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Optimal Strategies for Online Advance Selling With Random Rewards–Case From China

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ABSTRACT Advance selling helps retailers who often face a newsvendor problem to reduce demand uncertainty. With the development of e-commerce and online retailing, online retailers are no longer limited to adopting single advance selling strategy. With the support of information technology and online payment, the combined strategy of random reward promotion and advance selling becomes increasingly easy to apply. In this article, the model of online advance selling with random rewards strategy is developed based on single advance selling model. The result shows that there is a threshold which determines which of the two strategies is optimal and the numerical analysis further verifies the research results, through comparison and analysis of the advance selling price, consumer utility and retailer's total profit of single advance selling strategy and advance selling with random rewards strategy. Furthermore, the study also finds the effect of the expected utility of random rewards on the advance selling price.

INDEX TERMS Newsvendor, online advance selling, random rewards, risk aversion, strategic consumer.

I. INTRODUCTION

The retailers face newsvendor problems in a market with uncertain demands. Advance selling strategy is introduced in service industries with limited supply and uncertain demand, such as hotels and airlines, to reduce demand uncertainty. The early studies of Xie and Shugan [1] and Shugan and Xie [2] indicate that advance selling is generally applicable to markets with uncertain demand. Tang, Rajaram, Alptekinoğlu and Ou [3] investigate the market of perishable goods with a short life cycle and unpredictable demand and find that advance selling can help retailers to accurately predict the demand during the selling season.

It is estimated that the number of online shoppers in China have reached 533 million in 2017, an increase of 14.3% from 2016, with online shoppers accounting for 69.1 percent of the total Internet users, [4] which have supported the constant innovation of marketing models and promotion methods. Thus, advance selling has gained further attention and has been widely applied. For instance, in 2018, the transaction sum of China's double 11 shopping carnival, which came from 75 countries and regions, exceeded RMB 300 billion in one day with more than one billion packages. During this event, 180,000 merchants from famous e-commerce platforms such as Tmall, Taobao, Jing Dong, Su Ning and Pinduoduo adopted advance selling strategy. The advance selling period for Tmall and Taobao was from October 20 to November 10. Thus, advance selling has always been a popular research object for scholars and a business practice for continuous innovation.

As market competition intensifies, consumers' buying behaviour becomes increasingly rational. Thus, online retailers should consider a number of factors concerning marketing strategy, such as strategic consumer, risk aversion and shortsightedness, among others. Online retailers are no longer restricted to adopting single advance selling strategy and begin to explore the combination of advance selling strategies. Under such background, random rewards (or sweepstakes promotion) is the one that cater to the psychological needs of strategic consumers.

With the support of network and information technology, random rewards promotion becomes increasingly easy to apply. Random rewards exhibit functions on both platforms of WeChat and Alipay, registered users' number of which has reached 1 billion and 0.87 billion respectively. For example, the amount of money inside the 'red envelope' can be credited to a receiver's bank account through the function of WeChat's 'random red envelope'. Random rewards

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promotion, including WeChat 'random red envelope' that encourages interaction and participation, can increase the expected utility of consumers [5] and can consequently stimulate consumers' purchasing enthusiasm and increase market demand. Therefore, the advance selling strategy considering random draws is favoured by retailers. For instance, during the double 11 shopping carnival in 2018, Alibaba's E-commerce platforms, Tmall and Taobao, have nearly 200,000 merchants to implement a combination strategy of advance selling and random rewards. They announced that the total value of random rewards could reach one billion, and the wining amount of individual consumers could be as high as RMB 1,111 during the advance selling period from October 20 to November 10.

Although advance selling considering random rewards has been widely applied, research on academic circles and related literature focusing on such topic is rarely available.

This study focuses on whether there is threshold to determine which strategy online retailers should adopt to obtain the optimal profit by comparing and analysing the two strategies. For the first time, we find that thresholds $\bar{\mu}_A$ and $\bar{A}$ exist, where $\bar{\mu}_A$ and $\bar{A}$ are the thresholds of μ_A and A respectively. When $\mu_A > \bar{\mu}_A$ or A $<\bar{A}$, where μ_A is defined as expected number of consumers who arrive during the advance selling period and A is defined as the expected value of consumers randomly winning, retailers should adopt advance selling strategy considering random rewards; when $\mu_A < \bar{\mu}_A$ or A $> \bar{A}$, retailers should adopt a single advance selling strategy.

This study is structured as follows. After the introduction section, Section 2 presents a review of literature. Section 3 describes the model and lists the assumptions. Section 4 discusses advance selling without random rewards, whereas Section 5 presents models and optimal solutions for advance selling strategy considering random rewards. Section 6 compares and analyses the two strategies, and Section 7 provides a numerical analysis. Section 8 concludes the study. Lastly, the Appendix lists all proofs.

II. THE OUTLINE OF RELATED LITERATURE

In this section, the related work of online advance selling and random rewards are presented.

A. REVIEW ON THE ADVANCE SELLING

Shugan and Xie [6] prove that advance selling strategy can be applied as long as consumers are uncertain about the value of products or services. Following the study of Shugan, Xie and Shugan further investigate that advance selling profit is caused by reducing the uncertainty of demand and expanding market demand. Relevant literature focuses on aspects regarding forecasting demand, advance selling price, consumers' behaviour on the influence of advance selling and the combination and application of advance selling strategy.

Tang *et al.* [3] concluded that adjusting demand through advance selling information can accurately predict market demand to increase corporate profit. Boyaci and Özer [7] prove that the market information obtained from advance selling can guide optimal production. Based on the findings of Boyaci and Özer, Prasad and Stecke [8] demonstrate that advance selling reduces uncertainty of market demand and helps manufacturers to reasonably decide on the output to reduce out-of-stock risk. Moreover, advance selling can increase sales and profits by consumers' uncertainty of product valuations.

Gale and Holmes' [9], [10] early study on advance selling in the service industry mostly advocates discount pricing. For instance, airlines could encourage consumers with little difference in flight preferences to book ahead of time through a low-price advance selling strategy in which passengers can be transferred from peak routes to other flights to balance the demand for different flights. Weng and Parlar [11] find that the demand for products during the advance selling period considerably depends on the discount price of the advance selling period. You [12] analyses the ordering and pricing of service products in an advanced sales system and finds that the optimal ordering quantity and prices are derived via closed-form solutions. Nocke *et al.* [13] also support discount in advance selling, and they perceive that advance selling can maximise profits with price discrimination. Based on Xie and Shugan, Wang and Zeng [14] conduct an in-depth study of the applicable conditions for advance selling when capacity constraints and consumer heterogeneity to adopt low, premium or normal price are evident.

