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Output Tracking of Boolean Control Networks With Impulsive Effects

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ABSTRACT In this article, the output tracking of Boolean control networks with impulsive effects (BCNs-IE) is discussed. Based on structure matrices of BCNs-IE and controllability matrices with respect to state subsets, stable complex attractors are studied. By constructing an auxiliary BCN-IE and using stable complex attractors, a necessary and sufficient condition for the output tracking problem is proposed. In addition, an algorithm to design state feedback controllers for the output tracking problem is provided. Moreover, an example is given to show the validity of the obtained results.

INDEX TERMS Boolean control network, impulsive effect, output tracking, stable complex attractor, state feedback controller.

I. INTRODUCTION

To model genetic regulation networks, Kauffman [1] proposed Boolean networks (BNs) with variables taking values 0 or 1. Since then, BNs has been widely used in biological fields [2]–[5]. Consider the influence of external environment as a control variable of a BN, Boolean control networks (BCNs) [6] were introduced. As a creative mathematical tool for studying BNs and BCNs, the semi-tensor product (STP) of matrices was presented and applied to convert BNs and BCNs into linear (bilinear) discrete-time systems [7]. Thanks to the application of STP, many excellent works about BNs and BCNs are emerged, such as controllability and observability [8]–[10], stability and stabilization [11]–[13], optimal control problem [14]–[16] and other related problems [17]–[19].

As a significant issue in the control theory, the purpose of output tracking is to design suitable controllers that steer the output of a system to a given reference signal. In biological systems, the automated monitoring of cell populations in a high-throughput, high-content environment depends on accurate cell tracking of individual cell that display various behaviors. Reference [20] presented a cell tracking approach, which explicitly models cell behaviors in graph-theoretic frameworks. In addition, many modern live-cell imaging experiments are technologies that automatically track and analyze the motion of objects in time-lapse microscopy images [21].

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Moreover, reference [22] designed controllers to drive a certain amount of *Escherichia coli* to an desired state. There is no doubt that [22] provides an example of the output tracking of genetic regulatory networks.

Based on the analysis above, it is meaningful to concern the output tracking problem of BCNs. By constructing matrices reflecting the (output) reachability, the output regulation problem of BCNs was solved in [23]–[25]. In order to track the output of a time-varying reference signal, reference [26] established a bilinear equation, which reflects the relationship between states and outputs. And reference [27] introduced an auxiliary system, calculated the set of control attractors and control invariant subsets of this system. The method proposed in [27] was also used to handle the output tracking of BCNs driven by a constant reference signal [28]. Furthermore, inspired by cycles and control invariant subsets, reference [29] defined the concept of stable complex attractor, which is similar to cycles, and contained in control invariant subsets. Therefore, it is possible to investigate the output tracking problem via stable complex attractors.

Impulsive systems serve as basic models to research such dynamical process, whose states undergo abrupt changes at certain instants. Due to the extensive applications in many fields, impulsive systems have received considerable attention. For example, reference [30] studied the controllability of complex-valued impulsive systems with time-varying delay in control input. Reference [31] concerned the Lyapunov stability problem for impulsive systems via event-triggered

impulsive control. And other types of stability problem for impulsive systems were considered in [32]–[35].

In biological networks, to describe the dynamic process of sudden changes of states, BCNs with impulsive effects (BCNs-IE) were discussed firstly in [36]. Many related problems about BCNs-IE are solved, such as control problems [37]–[39], stability and stabilization [40]–[42], output tracking [43] and so on. Although the output tracking problem of BCNs-IE has been addressed in [43], stable complex attractors were not considered to deal with this problem, and the corresponding controller was not designed. Therefore, this article re-discusses the output tracking problem of BCNs-IE on the basis of stable complex attractors. The main contributions of this article are shown as follows.

(1) Stable complex attractors of BCNs-IE are defined. Based on structure matrices of BCNs-IE and controllability matrices with respect to state subsets, an algorithm to find all stable complex attractors of BCNs-IE is given.

(2) According to the original BCN-IE and the time-varying reference signal, an auxiliary BCN-IE is constructed. Motivated by the results proposed in [27] and [29], a necessary and sufficient condition for the output tracking problem of BCNs-IE is presented.

(3) Combined the necessary and sufficient condition for the output tracking problem with properties of stable complex attractors, an approach to design state feedback controllers for this problem is provided.

The rest of this article is organised as follows. Section II reviews some necessary notations and the algebraic form of BCNs-IE. In Section III, stable complex attractors are studied. A necessary and sufficient condition for the output tracking problem of BCNs-IE is proposed. Besides, a method is presented for the controller design of this problem. Section IV provides an example to show the effectiveness of our main results. Some concluding remarks are given in Section V.

II. PRELIMINARIES

In this section, we introduce some necessary preliminaries about STP and BCNs-IE, which will be used throughout this article.

A. NOTATIONS

To begin with, we provide a list of notations, most of which can be found in [44] and [45].

- \mathbb{Z}^+ = $\{n|n \text{ is a positive integer}\}$ and $\mathbb{N} = \mathbb{Z}^+ \cup \{0\}$.
- $\mathcal{M}_{m \times n}$ contains all $m \times n$ real matrices.
- Given a matrix $M \in \mathcal{M}_{m \times n}$. Denote the i -th row (column) of M by $Row_i(M)$ ($Col_i(M)$). For $\omega = \{i_j \in \mathbb{Z}^+ | j = 1, 2, \dots, l\}$ with $i_1 < i_2 < \dots < i_l$, denote $Col_\omega(M) = [Col_{i_1}(M), \dots, Col_{i_l}(M)]$ and $Row_\omega(M) = [Row_{i_1}(M), \dots, Row_{i_l}(M)]$. Particular, $M_{ij} = Row_i(Col_j(M))$ and $M|_\omega = Row_\omega(Col_\omega(M))$.

