IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received August 13, 2020, accepted August 28, 2020, date of publication September 3, 2020, date of current version September 16, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3021471

Performance Analysis of Networked Systems With Two-Channel Noise and Bandwidth Constraints

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This paper was partially supported by the National Natural Science Foundation of China under Grants 61971181 and 61602163, the Youth Technology Innovation Team of Hubei Province under Grant T201710, the Natural Science Foundation of Hubei Province under Grants 2017CFA034 and 2019CFB226.

ABSTRACT In this paper, the performance of networked systems with two-channel noise and bandwidth constraints is investigated. The given plant with non-minimum phase zero and unstable pole are considered in the system. Considering the white noise constraint in two channels (forward and feedback channels) and the bandwidth constraint in the feedback channel, the expression of performance limitations is obtained by using spectral decomposition technique and selecting the optimal single parameter. The obtained results demonstrate that the performance of networked systems depends on the intrinsic properties of the given plant (such as non-minimum phase zeros and unstable poles), and the network parameters (such as white noise, bandwidth and codec). Furthermore, the results also show the impacts of two-channel noise and the bandwidth constraints on the performance of networked systems. Finally, a classical example is presented to illustrate the theoretical results.

INDEX TERMS Networked systems, performance limitation, white noise, bandwidth constraint.

I. INTRODUCTION

With the rapid development of the Internet and communication technologies, it has become difficult for the traditional control systems to meet the requirements of the users, and the network has inextricably linked with the users. Therefore, the use of networks for traditional control systems is a development trend. The networked systems [1]–[7] have emerged as promising solution with advantages of strong flexibility, low cost, simple installation and maintenance, light weight, and low power consumption. The signal is converted between analog signal and digital signal in the network by encode-decode, so the design of encode-decode will inevitably affect the performance of the networked systems. However, due to the introduction of the network, constraints such as delay and noise are generated, which seriously affect the stability and performance of the networked systems.

In recent years, extensive research has been conducted on the stability analysis of networked systems with communication constraints, such as quantization, delay, bandwidth, and packet dropout. The stability of networked systems based on communication delay and bandwidth constraints has been studied in [8]. The stability of networked systems based on packet dropout and quantification has been explored in [9]. The stability of networked systems under the conditions of transmission delay and data packet dropout has been studied in [10]. The stability analysis of networked systems with the conditions of random packet dropout and network delay changes has been performed in [11].

However, studying only the stability of networked systems is not enough, meanwhile these also have many problems in terms of performance. Currently, several scholars have studied the performance of networked systems [12]–[20]. The optimal performance of networked control systems with the packet dropouts and channel noise has been explored in [21]. The optimal tracking performance of networked systems with bandwidth and network-induced delay constraints has been studied in [22]. The performance of networked systems with bandwidth constraints has been explored in [23]. The effect of the channel noise on the tracking performance of the networked system has been studied in [24]. The performance limitation problem in the multi-variable discrete networked system has been investigated in [25]. Recently, some scholars have also studied the performance of discrete-time systems with limited signal-to-noise ratio (SNR) [26] and LTI systems

The associate editor coordinating the review of this manuscript and approving it for publication was Liang Hu.

with limited power [27]. Most of the above-mentioned studies have considered the effect of one-way channel, while the problems of considering the effects of two-channel noise and bandwidth constraints have rarely been discussed. However, in networked systems, constraints such as delay, codec, white noise and bandwidth often appear simultaneously. These constraints will cause the performance of the system to decline or even make the system unstable. Therefore, it is essential to study the performance of networked systems with the constraints of two-channel noise and bandwidth.

In this paper, the performance of networked systems is analyzed based on two-channel noise and bandwidth constraints. Firstly, a networked system model based on two-channel noise and bandwidth constraints is proposed, that is, white noise exist in both the forward and the feedback channels, and bandwidth constraint in the feedback channel. Therefore, one contribution of this paper is to combine the performance of networked control systems under new constraints such as bandwidth, and obtain the expression of network system performance limitation through factor decomposition and spectral decomposition technology, which is the main difference from [28]. On the other hand, we quantitatively reveal how two-channel noise and bandwidth constraints affect the performance limitations of network systems. The above results have certain reference value for the optimization design of network communication control system and communication channel design.