B. REVIEW ON THE EXTENSION OF ADVANCE SELLING

Zhao and Stecke [15] consider loss-averse consumers and study how the optimal strategy is chosen among the three strategies, namely, no advance selling, moderate discount advance selling and deep discount advance selling wherein retailers release a new product. Swinney [16] notes that through the research on the sales strategy of enterprises, the value of quick response production model is generally lower in the face of strategic (forward-looking) consumers than of non-strategic consumers (short-sighted consumers). That is, consumers' decision-making patterns (whether strategic or not) can influence the choice of presale strategies. Nasiry and Popescu [17] discuss the advance selling model strategy and enterprise profit by considering repentance consumer behaviour. Lim and Tang [18] find the advance selling decision-making of monopoly retailers in the market including short-term consumers, forward-looking consumers and middlemen. Li [19] studies advance selling strategy for a retailer who sells a newsvendor-type of fashionable product in light of potential consumer opportunistic returns. Li and Shan [20] explore that consumers may overestimate the accuracy of their valuation forecasts and thus propose a decision-making model about overconfidence in valuation. Yu, Liu, Han and Chen [21] find that sellers' selling strategies are significantly affected by the relationship between valuation bias and strategic consumers' estimation of the bias.

Some scholars have studied the combination and expansion of advance selling with other strategies. For example, Cachon and Feldman [22] study the impact of competition on presale strategies among retailers. Tian and Wang [23] investigate the advance selling problem within the framework of rational expectation. In terms of extended applications, McCardle, Rajaram and Tang [24] add competitive factors among enterprises to discuss the conditions under which advance selling discount items are uniquely balanced. Cho and Tang [25] investigate the presale model of supply chain. Furthermore, Cachon and Feldman [22] study the impact of competition among retailers on advance selling strategies. Certain research considers optimal service capacity allocation policy in an advance selling environment in a continuous period [26].

C. REVIEW ON THE REWARDS PROMOTION AND ONLINE ADVANCE SELLING

The Marketing Theory of 4Ps proposed by McCarthy provides an effective tool for enterprises to analyse and solve marketing problems. Keller and Kottler [27] emphasised that common consumer promotional tools include raffle, sweepstake, special promotions and coupons. Promotion plays an important role in modern marketing strategies. Kahneman and Tversky [28] propose prospect theory that provides a theoretical framework for evaluating the attractiveness of promotional methods. Prospect theory holds that the value equation and the weight equation are independent of each other in the effect on utility, whereas the overestimation of small probability weight is the basic feature of weight equation [29]. Ward and Hill [30] define raffle promotion as an opportunity for consumers to win prizes designed to promote goods or services. Following the study of Ward and Hill, d'Astous and Landreville [31] study the factors affecting the raffle effect and find that the factors regarding prize attractiveness, prize and product match, prize valuation and winning probability will affect consumers' evaluation of the attractiveness and satisfaction and will consequently impact purchase decision. With the rapid internet-based development and online payment progress in China, online shopping has become an integral component in facilitating people's lives [4].

In addition to the promotion evaluation on economic value, Chandon *et al.* [32] find that consumers' evaluation on random raffle is based on instrumental or economic value, as well as entertainment value, which is an important aspect of promotional evaluation. Smith, Dickhaut, McCabe and Pardo [33] find that the value of a prize does not directly affect the weighting equation but affects a consumer's subjective estimation of the probability of winning by comparing the value of the prize with the value of the purchased product. Chen and Jia [34] study the effectiveness of two forms of promotion, namely, winning a grand prize with a small probability and winning a small prize with 100% probability. Khouja and Zhou [35] research on channel and pricing decisions in a supply chain with advance selling of gift cards. Wang examine optimal advance selling strategies using coupons in a monopoly market and find that when the seller's marginal cost is moderately small, the profit gains from coupon buyers exceed the profits loss from regular selling. [36] Recently, Yu *et al.* [37] examine online promotions with gift rewards for a Chinese tea retailer and illustrate how to improve gift allocation based on the robust inventory solutions to increase retailer's profits. Referring to their research, this study extends the sales strategy to the currently popular advance selling strategy and extends gift rewards into random rewards. These extensions are more in line with the facts of actual online promotion.

In summary, the contribution on single advance selling strategy has been extremely rich, and new progress in joint strategy with advance selling is evident. However, the study on advance selling strategy considering random rewards is relatively new. As far as we know, our work is the first to investigate on advance selling strategy considering random rewards.

III. MODEL DESCRIPTION AND ASSUMPTIONS

In line with the literature review, retailers' adoption of advance selling strategies is modelled in two periods of time. The first period is advance selling period, and the second is the spot selling period. To attract consumers to buy, retailers hold sweepstake promotions in the advance selling period to provide random rewards for consumers who buy in advance. In contrast, a retailer sells spot products in the selling period, which is also the consumption period. Figure 1 illustrates the entire process of the advance selling strategy.

During the advance selling period, a retailer announces the advance selling price of a, the selling price of p and random rewards. A retailer collects the money at the price of a but does not deliver the product during the advance selling period. A consumer pays but does not receive the product; thus, the valuation of the product is uncertain. A retailer delivers and sells products during the selling period. The number of customers arriving at the market during both stages is stochastic. A retailer calculates the total profit based on revenue and cost and deducts the total random rewards from the profit. The retailer and consumers' assumptions related to the model are detailed below. Table 1 lists the definitions of the variables associated with the model.

A. RETAILERS

Suppose that only one monopoly retailer exists in the market. The retailer's sale is divided into two periods; the first period is advance selling, which ends before the selling period begins. The retailer announces a and p to consumers at the beginning of the advance selling period. The retailer charges a from a consumer who buys in advance, but the product is not delivered yet. The second stage is the selling period, where the retailer delivers products to consumers who pay during the first period and sells the products at the price of p. The retailer pays c for unit product. For unsold goods at the end of the selling period, the retailer will receive a residual value of s per unit, apparently, $s < c < a < p$.

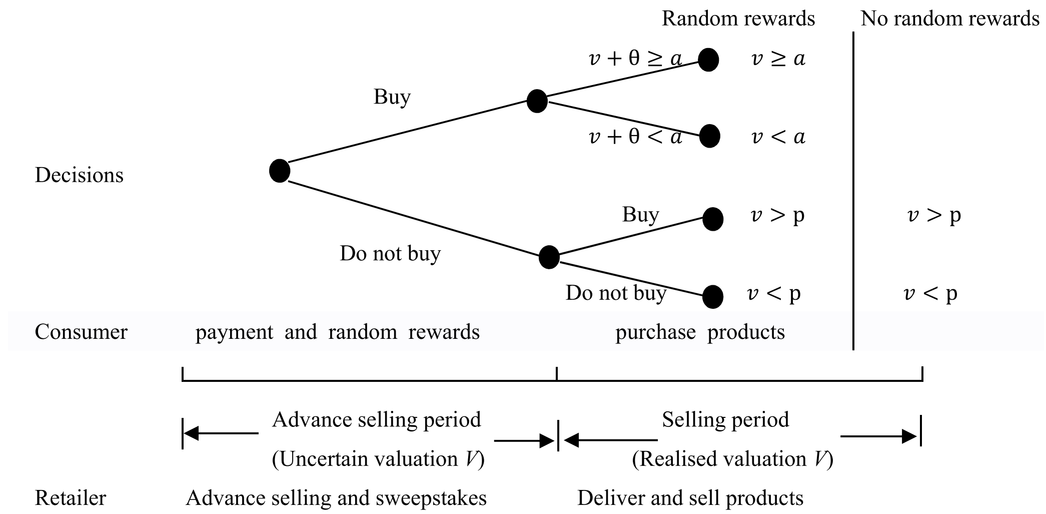

FIGURE 1. Sale process and surpluses for consumer decisions.

