

Received August 18, 2020, accepted August 28, 2020, date of publication September 1, 2020, date of current version September 15, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3021009

H_∞ Control for Stochastic Singular Systems With Time-Varying Delays

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This work was supported in part by the National Natural Science Foundation of China under Grant 61803275, Grant 61733008, and Grant 61673099; in part by the Liaoning Revitalization Talents Program under Grant XLYC1907044; in part by the Natural Science Foundation of Liaoning Province under Grant 2020-MS-218; and in part by Scientific Research Project of Liaoning Provincial Department of Education-Science and Engineering Basic Research Project under Grant No. Injc 202018.

ABSTRACT In this article, we study the H_∞ control problems for stochastic singular systems with time-varying delays. Firstly, a new Lyapunov-Krasovskii functional is constructed, employing the free weighting matrix technique and Jensen inequality, the stochastic admissibility criteria in the mean square for stochastic singular time-varying delay systems are proposed on the basis of the auxiliary vector function. Secondly, the state feedback controller is designed such that the resulting closed-loop system meets regular, impulse-free, stochastically stable in the mean square and has H_∞ performance γ . In the proof process, the dual equation is used to derive the conditions of stochastic admissibility in the mean square. Finally, a practical example of DC motor model is presented to show the validity of our proposed theoretical results.

INDEX TERMS Stochastic singular systems, time-varying delays, H_∞ control, stochastic admissibility.

I. INTRODUCTION

In the past few decades, singular systems play an important role in many scientific fields, such as biologic systems, circuit systems and power systems. Therefore, the singular systems have been investigated by many researchers and a lot of important results relating to such systems have been reported (see [1]–[6]). In recent years, a more general model than singular systems is the stochastic singular systems, which play a more extensive role in model analysis than singular systems. As is well known, stochastic singular systems, also known as generalized stochastic systems, refer to the generalized dynamic systems under stochastic interference. It is widely found in many fields such as industry, social economy, power systems, financial economy systems, aerospace systems, etc. Due to the complexity of the internal structure of the stochastic singular systems, especially the co-existence of the impulse problem and the stochastic disturbance factors in the systems, it is difficult to study the theory of the stochastic singular systems. Therefore, considering the stochastic characteristics of the systems, many scholars began to pay attention to the study of singular systems with stochastic characteristics, such as stability analysis

(see [7]–[11]), filtering problem (see [12]–[14]), controller design (see [15]–[17]).

On the other hand, time delays inevitably exist in a variety of practical systems, which can frequently lead to instability or significantly deteriorated performance, and greatly increase the difficulty of stability analysis and controller design (see [18]–[25]). For singular systems, compared with the previously studied time-invariant delay systems, in recent years, more and more attention has been paid to the study of singular time-varying delay systems. It should be noted that the study of singular systems with time-varying delays is much more complex than that of normal systems with time-varying delays, because it requires consideration not only of stability, but also of regularity, impulse-free or causality (for discrete time singular systems) under time-varying delay case. For example, Yue and Han [26] investigated the delay-dependent robust H_∞ controller design for uncertain singular systems with time-varying discrete and distributed delays. Xia *et al.* [27] studied about the problem of filtering for nonlinear singular Markovian jumping systems with interval time-varying delays. Especially, it increases the difficulty of stability and related control analysis of the systems when the singular systems have both time-varying delays and random disturbance, and gradually attracts more and more attention. Xing *et al.* [28] researched the stability

The associate editor coordinating the review of this manuscript and approving it for publication was Guangdeng Zong.

criteria for stochastic singular systems with time-varying delays and uncertain parameters. Li *et al.* [29] studied stability and stabilisation problems for a series of continuous stochastic singular systems with multiple time-varying delays via a delay-distribution-dependent Lyapunov functional, and so on. However, according to the author's grasp of the situation, the problem of H_∞ control for stochastic singular systems with time-varying delays has not been fully investigated yet. Therefore, the comprehensive and thorough study of the control problem of stochastic singular systems with time-varying delays is of great significance and necessity both in theory and practice, which motivates us to do this study.

In this work, we focus on studying the H_∞ control problem for stochastic singular systems with time-varying delays. In the first part, by constructing a new Lyapunov-Krasovskii functional, based on the auxiliary vector function, using the free weighting matrix technique and the improved Jensen inequality, we propose the stochastic admissibility criteria in the mean square for the systems consideration. In the second part, by designing the state feedback controller based on the stochastic admissibility criteria in the mean square, the corresponding closed-loop systems are regular, impulse-free and stochastically stable in the mean square. In the third part, in the process of designing the state feedback controller, the dual equation is adopted to derive the conditions of stochastic admissibility in the mean square of the resulting closed-loop systems. An example of DC motor model is given to show the effectiveness of the controller design method.

The main contributions of this study include: 1) We study the stability and H_∞ control for singular systems with time-varying delays and stochastic disturbances at the same time. Compared with singular systems [21], stochastic systems [36], the models we study are more general and have a wider range of applications. 2) A novel Lyapunov-Krasovskii functional is built, the stochastic admissibility conditions in the mean square of stochastic singular systems with time-varying delays are proposed. 3) We employ an auxiliary vector function and the new free-weighting-matrix approach to reduce the conservatism of the solution for the systems. 4) The dual equation is used in the proof process of designing the state feedback controller, which easily derive the conditions of stochastic admissibility in the mean square of the systems.

