infinity control approaches are combined using a fuzzy neural network (neuro-fuzzy) weighting method to properly control the movement of a flexible manipulator. The proposed weighting method provides a weighted combination of both controller outputs to maintain a smooth synthesis of sliding mode and H-infinity control approach rather than the drastic switching between the two controllers. The sliding mode control is a nonlinear approach that exerts substantial influence on accurate tracking and error cancellation, and hence the robust conduct of a nonlinear system. However, the use of a sliding mode controller increases the nonlinear dynamics of a flexible manipulator system resulting in noise appearance and a more complicated nonlinear system. Therefore, an H-infinity control approach is designed in this work to accompany the sliding mode controller (hybrid control approach) using a neuro-fuzzy weighting approach in order to mitigate the aforementioned defects with respect to a pure sliding mode controller. The designed H-infinity controller affects the system in the vicinity of equilibrium point and enhances the noise rejection capability of the system by reducing the total system nonlinearity. The hybrid sliding mode/H-infinity controller is proposed with the goal of mitigating system nonlinearity and improving noise cancellation capability, as will be shown later in Section V. In addition to the proposed hybrid controller, a fuzzy estimator is derived –independent from system dynamics– to characterize the unstructured nonlinear disturbances in flexible manipulator's systems. The main contribution of this work is to design a novel controller stabilizing flexible manipulator system while rejecting external noise at the same time. The rest of the paper is organized as follows: Section II explains the dynamic model of a flexible manipulator. The derivation of fuzzy estimator for unstructured disturbances is described in Section III. In Section IV, the proposed hybrid controller along with the weighting method is explained. Section V demonstrates the results and discussions for a numerical example, and finally, conclusions are drawn in Section VI.

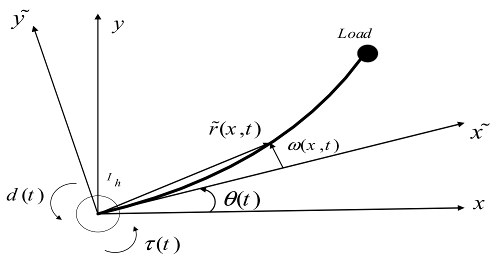

FIGURE 1. Schematic of single-link flexible manipulator.

II. DYNAMIC MODEL OF FLEXIBLE MANIPULATOR [20]

To model a flexible manipulator system, its workspace is assumed in a horizontal plane. Deviation from the rigid form of link is small. However, this deformation calls forth an unstructured disturbance exerted on the input torque [22]. Fig. 1 demonstrates a schematic of single-link flexible manipulator, and Table 1 describes pertinent parameters needed for deriving a dynamic model for such a manipulator.

TABLE 1. Model parameters for a single-link flexible manipulator.

Parameter list	Description
I_h	Inertia of the hub
L	Length of the beam
EL	Bending stiffness
ρ	Mass per unit length of the beam
$\theta(t)$	Joint angular trajectory
$\omega(x,t)$	Vibratory deflection at point P
$r(x,t)=x\theta(t)+\omega(x,t)$	Position of point P expressed in the fixed-base frame
m	Mass of the payload
$\tau(t)$	Torque input generated by joint motor
$d(t)$	Unstructured disturbance applied on input torque

The total kinetic energy, potential energy and the work originated from external forces are required to formulate the dynamic equations. The total kinetic energy E_k is the summation of joint kinetic energy $E_{k-joint}$, load kinetic energy E_{k-load} and link kinetic energy E_{k-link} and formulated as

$$E_{k-joint} = \frac{1}{2}I_h\dot{\theta}(t),$$

$$E_{k-load} = \frac{1}{2}m\dot{r}^2(L,t)$$

$$E_{l-link} = \frac{\rho}{2}\int_0^L \dot{r}^2 dx$$

$$\longrightarrow E_k = \frac{1}{2}I_h\dot{\theta}(t) + \frac{1}{2}m\dot{r}^2(L,t) + \frac{\rho}{2}\int_0^L \dot{r}^2 dx \quad (1)$$

Also, the total potential energy of the system E_p is

$$E_p = \frac{EI}{2}\int_0^L \dot{\omega}^2(x,t)dx \quad (2)$$

The work originated from the external forces can be formulated as

$$W_e = [\tau(t) + d(t)]\theta(t) \quad (3)$$

where $d(t)$ is the disturbance term resulted from the flexibility of manipulator and can be expressed as

$$d(t) = a + b\exp(-\alpha\dot{\theta}) + c\dot{\theta} \quad (4)$$

where a, b, and c are positive constants and $\alpha \in [0, 1]$ is an uncertain parameter.

Considering the infinite number of degree of freedom for flexible manipulators, the extended Hamilton principle can be written as [23]

$$\int_{t_1}^{t_2} (\delta E_k - \delta E_p + \delta W_e)dt = 0 \tag{5}$$

where $\delta E_k, \delta E_p$ and δW_e are partial variations of kinetic energy, potential energy and the work originated from external forces, respectively. Substituting the parameters from (1)-(4) into (5) holds:

$$\int_{t_1}^{t_2} A_1 \delta\theta(t)dt + \int_{t_1}^{t_2}\int_0^L A_2 \delta\omega(x,t)dxdt$$
$$+ \int_{t_1}^{t_2} A_3 \delta\omega(L,t)dt + \int_{t_1}^{t_2} A_4 \delta\omega_x(L,t)dt = 0 \tag{6}$$

where

$$\begin{aligned} A_1 &= \tau(t) + f(t) - [I_h\ddot{\theta}(t) + \rho\frac{L^3}{3}\ddot{\theta}(t) \\ &\quad + \rho\int_0^L x\ddot{\omega}(x,t)dx + mL^2\ddot{\theta}(t) + mL\ddot{\omega}(L,t)] \\ A_2 &= -[\rho(x\ddot{\theta}(t) + \ddot{\omega}(x,t)) + EI\omega_{xxxx}(x,t)] \\ A_3 &= EI\omega_{xxx}(L,t) - [m\ddot{\omega}(L,t) + mL\ddot{\theta}(t)] \\ A_4 &= \omega_{xx}(L,t). \end{aligned} \tag{7}$$

