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Robust Control for Trajectory Tracking and Balancing of a Ballbot

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ABSTRACT This article presents a robust control method for trajectory tracking and balancing of a ballbot with uncertainty. Since the ballbot is an underactuated system, previous studies have designed controllers using a hierarchical strategy and/or multi-loop approach. However, multi-loop control systems require several controllers and the hierarchical strategy has a local minimum problem that does not guarantee the convergence of all errors globally. To overcome these drawbacks, we introduce a virtual angle and design a sliding mode controller with a single-loop control system. As a result, the proposed controller is simple and can achieve simultaneous tracking and balancing of the ballbot. From Lyapunov stability theory, it is proven that the tracking and balancing errors of the ballbot are uniformly ultimately bounded and can be made arbitrarily small. Finally, simulation results are presented to verify the proposed control system.

INDEX TERMS Ballbot, trajectory tracking, balancing, underactuated system, virtual angle.

I. INTRODUCTION

A ballbot is a mobile robot consisting of a body mounted on a spherical wheel [1]. Unlike nonholonomic mobile robots, it can be moved in all directions and can be used even in a narrow space. Since it makes point contact with the ground, it has the advantages of low friction and low energy consumption for movement [2], [3]. Due to these advantages, a ballbot that can be boarded by humans has been implemented [4], and studies to control it have been recently proposed.

The ballbot is an underactuated system that has strong nonlinear couplings. Early studies used single-loop control methods such as proportional-integral-derivative (PID) [5], [6] and linear quadratic regulator (LQR) [7] based on the linearized model of the ballbot. However, linear control methods using a single-loop only focus on balance control. To achieve trajectory tracking, multi-loop control methods were proposed [8]–[12]. These methods design additional controllers in a single-loop control scheme to reduce angle errors and enable trajectory tracking. However, to be robust against model uncertainties and disturbances, it is necessary to additionally design a feedforward controller, which causes the disadvantage of designing multiple controllers. To deal with model uncertainties, fuzzy logic-based PID control [13] and LMI-based robust control [14] methods were proposed, but focus on balancing control still makes it difficult to track a desired trajectory.

To address these problems, hierarchical sliding mode control methods were recently proposed [15], [16]. To achieve trajectory tracking and balancing simultaneously, sliding surfaces for trajectory tracking and balancing are designed. By combining them, the underactuation problem can be solved. Because of the advantages of the sliding mode control method [17], it is robust against disturbances. However, due to the combination of independent sliding surfaces for trajectory tracking and balancing, it has a local minimum problem that does not guarantee the convergence of all errors globally [18]. This means that the ballbot may not achieve trajectory tracking and balancing simultaneously. Furthermore, even if it is outside the local minima, there are limitations in smoothly moving various trajectories consisting of straight and curved paths due to the nature of the hierarchical strategy. Therefore, for real applications, it is necessary to design the controller using a new control strategy different from the hierarchical strategy.

Motivated by these observations, this article proposes a new control strategy to address the underactuation problem and uncertainties different from the hierarchical strategy.

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To this end, a method using a virtual angle is proposed, and by using the virtual angle, trajectory tracking and balancing can be achieved with a single controller, unlike previous studies. Thus, the local minimum problem is solved and various trajectories consisting of straight and curved paths can be smoothly tracked. For robustness against external disturbances, a controller based on the sliding mode control (SMC) method is designed. From the Lyapunov stability theory, it is proven that all error signals in a closed-loop system are uniformly ultimately bounded. Finally, the performance of the proposed method is verified through the simulation results. The contributions of this article are as follows:

1) Unlike in previous studies using a multi-loop control system, it is possible to design a single controller that can simultaneously achieve trajectory tracking and balancing of the ballbot with a single-loop control system.

2) The local minimum problem is eliminated, and various trajectories consisting of straight and curved paths can be smoothly tracked.

This article is organized as follows. In Section II, the dynamics of the ballbot is described and the problem formulation is presented. In Section III, the robust controller for the trajectory tracking and balancing of a ballbot is designed. In Section IV, the simulation results are presented to demonstrate the performance of the proposed method. In Section V, the conclusion is given.

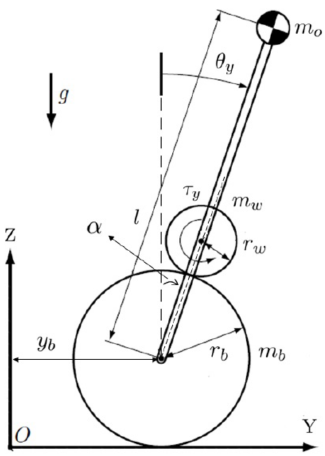

FIGURE 1. Ballbot model for the Y-Z plane.

