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Finite-Time Incremental Passivity and Output Tracking Control for Switched Nonlinear Systems

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ABSTRACT This paper studies the finite-time incremental passivity of switched nonlinear systems. Then, the established theory is applied to solve the finite-time output tracking control problem of switched nonlinear systems. First, finite-time incremental passivity is firstly defined for switched nonlinear systems. Each subsystem is finite-time incrementally passive during its active time interval. Unlike incremental passivity, the state trajectories of finite-time incrementally passive system with no external supplied energy can converge to each other in finite time. Second, the criterion of finite-time incremental passivity is established. Third, finite-time incremental passivity is shown to be preserved under the feedback interconnection. A composite switching law design method is provided. Under this switching law, the interconnected switched systems can switch asynchronously. Finally, the finite-time output tracking control problem was solved by the established finite-time incremental passivity theory of the switched nonlinear systems, even if the finite-time output tracking control of individual subsytem is not solvable. The effectiveness of the proposed method is verified by an example.

INDEX TERMS Switched nonlinear systems, Finite-time output tracking control, Finite-time incremental passivity.

I. INTRODUCTION

In the past few years, the output tracking control for nonlinear systems has been received increasing attention. There have been many research results on output tracking control [1], [2]. However, finite-time control can better meet the practical requirements. Compared with the traditional asymptotic control, the control precision, anti-interference and robustness properties of finite-time control are better. Hence, it is interesting to study the finite time output tracking problem [3], [4].

The passivity concept proposed by Willems [5] can also be useful for dealing with the output tracking control problem [6], [7], because one can take the storage function of a passive system as a Lyapunov function. Passivity was firstly extended to incremental passivity from the perspective of operator in [8]. For a system with an equilibrium point or not, the incremental passivity in stae space form was defined in [9]. Moreover, the incrementally passive interconnected systems were shown to be incrementally passive. In general, the trajectories of an incrementally passive nonlinear system without external supply of energy can converge to one another. Therefore, incremental passivity was often adopted to investigate the output tracking problems [9], [10]. The aforementioned control method can only achieve infinite-time output tracking. Nevertheless, finite time is a better performance indicator. In [11]–[13], a notion of finite-time passivity was proposed for nonlinear systems. Finite-time passivity was also applicated to synchronization [13].

On the other hand, switched systems have been widely studied in recent years because of thire great many applications in real world [14], [15]. Methods commonly used to deal with switched nonlinear systems include multiple Lyapunov function method [16], average dwell time method [15]–[17]. Research on output tracking control is also of great significance for switching nonlinear systems [17], [18]. However, few results on finite-time output tracking control of switched nonlinear systems have appeared [19], [20].

The passivity of switched nonlinear systems is also worth studing like non-switched systems. There have been many results on passivity of switched nonlinear systems

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reported [21]–[23]. In [23], passivity was applicated to solve the output tracking problem of switched nonlinear systems. Incremental passivity was expected to be helpful for switched nonlinear systems. Therefore, [24]–[26] established the incremental passivity theory of switched nonlinear systems and solved the output tracking problem. All works mentioned above studied passivity over infinite-time interval. Subsequently, finite-time passivity of switched nonlinear systems has been investigated in [27], [28]. So far, finite-time incremental passivity and ouput tracking problem have been not studied.

Motivated by the above disscusion, this paper will study the finite-time incremental passivity and the output tracking control for switched nonlinear systems. The contributions of this paper are threefold. First, finite-time incremental passivity concept is firstly defined. Unlike [24]–[26], the state trajectories can converge each other in finite time if there is no external supplied energy. Second, a state-dependent switching law is designed to achieve finite-time incremental passivity. In contrast to the well-known min-switching law [24], by the designed switching law, any subsystem corresponding to the smallest continuous function is actived instead of the smallest Lyapunov function. This provides more freedom for the design of the switching law. Finally, finite-time incremental passivity is shown to be preserved for the feedback interconnection system under a composite state-dependent switching law, which allows the interconnected switched system switch asynchronously.

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider a switched nonlinear system

$$\begin{aligned} \dot{x} &= f_\sigma(x, u_\sigma), \\ y &= h_\sigma(x), \end{aligned} \tag{1}$$

where $x \in R^n$ is the state, a piecewise constant function $\sigma : [0, \infty) \to I = \{1, 2, \cdots, M\}$ is a switching signal, M denotes the number of subsystems of system (1). $u_i \in R^m$ and $y \in R^m$ are the input and output vectors of the i-th subsystem, respectively. f_i, h_i are assumed to be smooth with $f_i(0, 0) = 0$ and $h_i(0) = 0$. The switching time sequence is described by

$$\sum = \{x_0; (i_0, t_0), (i_1, t_1), \ldots, (i_k, t_k), \ldots | i_k \in I, k \in Z_+\}, \tag{2}$$

in which x_0 denotes the initial state at the initial time, t_0 and Z_+ denotes the set of non-negative integers, respectively. (i_k, t_k) means the i_k-th subsystem is switching on at the k th switching time t_k. Namely, the switching signal is $\sigma(t) = i_k$ during $[t_k, t_{k+1})$. In addition, we assume that the state of system (1) does not jump at the switching instants. For any $j \in I$, let t_{j_k} denote the k-th switching times of the j-th subsystem when it is switched on and t_{j_k+1} denote the k-th switching times of the j-th subsystem, when it is switched off.

The main control objective is to sove the finite-time output tracking control problem for system (1) formulated as follows:

Given a bounded reference signal $y^*(t)$, design a switching signal σ and controllers u_i, $i \in I$ for system (1) such that

(1) all the state trajectories of the closed-loop system (1) are globally bounded.

(2) for every $x(t_0) \in R^n$, $\lim\limits_{t \to t_0 + T} \|y(t) - y^*(t)\| = 0$, i.e.

$$\left\|y(t) - y^*(t)\right\| = 0, \quad \forall t \geq t_0 + T(x_0), \tag{3}$$

where $T > 0$ is a settle time.

First, the assumption on the output tracking control is introduced.