Considering (5)-(7), the following results can be immediately derived:

$$\begin{aligned} A_1 &= A_2 = A_3 = A_4 = 0 \\ \tau(t) &= (I_h + \rho\frac{L^3}{3})\ddot{\theta}(t) + \rho\int_0^L x\ddot{\omega}(x,t)dx \\ &\quad + mL\ddot{r}(L,t) - d(t) \\ \rho\ddot{r}(x,t) &= -EI\omega_{xxxx}(x,t) \\ m\ddot{r}(L,t) &= EI\omega_{xxx}(L,t) \\ \omega_{xx}(L,t) &= 0. \end{aligned} \tag{8}$$

The vibratory deflection is assumed to be zero according to the Euler-Bernoulli beam theory [24] as

$$\omega(0,t) = 0, \quad \omega_{\mathrm{x}}(,t) = 0. \tag{9}$$

Now, by rephrasing (8), the manipulator dynamic model is expressed as:

$$\ddot{\theta} = \frac{1}{I_h}[\tau(t) + d(t) + EI\omega_{xx}(0,t)] \tag{10}$$

III. FUZZY ESTIMATOR FOR DISTURBANCE

To design a controller for the flexible manipulator described in Section II, an appropriate estimation of disturbances is needed. The disturbance in a flexible manipulator system was modeled as a nonlinear exponential term, as shown in (4). A fuzzy system is employed to estimate the nonlinear disturbance of (4). The estimator is designed intuitively using Mamdani fuzzy system with seven and three fuzzy membership functions for the estimator input($\dot{\theta}$) and estimator output ($\hat{d}$), respectively. The fuzzy rules are demonstrated in Table 2. It should be noted that the fuzzy rules are independent from the system dynamics. The Fuzzy membership functions for $\dot{\theta}$ (input estimator) and $\hat{d}$ (output estimator) are Gaussian functions, shown in Fig. 2. Also, the singleton fuzzifier, product inference engine, and the center of average deffuzzifier are used in the proposed estimator system. The disturbance term of (4) can be estimated with the designed fuzzy system as

TABLE 2. Fuzzy rules for the disturbance estimation.

$\dot{\theta}$	NB	NM	NS	Z	PS	PM	PB
$\hat{d}$	M	M	S	B	M	S	S

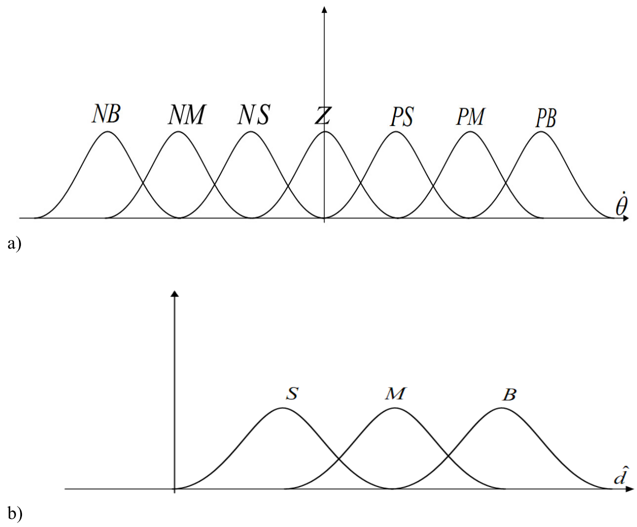

FIGURE 2. a) Fuzzy membership functions for a) input estimator ($\dot{\theta}$) and b) output estimator ($\hat{d}$).

$$\hat{d} = \frac{\sum_{l=1}^{N} \bar{y}^l \mu_{\dot{\theta}^l}}{\sum_{l=1}^{N} \mu_{\dot{\theta}^l}} \tag{11}$$

where $\hat{d}$ is the estimation of disturbance, $\mu_{\dot{\theta}^l}$ is the membership function of $\dot{\theta}$ and $\bar{y}^l$ is the center of membership function for $\hat{d}$.

IV. HYBRID SLIDING MODE/H-INFINITY CONTROL

A hybrid control approach combining the sliding mode and H-infinity control is designed for the flexible manipulator model described in Section II. The dynamic model has considerable degree of nonlinearity, as shown in (10), so a nonlinear sliding mode controller is designed to stabilize the system. In addition, an H-infinity controller is designed to accompany the sliding mode controller in order to mitigate the noise appearance and increase the overall performance of the control system. To avoid unintentional consequences of switching between the sliding mode and H-infinity controller, a fuzzy neural network weighting method is proposed providing a smooth synthesis of both controller outputs. The neuro-fuzzy weighting method applies a weighted combination of the two controller outputs to the manipulator system.

