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# Adaptive Fault-Tolerant Output Regulation of Linear Multi-Agent Systems With Sensor Faults

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**ABSTRACT** This article studies the problem of cooperative fault-tolerant output regulation of leader-follower multi-agent systems with sensor faults. To compensate for the faults existing in the followers, distributed observers based on relative output estimation errors are firstly designed. Then an adaptive fault-tolerant output regulation framework is built by solving the regulator equation. It is shown that stability of the closed-loop system can be ensured and that all tracking errors will converge to zero under the designed fault-tolerant controller. Finally, simulation results demonstrate the effectiveness of the proposed control law.

**INDEX TERMS** Fault-tolerant control, output regulation, multi-agent systems, sensor faults.

## I. INTRODUCTION

In recent years, cooperative control for multi-agent systems (MASs) has become a hot spot in the field of control, which has also been applied in multi-sensor networks [1], satellite networks [2] and cooperative vehicle infrastructure systems [3], etc. Output regulation theory [4] is used to solve several classes of consensus problems of MASs, which are also called as the cooperative output regulation problems (CORPs).

With the increasing complexity of MASs, the occurrence of faults is inevitable, such as actuator faults and sensor faults [5]–[8]. As pointed out by [9], it will lead to various problems such as system instability, inaccurate tracking, information distortion, resultant erroneous decision making and so on. For MASs formed by networking, the normal operation of the actuator and sensor of each agent enables the system to keep stable and consensus. If there exist faults in one or more agents, it will do harm to stability and consensus of the whole system. There are many studies of traditional fault diagnosis for sensor faults, e.g., [10]–[12]. Moreover, many studies [13]–[16] have been made in fault-tolerant consensus of MASs with faults. Most of them need the fault

diagnosis process [15] or only work for undirected communication topologies [16], which impose great limitations on practical applications.

In the control field, the main aim of CORP is to deal with a class of trajectory tracking problems such that all followers can retain stability and asymptotically track the leader, which is also called exosystem. If faults occur in parts of agents in CORP, the corresponding problem can be called fault-tolerant output regulation problem (FTORP). The main challenge of FTORP is how to find an appropriate solution of linear regulator equation as the feedforward information. Various results have been obtained to solve FTORP of MASs based on distributed control protocol [14]–[18]. Deng and Yang [15] studied the FTORP of linear MASs with faults by designing a distributed adaptive fault-tolerant control law. Zhang *et al.* [16] studied the similar FTORP of linear MASs with an undirected topology and multiple leaders, and adaptive observers were designed to estimate states and faults of followers. Note that these studies are based on an undirected graph and can not be used for directed topology. There are also some results for FTORP under directed topology. For example, Deng *et al.* [17] considered the FTORP of linear MASs with actuator faults and directed communication topology. Zhang *et al.* [18] considered the FTORP of linear MASs with process faults and directed topology by introducing

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adaptive observers. It should be noted that the above studies mainly focused on the FTORP of MASs with actuator faults or process faults. In particular, Qin *et al.* [14] studied the FTORP of linear MASs with sensor faults by introducing  $p$ -copy internal model principle. However, the result in [14] is under an undirected graph and it needs the fault diagnosis process. Zhang *et al.* [10] studied the problem of fault estimation and designed adaptive observer. Xiao *et al.* [11] studied the fault tolerant control for aircraft engine with sensors faults. However, they work for a single system.

As mentioned above, we will try to solve the FTORP of linear multi-agent systems with sensor faults and directed communication topology. The salient features of this article lie in three aspects:

(1) A distributed fault-tolerant control framework for directed topology is constructed, which is different from the existing results under undirected communication environment [15], [16]. Moreover, the fault detection process in [14] is not needed in this article by introducing distributed real-time fault observers.

(2) An adaptive observer is introduced to estimate the state of exosystem and provide the feedforward information. The limitation of global condition depending on the eigenvalues of Laplacian matrix associated with the network topology can be overcome by replacing the fixed parameters with adaptive gains.

(3) The fault-tolerant control problem of linear MASs is transformed into the FTORP of an augmented system by introducing an auxiliary system, which can also be extended to solve some similar tracking control problems of MASs without faults.

*Notations:* In this article,  $R^n$  and  $R^{n \times m}$  represent the real sets of  $n$ -vectors and  $n \times m$  matrices, respectively.  $\text{diag}\{a_1, \dots, a_n\}$  denotes the diagonal matrix composed by the entries  $a_i, i = 1, \dots, n$ .  $A > 0$  denotes that the matrix  $A$  is a symmetric positive definite matrix, whereas  $A < 0$  denotes that  $A$  is a symmetric negative definite matrix. The  $n$ -vectors  $\mathbf{1}_n$  has the same elements being 1.  $I_n$  is the  $n$ -dimensional real identity matrix. The term  $\lambda(A)$  denotes the spectrum of a square matrix  $A$ . In particular,  $\lambda_{\min}(A)$  is the minimum eigenvalue of  $A$  and  $\lambda_{\max}(A)$  is the maximum eigenvalue of  $A$ . The symbol  $\otimes$  denotes the Kronecker product.