Notations. The following symbols will be used throughout the work. \mathbb{R}^n represents the n -dimensional Euclidean space, and $\mathbb{R}^{n \times n}$ denotes the set of all $n \times n$ real matrices. The symbol $*$ represents transpose terms in a symmetric matrix and $\text{diag}\{\dots\}$ stands for a block-diagonal matrix. A^T is the transpose of matrix A . $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ are used to denote the maximum and minimum eigenvalue of A , respectively. $\mathcal{E}\{\cdot\}$ denotes the expectation operator. I is the identity matrix with the appropriate dimensions. If the dimensions of matrices are not explicitly specified, they are assumed to be algebraically compatible.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the stochastic singular time-varying delay system defined in a completely probability space $(\Omega, \mathcal{F}, \hat{P})$ as follows

$$\begin{aligned} E dx(t) &= (Ax(t) + A_d x(t - h(t)) + Bu(t) + B_v v(t))dt \\ &\quad + Jx(t)d\omega(t), \\ z(t) &= Cx(t), \\ x(t) &= \varphi(t), \quad t \in [-h_0, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t)$ is control input, $v(t)$ is external input disturbance signal, $z(t) \in \mathbb{R}^p$ is the measure output vector. The matrix $E \in \mathbb{R}^{n \times n}$ maybe singular and it is assumed that $\text{rank}(E) = r \leq n$. A, A_d, B, B_v, J, C are known real constant matrices with appropriate dimensions. $\omega(t)$ is one-dimensional standard Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \hat{P})$ satisfying $\mathcal{E}\{d\omega(t)\} = 0$, $\mathcal{E}\{d\omega(t)^2\} = dt$, $\varphi(t)$ is the initial condition defined on $[-h_0, 0]$, $h(t)$ is time-varying delay, which satisfies for all $t \geq 0$, $0 \leq h(t) \leq h_0$, $\dot{h}(t) \leq \mu \leq 1$, where h_0 and μ are scalars.

Next, the state feedback controller is designed as

$$u(t) = Kx(t), \tag{2}$$

where K is the state feedback gain matrix. Then, the closed-loop system is as follows

$$\begin{aligned} E dx(t) &= ((A + BK)x(t) + A_d x(t - h(t)) + B_v v(t))dt \\ &\quad + Jx(t)d\omega(t), \\ z(t) &= Cx(t). \end{aligned} \tag{3}$$

Below, introduced some preliminary works, which will be the basis of the main research results of this article.

Definition 1: [32]

(I) The matrix pair (E, A) is regular, if $\det(sE - A)$ is not identically zero.

(II) The matrix pair (E, A) is impulse-free, if $\deg(\det(sE - A)) = \text{rank}(E)$.

Definition 2: [33], [34]

(I) For a given scalar $h_0 > 0$ and any time-varying delay $h(t)$ satisfying $0 \leq h(t) \leq h_0$, if the pairs (E, A) and $(E, A + A_d)$ are regular and impulse-free, then the system (1) with $u(t) = 0$ and $v(t) = 0$ is regular and impulse-free.

(II) The system (1) with $u(t) = 0$ and $v(t) = 0$ is stochastically stable in the mean square, for $\forall \sigma > 0$, there exists a $\delta(\sigma) > 0$ such that $\mathcal{E}\|x(t)\|^2 < \sigma$, $t > 0$, when $\sup_{-h_0 \leq s \leq 0} \mathcal{E}\|\phi(s)\|^2 < \delta(\sigma)$.

Definition 3: If the system (1) is regular, impulse-free and stochastically stable in the mean square, then the system (1) is said to be stochastically admissible in the mean square.

Assumption 1: [18] $\text{rank}([E \ J]) = \text{rank}(E)$.

Under Assumption 1, if $\text{rank}(E) = r$, without loss of generality, we can decompose matrices in the form (1) as follows

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$

$$A_d = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix}, \quad J = \begin{bmatrix} J_{11} & J_{12} \\ 0 & 0 \end{bmatrix}, \quad (4)$$

and $x(t) = [x_1^T(t), x_2^T(t)]^T$, $A_{11} \in \mathbb{R}^{r \times r}$, $A_{d11} \in \mathbb{R}^{r \times r}$, $B_1 \in \mathbb{R}^{q \times q}$, $J_{11} \in \mathbb{R}^{r \times r}$, $x_1(t) \in \mathbb{R}^r$.

Furthermore, by the expression (4), system (1) with $v(t) = 0$ is restricted equivalent to the following dynamics decomposition form

$$\begin{aligned} dx_1(t) &= (A_{11}x_1(t) + A_{12}x_2(t) + A_{d11}x_1(t - h(t)) \\ &\quad + A_{d12}x_2(t - h(t)) + B_1u(t))dt + (J_{11}x_1(t) \\ &\quad + J_{12}x_2(t))d\omega(t), \\ 0 &= (A_{21}x_1(t) + A_{22}x_2(t) + A_{d21}x_1(t - h(t)) \\ &\quad + A_{d22}x_2(t - h(t)) + B_2u(t))dt, \\ z(t) &= C_1x_1(t) + C_2x_2(t). \end{aligned} \quad (5)$$

and $\varphi(t) = [\varphi_1^T(t) \quad \varphi_2^T(t)]^T$.