The remainder of this paper is organized as follows. In Section II, the problem statement is briefly introduced. A theorem is proposed to characterize the performance limitation with two-channel noise and bandwidth constraints in Section III. The simulations are provided in section IV, and the conclusions are presented in Section V.

II. PROBLEM STATEMENT

A networked system is established as depicted in Fig.1, where the objective is to investigate the performance limitation of the system with two-channel noise and bandwidth constraints.

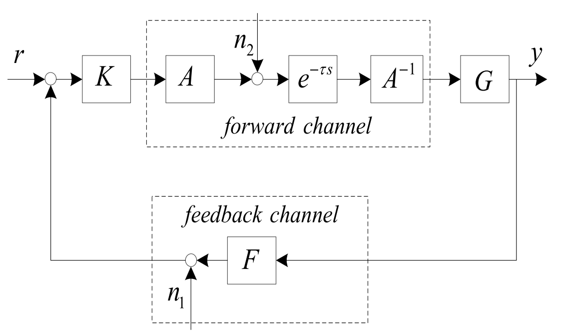

FIGURE 1. The networked system with two-channel noise constraints.

In Fig.1, r is the input signal, G and K represent the controlled plant and one-parameter compensator, whose transfer functions are $G(s)$ and $K(s)$, respectively. n_1 and n_2 represent the additive white Gaussian noise in the feedback and the forward channels, respectively. φ_2^2 and φ_3^2 denote the power spectral densities of n_1 and n_2, respectively. The reference signal r is considered a random signal, and the variance of the random process is φ_1^2.

According to Fig.1, following can be obtained:

$$y = e^{-\tau s}KGr - e^{-\tau s}KFGy - e^{-\tau s}KGn_1 + n_2e^{-\tau s}GA^{-1} \quad (1)$$

Furthermore:

$$y = \frac{e^{-\tau s}KGr}{1+e^{-\tau s}KFG} - \frac{e^{-\tau s}n_1KG}{1+e^{-\tau s}KFG} + \frac{e^{-\tau s}n_2GA^{-1}}{1+e^{-\tau s}KFG}$$

The tracking error of the system is:

$$e = r - y = \lambda_1 r + \lambda_2 n_1 - \lambda_3 n_2 \quad (2)$$

where

$$\lambda_1 = \frac{1}{1+e^{-\tau s}KFG}, \lambda_2 = \frac{e^{-\tau s}KG}{1+e^{-\tau s}KFG}, \lambda_3 = \frac{e^{-\tau s}GA^{-1}}{1+e^{-\tau s}KFG} \quad (3)$$

Adopting the method of coprime factorizations, the transfer function G can be expressed as:

$$FG = \frac{N}{M} \quad (4)$$

where $N, M \in \mathbb{R}\mathcal{H}_\infty$.

For some $X, Y \in \mathcal{H}_\infty$, and satisfy [29]:

$$MX + e^{-\tau s}NY = 1 \quad (5)$$

It is well known that the compensator $\mathcal{K}$ that makes the system stable can be expressed by Youla parameters [22]:

$$\mathcal{K} = \left\{K : K = \left(X - e^{-\tau s}NR\right)^{-1}(Y + MR), R \in \mathcal{H}_\infty\right\} \quad (6)$$

In the above set of compensator $\mathcal{K}$, N and M are non-minimum phase transfer functions, which can be decomposed into:

$$N = L_z N_m, M = B_z M_m \quad (7)$$

where N_m and M_m are the non-minimum phase parts; L_z and B_z are all-pass factors that include all non-minimum phase zeros $z_i \in C_+, i = 1, \ldots, n$ and all unstable poles $p_j \in C_+, j = 1, \ldots, m$ of the given plant, respectively. L_z and B_z can be expressed as:

$$L_z(s) = \prod_{i=1}^{n} \frac{s - z_i}{s + \bar{z}_i}, \quad B_z(s) = \prod_{j=1}^{m} \frac{s - p_j}{s + \bar{p}_j}$$

III. PERFORMANCE LIMITATIONS WITH TWO-CHANNEL NOISE AND BANDWIDTH CONSTRAINTS

The performance limitation of networked system is studied by measuring the minimum tracking error, which is defined as:

$$J^* = \inf_{K \in \mathcal{K}} J \quad (8)$$

where $J =: \varepsilon\left\{\|e(s)\|_2^2\right\} = \varepsilon\left\{\|y(s)-r(s)\|_2^2\right\}$, it is the tracking performance index.