The dynamic model of a flexible manipulator system has considerable degree of nonlinearity, as shown in (8). In the proposed sliding mode controller, a dynamic sliding surface based on the vector of tracking errors $[e, \dot{e}, \ddot{e}, \ldots, e^{(n-1)}]$ is introduced as

$$S(\theta, t) = (\frac{d}{dt} + \lambda)^{n-1} e = 0 \xrightarrow{\text{n=2}} S(\theta, t) = \frac{de}{dt} + \lambda e = 0 \quad (e = \theta_{desired} - \theta) \quad (12)$$

where λ is a positive constant and $\theta_{desired}$ is the desired trajectory of manipulator's joint. The goal is to derive a control law by which the vector of tracking errors is directed towards origin. As a result, the sliding condition is used to establish an appropriate constraint as

$$\begin{aligned} \frac{1}{2}\frac{d}{dt} S^2(\theta, t) &\leq -\eta \left| S(\theta, t) \right| \\ &\Rightarrow S(\theta, t)\dot{S}(\theta, t) \leq -\eta \left| S(\theta, t) \right| \\ &\Rightarrow X + \tau_s - \ddot{\theta}_{desired} + \lambda \dot{e} \leq -\eta sgn(S(\theta, t)) \\ &(\ddot{\theta} = X + \tau_s) \end{aligned} \quad (13)$$

where η is a positive constant. According to (13), the control law (τ_s) is presented as

$$\tau_s = S - ksgn(S) \quad (14)$$

$$k \geq X - \hat{X} \quad (15)$$

where k is a positive constant and X and $\hat{X}$ are uncertain dynamics of system and estimation of uncertain dynamics of system, respectively. Also, sgn is the sign function. To cancel the chattering phenomenon [25], the sign function is replaced with a saturation function as

$$\tau_s = S - ksat(S) \quad (16)$$

where *sat* is saturation function. As the uncertain dynamics are embodied in the disturbance term of the manipulator, (15) is rephrased as

$$k \geq d - \hat{d} \quad (17)$$

The nonlinear dynamic terms appear in the model of a flexible manipulator system, as shown in (10). The use of a pure sliding mode control law of (16) even increases the total nonlinearity of the manipulator system resulting in a more complicated system prone to noise appearance. In such circumstances, the overall performance of the sliding mode manipulator control system may not be satisfactory, especially in presence of noise as will be shown later in Section V. Therefore, an H-infinity control approach is proposed to accompany the sliding mode controller in order to efficiently decrease the nonlinearity and promote the performance of the manipulator control system, as will be later demonstrated in Section V. In the proposed method, the maximum feedback or feedforward gain is calculated to stabilize the overall system in presence of uncertainty. The proposed H-infinity controller linearizes the overall system in accordance with a weighting method. To design an H-infinity controller, linear state space equations and system transfer function are needed [26]. Therefore, the exponential disturbance of (4) is converted to linear terms using appropriate series. By linearizing the disturbance term as $\dot{\theta}$ approaches the origin, the model of (10) can be rewritten as:

$$\begin{aligned} &\ddot{\theta} \\ &= \frac{1}{I_h}[\tau(t) + a + b\exp(-\alpha\dot{\theta}) + c\dot{\theta} + +EI\omega_{xx}(0, t)] \\ &\xrightarrow{\text{linearized as } \dot{\theta} \text{ approaches origin (in the neighborhood of equilibirium point)}} \\ &\ddot{\theta} \\ &= \frac{1}{I_h}[\tau(t) + a + b(1 - \alpha\dot{\theta}) + c\dot{\theta} + +EI\omega_{xx}(0, t)] \end{aligned} \quad (18)$$

Based on the H-Infinity condition, the inequality of (19) must be met to provide stable system dynamics:

$$\left\| KT_{tf} \right\|_\infty \leq \left\| K \right\|_\infty \left\| T_{tf} \right\|_\infty \leq 1 \quad (19)$$

where T_{tf} is the linearized transfer functions and K is the constant state feedback or feedforward gain that forms the control loop.

Also, the switching operation between the sliding mode and H-infinity controller is replaced by a neuro-fuzzy weighting method so that unintentional consequences of switching are eliminated, and a smooth synthesis of both controller outputs is provided. In fact, the control law τ_{total} is defined as the weighted sum of control torque from the sliding mode controller τ_s and from the H-infinity controller τ_∞, where the weights (gains) k_s and k_i are determined by a neuro-fuzzy approach, i.e.,

$$\tau_{total} = k_s \tau_s + k_i \tau_\infty \quad (20)$$

To construct the neuro-fuzzy weighting system, a single layer neural network with one input is employed and synaptic weights are tuned by the fuzzy logic to appropriately compute the gains k_s and k_i. Fig. 3 shows the schematic of a general neuro-fuzzy system along with the configuration of the proposed neuro-fuzzy weighting method.

The neural network functions f_s and f_i in Fig. 4 are set to be:

$$\begin{cases} f_s = \dfrac{1 - \exp(-\omega_{sf}\hat{d})}{1 + \exp(-\omega_{sf}\hat{d})} \\ f_i = \dfrac{1 - \exp(-\omega_{if}\hat{d})}{1 + \exp(-\omega_{if}\hat{d})} \end{cases} \quad (21)$$

Also, the membership functions for ω_{sf} and ω_{if} are shown in Fig. 4 and the fuzzy rules for tuning of ω_{sf} and ω_{if} demonstrated in Table 3. In this learning logic, the singleton fuzzifier, product inference engine, and the center of average deffuzzifier are applied. The fuzzy system computation is:

$$\omega_{sf} = \omega_{if} = \frac{\sum_{l=1}^{N} \bar{p}^l \mu_{\dot{\theta}^l}}{\sum_{l=1}^{N} \mu_{\dot{\theta}^l}} \quad (22)$$