TABLE 1. Notation.

Decision variables	
a=	Price per unit during the advance selling period
Q=	Quantity of products that the retailer should order for the selling stage
Parameters/Variables concerning retailer	
c=	Purchase cost per unit of product for the retailer
p=	Market sales price per unit of product in the selling stage
s=	Salvage price per unit of product unsold at the end of selling stage
π=	Retailer's total profit in the advance selling and selling periods
Parameters/Variables related to consumers and markets	
V=	Consumer valuation per unit of product, a random variable with a probability density function $f(v)$, cumulative distribution function $F(v)$, $\bar{F} \equiv 1-F(v)$
v=	Determined valuation for the product at the selling stage
N_j =	Number of potential consumers in phase j, a random variable with a normal distribution, mean μ_j and standard deviation $\sigma_j, j \epsilon (A, S)$
N_1=	Number of consumers who purchase during the advance selling period, a random variable with mean μ_1 and standard deviation σ_1
N_2=	Number of consumers who purchase in the selling stage, a random variable with mean μ_2 and standard deviation σ_2
n_1=	Value realised for the random variable N_1 at the end of the advance selling period
ρ=	Correlation coefficient between the number of consumers (N_A) who arrived in the advance selling period and the number of consumers (N_S) who arrived in the selling period, $\rho \in [-1,1]$
U_A=	Expected utility of a consumer when he/she chooses to buy during the advance selling period
U_{NA}=	Expected utility of a consumer when he/she chooses to wait during the advance selling period

When adopting the advance selling strategy considering random rewards, the retailer provides a free random rewards opportunity to all consumers who buy in advance. The rewards are random cash θ, and the largest reward is $\max\theta = \theta_n$, whereas the smallest reward is $\min\theta = \theta_0 \geq 0$. Evidently, $\max\theta = \theta_n$ is in accordance with the relevant provisions of the antimonopoly law. Retailers pay the winning amount to customers on the spot through an electronic payment platform, such as ApplePay or Alipay. The total winning prizes will be deducted from the retailer's profit as a sales expense. The retailer also announces θ_0, θ_n to consumers at the beginning of the advance selling period.

The quantity of advance selling is marked by N_1 with mean μ_1 and deviation σ_1^2. The quantity in this period equals the quantity of consumers who arrive during the advance selling spot (online advance selling refers to consumers who receive advance selling news). This study assumes that a person buys one product. The sales volume of the retailer in the selling period is marked by N_2 with mean μ_2 and deviation σ_2^2. The retailer updates the market demand forecast on the basis of the advance selling information. Whether random rewards are considered, the retailer can ensure that product inventory meets market demand. The retailer decides on the variables a, Q based on the model.

B. CUSTOMERS

Suppose that consumers are strategic and homogeneous. If the expected utility of buying in advance is greater than the expected utility of waiting, then a consumer buys or chooses to wait. Consumers who participate in the two stages are stochastic.

Under the single advance selling strategy, consumers' valuations of products are uncertain because they cannot see and consume the product during the advance selling period. The valuation of product follows a normal distribution, denoted as V. The cumulative distribution function of V is $F(v)$, and the probability density function is $f(v)$, where $V \sim N(\mu_V, \sigma_V^2)$, $v \in [0, +\infty)$ and $\mu_V > c$. Consumers decide whether to buy in advance or not by comparing the expected utility of buying and waiting. A consumer who chooses to buy in advance takes the risk that the expected utility may be lower than the actual utility. If the consumer chooses to wait, then the risks that the product may not be available in the selling period due to shortages and the product price may be higher than the advance selling price are present. During the selling period, the value of the product can be verified because the consumer receives the product immediately. Consumers who do not buy in advance can decide whether to buy the product at price of p. The number of consumers arriving in the advance selling period is denoted as N_A, where $N_A \sim N(\mu_A, \sigma_A^2)$. The number of consumers who buy in advance is denoted as N_1, where $N_1 \sim N(\mu_1, \sigma_1^2)$. The number of consumers who buy the product during the selling period is expressed as N_S, where $N_S \sim N(\mu_s, \sigma_s^2)$. N_A, N_1 and N_S follow normal distribution, wherein the correlation coefficient of N_A and N_S is ρ.

Compared with the single advance selling strategy, advance selling strategy considering random rewards differs from the fact that consumers who buy in advance have the chance to win prizes from random rewards promotions. The random rewards are marked by θ, which follows an even distribution; the largest prize is θ_n, and the smallest prize is θ_0. The utility of consumers' winning prizes adopts the power function of risk aversion.

IV. ADVANCE SELLING WITHOUT RANDOM REWARDS

If a consumer chooses to buy during the advance selling period, then he/she must pay the selling price of a in advance. The product valuation of V becomes a definite value v during the selling season. When $v \geq a$, the consumer obtains a utility of $v - a$. In contrast, when $v < a$, the consumer loses $a - v$; thus, the expected utility of the consumer is U_A.

$$U_A = \int_a^{+\infty} (v-a) f(v)\, dv - \int_0^a (a-v) f(v)\, dv = \mu_V - a \tag{1}$$

If a consumer chooses to wait during the advance selling period, then he/she will make purchasing decisions until the selling period. Strategic consumers do not make decisions with negative utility. This assumption means that consumers make purchasing decisions only when $v \geq p$. If $v \geq p$, then consumers choose to buy at a utility of $v -$ p; otherwise, consumers will not buy at a utility of 0. The expected utility of consumers' choice to wait is U_{NA}.

$$U_{NA} = E[\max(\mathrm{V} - \mathrm{p}, 0)] = \int_P^{+\infty} (v-p) f(v) dv$$

Evidently, when $U_A \geq U_{NA}$, the consumer decides to buy in advance. Under the condition of $U_A \geq U_{NA}$, we obtain $a \leq \mu_V - \int_P^{+\infty} (v-p) f(v)\, dv$. The optimal advance selling price for the retailer should be a^*.

$$\begin{aligned} a^* &= \mu_V - \int_P^{+\infty} (v-p) f(v)\, dv \\ &= \int_0^P v f(v)\, dv + \int_p^{+\infty} \mathrm{p} f(v)\, dv \end{aligned} \tag{2}$$

Given that $\int_0^P v f(v)\, dv < \int_0^P \mathrm{p} f(v)\, dv$, both sides add $\int_p^{+\infty} \mathrm{p} f(v)\, dv$. Thus, we can obtain:

$$\begin{aligned} a^* &= \int_0^P v f(v)\, dv + \int_p^{+\infty} \mathrm{p} f(v)\, dv \\ &< \int_0^P \mathrm{p} f(v)\, dv + \int_p^{+\infty} \mathrm{p} f(v)\, dv = \mathrm{p}. \end{aligned}$$

The advance selling price of a is lower than the selling price of p, which is consistent with previous research.

