The Identification of Poverty Alleviation Targets Based on the Multiple Hybrid Decision-Making Algorithms

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ABSTRACT The poverty alleviation problem as one of the social evaluation applications has long been a major focus of social problems. As the basis and starting step of the poverty alleviation project, it is crucial to accurately identify the targets of poverty alleviation. Therefore, first of all, it is necessary to establish a scientific and reasonable indicators system and then evaluate all the indicator values respectively. However, in the process of data evaluation, we found that it is often hard to decide the unique valuation for some indicators because of the hesitation among different possible valuations in the mind. Different from traditional algorithms only using a single indicator valuation, the paper uses Pythagorean fuzzy sets (PFSs) to keep possible valuations from the positive and negative aspects and it can overcome the hesitation in the data evaluation process to a certain extent. The paper considers the problem of identifying the poverty alleviation targets as a multi-criteria decision making (MCDM) problem and then proposes a modified algorithm to solve the problem on the basis of several traditional algorithms. The algorithm work well and can obtain the maximum group utility and the minimum individual regret at the same time in the following experiments. The optimal poverty alleviation targets have been found and the poverty ranking list has also been obtained through the algorithm.

INDEX TERMS Pythagorean fuzzy sets, multi-criteria decision making problem, precise poverty alleviation, fuzzy decision making.

I. INTRODUCTION

With the rapid development of China's economy, the number of poor people is gradually decreasing. The large-scale poverty alleviation work has nearly finished, while, the precise poverty alleviation is further emphasized at present. As the basis and starting step of the precise poverty alleviation project, it is crucial to accurately identify the targets of poverty alleviation.

Precise identification of the poverty alleviation targets as the starting step and also the important basis of the poverty alleviation project had attracted wide attention of scholars at home and abroad in recent years. The empirical research taken by Yang (2010) based on the data modeling of family surveys showed that the specific characteristics of families

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had a significant impact on the poverty level in the statistical sense, and it was concluded that if it is desired to improve the effectiveness of poverty alleviation further, it was necessary to accurately identify the poverty alleviation targets. Based on the survey data in Gansu Province of China, Li (2015) established a logistic model for identifying the poverty alleviation targets, and he pointed out that the health status of family members and the number of minor children had a significant impact on the poverty level of the families. Zheng *et al.* (2016) analyzed precise poverty alleviation from another angle, he believed that the core of precise poverty alleviation lies on the precision, and the key point of the precise poverty alleviation lies on aiming at the corresponding targets of poverty alleviation, while, the loss of rural population had not only weakened the main body of production and construction, but also eroded the organizational basis of poverty alleviation. Golan *et al.* (2017) studied poverty alleviation policies,

he used the conventional criteria and the propensity matching score method to analyze the data of rural families, and studied the poverty alleviation effect and the target effectiveness of the related policies. The analysis results showed that the effect of the poverty alleviation policies was limited, and there were large number of target errors which made it impossible to achieve precise poverty alleviation. How to accurately identify the targets of poverty alleviation has become a worldwide difficult problem.

At present, the academia mainly adopts the qualitative evaluation method on the poverty assessment problem, while, different from traditional perspectives, the paper introduces the method of management and takes the precise identification of the poverty alleviation targets as a decision-making problem because of the similarities between them. Furthermore, the paper proposes a modified algorithm to solve the problem combined the VIKOR (Vise Kriterijumska Optimizacija Kompromisno Resenje) method, the TODIM(Tomada de Decisión Inerativa Multicritero) approach and the Pythagorean fuzzy sets processing algorithm. The main advantages between the proposed algorithm and others are that the representation form of the indicator data, the method of calculating indicator weights and the information aggregation method. Finally, several experiments are carried out to verify the effectiveness of the algorithm, it is proved that the algorithm work well and can obtain the maximum group utility and the minimum individual regret at the same time.

II. FUNDAMENTAL THEORY
A. THE DEFINITION OF FUZZY SETS
Zadeh first advanced the concept of fuzzy sets in 1965 [1], which is a powerful tool for dealing with the fuzzy problems. The membership degree of a fuzzy element is a real number between 0 and 1. The fuzzy sets are defined as follows:

$$A = \{< x, \mu_A(x) > | x \in X\} \tag{1}$$

where X is a nonempty set and $\mu_A(x)$ indicates the membership degree of the element x of the set X to the A, while $0 \leq \mu_A(x) \leq 1$.

However, we found that many experts often cannot give a single valuation to express the membership degree in the actual situation; they may hesitate among a series of valuations because of uncertainty in their minds. Based on these considerations, Torra(2010) advanced the definition of the hesitant fuzzy sets, which is an extension of the fuzzy sets, it allows the membership degree of each element can have several possible valuations. Xia and Xu (2011) first advanced the following mathematical definition to the hesitant fuzzy sets (HFSs) [2].

$$E = \{< x, h_E(x) > | x \in X\} \tag{2}$$

where $h_E(x)$ can include various numerical valuations which are all between 0 and 1, indicating the different possible membership degree. The hesitant fuzzy element $h_E(x)$ is also called h for short. If all the hesitant fuzzy elements include

only one membership degree valuation, then the hesitant fuzzy sets will degenerate into the fuzzy sets immediately [3].