It is assumed that there is no mutual interference between the input signal r and Gaussian white noise n_1 with n_2, then the tracking error J of the system can be expressed as:

$$J = \|\lambda_1\|_2^2\varphi_1^2 + \|\lambda_2\|_2^2\varphi_2^2 + \|\lambda_3\|_2^2\varphi_3^2 \tag{9}$$

According to Eqs. (2),(3),(4),(8) and (9), following can be obtained:

$$\begin{aligned} J^* = \inf_{R\in\mathcal{H}_\infty} \Big\{ & \left\|(X - e^{-\tau s}NR)M\right\|_2^2 \varphi_1^2 \\ & + \left\|(Y+MR)Ne^{-\tau s}F^{-1}\right\|_2^2 \varphi_2^2 \\ & + \left\|e^{-\tau s}N(X-e^{-\tau s}NR)F^{-1}A^{-1}\right\|_2^2 \varphi_3^2 \Big\} \end{aligned} \tag{10}$$

Theorem 1: Assuming that the networked system shown in Fig.1, has multiple non-minimum phase zeros $z_i \in C_+, i = 1, \ldots, n$, and multiple unstable poles $p_j \in C_+$, $j = 1, \ldots, m$. The performance limitation of networked system with two-channel noise and bandwidth constraints can be written as follow:

$$\begin{aligned} J^* > & \sum_{i=1}^{n} 2\,\mathrm{Re}(z_i)\varphi_1^2 \\ & + \sum_{j,i\in N} \frac{4\,\mathrm{Re}(p_j)\,\mathrm{Re}(p_i)}{(p_j+p_i)p_j p_i} \frac{(1-L_z^{-1}(p_i))(1-L_z^{-1}(p_j))^H}{b_i\bar{b}_j e^{-\tau(p_i+\bar{p}_j)}}\varphi_1^2 \\ & + \sum_{j,i\in N} \frac{4\,\mathrm{Re}(p_j)\,\mathrm{Re}(p_i)}{(p_j+p_i)p_j p_i} \frac{L_z^{-1}(p_j)L_z^{-1}(p_i)F^{-1}(z_i)}{b_i\bar{b}_j e^{-\tau(p_i+\bar{p}_j)}}\varphi_2^2 \\ & + \sum_{j,i\in N} \frac{4\,\mathrm{Re}(z_i)\,\mathrm{Re}(z_i)}{\bar{z}_j + z_i} \frac{N_m(z_i)N_m(z_j)F^{-1}(z_i)A^{-1}(z_i)}{M(z_i)M(z_j)l_i\bar{l}_j e^{-\tau(z_i+z_j)}}\varphi_3^2 \end{aligned}$$

where $b_j(s) = \prod_{\substack{i\in N\\ i\neq j}}^{m} \frac{p_i - p_j}{\bar{p}_i + p_j}$, $l_j(s) = \prod_{\substack{j\in N\\ i\neq j}}^{m} \frac{z_i - z_j}{\bar{z}_i + z_j}$.

Proof: See Appendix.

Comment 1: It can be obtained from Theorem 1 that networked control systems are affected by inherent characteristics such as unstable poles, non-minimum phase zeros, and channel constraints such as two-channel noise, bandwidth, delay, and encode-decode. Therefore, communication constraints are also an important factor in performance.

IV. ILLUSTRATIVE SIMULATION

The influence of different conditions on the performance of the networked system is analyzed, with an example. The given object model is considered as follows:

$$G(s) = \frac{s-k}{s(s-2)(s+3)} \tag{11}$$

In this model, it can be seen that the non-minimum phase zero and the unstable pole are located at $z_i = k$ and $p_j = 2$, respectively.