## II. PRELIMINARIES AND PROBLEM STATEMENT

### A. PRELIMINARIES

In this article, we use a digraph to represent the communication between the agents in MASs. Now we consider a digraph  $G = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ , which consists of  $\mathcal{V} = \{1, 2, \dots, N\}$  and  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ .  $\mathcal{V} = \{1, 2, \dots, N\}$  is the set of nodes representing the agents and  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  is the set of edges representing the information transmission. In order to describe the composition and structure of the system, we introduce the nonnegative adjacency matrix  $\mathcal{A} = (a_{ij})_{N \times N}$ , if there exists a path from node  $j$  to node  $i$ , i.e.,  $(j, i) \in \mathcal{E}$ , then  $a_{ij} > 0$ , which is usually set to 1, and node  $j$  is called a

neighbor of the node  $i$ . If there is no information transmission, i.e.,  $(j, i) \notin \mathcal{E}$ , then  $a_{ij} = 0$ . Moreover,  $a_{ii} = 0$ , which means that each node has no self-loop. In particular, for an undirected graph,  $\mathcal{A}$  is a symmetric matrix. Furthermore, we can also introduce the Laplacian matrix to describe the property of graph. Based on the definition of adjacency matrix, the Laplacian matrix of  $G$  can be defined as  $L = D - \mathcal{A}$ , where  $D = \text{diag}\{d_1, d_2, \dots, d_N\}$  with elements  $d_i = \sum_{j=1}^N a_{ij}$  is called degree matrix. As mentioned above,  $a_{ij}$  are elements of  $\mathcal{A}$ . If there exists a directed sequence  $(i, i_1), (i_1, i_2), \dots, (i_k, j) \in \mathcal{E}$ , where  $i_l, l = 1, 2, \dots, k$  represent nodes in the graph, then we call there exists a directed path from node  $i$  to node  $j$ . If there exists a path between any two nodes, we call the graph a connected one. In addition, if there exists a root node with paths between it and any other nodes, the graph contains a spanning tree.

### B. PROBLEM STATEMENT

Consider the following linear MAS including one leader and  $N$  followers with sensor faults

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t) + Df_i(t), \end{cases} \quad i = 1, 2, \dots, N \quad (1)$$

where  $x_i(t) \in R^n$ ,  $u_i(t) \in R^m$  and  $y_i(t) \in R^p$  are the state, input and output of the  $i$ th follower, respectively.  $f_i(t) \in R^r$  is the sensor fault in the  $i$ th follower. In particular, if  $f_i = 0$ , there is no fault in the system. In this article, it is assumed that the sensor faults are constants, i.e.,  $\dot{f}_i = 0, i = 1, 2, \dots, N$ .  $A, B, C$ , and  $D$  are known real matrices with appropriate dimensions and it is assumed that the matrix  $D$  is of full column rank.

The dynamics of the leader can be expressed as follows

$$\begin{cases} \dot{v}(t) = Sv(t) \\ y_r(t) = C_r v(t) \end{cases} \quad (2)$$

where  $v(t) \in R^d$  and  $y_r(t) \in R^p$  are the state and output of the leader, and  $v(t)$  represents the signal to be tracked. In the output regulation problem, the leader is also called exosystem.  $S$  and  $C_r$  are known real matrices of appropriate dimensions.

Then the following definition of the fault-tolerant output regulation problem is given.

*Definition 1:* Consider a linear MAS composed by (1) and (2) under a directed topology  $G$ . If there are sensor faults in followers, the FTORP is solved under a distributed fault-tolerant control law  $u(t)$  so that all tracking errors between followers and leader converge to zero as time goes to infinity, i.e.,

$$\lim_{t \rightarrow \infty} (y_i(t) - y_r(t)) = 0 \quad (3)$$

Similar to [10], define a new form of state  $x_{si}(t) \in R^p$  to represent a filtered version of  $y_i(t)$ . Then the fault vectors are

transferred to the following state equation

$$\dot{x}_{si}(t) = -A_s x_{si}(t) + A_s C x_i(t) + A_s D f_i(t) \quad (4)$$

where the matrix  $-A_s \in R^{p \times p}$  is stable.

By introducing an auxiliary system, an augmented system is established. Denote  $\bar{x}_i(t) = (x_i^T(t), x_{si}^T(t))^T$  and  $\bar{y}_i(t) = x_{si}(t)$ , and thus the augmented system can be written as

$$\begin{cases} \dot{\bar{x}}_i(t) = \bar{A} \bar{x}_i(t) + \bar{B} u_i(t) + \bar{D} f_i(t) \\ \bar{y}_i(t) = \bar{C} \bar{x}_i(t) \end{cases} \quad (5)$$

where  $\bar{A} \in R^{(n+p) \times (n+p)}$ ,  $\bar{B} \in R^{(n+p) \times m}$ ,  $\bar{C} \in R^{p \times (n+p)}$  and  $\bar{D} \in R^{(n+p) \times r}$ , and their specific forms are as follows

$$\bar{A} = \begin{bmatrix} A & 0 \\ A_s C & -A_s \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},$$

$$\bar{C} = [0 \ I_p], \quad \bar{D} = \begin{bmatrix} 0 \\ A_s D \end{bmatrix}.$$

Since  $(A, C)$  is observable, it can be obtained that  $(\bar{A}, \bar{C})$  is also observable.

Denote the state, output, input and fault of the whole system as follows

$$\begin{aligned} \bar{x}(t) &= (\bar{x}_1^T(t), \bar{x}_2^T(t), \dots, \bar{x}_N^T(t))^T \\ \bar{y}(t) &= (\bar{y}_1^T(t), \bar{y}_2^T(t), \dots, \bar{y}_N^T(t))^T \\ u(t) &= (u_1^T(t), u_2^T(t), \dots, u_N^T(t))^T \\ f(t) &= (f_1^T(t), f_2^T(t), \dots, f_N^T(t))^T \end{aligned}$$

From (5), the following closed-loop system is obtained

$$\begin{cases} \dot{\bar{x}}(t) = (I_N \otimes \bar{A}) \bar{x}(t) + (I_N \otimes \bar{B}) u(t) + (I_N \otimes \bar{D}) f(t) \\ \bar{y}(t) = (I_N \otimes \bar{C}) \bar{x}(t) \end{cases} \quad (6)$$

To solve the problem of fault-tolerant consensus of multi-agent systems with sensor faults, some basic assumptions are given.