On the basis of the hesitant fuzzy sets, Yager proposed the definition of the Pythagorean fuzzy sets recently which can be represented as the equation (3), where, the $\mu_p(x)$ and the $v_p(x)$ denote the membership degree and the non-membership degree respectively.

$$P = \{< x, (u_p(x), v_p(x)) > | x \in X\} \tag{3}$$

The Pythagorean fuzzy sets theory is discussed deeply and extensively and several aggregation operators have been proposed in recent years. Peng (2017) *et al.* initiated a new method to measure the Pythagorean fuzzy distance and then presented an algorithm to solve the stochastic MCDM problem [4]. Subsequently, Garg (2018) *et al.* modified some aggregation operators and then solved the MCDM problem under the Pythagorean fuzzy environment [5]. Shakeel(2019) *et al.* investigated the interval-valued Pythagorean trapezoidal fuzzy aggregation methods and defined some Einstein operational laws [6].

B. THE MULTI-CRITERIA DECISION MAKING PROBLEM
The solutions of MCDM problems can support decision-makers to make key decisions when facing complex issues. Especially, it does not exist the unique optimal solution, while it has to select the most suitable solution among different alternatives by using decision-maker's preferences [7].

In the process of dealing with MCDM problems, firstly, the experts evaluate the valuations of the alternatives according to various indicators. Experts give the valuations of the membership degree and the non-membership degree respectively, so we can construct the Pythagorean fuzzy decision matrix. Secondly, the alternatives will be ranked according to the Pythagorean fuzzy decision matrix and the optimal alternative will be obtained through aggregation algorithms [8].

Suppose there are m alternatives in total recorded as $X = \{x_1, x_2, \cdots, x_m\}$ and n decision indicators denoted as $A = \{A_1, A_2, \cdots, A_n\}$. The Pythagorean fuzzy element β_{ij} indicates the indicator valuation given by the i alternative under the j decision indicators. Then we can construct the following Pythagorean fuzzy decision matrix.

$$B = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1} & \beta_{m2} & \cdots & \beta_{mn} \end{bmatrix} \tag{4}$$

If all the indicator importance weights denoted as $w = (w_1, w_2 \cdots w_n)^T$ are known, we can use the existed Pythagorean fuzzy aggregation operators to calculate the synthesis values for each alternative and then rank all the alternatives according to them [9]. However, we find that it is often difficult to obtain all the weight information in the actual situation, frequently, we can only obtain the importance relationship among different indicators, and meanwhile, decision-makers sometimes have specific subjective

preferences for certain alternatives. So, how to obtain suitable indicator weights becomes urgent to be solved [10].

The solution of MCDM problem is also one of hot issues in recent research. Mishra (2017) *et al.* combined intuitionistic fuzzy weighted method with TOPSIS method for dealing with multi-criteria decision making problems [11]. In order to further extend the algorithm application scope, Asadabadi (2018) *et al.* modified a stratified multi-criteria decision making method [12]. Breedveld (2019) *et al.* introduced the latest multi-criteria decision making algorithms into the medical field, subsequently, several experiments were carried out and the results show that the algorithms can effectively assist doctors in making complex decisions [13].

C. THE VIKOR ALGORITHM
The VIKOR algorithm is one of common algorithms for solving the multi-criteria decision-making problems. It was originally designed to solve the decision problems with conflicting or multiple criteria, assuming the fact that the acceptable compromise solution exists. The compromise solution is the most suitable one which is closest to the ideal optimal solution compared with other alternatives. The definition of the compromise solution was firstly introduced into the MCDM field by Yu in 1973. The VIKOR algorithm can rank alternatives and obtain the compromise solution according to the calculation result of the equation (5).

$$L_{p,i} = \left(\sum_{j=1}^{n} \left(\frac{w_j(f_j^* - f_{ij})}{f_j^* - f_j^-} \right)^p \right)^{\frac{1}{p}}, 1 \le p \le \infty;$$

$$i = 1, 2, \cdots m \quad (5)$$

where the w_j is the weight of criterion C_j mentioned above. $f_i^* = \max f_{ij}$ $(i = 1, 2, \cdots m)$ and $f_i^- = \min f_{ij}$ $(i = 1, 2, \cdots m)$ are the best and the worst valuations of each column. $F^* = (f_1^*, f_2^*, \cdots, f_n^*)$ is the ideal optimal solution, while $F^- = (f_1^-, f_2^-, \cdots, f_n^-)$ is the ideal worst solution. The value of $L_{p,i}$ indicates the distance between the alternative x_i and the ideal solutions. Therefore, the most suitable solution is the alternative which is comprehensively closest to the ideal optimal solution F^* and farthest from the ideal worst solution F^-.

Scholars have carried out extensive discussions on this field in recent years. Dong (2017) *et al.* developed a new linguistic hesitant fuzzy VIKOR method for solving multi-criteria group decision making problems [14]. Ghadikolaei(2018) *et al.* proposed a qualitative multi-criteria group decision making approach based on the extended hesitant fuzzy linguistic distance [15]. Kim(2019) *et al.* introduced a new VIKOR algorithm which makes full use of incomplete criteria weights and then ranked alternatives by using aggregated scores [16].

D. THE TODIM ALGORITHM
The TODIM algorithm is mainly based on the pairwise comparison between alternatives, calculating the dominance of

one alternative over another by using the equations below under each criterion. The main steps are mathematically summarized by the following equations (6)–(8).