Remark 1: Under the above assumption, the Itô stochastic term does not affect the system structure. So, (E, A) and $(E, A + A_d)$ are regular and impulse-free, which can guarantee the existence and uniqueness of the solution of the system (1).

Lemma 1: [35] (**Jensen inequality**) For a positive definite symmetric matrix $Z \in \mathbb{R}^{r \times r}$ and scalars a, b ($0 < a < b$), there exists a vector function $x(t)$ satisfying

$$\int_b^a x^T(t)Zx(t)dt \geq \frac{1}{b-a} \int_b^a x^T(t)dtZ \int_b^a x(t)dt.$$

Lemma 2: Consider the stochastic singular system

$$Edx(t) = Ax(t)dt + Jx(t)d\omega(t). \quad (6)$$

Let $V(x(t)) = x^T(t)E^TPEx(t)$, P is invertible and $E^TPE \geq 0$.

Define an infinitesimal operator \mathcal{L} , then, the stochastic derivative of $V(x(t))$ is given by

$$dV(x(t)) = \mathcal{L}V(x(t))dt + 2x^T(t)E^TPEJx(t)d\omega(t), \quad (7)$$

where

$$\begin{aligned} \mathcal{L}V(x(t)) &= x^T(t)(A^TPE + E^TPA + J^T(E^+)^T \\ &\quad \times E^TPEE^+J)x(t), \end{aligned} \quad (8)$$

and E^+ is the generalized inverse of the matrix E .

Proof: Because $V(x(t)) = x^T(t)E^TPEx(t)$, by the Itô differential formula, combining with the Eq. (6), we have

$$dV(x(t)) = \mathcal{L}V(x(t))dt + V_x(x(t))Jx(t)d\omega(t),$$

where operator $\mathcal{L}V(x(t))$ is defined as

$$\begin{aligned} \mathcal{L}V(x(t)) &= V_t(x(t)) + V_x(x(t))Ax(t) \\ &\quad + \frac{1}{2} \text{tr}\{x^T(t)J^T V_{xx}(x(t))Jx(t)\}, \end{aligned}$$

with

$$\begin{aligned} V_t(x(t)) &= \frac{\partial V(x(t))}{\partial t}, \\ V_x(x(t)) &= \left(\frac{\partial V(x(t))}{\partial x_1}, \frac{\partial V(x(t))}{\partial x_2}, \dots, \frac{\partial V(x(t))}{\partial x_n} \right), \end{aligned}$$

$$V_{xx}(x(t)) = \left(\frac{\partial^2 V(x(t))}{\partial x_j \partial x_k} \right)_{n \times n},$$

by the calculation, one gets

$$\mathcal{L}V(x(t)) = 2x^T(t)E^TPEAx(t) + \text{tr}\{x^T(t)J^TE^TPEJx(t)\},$$

so, (8) holds, the proof is completed.

III. MAIN RESULTS

In this section, the stochastic admissibility condition in the mean square is firstly derived for the system (1). At first, introduce an auxiliary vector function $\eta_0(t)$.

$$\eta_0(t) = Ax(t) + A_dx(t - h(t)) + Bu(t) + B_vv(t). \quad (9)$$

Using the above formula and system (1), we can get

$$Edx(t) = \eta_0(t)dt + Jx(t)d\omega(t). \quad (10)$$

Then

$$\begin{aligned} Ex(t) - Ex(t - h(t)) &= \int_{t-h(t)}^t \eta_0(s)ds \\ &\quad + \int_{t-h(t)}^t Jx(s)d\omega(s). \end{aligned} \quad (11)$$

Theorem 1: For given scalars $h_0 > 0$, $\gamma > 0$, the system (1) is stochastically admissible in the mean square with the H_∞ performance index γ , if there exist matrices $P > 0$, $Q > 0$, $Z > 0$, $S, S_0, S_1, S_2, S_3, S_d$ such that the following matrix inequality hold.

$$\begin{bmatrix} \Lambda_1 & \tilde{\Theta}_{12} & X_3 & \tilde{\Theta}_{13} & S_2R^T & C^T & (PEE^+J)^T \\ * & \Lambda_2 & X_4 & X_5 & S_3R^T & 0 & 0 \\ * & * & \Psi & X_6 & 0 & 0 & 0 \\ * & * & * & \Psi_1 & 0 & 0 & 0 \\ * & * & * & * & \Psi_2 & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -P \end{bmatrix} < 0, \quad (12)$$

where

$$\begin{aligned} \Lambda_1 &= A^TPE + E^TPA + Q + SR^TA + A^TRST, \\ \tilde{\Theta}_{12} &= E^TPA + SR^TA_d + A^TRSD^T, \\ \tilde{\Theta}_{13} &= E^TPB_v - A^TRSD^T + SR^TB_v, \\ \Lambda_2 &= -(1 - \mu)Q + S_dR^TA_d + A_d^TRSD^T, \\ X_3 &= A^TRSD_0^T - SR^T, \quad X_4 = A_d^TRSD_0^T - S_dR^T, \\ X_5 &= -A_d^TRSD_1^T + S_dR^TB_v, \quad X_6 = S_0R^TB_v + RS_1^T, \\ \Psi &= h_0^2Z - S_0R^T - RS_0^T, \\ \Psi_1 &= -\gamma^2I - B_v^T - S_1R^TB_v, \\ \Psi_2 &= -Z - RS_3^T - S_3R^T, \end{aligned}$$

$R \in \mathbb{R}^{n \times (n-r)}$ is an arbitrary column full rank matrix satisfying $E^TR = 0$.

