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Robust Adaptive Control for Stochastic Discrete-Time Nonlinear Systems and Application to Gas Engine as an Electric Vehicle Extender

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ABSTRACT In this paper, the problem of adaptive control with disturbance attenuation is investigated for stochastic discrete-time nonlinear systems with Markovian jumping parameters and unknown system parameters. Using the proposed control law and the sufficient conditions, the closed-loop system achieved H_∞ performance. As a practical example of the considered problem, an air-fuel ratio controller for a gas engine intended as an electric vehicle extender is designed based on the proposed adaptive disturbance attenuation control algorithm. The air-fuel ratio controller is validated via a numerical simulation under three working conditions. The results of the numerical simulation show that the air-fuel ratio can be regulated into a range around its desired value by the proposed adaptive disturbance attenuation controller and that the adaptive law can be tuned to a steady value. The control performance indices of the proposed adaptive disturbance attenuation controller are smaller than those of the open-loop controller, which means that the proposed adaptive disturbance attenuation controller achieves a greater control effect.

INDEX TERMS Stochastic robust adaptive control, air-fuel ratio, Markovian switching, gas engine for electric vehicle extender.

I. INTRODUCTION

Research on robust and/or adaptive control algorithms for stochastic systems has become an interesting area in the theory of control. For the continuous-time case, the finite time fault tolerant adaptive control problem for nonlinear system with various faults was deduced in [1], using the backstepping and neural networks technologies. The neural adaptive backstepping controller design problem for a nonlinear system with non-strict feedback characteristics and consideration of input delay was investigated in [2]. The finite time adaptive tracking control algorithm for an indeterminate non-strict feedback nonlinear system with restriction of input has was reported by [3], and the unmeasured states were obtained by the observer. By applying dynamic programming with a neural network, the robust adaptive event-driven control problem of an indeterminate nonlinear system was researched in [4]. The robust adaptive fuzzy constructive on-line control algorithm design problem with consideration of the disturbances and uncertainties of a surface tracking vehicle was deduced in [5]. The resilient controller design problem for regulation of cross movement of an intelligent vehicle was investigated by [6]. In further development, stochastic systems with Markovian jumping parameters and unknown system parameters have attracted much attention, and the tracking control problem of an indeterminate switched nonlinear system with non-lower triangular characteristics using the adaptive neural control algorithm was researched in [7]. The stochastic neural adaptive tracking control problem of an indeterminate switched nonlinear system with a non-strict feedback characteristic was investigated in [8], using the average dwell time approach. The neural adaptive tracking control problem of an indeterminate multi-input multi-output switched nonlinear non-strict feedback system with dead zone inputs and constraints of outputs was deduced in [9]. The stochastic fuzzy adaptive output feedback tracking control problem of a switched nonlinear system with pure feedback characteristics was studied in [10] using

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backstepping technology. Output feedback static robust H_∞ control design of linear system with uncertainties of polytopic examined in [11].

For discrete-time stochastic systems with Markovian jumping parameters, adaptive controller design of a nonlinear system with discrete-time characteristics was researched in [12] using neural networks. A robust controller was proposed by [13] for a linear discrete-time system that contains Markovian jumping parameters, and the achievements of stochastic stability and H_∞ performance were reported. Robust control and stability analysis of the stochastic linear system with discrete-time characteristics and Markovian jumping parameters were researched in [14]. The robust control problem of a discrete-time linear system with Markovian jump parameters and time-delays of mode-dependent was investigated in [15]. The results of [13] were extended to a nonlinear system with discrete-time characteristics and Markovian jumping parameters by [16]. The problems of a nonlinear system with discrete-time characteristics and Markovian jump parameters were investigated by [17] using control design to restrict executor and the partially known probabilities of transition. Stochastic stability and stabilization with the partially known transition probabilities of finite-time for a linear Markovian jump system with discrete-time characteristics was researched by [18]. The problem of stabilization for a Markovian jump delay system with discrete-time characteristics and stochastic non-linearity was deduced by [19]. The work in [20] addressed the output feedback optimal control algorithm of a stationary dynamic for linear system with discrete-time characteristics, Markovian jumping parameters and additive standard disturbance. The H_∞ filter design problem for an indeterminate discrete-time system with the characteristics of packet dropouts and quantized measurements was proposed in [21]. A robust H_∞ controller design for a non-homogeneous Markovian jump linear system with discrete-time characteristics and Markovian jumping parameters was studied using a multi-step Lyapunov function approach by [22]. Research on robust regulation for a linear system with discrete-time characteristics and Markovian jumping parameters subjected to variation of the structured parameter was proposed in [23].