Firstly, the influence of bandwidth on the performance of networked system is analyzed. A first order low-pass filter is applied to establish the model of bandwidth $F(s)$, with given three different cut-off frequency values of 10 and 1, respectively, then:

$$F_1 = \frac{10}{p+10}, \quad F_2 = \frac{1}{p+1}$$

It is assumed that the values of several correlated quantities are: $\varphi_1^2 = \varphi_2^2 = \varphi_3^2 = 1$, and $\tau = 0.2$. According to Theorem 1, $J_1{}^*$ and $J_2{}^*$, which represent the performance limitation with bandwidth constraints of F_1 and F_2, respectively, can be expressed as:

$$\begin{aligned} J_1{}^* &= 2k + e^{0.8}\left(\frac{-2k}{2-k}\right)^2 + 1.2e^{0.8}\left(\frac{2+k}{2-k}\right)^2 \\ &\quad + 2.4ke^{0.4k} \\ J_2{}^* &= 2k + e^{0.8}\left(\frac{-2k}{2-k}\right)^2 + 3e^{0.8}\left(\frac{2+k}{2-k}\right)^2 \\ &\quad + 6ke^{0.4k} \end{aligned} \tag{12}$$

The performance limitations of networked system based on the influence of different bandwidths are shown in Fig.2.

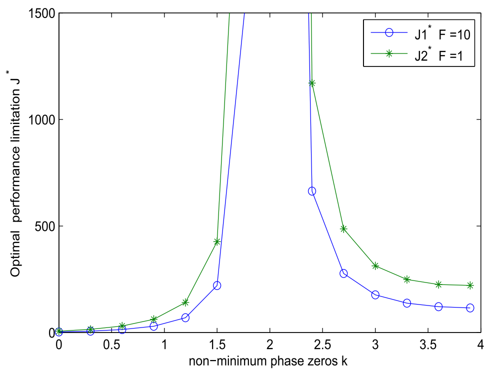

FIGURE 2. Performance limitations with different bandwidth constraints.

It can be seen from Fig. 2 that, the different values of bandwidth affect the performance of the networked system. The larger the bandwidth value, the better the performance and vice versa. It can also be seen that when the unstable pole is close to the non-minimum phase zero, the performance tends to infinity.

Then, based on the bandwidth constraints, the influence of codec on the performance of the networked system is analyzed. The encoding transfer function is given as:

$$A(s) = \frac{s+2}{s-5}$$

It is assumed that the cut-off frequency value of the model of bandwidth $F(s)$ is 10. Based on Eq. (12), we can get the performance limitation with codec, $J_3{}^*$ can be obtained as:

$$J_3^* = 2k + e^{0.8}\left(\frac{-2k}{2-k}\right)^2 + 1.2e^{0.8}\left(\frac{2+k}{2-k}\right)^2 + 2.4ke^{0.4k}\left(\frac{k-5}{k+2}\right)$$

Under the same bandwidth constraint, the performance limitations with and without codec are shown in Fig.3.

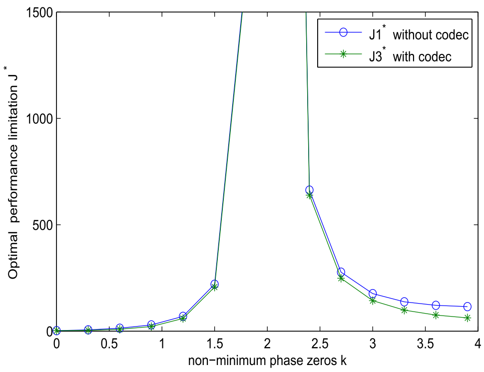

FIGURE 3. **Performance limitations with and without codec.**

It can be seen from Fig. 3 that the codec improves the performance of the networked system.

It is also assumed that the value of bandwidth $F(s)$ is 10, comparing with [28], which considers the impact of time-delay and codec constraints. As shown in Fig. 5, We can get the constraints of communication parameters increase, the performance of the system decreases.

Finally, the influence of noise in the two channels on the performance of the networked system is analyzed. Based on Eq. (11), assuming the values of several correlated quantities are: $k = 1, \varphi_1^2 = 1$, and $\tau = 0.2$.

According to Theorem 1, $J_4{}^*$, representing the performance limitation with noise in the two channels, can be expressed as:

$$J_4{}^* = 2 + 4e^{0.8} + 9e^{0.8}\varphi_2^2 + 2e^{0.4}\varphi_3^2$$

The performance limitation of networked system based on the influence of two-channel noise is shown in Fig.4.