Given that the advance selling price a meet $U_A \geq U_{NA}$, strategic consumers who arrive during the advance selling period will choose to buy in advance. At this time, $N_1 = N_A$; thus, $\mu_1 = \mu_A, \sigma_1 = \sigma_A$. The number of consumers entering the market in the selling period is assumed as N_S, where $N_S \sim N(\mu_S, \sigma_S^2)$. When $v \geq p$, consumers will choose to buy. Assume that the proportion of consumers who purchase products is $\bar{F}(p)$, then its quantity is $N_2 = \bar{F}(p) * N_S$, where $N_2 \sim N(\mu_2, \sigma_2^2)$. Thus, $\mu_2 = \bar{F}(p) * \mu_S, \sigma_2 = \bar{F}(p) * \sigma_S$.

The retailer knows the actual advance selling quantity after the advance selling period, namely, $N_1 = n_1$. The retailer can update the demand forecast for the selling period and takes advantage of conditional distribution $N_2^{'}$=($N_2|N_1$= n1). The updated demand in the selling period is marked as $N_2^{'}$,where $N_2^{'} \sim (\mu_2^{\prime}, \sigma_2^{\prime 2})$.

Lemma 1: If X $\sim$ N(μ_1, σ_1^2) and Y $\sim$ N(μ_2, σ_2^2) are two stochastic variables that follow a normal distribution with the correlation coefficient ρ, then the conditional stochastic variable (Y|X) is also as the normal distribution, with the mean $\mu_2 + \rho\left(\frac{\sigma_2}{\sigma_1}\right)(x-\mu_1)$ and the deviation $\sigma_2^2(1-\rho^2)$, namely, (X|Y =x) $\sim N[\mu_2 + \rho\left(\frac{\sigma_2}{\sigma_1}\right)(x-\mu_1), \sigma_2^2\left(1-\rho^2\right)]$ [38].

Based on Lemma 1, $\mu_2^{\prime}$ and $\sigma_2^{\prime}$ can be drawn as the following:

$$\mu_2^{\prime} = \bar{F}(\mathrm{p})\mu_S + \rho\frac{\bar{F}(P)\sigma_S}{\sigma_A}(n_1-\mu_A),$$
$$\sigma_2^{\prime} = \bar{F}(p)\sigma_s\sqrt{1-\rho^2},$$

$\mu_2^{\prime}$ and $\sigma_2^{\prime}$ are the mean and standard deviation of $N_2^{'}$, namely, $N_2^{'} \sim (\bar{F}(\mathrm{p})\mu_S + \rho\frac{\bar{F}(P)\sigma_S}{\sigma_A}(n_1-\mu_A), \bar{F}^2(p)\sigma_s^2 1-\rho^2)$.

The retailer's expected total profit is π, which is as follows.

$$\pi = \mathrm{E}_{N_1}\{aN_1 + max\ E_{N_2^{\prime}}[p\ min\left\{Q-n_1, N_2^{\prime}\right\} + s\max\{Q-n_1-N_2^{\prime}, 0\} - cQ]\}$$

This solution is a traditional newsvendor problem with normally distributed demand [39].

Following the standard solution method, the optimal order quantity Q and the optimal expected profit π are as follows.

$$\begin{aligned} \mathrm{Q}^* &= \mu_2^{\prime} + k\sigma_2^{\prime} + n_1 \\ &= \bar{F}(\mathrm{p})\mu_S + \rho\frac{\bar{F}(P)\sigma_S}{\sigma_A}(n_1-\mu_A) \\ &\quad + k\bar{F}(p)\sigma_s\sqrt{1-\rho^2} + n_1 \end{aligned} \tag{3}$$

$$\begin{aligned} \mathrm{E}\pi^* &= \mathrm{E}_{N_1}\left\{aN_1 + (p-c)\mu_2^{\prime} - (p-s)\varphi(\mathrm{k})\sigma_2^{\prime} - cn_1\right\} \\ &= (\mathrm{a}-\mathrm{c})\mu_A + (p-c)\bar{F}(\mathrm{p})\mu_S \\ &\quad - (\mathrm{p}-\mathrm{s})\varphi(\mathrm{k})\bar{F}(p)\sigma_s\sqrt{1-\rho^2}, \end{aligned} \tag{4}$$

where $k = \phi^{-1}(\frac{p-c}{p-s})$, $\phi(\cdot)$ and $\varphi(\cdot)$ are the distribution and the density functions of the standard normal distribution, respectively.

Substituting a^* into the profit expression, namely, substituting equation (2) into equation (4), the optimal expected profit is:

$$\mathrm{E}\pi^* = \left[\mu_V - \int_P^{+\infty}(v-p)f(v)\,dv - c\right]\mu_A + (p-c)\bar{F}(\mathrm{p})\mu_S - (\mathrm{p}-\mathrm{s})\varphi(\mathrm{k})\bar{F}(p)\sigma_s\sqrt{1-\rho^2} \tag{5}$$

V. ADVANCE SELLING STRAGEGY CONSIDERING RANDOM REWARDS

A. RANDOM REWARDS MECHANISM

The winning amount of the random rewards promotion is stochastic, and whether a consumer can win the prize is uncertain. The incentive design that strategic consumers prefer must satisfy its utility maximisation in the face of similar explicit factors, such as the promoted product, product quality and reward category.

The winning amount is evenly distributed in the interval of $[\theta_0, \theta_n]$, where $\theta \sim U[\theta_0, \theta_n]$ and $0 \leq \theta_0 < \theta_n$. Subsequently, a consumer's winning expectation is $\frac{\theta_n+\theta_0}{2}$, set A = E$\theta = \frac{\theta_n+\theta_0}{2}$. The total budget of retailers' random rewards promotion is A$*n_1$.

The winning prize utility of consumers adopts a power function of risk aversion:

U$(\theta) = \theta^{\lambda}$, where $0< \lambda < 1$ and λ is called risk aversion factor.