II. SYSTEM MODEL

Consider a ballbot equipped with three omni-wheel motors. The ballbot is represented by a combination of three independent planar models. Fig. 1 shows the Y-Z plane model. Since the X-Z plane model is similar to the Y-Z plane model, the figure of the X-Z plane model is omitted. The X-Y plane model represents rotating along the Z-axis, but it is not considered in this article because it is not related to the trajectory tracking and balancing of the ballbot.

Under the assumption that no slip occurs between the ball and the floor and between the ball and the omni-directional wheels, the dynamic equations of the ballbot can be given as follows [4]:

$$\begin{aligned} \ddot{y}_b &= f_{y1}(\theta_y, \dot{\theta}_y) + \Delta f_{y1}(\dot{y}_b, \dot{\theta}_y) + g_{y1}(\theta_y)\tau_y + \tau_{d1} \\ \ddot{\theta}_y &= f_{y2}(\theta_y, \dot{\theta}_y) + \Delta f_{y2}(\dot{y}_b, \dot{\theta}_y) + g_{y2}(\theta_y)\tau_y + \tau_{d2} \end{aligned} \tag{1}$$

$$\begin{aligned} \ddot{x}_b &= f_{x1}(\theta_x, \dot{\theta}_x) + \Delta f_{x1}(\dot{x}_b, \dot{\theta}_x) + g_{x1}(\theta_x)\tau_x + \tau_{d3} \\ \ddot{\theta}_x &= f_{x2}(\theta_x, \dot{\theta}_x) + \Delta f_{x2}(\dot{x}_b, \dot{\theta}_x) + g_{x2}(\theta_x)\tau_x + \tau_{d4} \end{aligned} \tag{2}$$

where

$$\begin{aligned} f_{y1} &= a_y^{-1} \sin\theta_y\{a_5(a_3\cos\theta_y - a_4) - (a_2 + I_y)a_3\dot{\theta}_y^2\} \\ \Delta f_{y1} &= a_y^{-1}\{-b_{ry}\dot{\theta}_y(a_3\cos\theta_y - a_4) - (a_2 + I_y)b_y\dot{y}_b)\} \\ f_{y2} &= a_y^{-1} \sin\theta_y\{a_3\dot{\theta}_y^2(a_4 - a_3\cos\theta_y) + a_1a_5\} \\ \Delta f_{y2} &= a_y^{-1}\{b_y\dot{y}_b(a_4 - a_3\cos\theta_y) - a_1b_{ry}\dot{\theta}_y\} \\ g_{y1} &= a_y^{-1}r_w^{-1}(a_2 + I_y + a_3r_b\cos\theta_y - a_4r_b) \\ g_{y2} &= a_y^{-1}r_w^{-1}(a_3\cos\theta_y - a_4 + a_1r_b) \\ a_y &= a_1(a_2 + I_y) - (a_4 - a_3\cos\theta_y)^2 \\ f_{x1} &= a_x^{-1} \sin\theta_x\{-a_5(a_3\cos\theta_x - a_4) + (a_2 + I_x)a_3\dot{\theta}_x^2\} \\ \Delta f_{x1} &= a_x^{-1}\{b_{rx}\dot{\theta}_x(a_3\cos\theta_x - a_4) - (a_2 + I_x)b_x\dot{x}_b)\} \\ f_{x2} &= a_x^{-1} \sin\theta_x\{a_3\dot{\theta}_x^2(a_4 - a_3\cos\theta_x) + a_1a_5\} \\ \Delta f_{x2} &= a_x^{-1}\{-b_x\dot{x}_b(a_4 - a_3\cos\theta_x) - a_1b_{rx}\dot{\theta}_x\} \\ g_{x1} &= a_x^{-1}r_w^{-1}(-a_2 - I_x - a_3r_b\cos\theta_x + a_4r_b) \\ g_{x2} &= a_x^{-1}r_w^{-1}(a_3\cos\theta_x - a_4 + a_1r_b) \\ a_x &= a_1(a_2 + I_x) - (a_4 - a_3\cos\theta_x)^2 \\ a_1 &= m_b + \frac{I_b}{r_b^2} + m_o + \frac{3I_w\cos^2\alpha}{2r_w^2} \\ a_2 &= m_ol^2 + \frac{3I_wr_b^2\cos^2\alpha}{2r_w^2} \\ a_3 &= m_ol, \quad a_4 = \frac{3r_bI_w\cos^2\alpha}{2r_w^2}, \ a_5 = m_ogl \end{aligned}$$

In these expressions, x_b and y_b denote the position of the ball along the X- and Y-axes, respectively, θ_x and θ_y are body angles along the X-Z and Y-Z planes, respectively, τ_x and τ_y are the control inputs of the three driving motors acting on the ball, m_b is the mass of the ball, m_o is the mass of the body, r_b is the radius of the ball, r_w is the radius of the omni-directional wheel, l is the distance between the center of the ball and the center of the mass of the body, I_x and I_y denote the moments of inertia of the body about the X- and Y-axes, respectively, I_b and I_w denote the moments of inertia of the ball and omni-directional wheel, respectively, α is the zenith angle, b_y, b_x, b_{ry}, and b_{rx} are the viscous damping coefficients that model the spherical wheel-ground friction, τ_{d1}, τ_{d2}, τ_{d3}, and τ_{d4} denote the external disturbances, and g is the gravitational acceleration. It is noted that a_y and a_x are invertible.