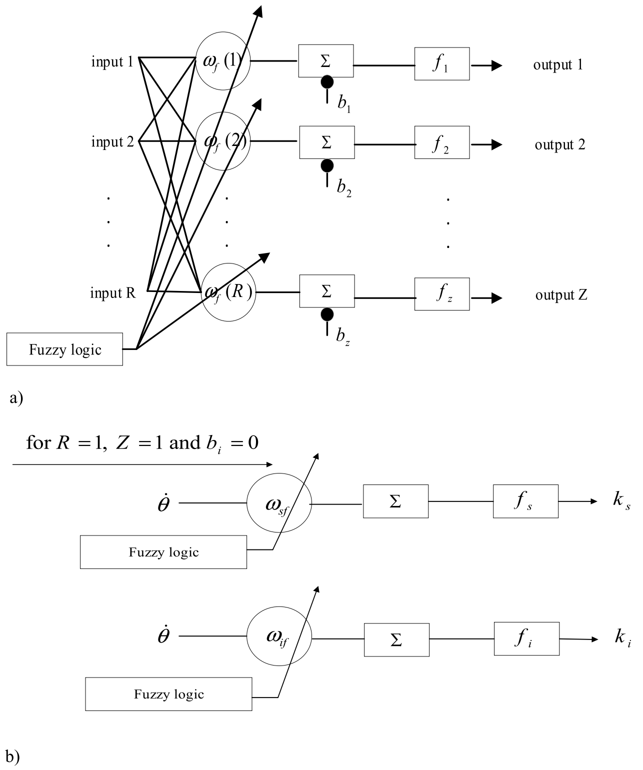

FIGURE 3. a) Schematic of a general neuro-fuzzy system b) Proposed neuro-fuzzy weighting method.

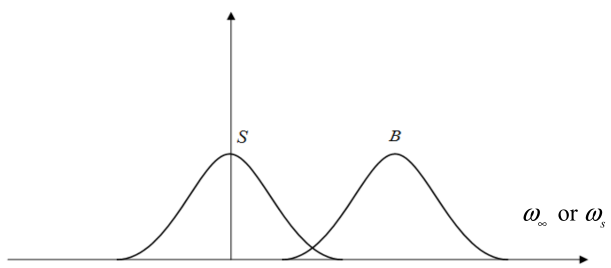

FIGURE 4. Membership functions for ω_{sf} and ω_{if}.

where ω_{sf} and ω_{if} are synaptic weights of NN learning process in sliding mode and H-infinity control, respectively. $\mu_{\dot{\theta}^l}$ is membership function of $\dot{\theta}$ and $\bar{p}^l$ is the center of membership functions for ω_{sf} and ω_{if}.

In summary, in the designed controller, the maximum feedback gain is calculated using the H-infinity method in a way that the system remains stable in presence of uncertainties, as shown in Eq. (19). This feedback gain along with the sliding mode controller, influences the nonlinear system (the two controllers are switched based on a fuzzy- Neural network weighting method).

Fig. 5 demonstrates the schematic of the overall system. As shown in the Fig. 5, a fuzzy estimator mechanism with derivative of joint trajectory as estimator input and disturbance as estimator output is designed to approximate unstructured disturbance. The result of this estimation is used in the

TABLE 3. Fuzzy rules for tuning ω_{sf} and ω_{if}.

$\dot{\theta}$	NB	NM	NS	Z	PS	PM	PB
ω_{sf}	B	B	B	S	B	B	B
ω_{if}	S	S	S	B	S	S	S

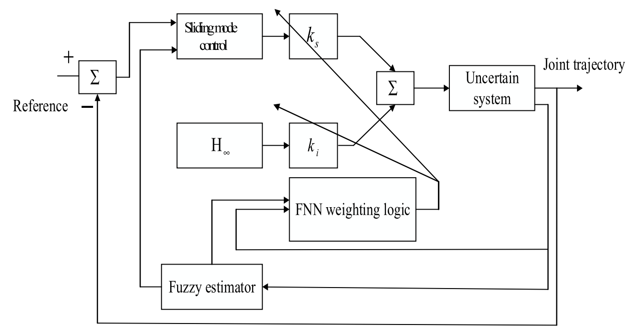

FIGURE 5. Overall system schematic.

process of sliding mode controller designing and learning procedure of neural network. Hybrid sliding mode/ H-infinity approach weighted based on FNN logic provides control signal for the system.

Stability analysis of the overall system is formulated via the dissipativity theory [27,28].

Definition 1: The supply rate function J, which is integrable and independent of the states, is dissipative (and system is stable) if there exists a continuously differentiable storage function $V \geq 0$ such that

$$\int_0^T J[\tau(t), y(t)]dt \geq V(\theta(t)), \quad \int_0^T J[\tau(t), y(t)]dt < +\infty \tag{23}$$

where $\tau(t)$ is the input torque and $y(t)$ is the system output (joint trajectory). The supply rate function is nominated as

$$J[\tau(t), y(t) = \theta(t)] = \|y(t)\|^2 + \|\tau(t)\|^2 \tag{24}$$

where $\|\,\|$ denotes the Euclidean norm. Also, the corresponding energy function for the manipulator is

$$V(\theta(t)) = \frac{1}{2}M\theta^2(t) \geq 0 \tag{25}$$

Therefore, based on the dissipativity theory, the inequality of (26) must be enforced to satisfy the stability condition:

$$\begin{aligned} &\int_0^T J[\tau(t), y(t)]dt \\ &\quad \geq V(\theta(t)) \\ &\quad \Rightarrow \int_0^T [\|y(t)\|^2 + \|\tau(t)\|^2]dt \geq V(\theta(t)) \\ &\quad \Rightarrow \int_0^T [\|y(t)\|^2 + \|\tau(t)\|^2]dt - V(\theta(t)) \geq 0 \end{aligned} \tag{26}$$