(1) Calculate the dominance of the criterion valuation A_i over the A_t under the criterion C_j by the equation (6).

$$\phi_j(A_i, A_t)$$
$$= \begin{cases} \sqrt{\dfrac{w_{jr} d(A_{ij}, A_{tj})}{\sum_{j=1}^{n} w_{jr}}}, & \text{if } score_{ij} > score_{tj} \\ 0, & \text{if } score_{ij} = score_{tj} \\ \dfrac{-1}{\lambda} \sqrt{(\sum_{j=1}^{n} w_{jr}) \dfrac{d(A_{ij}, A_{tj})}{w_{jr}}}, & \text{if } score_{ij} < score_{tj} \end{cases} \quad (6)$$

$$d(\beta_1, \beta_2)$$
$$= \frac{1}{2}\left(\left| \mu_{\beta_1}^2 - \mu_{\beta_2}^2 \right| + \left| v_{\beta_1}^2 - v_{\beta_2}^2 \right| + \left| \pi_{\beta_1}^2 - \pi_{\beta_2}^2 \right| \right) \quad (7)$$

$$\pi_\beta$$
$$= \sqrt{1 - \mu_\beta^2 - v_\beta^2} \quad (8)$$

where the $d(A_{ij}, A_{tj})$ means the distance between the two criterion valuations and the parameter λ represents the loss attenuation factor. We will obtain n comparison matrices in total which are shown in the equation (9), and the w_{jr} is the relative importance weight of the criterion C_j and w_r is the maximum value of all the importance weights, the specific calculation processes are shown in the equation (9) - (11).

$$\phi = [\phi_j(A_j, A_t)]_{m \times m} \quad (j = 1, 2, \ldots n) \quad (9)$$
$$w_r = \max_{j=1}^{n} w_j \quad (10)$$
$$w_{jr} = \frac{w_j}{w_r} \quad (11)$$

(2) Gather all the dominance of the criterion valuation A_i under all the criteria and obtain the total dominance $\psi_{it}(A_i, A_t)$ by the equation (12).

$$\psi_{it}(A_i, A_t) = \sum_{j=1}^{n} \phi_j(A_i, A_t), \quad (i, t = 1, 2, \cdots, m) \quad (12)$$

(3) The score value $\varphi(x_i)$ of each alternative $x_i(i = 1, 2, \cdots, m)$ will be calculated by the equation (13).

$$\varphi(x_i)$$
$$= \frac{\sum_{t=1}^{m} \psi(A_i, A_t) - \min_{i=1}^{m} \left(\sum_{t=1}^{m} \psi(A_i, A_t) \right)}{\max_{i=1}^{m} \left(\sum_{t=1}^{m} \psi(A_i, A_t) \right) - \min_{i=1}^{m} \left(\sum_{t=1}^{m} \psi(A_i, A_t) \right)} \quad (13)$$

The TODIM algorithm has been further developed recently. Yu (2017) *et al.* proposed a new method to deal with multi-criteria group decision making problems with unbalanced hesitant fuzzy linguistic term sets by considering the psychological behavior of the decision makers [17].

Wang (2018) *et al.* extended the original TODIM method to the 2-tuple linguistic fuzzy environment. The modified method has more advantage in considering the subjectivity of decision maker's behaviors [18]. A new definition of normalized Hamming distance was defined by Liang (2019) *et al.* Subsequently, they developed the multi-criteria proportional hesitant linguistic TODIM approach [19].

III. THE PYTHAGOREAN FUZZY VIKOR ALGORITHM BASED ON THE TODIM METHOD

In this section, we propose a new algorithm based on the VIKOR and the TODIM method under the Pythagorean fuzzy environment, it can make full use of their respective advantages. The flow chart of the algorithm proposed in the paper is shown as figure 1.
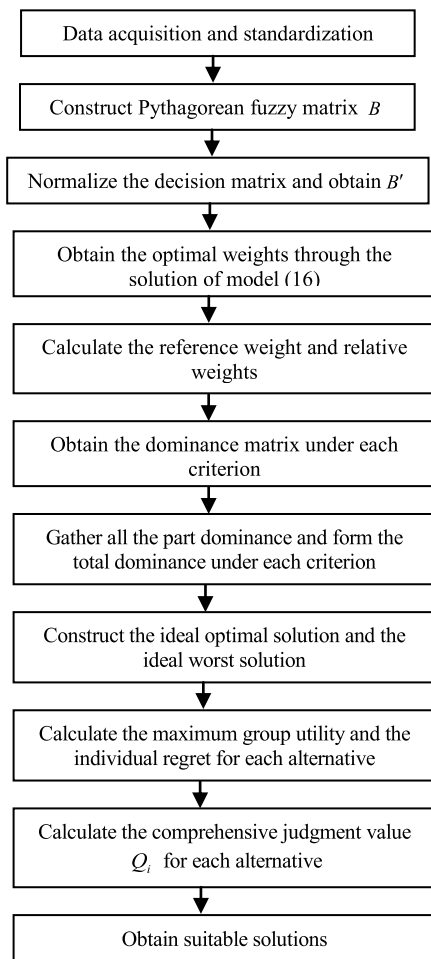

FIGURE 1. The flow chart of the algorithm proposed in the paper.

(1) Step 1: We obtain all the indicator valuations and transform them into Pythagorean fuzzy matrix elements. The final matrix form is shown in Table 1.

(2) Step 2: The criteria can be divided into benefit criteria and cost criteria according to the specific objective function [20]. We must normalize the decision matrix to ensure comparability amongst different criteria by the

TABLE 1. The Pythagorean fuzzy decision matrix *B*.