Assumption 1: The external disturbances τ_{di} for $i = 1, \ldots, 4$ are bounded such that $|\tau_{di}| \leq \tau_{d_M}$, where τ_{d_M} is a known positive constant.

Assumption 2: The viscous damping coefficients b_y, b_x, b_{ry}, and b_{rx} are unknown but bounded such that $|b_y| \leq b_{y,\max}$, $|b_x| \leq b_{x,\max}$, $|b_{ry}| \leq b_{ry,\max}$, and $|b_{rx}| \leq b_{rx,\max}$

where $b_{y,\max}$, $b_{x,\max}$, $b_{ry,\max}$, and $b_{rx,\max}$ are known positive constants.

It is noted that $\triangle f_{y1}$, $\triangle f_{y2}$, $\triangle f_{x1}$, and $\triangle f_{x2}$ are unknown due to the viscous damping coefficients.

The *control objective* is to design the controller for the trajectory tracking and balancing of the ballbot so that $y_b = y_r$, $x_b = x_r$, $\theta_y = 0$, and $\theta_x = 0$, where y_r and x_r denote the desired position.

Assumption 3: The time derivatives of y_r and x_r are available.

III. MAIN RESULTS

As shown in (1) and (2), the ballbot model is the underactuated system. This means that the desired position and angles cannot be achieved simultaneously using the control inputs. To address this problem, we introduce the virtual angles that are developed for the trajectory tracking of the ballbot and design the controller using the SMC method for robustness against external disturbances.

Let us define the following errors:

$$\begin{aligned} e_y &= y_b - y_r, \quad e_{\theta_y} = \tanh\theta_y - \theta_{f_y} \\ e_x &= x_b - x_r, \quad e_{\theta_x} = \tanh\theta_x - \theta_{f_x} \end{aligned} \tag{3}$$

where θ_{f_i} for $i = x, y$ is the signal passed through a second-order filter, i.e., $\sigma^2\ddot{\theta}_{f_i} + 2\sigma\dot{\theta}_{f_i} + \theta_{f_i} = \theta_{v_i}$ with $\sigma > 0$. In this expression, θ_{v_i} for $i = x, y$ is a virtual angle to deal with the underactuation problem and is given below:

$$\theta_{v_i} = \lambda_{\theta_i} s_i \tag{4}$$

where $s_i = \dot{e}_i + \lambda_i e_i$, λ_i and λ_{θ_i} are design parameters, and $i = x, y$.

To design the sliding mode controller, let us define the following sliding surfaces:

$$\begin{aligned} s_{\theta_y} &= \dot{e}_{\theta_y} + \lambda_1 e_{\theta_y} \\ s_{\theta_x} &= \dot{e}_{\theta_x} + \lambda_2 e_{\theta_x} \end{aligned} \tag{5}$$

where λ_1 and λ_2 are positive constants. Differentiating both sides of (5) along the solutions of (1), (2), and (3) yields

$$\begin{aligned} \dot{s}_{\theta_y} &= \mathrm{sech}^2\theta_y(f_{y2} + \triangle f_{y2} + g_{y2}\tau_y + \tau_{d2}) + \lambda_1\dot{e}_{\theta_y} - \ddot{\theta}_{f_y} \\ &\quad - 2\tanh\theta_y \mathrm{sech}^2\theta_y\dot{\theta}_y^2 \\ \dot{s}_{\theta_x} &= \mathrm{sech}^2\theta_x(f_{x2} + \triangle f_{x2} + g_{x2}\tau_x + \tau_{d4}) + \lambda_2\dot{e}_{\theta_x} - \ddot{\theta}_{f_x} \\ &\quad - 2\tanh\theta_x \mathrm{sech}^2\theta_x\dot{\theta}_x^2 \end{aligned} \tag{6}$$

Remark 1: From (6), it can be seen that the second derivatives of θ_{v_i} with respect to time are induced when we use the virtual angles θ_{v_i} in (3) instead of the filtered signals θ_{f_x} and θ_{f_y}. This makes the controller design and implementation difficult because the second derivatives of θ_{v_i} include the control inputs τ_x and τ_y. The filtered signals can be a solution to this problem. Since the second-order derivative of the filtered signals θ_{f_x} and θ_{f_y} is required in (6), we introduce the second-order filter and use the filtered signals θ_{f_x} and θ_{f_y} that only require the sliding surfaces s_{θ_y} and s_{θ_x} in designing the controller.