*Assumption 1:*  $(A, B)$  is stabilizable.

*Assumption 2:*  $(A, C)$  is observable.

*Assumption 3:*  $S$  has no eigenvalues with negative real parts.

*Assumption 4:*

$$\text{rank}(\bar{B}, \bar{D}) = \text{rank}(\bar{B}).$$

*Assumption 5:* Linear matrix equation

$$\begin{cases} X_N (I_N \otimes S) = \bar{A}_N X_N + \bar{B}_N U \\ 0 = \bar{C}_N X_N - C_r \end{cases} \quad (7)$$

has a solution pair  $(X_N, U)$ .

*Assumption 6:* The communication topology contains a spanning tree and node 0 is the root.

*Remark 1:* These are standard assumptions to solve the output regulation problem. Assumption 1 ensures that the system can be stabilized by a state feedback control law. Assumption 2 guarantees the existence of the observer. Since

in fact equation (7) is Sylvester equation, Assumption 3 is made to make sure that the linear matrix equation has a unique solution pair without loss of generality. Assumption 4 means that there exists a matrix  $\bar{B}^*$  that satisfies  $(I - \bar{B} \bar{B}^*) \bar{D} = 0$ , and details of the proof is presented in [19]. In Assumption 5, the equation (7) is called regulator equation which can guarantee the solvability of cooperative output regulation problem. Assumption 6 is a necessary condition for solving the FTORP.

Next the following lemma is given to solve the FTORP in this article.

*Lemma 1* [4]: Consider the following system

$$\begin{cases} \dot{x}_c = A_c x_c + B_c u + E_c w \\ e = C_c x_c + D_c u + F_c w \\ \dot{w} = S w \end{cases} \quad (8)$$

Assume that  $S$  satisfies Assumption 3 and  $A_c$  is Hurwitz, then

$$\lim_{t \rightarrow \infty} e(t) = 0 \quad (9)$$

if and only if there exists a unique matrix  $X_c$  that satisfies the following matrix equation

$$\begin{cases} X_c S = A_c X_c + B_c U + E_c \\ 0 = C_c X_c + D_c U + F_c \end{cases} \quad (10)$$

*Remark 2:* Equation (10) is the regulator equation in output regulation theory. Based on this, Lemma 1 is an important lemma to solve the output regulation problem. It means the output regulation problem can be solved by using the tool of linear algebra theory. It will be seen later that the equation required in the given system is a special form of (10).

### III. MAIN RESULTS

#### A. OBSERVER DESIGN

To estimate the states and outputs, we design the following observer

$$\begin{cases} \dot{\hat{x}}_i(t) = \bar{A} \hat{x}_i(t) + \bar{B} u_i(t) + \bar{D} \hat{f}_i(t) - L(\hat{y}_i(t) - \bar{y}_i(t)) \\ \hat{y}_i(t) = \bar{C} \hat{x}_i(t) \end{cases} \quad (11)$$

where  $\hat{x}_i(t) \in R^n$  is the observer state,  $\hat{y}_i(t) \in R^p$  is the observer output and  $\hat{f}_i(t) \in R^r$  is the estimation of  $f_i(t)$ . Since  $(\bar{A}, \bar{C})$  is observable, we can design the gain matrix  $L \in R^{(n+p) \times p}$  so that  $\bar{A} - L\bar{C}$  is stable. Define the following vector errors of states, outputs and faults

$$e_{xi}(t) = \hat{x}_i(t) - \bar{x}_i(t) \quad (12)$$

$$e_{yi}(t) = \hat{y}_i(t) - \bar{y}_i(t) \quad (13)$$

$$e_{fi}(t) = \hat{f}_i(t) - f_i(t) \quad (14)$$

From (6) and (11), we have

$$\begin{cases} \dot{e}_{xi}(t) = (\bar{A} - L\bar{C}) e_{xi}(t) + \bar{D} e_{fi}(t) \\ e_{yi}(t) = \bar{C} e_{xi}(t) \end{cases} \quad (15)$$

Define

$$\begin{aligned}\hat{x}(t) &= (\hat{x}_1^T(t), \hat{x}_2^T(t), \dots, \hat{x}_N^T(t))^T \\ \hat{y}(t) &= (\hat{y}_1^T(t), \hat{y}_2^T(t), \dots, \hat{y}_N^T(t))^T \\ \hat{f}(t) &= (\hat{f}_1^T(t), \hat{f}_2^T(t), \dots, \hat{f}_N^T(t))^T \\ e_x(t) &= (e_{x1}^T(t), e_{x2}^T(t), \dots, e_{xN}^T(t))^T \\ e_y(t) &= (e_{y1}^T(t), e_{y2}^T(t), \dots, e_{yN}^T(t))^T \\ e_f(t) &= (e_{f1}^T(t), e_{f2}^T(t), \dots, e_{fN}^T(t))^T\end{aligned}$$

and the error dynamics of the whole system is obtained as follows

$$\begin{cases} \dot{e}_x(t) = (I_N \otimes (\bar{A} - L\bar{C}))e_x(t) + (I_N \otimes \bar{D})e_f(t) \\ \dot{e}_y(t) = (I_N \otimes \bar{C})e_x(t) \end{cases} \quad (16)$$

The derivative of error vector  $e_f(t)$  with respect to time is as follows

$$\dot{e}_f(t) = \dot{\hat{f}}(t) - \dot{f}(t) \quad (17)$$

To solve the FTORP in this article, the following lemma is needed.

*Lemma 2* [20]: Given a scalar  $\mu > 0$  and a symmetric positive definite matrix  $P$ , then the inequality holds

$$2x^T y \leq \frac{1}{\mu} x^T P x + \mu y^T P^{-1} y, \quad x, y \in \mathbb{R}^n. \quad (18)$$

Based on above analysis, now we are ready to present the following observer of the fault-tolerant consensus problem. It provides the estimation of the sensor faults for control law.

