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Stochastic Differential Game Model Analysis of Emission-Reduction Technology Under Cost-Sharing Contracts in the Carbon Trading Market

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ABSTRACT Climate change and greenhouse gas emission reduction have become pressing concerns in recent years. Carbon trading systems and emission-reduction cost-sharing contracts are important emission-reduction measures, under the two mechanisms, this paper considers a dynamic emission-reduction technology investment decision-making problem in a dyadic supply chain consisting of a manufacturer and a retailer. In considering the influence of consumers' low-carbon preferences on market demand as well as the impact of uncertainty on carbon emission-reduction behaviour, this paper (1) constructs the investment game model under cost-sharing coordination between manufacturers and retailers; (2) adopts differential game and dynamic optimisation methods to obtain investment strategies for manufacturers and retailers under cost-sharing contracts.; and (3) uses a numerical simulation method to simulate the path evolution process of each state variable and, by analysing the sensitivity of various parameters, to determine the influence of various parameters on the decision making of emission reduction among stakeholders. The study finds that under the carbon trading system, cost-sharing contracts have a regulatory effect on enterprise emission-reduction investment and enterprise profits, and that the impact of regulatory effects increases over time. Likewise, the evolution path of the parameters used for various indicators presents a strengthened trend over time. The results show that it is necessary to enhance the cooperative development and exchange of carbon emission-reduction technology among enterprises.

INDEX TERMS Carbon emissions trading system, cost-sharing contracts, investment in emission-reduction technology, stochastic differential game.

I. INTRODUCTION

Global warming caused by greenhouse gas emissions has become a pressing concern, internationally, in recent years. From the macro-level to the micro level, all stakeholders have adopted measures to reduce greenhouse gas emissions, including structural adjustments, subsidies, innovation incentives, carbon tax, emission trading, and technological innovation. The control method has gradually shifted from total control to total control coupled with intensity control. Researchers are now studying the impact of these policies on emission-reduction behaviour. The present paper considers the impact of China's carbon emission trading and

emission-reduction technology investment subsidy system on the supply chain investment in emission reduction. With its rapid economic development, China has also become the largest carbon emitter. In response to climate change, China actively participates in various carbon emission reduction agreements. At the same time, to fulfil its responsibilities under the framework of the Paris Agreement, China has committed that "by 2030, China's carbon dioxide emissions per unit of GDP (Gross Domestic Product) will be 60% to 65% lower than 2005, and reach peak carbon emissions around 2030." For this reason, China has adopted supply-side reform and industrial restructuring policies from a macro perspective, while at the same time exploring and promoting, from a micro perspective, an emission trading system based on carbon quotas as well as an emission-reduction

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technology-innovation subsidy system. Carbon emission-reduction behaviour is affected by innovation behaviour and also external uncertainty, which has a strong influence on emission-reduction efforts. Understanding the impacts of this comprehensive policy background and also of the uncertainty associated with emission-reduction technology on supply chain emission-reduction behaviour has become the key to the effectiveness of micro-level policies in this domain. Accordingly, the present paper aims to illuminate the impacts in question.

II. LITERATURE REVIEW

Recently, domestic and foreign scholars have increasingly focused on issues related to carbon emission reduction. The impact of micro-policy on enterprises' investment behaviour with respect to carbon emission reduction began with the research of Benjaafar [1]. Benjaafar was the first to introduce carbon emission factors into the study of supply chain systems, modifying traditional models by associating carbon emission parameters with decision variables affecting supply chain enterprises. Furthermore, Hovelaque proposed a new model to reduce carbon emission, which considered the relationship between price, inventory policy, environmental-related demand, and total carbon emissions [2]. Hou *et al.* studied the decision-making problem in a dyadic supply chain consists of a manufacturer and a retailer for dynamic emission reduction technology investment, and the comparative analysis of four different decision-making circumstances, namely decentralized decision-making, centralized decision-making, coordinated decision-making and social welfare maximization, is obtained [3]. To study the production and emission control decision-making of the supply chain, Xu *et al.* considered how wholesale prices and cost-sharing contracts can lead to an ordered supply chain system: manufacturers use green production technology to reduce carbon emissions associated with unit products, and then cooperate with retailers through a contract to sell products to consumers with low-carbon preferences [4]. Toptal and Çetinkaya used Bayesian game theory to study the coordination between enterprises and suppliers, and analysed the impact of decentralised and centralised supply chain decision-making on total carbon emissions from the perspective of carbon footprints [5]. Considering competitive supply chains, which consist of a manufacturer and a retailer, Yang, drawing on ideas from game theory, discussed pricing and carbon emission-reduction decision-making processes from both a horizontal and a vertical perspective [6]. For their part, to study the decision-making of supply chain members vis-à-vis issues of social welfare, Du S F *et al.* established a "carbon constraint and trading" system composed of manufacturers and carbon trading permitting suppliers [7].

As for the related research on consumers' low-carbon consumption awareness and behaviour. Under the carbon trading market mechanism and consumers' low-carbon preference, aiming at the lack of funds of manufacturers, Liu *et al.* constructed a revenue-sharing contract in low-carbon service supply chain, and studied the feasible interval

of revenue-sharing contract and the influence of relevant factors on the feasible interval [8]. For a two stage supply chain with one dominant retailer and two manufacturers, considering the acceptance of the ordinary product by consumers with low-carbon preference, Xiong *et al.* constructed the profit models of ordinary product manufacturers under the scenarios of adopting or not adopting emission reduction technology, and obtained the optimal retail price and revenue under each scenario. They found that product selected to sell by the retailer is related to the customer's acceptance of the ordinary product [9]. Du *et al.* studied the impact of carbon trading systems and consumers' low-carbon preferences on monopolistic producer output and emission-reduction strategies, in a market where carbon emission reductions affect product demand [10]. Ji *et al.* argued that consumer preference is an essential factor affecting the carbon trading market. Therefore, they established a Stackelberg game model based on consumer preference to study the investment strategies of manufacturers and retailers [11]. Based on the Hotelling model, Huang *et al.* studied the influence of consumer preferences on the pricing strategy for new and used products and supply chain revenue in the closed-loop supply chain. The results show that the price of low-carbon new products and ordinary waste products is directly proportional to the consumer preference [12]. Du *et al.* introduced consumer behaviour into supply chain management, constructed a carbon-dependent demand function, analysed the impact of consumers' low-carbon awareness on emissions and supply chain performance, and designed a revenue-sharing contract and quantity-discount contract to coordinate supply chains [13]. Their findings suggest that, overall, consumer preference and low-carbon sensitivity have a positive impact on the supply chain's weak carbon economy.

Extending the research focus to consider innovation uncertainty, Gu Q studied the relationship between the uncertainty of external policies and the R & D (Research and Development) investment of enterprises; the research results showed that the uncertainty of economic policy promotes the R&D (research and development) investment of enterprises [14]. Tian W used the mean-variance model to analyse two-party game strategies in the supply chain system, in which the supplier plays the leading role, and the retailer makes the innovation investment; this study focused on the impact of innovation behaviour on innovation investment and the expected profit of the supply chain [15]. Lan Z R took the upstream and downstream enterprises in the supply chain system as the research object, analysed the factors that affected the innovation behaviour of enterprises, and established an evolutionary game model of green innovation behaviour with the goal of reducing carbon emissions. Based on the results of the game, optimisation suggestions were put forward to the enterprises [16].