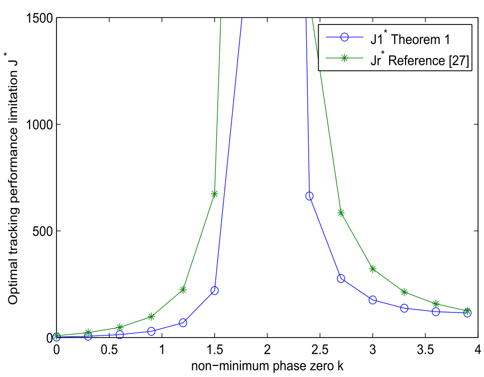

FIGURE 4. **Performance limitations with different communication parameters.**

It can be seen from Fig.5 that the noise of the forward and the feedback channels affect the performance limitations of the networked system, the larger the two-channel noise, the worse the performance.

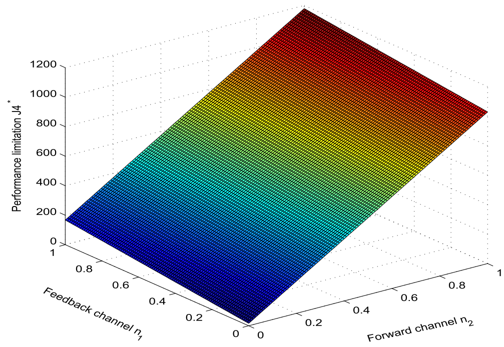

FIGURE 5. **Performance limitations with two-channel noise.**

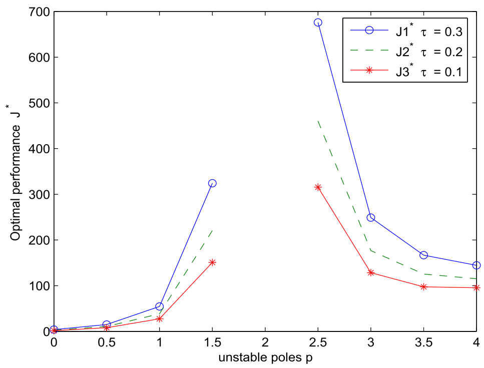

FIGURE 6. **Performance limitations with different delay values.**

When $\tau = 0.1$, $\tau = 0.2$ and $\tau = 0.3$ are respectively taken as different values. According to Theorem 1, we can get the influence of different delays on the performance limitations of the system. It can be seen from Fig. 6 that the performance limitations is worse when the delay is larger; when the delay tends to infinity, the system is unstable and there is no corresponding controller.

V. CONCLUSION

The performance of the networked systems with two-channel noise and bandwidth constraints is discussed in this paper. Specifically, white noise and codec in the forward channel; and white noise and bandwidth constraints in the feedback channel are considered. The H_2 norm and the spectral decomposition techniques are used to obtain the performance limitation value, which is mainly affected by the network parameters (such as white noise, bandwidth and codec), when the intrinsic properties of the given plant (such as non-minimum phase zeros and unstable poles) are determined. The obtained results quantitatively reveal the relationship between the system performance and the network parameters with intrinsic properties.

In the future, we will conduct in-depth research on how the tracking performance of the system changes according to the different locations of the encode-decode (forward channel or feedback channel). The work presented in this paper, can provide theoretical guidance for the research of the performance limitations of networked systems. Moreover, some parameter estimation approaches [30]–[35] can be combined with the method proposed in this article for network time-delay systems [36]–[42] with unknown parameters.

APPENDIX A
PROOF OF THEOREM 1

According to Eqs. (6) and (10), following can be obtained:

$$J^* = \inf_{R\in\mathcal{H}_\infty}\left\{\left\|(X-e^{-\tau s}NR)M\right\|_2^2\varphi_1^2 + \left\|(Y+MR)Ne^{-\tau s}F^{-1}\right\|_2^2\varphi_2^2 + \left\|(X-e^{-\tau s}NR)Ne^{-\tau s}F^{-1}A^{-1}\right\|_2^2\varphi_3^2\right\} \quad (13)$$