Alternatives	Criteria			
	C_1	C_2	$\cdots$	C_n
A_1	β_{11}	β_{12}	$\cdots$	β_{1n}
A_2	β_{21}	β_{22}	$\cdots$	β_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	β_{m1}	β_{m2}	$\cdots$	β_{mn}

following equation (14).

$$\beta'_{ij} = \begin{cases} \beta_{ij}, & benefit\ criterion \\ \beta^c_{ij}, & \cos t\ criterion \end{cases} \quad (14)$$

where the β^c_{ij} is the complement of the β_{ij} which can be calculated according to the equation (15), the normalized decision matrix is denoted as $B' = (\beta'_{ij})_{m \times n}$.

$$\beta^c_{ij} = \{< x_{ij}, (v_p(x_{ij}), \mu_p(x_{ij})) > x_{ij} \in X\} \quad (15)$$

(3) Step 3: Obtain the optimal weight denoted as $w^* = (w^*_1, w^*_2 \cdots, w^*_n)^T$. The w^* represents the importance degree of each indicator [21]. It is often difficult to obtain the exact weights, usually we can get some relationships or constraints information. Let the Ω be the total set of the indicator importance information provided by the decision makers. The Ω will be an empty set if it includes contradictory messages, the decision maker has to reconsider the weight constraints until the contradiction is eliminated when the contradictory messages exist. The importance information structure can be generally divided into the following categories. ① The weak Order, $\{w_i \geq w_j\}$; ② The strict order, $\{w_i - w_j > \partial_i\}$, while the ∂_i is a series of nonnegative constants; ③ The difference order, $\{w_i - w_j > w_k - w_l\}$, while $j \neq k \neq l$; ④ The multiplication order, $\{w_i \geq \partial_i w_j\}$; ⑤ The interval order, $\{\partial_i \leq w_i \leq \partial_i + \varepsilon_i\}$, while the ∂_i and ε_i are also a series of nonnegative constants [22].

The optimal weight w^* can be obtained through the solution of the model (16), as shown at the bottom of the next page, and the model can be solved efficiently by lingo which is a professional optimal solver in the linear and nonlinear domains [23], where $s(\beta_{ij})$ is the score function of the Pythagorean fuzzy elements which can be calculated by the equation (17), as shown at the bottom of the next page, the parameter $\theta(0 \leq \theta \leq 1)$ indicates the risk preference of the decision maker. It is an effective method to searching the optimal weight through maximizing satisfaction, and the method doesn't show any discrimination against all alternatives and has more comprehensively expressed the decision maker's cognitive information.

(4) Step 4: The reference criterion weight w_r can be calculated by the equation (10), and the relative weight of each criterion C_j denoted as w_{jr} can be calculated by the equation (11) mentioned above.

(5) Step 5: According to the TODIM algorithm mentioned above [24], the $\phi_j(A_i, A_t)$ represents the dominance of the alternative A_i over each alternative $A_t(t = 1, 2, \cdots, n)$ under the criterion C_j which can be calculated by the equation (6). So the whole dominance matrix $\phi_j = [\phi_j(A_i, A_t)]_{m \times m}(j = 1, 2, \cdots, n)$ under the criterion C_j can be obtained.

(6) Step 6: Gather all the part dominance and form the total dominance under each criterion denoted as $\psi_{ij}(A_i, A_j)$ which is showed on the table 2. Consequently, the total dominance matrix is finally constructed which is the basis of the following TODIM algorithm. Different from traditional algorithms, the elements of the dominance matrix are real numbers, so it has less calculation compared with other algorithms.

TABLE 2. The total dominance under each criterion.

Alternatives	Criteria			
	C_1	C_2	$\cdots$	C_n
A_1	$\sum_{t=1}^{m}\phi_1(A_1,A_t)$	$\sum_{t=1}^{m}\phi_2(A_1,A_t)$	$\cdots$	$\sum_{t=1}^{m}\phi_n(A_1,A_t)$
A_2	$\sum_{t=1}^{m}\phi_1(A_2,A_t)$	$\sum_{t=1}^{m}\phi_2(A_2,A_t)$	$\cdots$	$\sum_{t=1}^{m}\phi_n(A_2,A_t)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	$\sum_{t=1}^{m}\phi_1(A_m,A_t)$	$\sum_{t=1}^{m}\phi_2(A_m,A_t)$	$\cdots$	$\sum_{t=1}^{m}\phi_n(A_m,A_t)$

(7) Step 7: Construct the ideal optimal solution D^* and the ideal worst solution D^- through the total dominance matrix according to the equation (18) and (19).

$$D^* = (D_1^*, D_2^*, \cdots, D_n^*) = (\max_{j=1}^{m} \psi_{1j}, \cdots, \max_{j=1}^{m} \psi_{mj}) \quad (18)$$

$$D^- = (D_1^-, D_2^-, \cdots, D_n^-) = (\min_{j=1}^{m} \psi_{1j}, \cdots, \min_{j=1}^{m} \psi_{mj}) \quad (19)$$

(8) Step 8: Calculate the maximum group utility S_i and the individual regret R_i for each alternative respectively

according to the equation (20) - (23).