Finally, scholars have also carried out related research on the emission-reduction cooperation between upstream and downstream enterprises in the supply chain, Bhaskaran compared the advantages and disadvantages of task allocation

and cost allocation, and suggested scopes of application for these two cooperation modes [17]. Yu *et al.* constructed an investment game model based on cost-sharing coordination under a cost subsidy between manufacturers and retailers, and explored the effectiveness of supply chain enterprise behavior based on cost-sharing coordination under the cost subsidy [18]. Also using a Stackelberg game model, Wang K studied the optimal emission-reduction of the manufacturer, the optimal order quantity of the seller, and the profit of the participants under the constraint of carbon emission reduction with or without cost sharing and green financial loans [19]. Li Y D considered two types of contracts: sharing the increased income associated with reducing emissions, and sharing the cost of reducing investment. Via a Stackelberg game model involving retailers and suppliers, the study identified the optimal emission-reduction and the optimal sharing ratio of the two subjects under the two types of contracts, as well as the optimal profit value of the two subjects under the different contract forms [20].

In the context of research on green economies, Zhou Y J discussed supply chain equilibrium strategies and methods of coordination when retailers provide different joint R&D contracts to manufacturers responsible for green R&D cost investment. The results of the research show that, under certain conditions, the two-part cooperation mechanism can realize the win-win of greater profits for supply chain members and improved market demand for green products [21]. Zhi B D constructed a Stackelberg game model for low-carbon production by member enterprises in the supply chain, obtained the theoretical optimal carbon emission-reduction strategy, and found that joint decision-making helps promote carbon emission reduction and improve the overall performance of the supply chain, with cost-sharing contracts enabling the coordination of the supply chain [22]. Yu *et al.* constructed an investment game model between manufacturers and retailers under the two mechanisms of cost subsidy and cost-sharing coordination, and explored the effectiveness of supply chain enterprise behavior under the two mechanisms [18]. Chakraborty discussed the coordination mechanisms for revenue-sharing contracts [23]. Xia L J *et al.* studied supply chain coordination and decision-making when considering consumers' low-carbon preferences under compulsory emission-reduction regulations. In order to achieve Pareto improvement, the researchers designed a profit-sharing contract for purposes of coordinating the decision-making of manufacturers and retailers, and used Rubinstein's bargaining model to determine the optimal profit-sharing ratio for low carbon production [24]. In addition, in order to maximize profits, Lu *et al.* proposed a population-based hybrid evolutionary search algorithm [25], and a very effective hybrid dynamic programming and memory algorithm to obtain the optimal strategy [26].

In general, on the one hand, the current research mainly focuses on the R&D investment of manufacturers, and retailers' participation in emission reduction by adopting some kind of sharing contract, rarely considering the dual

background of carbon trading market and cost sharing; on the other hand, the current research in this general area is mostly based on an assumption of certainty with respect to carbon emission-reduction behaviour. Few studies have considered that the emission reduction of products will decline naturally over time, and then affect the initiative of enterprises to reduce emissions. Therefore, this paper uses a stochastic differential game method to expand the research methods used in this field, and fully considers the impact of uncertainty in connection with carbon emission-reduction behaviours. The other parts of the paper are arranged as follows: The third part of the article describes and models the impact of a carbon quota-based emission-trading system and an emission-reduction technology-innovation subsidy on technological investment when the behaviours associated with carbon emission reduction are uncertain. The fourth part then uses stochastic differential game theory to solve the Stackelberg equilibrium under a feedback strategy, with the fifth part using the stochastic differential Stackelberg game model to obtain the evolution path of expectation and variance vis-à-vis emission-reduction investment. The sixth part measures the influence of each parameter on the equilibrium evolution path, while the seventh part carries out a numerical simulation. The last part summarises the main conclusions of the study.

III. A STOCHASTIC DIFFERENTIAL GAME MODEL

A. PROBLEM DESCRIPTION

The supply chain of carbon emission reduction considered in this paper is composed of upstream manufacturers and downstream retailers. For the coordination of low-carbon supply chains, the coordination mechanism widely used in practice includes cooperative advertising contracts, revenue-sharing contracts, wholesale-price contracts, cost-sharing contracts, two-part contracts, and so forth. This paper chooses common scenarios in supply chain management practices to study the cooperation between upstream manufacturers and downstream retailers, in which upstream manufacturers carry out energy conservation and emission reduction, and downstream retailers share some of the costs of those initiatives. The scenarios capture a situation close to reality. In practice, manufacturers are the main agents of emission reduction, and manufacturers face greater pressure to reduce emissions than retailers. At the same time, manufacturers can transfer the cost of emission reduction to the downstream retailers through certain market forces, so manufacturers are chosen as the leader of the differential game, and retailers are chosen as followers who share some of the R&D costs of emission reduction. Generally speaking, a Stackelberg differential game scenario is formed between manufacturers and retailers. In the carbon trading system based on carbon quotas that was implemented in China, the government grants enterprises a certain carbon emission quota, and if they exceed the quota allocated by the government, they need to purchase from the society; at the same time, manufacturers reduce the carbon emissions generated in the production and use of

products through investment in emission-reduction technology. Against this policy backdrop, the present paper studies the impact of carbon emission-reduction technology investment.

The carbon emission-reduction behaviour of the supply chain consists of dynamic behaviour within a multi-stakeholders' game, where the stakeholders/participants include manufacturers, retailers, and consumer markets. The specific decision model is shown in Figure 1.

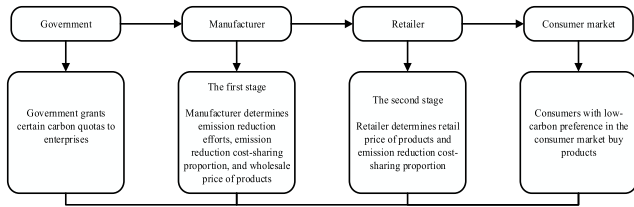


FIGURE 1. Decision diagram.

The present analysis includes the following considerations:

- 1) The government determines the carbon quota based on carbon emission intensity.
- 2) The manufacturer controls the carbon emissions associated with products and reduces emissions by investing in emission-reduction technologies.
- 3) The retailer purchases goods, at wholesale prices, from the manufacturer in purchase quantity q according to market demand and sells the product to the consumer at price p . In the game between manufacturers and retailers, manufacturers occupy an active position, the information of the two sides of the game is symmetrical, and decision-making is rationally, in order to maximise interests.
- 4) Consumers have low-carbon preferences; i.e., they have a relatively strong preference for products associated with lower carbon emissions.
- 5) To describe the problem, we create a list of parameters, as shown in Table 1.