Assumption 1 [24]: $y^*(t), \forall t \geq t_0$ is assumed to be a bounded reference trajectory.

Next, we will review some definitions and lemmas that will be used in the following.

Definition 1 [29]: A continuous function γ: [0, a) $\to$ R $\geq$ 0 is called a class $\mathcal{K}$ function if it is strictly increasing and $\gamma(0) = 0$. If in addition, γ is unbounded, it is of class $\mathcal{K}_\infty$ functions.

Lemma 1 [12]: Assume that $\gamma_i : R_{\geq 0} \to R_{\geq 0}, i = 1, 2, \cdots n$ are class $\mathcal{K}$ functions. If there exist $\varepsilon_i \geq 0, i = 1, 2, \cdots, n$ such that $\int_0^{\varepsilon_i} \frac{dz}{\gamma_i(z)} < +\infty$ then

i) $\gamma(z) = \min\left\{\gamma_i\left(\frac{z}{n}\right), i = 1, 2, \cdots, n\right\}, z \in R_{\geq 0}$ is a class $\mathcal{K}$ function.

ii) $\sum\limits_{i=1}^{n} \gamma_i(z_i) \geq \gamma\left(\sum\limits_{i=1}^{n} z_i\right)$ for any $z_i \in R_{\geq 0}, i = 1, 2\vdots n$.

iii) $\int_0^{\varepsilon} \frac{dz}{\gamma(z)} < +\infty$, where $\varepsilon = \min\{n\varepsilon_i, i = 1, 2 \cdots n\}$.

Finite time convergence of switched nonlinear systems is defined in the following.

Definition 2: System $\dot{x} = f_\sigma(x)$ is said to be globally uniformly finite time convergent, if for any given switching signal $\sigma(t)$ and all $x_0 \in R^n$, there exists an unique bounded solution $x^*(t)$ on R and $0 \leq T(x_0) < \infty$ satisfying

$$\lim_{t \to T(x_0)} x(t) = x^*(t).$$

III. FINITE TIME INCREMENTAL PASSIVITY

In this section, finite-time incremental passivity of switched nonlinear systems theory will be established.

A. FINITE-TIME INCREMENTAL PASSIVITY DEFINITION

First, we define the finite-time incremental passivity of system (1) as follows:

Definition 3: System (1) is said to be finite-time incrementally passive, if for a given switching signal $\sigma(t)$, there are C^1 (i.e. continuously differentiable) nonnegative continuous functions $V_i\left(x, \hat{x}\right) : R^n \times R^n \to R^+, i \in I$, called incremental storage functions, and class $\mathcal{K}$ functions $\gamma_i(*)$: $R \geq 0 \to R \geq 0$, and some $\varepsilon_i > 0, i \in I$, such that the following inequalities hold on $[t_k, t_{k+1})$ for any two inputs u_i, $\hat{u}_i$, any two solutions of system (1) x, $\hat{x}$ corresponding to these inputs, the respective outputs $y = h_i(x)$ and $\hat{y} = h_i(\hat{x})$

$$\dot{V}_{i_k}(x(t), \hat{x}(t)) \leq -\gamma_{i_k}(V_{i_k}(x(t), \hat{x}(t))) + \left(y^T - \hat{y}^T\right)(u_{i_k} - \hat{u}_{i_k}), \quad (4)$$

$$V_{i_k}(x(t_k), \hat{x}(t_k)) \leq V_{i_{k-1}}(x(t_k), \hat{x}(t_k)), \quad i_k \in I, \quad (5)$$

$$\int_0^{\varepsilon_i} \frac{dz}{\gamma_i(z)} < +\infty. \quad (6)$$

Remark 2: (4) means that finite-time incremental passivity inequality only holds on the corresponding active time interval $[t_k, t_{k+1})$. Thus, (4) and (5) implies the dissipated energy in the whole switched systems is no more than the supplied energy outside. Hence, Definition 3 is a generalization of conventional incremental passivity in [9].

Remrk 3: For system (1) with equilibrium (0, 0), finite-time incrementally passive system must be finite-time passive by setting $\hat{x} = 0, \hat{u}_i = 0$ [27], while finite time passive system may be not finite time incrementally passive. Compared with [24], the energy at each switching time is allowed to decrease. If there exists the common storage function $V_i = V$, then system (1) is finite-time incrementally passive under arbitrary switching signal.

B. SUFFICIENT CONDITIONS OF FINITE-TIME INCREMENTAL PASSIVITY

We will provide some conditions of finite-time incremental passivity for system (1) and a state-dependent switching law design method.

Theorem 1: Assume that there exist continuous functions $S_i(x, \hat{x})$, $\beta_{ij}(x, \hat{x}) \leq 0$, $\eta_{ij}(x, \hat{x}) \neq 0$, nonnegative smooth functions $V_i(x, \hat{x})$, class $\mathcal{K}$ functions $\gamma_i(*)$: $R \geq 0 \to R \geq 0$, and constants $\varepsilon_i > 0$, $i, j \in I$ such that (6) and

$$\frac{\partial V_i}{\partial x} f_i(x, u_i) + \frac{\partial V_i}{\partial \hat{x}} f_i(\hat{x}, \hat{u}_i) + \sum_{j=1}^{M} \beta_{ij}(x, \hat{x})\left(S_i(x, \hat{x}) - S_j(x, \hat{x})\right) \leq -\gamma_i(V_i(x, \hat{x})) + +(y^T - \hat{y}^T)(u_i - \hat{u}_i) \quad (7)$$

$$S_i(x, \hat{x}) - S_j(x, \hat{x}) = \eta_{ij}(x, \hat{x})\left(V_i(x, \hat{x}) - V_j(x, \hat{x})\right) \quad (8)$$

hold. Then, system (1) is finite time incrementally passive under the switching law

$$\sigma(t) = \arg\min_{i \in I} \left\{S_i(x, \hat{x})\right\}. \quad (9)$$

Proof: According to the switching law (9), the switching sequence can be described as (2). When $t \in [t_k, t_{k+1})$, the i_k-th subsystem is active, Thus, we can obtain

$$S_{i_{k+1}}\left(x(t_{k+1}), \hat{x}(t_{k+1})\right) = S_{i_k}\left(x(t_{k+1}), \hat{x}(t_{k+1})\right). \quad (10)$$

From (8), we have

$$V_{i_{k+1}}\left(x(t_{k+1}), \hat{x}(t_{k+1})\right) = V_{i_k}\left(x(t_{k+1}), \hat{x}(t_{k+1})\right). \quad (11)$$

(7) implies

$$\dot{V}_{i_k} \leq -\gamma_{i_k}(V_{i_k}(x, \hat{x})) + (y^T - \hat{y}^T)(u_{i_k} - \hat{u}_{i_k}). \quad (12)$$

Therefore, system (1) is finite-time incrementally passive.