Combining Eq. (7), we can get:

$$J^* = \inf_{R\in\mathcal{H}_\infty}\left\{\left\|(XM-e^{-\tau s}L_zN_mRB_zM_m)\right\|_2^2\varphi_1^2 + \left\|\frac{YL_zN_m+L_zN_mRB_zM_m}{e^{\tau s}F}\right\|_2^2\varphi_2^2 + \left\|\frac{L_zN_mX-e^{-\tau s}L_zN_mL_zN_mR}{e^{\tau s}FA}\right\|_2^2\varphi_3^2\right\}$$

Because L_z is an all-pass factor, then:

$$J^* = \inf_{R\in\mathcal{H}_\infty}\left\{\left\|(L_z^{-1}MX-e^{-\tau s}N_mRB_zM_m)\right\|_2^2\varphi_1^2 + \left\|\frac{YN_m+N_mRB_mM_m}{e^{\tau s}F}\right\|_2^2\varphi_2^2 + \left\|\frac{L_z^{-1}N_mX-e^{-\tau s}N_mN_mR}{e^{\tau s}FA}\right\|_2^2\varphi_3^2\right\} \quad (14)$$

According to Eqs. (5) and (7), a simple calculation will provide,

$$L_z^{-1}MX - L_z^{-1} = -e^{-\tau s}N_mY \quad (15)$$

Substituting Eq. (15) into (14),

$$\begin{aligned}J^* &= \inf_{R\in\mathcal{H}_\infty}\left\{\left\|(L_z^{-1}-\frac{N_nY}{e^{\tau s}}-\frac{N_mRB_zM_m}{e^{\tau s}})\right\|_2^2\varphi_1^2 + \left\|\frac{YN_m+N_mRB_zM_m}{e^{\tau s}F}\right\|_2^2\varphi_2^2 + \left\|(\frac{L_z^{-1}N_mX-e^{-\tau s}N_mN_mR}{e^{\tau s}FA})\right\|_2^2\varphi_3^2\right\}\\ &= \inf_{R\in\mathcal{H}_\infty}\left\{\left\|\frac{1-L_z}{L_z}+\frac{e^{\tau s}-N_mY}{e^{\tau s}}-\frac{N_mRB_zM_m}{e^{\tau s}}\right\|_2^2\varphi_1^2 + \left\|\frac{YN_m+N_mRB_zM_m}{e^{\tau s}F}\right\|_2^2\varphi_2^2 + \left\|(\frac{L_z^{-1}N_mX-e^{-\tau s}N_mN_mR}{e^{\tau s}FA})\right\|_2^2\varphi_3^2\right\}\end{aligned}$$

Furthermore:

$$J^* = \left\|(L_z^{-1}-1)\right\|_2^2\varphi_1^2 + \inf_{R\in\mathcal{H}_\infty}\left\{\left\|\frac{e^{\tau s}-N_mY}{e^{\tau s}}-\frac{N_mRB_zM_m}{e^{\tau s}}\right\|_2^2\varphi_1^2 + \left\|\frac{YN_m+N_mRB_zM_m}{e^{\tau s}F}\right\|_2^2\varphi_2^2 + \left\|(\frac{L_z^{-1}N_mX-e^{-\tau s}N_mN_mR}{e^{\tau s}FA})\right\|_2^2\varphi_3^2\right\}$$

Because B_z with $e^{\tau s}$ is the all-pass factor part, following can be obtained:

$$J^* = \left\|(L_z^{-1}-1)\right\|_2^2\varphi_1^2 + \inf_{R\in\mathcal{H}_\infty}\left\{\left\|\frac{e^{\tau s}-N_mY}{B_z}-N_mRM_m\right\|_2^2\varphi_1^2 + \left\|(\frac{YN_m}{B_z}+N_mRM_m)F^{-1}\right\|_2^2\varphi_2^2 + \left\|(\frac{N_mXe^{\tau s}}{L_z}-N_mN_mR)F^{-1}A^{-1}\right\|_2^2\varphi_3^2\right\} \quad (16)$$