Remark 2: The convergence of s_{θ_x} and s_{θ_y} to zero implies that θ_{f_x} and θ_{f_y} are bounded by the definitions of e_{θ_x} and e_{θ_y}. This leads to the boundedness of θ_{v_i}. From (4), the tracking errors e_y and e_x are bounded, and thus, the underactuation problem can be solved when the convergence of s_{θ_x} and s_{θ_y} is guaranteed.

Consider the Lyapunov function candidate as

$$V = \frac{1}{2}(s_{\theta_y}^2 + s_{\theta_x}^2) \tag{7}$$

Differentiating both sides of (7) along the solution of (6) yields

$$\begin{aligned} \dot{V} &= s_{\theta_y}\Big\{\mathrm{sech}^2\theta_y(f_{y2} + \triangle f_{y2} + g_{y2}\tau_y + \tau_{d2}) + \lambda_1\dot{e}_{\theta_y} \\ &\quad - \ddot{\theta}_{f_y} - 2\tanh\theta_y\mathrm{sech}^2\theta_y\dot{\theta}_y^2\Big\} \\ &\quad + s_{\theta_x}\Big\{\mathrm{sech}^2\theta_x(f_{x2} + \triangle f_{x2} + g_{x2}\tau_x + \tau_{d4}) + \lambda_2\dot{e}_{\theta_x} \\ &\quad - \ddot{\theta}_{f_x} - 2\tanh\theta_x\mathrm{sech}^2\theta_x\dot{\theta}_x^2\Big\} \end{aligned} \tag{8}$$

By Assumption 2, the following inequalities are satisfied:

$$\begin{aligned} |\triangle f_{y2}| &\le b_{y,\max}|a_y^{-1}\dot{y}_b(a_4 - a_3\cos\theta_y)| + b_{ry,\max}|a_y^{-1}a_1\dot{\theta}_y| \\ |\triangle f_{x2}| &\le b_{x,\max}|a_x^{-1}\dot{x}_b(a_4 - a_3\cos\theta_x)| + b_{rx,\max}|a_x^{-1}a_1\dot{\theta}_x| \end{aligned} \tag{9}$$

From (8), we choose the actual controls τ_y and τ_x to be

$$\begin{aligned} \tau_y &= \frac{1}{g_{y2}}\Big[-f_{y2} - \zeta_1\mathrm{sgn}(s_{\theta_y}) + \cosh^2\theta_y\big\{-\lambda_1\dot{e}_{\theta_y} + \ddot{\theta}_{f_y} \\ &\quad + 2\tanh\theta_y\mathrm{sech}^2\theta_y\dot{\theta}_y^2 - k_1 s_{\theta_y}\big\}\Big] \\ \tau_x &= \frac{1}{g_{x2}}\Big[-f_{x2} - \zeta_2\mathrm{sgn}(s_{\theta_x}) + \cosh^2\theta_x\big\{-\lambda_2\dot{e}_{\theta_x} + \ddot{\theta}_{f_x} \\ &\quad + 2\tanh\theta_x\mathrm{sech}^2\theta_x\dot{\theta}_x^2 - k_2 s_{\theta_x}\big\}\Big] \end{aligned} \tag{10}$$

where k_1 and k_2 are positive design constants, $\mathrm{sgn}(\cdot)$ is a signum function, and ζ_1 and ζ_2 are selected as follows:

$$\begin{aligned} \zeta_1 &= \eta_{y1}|a_y^{-1}\dot{y}_b(a_4 - a_3\cos\theta_y)| + \eta_{y2}|a_y^{-1}a_1\dot{\theta}_y| + \eta \\ \zeta_2 &= \eta_{x1}|a_x^{-1}\dot{x}_b(a_4 - a_3\cos\theta_x)| + \eta_{x2}|a_x^{-1}a_1\dot{\theta}_x| + \eta \end{aligned}$$

with positive constants η_{y1}, η_{y2}, η_{x1}, η_{x2}, and η.

Remark 3: In (10), τ_y and τ_x are designed to control the Y-axis and X-axis movement of the ballbot, respectively. Since the ballbot is moved by the three driving motors acting on the ball, it is necessary to convert the control inputs τ_y and τ_x to the torques of the actual motors for implementation (see details in [4]).