Proof: Firstly, we prove the system (1) with $u(t) = 0$ and $v(t) = 0$ is regular and impulse-free.

From $\text{rank}(E) = r \leq n$, there exist two invertible matrices $G, H \in \mathfrak{R}^{n \times n}$ such that

$$\begin{aligned} \bar{E} &= GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{A} = GAH = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \\ \bar{B} &= GB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad \bar{A}_d = GA_dH = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix}, \\ \bar{P} &= G^{-T}PG^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, \quad \bar{S} = H^T S = \begin{bmatrix} S_{11} \\ S_{21} \end{bmatrix}. \end{aligned}$$

It is noted that $E^T R = 0$, set $\bar{R} = G^{-T}R = \begin{bmatrix} 0 \\ \kappa \end{bmatrix}$. Here, $\kappa \in \mathfrak{R}^{(n-r) \times (n-r)}$ is a nonsingular matrix, it is obvious that $\bar{E}^T \bar{R} = 0$.

From (12), we have

$$\Lambda_1 < 0. \tag{13}$$

Because $Q > 0$, we have

$$\Pi = A^T P E + E^T P A + A^T R S^T + S R^T A < 0. \tag{14}$$

Before and after multiplying (14) by H^T and H , respectively, it has

$$\begin{aligned} \bar{\Pi} &= \bar{A}^T \bar{P} \bar{E} + \bar{E}^T \bar{P} \bar{A} + \bar{A}^T \bar{R} \bar{S}^T + \bar{S} \bar{R}^T \bar{A} \\ &= \begin{bmatrix} \otimes & \tilde{\otimes} \\ * & A_{22}^T \kappa S_{21}^T + S_{21} \kappa^T A_{22} \end{bmatrix} < 0. \end{aligned} \tag{15}$$

Since \otimes and $\tilde{\otimes}$ are independent of the results discussed below, the real expressions of these two irrelevant are omitted. According to (15), we get $A_{22}^T \kappa S_{21}^T + S_{21} \kappa^T A_{22} < 0$, it is easy to verify A_{22} is a nonsingular matrix.

Thus,

$$\begin{aligned} \det(sE - A) &= \det(G^{-1}) \det(sE - A) \det(H^{-1}) \\ &= \det(-A_{22}) \det(I_r - (A_{11} - A_{21}^{-1} A_{21})) \det(G^{-1}) \\ &\quad \times \det(H^{-1}). \end{aligned} \tag{16}$$

According to formula (16), it has $\det(sE - A) \neq 0$, $\deg(\det(sE - A)) = \text{rank}(E)$. Thus, the pair (E, A) is regular and impulse-free.

Additionally, from (12), we can also obtain the following matrix inequality

$$\begin{bmatrix} \Lambda_1 & \tilde{\Theta}_{12} \\ * & \Lambda_2 \end{bmatrix} < 0. \tag{17}$$

Then, before and after multiplying (17) by $[I, I]$ and $[I, I]^T$, respectively, we can obtain

$$(A + A_d)^T P E + E^T P (A + A_d) + (A^T + A_d^T) R S^T + S R^T (A^T + A_d^T) < 0.$$

Using the same approach as above, we have

$$\begin{bmatrix} \odot & \tilde{\odot} \\ * & (A_{22}^T + A_{d22}^T) \kappa S_{21}^T + S_{21} \kappa^T (A_{22} + A_{d22}) \end{bmatrix} < 0. \tag{18}$$

From (18), we can easily see that

$$(A_{22}^T + A_{d22}^T) \kappa S_{21}^T + S_{21} \kappa^T (A_{22} + A_{d22}) < 0.$$

This implies that the pair $(E, A + A_d)$ is regular and impulse-free. According to Definition 2, the system (1) with $u(t) = 0$ and $v(t) = 0$ is regular and impulse-free.

Next, we prove the system (1) with $u(t) = 0$ and $v(t) = 0$ is stochastically stable in the mean square.

Constructing a Lyapunov-Krasovskii functional as follows

$$\begin{aligned} V(t) &= V_1(t) + V_2(t) + V_3(t), \\ V_1(t) &= x^T(t) E^T P E x(t), \\ V_2(t) &= \int_{t-h(t)}^t x^T(s) Q x(s) ds, \\ V_3(t) &= h_0 \int_{-h_0}^0 \int_{t+\theta}^t \eta_0^T(s) Z \eta_0(s) ds d\theta. \end{aligned} \tag{19}$$