- $\mathcal{D} = \{0, 1\}$.
- $\mathcal{B}_{m \times n} := \{B \in \mathcal{M}_{m \times n} | B_{ij} \in \mathcal{D}\}$ includes all $m \times n$ Boolean matrices.

- $I_n = diag\{1, \dots, 1\}$ and $\Delta_n = \{\delta_n^i | \delta_n^i = Col_i(I_n), i = 1, 2, \dots, n\}$. Denote $\Delta := \Delta_2$.

- $\mathbf{1}_{m \times n} = [\mathbf{1}_m, \dots, \mathbf{1}_m] \in \mathcal{M}_{m \times n}$, where $\mathbf{1}_m := \sum_{i=1}^m \delta_m^i$.

- $\mathbf{0}$ is a zero matrix.

- $\mathcal{L}_{m \times n} := \{L \in \mathcal{B}_{m \times n} | Col_i(L) \in \Delta_m, i = 1, 2, \dots, n\}$ is the set of logical matrices. Denote logical matrix $[\delta_{2^n}^{i_1}, \dots, \delta_{2^n}^{i_l}]$ by $\delta_{2^n}[i_1, \dots, i_l]$. Denote $\omega = \{i_j \in \mathbb{Z}^+ | j = 1, 2, \dots, l\}$, then $\delta_{2^n}^\omega = \{\delta_{2^n}^{i_j} | i_j \in \omega\}$.

- $Sgn(\cdot)$ is the sign function. Given a matrix $M \in \mathcal{M}_{m \times n}$. Define $(Sgn(M))_{ij} := Sgn(M_{ij})$.

- Given two matrices $P \in \mathcal{M}_{m \times n}$ and $Q \in \mathcal{M}_{p \times q}$. The STP of P and Q is defined as

$$P \times Q := (P \otimes I_{t/n})(Q \otimes I_{t/p}),$$

where \otimes is the Kronecker product, t is the least common multiple of n and p . Without confusion, the symbol \times is omitted.

To expressed logical functions in algebraic forms, two lemmas are presented.

Lemma 1 [44]: For an n -ary Boolean function $f(x_1, x_2, \dots, x_n)$, there exists a unique structure matrix $L_f \in \mathcal{L}_{2 \times 2^n}$ such that $f(x_1, x_2, \dots, x_n)$ is expressed in vector form as

$$f(x_1, x_2, \dots, x_n) = L_f \times_{i=1}^n x_i.$$

Lemma 2 [44]: Assume

$$\begin{cases} y = M_y \times_{i=1}^n x_i, \\ z = M_z \times_{i=1}^n x_i, \end{cases}$$

where $x_i \in \Delta, i = 1, 2, \dots, n, M_y, M_z \in \mathcal{L}_{2 \times 2^n}$. Then

$$yz = (M_y * M_z) \times_{i=1}^n x_i,$$

where $M_y * M_z = [Col_1(M_y) \otimes Col_1(M_z), \dots, Col_{2^n}(M_y) \otimes Col_{2^n}(M_z)]$.

B. MATRIX EXPRESSION OF BCNS-IE

A BCN-IE is described as

$$\begin{cases} x_1(t+1) = f_1(u_1(t), \dots, u_m(t), x_1(t), \dots, x_n(t)), \\ \vdots \\ x_n(t+1) = f_n(u_1(t), \dots, u_m(t), x_1(t), \dots, x_n(t)), \\ t \neq t_k, k \in \mathbb{Z}^+, \\ x_1(t_k+1) = g_1(x_1(t_k), \dots, x_n(t_k)), \\ \vdots \\ x_n(t_k+1) = g_n(x_1(t_k), \dots, x_n(t_k)), \\ y_1(t) = h_1(x_1(t), \dots, x_n(t)), \\ \vdots \\ y_p(t) = h_p(x_1(t), \dots, x_n(t)), \end{cases} \quad (1)$$

where $\{t_k | t_k < t_{k+1}, k \in \mathbb{Z}^+\} \subset \mathbb{Z}^+$ is an impulsive time sequence. f_i, g_i and h_j are Boolean functions. $x_i \in \mathcal{D}, u_j \in \mathcal{D}$ and $y_k \in \mathcal{D}$ are state variables, input variables and output variables, respectively.

Identify $1 \sim \delta_2^1$ and $0 \sim \delta_2^2$. Set $x = \times_{i=1}^n x_i, u = \times_{i=1}^m u_i$ and $y = \times_{i=1}^p y_i$. Combined with Lemma 1, BCN-IE (1) is written as

$$\begin{cases} x_1(t+1) = M_1 u(t)x(t), \\ \vdots \\ x_n(t+1) = M_n u(t)x(t), \\ t \neq t_k, k \in \mathbb{Z}^+, \\ x_1(t_k+1) = G_1 x(t_k), \\ \vdots \\ x_n(t_k+1) = G_n x(t_k), \\ y_1(t) = H_1 x(t), \\ \vdots \\ y_p(t) = H_p x(t), \end{cases}$$

where $M_i \in \mathcal{L}_{2 \times 2^{m+n}}$, $G_i \in \mathcal{L}_{2 \times 2^n}$ and $H_j \in \mathcal{L}_{2 \times 2^n}$. From Lemma 2, the matrix expression of system (1) is derived

$$\begin{cases} x(t+1) = L_1 u(t)x(t), \\ t \neq t_k, k \in \mathbb{Z}^+, \\ x(t_k+1) = L_2 x(t_k), \\ y(t) = Hx(t), \end{cases} \quad (2)$$

where $L_1 \in \mathcal{L}_{2^n \times 2^{m+n}}$, $L_2 \in \mathcal{L}_{2^n \times 2^n}$ and $H \in \mathcal{L}_{2^n \times 2^n}$.