B. MODEL ASSUMPTIONS

The model is designed to simplify some complex conditions without changing the basic nature of the problem, while also drawing on existing research [12]–[15], the model assumptions are as follows:

Assumption 1: We assume that the supply chain consists of two enterprises: an upstream manufacturer and a downstream retailer, in accordance with the studies cited in reference [7]. By introducing new technologies and transforming production processes, the manufacturer invests in emission reduction technology, reducing carbon emissions during the production phase. The government allocates a certain emissions quota based on the nature of the enterprise, and any excess emissions, beyond the quota, need to be purchased from the society.

TABLE 1. List of parameters.

variable	description	variable	description
p	product retail price	w	manufacturer's wholesale price
a	market size of the product	b	marginal demand for the product
Q_E	quantity demanded given consumers' low-carbon preferences	Z_M	emission reduction effort by the manufacturer
μ_M	manufacturer's abatement cost coefficient	E	carbon emission reduction
α	impact coefficient of the manufacturer's emission-reduction effort	σ	relative attenuation rate for the product emission reduction function
g_M	per unit product emissions quotas set by government regulations	e_M	level of carbon emission per unit product without investment in carbon emission reduction
c	manufacturer's unit production cost	ρ	discount rate
J_M	profits of the manufacturer	J_R	profits of the retailer
ζ	retailer emission-reduction cost-sharing proportion	k	Consumer's low-carbon sensitivity coefficient
δ	fluctuation coefficient of carbon emission reduction	p_e	carbon trading price

Assumption 2: The scale of China's carbon trading market is huge, and the carbon trading price p_e is affected by factors such as climate, supply and demand, and the macroeconomic environment. The supply chain has basically no influence on the carbon trading price, so the carbon trading price is an exogenous variable in the model.

Assumption 3: The upstream manufacturer has the same interest intensity as the downstream retailer ρ , and $\rho > 0$.

Assumption 4: Based on the practices of Laroche et al. [27] and Ouardighi [28], [29], this paper divides the market demand factors into price factors and non-price factors, and considers that the two factors have an impact on market demand in the form of separable multiplication. We assume the price is $p = a - bq$ (a is the market size, b is the marginal demand of the product, q is quantity demanded), $E(t)$ is the carbon emission reduction at time t . The demand function can then be expressed as follows:

$$Q_E(t) = (a - bp(t))kE(t) \tag{1}$$

Assumption 5: Drawing on Sherrill's assumption [30] concerning the innovation cost function, without considering inventory and shortages, this paper assumes that the manufacturer's abatement cost function is the convex function of emission-reduction efforts. Then the manufacturer's abatement cost at time t is:

$$C(Z_M(t)) = \frac{\mu_M}{2} Z_M^2(t) \tag{2}$$

The investment system for carbon emission-reduction technology is itself a dynamic process. The process of carbon emission reduction is affected by the agent making the emission-reduction investment, facility maintenance,

the environmental protection awareness of consumers, and various uncontrollable factors; hence, it is a random process. According to the study results of Kalish [31] and other researchers [32], [33], this process can be described in terms of the following assumption:

Assumption 6: The random process of carbon emission reduction can be analysed into three kinds of factors. One is the random factor that can be described in terms of the standard Wiener process, $dE(t) = \delta\sqrt{E(t)}dz(t)$, where δ is the emission-reduction fluctuation parameter. Another is the emission reduction brought about by the R&D investment of the agent initiating the reduction, $dE(t) = \alpha Z_M(t)dt$, where $Z_M(t)$ is the emission reduction efforts. The third is the influence of changing levels of environmental awareness as well as equipment aging, $dE(t) = -\sigma E(t)dt$, where σ is the emission-reduction coefficient. The total dynamic equation is thus the sum of three parts:

$$dE(t) = (\alpha Z_M(t) - \sigma E(t))dt + \delta\sqrt{E(t)}dz(t) \quad (3)$$

C. TARGET PROFIT FUNCTIONS FOR THE MANUFACTURER AND THE RETAILER

The quota-based carbon trading system is essentially an intensity control method [34]. This paper assumes that the carbon quota per unit of product allocated by the government to the enterprise is g_M , that the carbon emission per unit of product when the enterprise does not invest in carbon emission reduction is e_M , and that the unit's carbon allowance and carbon emissions per unit of product allocated by the government are constant for a certain period. The carbon emissions trading costs are as follows:

$$E_{MT(t)} = p_e[g_M Q_E(t) - (e_M Q_E(t) - E(t))] \quad (4)$$

In the actual operation process, for a carbon emission-reduction supply chain regulated by the government, the manufacturer's profit function is composed of three parts: sales revenue, carbon emission-reduction cost, and carbon-quota transaction cost. Meanwhile, considering that the retailer will share part of the manufacturer's carbon emission-reduction cost, the retailer's profit function consists of two parts: sales revenue and carbon emission-reduction cost. The goal of the game between the manufacturer and the retailer is to maximise the overall profit within the planning period. For the sake of convenience, the time variable t is omitted below.

Based on these considerations, the target profit function for the manufacturer can be represented as:

$$\begin{aligned} & \max E \{J_M [\omega, Z_M, \xi]\} \\ & = \max E \left\{ \int_0^\infty e^{-\rho t} \{(w-c)Q_E - (1-\xi)\frac{\mu_M}{2}Z_M^2 + p_e(g_M Q_E - (e_M Q_E - E))\} dt \right\} \end{aligned} \quad (5)$$

The target profit function for the retailer is:

$$\begin{aligned} & \max E \{J_R [p, \xi]\} \\ & = \max E \left\{ \int_0^\infty e^{-\rho t} \{(p-w)Q_E - \xi\frac{\mu_M}{2}Z_M^2\} dt \right\} \end{aligned} \quad (6)$$

D. A STOCHASTIC DIFFERENTIAL GAME MODEL

Assuming that the manufacturer and retailer participate in a manufacture-led, retailer-following Stackelberg game, the Stackelberg stochastic differential game model of the supply chain can be expressed as follows:

$$\begin{cases} \max_{\omega, Z_M, \xi} J_M(\omega, Z_M, \xi; p), \\ s.t. \max_{p, \xi} J_R(\omega, Z_M, \xi; p); \\ dE(t) = (\alpha Z_M(t) - \sigma E(t))dt + \delta\sqrt{E(t)}dz(t), E(0) = E_0. \end{cases} \quad (7)$$

IV. EQUILIBRIUM STRATEGY

The stochastic differential game theory is used to analyse the game process involving the manufacturer and the retailer, allowing the feedback equilibrium between the manufacturer and the retailer to be obtained. First, the manufacturer determines the wholesale price of the product and the carbon emission-reduction effort at each moment, and determines the cost-sharing ratio with the retailer. Secondly, the retailer determines the retail price of the product at each moment, and determines the cost-sharing ratio with the manufacturer.