By Lemma 2, computing the stochastic derivative of $V(t)$ along the trajectory of the system (1), one has

$$dV(t) = \mathcal{L}V(t)dt + 2x^T(t)E^T P J x(t)d\omega(t), \tag{20}$$

where

$$\mathcal{L}V(t) = \mathcal{L}V_1(t) + \mathcal{L}V_2(t) + \mathcal{L}V_3(t),$$

and

$$\begin{aligned} \mathcal{L}V_1(t) &= x^T(t)(E^T P A + A^T P E + J^T(E^+)^T E^T P E E^+ J) \\ &\quad \times x(t) + x^T(t) E^T P A_d x(t - h(t)) \\ &\quad + x^T(t - h(t)) A_d^T P E x(t), \\ \mathcal{L}V_2(t) &= x^T(t) Q x(t) - (1 - \dot{h}(t)) x^T(t - h(t)) \\ &\quad \times Q x(t - h(t)) \\ &\leq x^T(t) Q x(t) - (1 - \mu) x^T(t - h(t)) Q x(t - h(t)), \\ \mathcal{L}V_3(t) &= h_0^2 \eta_0(t)^T Z \eta_0(t) - h_0 \int_{t-h(t)}^t \eta_0^T(s) Z \eta_0(s) ds \\ &\leq h_0^2 \eta_0(t)^T Z \eta_0(t) - \int_{t-h(t)}^t \eta_0^T(s) ds Z \int_{t-h(t)}^t \eta_0(s) ds. \end{aligned} \tag{21}$$

When $u(t) = 0$, from (9), one has

$$2(Ax(t) + A_d x(t - h(t))) + B_v v(t) - \eta_0(t) = 0.$$

So, there exist free weighting matrices S, S_d, S_0, S_1 , it has

$$\begin{aligned} &2(Ax(t) + A_d x(t - h(t))) + B_v v(t) - \eta_0(t)^T R \\ &\quad (S^T x(t) + S_d^T x(t - h(t))) + S_0^T \eta_0(t) - S_1^T v(t)) \\ &= x^T(t)(A^T R S^T + S R^T A)x(t) + 2x^T(t)A^T R S_d^T \\ &\quad \times x(t - h(t)) + 2x^T(t)A^T R S_0^T \eta_0(t) - 2x^T(t)A^T R S_1^T v(t) \\ &\quad + 2x^T(t - h(t))A_d^T R S^T x(t) - 2x^T(t - h(t))A_d^T R S_1^T v(t) \\ &\quad + x^T(t - h(t))(S_d R^T A_d + A_d^T R S_d^T)x(t - h(t)) \\ &\quad + 2x^T(t - h(t))A_d^T R S_0^T \eta_0(t) + 2v^T(t)B_v^T R S^T x(t) \\ &\quad + 2v^T(t)B_v^T R S_d^T x(t - h(t)) + 2v^T(t)B_v^T R S_0^T \eta_0(t) \\ &\quad - v^T(t)(B_v^T R S_1^T + S_1 R^T B_v)v(t) - 2\eta_0^T(t)R S^T x(t) \\ &\quad - 2\eta_0^T(t)R S_d^T x(t - h(t)) - \eta_0^T(t)(R S_0^T + S_0 R^T)\eta_0(t) \\ &\quad + 2\eta_0^T(t)R S_1^T v(t) = 0. \end{aligned} \tag{22}$$

Because $E^T R = 0$, from (11), there exist free weighting matrices S_2, S_3 , we have

$$\begin{aligned} & 2x^T(t)E^T R(S_2^T x(t) - S_3^T \int_{t-h(t)}^t \eta_0(s)ds) \\ &= 2 \int_{t-h(t)}^t \eta_0^T(s)ds R S_2^T x(t) - \int_{t-h(t)}^t \eta_0^T(s)ds \\ & \quad \times (R S_3^T + S_3 R^T) \int_{t-h(t)}^t \eta_0(s)ds \\ & \quad + 2 \left[\int_{t-h(t)}^t Jx(s)d\omega(s) \right]^T R(S_2^T x(t) - S_3^T \int_{t-h(t)}^t \eta_0(s)ds) \\ &= 0. \end{aligned} \tag{23}$$

From (20), (22) and (23), we obtain

$$\begin{aligned} dV(t) &= \mathcal{L}\tilde{V}(t)dt + 2x^T(t)E^T P Jx(t)d\omega(t) \\ & \quad + 2 \left[\int_{t-h(t)}^t Jx(s)d\omega(s) \right]^T R(S_2^T x(t) - S_3^T \\ & \quad \times \int_{t-h(t)}^t \eta_0(s)ds), \end{aligned} \tag{24}$$

where

$$\begin{aligned} \mathcal{L}\tilde{V}(t) &= \mathcal{L}V(t) + 2(Ax(t) + A_d x(t-h(t)) + B_v v(t) \\ & \quad - \eta_0(t))^T R(S^T x(t) + S_d^T x(t-h(t)) \\ & \quad + S_0^T \eta_0(t) - S_1^T v(t)) \\ & \quad + 2x^T(t)E^T R(S_2^T x(t) - S_3^T \int_{t-h(t)}^t \eta_0(s)ds). \end{aligned} \tag{25}$$

When $v(t) = 0$, let

$$\xi^T(t) = [x^T(t) \quad x^T(t-h(t)) \quad \eta_0^T(t) \quad \int_{t-h(t)}^t \eta_0^T(s)ds].$$

Then, we have

$$\mathcal{L}\tilde{V}(t) < \xi^T(t)\Phi\xi(t), \tag{26}$$

where

$$\Phi = \begin{bmatrix} \bar{\Lambda}_1 & \tilde{\Theta}_{12} & X_3 & S_2 R^T \\ * & \Lambda_2 & X_4 & S_3 R^T \\ * & * & \Psi & 0 \\ * & * & * & \Psi_2 \end{bmatrix},$$

and

$$\begin{aligned} \bar{\Lambda}_1 &= A^T P E + E^T P A + Q + S R^T A + A^T R S^T \\ & \quad + J^T (E^+)^T E^T P E E^+ J. \end{aligned}$$

For the condition (20), by the Schur complement lemma, we have $\Phi < 0$. Thus,

$$\begin{aligned} \mathcal{E}\{\mathcal{L}V(t)\} &\leq \mathcal{E}\{\mathcal{L}\tilde{V}(t)\} \\ &\leq \lambda_{\max}(\Phi)\mathcal{E}\|\xi(t)\|^2 \leq \lambda_{\max}(\Phi)\mathcal{E}\|x(t)\|^2. \end{aligned}$$

Therefore, according to Definition 2, system (1) with $u(t) = 0$ and $v(t) = 0$ is stochastically stable in the mean square. It follows from Definition 3, we have the system (1) is stochastically admissible in the mean square.