Consumers' expected utility of winning prize in random rewards promotion is

$$\begin{aligned} \mathrm{EU}(\theta) &= \int_{\theta_0}^{\theta_n}\frac{1}{\theta_n-\theta_0}\theta^{\lambda}d\theta(0<\lambda<1) \\ &= \frac{\theta_n^{\lambda+1}-\theta_0^{\lambda+1}}{(\theta_n-\theta_0)(\lambda+1)}. \end{aligned} \tag{6}$$

B. ADVANCE SELLING WITH RANDOM REWARDS

Under the advance selling strategy considering random rewards, consumers can obtain a random rewards opportunity for free if they buy a product in advance. The wining prize is random cash that can be encashed by consumers on the spot through an electronic payment platform, such as WeChat or Alipay. Thus, compared with single advance selling strategy, the expected utility of consumers who choose to buy in advance consists of two parts, namely, product valuation utility and random rewards utility. The expected utility of the consumer can be expressed as

$$\begin{aligned} \tilde{U}_A &= \int_a^{+\infty}(v-a)f(v)\,dv - \int_0^a(a-v)f(v)\,dv \\ &\quad + \int_{\theta_0}^{\theta_n}\frac{1}{\theta_n-\theta_0}\theta^{\lambda}d\theta(0<\lambda<1) \\ &= \mu_V - a + \frac{\theta_n^{\lambda+1}-\theta_0^{\lambda+1}}{(\theta_n-\theta_0)(\lambda+1)}. \end{aligned} \tag{7}$$

A retailer sells spot goods during the selling period, during which consumers can access the product; thus, the value of the product can be determined. Strategic consumers will not decide based on negative utility. When $v \geq p$, the consumer will choose to buy. Therefore, the expected utility of the consumer when he/she chooses to wait can be expressed as:

$$\tilde{U}_{NA} = E[\max(\mathrm{V}-\mathrm{p}, 0)] = \int_P^{+\infty}(v-p)f(v)dv \tag{8}$$

The condition for consumers to purchase in advance is $\tilde{U}_A \geq \tilde{U}_{NA}$.

Given that $\tilde{U}_A \geq \tilde{U}_{NA}$, then we obtain $a \leq \mu_V + \frac{\theta_n^{\lambda+1}-\theta_0^{\lambda+1}}{(\theta_n-\theta_0)(\lambda+1)} - \int_P^{+\infty}(v-p)f(v)\,dv$.

The optimal advance selling price, which maximises the retailers' profit can be marked as $\tilde{a}^*$.

$$\tilde{a}^* = \int_0^P vf(v)\,dv + \int_p^{+\infty} pf(v)\,dv + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n - \theta_0}\theta^\lambda d\theta \tag{9}$$

Evidently, $\int_0^P vf(v)\,dv + \int_p^{+\infty} pf(v)\,dv + \frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)} > 0$.

In the ends of $\int_0^P vf(v)\,dv < \int_0^{\mathrm{p}} pf(v)\,dv$, add $\int_p^{+\infty} pf(v)\,dv + \frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)}$, then we can obtain the following equation:

$$\begin{aligned}\int_0^P vf(v)\,dv + \int_p^{+\infty} pf(v)\,dv + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n - \theta_0}\theta^\lambda d\theta \\ < \int_0^P pf(v)\,dv + \int_p^{+\infty} pf(v)\,dv + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n - \theta_0}\theta^\lambda d\theta \\ = \mathrm{p} + \frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)}.\end{aligned}$$

Consequently, we can obtain

$$0 < \tilde{a}^* < \mathrm{p} + \frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)}. \tag{10}$$

When the advance selling price is set as $\tilde{a}^*$, the expected utility of the consumer who buys in advance is greater than that of the consumer who chooses to wait. Therefore, all strategic consumers who enter the market during the advance selling period will choose to buy. If $\mathrm{N_1} = \mathrm{N_A}$, then $(\mu_1 = \mu_{\mathrm{A}}, \sigma_1 = \sigma_{\mathrm{A}})$ can be obtained.

During the selling period, the number of consumers entering the market is $\mathrm{N_S}$, which is $\mathrm{N_S} \sim N(\mu_{\mathrm{S}}, \sigma_{\mathrm{S}}^2)$. When $v \geq p$, a portion of consumers arriving during the selling period will choose to buy. Assuming that the proportion of consumers who purchase the product is $\bar{F}(\mathrm{p})$, we can obtain

$$N_2 = N_S \cdot \bar{F}(p),\ \mu_2 = \bar{F}(\mathrm{p}) * \mu_{\mathrm{S}},\quad \sigma_2 = \bar{F}(\mathrm{p}) * \sigma_{\mathrm{S}}.$$

During the end of the advance selling period, the retailer can obtain the actual quantity of the advance selling $\mathrm{n_1}$, namely, $\mathrm{N_1} = \mathrm{n_1}$. The retailer can update the demand forecast during the selling period through the advance selling data. The updated demand forecast is set as $\mathrm{N_2'}$, where $\mathrm{N_2'} \sim N(\mu_2', \sigma_2^{2'})$.

$$\begin{aligned}\mu_2' &= \bar{F}(\mathrm{p})\,\mu_S + \rho\frac{\bar{F}(P)\,\sigma_S}{\sigma_{\mathrm{A}}}(n_1 - \mu_{\mathrm{A}}) \\ \sigma_2' &= \bar{F}(p)\sigma_s\sqrt{1-\rho^2}\end{aligned}$$

The total amount of random rewards paid to consumers is offset from the retailer's profits as sales expenses. Therefore, the total expected profit under the advance selling strategy considering random rewards can be expressed as below:

$$\begin{aligned}\mathrm{E}\tilde{\pi} = \mathrm{E}_{N_1}\{aN_1 + max E_{N_2'}[-cQ + p\,min\{Q - n_1, N_2'\} \\ + s\,max\{Q - n_1 - N_2', 0\}]\} - A * n_1.\end{aligned}$$

The optimal order quantity under the advance selling strategy considering random rewards is set as $\tilde{Q}^*$. The following results can be obtained based on the newsvendor model.

$$\begin{aligned}\tilde{Q}^* &= \mu_2' + k\sigma_2' + n_1 \\ &= \bar{F}(\mathrm{p})\,\mu_S + \rho\frac{\bar{F}(P)\,\sigma_S}{\sigma_{\mathrm{A}}}(n_1 - \mu_{\mathrm{A}}) \\ &\quad + k\bar{F}(p)\,\sigma_s\sqrt{1-\rho^2} + n_1.\end{aligned} \tag{11}$$

The optimal expected total profit can be expressed as follows.

$$\begin{aligned}\mathrm{E}\tilde{\pi}^* &= \mathrm{E}_{N_1}\{aN_1 + (p-c)\,\mu_2' - (p-s)\,\varphi(\mathrm{k})\,\sigma_2' \\ &\quad - cn_1\} - A * n_1 \\ &= (\mathrm{a}-\mathrm{c})\,\mu_A + (p-c)\,\bar{F}(\mathrm{p})\,\mu_S \\ &\quad - (\mathrm{p}-\mathrm{s})\,\varphi(\mathrm{k})\,\bar{F}(p)\,\sigma_s\sqrt{1-\rho^2} - A * n_1,\end{aligned} \tag{12}$$

where $k = \phi^{-1}(\frac{p-c}{p-s})$, $\phi(\cdot)$ and $\varphi(\cdot)$ are the distribution and the density functions of the standard normal distribution, respectively.