However, no robust adaptive control algorithms have been reported for stochastic discrete-time nonlinear systems with Markovian jumping parameters and unknown system parameters, even though those of stochastic continuous-time nonlinear systems with Markovian jumping parameters and unknown system parameters are well researched. Indeed, the methodological systems in stochastic robust adaptive control algorithm design for the continuous-time case and the discrete-time case are dissimilar, e.g., the method of solving the Lyapunov function in the continuous-time case is based on differential theory, whereas that in the discrete-time case is based on difference theory, which leads to different processing technologies.

The robust adaptive air-fuel ratio control problem of a gas engine intended as an electric vehicle extender is a practical application of the proposed robust adaptive control problem of stochastic discrete-time nonlinear systems with Markovian jumping parameters and unknown system parameters. Indeed, the control problems of gas engines have been widely researched. The variation in the cyclic cylinder pressure of gas engines relative to the lean burn operating mode was researched in [24]. In [25], the results of an experiment on gas engines with direct injection were investigated by improvement of the gasoline engine with port injection systems. For calculation of the equivalence ratio of the pre-combustion chamber, a model was established under various mixtures and fuel flow proportions in [26]. In [27], an experimental study was performed on the effect of compression ratio on the performances of combustion and emission which are affected by the compression proportion for gas engines with enriched hydrogen under various air-fuel ratios. The research results related to the effect of the natural gas composition on the combustion and emission performances of internal combustion engines fueled by natural gas were reviewed by [28]. Cyclic variation of combustion in gas engines with pre-mixed and lean-burn characteristics was investigated in [29]. Improvement of the thermal efficiency by increasing the compression ratio for acquisition of a higher expansion proportion was validated by the experiments in [30]. Compressed natural gas and air mixed fired by a laser was investigated in experiments under various compression proportions and excess air for the purpose of fully utilizing compressed natural gas was discussed in [31]. It should be noted that the cyclic mass of the intake air in the gas engines is uncertain even at steady working conditions, which consist of an unknown nominal component and a disturbance component. Moreover, cyclic transmutation of the residual gas fraction which reflects the level of the residual gas, obeys the Markov property [32]. The above factors have a large influence on the air-fuel ratio control performance of gas engines, and the proposed robust adaptive control design technology of stochastic discrete-time nonlinear systems with Markovian jumping parameters and unknown system parameters is a suitable approach to solving the problem.

Based on the results of [13] and [33], the robust adaptive controller, the adaptive law and the sufficient conditions of the stochastic nonlinear discrete-time systems with Markovian jumping parameters and unknown system parameters are given in this paper by solving a discrete-time Lyapunov function. Robust adaptive air-fuel ratio control of the gas engine for an electric vehicle extender, which meets the form of the considered problem, is used as a practical application of the proposed control law. From the results of the numerical simulation, we observe that the proposed control law is effective under various working conditions.

The contributions of this paper can be summarized in the following two points. First, the robust adaptive control algorithm is designed for stochastic discrete-time nonlinear systems with Markovian jumping parameters and unknown system parameters by solving a discrete-time Lyapunov function. Second, the proposed control algorithm is applied to air-fuel ratio control of a gas engine intended as an electric vehicle extender to overcome the influence of the intake air and residual gas on the control accuracy of the air-fuel ratio.

The rest parts of the paper are organized as follows. The problem formulation and controller design are described in Section II. Section III presents application to gas engine as an electric vehicle extender. The numerical simulation is demonstrated in Section IV. Lastly, Section V summarizes the conclusions of this paper.