Now, consider a swiched affine nonlinear systems:

$$\dot{x} = f_\sigma(x) + g_\sigma(x) u_\sigma, \quad y = h_\sigma(x), \quad (13)$$

where $f_i(x)$, $g_i(x)$ and $h_i(x)$ are smooth with $f_i(0) = 0$ and $h_i(0) = 0$.

Theorem 2: Assume that there exist nonnegative smooth functions $V_i(x, \hat{x})$, continuous functions $S_i(x, \hat{x})$, $\beta_{ij}(x, \hat{x}) \leq 0$, $\eta_{ij}(x, \hat{x}) \neq 0$, and class $\mathcal{K}$ functions $\gamma_i(*)$: $R \geq 0 \to R \geq 0$, and some $\varepsilon_i > 0$, $i, j \in I$ such that (6) and

$$\frac{\partial V_i}{\partial x} f_i(x) + \frac{\partial V_i}{\partial \hat{x}} f_i(\hat{x}) + \sum_{j=1}^{M} \beta_{ij}(x, \hat{x})(S_i(x, \hat{x}) - S_j(x, \hat{x})) \leq -\gamma_i(V_i(x, \hat{x})), \quad (14)$$

$$\frac{\partial V_i}{\partial x} g_i(x) - (h_i^T(x) - h_i^T(\hat{x})) = 0, \quad \frac{\partial V_i}{\partial \hat{x}} g_i(\hat{x}) - (h_i^T(\hat{x}) - h_i^T(x)) = 0 \quad (15)$$

hold. Then, system (13) is finite-time incrementally passive under switching law (9).

Proof: Consider an augumented system

$$\dot{x} = f_\sigma(x) + g_\sigma(x) u_\sigma, \quad \dot{\hat{x}} = f_\sigma(\hat{x}) + g_\sigma(\hat{x}) \hat{u}_\sigma, \quad y = h_\sigma(x), \hat{y} = h_\sigma(\hat{x}). \quad (16)$$

Since (14), (15) and (16) hold, the derivative of $V_i(x, \hat{x})$ is

$$\begin{aligned}\dot{V}_i &= \frac{\partial V_i}{\partial x} f_i(x) + \frac{\partial V_i}{\partial \hat{x}} f_i(\hat{x}) + \frac{\partial V_i}{\partial x} g_i(x) u_i + \frac{\partial V_i}{\partial \hat{x}} g_i(\hat{x}) \hat{u}_i \\ &\leq \frac{\partial V_i}{\partial x} f_i(x) + \frac{\partial V_i}{\partial \hat{x}} f_i(\hat{x}) + (h_i^T(x) - h_i^T(\hat{x})) u_i + (h_i^T(\hat{x}) - h_i^T(x)) \hat{u}_i \\ &\leq -\gamma_i(V_i(x, \hat{x})) + (y^T - \hat{y}^T)(u_i - \hat{u}_i) - \sum_{j=1}^{M} \beta_{ij}(x, \hat{x})(S_i(x, \hat{x}) - S_j(x, \hat{x})). \end{aligned} \quad (17)$$

According to Theorem 1, Theorem 2 holds.

Remark 4: Theorem 1 tells us that a switched system is finite-time incrementally passive by the design of switching law, even if each subsystem is non-finite-time incrementally passive. If (14) and (15) hold with $\beta_{ij} = 0$, then each subsystem is finite time incrementallly passive. Since the switching law (9) can degenerate into the well-known ''min-switching'' law in [24] by setting $V_i = S_i$.

C. FEEDBACK INTERCONNECTION

This section will analyze the invariance properties of the finite-time incremental passivity of the feedback interconnected switched nonlinear systems.

Consider a feedback interconnection system H formed by the feedback interconnection of two switched systems H_1

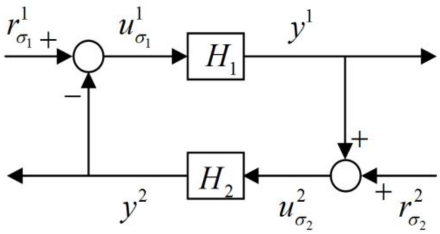

FIGURE 1. Feedback interconnection system.

and H_2 depicted in Fig. 1

$$H_1: \dot{x}^1 = f^1_{\sigma_1}\left(x^1, u_{\sigma_1}\right), \quad y^1 = h^1_{\sigma_1}\left(x^1\right), \tag{18}$$

where the state is $x^1 \in R^{n_1}, \sigma_1(t) : [0,\infty) \to I_1 = \{1, 2, \cdots M_1\}$ is the switching signal with the switching sequence

$$\Sigma_1 = \{(i^1_0, t^1_0), (i^1_1, t^1_1), \ldots (i^1_{j^1}, t^1_{j^1}), \ldots \left| i^1_{j^1} \in I_1, j^1 \in N\right.\}$$

and

$$H_2: \dot{x}^2 = f^2_{\sigma_2}\left(x^2, u^2_{\sigma_2}\right), \quad y^2 = h^2_{\sigma_2}\left(x^2\right) \tag{19}$$

where the state is $x^2 \in R^{n_2}, \sigma_2(t) : [0,\infty) \to I_2 = \{1, 2, \cdots M_2\}$ is the switching signal with the switching sequence

$$\Sigma_2 = \{(i^2_0, t^2_0), (i^2_1, t^2_1), \ldots, (i^2_{j^2}, t^2_{j^2}), \ldots \left| i^2_{j^2} \in I_2, j^2 \in N\right.\}.$$