Theorem 1: The decision-making goals of the manufacturer and retailer are to maximise their respective profits, and the equilibrium strategies of the game are as follows:

$$w^* = \frac{1}{2b}(a + bc + bp_e e_M - bp_e g_M) \quad (8)$$

$$Z_M^* = \frac{V_M^{S'}}{(1-\xi)\mu_M} \alpha \quad (9)$$

$$p^* = \frac{1}{4b}(3a + bc + bp_e e_M - bp_e g_M) \quad (10)$$

$$\xi^* = \frac{2V_R^{S'} - V_M^{S'}}{2V_R^{S'} + V_M^{S'}} \quad (11)$$

Proof: For any moment $t \in [0, \infty)$, given the manufacturer's wholesale price $\omega(t)$ and carbon emission-reduction effort $Z_M(t)$, using the inverse induction method and continuous-time dynamic programming theory, the HJB (Hamilton-Jacobi-Bellman) equation that the retailer's optimal retail price and optimal cost-sharing ratio should satisfy is:

$$\begin{aligned} \rho V_R^S = \max_{p, \xi} \{ & (p-w)(a-bp)kE - \xi\frac{\mu_M}{2}Z_M^2 \\ & + V_R^{S'}(\alpha Z_M - \sigma E)\} + \frac{1}{2}\delta^2 E V_R^{S''} \end{aligned} \quad (12)$$

where V_R^S represents the optimal value function for the retailer.

Taking the first derivative of the retailer's function with respect to the p , and make it equal to zero, we obtain:

$$p = \frac{a + bw}{2b} \quad (13)$$

In the context of continuous-time dynamic programming theory, the manufacturer's optimal strategies should satisfy

the following HJB equation:

$$\begin{aligned} &\rho V_M^S \\ &= \max_{Z_M \geq 0} \{ (w - c - p_e e_M + p_e g_M)(a - bp)kE - (1 - \xi) \frac{\mu_M}{2} Z_M^2 \\ &\quad + p_e E + V_M^{S'}(\alpha Z_M - \sigma E) \} + \frac{1}{2} \delta^2 E V_M^{S''} \end{aligned} \quad (14)$$

Substituting into Formula (14) the retail price response strategy for the retailer product obtained in Formula (13), we solve the equation as follows:

$$\begin{aligned} &\rho V_M^S \\ &= \max_{Z_M \geq 0} \{ (w - c - p_e e_M + p_e g_M) \left(\frac{a - bw}{2} \right) kE - (1 - \xi) \frac{\mu_M}{2} Z_M^2 \\ &\quad + p_e E + V_M^{S'}(\alpha Z_M - \sigma E) \} + \frac{1}{2} \delta^2 E V_M^{S''} \end{aligned} \quad (15)$$

where V_M^S represents the optimal value function for the manufacturer.

Taking the first derivative of the manufacturer's optimal value function with respect to ω and Z_M , and make them equal to zero respectively, we obtain:

$$\begin{cases} w^* = \frac{1}{2b}(a + bc + bp_e e_M - bp_e g_M) \\ Z_M^* = \frac{V_M^{S'} \alpha}{(1 - \xi) \mu_M} \end{cases} \quad (16)$$

By substituting Formula (16) into Formula (13), the retailer's optimal product retail price strategy can be obtained:

$$p^* = \frac{1}{4b}(3a + bc + bp_e e_M - bp_e g_M) \quad (17)$$

Then, substituting into Formula (12) the strategy for carbon emission-reduction efforts represented in Formula (16), taking the first derivative of the retailer's function with respect to ξ , and make it equal to zero, we obtain:

$$\xi^* = \frac{2V_R^{S'} - V_M^{S'}}{2V_R^{S'} + V_M^{S'}} \quad (18)$$

In order to use Formula (16), Formula (17), and Formula (18) to obtain the optimal strategies for the manufacturer and the retailer, it is also necessary to determine the optimal value function for both. Accordingly, substituting Formula (16), Formula (17), and Formula (18) into Formula (12) and Formula (14), respectively, the optimal value functions for the manufacturer and the retailer satisfy:

$$\begin{aligned} &\rho V_M^S \\ &= \max_{Z_M \geq 0} \left\{ \left[\frac{1}{8b}(a - bc - bp_e e_M + bp_e g_M)^2 k - V_M^{S'} \sigma \right] \right. \\ &\quad \left. + p_e + \frac{1}{2} \delta^2 V_M^{S''} \right\} E \\ &\quad + \frac{\alpha^2 V_M^{S'} (2V_R^{S'} + V_M^{S'})}{4\mu_M} \end{aligned} \quad (19)$$

$$\begin{aligned} &\rho V_R^S \\ &= \max_{p, \xi} \left\{ \left[\frac{1}{16b}(a - bc - bp_e e_M + bp_e g_M)^2 k \right. \right. \\ &\quad \left. \left. - V_R^{S'} \sigma + \frac{1}{2} \delta^2 V_R^{S''} \right] E \right. \\ &\quad \left. + \frac{V_R^{S'} \alpha^2 (2V_R^{S'} + V_M^{S'})}{2\mu_M} \right. \\ &\quad \left. - \frac{\alpha^2 (2V_R^{S'} - V_M^{S'}) (2V_R^{S'} + V_M^{S'})}{8\mu_M} \right\} \end{aligned} \quad (20)$$

In turn, solving the differential equation systems represented by Formula (19) and Formula (20) means solving the optimal value function for the manufacturer and retailer that satisfy them. According to the structure of the problem and the intrinsic function relationship, it is assumed that the optimal value function forms for the manufacturer and the retailer are, respectively:

$$\begin{cases} V_M^S = f_1 E^2 + f_2 E + f_3 \\ V_R^S = g_1 E^2 + g_2 E + g_3 \end{cases} \quad (21)$$

This means that the first derivative and the second derivative of the optimal value functions for the manufacturer and retailer are:

$$\begin{cases} V_M^{S'} = 2f_1 E + f_2; V_M^{S''} = 2f_1 \\ V_R^{S'} = 2g_1 E + g_2; V_R^{S''} = 2g_1 \end{cases} \quad (22)$$

In order for the optimal value function for the manufacturer and retailer assumed by Formula (21) to be the solution of Formula (19) and Formula (20), the value of the coefficient $f_i, g_i (i = 1, 2, 3)$ in Formula (21) should be determined. Therefore, substituting Formula (21) and Formula (22) into Formula (19) and Formula (20), respectively, the results are as follows:

$$\begin{cases} \rho f_1 = \frac{\alpha^2 (2f_1 g_1 + f_1^2)}{\mu_M} - 2\sigma f_1 \\ \rho f_2 = \frac{(a - bc - bp_e e_M + bp_e g_M)^2 k + 8bp_e + 8b\delta^2 f_1 - 8b\sigma f_2}{8b} \\ \quad + \frac{\alpha^2 (f_1 g_2 + f_2 g_1 + f_1 f_2)}{\mu_M} \\ \rho f_3 = \frac{\alpha^2 f_2 (2g_2 + f_2)}{4\mu_M} \end{cases} \quad (23)$$

$$\begin{cases} \rho g_1 = \frac{\alpha^2 (4g_1 + 2f_1)^2}{8\mu_M} - 2\sigma g_1 \\ \rho g_2 = \frac{(a - bc - bp_e e_M + bp_e g_M)^2 k + 16b\delta^2 g_1 - 16b\sigma g_2}{16b} \\ \quad + \frac{\alpha^2 (2g_2 + f_2) (2g_1 + f_1)}{2\mu_M} \\ \rho g_3 = \frac{\alpha^2 (2g_2 + f_2)^2}{8\mu_M} \end{cases} \quad (24)$$

Formula (23) and Formula (24) hold all possible values for carbon emission reduction, which indicates that the coefficients of E^2 and E , and constant terms on both sides of

the equation, are correspondingly equal. As a consequence, further results can be obtained in (25) and (26), as shown at the bottom of the page.