From Formula (2), it is not hard to find that a BCN-IE can be considered as a switched BCN with a specific switching signal. If BCN-IE (2) is controllable, then there exists a proper control sequence such that initial state $\delta_{2^n}^i$ reaches each state in time 2^{n+1} . This result has been given in [46].

Lemma 3 [46]: Consider BCN-IE (2) as a switched BCN with a specific switching signal. Then the system is controllable iff each state of the system reaches any state in time 2^{n+1} with proper controls.

Remark 1: By analyzing BCN-IE (2), the state changes abruptly at a prescribed time. It shows that the impulsive effect this article studied is triggered by time. But reference [42] concerned the other impulsive effect, which is triggered by system states. It is the first difference between these two impulsive effects. In addition, as shown in Lemma 3, BCN-IE (2) can be viewed as a switched BCN with a specific switching signal. However, states of state-triggered impulsive BNs may jump successively at a single time instant. Reference [42] points out that it makes state-triggered impulsive BNs different from switched systems.

Let $x(t; x_0, u(t))$ ($y(t; x_0, u(t))$) represent the state (output) at time t with initial state x_0 under control input $u(t)$.

III. MAIN RESULTS

This section discusses stable complex attractors of BCNs-IE, considers the output tracking of BCNs-IE, and designs state feedback controllers for the output tracking problem.

A. STABLE COMPLEX ATTRACTORS

To begin with, the definition of stable complex attractor is proposed.

Definition 1: A set $\Omega \subset \Delta_{2^n}$ is called a complex attractor of BCN-IE (2), if for $\forall x_i, x_j \in \Omega$, there exist an integer T_{ij} and a control sequence $\{u_{ij}(t)|t \in \mathbb{N}\}$ such that $x(T_{ij}; x_i, u_{ij}(T_{ij})) = x_j$ and $x(t; x_i, u_{ij}(t)) \in \Omega, t \in [0, T_{ij}]$.

A set $\Omega \subset \Delta_{2^n}$ is said to be stable, if $\forall x_i \in \Omega$ and $\forall x_k \notin \Omega$, x_k is not reachable from x_i .

As shown in Section I, stable complex attractors are similar to cycles and contained in control invariant subsets. Some explication are given in the following.

If Ω is a cycle, then for $\forall x_i, x_j \in \Omega$, there exists an integer T_{ij} such that $x(T_{ij}; x_i) = x_j$ and $x(t; x_i) \in \Omega, t \in [0, T_{ij}]$. Because the concept of cycle is defined in BNs, one knows that $x(t; x_i) \in \Omega$ holds all the time for $\forall x_i \in \Omega$. From the definition of stable complex attractor, one sees the similarity between cycles and stable complex attractors.

A subset Ω is called a control invariant subset, if there exist a control sequence $\{u(t)|t \in \mathbb{N}\}$ such that $x(t; x_i, u(t)) \in \Omega$ holds for $\forall x_i \in \Omega, t \geq 1$. It is easy to find that stable complex attractors are control invariant subsets. But a control invariant subset may not be a stable complex attractor. The reasons are as follows. State $x_i \in \Omega$ may reach state $x_j \notin \Omega$. And state $x_i \in \Omega$ may not be reachable from $x_k \in \Omega$. Thus, stable complex attractors are contained in control invariant subsets.

Defined $\hat{M} = \sum_{i=1}^{2^m} L_1 \delta_{2^m}^i$ and $M = \text{Sgn}(\hat{M})$. Obviously, $M_{ij} = 1$ implies that there exists an input $u(t-1)$ such that BCN-IE (2) reaches $x(t) = \delta_{2^n}^i$ from $x(t-1) = \delta_{2^n}^j (t \neq t_k, k \in \mathbb{Z}^+)$. If $t = t_k, k \in \mathbb{Z}^+$, then $(L_2)_{ij} = 1$ means that BCN-IE (2) can reach $x(t_k+1) = \delta_{2^n}^i$ from $x(t_k) = \delta_{2^n}^j$. Based on matrices M and L_2 , the following proposition is provided.

Proposition 1: The following statements are equivalent:

(1) A set $\Omega = \delta_{2^n}^\omega = \{\delta_{2^n}^i | i_j \in \omega\} \subset \Delta_{2^n}$ is stable in BCN-IE (2).

(2) $\text{Row}_{\bar{\omega}}(\text{Col}_\omega(M)) = \mathbf{0}$ and $\text{Row}_{\bar{\omega}}(\text{Col}_\omega(L_2)) = \mathbf{0}$, where $\bar{\omega} = \{i_k | i_k \notin \omega\}$.

(3) $\sum_{i=1}^l \text{Row}_i(\hat{M}|_\omega) = 2^m \mathbf{1}_l^T$ and $L_2|_\omega \in \mathcal{L}_{l \times l}$.

Proof 1: By Definition 1, Ω is stable in BCN-IE (2), if and only if for $\forall x_i \in \Omega$ and $\forall x_k \notin \Omega$, x_k is not reachable from x_i . It is equivalent to $\text{Row}_{\bar{\omega}}(\text{Col}_\omega(M)) = \mathbf{0}$ and $\text{Row}_{\bar{\omega}}(\text{Col}_\omega(L_2)) = \mathbf{0}$. That is $L_2|_\omega \in \mathcal{L}_{l \times l}$. Since $M = \text{Sgn}(\hat{M})$, we also get $\sum_{i=1}^l \text{Row}_i(\hat{M}|_\omega) = 2^m \mathbf{1}_l^T$.