Seen from Figure 1, $u^1_{\sigma_1} = r^1_{\sigma_1} - y^2, u^2_{\sigma_2} = r^2_{\sigma_2} + y^1$. $\dim r^2_{\sigma_2} = \dim h^1_{\sigma_1} = \dim u^2_{\sigma_2}$ and $\dim r^1_{\sigma_1} = \dim h^2_{\sigma_2} = \dim u^1_{\sigma 1}$. The input of system H is $u_\sigma = \begin{pmatrix} r^1_{\sigma_1} \\ r^2_{\sigma_2} \end{pmatrix}$ and the output of system H is $y = \begin{pmatrix} y^1 \\ y^2 \end{pmatrix}$. By the merging switching signal technique, the switching signal is defined as $\sigma = \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} : [0,\infty) \to I = I_1 \times I_2$ with the switching sequence described as

$$\Sigma = \left\{(i_0, t_0), (i_1, t_1), \cdots (i_j, t_j), \cdots \left| i_j \in I, j \in N\right.\right\}, \tag{20}$$

where $t_0 = t^1_0 = t^2_0, i_j = (\sigma_1(t_j), \sigma_2(t_j)) = \left(i^1_{j^1}, i^2_{j^2}\right)$.

Now, we study invariance properties of the finite-time incremental passivity for system H.

Theorem 3: Assume that there exist nonnegative smooth functions $V^1_{i^1}(x^1, \hat{x}^1)$ and $V^2_{i^2}(x^2, \hat{x}^2)$, continuous functions $S^1_{i^1}(x^1, \hat{x}^1)$ and $S^2_{i^2}(x^2, \hat{x}^2)$, functions $\beta^1_{i^1j^1}(x^1, \hat{x}^1) \leq 0$, $\beta^2_{i^2j^2}(x^2, \hat{x}^2) \leq 0$, $\eta^1_{i^1j^1}(x^1, \hat{x}^1) \neq 0$, $\eta^2_{i^2j^2}(x^2, \hat{x}^2) \neq 0$, class $\mathcal{K}$ functions $\gamma^1_{i^1}(*)$, $\gamma^2_{i^2}(*)$ and constants $\varepsilon^1_{i^1} > 0$, $\varepsilon^2_{i^2} > 0$ such that for $i^q, j^q \in I_q, q = 1, 2$

$$\begin{aligned} &\frac{\partial V^1_i}{\partial x^1} f^1_{i^1}\left(x^1, u^1_{i^1}\right) + \frac{\partial V^1_i}{\partial \hat{x}^1} f^1_{i^1}\left(\hat{x}^1, \hat{u}^1_{i^1}\right) \\ &\quad + \sum_{j^1=1}^{M_1} \beta^1_{i^1j^1}\left(S^1_{i^1}\left(x^1, \hat{x}^1\right) - S^1_{j^1}\left(x^1, \hat{x}^1\right)\right) \\ &\leq -\gamma^1_{i^1}\left(V^1_{i^1}\left(x^1, \hat{x}^1\right)\right) + \left(u^1_{i^1} - \hat{u}^1_{i^1}\right)^T\left(y^1 - \hat{y}^1\right), \end{aligned} \tag{21}$$

$$\begin{aligned} &\frac{\partial V^2_{i^2}}{\partial x^2} f^2_{i^2}\left(x^2, u^2_{i^2}\right) + \frac{\partial V^2_{i^2}}{\partial \hat{x}^2} f^2_{i^2}\left(\hat{x}^2, \hat{u}^2_{i^2}\right) \\ &\quad + \sum_{j^2=1}^{M_2} \beta^2_{i^2j^2}\left(S^2_{i^2}\left(x^2, \hat{x}^2\right) - S^2_{j^2}\left(x^2, \hat{x}^2\right)\right) \\ &\leq -\gamma^2_{i^2}\left(V^2_{i^2}\left(x^2, \hat{x}^2\right)\right) + \left(u^2_{i^2} - \hat{u}^2_{i^2}\right)^T\left(y^2 - \hat{y}^2\right), \end{aligned} \tag{22}$$

$$\begin{aligned} &V^1_{i^1}\left(x^1, \hat{x}^1\right) - V^1_{j^1}\left(x^1, \hat{x}^1\right) \\ &= \eta^1_{i^1j^1}\left(S^1_{i^1}\left(x^1, \hat{x}^1\right) - S^1_{j^1}\left(x^1, \hat{x}^1\right)\right), \\ &V^2_{i^2}\left(x^2, \hat{x}^2\right) - V^2_{j^2}\left(x^2, \hat{x}^2\right) \\ &= \eta^2_{i^2j^2}\left(S^2_{i^2}\left(x^2, \hat{x}^2\right) - S^2_{j^2}\left(x^2, \hat{x}^2\right)\right) \end{aligned} \tag{23}$$

$$\int_0^{\varepsilon^1_i} \frac{dz}{\gamma^1_i(z)} < +\infty, \quad \int_0^{\varepsilon^2_{i^2}} \frac{dz}{\gamma^2_{i^2}(z)} < +\infty \tag{24}$$

hold. Design the composite state-dependent switching law as

$$\sigma(t) = (\sigma_1(t), \sigma_2(t)), \tag{25}$$

where

$$\begin{aligned} \sigma_1\left(x^1, \hat{x}^1\right) &= \arg\min_{i \in I_1}\left\{S^1_i\left(x^1, \hat{x}^1\right)\right\}, \\ \sigma_2\left(x^2, \hat{x}^2\right) &= \arg\min_{\hat{i} \in I_2}\left\{S^2_{i^2}\left(x^2, \hat{x}^2\right)\right\} \end{aligned}$$

Then, system H is finite time incrementally passive under the switching law (24).