$$S_i = \sum_{j=1}^{n} w_j \frac{d(D_j^*, D_{ij})}{d(D_j^*, D_j^-)} \quad (20)$$

$$R_i = \max_{j=1}^{n} w_j \frac{d(D_j^*, D_{ij})}{d(D_j^*, D_j^-)} \quad (21)$$

$$d(D_j^*, D_{ij}) = \max_{k=1}^{m} \psi_{jk} - \psi_{ij} \quad (22)$$

$$d(D_j^*, D_j^-) = \max_{k=1}^{m} \psi_{jk} - \min_{k=1}^{m} \psi_{jk} \quad (23)$$

(9) Step 9: Calculate the comprehensive judgment value Q_i for each alternative respectively according to the equation (24)-(28). The parameter ρ is the importance weight which can adjust the influence proportion between the maximum group utility and the individual regret. Generally, the value is set to 0.5.

$$Q_i = \rho \frac{S_i - S^-}{S^* - S^-} + (1 - \rho) \frac{R_i - R^-}{R^* - R^-} \quad (24)$$

$$S^- = \min_{i=1}^{m} S_i \quad (25)$$

$$S^* = \max_{i=1}^{m} S_i \quad (26)$$

$$R^- = \min_{i=1}^{m} R_i \quad (27)$$

$$R^* = \max_{i=1}^{m} R_i \quad (28)$$

(10) Step 10: According to the values of S_i, R_i and Q_i, rank the alternatives respectively in ascending order. So, we can get three ranking lists for the alternatives.

(11) Step 11: The alternative $A^{(1)}$ which has the minimum value of Q will be the most suitable solution if it meets the following two conditions.

1) Condition I: The alternative $A^{(1)}$ also has the minimum value of R and S at the same time.

2) Condition II: The inequality is satisfied which is $Q(A^{(2)}) - Q(A^{(1)}) \geq \frac{1}{m-1}$, the $A^{(1)}$ and $A^{(2)}$ are the alternatives which are in the first and second positions on the Q ranking list. The m is the total number of alternatives.

$$z = \max \sum_{i=1}^{m} \frac{(1 - \theta)(1 + \sum_{j=1}^{n} w_j s(\beta_{ij}))}{\theta[\sum_{j=1}^{n} w_j(1 - s(\beta_{ij}))] + (1 - \theta)(1 + \sum_{j=1}^{n} w_j s(\beta_{ij}))} \quad (16)$$

$$s.t. \ w = (w_1, w_2, \cdots, w_n)^T \in \Omega$$

$$0 \leq w_j \leq 1 \ (j = 1, 2, \cdots, n)$$

$$\sum_{j=1}^{n} w_j = 1$$

$$s(\beta_{ij}) = \mu_\beta^2 - v_\beta^2 \quad (17)$$

While, a set of suitable solutions will be obtained if one of the condition mentioned above cannot be met. It can be divided into the following two situations.

1) All the $A^{(1)}$ and $A^{(2)}$ are the suitable solutions if only the condition I is not met.

2) We can obtain the maximum value of c through the calculation of the inequality (29), all the alternatives $A^{(i)}(i = 1, 2, \cdots, c)$ will be suitable solutions because they are too close to distinguish.

$$Q(A^{(c)}) - Q(A^{(1)}) \le \frac{1}{m - 1} \qquad (29)$$

IV. THE IDENTIFICATION OF POVERTY ALLEVIATION TARGETS

Suppose there are five poor families with various conditions and we must rank them according to their poverty level which is hard to achieve. According to the algorithm in this article, firstly, we must choose scientific and reasonable measurement indicators. There are many indicators can be collected with the support of the large data and artificial intelligence technology. While, the selection of the appropriate measurement indicators is crucial to accurately identify the most needed poverty alleviation targets. The paper is going to use the following key indicators which are listed on the table 3 after preliminary investigations. At present we can only conclude that the importance relationship of the five key indicators is $C_3 \ge C_4 \ge C_2 \ge C_1 \ge C_5$, and each weight is greater than or equal to 0.05 which means $w_3 \ge w_4 \ge w_2 \ge w_1 \ge w_5 \ge 0.05$. The importance weight of the disease severity of family members (C_3) is greater than or equal to 0.4 and less than or equal to 0.5, that means $0.5 \ge w_3 \ge 0.4$.

TABLE 3. The criteria used in the paper.

Criterion	Property
The per-capita net incomes(C_1)	Cost criterion
The total number of minor children(C_2)	Benefit criterion
Disease severity of family members(C_3)	Benefit criterion
Housing conditions(C_4)	Cost criterion
Evaluation of the poverty alleviation staff(C_5)	Benefit criterion

(1) Step 1: The original indicator data are obtained through background visits, family surveys, data collation and subjective analysis of the investigators. The original Pythagorean fuzzy decision matrix B is listed on the table 4 after data standardization processing.

(2) Step 2: We find that the criteria of the per-capita net incomes (C_1) and housing conditions (C_4) are cost criteria. So, the original Pythagorean fuzzy decision matrix will be processed according to the equation (14), and the normalized Pythagorean fuzzy decision matrix is showed on the table 5.

(3) Step 3: The score matrix S is calculated according to the equation (17) and the optimal weights of the criteria

TABLE 4. The original Pythagorean fuzzy decision matrix.