Let $\Omega = \delta_{2^n}^\omega = \{\delta_{2^n}^i | i_j \in \omega\} \subset \Delta_{2^n}$. Now we construct such a matrix \mathcal{C}_Ω , which reflects the controllability of states in Ω . Suppose $t_q \leq 2^{n+1} < t_{q+1}$, where $q \in \mathbb{Z}^+$ is a constant. A sequence of matrices $\Gamma_r^\Omega \subset \mathcal{B}_{l \times l}, r = 1, 2, \dots, q+1$ is calculated

$$\begin{cases} \Gamma_1^\Omega = \sum_{i=1}^{t_1} (M|_\omega)^i, \\ \Gamma_2^\Omega = \sum_{i=0}^{t_2-t_1-1} (M|_\omega)^{t_1} L_2|_\omega (M|_\omega)^i, \\ \dots \\ \Gamma_q^\Omega = \sum_{i=0}^{t_q-t_{q-1}-1} ((M|_\omega)^{t_1} L_2|_\omega \prod_{l=2}^{q-1} (M|_\omega)^{t_l-t_{l-1}-1} L_2|_\omega) (M|_\omega)^i, \\ \Gamma_{q+1}^\Omega = \sum_{i=0}^{2^{n+1}-t_q} ((M|_\omega)^{t_1} L_2|_\omega \prod_{l=2}^q (M|_\omega)^{t_l-t_{l-1}-1} L_2|_\omega) (M|_\omega)^i. \end{cases}$$

Then the controllability matrix with respect to Ω is

$$C_{\Omega} = Sgn\left(\sum_{r=1}^{q+1} \Gamma_r^{\Omega}\right). \quad (3)$$

With no doubt, $(C_{\Omega})_{kj} = 1$ implies that system (2) reaches state $\delta_{2^n}^{ik}$ from state $\delta_{2^n}^{ij}$. If $\Omega = \Delta_{2^n}$, then $C_{\Delta_{2^n}}$ is the controllability matrix of BCN-IE (2).

According to the controllability matrix with respect to Ω , the following result is obtained.

Proposition 2: A set $\Omega = \delta_{2^n}^{\omega} = \{\delta_{2^n}^{ij} | i, j \in \omega\} \subset \Delta_{2^n}$ is a complex attractor of BCN-IE (2), if and only if $C_{\Omega} = \mathbf{1}_{l \times l}$.

Proof 2: According to Definition 1, it is easy to see that this result holds.

By Propositions 1 and 2, an approach to find all stable complex attractors of BCNs-IE is given.

Algorithm 1: Consider BCN-IE (2).

Step 1: Compute the controllability matrix $C_{\Delta_{2^n}}$. If $C_{\Delta_{2^n}} = \mathbf{1}_{2^n \times 2^n}$, then Δ_{2^n} is the unique stable complex attractor of BCN-IE (2). Otherwise, go to the next step.

Step 2: On the basis of Proposition 1, find ω_i satisfying $\sum_{i=1}^l \text{Row}_i(\hat{M}|_{\omega_i}) = 2^m \mathbf{1}_l^T$ and $L_2|_{\omega_i} \in \mathcal{L}_{l_i \times l_i}$.

Step 3: Denote $\Omega_i = \delta_{2^n}^{\omega_i}$. From Proposition 2, calculate the controllability matrix C_{Ω_i} . If $C_{\Omega_i} = \mathbf{1}_{l_i \times l_i}$, then Ω_i is a stable complex attractor of BCN-IE (2).

To provide a necessary and sufficient condition for the output tracking problem, a useful result about stable complex attractors is presented.

Proposition 3: Consider BCN-IE (2). Each state of BCN-IE (2) can reach one of stable complex attractors.

Proof 3: Suppose $\Omega_1, \dots, \Omega_{\nu}$ are all stable complex attractors of BCN-IE (2). We only need to prove that each state of $\Delta_{2^n} \setminus (\bigcup_{i=1}^{\nu} \Omega_i)$ can reach one of stable complex attractors. Assume $\delta_{2^n}^{i_0} \in \Delta_{2^n} \setminus (\bigcup_{i=1}^{\nu} \Omega_i)$ satisfies $x(t; \delta_{2^n}^{i_0}, u(t)) \notin \Omega_i, i = 1, 2, \dots, \nu$ for $\forall t \geq 1$. If $\delta_{2^n}^{i_0}$ only reaches state $\delta_{2^n}^{j_1} \in \Delta_{2^n} \setminus (\bigcup_{i=1}^{\nu} \Omega_i), j = 1, 2, \dots, k$, then $\delta_{2^n}^{j_1}, j = 1, 2, \dots, k$ satisfies $x(t; \delta_{2^n}^{j_1}, u(t)) \notin \Omega_i, i = 1, 2, \dots, \nu$ for $\forall t \geq 1$. It means that $\{\delta_{2^n}^{i_0}, \delta_{2^n}^{j_1}, \dots, \delta_{2^n}^{k_1}\}$ is stable. And there is at least one stable complex attractor contained in $\{\delta_{2^n}^{i_0}, \delta_{2^n}^{j_1}, \dots, \delta_{2^n}^{k_1}\}$. It is a contradiction. Hence, each state of BCN-IE (2) can reach one of stable complex attractors.