Substituting (9) and (10) into (8) yields

$$\dot{V} \le -k_1 s_{\theta_y}^2 - k_2 s_{\theta_x}^2 + \mathrm{sech}^2\theta_y|s_{\theta_y}|h_y + \mathrm{sech}^2\theta_x|s_{\theta_x}|h_x \tag{11}$$

where

$$h_y = (b_{y,\max} - \eta_{y1})|a_y^{-1}\dot{y}_b(a_4 - a_3\cos\theta_y)|$$

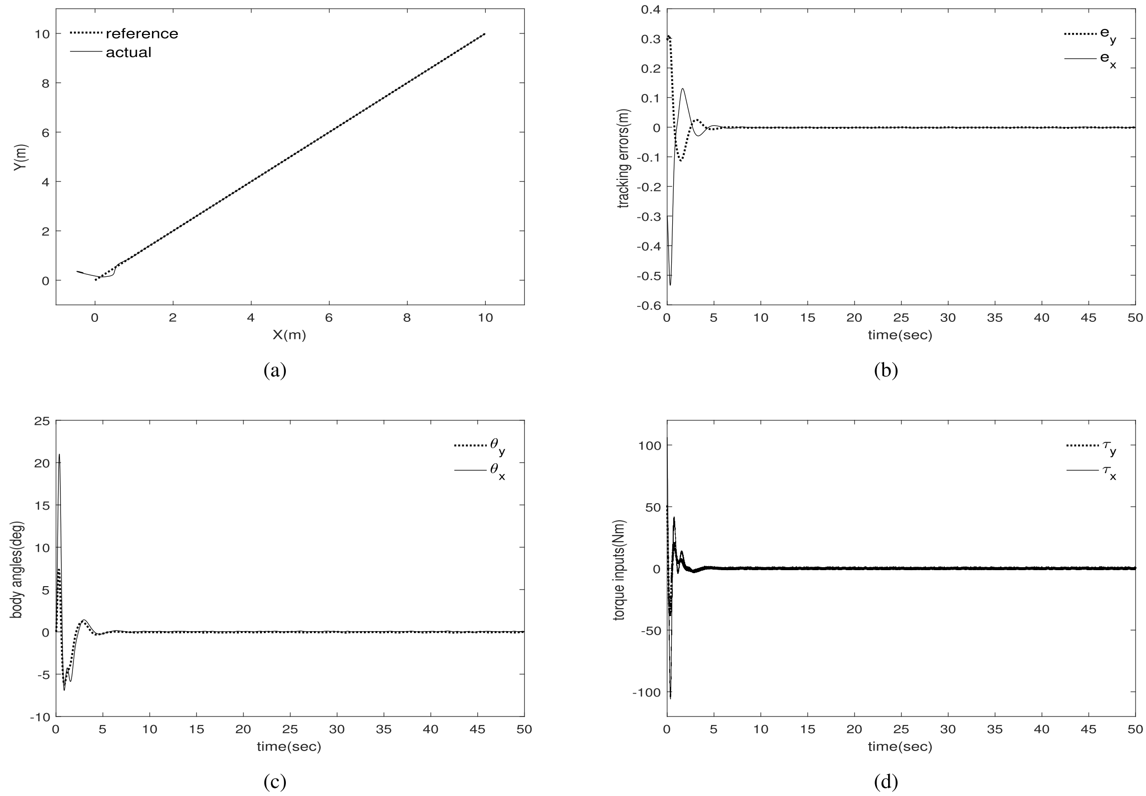

FIGURE 2. Simulation results of Scenario 1 (a) trajectory (b) tracking errors (c) body angles (d) control inputs.

$$
\begin{aligned}
&+(b_{ry,\max}-\eta_{y2})|a_y^{-1}a_1\dot{\theta}_y|+\tau_{d_M}-\eta \\
h_x &= (b_{x,\max}-\eta_{x1})|a_x^{-1}\dot{x}_b(a_4-a_3\cos\theta_x)| \\
&+(b_{rx,\max}-\eta_{x2})|a_x^{-1}a_1\dot{\theta}_x|+\tau_{d_M}-\eta
\end{aligned}
$$

Remark 4: Unlike in previous studies [15], [16] using hierarchical sliding mode control (HSMC) methods, the proposed method uses a virtual angle to eliminate a local minimum problem caused by the combination of independent sliding surfaces in designing the controller. In addition, the controller is simple by designing the controller in a single-loop compared to the methods [8]–[12] using the double-loop approach. Therefore, the proposed method has the advantages that the structure of the control system is simple, and it is possible to achieve trajectory tracking and balancing of the ballbot simultaneously without causing the local minimum problem.

The following theorem shows the main result of this article.

Theorem 1: Consider the ballbot models in (1) and (2). Under Assumptions 1-3, if the actual controls (10) are applied to these models, then all error signals in the closed-loop system are uniformly ultimately bounded and can be made arbitrarily small.