*Theorem 1*: Under Assumptions 1–6 and given scalars  $\sigma, \mu > 0$ , if there exist matrices  $Y \in \mathbb{R}^{(n+p) \times p}$ ,  $F \in \mathbb{R}^{r \times p}$  and symmetric positive definite matrices  $P \in \mathbb{R}^{(n+p) \times (n+p)}$  and  $G \in \mathbb{R}^{r \times r}$  so that the following two conditions hold

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} < 0 \quad (19)$$

$$\bar{D}^T P = F C \quad (20)$$

where

$$\begin{aligned}\Sigma_{11} &= I_N \otimes (P\bar{A} + \bar{A}^T P - Y\bar{C} - \bar{C}^T Y^T) \\ \Sigma_{12} &= -\frac{1}{\sigma} (I_N \otimes (\bar{A}P\bar{D} - \bar{C}^T Y^T \bar{D})) \\ \Sigma_{22} &= -\frac{2}{\sigma} (I_N \otimes \bar{D}^T P \bar{D}) + \frac{1}{\sigma \mu} (I_N \otimes G)\end{aligned}$$

and  $Y = PL$  and  $*$  denotes the symmetric elements, then, the fault estimation algorithm

$$\dot{\hat{f}}(t) = -(I_N \otimes \Gamma F)(\dot{e}_y(t) + \sigma e_y(t)) \quad (21)$$

can realize convergence of  $e_x(t)$  and  $e_f(t)$ , where  $\Gamma^{-1} = \text{diag}(r_1, r_2, \dots, r_q)$  and positive constant  $r_i$  represents the learning rate.

Consider the following Lyapunov function

$$V(t) = e_x^T(t)(I_N \otimes P)e_x(t) + \frac{1}{\sigma} e_f^T(t)(I_N \otimes \Gamma^{-1})e_f(t) \quad (22)$$

Form (16), (17) and (21), the derivative of (22) with respect to time is

$$\begin{aligned}\dot{V}(t) &= \dot{e}_x^T(t)(I_N \otimes P)e_x(t) + e_x^T(t)(I_N \otimes P)\dot{e}_x(t) \\ &\quad + 2\frac{1}{\sigma} e_f^T(t)(I_N \otimes \Gamma^{-1})\dot{e}_f(t) \\ &= e_x^T(t)(I_N \otimes (P(\bar{A} - L\bar{C}) + (\bar{A} - L\bar{C})^T P))e_x(t) \\ &\quad - \frac{2}{\sigma} e_f^T(t)(I_N \otimes F)(\dot{e}_y(t) + \sigma e_y(t)) \\ &\quad - \frac{2}{\sigma} e_f^T(t)(I_N \otimes \Gamma^{-1})\dot{f}(t) \\ &\quad + 2e_x^T(t)(I_N \otimes P\bar{D})e_f(t)\end{aligned} \quad (23)$$

From (16), (20) and (23), we have

$$\begin{aligned}\dot{V}(t) &= e_x^T(t)(I_N \otimes (P(\bar{A} - L\bar{C}) + (\bar{A} - L\bar{C})^T P))e_x(t) \\ &\quad - \frac{2}{\sigma} e_f^T(t)(I_N \otimes \bar{D}^T P(\bar{A} - L\bar{C}))e_x(t) \\ &\quad - \frac{2}{\sigma} e_f^T(t)(I_N \otimes \bar{D}^T P\bar{D})e_f(t) \\ &\quad - \frac{2}{\sigma} e_f^T(t)(I_N \otimes \Gamma^{-1})\dot{f}(t)\end{aligned} \quad (24)$$

By Lemma 2, we can obtain that

$$\begin{aligned}& -\frac{2}{\sigma} e_f^T(t)(I_N \otimes \Gamma^{-1})\dot{f}(t) \\ & \leq \frac{1}{\sigma \mu} e_f^T(t)(I_N \otimes G)e_f(t) \\ & \quad + \frac{\mu}{\sigma} \dot{f}^T(t)(I_N \otimes \Gamma^{-1} G^{-1} \Gamma^{-1})\dot{f}(t) \\ & \leq \frac{1}{\sigma \mu} e_f^T(t)(I_N \otimes G)e_f(t) \\ & \quad + \frac{\mu}{\sigma} f_1^2 \lambda_{\max}(I_N \otimes \Gamma^{-1} G^{-1} \Gamma^{-1})\end{aligned} \quad (25)$$

From (24) and (25), we have

$$\dot{V}(t) \leq \zeta^T(t) \Sigma \zeta(t) + \delta \quad (26)$$

where

$$\begin{aligned}\Sigma &= \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} \\ \Sigma_{11} &= I_N \otimes (P(\bar{A} - L\bar{C}) + (\bar{A} - L\bar{C})^T P) \\ \Sigma_{12} &= -\frac{1}{\sigma} (I_N \otimes (\bar{A} - L\bar{C})^T P \bar{D}) \\ \Sigma_{22} &= -\frac{2}{\sigma} (I_N \otimes \bar{D}^T P \bar{D}) + \frac{1}{\sigma \mu} (I_N \otimes G) \\ \zeta(t) &= (e_x^T(t), e_f^T(t))^T \\ \delta &= \frac{\mu}{\sigma} f_1^2 \lambda_{\max}(I_N \otimes \Gamma^{-1} G^{-1} \Gamma^{-1}) \\ f_1 &= \dot{f}(t)\end{aligned}$$

Since  $\bar{D}$  is of column rank, we can obtain  $\dot{V}(t) \leq -\varepsilon \|\zeta^T(t)\|^2 + \delta$  when  $\Sigma < 0$ , where  $\varepsilon = \lambda_{\min}(-\Sigma)$ . Thus, if  $\varepsilon \|\zeta^T(t)\|^2 \geq \delta$ ,  $\dot{V}(t) \leq 0$ . By Lyapunov stability theory,  $e_x(t)$  and  $e_f(t)$  converge to zero, which means the estimation errors of states and faults are bounded.