B. OUTPUT TRACKING OF BCNS-IE

Suppose the time-varying reference signal is generated by the following BN:

$$\begin{cases} \hat{x}_1(t+1) = \hat{f}_1(\hat{x}_1(t), \dots, \hat{x}_{\hat{n}}(t)), \\ \vdots \\ \hat{x}_{\hat{n}}(t+1) = \hat{f}_{\hat{n}}(\hat{x}_1(t), \dots, \hat{x}_{\hat{n}}(t)), \\ \hat{y}_1(t) = \hat{h}_1(\hat{x}_1(t), \dots, \hat{x}_{\hat{n}}(t)), \\ \vdots \\ \hat{y}_p(t) = \hat{h}_p(\hat{x}_1(t), \dots, \hat{x}_{\hat{n}}(t)), \end{cases} \quad (4)$$

where \hat{f}_i and \hat{h}_j are Boolean functions, \hat{x}_i and \hat{y}_j are state variables and output variables, respectively.

Set $\hat{x} = \times_{i=1}^{\hat{n}} \hat{x}_i$ and $\hat{y} = \times_{i=1}^p \hat{y}_i$. By Lemmas 1 and 2, the matrix expression form of BN (4) is provided:

$$\begin{cases} \hat{x}(t+1) = \hat{L}\hat{x}(t), \\ \hat{y}(t) = \hat{H}\hat{x}(t), \end{cases} \quad (5)$$

where $\hat{L} \in \mathcal{L}_{2^{\hat{n}} \times 2^{\hat{n}}}$ and $\hat{H} \in \mathcal{L}_{2^p \times 2^{\hat{n}}}$.

The definition of output tracking is introduced in the following.

Definition 2: The output of BCN-IE (2) is said to track the output of time-varying reference signal (5), if there exist an integer T and a control sequence $\{u(t) | t \in \mathbb{N}\}$ such that $y(t; x(0), u(t)) = \hat{y}(t; \hat{x}(0))$ holds for $\forall x(0) \in \Delta_{2^n}, \hat{x}(0) \in \Delta_{2^{\hat{n}}}$ and $\forall t \geq T$.

Now we construct an auxiliary BCN-IE to study the solvability of the output tracking problem. Consider BCN-IE (2) and time-varying reference signal (5). Denote $\bar{x}(t) = x(t)\hat{x}(t)$ and $\bar{y}(t) = y(t)\hat{y}(t)$. By Lemma 2, the following BCN-IE is obtained

$$\begin{cases} \bar{x}(t+1) = \bar{L}_1 u(t) \bar{x}(t), \\ t \neq t_k, k \in \mathbb{Z}^+ \\ \bar{x}(t_k+1) = \bar{L}_2 \bar{x}(t_k), \\ \bar{y}(t) = \bar{H} \bar{x}(t), \end{cases} \quad (6)$$

where $\bar{L}_1 = (L_1 \otimes \mathbf{1}_{2^{\hat{n}}}^T) * (\mathbf{1}_{2^{m+\hat{n}}}^T \otimes \hat{L})$, $\bar{L}_2 = (L_2 \otimes \mathbf{1}_{2^{\hat{n}}}^T) * (\mathbf{1}_{2^{\hat{n}}}^T \otimes \hat{L})$ and $\bar{H} = (H \otimes \mathbf{1}_{2^{\hat{n}}}^T) * (\mathbf{1}_{2^{\hat{n}}}^T \otimes \hat{H})$.

Define $\Lambda = \{\delta_{2^{m+\hat{n}}}^i \times \delta_{2^{\hat{n}}}^j | i = 1, 2, \dots, 2^p\}$ and $\Theta = \{\delta_{2^{m+\hat{n}}}^j | \text{Col}_j(\bar{H}) \in \Lambda\}$. From Definition 2, the output of BCN-IE (2) tracks the output of reference signal (5), if and only if there exist an integer T and a control sequence $\{u(t) | t \in \mathbb{N}\}$ such that $\bar{y}(t; \bar{x}(0), u(t)) \in \Lambda$ holds for $\forall \bar{x}(0) \in \Delta_{2^{m+\hat{n}}}, \forall t \geq T$. Thus, it equals to the conclusion that there exist an integer T and a control sequence $\{u(t) | t \in \mathbb{N}\}$ such that $\bar{x}(t; \bar{x}(0), u(t)) \in \Theta$ holds for $\forall \bar{x}(0) \in \Delta_{2^{m+\hat{n}}}, \forall t \geq T$. It means that Θ contains one complex attractor at least. Therefore, the output tracking problem of BCN-IE (2) can be discussed by using stable complex attractors of BCN-IE (6).

Theorem 1: The output of BCN-IE (2) tracks the output of reference signal (5), if and only if $\Omega_i \cap \Theta \neq \emptyset, i = 1, 2, \dots, \nu$, where $\Omega_1, \dots, \Omega_{\nu}$ are all stable complex attractors of BCN-IE (6).

Proof 4: Based on the analysis above, we only need to prove that $\Omega_i \cap \Theta \neq \emptyset, i = 1, 2, \dots, \nu$, if and only if there exist an integer T and a control sequence $\{u(t) | t \in \mathbb{N}\}$ such that $\bar{x}(t; \bar{x}(0), u(t)) \in \Theta$ holds for $\forall \bar{x}(0) \in \Delta_{2^{m+\hat{n}}}, \forall t \geq T$.