According to partial factorization,

$$\frac{e^{\tau s}-N_mY}{B_z} = \sum_{j\in N}\frac{s+\bar{p}_j}{s-p_j}\frac{e^{\tau p_j}-N_m(p_j)Y(p_j)}{b_j}+\zeta_1 \quad (17)$$

$$\frac{YN_m}{B_z} = \sum_{j\in N}\frac{s+\bar{p}_j}{s-p_j}\frac{Y(p_j)N_n(p_j)}{b_j}+\zeta_2 \quad (18)$$

$$\frac{N_mXe^{\tau s}}{L_z} = \sum_{i\in N}\frac{s+z_i}{s-z_i}\frac{N_m(z_i)X(z_i)F^{-1}(z_i)A^{-1}(z_i)e^{\tau z_i}}{l_i}+\zeta_3 \quad (19)$$

where $\zeta_1(s), \zeta_2(s), \zeta_3(s) \in \mathbb{R}\mathcal{H}_\infty$, and $b_j(s) = \prod_{\substack{i\in N\\ i\neq j}}^{m}\frac{p_i-p_j}{\bar{p}_i+p_j}$, $l_j(s) = \prod_{\substack{j\in N\\ i\neq j}}^{m}\frac{z_i-z_j}{\bar{z}_i+z_j}$.

According to Eq. (7): $M(p_j) = B_z(p_j)M_m = 0$, then:

$$Y(p_j)N_m(p_j) = L_z^{-1}(p_j)e^{\tau p_j}$$
$$N_m(z_i)X(z_i) = N_m(z_i)M^{-1}(z_i)$$

Combining Eqs. (17),(18) and (19),

$$\frac{e^{\tau s}-N_mY}{B_z} = \sum_{j\in N}\frac{s+\bar{p}_j}{s-p_j}\frac{e^{\tau p_j}(1-L_z^{-1}(p_j))}{b_j}+\zeta_1$$

$$\frac{YN_m}{B_z} = \sum_{j\in N}\frac{s+\bar{p}_j}{s-p_j}\frac{L_z^{-1}(p_j)e^{\tau p_j}}{b_j}+\zeta_2$$

$$\frac{N_mXe^{\tau s}}{L_z} = \sum_{i\in N}\frac{s+z_i}{s-z_i}\frac{N_m(z_i)F^{-1}(z_i)A^{-1}(z_i)e^{\tau z_i}}{M(z_i)l_i}+\zeta_3$$

Therefore, Eq. (16) can be expressed as:

$$\begin{aligned}
J^* &= \left\|(L_z^{-1}-1)\right\|_2^2 \varphi_1^2 + \inf_{R\in\mathcal{H}_\infty} \\
&\left\{ \left\|(\sum_{j\in N} \frac{s+\bar{p}_j}{s-p_j}\frac{L_z(p_j)-1}{L_z(p_j)b_j}e^{\tau p_j} + \zeta_1 - N_m R M_m)\right\|_2^2 \varphi_1^2 \right.\\
&+ \left\|(\sum_{j\in N}\left(\frac{s+\bar{p}_j}{s-p_j}\right)\frac{L_z^{-1}(p_j)e^{\tau p_j}}{b_j} + \zeta_2 + N_m R M_m)F^{-1}\right\|_2^2 \varphi_2^2 \\
&\left. + \left\|(\sum_{i\in N}\left(\frac{s+z_i}{s-z_i}\right)\frac{N_m(z_i)e^{\tau z_i}}{M(z_i)l_i} + \zeta_3 - \frac{N_m R N_m}{A})\right\|_2^2 \varphi_3^2 \right\} \\
&= \left\|(L_z^{-1}-1)\right\|_2^2 \varphi_1^2 \\
&+ \left\|\sum_{j\in N}\left(\frac{\bar{p}_j+s}{p_j-s}-1\right)\frac{e^{\tau p_j}(1-L_z^{-1}(p_j))}{b_j s}\right\|_2^2 \varphi_1^2 \\
&+ \left\|\sum_{j\in N}\left(\frac{s+\bar{p}_j}{s-p_j}-1\right)\frac{L_z^{-1}(p_j)e^{\tau p_j}}{b_j s}F^{-1}\right\|_2^2 \varphi_2^2 \\
&+ \left\|\sum_{i\in N}\left(\frac{s+z_i}{s-z_i}-1\right)\frac{N_m(z_i)e^{\tau z_i}}{M(z_i)l_i s}\right\|_2^2 \varphi_3^2 \\
&+ \inf_{R\in\mathcal{H}_\infty}\left\|(\zeta_1 + \frac{e^{\tau p_j}(1-L_z^{-1}(p_j))}{b_j} - N_m R M_m)\right\|_2^2 \varphi_1^2 \\
&+ \inf_{R\in\mathcal{H}_\infty}\left\|(\zeta_2 + \frac{L_z^{-1}(p_j)e^{\tau p_j}}{b_j} + N_m R M_m)\right\|_2^2 \varphi_2^2 \\
&+ \inf_{R\in\mathcal{H}_\infty}\left\|(\zeta_3 + \frac{N_m(z_i)e^{\tau z_i}}{M(z_i)l_i} - \frac{N_m R N_m}{A})\right\|_2^2 \varphi_3^2 \qquad (20)
\end{aligned}$$