B	C_1	C_2	C_3	C_4	C_5
A_1	(0.1,0.8)	(0.3,0.6)	(0.8,0.1)	(0.8,0.2)	(0.8,0.2)
A_2	(0.2,0.7)	(0.2,0.7)	(0.2,0.8)	(0.1,0.8)	(0.8,0.1)
A_3	(0.2,0.8)	(0.8,0.1)	(0.3,0.7)	(0.7,0.2)	(0.9,0.2)
A_4	(0.6,0.3)	(0.2,0.4)	(0.8,0.2)	(0.1,0.7)	(0.9,0.1)
A_5	(0.5,0.4)	(0.9,0.1)	(0.8,0.3)	(0.8,0.2)	(0.8,0.3)

TABLE 5. The normalized Pythagorean fuzzy decision matrix.

B′	C_1	C_2	C_3	C_4	C_5
A_1	(0.8,0.1)	(0.3,0.6)	(0.8,0.1)	(0.2,0.8)	(0.8,0.2)
A_2	(0.7,0.2)	(0.2,0.7)	(0.2,0.8)	(0.8,0.1)	(0.8,0.1)
A_3	(0.8,0.2)	(0.8,0.1)	(0.3,0.7)	(0.2,0.7)	(0.9,0.2)
A_4	(0.3,0.6)	(0.2,0.4)	(0.8,0.2)	(0.7,0.1)	(0.9,0.1)
A_5	(0.4,0.5)	(0.9,0.1)	(0.8,0.3)	(0.2,0.8)	(0.8,0.3)

TABLE 6. The calculation and ranking order results.

Alternatives	S_i	S_i Order	R_i	R_i Order	Q_i	Q_i Order
A_1	0.6298	3	0.42	4	0.8478	3
A_2	**0.4519**	**1**	**0.18**	**1**	**0**	**1**
A_3	0.5046	2	0.306	2	0.3879	2
A_4	0.7391	5	0.357	3	0.8429	4
A_5	0.6689	4	0.42	5	0.9022	5

$w^* = (0.14, 0.16, 0.18, 0.42, 0.10)$ can be obtained according to the model (16).

$$S = \begin{bmatrix} 0.63 & -0.27 & 0.63 & -0.6 & 0.6 \\ 0.45 & -0.45 & -0.6 & 0.63 & 0.63 \\ 0.60 & 0.63 & -0.4 & -0.45 & 0.77 \\ -0.27 & -0.12 & 0.6 & 0.48 & 0.8 \\ -0.09 & 0.8 & 0.55 & -0.6 & 0.55 \end{bmatrix} \qquad (30)$$

(4) Step 4: The reference weight is $w_r = 0.42$ according to the equation (10), and the relative importance weights $w_{jr} = (0.33, 0.38, 0.43, 1, 0.24)$ can be obtained according to the equation (11).

(5) Step 5: The dominance of the alternative A_i over the alternative A_t under each criterion is calculated respectively according to the equation (6)-(8). The matrices (ϕ_1-ϕ_5) are showed as follows which indicate the part dominance under the criteria 1 to 5 respectively.

$$\phi_1 = \begin{bmatrix} 0 & 0.145 & 0.065 & 0.277 & 0.259 \\ -0.414 & 0 & -0.414 & 0.237 & 0.215 \\ -0.185 & 0.145 & 0 & 0.277 & 0.259 \\ -0.793 & -0.676 & -0.793 & 0 & -0.355 \\ -0.741 & -0.614 & -0.741 & 0.124 & 0 \end{bmatrix}$$

TABLE 7. The different ranking results with the change of the parameter ρ.

Parameter value	The optimal solution	The ranking result	Remarks
$\rho = 0.5$	A_2	$A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$	Group utility and individual regret equally
$\rho = 1$	A_2, A_3	$A_2 \succ A_3 \succ A_1 \succ A_5 \succ A_4$	Group utility
$\rho = 0$	A_2	$A_2 \succ A_3 \succ A_4 \succ A_1 \sim A_5$	Individual regret

$$\phi_2 = \begin{bmatrix} 0 & 0.144 & -0.742 & -0.5 & -0.849 \\ -0.361 & 0 & -0.775 & -0.574 & -0.878 \\ 0.297 & 0.31 & 0 & 0.31 & -0.412 \\ 0.2 & 0.23 & -0.775 & 0 & -0.878 \\ 0.339 & 0.351 & 0.165 & 0.351 & 0 \end{bmatrix}$$

$$\phi_3 = \begin{bmatrix} 0 & 0.337 & 0.315 & 0.073 & 0.12 \\ -0.748 & 0 & -0.365 & -0.73 & -0.73 \\ -0.7 & 0.164 & 0 & -0.7 & -0.7 \\ -0.163 & -0.73 & -0.7 & 0 & -0.211 \\ -0.267 & 0.329 & 0.315 & -0.211 & 0 \end{bmatrix}$$

$$\phi_4 = \begin{bmatrix} 0 & -0.49 & -0.24 & -0.49 & 0 \\ 0.514 & 0 & 0.502 & 0.251 & 0.514 \\ 0.251 & -0.478 & 0 & -0.428 & 0.251 \\ -0.49 & -0.24 & 0.449 & 0 & -0.49 \\ 0 & -0.49 & -0.24 & -0.49 & 0 \end{bmatrix}$$

$$\phi_5 = \begin{bmatrix} 0 & -0.219 & -0.522 & -0.522 & 0.071 \\ 0.055 & 0 & -0.566 & -0.522 & 0.09 \\ 0.13 & 0.141 & 0 & -0.22 & 0.13 \\ 0.13 & 0.13 & 0.055 & 0 & 0.13 \\ -0.283 & -0.358 & -0.522 & -0.522 & 0 \end{bmatrix}$$