Proof: If η_{y1}, η_{y2}, η_{x1}, η_{x2}, and η are chosen to satisfy that $\eta_{y1} > b_{y,\max}$, $\eta_{y2} > b_{ry,\max}$, $\eta_{x1} > b_{x,\max}$, $\eta_{x2} > b_{rx,\max}$, and $\eta > \tau_{d_M}$, then (8) is represented as

$$\dot{V} \le -k_1 s_{\theta_y}^2 - k_2 s_{\theta_x}^2 \tag{12}$$

Since $\dot{V}$ is negative definite, s_{θ_y} and s_{θ_x} converge to zero. Thus, the asymptotic stability of e_{θ_y} and e_{θ_x} is guaranteed by (5) and the filtered signals θ_{f_y} and θ_{f_x} are bounded by (3). This implies that the boundedness of θ_{v_i}. Integrating both sides of (4), we have

$$e_i(t) \le \left(e_i(0) - \frac{c_i}{\lambda_i}\right)e^{-\lambda_i t} + \frac{c_i}{\lambda_i} \tag{13}$$

where c_i is the maximum value of $|\theta_{v_i}/\lambda_{\theta_i}|$ and $i = x, y$. Therefore, all error signals in the closed-loop system are uniformly ultimately bounded and can be made arbitrarily small by properly choosing the design parameters. This completes the proof. ■

Remark 5: If the tracking errors converge to zero, i.e, $s_y = s_x = 0$, the convergence of e_{θ_y} and e_{θ_x} leads to $\theta_y = \theta_x = 0$ from (3) and (4). Therefore, trajectory tracking and balancing of the ballbot are achieved simultaneously. However, the boundedness of tracking errors leads to the boundedness of angle errors. This is reasonable because the body angles θ_y and θ_x must have a nonzero value to reduce the tracking errors. As proved in Theorem 1, the bounds of all errors can be made arbitrarily small by properly choosing the design parameters.

Remark 6: Practical ballbot systems suffer from unmeasured velocities and input saturation. To address these problems, output feedback control methods considering actuator constraints can be used. Recently, quasi-velocities

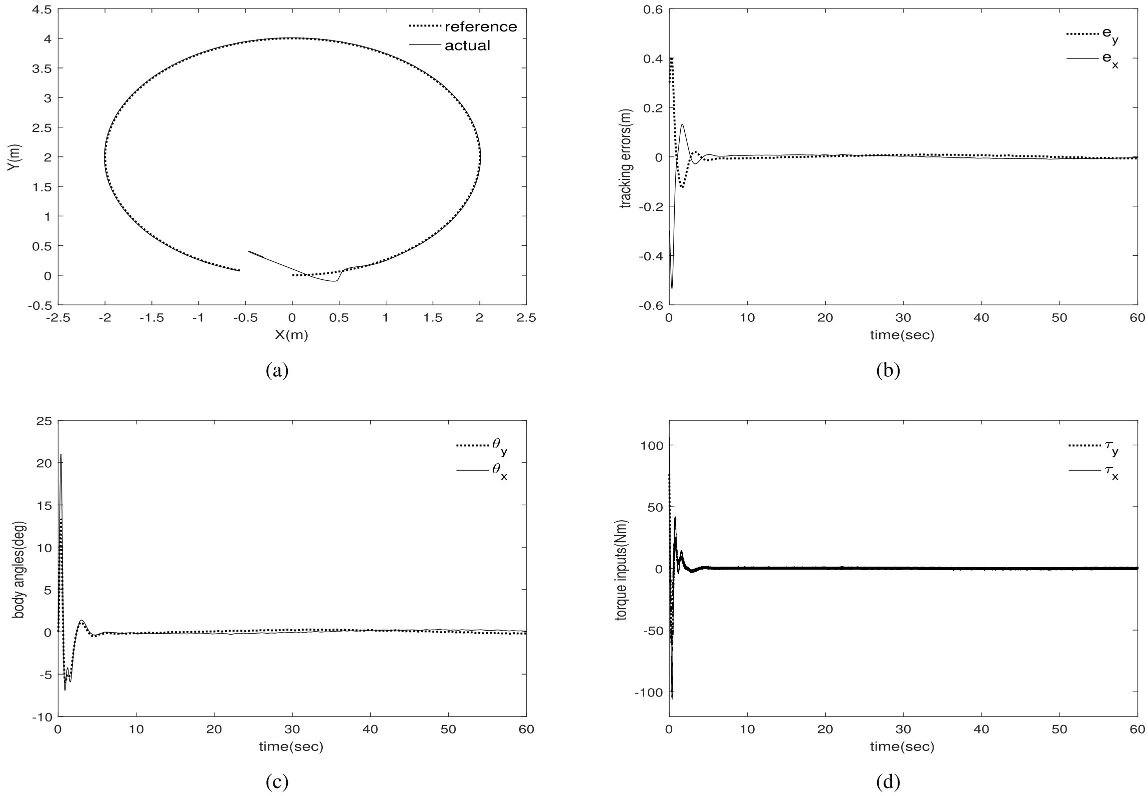

FIGURE 3. Simulation results of Scenario 2 (a) trajectory (b) tracking errors (c) body angles (d) control inputs.

and saturation functions were utilized to provide effective solutions for crane systems [19], [20]. However, to apply these methods to ballbot systems, the controller design and stability analysis must be renewed. Since we focus on the controller design for trajectory tracking and balancing of the ballbot with uncertainty, considering unmeasured velocities and input saturation remains a meaningful subject for future research.