V. NUMERICAL EXAMPLE

In this Section, performance of the proposed hybrid control method is tested via a numerical example. Consider a flexible manipulator system presented in Section II and assume the model parameters to be:

$$\begin{aligned} I_h &= 0.5\text{kg.m}^2, EI = 2\text{ N.m}^2 \\ \rho &= 0.2\text{ kg.m}^{-1}, \quad m = 0.6\text{ kg}, L = 1\text{ m} \end{aligned} \tag{27}$$

and the disturbance parameters to be given as

$$\begin{aligned} &d(t) = 0.2 + 0.2\exp(-\alpha\dot{\theta}) + 0.02\dot{\theta} \\ &\qquad (\text{a} = 0.2, \text{b} = 0.2, \text{c} = 0.02 \text{ and } \alpha \in [-1,1]). \end{aligned} \tag{28}$$

To model the disturbance, the following parameters are set for the fuzzy estimator:

$$\begin{aligned} &\text{membership function variance for } \dot{\theta}(t) = 0.02 \\ &\text{membership function variance for } \hat{d} = 0.1 \\ &\dot{\theta}_{\min}(t) = -1, \\ &\dot{\theta}_{\max}(t) = 1, \hat{d}_{\min} = 0.3, \\ &\hat{d}_{\max} = 0.7, N = 7 \end{aligned} \tag{29}$$

For the sliding mode controller, the parameters of (12) are considered: $\lambda = 20, n = 2$, so k is obtained from (17) as $k_{\min} = 0.4$

The H-infinity controller parameter $\|K\|_\infty$ is computed from (19) as

$$\begin{aligned} \|KT_{tf}\|_\infty &\le \|K\|_\infty \|T_{tf}\|_\infty \le 1 \\ &\Rightarrow \|K\|_\infty \le \left\| \frac{s}{s^2 + 0.2\alpha - 0.02} \right\|_\infty^{-1} \\ &\Rightarrow \|K\|_\infty \le \left\| \frac{s}{s^2 - 0.02} \right\|_\infty^{-1} + \|0.2\alpha\|_\infty \\ &\Rightarrow \|K\|_\infty \le 0.22 \end{aligned} \tag{30}$$

which is the robust state feedback gain. This gain (H-infinity control gain) is restricted in a way that the whole system becomes stable. The parameters of the neuro-fuzzy weighting method, is set as follows:

$$\begin{aligned} &\text{membership function variance for } \dot{\theta}(t) = 0.02 \\ &\text{membership function variance for } \omega_{sf}, \omega_{if} = 10 \\ &\dot{\theta}_{\min}(t) = -1, \dot{\theta}_{\max}(t) = 1 \\ &\omega_{sf\ \min} = \omega_{if\ \min} = 0, \\ &\omega_{sf\ \max} = \omega_{if\ \max} = 20, \\ &N = 2 \end{aligned} \tag{31}$$

The fuzzy estimation error ($\hat{e} = d - \hat{d}$) is shown in Fig. 6.a. which is properly bounded in the vicinity of origin (Fig. 6.a). The reference trajectory tracking for a linear H-infinity controller is depicted in Fig. 6.b. As clearly shown in Fig. 6.b, the linear H-infinity controller alone cannot properly stabilize the nonlinear manipulator system. There-fore, a nonlinear method based on the sliding mode control is nominated to stabilize the system. On the other hand, a pure sliding mode controller makes the system vulnerable to the noise appearance, as will be shown later in Fig. 7. Therefore, the proposed hybrid controller (sliding mode combined with H-infinity) is designed to provide satisfactory stability and performance requirements for the manipulator system.

According to (30), the H-infinity feedback gain needs to be less than or equal to 0.22. To visually demonstrate how this gain might affect the hybrid controller results, two values of H-infinity feedback gain are chosen: $\|K\|_\infty = 2$(outside of the calculated range) and $\|K\|_\infty = 0.22$. The corresponding sinusoidal reference trajectory tracking for the gains are shown in Figs 6.c and 6.d. The gain computed by (30) ($\|K\|_\infty = 0.22$) leads to more efficient tracking and will be used in the rest of the calculations. Fig. 6.e demonstrates the reference trajectory tracking of the hybrid controller for the square pulse reference signal. The designed hybrid controller causes the flexible manipulator to appropriately track the reference (desired) trajectory (Figs. 6.d and 6.e), which are a sinusoidal path and square pulse with the amplitude of 1 and frequency of 1 rad.s^{-1} and 0.05s^{-1}, respectively.

The total control torque for the manipulator hybrid control system is shown in Fig. 6.f. The total torque, i.e., weighted combination of sliding mode and H-infinity control, smoothly changes within a certain range in time and avoids any drastic shifting or switching. Fig. 6.g demonstrates the stability substantiation curve of the system for the hybrid control method. The stability curve, the difference between the storage function and the nominated supply rate, is an absolute positive number, as shown in Fig. 6.f. According to the dissipativity approach, the stability equation (inequality of (26)) must be a positive number to satisfy stability and the curve shown in Fig. 6.g is absolutely positive (converging to infinity) verifying the system stability (for $M = 1$).

As flexible manipulators may frequently encounter external noises in their application [1,3], the performance of the proposed hybrid control method in presence of noise is assessed by applying a white noise with the power of 0.001 to the manipulator system. The results of reference trajectory tracking for a pure sliding mode control approach and a hybrid sliding mode/H-infinity control method are depicted in Figs. 7.a and 7.b, respectively. The system is vulnerable to noise acquisition in case of pure sliding mode controller, as demonstrated by Fig.7.a. By contrast, the hybrid sliding mode/H-infinity controller possesses a lower degree of sensitivity to the noise appearance, Fig. 7.b.