(6) Step 6: The total dominance under all criteria is calculated according to the equation (12) by using the part dominance calculation results of the step 5.

$$\psi = \begin{bmatrix} 0.746 & -1.947 & 0.845 & -1.22 & -1.192 \\ -0.376 & -2.588 & -2.573 & 1.781 & -0.943 \\ 0.496 & 0.505 & -1.936 & -0.404 & 0.181 \\ -2.617 & -1.223 & -1.804 & -0.771 & 0.445 \\ -1.972 & 1.206 & 0.166 & -1.22 & -1.685 \end{bmatrix}$$

(7) Step 7: The ideal optimal solution and the ideal worst solution are obtained according to the calculation results of the step 6. The ideal optimal solution is composed by the best values in each column, while the ideal worst solution is composed by the worst values in each column.

$$D* = (0.746, 1.206, 0.845, 1.781, 0.445)$$

$$D^- = (-2.617, -2.588, -2.573, -1.22, -1.685)$$

(8) Step 8-10: The maximum group utility S_i and the individual regret R_i of each alternative are calculated

respectively according to the equation (20) - (23). The comprehensive judgment values Q_i are also calculated according to the equation (24)-(28) and the parameter ρ is set to 0.5. We can rank the alternatives respectively according to the values of the S_i, R_i and Q_i in ascending order, the results are listed on the table 6.

(9) Step 11: The alternative A_2 is in the first position on the ranking list of Q_i and also in the first position on the ranking list of S_i and R_i which means it satisfy the first condition. The difference between the $Q(A^{(2)})$ and the $Q(A^{(1)})$ which are in the second and first position respectively on the ranking list of Q_i is greater than $\frac{1}{m-1}$ and the computational process is showed in the following formula, that means the alternative A_2 is also satisfy the second condition. So, the alternative A_2 is the unique optimal solution according to the TODIM algorithm.

$$Q(A^{(2)}) - Q(A^{(1)}) = 0.387948 > 1/(5-1) = 0.25$$

In the above experiment the parameter ρ is set to 0.5, which means that we consider the effect of the group utility and the individual regret equally. While we are going to carry out two other experiments, the first one is that we only consider the effect of the group utility and the parameter ρ is set to 1, the second one is that we only consider the effect of the individual regret and the parameter ρ is set to 0. The different ranking results with the change of the parameter ρ are showed on the table 7. We can find the fact that the alternative A_2 is included in the optimal solutions in the three cases, so the alternative A_2 will be the optimal solution undoubtly. It means that the second family needs the help of poverty alleviation most.

The table 8 lists the execution results of different algorithms, we find that the algorithm proposed by Yager cannot distinguish the alternative A_2 and the alternative A_4, that is mainly because the Pythagorean fuzzy sets is not used in the algorithm to collect data, the description of data details is not enough; if we only use TODIM approach, the alternative A_2 and the alternative A_3 cannot be distinguished, one of the main reasons is that the weight of indicators can't be set to the most appropriate values and the effect of core indicators has not been brought into fully play; if we only use VIKOR method, the alternative A_2, the alternative A_3 and the alternative A_1 cannot be distinguished, the main disadvantages of this algorithm are that the alternative ranking method is too simple and not enough consideration was given to various situations. While we find that the algorithm proposed in

TABLE 8. Comparison among different algorithms.

Algorithm	The ranking result
The algorithm proposed by Yager[25]	$A_3 \succ A_2 \sim A_4 \succ A_1 \sim A_5$
Only TODIM	$A_2 \sim A_3 \succ A_1 \succ A_4 \sim A_5$
Only VIKOR	$A_2 \sim A_3 \sim A_1 \succ A_4 \sim A_5$
The multiple hybrid algorithm proposed in the paper	$A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$

the paper has strongest discrimination ability and can rank alternatives using small differences among data compared with other algorithms.

V. CONCLUSION

The TODIM approach is an effective tool to obtain the compromise solution. The VIKOR algorithm can rank alternatives in limited time and can both consider the effect of the group utility and the individual regret. The Pythagorean fuzzy sets can effectively store detail information of indicators. The paper considers the problem of identifying the poverty alleviation targets as a multi-criteria decision making problem. The main contribution of the paper is that it proposes an algorithm which integrate the TODIM approach, the VIKOR algorithm and the Pythagorean fuzzy sets method together to solve the problem of poverty alleviation target identification. We obtain the optimal importance weights by constructing a goal optimization model. The introduction of the Pythagorean fuzzy sets has greatly improved the ability of collecting data details; the efficiency of the algorithm is increased with the help of VIKOR algorithm; the diversity of feasible solutions is guaranteed by the idea of the TODIM approach. In the end, several experiment results show that the algorithm works well and can solve the problem effectively. While, the algorithm also has two main limitations, the first limitation is that the results are difficult to verify with digital evidence, the other one is that it is difficult for different data collectors to have the unified objectivity in the process of data collection, data digitization and data standardization, in particular, the validity and authenticity of primary data will be an important research direction in the future.

REFERENCES

[1] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.

[2] M. M. Xia and Z. S. Xu, "Hesitant fuzzy information aggregation in decision making," *Int. J. Approx. Reason*, vol. 52, no. 3, pp. 395–407, 2011.

[3] S. Greco, B. Matarazzo, and R. Slowinski, "Rough sets theory for multicriteria decision analysis," *Eur. J. Oper. Res.*, vol. 129, no. 1, pp. 1–47, Feb. 2001.