(Necessity.) Assume $\Omega_i \cap \Theta \neq \emptyset, i = 1, 2, \dots, \nu$. Take an initial state $\bar{x}(0) \in \Omega_i, i = 1, 2, \dots, \nu$. Since $\Omega_1, \dots, \Omega_{\nu}$ are stable complex attractors of BCN-IE (6), there exist an integer T_i and a control sequence $\{u_i(t) | t \in \mathbb{N}\}$ such that $\bar{x}(t; \bar{x}(0), u_i(t)) \in \Theta$ holds for $\forall t \geq T_i$. Take an initial state $\bar{x}(0) \in \Delta_{2^{m+\hat{n}}} \setminus (\bigcup_{i=1}^{\nu} \Omega_i)$. Based on Proposition 3, by selecting control sequence $\{u'_{\nu+1}(t) | t \in \mathbb{N}\}$, there exists a stable

complex attractor Ω_i such that $\bar{x}(t; \bar{x}(0), u'_{v+1}(t)) \in \Omega_i$ holds. From the first case, there exist an integer T_{v+1} and a control sequence $\{u_{v+1}(t)|t \in \mathbb{N}\}$ such that $\bar{x}(t; \bar{x}(0), u_{v+1}(t)) \in \Theta$ holds for $\forall t \geq T_i$.

(Sufficiency.) Assume there exists a stable complex attractor Ω_i such that $\Omega_i \cap \Theta = \emptyset$. Then one gets $\bar{x}(t; \bar{x}(0), u(t)) \notin \Theta$ for each initial state $\bar{x}(0) \in \Delta_{2^{n+\hat{n}}}$ satisfying $\bar{x}(t; \bar{x}(0), u(t)) \in \Omega_i$. It is a contradiction. Hence, for each stable complex attractor $\Omega_i, \Omega_i \cap \Theta \neq \emptyset$.

Remark 2: Consider a constant reference signal $y_r = \delta_{2^p}^\alpha, 1 \leq \alpha \leq 2^p$. It is easy to see that $\Theta = \{\delta_{2^n}^j | Col_j(H) = y_r\}$. According to the analysis above, the output of BCN-IE (2) tracks the constant reference signal y_r , if and only if there are an integer T and a control sequence $\{u(t)|t \in \mathbb{N}\}$ such that $x(t; x(0), u(t)) \in \Theta$ holds for $\forall x(0) \in \Delta_{2^n}, \forall t \geq T$. From the proof of Theorem 1, it equals to $\Omega_i \cap \Theta \neq \emptyset, i = 1, 2, \dots, v$, where $\Omega_1, \dots, \Omega_v$ are all stable complex attractors of BCN-IE (2). Hence, Theorem 1 can be used to solve the output tracking problem driven by a constant reference signal.

Remark 3: This article studies the output tracking problem of BCNs-IE, while this problem of BCNs has been discussed in [23]–[28]. A time-varying reference output trajectory is concerned in this article. But the tracking object [23], [25], [28] considered is a constant reference signal, which is a special case of time-varying reference signal. References [23]–[28] do not use stable complex attractors to solve the output tracking problem. And the methods they proposed have been introduced in Section I.

Although [43] and this article investigate the output tracking problem of BCNs-IE, there are some differences between these two papers. From Theorems 3.7 and 3.8 of [43], the corresponding results hold under the assumption that Θ is an \bar{L}_1 -invariant set and an \bar{L}_2 -invariant set. However, Theorem 1 of this article holds without any preconditions. The control sequence required in Theorem 3.7 (3.8) is a state (an output) feedback control sequence, while Theorem 1 does not have this requirement. In addition, [43] does not use stable complex attractors to address the output tracking problem. And the corresponding controller is not designed in [43].

C. DESIGN OF STATE FEEDBACK CONTROLLERS

In order to design the state feedback controller for the output tracking problem of BCNs-IE, we prove the following theorem firstly.

Theorem 2: The output of BCN-IE (2) tracks the output of reference signal (5), if and only if the output of BCN-IE (2) tracks the output of reference signal (5) under the state feedback control $u(t) = Kx(t)\hat{x}(t)$.

Proof 5: The sufficiency is obvious, we only need to prove the necessity.

Define $\bar{x}(t) = x(t)\hat{x}(t)$ and $\bar{y}(t) = y(t)\hat{y}(t)$, BCN-IE (6) can be derived. Suppose $\Omega_1, \dots, \Omega_v$ are all stable complex attractors of BCN-IE (6). Denote the domain of attraction of stable

complex attractor Ω_i by $\Upsilon'_i = \{\bar{x}|\exists T_{\bar{x}} \text{ and } u(\cdot), \bar{x}(T_{\bar{x}}; \bar{x}, u) \in \Omega_i\}$. The following operators are done to make sure that $\{\Upsilon_i|i = 1, \dots, v\}$ is a partition of $\Delta_{2^{n+\hat{n}}}$:

$$\begin{cases} \Upsilon_1 = \Upsilon'_1, \\ \Upsilon_i = \Upsilon'_i \setminus \left(\bigcup_{j=1}^{i-1} \Upsilon_j\right), \quad j = 2, \dots, v. \end{cases} \quad (7)$$

According to Theorem 1, The output of BCN-IE (2) tracks the output of reference signal (5), if and only if $\Omega_i \cap \Theta \neq \emptyset, i = 1, 2, \dots, v$. From this result and $\{\Upsilon_i|i = 1, \dots, v\}$, the state feedback matrix K can be gotten by the following steps.