Because N_m and M_m both are the non-minimum phase parts, by choosing the appropriate R, following is obtained:

$$\left\|(\zeta_1 + \frac{e^{\tau p_j}(1-L_z^{-1}(p_j))}{b_j} - N_m R M_m)\right\|_2^2 \varphi_1^2 = 0$$

$$\left\|(\zeta_2 + \frac{L_z^{-1}(p_j)e^{\tau p_j}}{b_j} + N_m R M_m)\right\|_2^2 \varphi_2^2 = 0$$

$$\left\|(\zeta_3 + \frac{N_m(z_i)e^{\tau z_i}}{M(z_i)l_i} - \frac{N_m R N_m}{A})\right\|_2^2 \varphi_3^2 = 0$$

Therefore, the above three equations cannot be true at the same time, and combining Eq. (20), we will provide

$$\begin{aligned}
J^* &> \left\|(L_z^{-1}-1)\right\|_2^2 \varphi_1^2 \\
&+ \left\|\sum_{j\in N}\left(\frac{s+\bar{p}_j}{s-p_j}-1\right)\frac{e^{\tau p_j}(1-L_z^{-1}(p_j))}{b_j s}\right\|_2^2 \varphi_1^2 \\
&+ \left\|\sum_{j\in N}\left(\frac{s+\bar{p}_j}{s-p_j}-1\right)\frac{L_z^{-1}(p_j)e^{\tau p_j}F^{-1}}{b_j s}\right\|_2^2 \varphi_2^2 \\
&+ \left\|\sum_{i\in N}\left(\frac{s+z_i}{s-z_i}-1\right)\frac{N_m(z_i)e^{\tau z_i}}{M(z_i)l_i s}\right\|_2^2 \varphi_3^2
\end{aligned}$$

According to [28],

$$\left\|(L_z^{-1}-1)\right\|_2^2 \varphi_1^2 = \sum_{i=1}^{n} 2\,\mathrm{Re}(z_i)\varphi_1^2$$

Then:

$$\begin{aligned}
J^* &> \sum_{i=1}^{n} 2\,\mathrm{Re}(z_i)\varphi_1^2 \\
&+ \sum_{j,i\in N}\frac{4\,\mathrm{Re}(p_j)\,\mathrm{Re}(p_i)}{(p_j+p_i)p_j p_i}\frac{(1-L_z^{-1}(p_i))(1-L_z^{-1}(p_j))^H}{b_i\bar{b}_j e^{-\tau(p_i+\bar{p}_j)}}\varphi_1^2 \\
&+ \sum_{j,i\in N}\frac{4\,\mathrm{Re}(p_j)\,\mathrm{Re}(p_i)}{(p_j+p_i)p_j p_i}\frac{L_z^{-1}(p_j)L_z^{-1}(p_i)F^{-1}(z_i)}{b_i\bar{b}_j e^{-\tau(p_i+\bar{p}_j)}}\varphi_2^2 \\
&+ \sum_{j,i\in N}\frac{4\,\mathrm{Re}(z_i)\,\mathrm{Re}(z_i)}{\bar{z}_j+z_i}\frac{N_m(z_i)N_m(z_j)F^{-1}(z_i)A^{-1}(z_i)}{M(z_i)M(z_j)l_i\bar{l}_j e^{-\tau(z_i+z_j)}}\varphi_3^2
\end{aligned}$$

where $b_j(s) = \prod_{\substack{i\in N\\ i\neq j}}^{m} \frac{p_i-p_j}{\bar{p}_i+p_j}$, $l_j(s) = \prod_{\substack{j\in N\\ i\neq j}}^{m} \frac{z_i-z_j}{\bar{z}_i+z_j}$.

The proof is completed.

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