II. PROBLEM FORMULATION AND CONTROLLER DESIGN

For the stochastic nonlinear system with discrete-time characteristics, Markovian jumping parameters and unknown system parameters, the following applies:

$$x(k+1) = A(r(k))x(k) + B_1(r(k))\omega(k) + \alpha(x(k))\theta + B_2(r(k))u(k),\ y(k) = C(r(k))x(k), \tag{1}$$

where $x(k) \in R^n$ denotes the state of the system, $u(k) \in R^p$ denotes the control, $y(k) \in R^m$ denotes the output, $\theta \in R^r$ denotes the unknown parameter vector, $\alpha(\cdot)$ denotes the known positive bounded function of the appropriate dimensions, $\omega(k) \in R^q$ denotes the disturbance input that belongs to $L_2[0,\infty]$, the discrete-time homogeneous Markov chain, which is denoted by $r(k)$, taking values into the set $S = \{s_1, \cdots, s_n\}$, and the one-step transition probability p_{ij},

$$p_{ij} = P\left(r(k+1) = s_j \mid r(k) = s_i\right). \tag{2}$$

For $r(k) = s_i$, the matrices $A(s_i)$, $B_1(s_i)$, $B_2(s_i)$ and $C(s_i)$ are constant matrices with the appropriate dimensions.

The control objective is to ensure that the system (1) stochastically stable when $\omega(k)$ is 0 and has H_∞ performance γ from the disturbance input $\omega(k)$ to the output $y(k)$ over $[0,\infty]$, i.e.,

$$\mathrm{E}\left(\sum_{k=0}^{\infty} y^T(k)y(k)\right) \le \gamma \sum_{k=0}^{\infty} \omega^T(k)\,\omega(k), \tag{3}$$

where E denotes the operator of expectation, $y^T(k)$ denotes the transpose of $y(k)$, and γ is a given scaler.

The definition of the stochastically stable and stability theorem is shown first in this section [34].

Definition 1: If for every initial state $(x(0), r(0))$, a finite number $M(x(0), r(0)) > 0$ exists, such that

$$\mathrm{E}\left\{\sum_{k=0}^{\infty} x^T(k)\,x(k)\,|x(0), r(0)\right\} < M(x(0), r(0)), \tag{4}$$

then system (1) in the case of $\omega(k) = 0$, $u(k) = 0$ and $\alpha(k) = 0$ is said to be stochastically stable.

Lemma 1: If for any given set of $\{W(s_i) > 0, i = 1, \cdots, N\}$, a series of appropriate dimension matrices $\{\chi(s_i) > 0, i = 1, \cdots, N\}$ exists, such that

$$\sum_{j=1}^{N} p_{ij}A^T(s_i)\,\chi(s_j)\,A(s_i) - \chi(s_i) = -W(s_i), \tag{5}$$

then system (1) in the case of $\omega(k) = 0$, $u(k) = 0$ and $\alpha(k) = 0$ is stochastically stable.

The design process of the control algorithm is expressed in the following:

Theorem 1: Set a scaler γ, if a series of appropriate dimension matrices $\{\chi(s_i) > 0, i = 1, \cdots, N\}$ exists, satisfying

$$\begin{aligned}\gamma >\ & B_1^T(s_i)\bar{\chi}(s_i)B_1(s_i) \\ & + \left(\left(\alpha^T(x(k))\,\alpha(x(k))\right)^{-1}\alpha^T(x(k))\,B_1(s_i)\right)^T\Gamma \\ & \times \left(\left(\alpha^T(x(k))\alpha(x(k))\right)^{-1}\alpha^T(x(k))B_1(s_i)\right),\end{aligned} \tag{6}$$