Proof: Define the storage function of system H as

$$V_{(i^1,i^2)}\left(x^1, \hat{x}^1, x^2, \hat{x}^2\right) = V^1_{i^1}\left(x^1, \hat{x}^1\right) + V^2_{i^2}\left(x^2, \hat{x}^2\right), \left(i^1, i^2\right) \in I.$$

The derivative of $V_{(i^1,i^2)}(x^1, \hat{x}^1, x^2, \hat{x}^2)$ is

$$\begin{aligned} \dot{V}_{(i^1,i^2)} &= \frac{\partial V^1_{i^1}}{\partial x^1} f^1_{i^1}\left(x^1, u^1_{i^1}\right) + \frac{\partial V^1_{i^1}}{\partial \hat{x}^1} f^1_{i^1}\left(\hat{x}^1, \hat{u}^1_{i^1}\right) \\ &\quad + \frac{\partial V^2_{i^2}}{\partial x^2} f^2_{i^2}\left(x^2, u^2_{i^2}\right) + \frac{\partial V^2_{i^2}}{\partial \hat{x}^2} f^2_{i^2}\left(\hat{x}^2, \hat{u}^2_{i^2}\right) \\ &\leq -\sum_{j^1=1}^{M_1} \beta^1_{i^1j^1}\left(S^1_{i^1}\left(x^1, \hat{x}^1\right) - S^1_{j^1}\left(x^1, \hat{x}^1\right)\right) \\ &\quad -\sum_{j^2=1}^{M_2} \beta^2_{i^2j^2}\left(S^2_{i^2}\left(x^2, \hat{x}^2\right) - S^2_{j^2}\left(x^2, \hat{x}^2\right)\right) \\ &\quad -\gamma^1_{i^1}\left(V^1_{i^1}\left(x^1, \hat{x}^1\right)\right) + \left(u^1_{i^1} - \hat{u}^1_{i^1}\right)^T\left(y^1 - \hat{y}^1\right) \\ &\quad -\gamma^2_{i^2}\left(V^2_{i^2}\left(x^2, \hat{x}^2\right)\right) + \left(u^2_{i^2} - \hat{u}^2_{i^2}\right)^T\left(y^2 - \hat{y}^2\right), \end{aligned} \tag{26}$$

Let $\gamma_{i^1 i^2}(z) = \min\left\{\gamma_i^1(z/2), \gamma_{i^2}^2(z/2)\right\}$. By Lemma 1, we obtain that

$$\gamma_{i^1 i^2}\left(V_{i^1}^1 + V_{i^2}^2\right) \leq \gamma_{i^1}^1\left(V_{i^1}^1\right) + \gamma_{i^2}^2\left(V_{i^2}^2\right). \tag{27}$$

By substituting $u_{i^1}^1 = r_{i^1}^1 - y^2, u_{i^2}^2 = r_{i^2}^2 + y^1$ into (26) together with (27), we have

$$\begin{aligned}\dot{V}_{(i^1,i^2)} \leq & -\gamma_{i^1 i^2}\left(V_i^1 + V_{i^2}^2\right) + (r-\hat{r})^T(y-\hat{y})^T \\ & - \sum_{j^1=1}^{M_1} \beta_{i^1 j^1}^1 \left(S_{i^1}^1\left(x^1, \hat{x}^1\right) - S_{j^1}^1\left(x^1, \hat{x}^1\right)\right) \\ & - \sum_{j^2=1}^{M_2} \beta_{i^2 j^2}^2 \left(S_{i^2}^2\left(x^2, \hat{x}^2\right) - S_{j^2}^2\left(x^2, \hat{x}^2\right)\right)\end{aligned}$$

with $r = \left(r_{i^1}^1, r_{i^2}^2\right)^T, y = \left(y^1, y^2\right)$.

The switching law (25) implies the following equation

$$V_{\sigma(t)}\left(x^1(t), x^2(t)\right) \leq -r_{\sigma(\mathrm{t})}\left(\mathrm{V}_{\sigma(\mathrm{t})}\left(x^1(\mathrm{t}), x^2(\mathrm{t})\right)\right) + (r-\hat{r})^{\mathrm{T}}(y-\hat{y})^{\mathrm{T}}.$$

and

$$\begin{aligned}& V_{\sigma(t_k)}\left(x^1(t_k), x^2(t_k), \hat{x}^1(t_k), \hat{x}^2(t_k)\right) \\ &= V_{i_k^1}^1\left(x^1(t_k), \hat{x}^1(t_k)\right) + V_{i_k^2}^2\left(x^2(t_k), \hat{x}^2(t_k)\right) \\ &= V_{i_{k-1}^1}^1\left(x^1(t_k), \hat{x}^1(t_k)\right) + V_{i_{k-1}^2}^2\left(x^2(t_k), \hat{x}^2(t_k)\right) \\ &= V_{\sigma(t_{k-1})}\left(x^1(t_k), x^2(t_k), \hat{x}^1(t_k), \hat{x}^2(t_k)\right)\end{aligned} \tag{28}$$

Let $\varepsilon_{i^1 i^2} = \min\left\{2\varepsilon_{i^1}^1, 2\varepsilon_{i^2}^2\right\}$. It follows from Lemma 1 and (24) that

$$\begin{aligned}\int_0^{\varepsilon_{\hat{i}\hat{i}}} \frac{dz}{\gamma_{i^1 i^2}(z)} &\leq \int_0^{\varepsilon_{i^1 i^2}} \left(\frac{1}{\gamma_{i^1}^1(z)} + \frac{1}{\gamma_{i^2}^2(z)}\right) dz \\ &\leq \int_0^{2\varepsilon_{i^1}^1} \frac{dz}{\gamma_{i^1}^1(z)} + \int_0^{2\varepsilon_{i^2}^2} \frac{dz}{\gamma_{i^2}^2(z)} \\ &< +\infty\end{aligned} \tag{29}$$

Hence, system H is finite time incrementally passive.