Substituting the value of the optimal advance selling price $\tilde{a}^*$ into the profit expression, namely, substituting equation (9) into equation (12), we mark the following expression for optimal expected profit as $\tilde{\pi}^*$.

$$\begin{aligned}&\mathrm{E}\tilde{\pi}^* \\ &= \left[\mu_V + \frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)} - \int_P^{+\infty}(v-p)f(v)\,dv - c\right] \\ &\quad \times \mu_A + (p-c)\,\bar{F}(\mathrm{p})\,\mu_S - (\mathrm{p}-\mathrm{s})\,\varphi(\mathrm{k})\,\bar{F}(p)\,\sigma_s \\ &\quad \times\sqrt{1-\rho^2} - A * n_1\end{aligned} \tag{13}$$

VI. COMPARISON AND ANALYSIS RESULTS OF THE TWO STRATEGIES

The following interesting results are obtained by comparing and analysing the relevant variables of the two strategies.

Proposition 1: When the online advance selling strategy considering random rewards is adopted, the expected utility of consumers during the advance selling period is greater than that of consumers when a single advance selling strategy is adopted, which can be denoted as $\tilde{U}_A > U_A$.

Proposition 2: If the advance selling discount amount under the single advance selling strategy is larger than the consumer's winning expectation, then under the advance selling strategy considering random rewards the online advance selling price is smaller than the sales price; On the contrary, the online advance selling price is greater than the sales price, which can be expressed as $\tilde{a}^* - \mathrm{p} = (a^* - p) + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n - \theta_0}\theta^\lambda d\theta$.

Proposition 3: When the advance selling strategy considering random rewards is adopted, the advance selling price set by the retailer is higher than that when a single advance selling strategy is adopted, which can be represented as $\tilde{a}^* > a^*$.

Proposition 4: A threshold $\bar{\mu}_A = \frac{An_1(\theta_n - \theta_0)(\lambda+1)}{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}$ exists. When $\mu_A > \bar{\mu}_A$, retailers should adopt advance selling

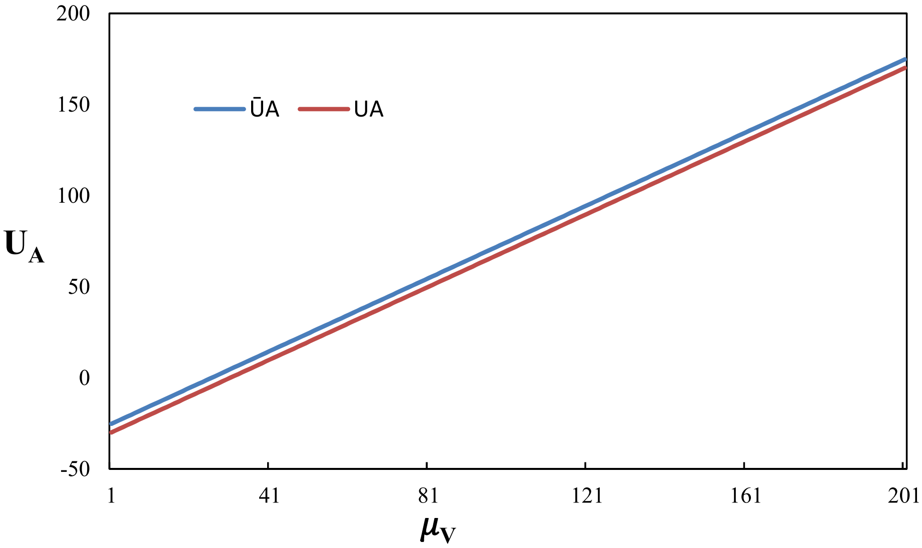

FIGURE 2. Expected utility of a consumer changes as μ_A changes.

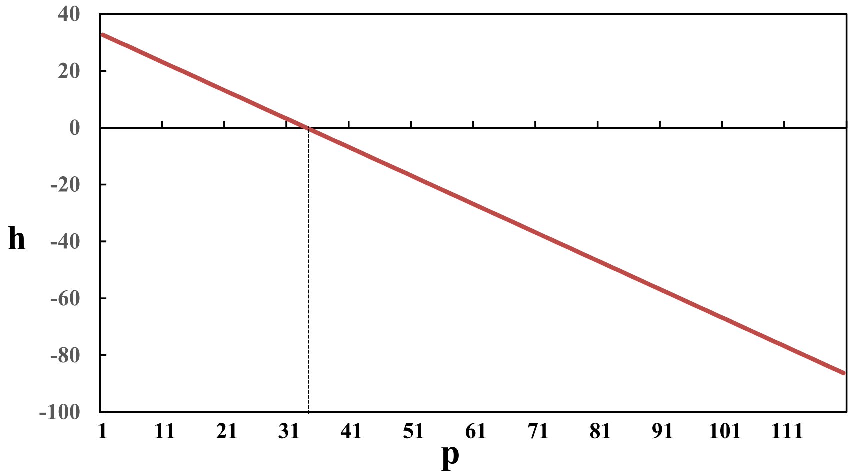

FIGURE 3. Discount under the strategy considering random rewards changes with *p*.

strategy considering random rewards, whereas when $\mu_A < \bar{\mu}_A$, the retailers should adopt a single advance selling strategy.

Proposition 5: A threshold $\bar{A} = \frac{u_A}{n_1}\frac{\theta_n^{\lambda+1}-\theta_0^{\lambda+1}}{(\theta_n-\theta_0)(\lambda+1)}$ exists. When A > $\bar{A}$, retailers should adopt a single advance selling strategy, whereas when A <$\bar{A}$, retailers should adopt advance selling strategy considering random rewards.

VII. NUMERICAL ANLYSIS

In this section, we examine the results of two different strategies. To do so, numerical tests are constructed to illustrate retailer will choose a strategy that is optimal.

We use the following values: $\mu_V = 30$, $p = 35$, $c = 17$, s = 3, a^*= 30, θ_n= 30, θ_0= 0.5, $\lambda = 0.5$,$\sigma_s = 1.5$, $A = 15$, $U_{AN} = 2$, $\mu_s = 37$, $\mu_A = 55$, $n_1 = 50$, $\rho = 0.4$, $\bar{F}(p) = 0.85$ and φ (k) = 0.0032. $f(v)$ is the probability density function of V, where $k = \phi^{-1}(\frac{p-c}{p-s})$, $\phi(\cdot)$ and $\varphi(\cdot)$ are the distribution and the density functions of the standard normal distribution, respectively.

(1) The consumer's expected utility of two strategies as the change of product valuation is observed. Figure 2 illustrates that the real line represents the $\tilde{U}_A$, and the dotted line represents U_A.

Keeping the other parameters constant and allowing μ_V to change from 1 to 200, then we can observe in Figure 2 that line $\tilde{U}_A$ is constantly above line U_A. Evidently, $\tilde{U}_A$ is constantly greater than U_A.