$$R(s_i) < 0, \tag{7}$$

where

$$\bar{\chi}(s_i) = \sum_{j=1}^{N} \chi(s_j)\,p_{ij}, \tag{8}$$

$$\begin{aligned}R(s_i) =\ & A^T(s_i)\,\bar{\chi}(s_i)\,A(s_i) - \chi(s_i) + C^T(s_i)\,C(s_i) \\ & + \varepsilon_1^{-1}A^T(s_i)\,\bar{\chi}(s_i)\,B_1(s_i)\,\Omega_1(s_i)^{-1} \\ & \times B_1^T(s_i)\,\bar{\chi}(s_i)\,A(s_i) + \varepsilon_4^{-1}A^T(s_i)\,\bar{\chi}(s_i)\,\alpha(x(k)) \\ & \times \Omega_2(s_i)^{-1}\alpha^T(x(k))\,\bar{\chi}(s_i)\,A(s_i) \\ & - A^T(s_i)\,\bar{\chi}(s_i)\left(\bar{\chi}(s_i) + \varepsilon_2^{-1}\bar{\chi}(s_i)\,B_1(s_i)\right. \\ & \times \Omega_1(s_i)^{-1}B_1^T(s_i)\bar{\chi}(s_i) + \varepsilon_5^{-1}\bar{\chi}(s_i)\,\alpha(x(k)) \\ & \left.\times \Omega_2^{-1}(s_i)\,\alpha^T(x(k))\,\bar{\chi}(s_i)\right)^{-1}\bar{\chi}(s_i)\,A(s_i),\end{aligned} \tag{9}$$

where

$$\begin{aligned}\Omega_1(s_i) =\ & \gamma - B_1(s_i)^T\,\bar{\chi}(s_i)\,B_1(s_i) \\ & - \left(\left(\alpha^T(x(k))\,\alpha(x(k))\right)^{-1}\alpha^T(x(k))\,B_1(s_i)\right)^T\Gamma \\ & \times \left(\left(\alpha^T(x(k))\alpha(x(k))\right)^{-1}\alpha^T(x(k))B_1(s_i)\right),\end{aligned} \tag{10}$$

$$\begin{aligned}\Omega_2(s_i) =\ & \Gamma - \alpha^T(x(k))\bar{\chi}(s_i)\,\alpha(x(k)) - \varepsilon_3^{-1}\alpha^T(x(k)) \\ & \times \bar{\chi}(s_i)\,B_1(s_i)\,\Omega_1^{-1}(s_i)\,B_1^T(s_i)\bar{\chi}(s_i)\,\alpha(x(k)),\end{aligned} \tag{11}$$

and $\Gamma > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > 0$, $\varepsilon_5 > 0$ are design parameters satisfying

$$\begin{aligned}\Omega_2(s_i) &> 0, \\ \varepsilon_1 + \varepsilon_2 + \varepsilon_3 &= 1, \\ \varepsilon_4 + \varepsilon_5 &= 1.\end{aligned} \tag{12}$$

There exists a stochastic adaptive control algorithm and the adaptive law:

$$\begin{aligned}u(k) =\ & -B_2^T(k)\left(B_2(k)\,B_2^T(k)\right)^{-1}\left(\alpha(k)\,\hat{\theta}(k)\right. \\ & + \left(\bar{\chi}(s_i) + \varepsilon_2^{-1}\bar{\chi}(s_i)\,B_1(s_i)\,\Omega_1(s_i)^{-1}\right. \\ & \times B_1^T(s_i)\,\bar{\chi}(s_i) + \varepsilon_5^{-1}\bar{\chi}(s_i)\,\alpha(x(k))\,\Omega_2^{-1} \\ & \left.\times (s_i)\,\alpha^T(x(k))\,\bar{\chi}(s_i)\right)^{-1} \\ & \left.\times\ \bar{\chi}(s_i)\,A(s_i)\,x(k)\right),\end{aligned} \tag{13}$$

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \left(\alpha^T(x(k))\alpha(x(k))\right)^{-1}\alpha^T(x(k)) \times \left(x(k+1) - \hat{x}(k+1)\right), \quad (14)$$

where

$$\hat{x}(k+1) = A(r(k))x(k) + \alpha(x(k))\hat{\theta}(k) + B_2(s_i)u(k), \quad (15)$$

where $\hat{x}(k)$ and $\hat{\theta}(k)$ are the estimated values of $x(k)$ and θ, respectively, such that the system (1), (13) and (13) is stochastically stable and has L_2-gain performance γ over $[0, \infty]$.