IV. FINITE-TIME OUTPUT TRACKING CONTROL

This section will solve the finite-time output tracking problem using the established finite-time incremental passivity theory.

Theorem 4: Consider a finite-time incrementally passive system (1) with storage functions $V_i(x, \hat{x})$ under a switching signal $\sigma(t)$. Suppose that for bounded inputs $u_i = \bar{u}_i(t)$, there exists a bounded solution $\bar{x}(t), t \geq t_0$ of system (1) satisfying $h_\sigma(\bar{x}(t)) = y^*(t), t \geq t_0$. If in addition, $\alpha_1\left(\left\|x-\hat{x}\right\|\right) \leq V_i(x,\hat{x}) \leq \alpha_2\left(\left\|x-\hat{x}\right\|\right)$ holds with class K_∞ functions $\alpha_1(\cdot), \alpha_2(\cdot)$, then there exists controllers $u_i = \bar{u}_i - K_i(y-y^*)$ with positive definite matrices $K_i, i \in I$ such that the finite time output tracking problem is solvable

Proof: For $t_0 < t < \infty$, we can find $k \in N$ satisfying $t \in [t_k, t_{k+1})$. Since system (1) is finite-time incrementally passive, for $t \in [t_k, t_{k+1})$, we have

$$\dot{V}_{i_k}(x(t), \hat{x}(t)) \leq -\gamma_{i_k}(V_{i_k}(x(t), \hat{x}(t))) + \left(u_{i_k}^T - \hat{u}_{i_k}\right)(y-\hat{y}). \tag{30}$$

Substituting $\bar{x}(t), y^*(t), \bar{u}_i$ for $\hat{x}, \hat{y}, \hat{u}_i$ into the above inequality gives

$$\dot{V}_{i_k}(x(t), \bar{x}(t)) \leq -\gamma_{i_k}(V_{i_k}(x(t), \bar{x}(t))) + \left(u_{i_k}^T - \bar{u}_{i_k}\right)(y-y^*). \tag{31}$$

Designing the controllers as $u_i = \bar{u}_i - K_i(y-y^*)$ yields

$$\dot{V}_{i_k}(x(t), \bar{x}(t)) \leq -\gamma_{i_k}(V_{i_k}(x(t), \bar{x}(t))) < 0. \tag{32}$$

From (32), we have

$$V_{i_k}(x(t_k), \bar{x}(t_k)) \leq V_{i_{k-1}}(x(t_k), \bar{x}(t_k)), \quad i_k \in I. \tag{33}$$

Since (32) and (33) hold, we can obtain that

$$\begin{aligned}& V_{i_k}(x(t), \bar{x}(t)) - V_{i_0}(x(t_0), \bar{x}(t_0)) \\ &= V_{i_k}(t) - V_{i_k}(t_k) \\ &\quad + \sum_{p=1}^{k} V_{i_p}(t_p) - V_{i_{p-1}}(t_{p-1}) + \sum_{p=1}^{k} V_{i_k}(t_p) - V_{i_{k-1}}(t_p) \\ &\leq -\int_{t_0}^{t} \gamma_{\sigma(\tau)}(V_{\sigma(\tau)}(x(\tau), \bar{x}(\tau)))d\tau.\end{aligned} \tag{34}$$

Therefore,

$$V_{i_k}(x(t), \bar{x}(t)) \leq V_{i_0}(x(t_0), \bar{x}(t_0)).$$

Since $\alpha_1\left(\left\|x-\hat{x}\right\|\right) \leq V_i(x,\hat{x}) \leq \alpha_2\left(\left\|x-\hat{x}\right\|\right)$ holds, we have

$$\begin{aligned}\alpha_1(\|x(t)-\bar{x}(t)\|) &\leq V_{i_k}(x(t), \bar{x}(t)) \\ &\leq V_{i_0}(x(t_0), \bar{x}(t_0)) \leq \alpha_2(\|x(t_0)-\bar{x}(t_0)\|).\end{aligned} \tag{35}$$

Since $\bar{x}(t)$ is bounded, (35) implies that $x(t)$ is also bounded. Thus, $\dot{x}(t), \dot{\bar{x}}(t)$ is bounded, because the input signal $\bar{u}_i$ are bounded and f_i and g_i are continuous. Hence, the boundeness and uniform continuous property of $\|x(t)-\bar{x}(t)\|$ is obtained for $t \geq t_0$. Let $\gamma(z) = \min_{i\in I}\{\gamma_i(z)\}$. Therefore, $\gamma(\alpha_1(\cdot))$ is positive definite and uniformly continuous, which implies $\gamma(\alpha_1(\|x(t)-\bar{x}(t)\|))$ is uniformly continuous.

From (32), we have

$$\begin{aligned}&\int_{t_0}^{t} \gamma(\alpha_1(\|x(\tau)-\bar{x}(\tau)\|))d\tau \\ &\qquad \leq \int_{t_0}^{t} \gamma_{\sigma(\tau)}(V_{\sigma(\tau)}(x(\tau), \bar{x}(\tau)))d\tau < \infty.\end{aligned} \tag{36}$$

According to Barbalat's lemma, we have $\lim_{t\to\infty} \gamma(\alpha_1(\|x(t)-\bar{x}(t)\|)) = 0$. Thus, $\lim_{t\to\infty} \|x(t)-\bar{x}(t)\| = 0$.