The fact that the hybrid controller decreases the total nonlinearity of the system due to combining H-infinity with the sliding mode leads to the higher performance with respect to noise cancellation. In fact, the hybrid controller reduces the excess use of sliding mode and its derivative operator (derivative intensifies the noise magnitude, (see (12)). Consequently, the controller affects the system like a low-pass filter. The trajectory tracking errors for the two control approaches are shown in Figs. 7.c and 7.d. The mean, standard deviation, and the mean squared error of trajectory tracking are used as the performance measures for comparison of controllers.

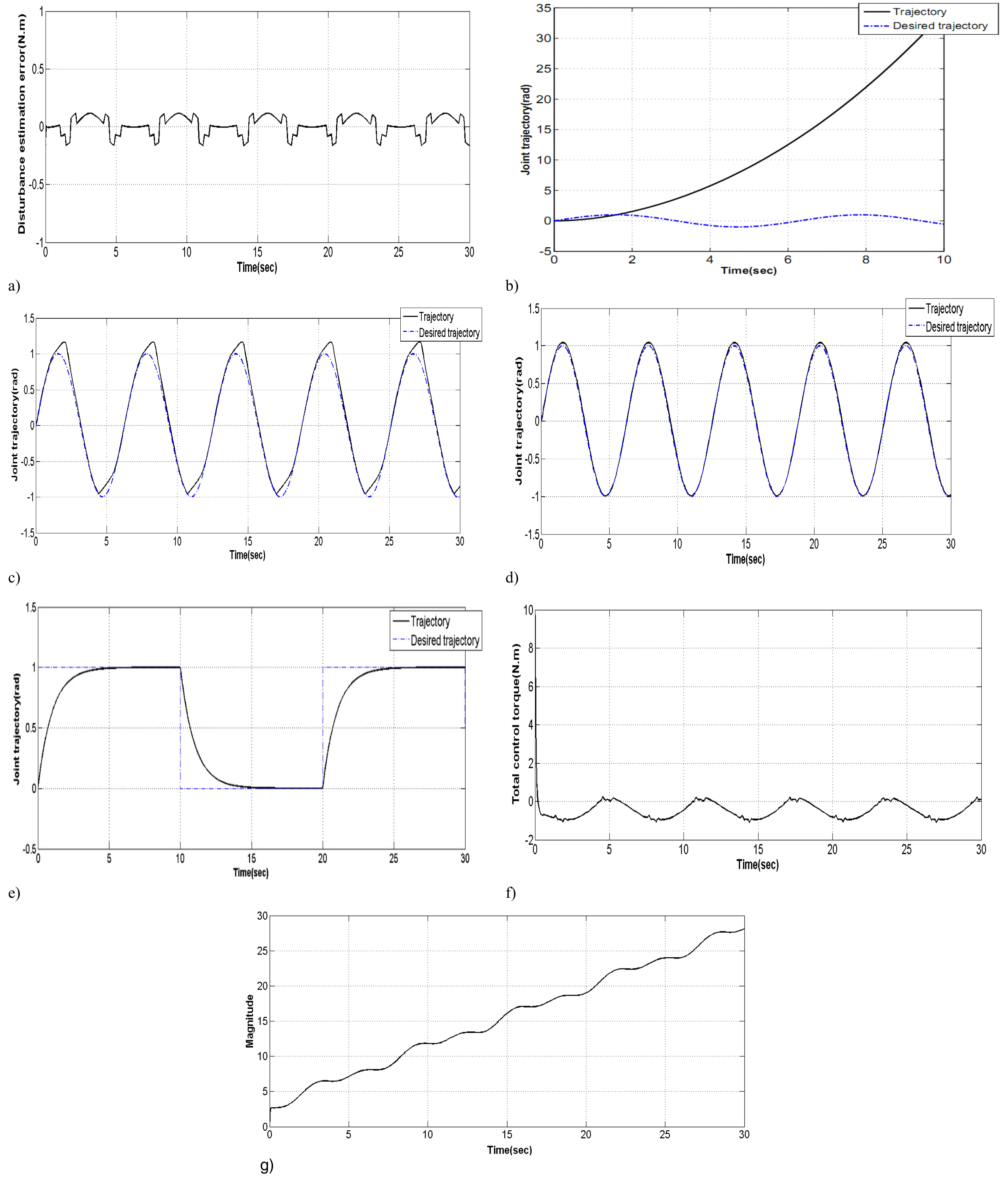

FIGURE 6. a) Fuzzy estimation error b) Reference trajectory tracking for H-infinity control method (sinusoidal reference) c) Reference trajectory tracking for hybrid control method when feedback gain for H-infinity controller is out of calculated range (for $\|K\|_\infty = 2$, sinusoidal reference) d) Reference trajectory tracking for hybrid control method (for $\|K\|_\infty = 0.22$, sinusoidal reference) e) Reference trajectory tracking for hybrid control method (for $\|K\|_\infty = 0.22$, square pulse reference) f) Total control torque for hybrid control method g) Stability substantiation curve for hybrid control method.

The mean and standard deviation of error for the pure sliding mode and the hybrid controller are (0.04 ± 0.03) and (0.02 ± 0.01), respectively. Also, Figs. 7.e and 7.f demonstrate the mean squared error for trajectory tracking in