Denote $\Gamma_{i_1} = \Omega_i \cap \Theta$. Since $\Gamma_{i_1} \subset \Omega_i \subset \Upsilon_i$, we have $\Gamma_{i_2} = \{\bar{x}|\bar{L}_1 u \bar{x} \in \Gamma_{i_1}\} \setminus \Gamma_{i_1}$. If $\Gamma_{i_1} \cup \Gamma_{i_2} \neq \Upsilon_i$, then denote $\Gamma_{i_3} = \{\bar{x}|\bar{L}_1 u \bar{x} \in \Gamma_{i_2}\} \setminus \left(\bigcup_{j=1}^2 \Gamma_{i_j}\right)$. Repeat this process until

$\bigcup_{j=1}^{\mu_i} \Gamma_{i_j} = \Upsilon_i$. Take a state $\bar{x} = \delta_{2^{n+\hat{n}}}^\alpha \in \Delta_{2^{n+\hat{n}}}$. If $\delta_{2^{n+\hat{n}}}^\alpha \in \Gamma_{i_1}$,

then $Col_\alpha(K) \in \{\delta_{2^m}^\beta | Col_\alpha(\bar{L}_1 \delta_{2^m}^\beta) \in \Gamma_{i_1}\}$. If $\delta_{2^{n+\hat{n}}}^\alpha \in \Gamma_{i_j}, j = 2, \dots, \mu_i$, then $Col_\alpha(K) \in \{\delta_{2^m}^\beta | Col_\alpha(\bar{L}_1 \delta_{2^m}^\beta) \in \Gamma_{i_{j-1}}\}$. Do this process for each state subset $\Upsilon_i, i = 1, 2, \dots, v$, the state feedback matrix K is obtained.

Based on the results above, an approach to design the state feedback controller is proposed.

Algorithm 2: Consider BCN-IE (2) and reference signal (5). Design the state feedback control $u(t) = Kx(t)\hat{x}(t)$ such that the output of BCN-IE (2) tracks the output of reference signal (5).

Step 1: Compute matrices \bar{L}_1, \bar{L}_2 and \bar{H} , gain BCN-IE (6), the output subset Λ and the state subset Θ .

Step 2: Find all stable complex attractors $\Omega_i, i = 1, 2, \dots, v$ of BCN-IE (6).

Step 3: Compute $\Omega_i \cap \Theta, i = 1, 2, \dots, v$. If there exists Ω_i such that $\Omega_i \cap \Theta = \emptyset$, then the output of BCN-IE (2) does not track the output of reference signal (5). Otherwise, go to the next step.

Step 4: According to (7), calculate $\Upsilon_i, i = 1, 2, \dots, v$. Denote $\Gamma_{i_1} = \Omega_i \cap \Theta$. Compute $\Gamma_{i_j} = \{\bar{x}|\bar{L}_1 u \bar{x} \in \Gamma_{i_{j-1}}\}$ until $\bigcup_{j=1}^{\mu_i} \Gamma_{i_j} = \Upsilon_i$.

Step 5: For each state $\delta_{2^{n+\hat{n}}}^\alpha \in \Upsilon_i, i = 1, \dots, v$, the state feedback matrix K is obtained by the following formula

$$Col_\alpha(K) \in \begin{cases} \{\delta_{2^m}^\beta | Col_\alpha(\bar{L}_1 \delta_{2^m}^\beta) \in \Gamma_{i_1}\}, & \text{if } \delta_{2^{n+\hat{n}}}^\alpha \in \Gamma_{i_1}, \\ \{\delta_{2^m}^\beta | Col_\alpha(\bar{L}_1 \delta_{2^m}^\beta) \in \Gamma_{i_{j-1}}\}, & \text{if } \delta_{2^{n+\hat{n}}}^\alpha \in \Gamma_{i_j}, j \neq 1. \end{cases}$$

IV. AN ILLUSTRATE EXAMPLE

In this section, an example is provided to show the effectiveness of our main results.

Example 1: Consider the following BCN-IE

$$\begin{cases} x_1(t+1) = (u(t) \wedge (x_1(t) \vee x_2(t))) \vee (\neg u(t) \\ \quad \wedge \neg(x_1(t) \vee x_2(t))), \\ x_2(t+1) = (u(t) \wedge x_1(t)) \vee (\neg u(t) \wedge \neg x_1(t)), \\ \quad t \neq t_k, t_k = 3k, k \in \mathbb{Z}^+ \\ x_1(t_k+1) = x_1(t_k) \vee (\neg x_1(t_k) \wedge \neg x_2(t_k)), \\ x_2(t_k+1) = (x_1(t_k) \wedge x_2(t_k)) \vee x_1(t_k), \\ y(t) = x_1(t) \vee x_2(t), \end{cases} \quad (8)$$

and the time-varying reference signal

$$\begin{cases} \hat{x}_1(t+1) = (\hat{x}_1(t) \wedge \hat{x}_2(t)) \vee \neg \hat{x}_1(t), \\ \hat{x}_2(t+1) = \hat{x}_1(t) \vee (\neg \hat{x}_1(t) \wedge \neg \hat{x}_2(t)), \\ \hat{y}(t) = \hat{x}_1(t) \vee (\neg \hat{x}_1(t) \wedge \neg \hat{x}_2(t)). \end{cases} \quad (9)$$

Set $x(t) = x_1(t) \times x_2(t)$ and $\hat{x}(t) = \hat{x}_1(t) \times \hat{x}_2(t)$. Matrix expressions of systems (8) and (9) can be derived:

$$\begin{cases} x(t+1) = L_1 u(t)x(t), t \neq t_k, \\ x(t_k+1) = L_2 x(t_k), \\ y(t) = Hx(t), \end{cases} \quad (10)$$

and

$$\begin{cases} \hat{x}(t+1) = \hat{L}\hat{x}(t), \\ \hat{y}(t) = \hat{H}\hat{x}(t), \end{cases} \quad (11)$$

where $L_1 = \delta_4[1\ 1\ 2\ 4\ 4\ 3\ 1]$, $L_2 = \delta_4[1\ 2\ 3\ 1]$, $H = \delta_2[1\ 1\ 1\ 2]$, $\hat{L} = \delta_4[1\ 3\ 2\ 1]$ and $\hat{H} = \delta_2[1\ 1\ 2\ 1]$.

In the following, the state feedback controller can be designed according to Algorithm III-C.