When calculating these coefficients, according to the given parameters, we first calculate the coefficient f_1 and then calculate other undetermined coefficients. Next, we obtain the optimal value function from Formula (21). Finally, we use Theorem 1 to obtain the optimal equilibrium strategy of the manufacturer and the retailer.

V. EVOLUTION CHARACTERISTICS OF CARBON EMISSION REDUCTION

In order to grasp the statistical characteristics of random carbon emission reduction, the present paper analyses the expectations and variances associated with this emission reduction. The specific conclusions are as follows.

Theorem 2: The expectation and expectation limit of random carbon emission reduction are respectively:

$$E(E) = e^{Mt} (E_0 + NM^{-1} - NM^{-1}e^{-Mt}),$$

$$\lim_{t \rightarrow \infty} E(E) = -NM^{-1}$$

The variance and variance limit of random carbon emission reduction are respectively:

$$D(E) = e^{2Mt} (E_0 + NM^{-1} - NM^{-1}e^{-Mt})^2$$

$$- e^{2Mt} (E_0^2 + (ME_0 + N)(2N + \delta^2)M^{-2}$$

$$- N(2N + \delta^2)(2M^2)^{-1})$$

$$- e^{Mt} (ME_0 + N)(2N + \delta^2)M^{-2}$$

$$+ N(2N + \delta^2)(2M^2)^{-1}$$

$$\lim_{t \rightarrow \infty} D(E) = N(2N + \delta^2)(2M^2)^{-1} - N^2M^{-2}$$

where $M = -(3\sigma + \rho)$, $N = \alpha^2(2g_2 + f_2)(2\mu_M)^{-1}$.

Proof: By substituting the manufacturer’s carbon emission-reduction effort in Theorem 1 into Formula (3),

which is the random process of carbon emission reduction, and solving, we obtain:

$$dE(t) = [\alpha Z_M(t) - \sigma E(t)] dt + \delta \sqrt{E(t)} dZ(t), \quad E(0) = E_0 \tag{27}$$

Integrating on both sides and using boundary conditions, we then get:

$$E = E_0 + \int_0^t (ME + N) dt + \int_0^t \delta \sqrt{E(t)} dZ(t) \tag{28}$$

Taking expectations on both sides, and taking advantage of the zero-mean property of the Wiener process, we next obtain:

$$E(E) = E_0 + \int_0^t (ME(E) + N) dt \tag{29}$$

Via further integration, the expectation of random carbon emission reduction can be represented as:

$$E(E) = e^{Mt} (E_0 + NM^{-1} - NM^{-1}e^{-Mt}) \tag{30}$$

When $t \rightarrow \infty$ and $M < 0$ is known from $M = -(3\sigma + \rho)$, then the expectation limit of random carbon emission reduction is:

$$\lim_{t \rightarrow \infty} E(E) = -NM^{-1} \tag{31}$$

Next, the random change process of the square of carbon emission reduction is analysed via the ITO lemma:

$$dE^2 = [2ME^2 + (2N + \delta^2)E] dt + 2\delta E \sqrt{E(t)} dZ(t),$$

$$E^2(0) = E_0^2 \tag{32}$$

Integrating on both sides and using boundary conditions, we get:

$$E^2 = E_0^2 + \int_0^t (2ME^2 + (2N + \delta^2)E) dt$$

$$+ \int_0^t 2E \delta \sqrt{E(t)} dZ(t) \tag{33}$$

$$\begin{cases} f_1 = -\frac{2(2\sigma + \rho)\mu_M}{\alpha^2} \\ f_2 = \frac{[(a - bc - bp_c e_u + bp_c g_M)^2 k \alpha^2 (\sigma + \rho) + 8b \alpha^2 p_c (3\sigma + 2\rho) - 8b \delta^2 (2\sigma + \rho)(8\sigma + 5\rho)\mu_M]}{4b \alpha^2 [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} \\ f_3 = \frac{\alpha^2 g_2 f_2}{2\mu_M \rho} + \frac{\alpha^2 f_2^2}{4\mu_M \rho} \\ g_1 = \frac{(2\sigma + \rho)\mu_M}{2\alpha^2} \\ g_2 = \frac{[(a - bc - bp_e e_M + bp_e g_M)^2 k \alpha^2 (4\sigma + 3\rho) - 16b \alpha^2 p_e (2\sigma + \rho) + 8b \delta^2 (2\sigma + \rho)(16\sigma + 9\rho)\mu_M]}{16b \alpha^2 [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} \\ g_3 = \frac{\alpha^2 (4g_2^2 + f_2^2 + 4g_2 f_2)}{8\mu_M \rho} \end{cases} \tag{25}$$

$$\begin{cases} g_1 = \frac{(2\sigma + \rho)\mu_M}{2\alpha^2} \\ g_2 = \frac{[(a - bc - bp_e e_M + bp_e g_M)^2 k \alpha^2 (4\sigma + 3\rho) - 16b \alpha^2 p_e (2\sigma + \rho) + 8b \delta^2 (2\sigma + \rho)(16\sigma + 9\rho)\mu_M]}{16b \alpha^2 [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} \\ g_3 = \frac{\alpha^2 (4g_2^2 + f_2^2 + 4g_2 f_2)}{8\mu_M \rho} \end{cases} \tag{26}$$

Taking expectations on both sides, and taking advantage of the zero-mean property of the Wiener process, we next obtain:

$$E(E^2) = E_0^2 + \int_0^t (2ME(E) + (2N + \delta^2)E(E))dt \tag{34}$$

Substituting Formula (30) into Formula (34) yields the following:

$$E(E^2) = e^{2Mt} \left(\frac{E_0^2 + \frac{(ME_0 + N)(2N + \delta^2)}{M^2}}{\frac{N(2N + \delta^2)}{2M^2}} \right) - e^{Mt} \frac{(ME_0 + N)(2N + \delta^2)}{M^2} + \frac{N(2N + \delta^2)}{2M^2} \tag{35}$$

Using the variance formula in statistics, i.e., $D(E) = E(E^2) - [E(E)]^2$, the variance of carbon emission reduction can then be obtained:

$$D(E) = e^{2Mt} (E_0 + NM^{-1} - NM^{-1}e^{-Mt})^2 - e^{2Mt} (E_0^2 + (ME_0 + N)(2N + \delta^2)M^{-2} - N(2N + \delta^2)(2M^2)^{-1}) - e^{Mt} (ME_0 + N)(2N + \delta^2)M^{-2} + N(2N + \delta^2)(2M^2)^{-1} \tag{36}$$

Considering that $M < 0$, $\lim_{t \rightarrow \infty} E(E^2) = N(2N + \delta^2) / (2M^2)$. Furthermore, the variance limit of carbon emission reduction is $\lim_{t \rightarrow \infty} D(E) = \lim_{t \rightarrow \infty} E(E^2) - \lim_{t \rightarrow \infty} [E(E)]^2$, $\lim_{t \rightarrow \infty} D(E) = N(2N + \delta^2) / (2M^2) - N^2 / M^2$. For the case where there is no random interference, $\delta = 0$, $\lim_{t \rightarrow \infty} E(E^2) = N^2 / M^2$, and therefore $D(E) = E(E^2) - [E(E)]^2 = 0$.