Next, the system (1) with the performance H_∞ index γ is analyzed.

Set

$$J_T = \mathcal{E}\left\{ \int_0^\infty (z^T(t)z(t) - \gamma^2 v^T(t)v(t))dt \right\}. \tag{27}$$

Let

$$\zeta^T(t) = [x^T(t) \quad x^T(t-h(t)) \quad \eta_0^T(t) \quad v^T(t) \quad \int_{t-h(t)}^t \eta_0^T(s)ds].$$

We have

$$J_T \leq \zeta^T(t)\hat{\Phi}\zeta(t), \tag{28}$$

where

$$\begin{aligned} \hat{\Phi} &= \begin{bmatrix} \bar{\Lambda}_1 & \tilde{\Theta}_{12} & X_3 & \tilde{\Theta}_{13} & S_2 R^T \\ * & \Lambda_2 & X_4 & X_5 & S_3 R^T \\ * & * & \Psi & X_6 & 0 \\ * & * & * & \Psi_1 & 0 \\ * & * & * & * & \Psi_2 \end{bmatrix}, \\ \bar{\Lambda}_1 &= A^T P E + E^T P A + Q + S R^T A + A^T R S^T, \\ & \quad + J^T (E^+)^T E^T P E E^+ J + C^T C. \end{aligned}$$

From (12), by Schur complement lemma, one has $J_T < 0$. Therefore, the system (1) is stochastically admissible in the mean square and has H_∞ performance γ . This completes the proof.

Remark 2: When $J = 0$, the system (1) is reduced to the singular systems discussed by Wu *et al.* [22], however, they investigated the time-invariant delay systems. Here, we study time-varying delay systems. When $E = 0$, system (1) is reduced to the stochastic systems with time-varying delays studied by Xia *et al.* [36], the model we studied is more complex and has extensive applications.

Next, we give the state feedback controller design method of the system (3), in order to analyse conveniently, the closed-loop system (3) can be written the following equivalent form

$$\begin{aligned} \tilde{E}d\tilde{x}(t) &= (\tilde{A}\tilde{x}(t) + \tilde{A}_d\tilde{x}(t-h(t)) + \tilde{B}_v v(t))dt \\ & \quad + \tilde{J}x(t)d\omega(t), \\ Z(t) &= \tilde{C}\tilde{x}(t), \end{aligned} \tag{29}$$

among them

$$\begin{aligned} \tilde{E} &= \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} 0 & I \\ A + BK & -I \end{bmatrix}, \\ \tilde{A}_d &= \begin{bmatrix} 0 & 0 \\ 0 & A_d \end{bmatrix}, \quad \tilde{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \\ \tilde{x}(t-h(t)) &= \begin{bmatrix} 0 \\ x(t-h(t)) \end{bmatrix}, \quad \tilde{B}_v = \begin{bmatrix} 0 \\ B_v \end{bmatrix}, \\ \tilde{C} &= [C \quad 0], \quad \tilde{J} = \begin{bmatrix} 0 & 0 \\ J & 0 \end{bmatrix}. \end{aligned} \tag{30}$$

Therefore, we only need to prove the system (29) is stochastically admissible in the mean square and has H_∞ performance index γ .

Theorem 2: For given two scalars $h_0 > 0, \gamma > 0$, the closed-loop system (29) is stochastically admissible in

the mean square with H_∞ performance index γ . If there exist positive matrices $P > 0, Q > 0, Z > 0$, matrices X, Y, S, S_0, S_2, S_3 , such that the following matrix inequality holds.

$$\begin{bmatrix} \Xi_1 & \Xi_2 \\ * & \Xi_3 \end{bmatrix} < 0, \quad (31)$$

where

$$\begin{aligned} \Xi_1 &= \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \\ * & \Upsilon_3 \end{bmatrix}, \\ \Upsilon_1 &= \begin{bmatrix} \Omega_1 & \Omega_2 \\ * & -X - X^T \end{bmatrix}, \\ \Upsilon_2 &= \begin{bmatrix} X^T A_d^T & \Omega_3 & \Omega_4 \\ X^T A_d^T & RS_0^T - X & -X - X^T \end{bmatrix}, \\ \Upsilon_3 &= \begin{bmatrix} -(1-\mu)Q & A_d X & A_d X \\ * & \Psi & -X^T \\ * & * & -X - X^T \end{bmatrix}, \\ \Xi_2 &= \begin{bmatrix} X^T C^T & S_2 R^T & X^T & B_v & JE^+ EP \\ X^T C^T & 0 & X^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ X^T C^T & 0 & 0 & 0 & 0 \\ X^T C^T & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Xi_3 &= \begin{bmatrix} -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & \Omega_5 & -X^T & 0 & 0 \\ * & * & -X - X^T & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -P \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \Omega_1 &= Q + AX + BY + X^T A^T + Y^T + B^T, \\ \Omega_2 &= EP + SR^T - X^T + AX + BY, \\ \Omega_3 &= AX + BY - SR^T, \quad \Omega_4 = AX + BY - X^T, \\ \Omega_5 &= -Z - S_3 R^T - RS_3^T, \\ \Psi &= h_0^2 Z - S_0 R^T - RS_0^T, \end{aligned}$$

$R \in \mathfrak{R}^{n \times (n-r)}$ is an arbitrary column full rank matrix satisfying $E^T R = 0$.