Step 1: Denote $\bar{x}(t) = x(t) \times \hat{x}(t)$ and $\bar{y}(t) = y(t) \times \hat{y}(t)$. From Lemma 2, the following BCN-IE is derived

$$\begin{cases} \bar{x}(t+1) = \bar{L}_1 u(t)\bar{x}(t), t \neq t_k \\ \bar{x}(t_k+1) = \bar{L}_2 \bar{x}(t), \\ \bar{y}(t) = \bar{H}\bar{x}(t), \end{cases} \quad (12)$$

where $\bar{L}_1 = \delta_{16}[1\ 3\ 2\ 1\ 1\ 3\ 2\ 1\ 5\ 7\ 6\ 5\ 13\ 15\ 14\ 13\ 13\ 15\ 14\ 13\ 13\ 15\ 14\ 13\ 9\ 11\ 10\ 9\ 1\ 3\ 2\ 1]$, $\bar{L}_2 = \delta_{16}[1\ 3\ 2\ 1\ 5\ 7\ 6\ 5\ 9\ 11\ 10\ 9\ 1\ 3\ 2\ 1]$ and $\bar{H} = \delta_4[1\ 1\ 2\ 1\ 1\ 1\ 2\ 1\ 1\ 1\ 2\ 1\ 3\ 3\ 4\ 1]$. The output subset Λ and the state subset Θ are gotten $\Lambda = \{\delta_4^1, \delta_4^4\}$, $\Theta = \{\delta_{16}^1, \delta_{16}^2, \delta_{16}^4, \delta_{16}^5, \delta_{16}^6, \delta_{16}^8, \delta_{16}^9, \delta_{16}^{10}, \delta_{16}^{12}, \delta_{16}^{15}, \delta_{16}^{16}\}$.

Step 2: Define $M = \bar{L}_1 \delta_2^1 + \bar{L}_1 \delta_2^2 \in \mathcal{B}_{16 \times 16}$. From Proposition 1, $\Omega_1 = \delta_{16}^{\omega_1} = \{\delta_{16}^1, \delta_{16}^{13}\}$, $\Omega_2 = \delta_{16}^{\omega_2} = \{\delta_{16}^2, \delta_{16}^3, \delta_{16}^{14}, \delta_{16}^{15}\}$, $\Omega_3 = \delta_{16}^{\omega_3} = \{\delta_{16}^4, \delta_{16}^5, \delta_{16}^6, \delta_{16}^8, \delta_{16}^{13}, \delta_{16}^{16}\}$ and $\Omega_4 = \delta_{16}^{\omega_4} = \{\delta_{16}^9, \delta_{16}^{10}, \delta_{16}^{12}, \delta_{16}^{14}, \delta_{16}^{15}\}$ are stable in BCN-IE (12). Since $M|_{\omega_1} = \mathbf{1}_{2 \times 2}$, $M|_{\omega_2} = \mathbf{1}_{4 \times 4}$, $M|_{\omega_3} \neq \mathbf{1}_{6 \times 6}$ and $M|_{\omega_4} \neq \mathbf{1}_{6 \times 6}$, by Proposition 2, Ω_1 and Ω_2 are stable complex attractors.

Step 3: Because $\Theta \cap \Omega_1 = \{\delta_{16}^1\} \neq \emptyset$ and $\Theta \cap \Omega_2 = \{\delta_{16}^2, \delta_{16}^{15}\} \neq \emptyset$, the output of BCN-IE (10) can track the output of reference signal (11) according to Theorem 1.

Step 4: Based on formula (7), one obtains $\Upsilon_1 = \{\delta_{16}^1, \delta_{16}^4, \delta_{16}^5, \delta_{16}^8, \delta_{16}^{13}, \delta_{16}^{16}\}$, $\Upsilon_2 = \{\delta_{16}^2, \delta_{16}^3, \delta_{16}^6, \delta_{16}^7, \delta_{16}^9, \delta_{16}^{10}, \delta_{16}^{12}, \delta_{16}^{14}, \delta_{16}^{15}\}$. Thus, a sequence of state subsets is computed: $\Gamma_{11} = \Omega_1 \cap \Theta = \{\delta_{16}^1\}$, $\Gamma_{12} = \{\delta_{16}^4, \delta_{16}^5, \delta_{16}^8, \delta_{16}^{13}, \delta_{16}^{16}\}$, $\Gamma_{13} = \{\delta_{16}^9, \delta_{16}^{12}\}$, $\Gamma_{21} = \Omega_2 \cap \Theta = \{\delta_{16}^2, \delta_{16}^{15}\}$, $\Gamma_{22} = \{\delta_{16}^3, \delta_{16}^6, \delta_{16}^7, \delta_{16}^{14}\}$ and $\Gamma_{23} = \{\delta_{16}^{10}, \delta_{16}^{11}\}$.

Step 5: For each state $\delta_{16}^\alpha \in \Upsilon_i, i = 1, 2$, the state feedback matrix K is obtained: $K = \delta_1[1\ 2\ 1\ 1\ 2\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 2\ 1\ 2\ 2]$.

V. CONCLUSION

In this article, the output tracking of BCNs-IE has been considered. To give a necessary and sufficient condition for this problem, an auxiliary BCN-IE has been constructed, and stable complex attractors of BCNs-IE have been defined. According to structure matrices of BCNs-IE and controllability matrices with respect to state subsets, a method to find all stable complex attractors has been proposed. In addition, an approach has been provided to determine state feedback controllers for the output tracking of BCNs-IE. Moreover, an example has been given to illustrate the validity of main results.

Reference [29] points out that stable complex attractors are more general stable structures. Many problems can be solved by using this structure, such as set stabilizability, output tracking, synchronization and so on. Hence, we will use stable complex attractors to deal with more problems in the future.

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