*Remark 3*: From (26), if  $\dot{f}(t) = 0$ , i.e.,  $f_1 = 0$ , then the designed observer can also estimate the state and fault.

So it should be noted that the observer is valid for constant faults. Furthermore, by integration we can know that the fault estimation combines proportional term and integral term

$$\hat{f}(t) = -(I_N \otimes \Gamma F)(e_y(t) + \sigma \int_{t_f}^t e_y(\tau) d\tau) \quad (27)$$

where the proportional term plays an important role in improving the speed of fault estimation.

*Remark 4:* Now we consider how to solve the FTORP under the two conditions in Theorem 1. Equation (19) is a linear matrix inequality (LMI) and it is easy to solve by Matlab LMI Solvers. However, it is difficult to solve (20) at the same time. Therefore, we can transform this problem to an optimization problem as follows:

Minimize  $a$  subject to (19) and

$$\begin{bmatrix} aI & \bar{D}^T P - FC \\ (\bar{D}^T P - FC)^T & aI \end{bmatrix} > 0 \quad (28)$$

### B. FAULT-TOLERANT CONTROLLER DESIGN

In order to compensate for the impact of faults on the system and due to the fact that not all followers can directly get information from the leader, the following distributed adaptive fault-tolerant controller and exosystem observer are given

$$\begin{cases} u_i(t) = K_1 \hat{x}_i(t) + K_2 \eta_i(t) - \bar{B}^* \bar{D}_N^* \hat{f}_i(t) \\ \dot{\eta}_i(t) = S \eta_i(t) + \mu_i(t) \phi_i(t) \\ \phi_i(t) = \sum_{j \in N_i} a_{ij} (\eta_j(t) - \eta_i(t)) + a_{i0} (v(t) - \eta_i(t)) \end{cases} \quad (29)$$

where  $K_1 \in R^{m \times n}$  and  $K_2 \in R^{m \times d}$  are gain matrices to be designed,  $\eta_i(t) \in R^d$  is the estimation of  $v(t)$ ,  $\mu_i(t)$  is a positive function representing adaptive gain,  $N_i$  are neighbors of the node  $i$ ,  $a_{ij}$  is the element in Laplacian matrix  $L_G$  of  $G$ , and  $a_{i0}$  represents the information transmission between the  $i$ th follower and the leader. If there exists information transmission,  $a_{i0} = 1$ , otherwise  $a_{i0} = 0$ .

Let

$$\begin{aligned} \bar{B}_N^* &= I \otimes \bar{B}^* \\ \hat{v}(t) &= 1_N \otimes v(t) \\ H &= L_G + \Delta \\ \Delta &= \text{diag}(a_{10}, a_{20}, \dots, a_{N0}) \\ \eta(t) &= (\eta_1^T(t), \eta_2^T(t), \dots, \eta_N^T(t))^T \\ K_{1N} &= \text{diag}(K_1, K_1, \dots, K_1) \\ K_{2N} &= \text{diag}(K_2, K_2, \dots, K_2) \end{aligned}$$

then the controller (29) can be rewritten as

$$\begin{cases} u(t) = K_{1N} \hat{x}(t) + K_{2N} \eta(t) - \bar{B}_N^* \bar{D}_N^* \hat{f}(t) \\ \dot{\eta}(t) = ((I_N \otimes S) - \mu(t)(H \otimes I_d)) \eta(t) \\ \quad + \mu(t)(H \otimes I_d) \hat{v}(t) \\ \mu(t) = \alpha \phi^T(t)(H \otimes I_d) \phi(t) \\ \phi(t) = -(H \otimes I_d)(\eta(t) - \hat{v}(t)) \end{cases} \quad (30)$$

Let  $\bar{A}_N = I_N \otimes \bar{A}$ ,  $\bar{B}_N = I_N \otimes \bar{B}$ ,  $\bar{C}_N = I_N \otimes \bar{C}$  and  $\bar{D}_N = I_N \otimes \bar{D}$ , then the system (6) can be rewritten as

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}_N \bar{x}(t) + \bar{B}_N u(t) + \bar{D}_N f(t) \\ \bar{y}(t) = \bar{C}_N \bar{x}(t) \end{cases} \quad (31)$$

From (30) and (31), we have

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}_N \bar{x}(t) + \bar{B}_N u(t) + \bar{D}_N f(t) \\ &= \bar{A}_N \bar{x}(t) + \bar{B}_N (K_{1N} \hat{x}(t) + K_{2N} \eta(t) \\ &\quad - \bar{B}_N^* \bar{D}_N^* \hat{f}(t)) + \bar{D}_N f(t) \\ &= (\bar{A}_N + \bar{B}_N K_{1N}) \bar{x}(t) + \bar{B}_N K_{2N} \eta(t) \\ &\quad + \bar{B}_N K_{2N} \eta(t) - \bar{D}_N e_f(t) \end{aligned} \quad (32)$$

Define  $x_c(t) = (\bar{x}^T(t), \eta^T(t))^T$  and  $\xi_f(t) = (e_x^T(t), e_f^T(t))^T$ .