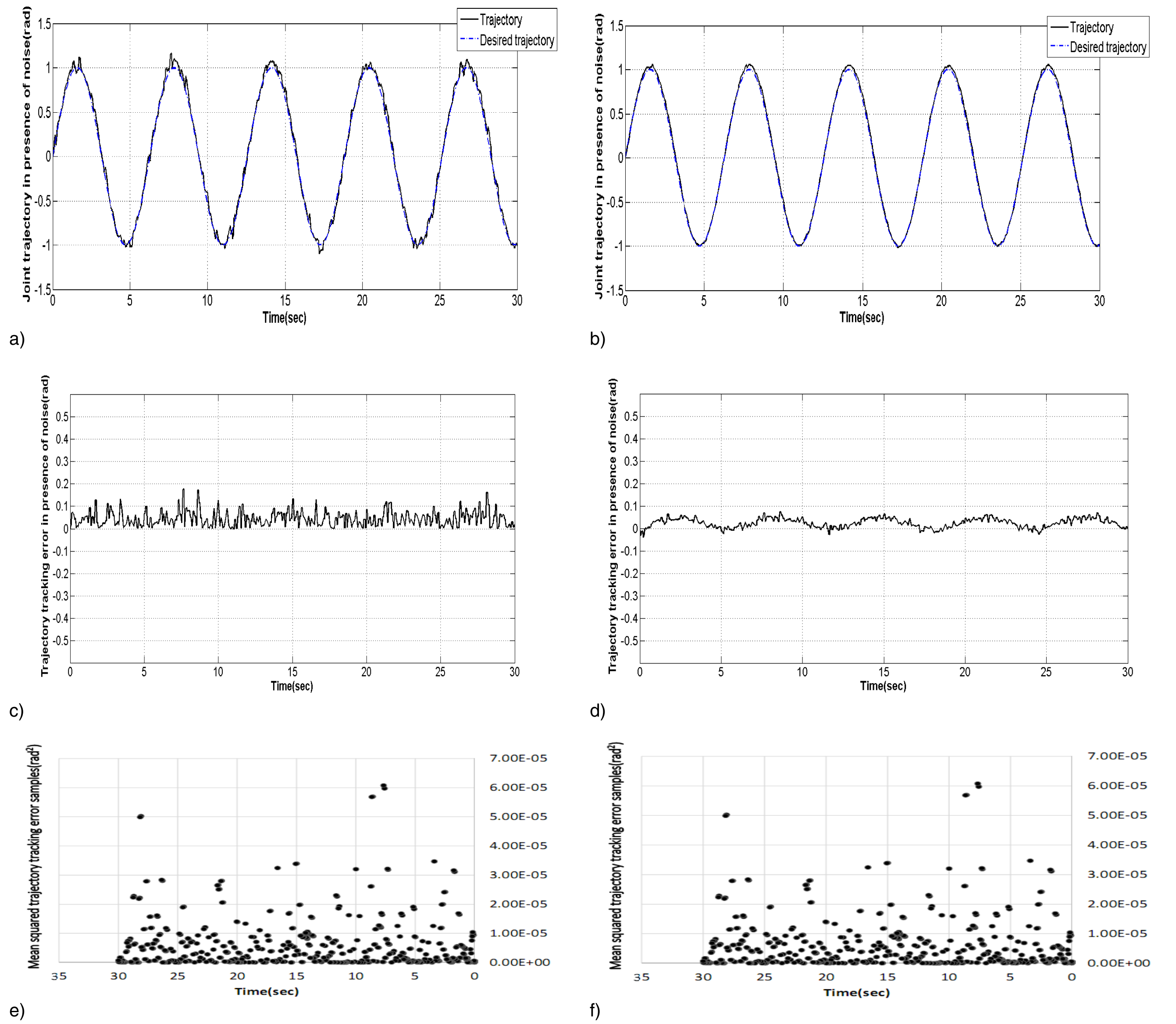

FIGURE 7. a) Reference trajectory tracking in presence of noise for pure sliding mode controller b) Reference trajectory tracking in presence of noise for hybrid sliding mode/H-infinity controller c) Trajectory tracking error in presence of noise for pure sliding mode controller d) Trajectory tracking error in presence of noise for hybrid sliding mode/H-infinity controller e) Mean squared trajectory tracking error samples in presence of noise for pure sliding mode controller f) Mean squared trajectory tracking error samples in presence of noise for hybrid sliding mode/H-infinity controller.

presence of noise for the pure sliding mode and hybrid controller, respectively. Comparing the performance measures clearly indicates a higher performance (less error) for the hybrid controller. Results of Fig. 7 verifies the efficiency of the proposed hybrid sliding mode/ H-infinity controller in terms of noise cancellation, compared to a pure sliding mode controller.

VI. CONCLUSION

In this article, a new control approach for stabilizing a single-link flexible manipulator was presented. The proposed approach was a hybrid control method combining the sliding mode and H-infinity control using a neuro-fuzzy based weighting method. While the sliding mode has a descent performance in stabilizing the nonlinear manipulator system, the accompanying H-infinity controller reduces the total nonlinearity, hence promoting the overall performance of the system, especially in rejection of external noises. Also, a fuzzy logic system was designed to estimate the unstructured nonlinear disturbances in flexible manipulator systems. The stability as well as efficiency of the proposed hybrid controller was verified through a numerical example. Also, the superior performance of the proposed controller in terms of noise cancellation was demonstrated against a pure sliding mode controller. The practical implementation of the hybrid controller and the effect of parameter variations on the system dynamic response need to be further investigated, as future research directions.