Assuming that the system carbon emission reduction follows a normal distribution, then the confidence interval of the system for carbon emission reduction when the confidence level is 95% is $(E(E(t)) - 1.96\sqrt{D(E(t))}, E(E(t)) + 1.96\sqrt{D(E(t))})$. This gives managers a very important piece of information: Although the actual carbon emission reduction is disturbed by random factors and deviates from the expectation value of the emission reduction, it can be determined that the system for carbon emission reduction in the planning period always fluctuates within a certain range around the expectation value. In other words, in the actual situation, although managers cannot accurately judge the real state per se, they can accurately grasp the expectation value for the real state. As a result, they can engage in corresponding decision-making processes within an acceptable error range, acting on the assumption that the expected goals set during the planning period can, in fact, be achieved.

TABLE 2. Sensitivity analysis of the parameters.

	$E(t)^*$	Z_M^*	w^*	p^*
k	↑	↑	→	→
α	↑	↑	→	→
μ_M	↓	↓	→	→
p_e	↑	↑	↓	↓

VI. SENSITIVITY ANALYSIS OF MAIN PARAMETERS

The consumer’s low-carbon preference coefficient is k , the coefficient of the impact of the manufacturer’s emission reduction efforts is α , the emission-reduction cost coefficient is μ_M , and the carbon trading price is p_e . We calculated the relative impact of these parameters on the expected carbon emission reduction of the system $E(E(t)^*)$, the optimal emission-reduction effort of the equilibrium strategy Z_M^* , the wholesale price w^* , and the retail price p^* . Results are as follows:

Observation 1: As consumers’ low-carbon sensitivity coefficient k increases, while the carbon emission reduction of the system and the manufacturer’s emissions-reduction efforts increase, the manufacturer’s wholesale prices and the retailer’s product pricing remain unchanged.

Proof:

$$\begin{aligned} \frac{\partial p^*}{\partial k} &= 0 \\ \frac{\partial w^*}{\partial k} &= 0 \\ \frac{\partial Z_M^*}{\partial k} &= \frac{(2\sigma + \rho) dE(E)}{\alpha \frac{dk}{dk}} + \frac{(a - bc - bp_e e_M + bp_e g_M)^2 \alpha (6\sigma + 5\rho)}{16b\mu_M [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} > 0 \\ \frac{\partial M}{\partial k} &= 0 \\ \frac{\partial N}{\partial k} &= \frac{(a - bc - bp_e e_M + bp_e g_M)^2 \alpha^2 (6\sigma + 5\rho)}{16b\mu_M [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} > 0 \\ \frac{\partial E(E(t)^*)}{\partial k} &= \frac{(e^{Mt} - 1) \partial N}{M \partial k} > 0 \end{aligned}$$

Observation 2: As the manufacturer’s emission-reduction efforts’ influence coefficient α increases, the carbon emission reduction of the system and the manufacturer’s emissions-reduction efforts increase, while the manufacturer’s wholesale prices and the retailer’s product pricing remain unchanged.

Proof: $\frac{\partial p^*}{\partial \alpha}$, $\frac{\partial w^*}{\partial \alpha}$, $\frac{\partial Z_M^*}{\partial \alpha}$, $\frac{\partial M}{\partial \alpha}$, and $\frac{\partial N}{\partial \alpha}$, as shown at the bottom of the next page.

Observation 3: As the emission-reduction cost coefficient μ_m increases, the carbon emission reduction of the system and the manufacturer’s emissions-reduction efforts decrease, while the manufacturer’s wholesale prices and the product pricing remain unchanged.

Proof: $\frac{\partial p^*}{\partial \mu_M}, \frac{\partial w^*}{\partial \mu_M}, \frac{\partial Z_M^*}{\partial \mu_M}, \frac{\partial M}{\partial \mu_M}$, and $\frac{\partial N}{\partial \mu_M}$, as shown at the bottom of the page.

Observation 4: As the carbon trading price p_e increases, the carbon emission reduction of the system and the manufacturer’s emissions-reduction efforts increase, while the manufacturer’s wholesale prices and the retailer’s product pricing decrease.

Proof: $\frac{\partial p^*}{\partial p_e}, \frac{\partial w^*}{\partial p_e}, \frac{\partial Z_M^*}{\partial p_e}, \frac{\partial M}{\partial p_e}$, and $\frac{\partial E(E(t)^*)}{\partial p_e}$, as shown at the bottom of the next page.

VII. NUMERICAL EXAMPLE

A. EVOLUTION PATH ANALYSIS

To develop an intuitive analysis of the optimal strategy trajectory for the supply chain under the carbon trading and cost-sharing scenarios, and to map the evolution trend of carbon emission reduction expectation, manufacturer’s profit, and the retailer’s profit, we use the method of numerical simulation to express the evolution path. Drawing on existing research [3]–[7], we set the parameters as follows: $\rho = 0.3$, $\sigma = 0.2$, $a = 4.5$, $b = 1$, $c = 3$, $\alpha = 0.8$, $p_e = 0.02$, $k = 0.6$, $\mu_M = 1$, $e_M = 0.5$, $g_M = 2$, $E(0)=0$,

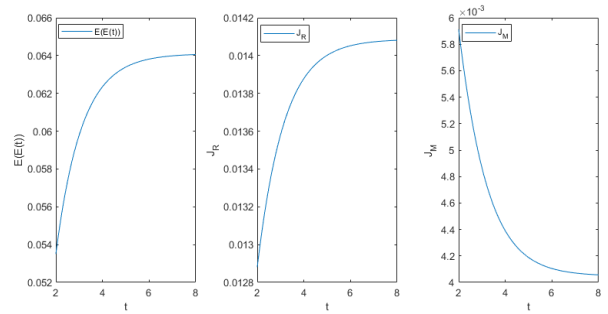


FIGURE 2. State variable trajectory under a cost-sharing contract.

and $\delta = 0.01$. In this way, as shown in Fig. 2, the trajectory of the manufacturer’s profit, the retailer’s profit, and the carbon emission reduction expectation evolution under the scenario of cost-sharing contracts can be obtained.