From (30) and (32), it is obtained that

$$\dot{x}_c(t) = A_c x_c(t) + B_c \xi_f(t) + C_c \hat{v}(t) \quad (33)$$

where

$$\begin{aligned} A_c &= \begin{pmatrix} \bar{A}_N + \bar{B}_N K_{1N} & \bar{B}_N K_{2N} \\ 0 & (I_N \otimes S) - \mu(t)(H \otimes I_d) \end{pmatrix}, \\ B_c &= \begin{pmatrix} \bar{B}_N K_{1N} & -\bar{D}_N \\ 0 & 0 \end{pmatrix}, \quad C_c = \begin{pmatrix} 0 \\ \mu(t)(H \otimes I_d) \end{pmatrix}. \end{aligned}$$

Now we prove that  $\eta_i(t)$  is the estimation of  $v(t)$ , and the adaptive gain  $\mu(t)$  approaches to constant as time goes to infinity and the system is stable.

In order to prove that the whole system is stable, the dynamics of  $\phi(t)$  is given as follows

$$\begin{aligned} \dot{\phi}(t) &= -(H \otimes I_d)((I_N \otimes S) - \mu(t)(H \otimes I_d)) \eta(t) \\ &\quad + \mu(t)(H \otimes I_d) \hat{v}(t) - (I_N \otimes S) \hat{v}(t) \\ &= -(H \otimes I_d)((I_N \otimes S) - \mu(t)(H \otimes I_d))(\eta(t) - \hat{v}(t)) \\ &= -((I_N \otimes S) - \mu(t)(H \otimes I_d))(H \otimes I_d)(\eta(t) - \hat{v}(t)) \\ &= ((I_N \otimes S) - \mu(t)(H \otimes I_d)) \phi(t) \end{aligned} \quad (34)$$

Define  $\tilde{\mu}(t) = \mu(t) - \mu$ , where  $\mu$  is a constant to be determined. We can construct the following Lyapunov function

$$V_1(t) = \frac{1}{2} \phi^T(t) \phi(t) + \frac{1}{2\alpha} \sum_{i=1}^N \tilde{\mu}_i^2(t) \quad (35)$$

then the derivative of  $V_1(t)$  with respect to time is

$$\begin{aligned} \dot{V}_1(t) &= \phi^T(t)((I_N \otimes S) - \mu(t)(H \otimes I_d)) \phi(t) \\ &\quad + \frac{1}{\alpha} \sum_{i=1}^N \tilde{\mu}_i(t) \dot{\tilde{\mu}}_i(t) \\ &= \phi^T(t)((I_N \otimes S) - \mu(t)(H \otimes I_d)) \phi(t) \\ &\quad + \tilde{\mu}(t) \phi^T(t)(H \otimes I_d) \phi(t) \\ &= \phi^T(t)((I_N \otimes S) - \mu(H \otimes I_d)) \phi(t) \\ &\quad - \phi^T(t) \tilde{\mu}(t)(H \otimes I_d) \phi(t) + \tilde{\mu}(t) \phi^T(t)(H \otimes I_d) \phi(t) \\ &= \phi^T(t)((I_N \otimes S) - \mu(H \otimes I_d)) \phi(t) \end{aligned} \quad (36)$$

The eigenvalues of  $(I_N \otimes S) - \mu(H \otimes I_d)$  are  $\lambda_i(S) - \mu \lambda_j(H)$ ,  $i = 1, 2, \dots, d, j = 1, 2, \dots, N$ , where  $\lambda_i(S)$  and

$\lambda_j(H)$  are the eigenvalues of  $S$  and  $H$ , respectively. The real parts of  $\lambda_j(H)$  are positive, so there exists a sufficient large constant  $\mu$  so that  $\dot{V}_1(t) < 0$ . Thus, when  $t \rightarrow \infty$ , it can be obtained that  $\phi(t) \rightarrow 0$  and  $\tilde{\mu}(t) = \mu(t) - \mu \rightarrow 0$ . Therefore  $\eta(t)$  is the estimation of  $\hat{v}(t)$  and  $\mu(t)$  converges to constant.

According to Assumption 1, we know that  $(\bar{A}, \bar{B})$  and  $(\bar{A}_N, \bar{B}_N)$  are stabilizable. Thus, there exists  $K_{1N}$  so that  $\bar{A}_N + \bar{B}_N K_{1N}$  is stable. As proved before,  $\phi(t) \rightarrow 0$ , so  $(I_N \otimes S) - \mu(t)(H \otimes I_d)$  is stable. Since the block (2,2) in  $A_c$  has the same structure with  $\phi(t)$ , we can conclude that the system is stable.

*Remark 5:* In some existing studies such as [21], the gain matrix in control law depends on eigenvalues of Laplacian matrix, which contains a global information. In this article, the global information is not needed by giving adaptive gain. Thus, even if all the information of the system can not be obtained, the controller can still be designed.

Next, we consider how to solve the FTORP with tracking errors under sensor faults using output regulation theory.

*Remark 6:* Since  $(\bar{A}_N, \bar{B}_N)$  is stabilizable, we choose  $K_{1N}$  that makes  $\bar{A}_N + \bar{B}_N K_{1N}$  stable. Select  $K_{2N}$  as

$$K_{2N} = U - K_{1N}X_N \quad (37)$$

From (7) and (37), we have

$$\begin{aligned} X_N(I_N \otimes S) &= \bar{A}_N X_N + \bar{B}_N(K_{1N}X_N + K_{2N}) \\ &= (\bar{A}_N + \bar{B}_N K_{1N})X_N + \bar{B}_N K_{2N} \end{aligned} \quad (38)$$

Based on the above analysis, we have provided control law with gain matrices and control gains from the regulator equation, then the following theorem is given:

*Theorem 2:* Based on Assumptions 3 to 5, select  $K_{1N}$  such that  $\bar{A}_N + \bar{B}_N K_{1N}$  is stable and  $K_{2N}$  as (37), then the FTORP can be solved by the controller (30).