REFERENCES

[1] Y. Gao, F. Y. Wang, and Z. Q. Xiao, "Past and recent works," in *Flexible Manipulators*, 1st ed. Whaltham, MA, USA: Science Direct, 2012, pp. 5–11.
[2] S. C. P. Gomes, V. S. da Rosa, and B. C. Albertini, "Active control to flexible manipulators," *IEEE/ASME Trans. Mechatronics*, vol. 11, no. 1, pp. 75–83, Feb. 2006.
[3] M. Benosman and G. Le Vey, "Control of flexible manipulators: A survey," *Robotica*, vol. 22, no. 5, pp. 533–545, Sep. 2004.
[4] A. DeLuca, "Trajectory control of flexible manipulators," *Control Problems Robot. Automat.*, vol. 230, pp. 88–104, Jun. 2005.
[5] C. T. Kiang, A. Spowage, C. K. Yoong, "Review of control and sensor system of flexible manipulator," *J. Intell. Robotic Syst.*, vol. 77, pp. 187–213, Jul. 2015.
[6] M. Sayahkarajy, Z. Mohamed, and A. A. Mohd Faudzi, "Review of modelling and control of flexible-link manipulators," *Proc. Inst. Mech. Eng., I, J. Syst. Control Eng.*, vol. 230, no. 8, pp. 861–873, 2016.
[7] M. Sabatini, P. Gasbarri, R. Monti, and G. B. Palmerini, "Vibration control of a flexible space manipulator during on orbit operations," *Acta Astronautica*, vol. 73, pp. 109–121, Apr. 2012.
[8] G. R. Vossoughi and A. Karimzadeh, "Impedance control of a two degree-of-freedom planar flexible link manipulator using singular perturbation theory," *Robotica*, vol. 24, no. 2, pp. 221–228, Mar. 2006.
[9] H. A. Malki, D. Misir, D. Feigenspan, and G. Chen, "Fuzzy PID control of a flexible-joint robot arm with uncertainties from time-varying loads," *IEEE Trans. Control Syst. Technol.*, vol. 5, no. 3, pp. 371–378, May 1997.
[10] R. M. Mahamood and J. O. Pedro, "Hybrid PD/PID controller design for two-link flexible manipulators," in *Proc. 8th Asian Control Conf. (ASCC)*, Kaohsiung, Taiwan, May 2011, pp. 1358–1363.
[11] A. Mujumdar, S. Kurode, and B. Tamhane, "Fractional order sliding mode control for single link flexible manipulator," in *Proc. IEEE Int. Conf. Control Appl. (CCA)*, Hyderabad, India, Aug. 2013, pp. 288–293.
[12] Y. P. Chen and K. S. Yeung, "Sliding-mode control of multi-link flexible manipulators," *Int. J. Control*, vol. 54, no. 2, pp. 257–278, Aug. 1991.
[13] K. Lochan, B. K. Roy, and B. Subudhi, "SMC controlled chaotic trajectory tracking of two-link flexible manipulator with PID sliding surface," *IFAC-PapersOnLine*, vol. 49, no. 1, pp. 219–224, 2016.
[14] K. Lochan, B. K. Roy, and B. Subudhi, "Robust tip trajectory synchronisation between assumed modes modelled two-link flexible manipulators using second-order PID terminal SMC," *Robot. Auto. Syst.*, vol. 97, pp. 108–124, Nov. 2017.
[15] K. Lochan, J. P. Singh, B. K. Roy, and B. Subudhi, "Adaptive time-varying super-twisting global SMC for projective synchronisation of flexible manipulator," *Nonlinear Dyn.*, vol. 93, no. 4, pp. 2071–2088, Sep. 2018.
[16] K. Lochan, B. K. Roy, and B. Subudhi, "Generalized projective synchronization between controlled master and multiple slave TLFMs with modified adaptive SMC," *Trans. Inst. Meas. Control*, vol. 40, no. 4, pp. 1049–1071, Feb. 2018.
[17] M. Z. M. Zain, M. O. Tokhi, and Z. Mohamed, "Hybrid learning control schemes with input shaping of a flexible manipulator system," *Mechatronics*, vol. 16, nos. 3–4, pp. 209–219, Apr. 2006.
[18] Z. Liu and J. Liu, "Adaptive iterative learning boundary control of a flexible manipulator with guaranteed transient performance," *Asian J. Control*, vol. 20, no. 3, pp. 1027–1038, May 2018.
[19] Z. Mohamed and M. A. Ahmad, "Hybrid input shaping and feedback control schemes of a flexible robot manipulator," in *Proc. 17th World Congr. Int. Fed. Autom. Control*, 2008, pp. 11714–11719.
[20] H.-J. Yang and M. Tan, "Sliding mode control for flexible-link manipulators based on adaptive neural networks," *Int. J. Autom. Comput.*, vol. 15, no. 2, pp. 239–248, Apr. 2018.
[21] L. Zhang and S. Liu, "Iterative learning control for flexible manipulator using Fourier basis function," *Int. J. Autom. Comput.*, vol. 12, no. 6, pp. 639–647, Dec. 2015.
[22] T. Jiang, J. Liu, and W. He, "Boundary control for a flexible manipulator based on infinite dimensional disturbance observer," *J. Sound Vib.*, vol. 348, pp. 1–14, Jul. 2015.
[23] J. Ma, D. Jin, Z. Wei, T. Chen, and H. Wen, "Boundary control of a flexible manipulator based on a high order disturbance observer with input saturation," *Shock Vib.*, vol. 2018, pp. 1–10, 2018.
[24] O. A. Bauchau and J. I. Craig, "Euler-Bernoulli beam theory," in *Structural Analysis*, 1st ed. Atlanta, GA, USA: Springer, 2009, pp. 173–221.
[25] H. K. Khalil, "Nonlinear design tools," in *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002, pp. 551–625.
[26] F. Aghili, M. Buehler, and J. M. Hollerbach, "Motion control systems with H_∞ positive joint torque feedback," *IEEE Trans. Control Syst. Technol.*, vol. 9, no. 5, pp. 685–695, Sep. 2001.
[27] J. Zhao and D. J. Hill, "Dissipativity theory for switched systems," *IEEE Trans. Autom. Control*, vol. 53, no. 4, pp. 941–953, May 2008.
[28] M. Arcak, A. Packard, and C. Meissen, "Dissipativity with dynamic supply rates," in *Netw. Dissipative Syst.*, 1st ed. Berkeley, CA, USA: Springer, 2016, pp. 63–72.

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