IV. SIMULATION RESULTS

In this section, simulation results on the ballbot using the proposed method are presented. The model parameters of the ballbot are taken from [15]. The design parameters are chosen as $k_1 = k_2 = 5$, $\lambda_1 = \lambda_2 = 5$, $\lambda_{\theta_x} = -0.25$, $\lambda_{\theta_y} = 0.25$, $\lambda_x = \lambda_y = 2$, $\eta_{y1} = \eta_{y2} = \eta_{x1} = \eta_{x2} = 10$, $\eta = 0.15$, and $\sigma = 0.1$. The initial posture of the ballbot is taken as $[y_b, \theta_y] = [0.3, 0]^T$ and $[x_b, \theta_x] = [-0.3, 0]^T$. The external disturbances τ_{d_i} for $i = 1, \ldots, 4$ are chosen to be Gaussian random noise with mean 0 and variance 0.01, and the upper bounds of disturbances are assumed to be $\tau_{d_M} = 0.1$. To demonstrate the performance of the proposed method, we simulate several scenarios: (1) straight line, (2) circle, and (3) path with a straight line and a curve.

Fig. 2 shows the results for Scenario 1, where it is shown that a straight line can be exactly followed as tracking errors and body angles converge to zero. It demonstrates that the trajectory tracking and balancing of the ballbot are achieved simultaneously, as mentioned in Remark 5 for the straight line.

Fig. 3 shows that trajectory tracking and balancing are successfully achieved for a circle in Scenario 2. The tracking and angle errors are bounded but become sufficiently small, as shown in Figs. 3(b) and (c). This verifies that the proposed method does not guarantee the convergence of errors to zero for the curve but can make the errors sufficiently small.

Fig. 4 shows the tracking performance for the path with a straight line and a curve in Scenario 3. As demonstrated in Scenarios 1 and 2, the errors converge to zero on the straight line and bounded on the curve. The reason for the relatively large errors is that the reference velocities of Scenario 3 are faster than those of Scenarios 1 and 2.

For comparison results, we simulate Scenario 3 with the HSMC method proposed in [15]. Fig. 5 shows the simulation results. As seen from Figs. 5(b)-(d), the tracking and balancing errors do not converge to zero, although the sliding surfaces converge to zero. This shows that the HSMC method can suffer from the local minimum problem, as mentioned in Remark 4. On the other hand, the proposed method can solve the local minimum problem, as shown in the simulation results.

From the simulation results, we can conclude that the proposed method is effective in achieving the trajectory tracking and balancing of the ballbot for various paths.

Remark 7: The signum functions in the control inputs cause the chattering phenomenon. The saturation functions