The output of the leader is

$$y_r(t) = C_r \hat{v} \quad (39)$$

Then the tracking error can be expressed as

$$\begin{aligned} e(t) &= \bar{y}(t) - y_r(t) \\ &= \bar{C}_N \bar{x}(t) - C_r \hat{v} \end{aligned} \quad (40)$$

In this article, faults are assumed to be constants, i.e.,  $\dot{f}(t) = 0$ , then from (16), (17) and (21), the derivative of  $\zeta(t)$  with respect to time is

$$\dot{\zeta}(t) = D_c \zeta(t) \quad (41)$$

where

$$D_c = \begin{pmatrix} I_N \otimes (\bar{A} - L\bar{C}) & I_N \otimes \bar{D} \\ -I_N \otimes \Gamma F \bar{C} (\bar{A} - L\bar{C}) + \sigma I & -I_N \otimes \Gamma F \bar{C} \bar{D} \end{pmatrix}$$

Define  $\bar{x}(t) = (x_c^T(t), \zeta^T(t))^T$ , and then from (33) and (41) it is obtained that

$$\dot{\bar{x}}(t) = A_R \bar{x}(t) + B_R \hat{v}(t) \quad (42)$$

where

$$A_R = \begin{pmatrix} A_c & B_c \\ 0 & D_c \end{pmatrix} \quad \text{and} \quad B_R = \begin{pmatrix} C_c \\ 0 \end{pmatrix}.$$

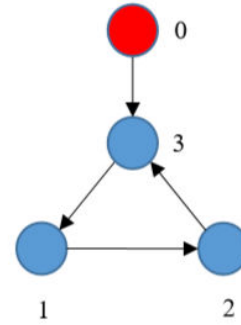


FIGURE 1. Communication topology.

The tracking error can be expressed as

$$e = C_R x_c(t) - C_r \hat{v} \quad (43)$$

where

$$C_R = (\bar{C}_N \quad 0)$$

Thus, the system can be written as

$$\begin{cases} \dot{\bar{x}}(t) = A_R \bar{x}(t) + B_R \hat{v}(t) \\ e(t) = C_R x_c(t) - C_r \hat{v} \end{cases} \quad (44)$$

Let  $X_I = (X_1^T, 0)^T$ , then

$$A_R X_I + B_R = \begin{pmatrix} A_c X_1 + C_c \\ 0 \end{pmatrix} \quad (45)$$

where  $X_1 = (X_N^T, I)^T$ , and

$$\begin{aligned} A_c X_1 + C_c &= \begin{pmatrix} (\bar{A}_N + \bar{B}_N K_{1N})X_N + \bar{B}_N K_{2N} \\ (I_N \otimes S) - \mu(t)(H \otimes I_d) + \mu(t)(H \otimes I_d) \end{pmatrix} \\ &= \begin{pmatrix} X_N(I_N \otimes S) \\ I_N \otimes S \end{pmatrix} \\ &= X_1(I_N \otimes S) \end{aligned} \quad (46)$$

$$C_R X_1 - C_r = \bar{C}_N X_N - C_r = 0 \quad (47)$$

Hence according to Lemma 1,  $X_I$  is a solution to the matrix equation corresponding to (44). Furthermore, based on Assumption 3,  $(I_N \otimes S)$  and  $A_R$  have no common eigenvalues, so  $X_1$  is the unique matrix satisfying (46) and then  $X_I$  is the unique solution. Thus,  $X_I$  is the unique solution to the matrix equation corresponding to (44). By Lemma 1,  $\lim_{t \rightarrow \infty} e(t) = 0$ .

#### IV. SIMULATION

In this section, simulation results are given to verify the validity of the algorithm. Consider the following multi-agent system including one leader and three followers

$$\begin{aligned} A &= \begin{pmatrix} 0 & 1 \\ 0 & -0.5 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C = (1 \quad 0), \\ D &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad S = \begin{pmatrix} 0 & 0.05 \\ -0.05 & 0 \end{pmatrix}, \quad C_r = (1 \quad 0). \end{aligned}$$

The communication topology is shown in Fig.1. The adjacency matrix and the diagonal matrix representing communication information between the leader and followers are as

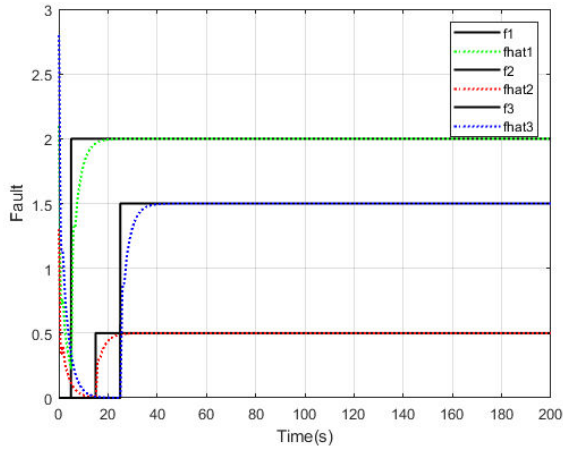


FIGURE 2. Estimations of sensor faults.

follows

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Consider the following sensor faults

$$f_1(t) = \begin{cases} 0 & 0 < t < 5 \\ 2 & t \geq 5 \end{cases}$$

$$f_2(t) = \begin{cases} 0 & 0 < t < 15 \\ 0.5 & t \geq 15 \end{cases}$$

$$f_3(t) = \begin{cases} 0 & 0 < t < 25 \\ 1.5 & t \geq 25 \end{cases}$$

The initial values of system and observer are

$$v(t) = \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix}, \quad x_1(t) = \begin{pmatrix} 0.6 \\ 0.5 \\ 0.4 \end{pmatrix}, \quad x_2(t) = \begin{pmatrix} 0.8 \\ 0.9 \\ 0.6 \end{pmatrix},$$

$$x_3(t) = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix}, \quad \hat{x}_1(t) = \hat{x}_2(t) = \hat{x}_3(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\eta_1(t) = \eta_2(t) = \eta_3(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$\hat{f}(t) = \begin{pmatrix} 2.1 \\ 1.3 \\ 2.8 \end{pmatrix}, \quad \mu(t) = \begin{pmatrix} 1.2 \\ 0.8 \\ 0.3 \end{pmatrix}.$$