Figure 2 indicates that $\tilde{U}_A$ and U_A likewise increase with the increase of μ_V. Thus, an increase in product valuation reflects an increase of expected utility of a consumer who buys in advance. In addition, we can observe that line $\tilde{U}_A$ is parallel to line U_A, which shows the similar marginal utility under the two strategies. Only one constant difference between $\tilde{U}_A$ and U_A is evident, which is consistent with proposition 1 in this paper.

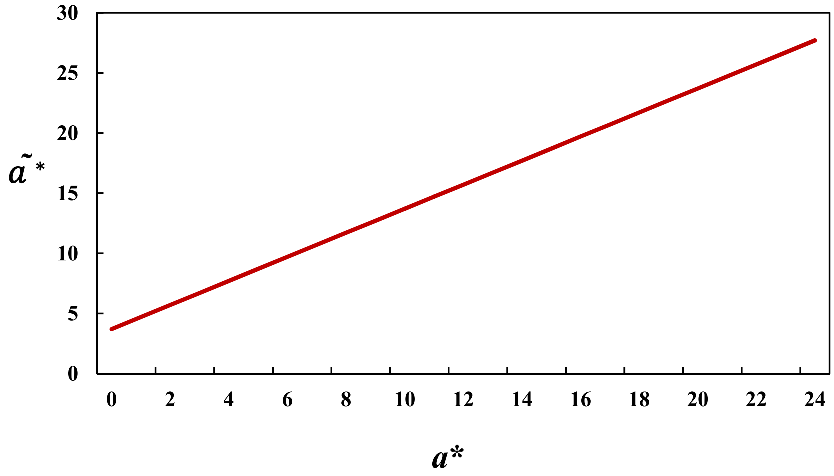

FIGURE 4. Advance selling price under the strategy of considering random rewards changes with a^*.

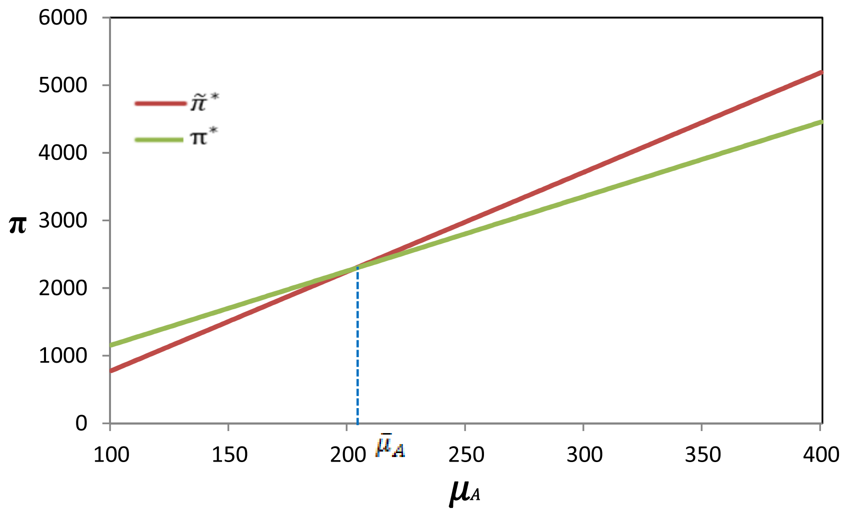

FIGURE 5. Profit changes with μ_A.

(2) We let h $=\tilde{a}^*-$p $= (a^* - p) + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta$. Figure 3 shows that curve h is changed as p changes, when the rest of the parameters remain unchanged.

Let p change gradually from 1 to 111. Figure 3 shows that h decreases with the increase of p. If $\int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta$ and a^* are constant when $p < 34$, then $h > 0$. Therefore, retailers do not take discount sales during the advance selling period under the advance selling strategy considering random rewards. If $p > 34$, then $h < 0$. Therefore, retailers take discount sales during the advance selling period under the advance selling strategy considering random rewards, and $h < 0$ indicates that $-(a^* - p) > \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta$. The numerical analysis shows that the relationship between the amount of discount and the size of the winning prize expectation under single advance selling strategy determines whether the retailer takes discount sales under the advance selling strategy considering random rewards, thereby validating proposition 2.

(3) Let $\tilde{a}^* = a^* + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta (\theta > 0, 0 < \lambda < 1)$. Figure 4 illustrates that curve $\tilde{a}^*$ changes as a^* changes with the rest of the parameters unchanged.

Figure 4 shows that the curve $\tilde{a}^*$ increases with the increase of a^*. The slope of curve $\tilde{a}^*$ is positive, thereby indicating that $\tilde{a}^*$ is always greater than a^*. That is, $\tilde{a}^* > a^*$. This finding is consistent with proposition 3.

(4) The change of profit as product valuation changes under the two strategies is analysed numerically. When other parameter values remain unchanged, let $\mu_A$change gradually from 100 to 400. The solid line represents $\tilde{\pi}^*$, whereas the dotted line represents π^*.

Figure 5 illustrates that when $\mu_A < 202$, then $\tilde{\pi}^* < \pi^*$; when $\mu_A > 202$, then $\tilde{\pi}^* > \pi^*$. Notably, 202 is a threshold. Figure 5 shows that a threshold $\bar{\mu}_A = \frac{An_1(\theta_n-\theta_0)(\lambda+1)}{\theta_n^{\lambda+1}-\theta_0^{\lambda+1}}$ exists. When $\mu_A > \bar{\mu}_A$, retailers should adopt an advance selling strategy considering random rewards, whereas when

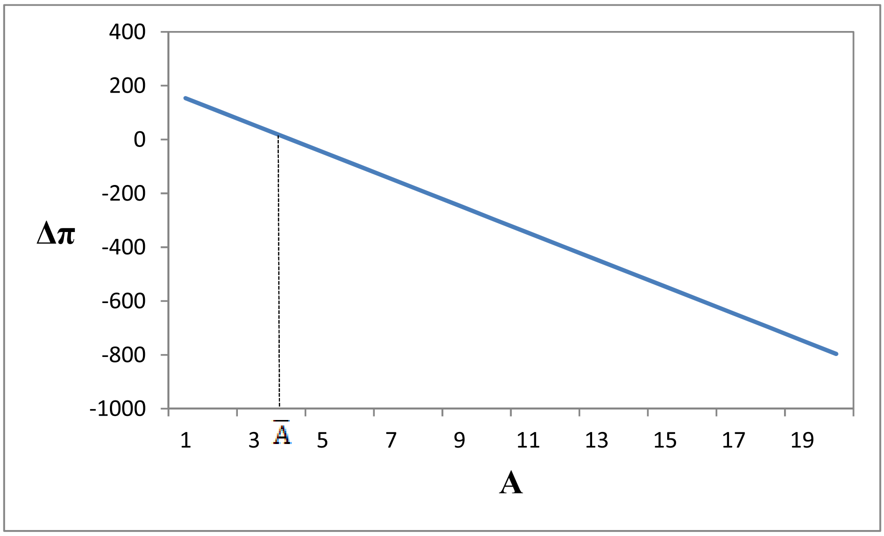

FIGURE 6. Profit changes with *A*.