Next, we will verify that there exists a settling time $0 < T < \infty$ such that $\lim_{t\to t_0+T} \|x(t) - \bar{x}(t)\| = 0, T > t_0$, i.e.

$$\|x(t) - \bar{x}(t)\| = 0, \quad t \geq t_0 + T.$$

We only need to verify that

$$V_{i_k}(x(t), \bar{x}(t)) = 0, \quad t \geq t_0 + T.$$

The transformation of variables:

$$[t_0, t_0 + T] \to \left[V_{i_0}(x(t_0), \bar{x}(t_0)), 0\right]$$

was given by $z = V_\sigma(x(t), \bar{x}(t))$. From (32) and (33), we have

$$\begin{aligned}
T &= \int_{t_0}^{t_1} d\tau + \int_{t_1}^{t_2} d\tau + \cdots \int_{t_k}^{t_k+T} d\tau \\
&\leq \int_{t_0}^{t_1} \frac{\dot{V}_{i_0}}{-\gamma_{i_0}(V_{i_0})} d\tau + \int_{t_1}^{t_2} \frac{\dot{V}_{i_1}}{-\gamma_{i_1}(V_{i_1})} d\tau + \cdots \\
&\quad + \int_{t_k}^{t_0+T} \frac{\dot{V}_{i_k}}{-\gamma_{i_k}(V_{i_k})} d\tau \\
&= \int_{V_{i_0}(t_0)}^{V_{i_0}(t_1)} \frac{dV_{i_0}}{-\gamma_{i_0}(V_{i_0})} + \int_{V_{i_1}(t_1)}^{V_{i_1}(t_2)} \frac{dV_{i_1}}{-\gamma_{i_1}(V_{i_1})} + \cdots \\
&\quad + \int_{V_{i_k}(t_k)}^{V_{i_k}(t_0+T)} \frac{dV_{i_k}}{-\gamma_{i_k}(V_{i_k})} \\
&= \int_{V_{i_0}(t_0)}^{V_{i_0}(t_1)} \frac{dz}{-\gamma_{i_0}(z)} + \int_{V_{i_1}(t_1)}^{V_{i_1}(t_2)} \frac{dV_{i_1}}{-\gamma_{i_1}(z)} + \cdots \\
&\quad + \int_{V_{i_k}(t_k)}^{V_{i_k}(t_0+T)} \frac{dz}{-\gamma_{i_k}(z)} \\
&= \int_{V_{i_0}(t_1)}^{V_{i_0}(t_0)} \frac{dz}{\gamma_{i_0}(z)} + \int_{V_{i_1}(t_2)}^{V_{i_1}(t_1)} \frac{dV_{i_1}}{\gamma_{i_1}(z)} + \cdots + \int_{V_{i_k}(t_0+T)}^{V_{i_k}(t_k)} \frac{dz}{\gamma_{i_k}(z)} \\
&\leq \int_{V_{i_k}(t_0+T)}^{V_{i_0}(t_0)} \frac{dz}{\gamma(z)} = \int_{0}^{V_{i_0}(t_0)} \frac{dz}{\gamma(z)}.
\end{aligned}$$

Since $\int_0^{\varepsilon_i} \frac{dz}{\gamma_i(z)} < \infty$ holds for some $\varepsilon_i > 0, i \in I$, we have

$$\int_0^{\varepsilon} \frac{dz}{\gamma(z)} \leq \sum_{i=1}^{M} \int_0^{\varepsilon_i} \frac{dz}{\gamma_i(z)} < \infty,$$

where $\varepsilon = \min_{i\in I}\{\varepsilon_i\}$. Therefore, when $V_{i_0}(x(t_0), \bar{x}(t_0)) \leq \varepsilon$, it holds that $T < \infty$. Therefore, all solutions of system (1) convergent to $\bar{x}(t)$ in finite time, which implies

$$\lim_{t\to t_0+T} \left\|y(t) - y^*(t)\right\| = 0.$$

This completes proof.

Combining Theorem 1 with Theorem 4 gives as follows:

Theorem 5: Suppose that there exist C^1 nonnegative functions $V_i(x, \hat{x})$, continuous functions $S_i(x, \hat{x})$, functions $\beta_{ij}(x, \hat{x}) \leq 0$, $\eta_{ij}(x, \hat{x}) \neq 0$, and class K_∞ functions $\gamma_i(*)$: $R \geq 0 \to R \geq 0$, $\alpha_1(\cdot)$, $\alpha_2(\cdot)$ and constants $\varepsilon_i > 0, i, j \in I$ such that $\alpha_1(\|x - \hat{x}\|) \leq V_i(x, \hat{x}) \leq \alpha_2(\|x - \hat{x}\|)$ and (6)-(8) hold. If in addition,

$$y^*(t) - h_i(\bar{x}(t)) + \max_{j\in I}\left\{V_i(x, \bar{x}) - V_j(x, \bar{x})\right\} = 0 \quad (37)$$

holds for all $i, j \in I$, where for $t \geq t_0$, $\bar{x}(t)$ is a bounded solution of system (1) with the bounded inputs $u_i = \bar{u}_i(t)$, then the finite time output tracking control problem of system (1) is solvable by $u_i = \bar{u}_i - K_i(y - y^*)$ under the switching law (9) with $x_2 = \bar{x}$, where $K_i, i \in I$ are positive definite matrices.

Proof: Theorem 1 implies that system (1) is finite time incrementally passive. On the other hand, from (37), there exists a bounded solution $\bar{x}(t)$ of system (1) such that $h_\sigma(\bar{x}(t)) = y^*(t)$ for $t \geq t_0$. Based on Theorem 4, the finite time output tracking control problem of system (1) is solvable.

V. EXAMPLE

This section will verify the effectiveness of the results by a numerical example.

Consider system (1) described by

$$\dot{x}_1 = -x_1^3 - 4x_1 + \frac{1}{2}x_2 + 5 - x_1^{\frac{3}{5}} + u_1,$$

subsystem1:

$$\begin{aligned}
\dot{x}_2 &= \frac{1}{2}x_1 + x_2 - 2.5 + 3^{\frac{4}{5}}.2^{\frac{3}{5}} - 3^{\frac{4}{5}}x_2^{\frac{3}{5}} + \frac{1}{3}u_1. \\
\dot{x}_1 &= x_1 + x_2 - 3 + 2^{-\frac{1}{5}} + \frac{1}{2}u_2 - 2^{-\frac{1}{5}}x_1^{\frac{3}{5}},
\end{aligned}$$

Subsystem 2:

$$\dot{x}_2 = 2x_1 - 10x_2 + 18 - \frac{1}{2}x_2^{\frac{3}{5}} + u_2 + 2^{-\frac{2}{5}} \quad (38)$$

with the outputs $y_1 = x_1 + x_2$, $y_2 = 2x_2 - x_1$. The reference signal is given as $y^* = 3$.