As revealed in Fig. 2, the carbon emission-reduction expectation and the retailer’s profit show a non-linear upward trend, while the manufacturer’s profit shows a non-linear descending trend. All of these trends eventually reach stable levels.

$$\begin{aligned} \frac{\partial p^*}{\partial \alpha} &= 0 \\ \frac{\partial w^*}{\partial \alpha} &= 0 \\ \frac{\partial Z_M^*}{\partial \alpha} &= \frac{(2\sigma + \rho) \left(E - \alpha \frac{dE(E)}{d\alpha} \right)}{\alpha^2} \\ &\quad + \frac{8b\delta^2\rho(2\sigma + \rho)}{16b\alpha^2 [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} > 0 \\ \frac{\partial M}{\partial \alpha} &= 0 \\ \frac{\partial N}{\partial \alpha} &= \frac{(a - bc - bp_e e_M + bp_e g_M)^2 k \alpha (6\sigma + 5\rho) + 16b\alpha p_e (\sigma + \rho)}{8b\mu_M [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} > 0 \\ \frac{\partial E(E(t)^*)}{\partial \alpha} &= \frac{(e^{Mt} - 1)}{M} \frac{\partial N}{\partial \alpha} > 0 \\ \frac{\partial p^*}{\partial \mu_M} &= 0 \\ \frac{\partial w^*}{\partial \mu_M} &= 0 \\ \frac{\partial Z_M^*}{\partial \mu_M} &= -\frac{(2\sigma + \rho) \frac{dE(E)}{d\mu_M}}{\alpha} \\ &\quad - \frac{(a - bc - bp_e e_M + bp_e g_M)^2 k \alpha (6\sigma + 5\rho) + 16b\alpha p_e (\sigma + \rho)}{16b\mu_M^2 [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} < 0 \\ \frac{\partial M}{\partial \mu_M} &= 0 \\ \frac{\partial N}{\partial \mu_M} &= -\frac{(a - bc - bp_e e_M + bp_e g_M)^2 k \alpha^2 (6\sigma + 5\rho) + 16b\alpha^2 p_e (\sigma + \rho)}{16b\mu_M^2 [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} < 0 \\ \frac{\partial E(E(t)^*)}{\partial \mu_M} &= \frac{(e^{Mt} - 1)}{M} \frac{\partial N}{\partial \mu_M} < 0 \end{aligned}$$

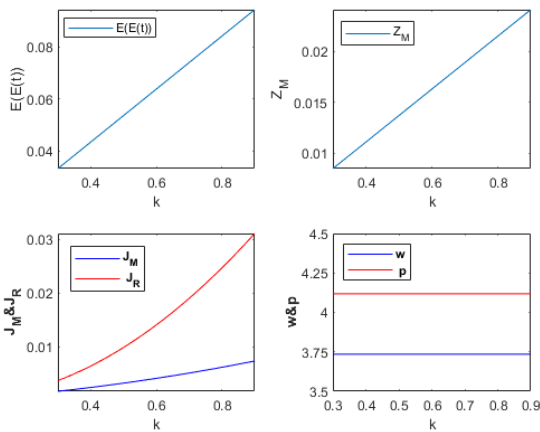


FIGURE 3. Sensitivity analysis of consumers' carbon sensitivity coefficient.

B. SENSITIVITY ANALYSIS

1) CONSUMERS' LOW-CARBON SENSITIVITY COEFFICIENT UNDER A COST-SHARING CONTRACT

Assuming other parameters remain unchanged, k changes from 0.4 to 0.9. The numerical simulation results are shown in Fig. 3.

Fig. 3 reveals that with the increase of k , the emission reduction efforts of the manufacturer and the reduction of carbon emissions associated with products both show a linear upward trend; furthermore, the increase in carbon emission reductions is faster. Product pricing and wholesale price remain unchanged, with product pricing being significantly higher than wholesale prices. The manufacturer's profit and retailer's profit show a non-linear upward trend, with the retailer's profit increasing at a higher rate over time. When consumers' low-carbon sensitivity coefficient is small, the retailer's profits are slightly larger than the manufacturer's profits. However, the retailer's profits rise faster than the manufacturer's profits; hence k is more significant the greater the difference between the manufacturer's profits and the retailer's profits. Finally, the retailer's profits are

significantly higher than the manufacturer's profits. Thus, it can be seen, under the dual mechanism of carbon trading systems and emission-reduction cost-sharing contracts, consumers' low carbon preference can effectively promote the investment of emission reduction technology in the supply chain, and improve the supply chain profits.

2) THE IMPACT COEFFICIENT OF THE MANUFACTURER'S EMISSION REDUCTION EFFORT UNDER A COST-SHARING CONTRACT

Assuming that other parameters remain unchanged, α changes from 0.4 to 1.2. The numerical simulation results are shown in Fig. 4.

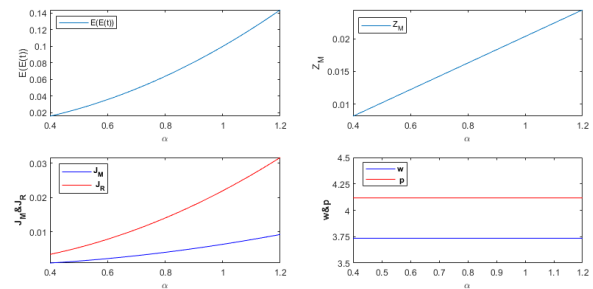


FIGURE 4. Sensitivity analysis of the impact coefficient of the manufacturer's emission reduction efforts.

Fig. 4 reveals that with the increase of α , the manufacturer's emission-reduction effort shows a linear upward trend. The manufacturer's profit, the retailer's profit, and the carbon emission reduction all show a non-linear upward trend. The manufacturer's wholesale price and the retailer's product pricing remain unchanged, and the product pricing is significantly higher than the wholesale price. Of these parameters, the trend of the change in the manufacturer's profits and the retailer's profits due to the change of α is the same as the trend due to the change of k . Thus, it can be seen, under the dual mechanism of carbon trading systems and emission-reduction cost-sharing contracts, the greater the manufacturer's efforts to reduce emissions, the more vigorous

$$\begin{aligned} \frac{\partial p^*}{\partial p_e} &= \frac{1}{4}(e_M - g_M) < 0 \\ \frac{\partial w^*}{\partial p_e} &= \frac{1}{2}(e_M - g_M) < 0 \\ \frac{\partial Z_M^*}{\partial p_e} &= -\frac{(2\sigma + \rho) dE(E)}{\alpha dp_e} \\ &\quad + \frac{8\alpha(\sigma + \rho) + (a - bc - bp_e e_M + bp_e g_M)k\alpha(g_M - e_M)(6\sigma + 5\rho)}{8\mu_M [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} \\ \frac{\partial M}{\partial p_e} &= 0 \\ \frac{\partial N}{\partial p_e} &= \frac{8\alpha^2(\sigma + \rho) + (a - bc - bp_e e_M + bp_e g_M)k\alpha^2(g_M - e_M)(6\sigma + 5\rho)}{8\mu_M [(3\sigma + 2\rho)(8\sigma + 5\rho) - 2(2\sigma + \rho)^2]} \\ \frac{\partial E(E(t)^*)}{\partial p_e} &= \frac{(e^{Mt} - 1)}{M} \frac{\partial N}{\partial p_e} > 0 \end{aligned}$$

investment of emission reduction technology in the supply chain, and the greater the profits of supply chain.

3) THE MANUFACTURER'S ABATEMENT COST COEFFICIENT UNDER A COST-SHARING CONTRACT

Assuming that other parameters remain unchanged, μ_m changes from 0.5 to 2. The numerical simulation results are shown in Fig. 5.