Let  $\sigma = 1$ , then by solving equations (7) and (37), the gain matrices can be selected as follows:

$$L = \begin{pmatrix} 1.7218 \\ -0.0214 \\ 2.1167 \end{pmatrix},$$

$$K_1 = \begin{pmatrix} -28.1640 & -9.1570 & -16.5341 \end{pmatrix},$$

$$K_2 = \begin{pmatrix} 16.5341 & 0.0000 \end{pmatrix}.$$

The results are shown in Figs. 2-5. Fig. 2 shows the estimation results of followers subject to sensor faults, which can

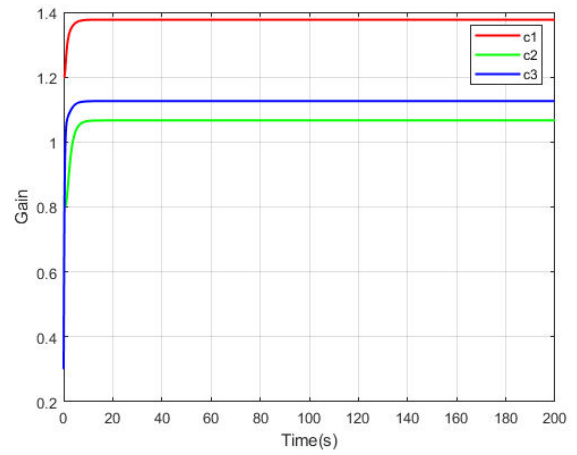


FIGURE 3. Adaptive gains.

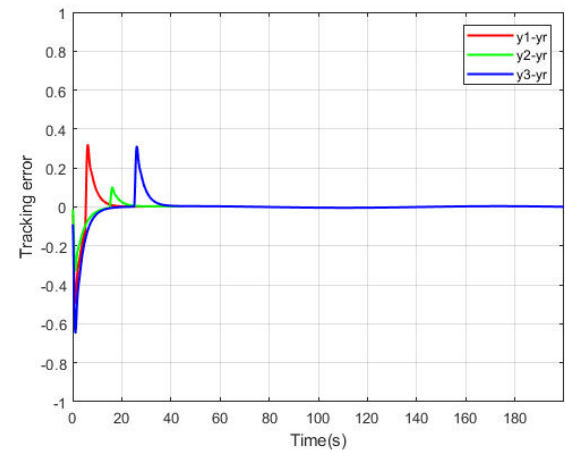


FIGURE 4. Tracking errors.

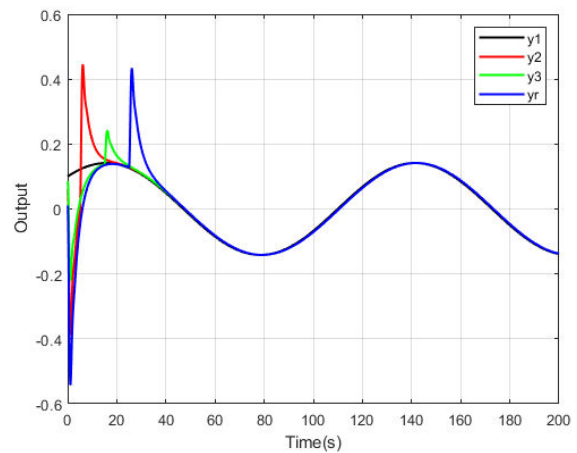


FIGURE 5. Outputs of the leader and followers.

converge to the real values of faults. Fig. 3 shows the adaptive gains, and it can be seen that they will converge to some constants. Fig. 4 shows that all tracking errors will converge to zero as time goes to infinity. Fig. 5 is the outputs of

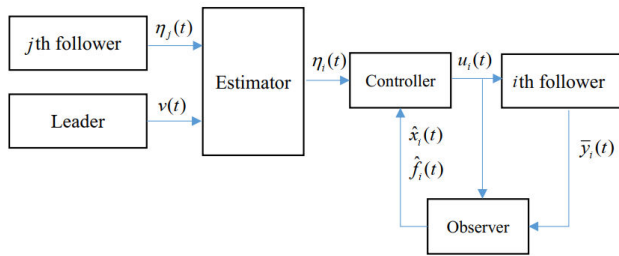


FIGURE 6. Fault-tolerant output regulation scheme.

the leader and followers. From the above simulation results, we can conclude that the outputs of followers can track the leader and then the consensus is achieved under sensor faults and directed topology.

## V. CONCLUSION

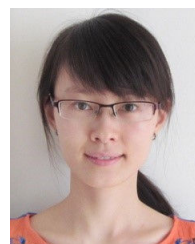
This article has investigated the distributed FTORP of linear leader-follower MASs with sensor faults under directed topology via output regulation theory. Fig. 6 shows the fault-tolerant output regulation scheme. We have constructed an augmented system by introducing an auxiliary system, and the fault-tolerant output regulation framework has been proposed based on the augmented system. Distributed observers have been designed to estimate the state and fault, which can compensate for effects of sensor faults existing in the followers. It has been proved that the designed distributed fault-tolerant controller can guarantee the stability and convergence of tracking errors regardless of sensor faults. Finally the simulation results have verified the validity of the algorithm. In the future, we will consider the distributed fault-tolerant control problem of nonlinear leader-follower MASs with time-varying faults.

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