First, we select the storage functions as

$$V_1(x, \hat{x}) = \frac{1}{2}(x - \hat{x})^T P_1 (x - \hat{x})$$

and

$$V_2(x_1, x_2) = \frac{1}{2}(x_1 - x_2)^T P_2 (x_1 - x_2),$$

where $P_1 = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$. Differentiating V_i gives that

$$\begin{aligned}
\dot{V}_1 &\leq -\beta_{12}(V_1 - V_2) - 1.2(V_1)^{0.8} + (u_1 - \hat{u}_1)(y_1 - \hat{y}_1), \\
\dot{V}_2 &\leq -\beta_{21}(V_2 - V_1) - 0.6V_2 + (u_2 - \hat{u}_2)(y_1 - \hat{y}_1).
\end{aligned}$$

where $\beta_{12} = -3.5$, $\beta_{21} = -7$.

By Theorem 1, system (38) is finite-time incrementally passive under switching law (9).

There is a bounded solution $\bar{x}(t) = [2, 1]^T$ of closed-loop system (23) with input $u_i = 0$ and $y = h_i(\bar{x}) = y^*$, $i = 1, 2$.

Design the feedback controllers as

$$u_1 = -(y_1 - 3), \quad u_2 = -2(y_2 - 3). \quad (39)$$

According to Theorem 3, the finite-time output tracking problem for closed-loop system (38) is solvable under the switching law $\sigma(t) = \arg\min_{i\in I}\{V_i(x, \bar{x})\}$.

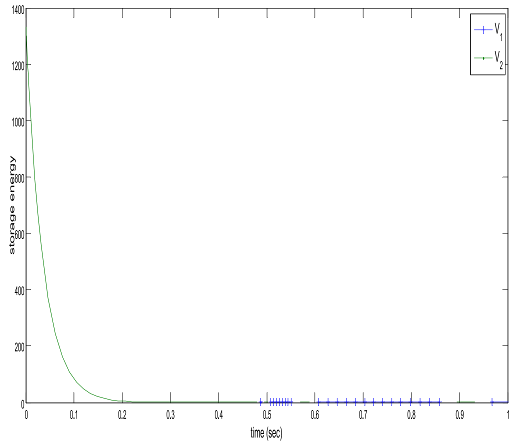

FIGURE 2. The stored energy of the switched system (23).

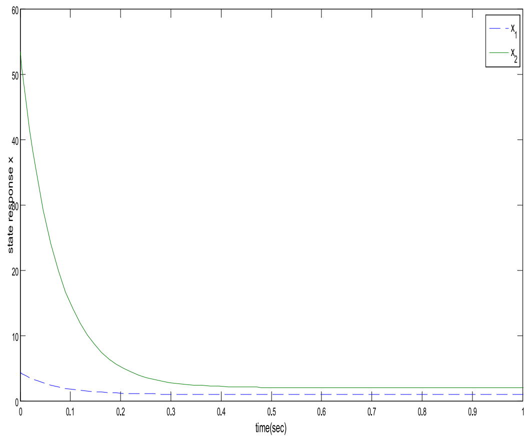

FIGURE 3. State response of the switched system.

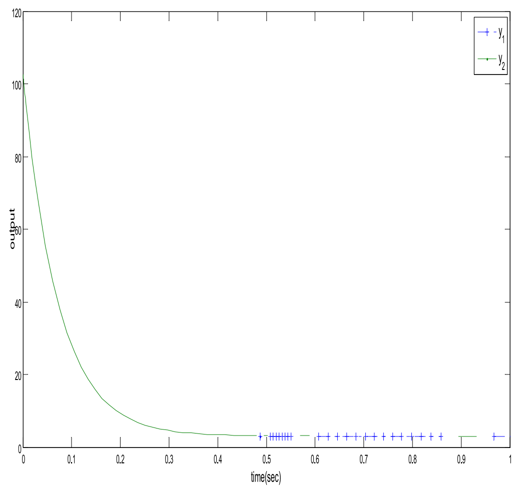

FIGURE 4. Outputs of the switched system.

The simulation was performed with the initial state $x(0) = (4.3, 53.4)$. The simulation results are presented in Figs. 1-4. Figure 1 describes the stored energy of system (38) with the controllers (39) under the switching law as shown in Figure 4. In Figure 1, since the energy is descreasing and degenerate into zero in finite time and the energy drops at each switching

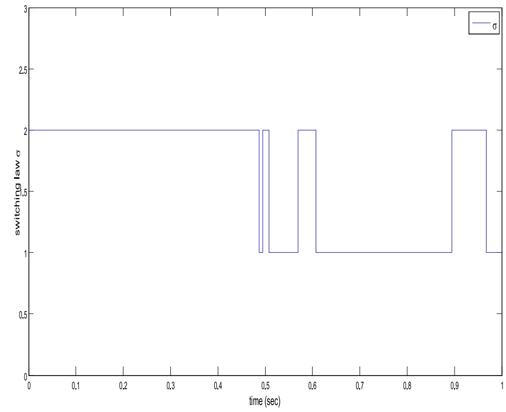

FIGURE 5. Switching law.

timet, this verifies finite time incremental passivity definition. The state response of the switched system convergent into bounded solution $\bar{x}$ in finite time as shown in Figure 2. Thus, the state is bounded. In Figure 3, the outputs of the switched system (38) track the reference signal in finite time. Therefore, the finite time output tracking control problem of system (38) is solvable. The simulation results well verified the effectiveness of the proposed approach.

VI. CONCLUSION

This paper has studied finite-time incremental passivity for switched nonlinesr systems. Then, the established finite-time incremental passivity theory was applied to solve the finite-time output tracking problem of switched nonlinear systems. A more general switching law design method was proposed. There are some interesting problems that need to be addressed. One of the problems is to study the relationship between finite time incremental passivity and finite time incremental stabilityswitched nonlinear systems.

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