$\mu_A < \bar{\mu}_A$, retailers should adopt a single advance selling strategy. This finding is consistent with proposition 4.

(5) We keep the other parameter values constant and observe the change of curve $\Delta\pi$ as A changes from 1 to 20, according to $\Delta\pi = \tilde{\pi}^* - \pi^* = \mu_A \frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)} - A \cdot n_1$. Figure 6 demonstrates this profit change.

Let A change gradually from 1 to 20. Figure 6 illustrates that $\Delta\pi$ decreases with the increase of A, and the slope of curve $\Delta\pi$ is negative. When $A < 4.2$, then $\Delta\pi > 0$. Thus, the profit under the advance selling strategy considering random rewards is greater than that under single advance selling strategy. When $A > 4.2$, then $\Delta\pi < 0$. Thus, the profit under the advance selling strategy considering random rewards is less than that under the single advance selling strategy. Numerical analysis shows that threshold $\bar{A}$ exists. When A $> \bar{A}$, retailers should adopt a single advance selling strategy, whereas when A $< \bar{A}$, retailers should adopt an advance selling strategy considering random rewards. This finding is consistent with proposition 5.

VIII. CONCLUSION AND FUTURE WORK

Advance selling can help retailers reduce the uncertainty of demand, while online advance selling with random rewards, such as sweepstakes and WeChat's 'random red envelope', further cater to the utility maximization pursuit of strategic consumers and are increasingly favoured by online retailers. Therefore, online advance selling with random rewards strategy is more popular than single advance selling strategy. With the attempt to set up the random rewards mechanism as realistic as possible by adopting power function of risk aversion for the utility of consumers winning prize, we establish the model of advance selling strategy considering random rewards based on the single advance selling model, subsequently solving the variables under the two strategy models. This study aims to compare and analyse the two strategies to provide valuable managerial insights for retailers who practice advance selling strategy considering random rewards.

We find that thresholds $\bar{\mu}_A = \frac{An_1(\theta_n - \theta_0)(\lambda+1)}{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}$ and $\bar{A} = \frac{u_A}{n_1}\frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)}$ exist. When $\mu_A > \bar{\mu}_A$ and A $< \bar{A}$, retailers should adopt advance selling strategy considering random rewards. When $\mu_A < \bar{\mu}_A$ and A $> \bar{A}$, retailers should adopt a single advance selling strategy.

Results comparison and analysis reveal that when the advance selling strategy considering random rewards is adopted, the advance selling price set by the retailer is higher than that when a single advance selling strategy is adopted. This result is consistent with the expected utility of consumers. Moreover, the relationship between the amount of discount and the size of the winning prize expectation under single advance selling strategy determines whether a retailer takes discount sales under the advance selling strategy considering random rewards.

Several research issues deserve further examine, such as setting the winning amount and advance selling price as decision variables. Furthermore, future research should examine the complex relationships among retailers, including multiple retailers who compete with one another and provide different random rewards.

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APPENDIX: PROOFS A

PROOF OF PROPOSITION 1

Given that $U_A = \mu_V - a$ and $\tilde{U}_A = \mu_V - a + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n - \theta_0}\theta^{\lambda} d\theta$, we have $\tilde{U}_A - U_A = \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n - \theta_0}\theta^{\lambda} d\theta$. Given that $\int_{\theta_0}^{\theta_n} \frac{1}{\theta_n - \theta_0}\theta^{\lambda} d\theta = \frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)} > 0 (\theta > 0, 0 < \lambda < 1)$; thus, $\tilde{U}_A > U_A$.

APPENDIX B
PROOF OF PROPOSITION 2

From Eq. (9) $\tilde{a}^* = \int_0^P vf(v)\,dv + \int_p^{+\infty} pf(v)\,dv + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta$ and p $= \int_0^P pf(v)\,dv + \int_p^{+\infty} pf(v)\,dv$, then we obtain $\tilde{a}^* - p = \int_0^P vf(v)\,dv + \int_p^{+\infty} pf(v)\,dv + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta - \int_0^P pf(v)\,dv - \int_p^{+\infty} pf(v)\,dv$,

where

$$\begin{aligned}\tilde{a}^* - p &= \int_0^P vf(v)\,dv - \int_0^p pf(v)\,dv + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta \\ &= \int_0^P (v-p)f(v)\,dv + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta \\ &= (a^*-p) + \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta.\end{aligned}$$

When $p - a^* = \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta$, we have $\tilde{a}^* = p$.

When $p - a^* < \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta$, we have $\tilde{a}^* > p$.

When $p - a^* > \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta$, we have $\tilde{a}^* < p$.

APPENDIX C
PROOF OF PROPOSITION 3

Based on Eq. (2) and Eq. (9) we obtain $\tilde{a}^* - a^* = \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta = \frac{\theta_n^{\lambda+1}-\theta_0^{\lambda+1}}{(\theta_n-\theta_0)(\lambda+1)}$, where $\theta > 0, 0 \le \theta_0 \le \theta_n, 0 < \lambda < 1$.

Given that we have $\theta_n^{\lambda+1} - \theta_0^{\lambda+1} > 0$, $(\theta_n - \theta_0)(\lambda+1) \ge 0$, and $\frac{\theta_n^{\lambda+1}-\theta_0^{\lambda+1}}{(\theta_n-\theta_0)(\lambda+1)} > 0$; hence, we have $\tilde{a}^* > a^*$.

APPENDIX D
PROOF OF PROPOSITION 4

Set $\Delta\pi$ as the difference in profit between the two strategies, then

$$\Delta\pi = \tilde{\pi}^* - \pi^* = \mu_A \frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)} - A \cdot n_1.$$

Let $\bar{\mu}_A = \frac{An_1(\theta_n-\theta_0)(\lambda+1)}{\theta_n^{\lambda+1}-\theta_0^{\lambda+1}}$.

When $\mu_A > \bar{\mu}_A$, then $\Delta\pi > 0$. Therefore, retailers should adopt advance selling strategy considering random rewards. When $\mu_A < \bar{\mu}_A$, then $\Delta\pi < 0$. Therefore, retailers should adopt a single advance selling strategy.

APPENDIX E
PROOF OF PROPOSITION 5

Set $\Delta\pi$ as the difference in profit between the two strategies, then

$$\Delta\pi = \tilde{\pi}^* - \pi^* = \mu_A \frac{\theta_n^{\lambda+1} - \theta_0^{\lambda+1}}{(\theta_n - \theta_0)(\lambda+1)} - A \cdot n_1$$

Let $\bar{A} = \frac{u_A}{n_1} \int_{\theta_0}^{\theta_n} \frac{1}{\theta_n-\theta_0}\theta^\lambda d\theta$.

When A $<\bar{A}$, then $\Delta\pi > 0$. Therefore, retailers should adopt advance selling strategy considering random rewards.

When A $>\bar{A}$, then $\Delta\pi < 0$. Therefore, retailers should adopt a single advance selling strategy.

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