Then the state feedback control law is given by

$$u(t) = Kx(t) = YX^{-1}x(t). \quad (32)$$

Proof: Denote

$$\begin{aligned} \tilde{P} &= \begin{bmatrix} P & 0 \\ 0 & \lambda I \end{bmatrix}, \quad \tilde{Q} = \begin{bmatrix} Q & 0 \\ 0 & \lambda I \end{bmatrix}, \quad \tilde{R} = \begin{bmatrix} R & 0 \\ 0 & X \end{bmatrix}, \\ \tilde{S}_0 &= \begin{bmatrix} S_0 & I \\ 0 & I \end{bmatrix}, \quad \tilde{S} = \begin{bmatrix} S & I \\ 0 & I \end{bmatrix}, \quad \tilde{S}_d = \tilde{S}_1 = 0, \\ \tilde{S}_2 &= \begin{bmatrix} S_2 & I \\ 0 & I \end{bmatrix}, \quad \tilde{S}_3 = \begin{bmatrix} S_3 & I \\ 0 & I \end{bmatrix}, \end{aligned} \quad (33)$$

where $X \in \mathfrak{R}^{n \times n}$ is an arbitrary nonsingular matrix. The $\tilde{P}, \tilde{Q}, \tilde{R}, \tilde{S}, \tilde{S}_d, \tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_0$ in (33) is used to replace the $P, Q, R, S, S_d, S_1, S_2, S_3, S_0$ in formula (12) of Theorem 1. By Schur complement lemma, and let $\lambda \rightarrow 0$, we get

$$\begin{bmatrix} \hat{\Xi}_1 & \hat{\Xi}_2 \\ * & \Xi_3 \end{bmatrix} < 0, \quad (34)$$

where

$$\begin{aligned} \hat{\Xi}_1 &= \begin{bmatrix} \hat{\Upsilon}_1 & \hat{\Upsilon}_2 \\ * & \hat{\Upsilon}_3 \end{bmatrix}, \\ \hat{\Upsilon}_1 &= \begin{bmatrix} \bar{\Omega}_1 & \bar{\Omega}_2 \\ * & -X - X^T \end{bmatrix}, \\ \hat{\Upsilon}_2 &= \begin{bmatrix} X^T A_d^T & \bar{\Omega}_3 & \bar{\Omega}_4 \\ X^T A_d^T & RS_0^T - X & -X - X^T \end{bmatrix}, \\ \hat{\Upsilon}_3 &= \begin{bmatrix} -(1-\mu)Q & A_d^T X & A_d^T X \\ * & \Psi & -X^T \\ * & * & -X - X^T \end{bmatrix}, \\ \hat{\Xi}_2 &= \begin{bmatrix} X^T B_v & S_2 R^T & X^T & C^T & (PEE^+ J)^T \\ X^T B_v & 0 & X^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ X^T B_v & 0 & 0 & 0 & 0 \\ X^T B_v & 0 & 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \bar{\Omega}_1 &= Q + X^T(A + BK) + (A + BK)^T X, \\ \bar{\Omega}_2 &= E^T P + SR^T - X^T + (A + BK)^T X, \\ \bar{\Omega}_3 &= (A + BK)^T X - SR^T, \\ \bar{\Omega}_4 &= (A + BK)^T X - X^T, \\ \Psi &= h_0^2 Z - S_0 R^T - RS_0^T. \end{aligned}$$

We know, in terms of the regularity, the absence of impulse and stochastic stability of the stochastic singular systems, the system (29) can be written in the following dual equation form

$$\begin{aligned} \tilde{E}d\tilde{x}(t) &= ((A + BK)^T \tilde{x}(t) + \tilde{A}_d^T \tilde{x}(t - h(t)) + C^T \zeta(t))dt \\ &\quad + \tilde{J}_v^T \tilde{x}(t)d\omega(t), \\ Z(t) &= \tilde{B}_v^T \tilde{x}(t). \end{aligned} \quad (35)$$

In Eq. (35), we use $\tilde{E}^T, (A + BK)^T, \tilde{A}_d^T, \tilde{B}_v^T, \tilde{J}^T, \tilde{C}^T$ instead of $E, (A + BK), A_d, B_v, J, C$, respectively, and let $Y = KX$, thus we can prove that the closed-loop system (29) is stochastically admissible in the mean square and has H_∞ performance index γ . This completes the proof.

IV. ILLUSTRATIVE EXAMPLES

Let's give a practical example to demonstrate the effectiveness of the proposed method.