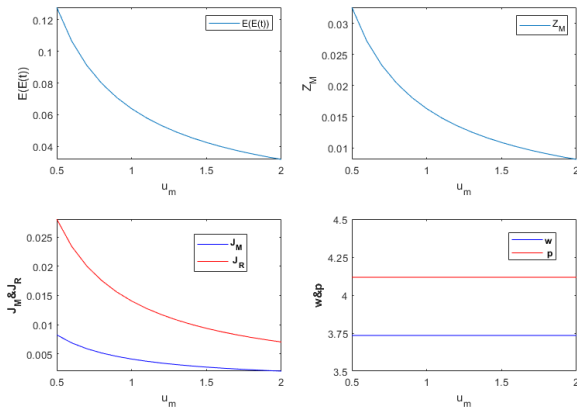


FIGURE 5. Sensitivity analysis of the cost coefficient of the manufacturer's emission reduction efforts.

Fig. 5 reveals that with the increase of μ_m , the carbon emission reduction, the manufacturer's emission-reduction effort, the manufacturer's profit, and the retailer's profit all show a non-linear downward trend. When the manufacturer's abatement cost coefficient is small, the retailer's profits are significantly higher than the manufacturer's profits. In addition, the rate of decline of the retailer's profits is always higher than the rate of decline of the manufacturer's profits; hence, the more substantial the μ_m , the smaller the difference between the manufacturer's profits and the retailer's profits. However, the profits of the retailer are always higher than the profits of the manufacturer. Finally, μ_m has no effect on product pricing and wholesale price, and product pricing is higher than the wholesale price. Thus, it can be seen, under the dual mechanism of carbon trading systems and emission-reduction cost-sharing contracts, consumers' low carbon preference can effectively promote the investment of emission reduction technology in the supply chain, and improve the supply chain profits. Thus, it can be seen, under the dual mechanism of carbon trading systems and emission-reduction cost-sharing contracts, cost of the manufacturer's emission reduction efforts will restrain the investment of emission reduction technology in the supply chain, and reduce the supply chain profits.

4) THE CARBON TRADING PRICE UNDER A COST-SHARING CONTRACT

Assuming that other parameters remain unchanged, p_e changes from 0.05 to 0.25. The numerical simulation results are shown in Fig. 6.

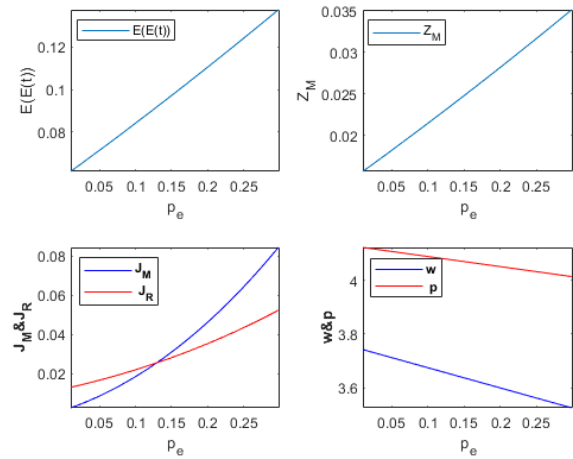


FIGURE 6. Sensitivity analysis of carbon trading price.

Fig. 6 reveals that with the increase of p_e , the carbon emission reduction and manufacturer's emission-reduction effort all show a linear upward trend. The manufacturer's profit and the retailer's profit show a non-linear upward trend. When the carbon trading price is small, the retailer's profit is higher than the manufacturer's profit. However, the profit of the manufacturer grows faster than that of the retailer, and eventually exceeds it. The manufacturer's wholesale price and the retailer's product pricing show a linear downward trend, and product pricing is significantly higher than the wholesale price. Thus, it can be seen, under the dual mechanism of carbon trading systems and emission-reduction cost-sharing contracts, the increase of carbon trading price can effectively promote the investment of emission reduction technology in the supply chain, and improve the supply chain profits.

VIII. CONCLUSION

This paper analyses a dynamic supply chain system composed of a leading manufacturer and a retailer. It considers the low-carbon preference of consumers and the randomness of the emission-reduction process, and uses differential game and dynamic optimisation methods to study the impact of a carbon trading system, based on carbon quotas and a cost-sharing coordination mechanism, on supply chain emission-reduction technology investment behaviour.

To characterise the impact of investment t by emission-reduction agents, the cost of facility maintenance, the environmental awareness of consumers, and various uncontrollable factors on the carbon emission-reduction process, the zero-mean nature of the Wiener process and the ITO lemma are used to describe the evolution process of carbon emission reduction under a random system scenario. In accordance with their respective profit structures, profit-target functions are constructed for the manufacturer and the retailer. Based on the evolution process of carbon emission reduction and the profit-target functions for the manufacturer and the retailer, we then constructed a Stackelberg stochastic differential game model for a dynamic supply

chain system. Using stochastic differential game theory and dynamic optimisation theory, we obtain the manufacturer's emission-reduction efforts and the equilibrium wholesale price, the retailer's equilibrium product retail price and the carbon emission-reduction cost sharing ratio, and the optimal value function of the manufacturer and the retailer.

At the same time, this paper analyses the impact of different parameters on the optimal strategies of the manufacturer and the retailer. The study finds that the consumer's low-carbon preference coefficient, manufacturer's emission-reduction efforts' influence coefficient, and the carbon trading price positively impact emission reduction, the manufacturer's profit, and the retailer's profit. But the impacts vary. The emission-reduction cost coefficient under the cost-sharing contract mechanism negatively impacts emission reduction, manufacturer's profit, and retailer's profit. Furthermore, in order to grasp the statistical characteristics of the system's stochastic carbon emission reduction, this paper also analyses the statistical properties of the stochastic carbon emission-reduction expectation and variance. It also uses numerical examples to verify the influence of the changes of various parameters on the strategy of the behaviour agents.

In short, this paper considers a dynamic emission-reduction technology investment decision-making problem in a dyadic supply chain consisting of a manufacturer and a retailer under carbon trading system and cost-sharing contracts. The results show that the dual mechanism of carbon trading systems and emission-reduction cost-sharing contracts can effectively promote the investment of emission reduction technology in the supply chain, and improve the supply chain profits. Therefore, it is necessary to strengthen the cooperation, development and exchange of carbon emission reduction technologies among enterprises. The contribution of this paper is to analyze the optimal decision-making of supply chain enterprises under carbon trading systems and emission-reduction cost-sharing contracts, and the impact of these two mechanisms on the emission reduction behavior of supply chain enterprises on the basis of fully considering the low-carbon sensitive preference of consumers and the randomness of emission reduction process. In addition, the research results have important guiding significance for supply chain enterprises to make decision-making of emission-reduction technology investment and to seek win-win cooperation between supply chain enterprises. Innovation activities in this domain are also influenced by other factors, including stochasticity, which should be considered in future research. In addition, in considering these other factors, it is worth studying how various government subsidy policies, such as tax reductions and product subsidies, affect multi-level supply chain emission reductions, as well as the impact of emission-reduction technology spillover.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this article.

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