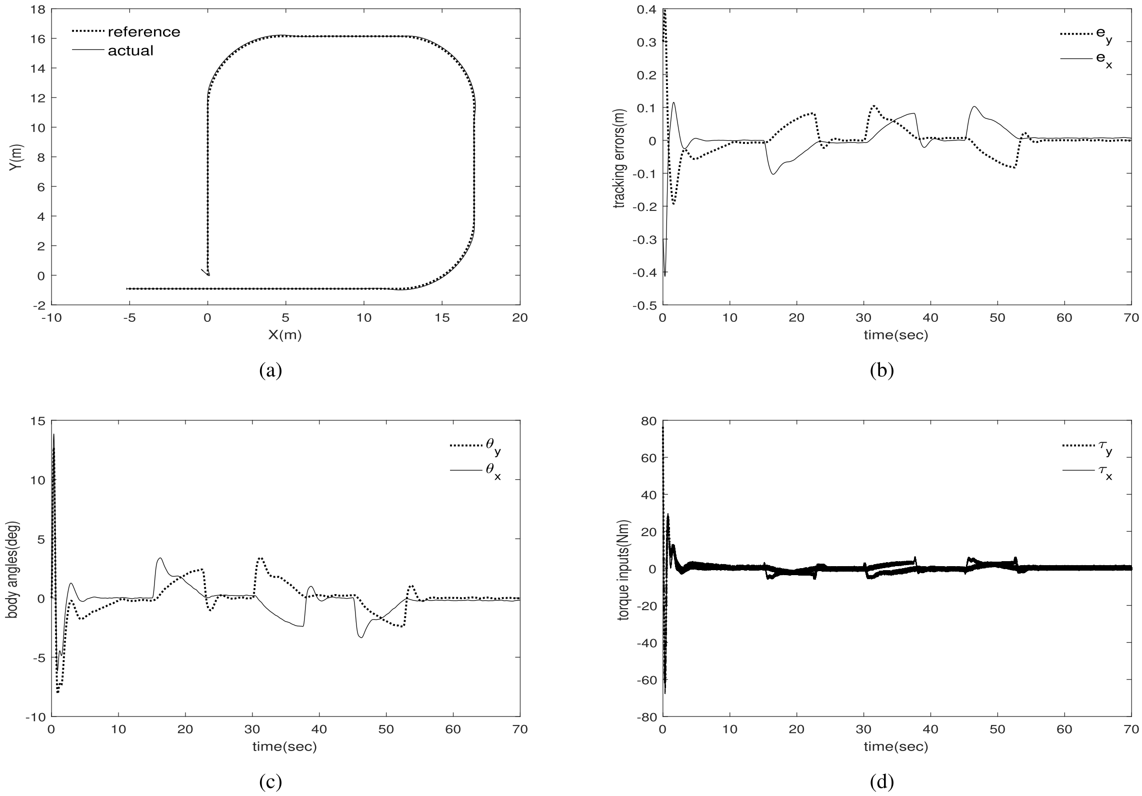

FIGURE 4. **Simulation results of Scenario 3 (a) trajectory (b) tracking errors (c) body angles (d) control inputs.**

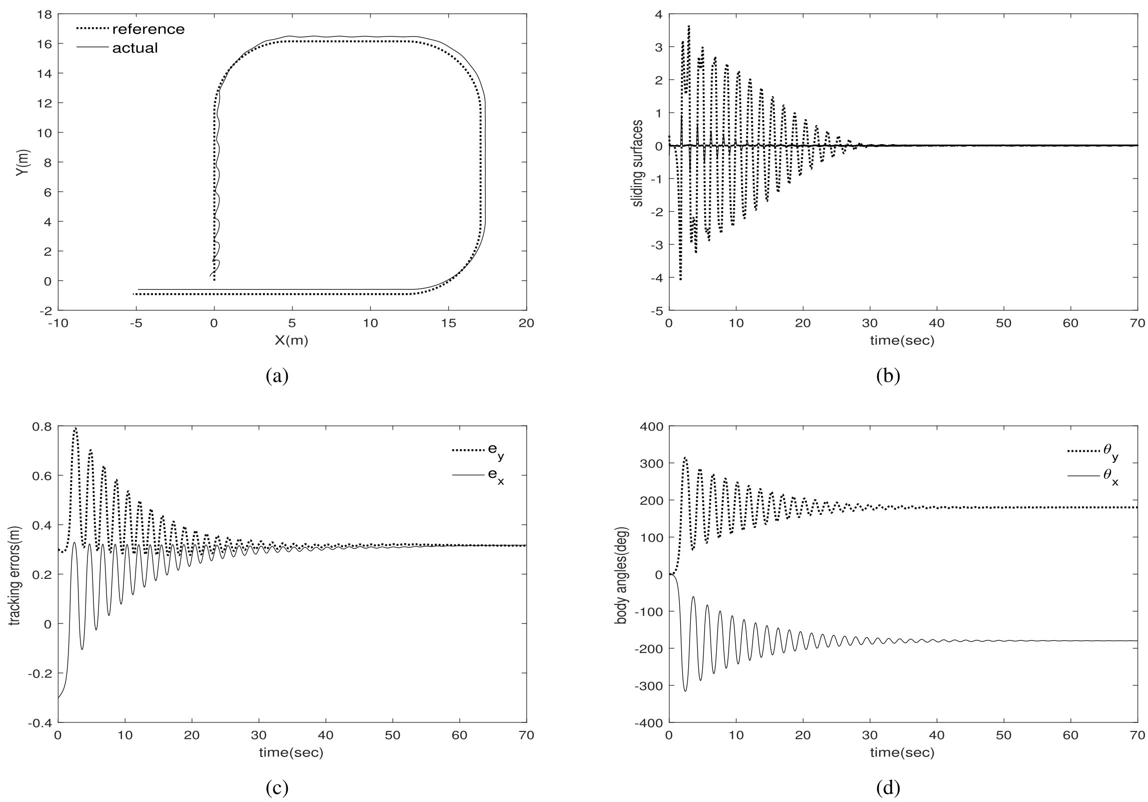

FIGURE 5. **Simulation results of the HSMC method proposed in [15] (a) trajectory (b) sliding surfaces (c) tracking errors (d) body angles.**

can be a solution. When the saturation functions are used in the control inputs instead of signum functions, all error signals are bounded and can be easily proven.

V. CONCLUSION

A robust control method was presented for trajectory tracking and balancing of the ballbot with uncertainties. The primary contribution of this article is that the proposed method can solve the underactuation problem without requiring several controllers and the local minimum problem of the HSMC method. To achieve this contribution, a virtual angle is introduced and a single controller robust to uncertainty is designed by combining with the sliding mode control technique. The stability of the proposed control system is proved in the Lyapunov sense and the simulation results are provided to verify the performance of the proposed control system.

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