Example 1: Considering the following DC motor model [37].

$$\begin{aligned} dx_i(t) &= \frac{b_i}{J_i} x_1(t) + \frac{K_t}{J_i} x_2(t)dt + Jx(t)d\omega(t), \\ u(t) &= K_v x_1(t) + \bar{R}x_2(t), \end{aligned}$$

where the state vector $x_1(t) = v(t)$ stands for speed, and the state vector $x_2(t) = i(t)$ refers to current. K_v, K_t , and \bar{R} refer to electromotive force, torque constant, and electric resistor, respectively. Here, J_i and b_i are defined as follows

$$\begin{aligned} J_i &= J_m + \frac{J_{ci}}{n^2}, \\ b_i &= b_m + \frac{b_{ci}}{n^2}. \end{aligned}$$

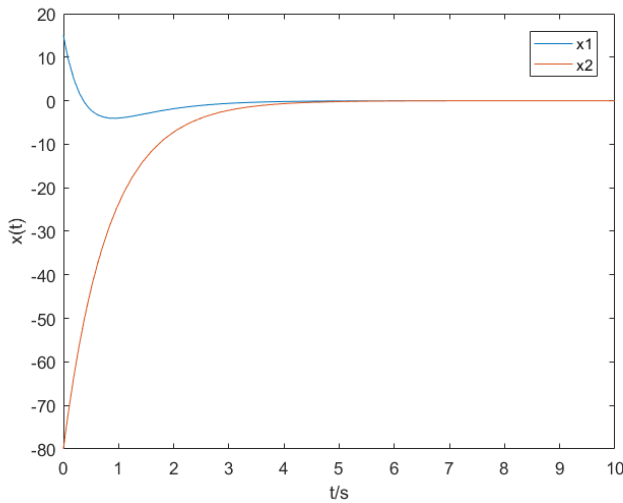


FIGURE 1. Stochastic stability of system (3) without external disturbance.

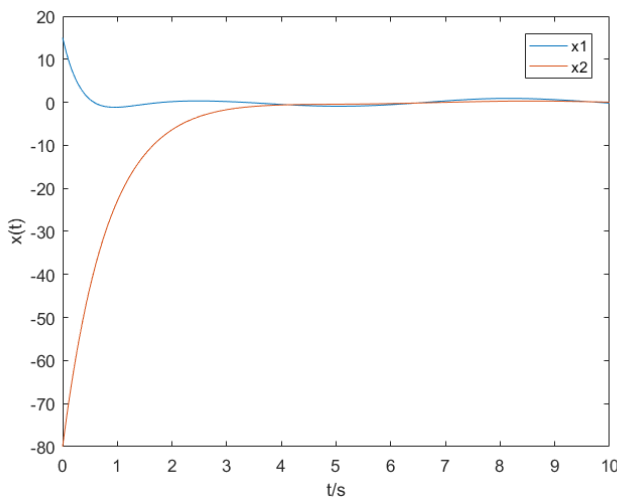


FIGURE 2. Stochastic stability of system (3) with external disturbance.

Set $J_m = 0.5\text{kgm}$, $J_{c1} = 50\text{kgm}$, $J_{c2} = 150\text{kgm}$, $b_{c1} = 100$, $b_{c2} = 240$, $\bar{R} = 1\Omega$, $b_m = 1$, $K_t = 3N \cdot m/A$, $K_v = 1\text{Vs/rad}$, and $n = 2$.

According to the modeling idea in [37], and the time of the trigger is out of control will cause delay phenomenon in a DC input and output, then the system parameters are given as

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -2 & 0.2 \\ 1 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, A_d = \begin{bmatrix} -0.7 & 0.1 \\ 1 & 1 \end{bmatrix}.$$

In addition, DC motor often be affected by temperature, humidity, the complexity and uncertainty of the external environment and other factors, there exist stochastic disturbance $\omega(t)$, external input disturbance $v(t)$ in the DC motor model, the parameters are given as follows

$$J = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0 \end{bmatrix},$$

$$B_v = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}.$$

Design the state feedback controller (2), set $h_0 = 0.3$, $\mu = 0.1$. For any delay $0 \leq h(t) \leq 0.3$, by solving (31) in Theorem 2, the corresponding state feedback gain is given by

$$K = \begin{bmatrix} -1.6245 & -2.8310 \end{bmatrix}.$$

When $\omega(t) = t - 0.3 * \sin(t)$ is considered, we can obtain the following plot.

Figs. 1 and 2 plot state response $x(t)$ of the closed-loop system (3), respectively, from which, we can see that the state $x(t)$ meets stochastic admissibility in the mean square of the studied system.

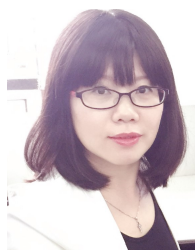
V. CONCLUSION

In this work, we discuss the stochastic admissibility in the mean square for singular systems with stochastic disturbance and time-varying delays. By constructing a new Lyapunov-Krasovskii functional, depending on the auxiliary vector function and using the free weighting matrix technique and the improved Jensen inequality, the stochastic admissibility criteria in the mean square for the systems consideration are proposed. In the process of designing the controller, the dual equation is adopted to derive the stochastic admissibility conditions in the mean square easily. Finally, an example of DC motor model is given to verify the validity of the theoretical results. Our conclusion can be further extended to stochastic singular systems with time-varying delays and parameter uncertainties.

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