[4] X. Peng, H. Yuan, and Y. Yang, "Pythagorean fuzzy information measures and their applications," *Int. J. Intell. Syst.*, vol. 32, no. 10, pp. 991–1029, Oct. 2017.

[5] H. Garg, "Linguistic pythagorean fuzzy sets and its applications in multiattribute decision-making process," *Int. J. Intell. Syst.*, vol. 33, no. 6, pp. 1234–1263, Jun. 2018.

[6] M. Shakeel, S. Abdullah, M. Shahzad, F. Amin, T. Mahmood, and N. Amin, "Pythagorean trapezoidal fuzzy geometric aggregation operators based on einstein operations and their application in group decision making," *J. Intell. Fuzzy Syst.*, vol. 36, no. 1, pp. 309–324, Feb. 2019.

[7] F. Feng, H. Fujita, M. I. Ali, R. R. Yager, and X. Liu, "Another view on generalized intuitionistic fuzzy soft sets and related multiattribute decision making methods," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 3, pp. 474–488, Mar. 2019.

[8] Z. Kong, J. Ai, L. Wang, P. Li, L. Ma, and F. Lu, "New normal parameter reduction method in fuzzy soft set theory," *IEEE Access*, vol. 7, pp. 2986–2998, 2019.

[9] S. Das, D. Malakar, S. Kar, and T. Pal, "Correlation measure of hesitant fuzzy soft sets and their application in decision making," *Neural Comput. Appl.*, vol. 31, no. 4, pp. 1023–1039, Apr. 2019.

[10] A. Khan and Y. Zhu, "A novel approach to parameter reduction of fuzzy soft set," *IEEE Access*, vol. 7, pp. 128956–128967, 2019.

[11] A. R. Mishra, P. Rani, and D. Jain, "Information measures based TOPSIS method for multicriteria decision making problem in intuitionistic fuzzy environment," *Iranian J. Fuzzy Syst.*, vol. 14, no. 6, pp. 41–63, 2017.

[12] M. R. Asadabadi, "The stratified multi-criteria decision-making method," *Knowl.-Based Syst.*, vol. 162, pp. 115–123, Dec. 2018.

[13] S. Breedveld, "Multi-criteria optimization and decision-making in radiotherapy," *Eur. J. Oper. Res.*, vol. 277, no. 1, pp. 1–19, 2019.

[14] J.-Y. Dong, F.-F. Yuan, and S.-P. Wan, "Extended VIKOR method for multiple criteria decision-making with linguistic hesitant fuzzy information," *Comput. Ind. Eng.*, vol. 112, pp. 305–319, Oct. 2017.

[15] A. S. Ghadikolaei, M. Madhoushi, and M. Divsalar, "Extension of the VIKOR method for group decision making with extended hesitant fuzzy linguistic information," *Neural Comput. Appl.*, vol. 30, no. 12, pp. 3589–3602, Dec. 2018.

[16] J. H. Kim and B. S. Ahn, "Extended VIKOR method using incomplete criteria weights," *Expert Syst. Appl.*, vol. 126, pp. 124–132, Jul. 2019.

[17] W. Yu, Z. Zhang, Q. Zhong, and L. Sun, "Extended TODIM for multi-criteria group decision making based on unbalanced hesitant fuzzy linguistic term sets," *Comput. Ind. Eng.*, vol. 114, pp. 316–328, Dec. 2017.

[18] J. Wang, G. Wei, and M. Lu, "TODIM method for multiple attribute group decision making under 2-Tuple linguistic neutrosophic environment," *Symmetry*, vol. 10, no. 10, p. 486, Oct. 2018.

[19] Y. Liang, Y. Tu, Y. Ju, and W. Shen, "A multi-granularity proportional hesitant fuzzy linguistic TODIM method and its application to emergency decision making," *Int. J. Disaster Risk Reduction*, vol. 36, May 2019, Art. no. 101081.

[20] H. Liao, Z. Xu, and M. Xia, "Multiplicative consistency of interval-valued intuitionistic fuzzy preference relation," *J. Intell. Fuzzy Syst.*, vol. 27, no. 6, pp. 2969–2985, 2014.

[21] H. Liao and Z. Xu, "Subtraction and division operations over hesitant fuzzy sets," *J. Intell. Fuzzy Syst.*, vol. 27, no. 1, pp. 65–72, 2014.

[22] H. Liao and Z. Xu, "A VIKOR-based method for hesitant fuzzy multi-criteria decision making," *Fuzzy Optim. Decis. Making*, vol. 12, no. 4, pp. 373–392, Dec. 2013.

[23] V. Torra and Y. Narukawa, "On hesitant fuzzy sets and decision," in *Proc. IEEE Int. Conf. Fuzzy Syst.*, Aug. 2009, pp. 1378–1382.

[24] V. Torra, "Hesitant fuzzy sets," *Int. J. Intell. Syst.*, vol. 25, no. 6, pp. 529–539, 2010.

[25] R. R. Yager, N. Alajlan, and Y. Bazi, "Aspects of generalized orthopair fuzzy sets," *Int. J. Intell. Syst.*, vol. 33, no. 11, pp. 2154–2174, Nov. 2018.

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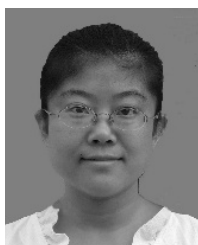
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