Proof: Choose a stochastic Lyapunov function as

$$V(k, r(k) = s_i) = x^T(k)\chi(s_i)x(k) + \tilde{\theta}^T(k)\Gamma\tilde{\theta}(k), \quad (16)$$

where $\tilde{\theta}(k) = \theta - \hat{\theta}(k)$. By differential calculus, we have

$$\begin{aligned}\Delta V(k, r(k) = s_i) &= \mathrm{E}[V(k+1, r(k+1)) | r(k) = s_i] - V(k, r(k) = s_i) \\ &= E\Big[x^T(k+1)\chi(r(k+1))x(k+1) + \tilde{\theta}^T(k+1)\Gamma\tilde{\theta}(k+1)\Big] \\ &\quad - x^T(k)\chi(s_i)x(k) - \tilde{\theta}^T(k)\Gamma\tilde{\theta}(k). \end{aligned} \quad (17)$$

Substituting (1) into (17), we obtain

$$\begin{aligned}\Delta V(k, r(k) = s_i) = \mathrm{E}\Big[&(A(r(k))x(k) + B_1(r(k))\omega(k) \\ &+ \alpha(x(k))\theta + B_2(r(k))u(k))^T \\ &\times \chi(r(k+1))(A(r(k))x(k) \\ &+ B_1(r(k))\omega(k) + \alpha(x(k))\theta \\ &+ B_2(r(k))u(k)) + \tilde{\theta}^T(k+1) \\ &\times \Gamma\tilde{\theta}(k+1)\Big] - x^T(k)\chi(s_i)x(k) \\ &- \tilde{\theta}^T(k)\Gamma\tilde{\theta}(k), \end{aligned} \quad (18)$$

and based on (18), we can obtain

$$\begin{aligned}&\Delta V(k, r(k) = s_i) \\ &= \mathrm{E}[(A(r(k))x(k) + B_1(r(k))\omega(k) \\ &\quad + \alpha(x(k))\theta + B_2(r(k))u(k))^T\chi(r(k+1)) \\ &\quad (A(r(k))x(k) + B_1(r(k))\omega(k) \\ &\quad + \alpha(x(k))\theta + B_2(r(k))u(k)) \\ &\quad + \tilde{\theta}^T(k+1)\Gamma\tilde{\theta}(k+1)\Big] \\ &\quad - x^T(k)\chi(s_i)x(k) - \tilde{\theta}^T(k)\Gamma\tilde{\theta}(k) \\ &\quad + y^T(k)y(k) - y^T(k)y(k) \\ &\quad + \gamma\omega^T(k)\omega(k) - \gamma\omega^T(k)\omega(k). \end{aligned} \quad (19)$$

Rearranging (19), we have

$$\begin{aligned}&\Delta V(k, r(k) = s_i) \\ &= \Big[(x^T(k)A^T(r(k))\bar{\chi}(s_i)A(r(k))x(k) \\ &\quad + \omega^T(k)B_1^T(r(k))\bar{\chi}(s_i)B_1(r(k))\omega(k) \\ &\quad + \theta^T\alpha^T(x(k))\bar{\chi}(s_i)\alpha(x(k))\theta \\ &\quad + u^T(k)B_2^T(r(k))\bar{\chi}(s_i)B_2(r(k))u(k) \\ &\quad + x^T(k)A^T(r(k))\bar{\chi}(s_i)B_1(r(k))\omega(k) \\ &\quad + \omega^T(k)B_1^T(r(k))\bar{\chi}(s_i)A(r(k))x(k) \\ &\quad + x^T(k)A^T(r(k))\bar{\chi}(s_i)\alpha(x(k))\theta \\ &\quad + \theta^T\alpha^T(x(k))\bar{\chi}(s_i)A(r(k))x(k) \\ &\quad + x^T(k)A^T(r(k))\bar{\chi}(s_i)B_2(r(k))u(k) \\ &\quad + u^T(k)B_2^T(r(k))\bar{\chi}(s_i)A(r(k))x(k) \\ &\quad + \omega^T(k)B_1^T(r(k))\bar{\chi}(s_i)\alpha(x(k))\theta \\ &\quad + \theta^T\alpha^T(x(k))\bar{\chi}(s_i)B_1(r(k))\omega(k) \\ &\quad + \omega^T(k)B_1^T(r(k))\bar{\chi}(s_i)B_2(r(k))u(k) \\ &\quad + u^T(k)B_2^T(r(k))\bar{\chi}(s_i)B_1(r(k))\omega(k) \\ &\quad + \theta^T\alpha^T(x(k))\bar{\chi}(s_i)B_2(r(k))u(k) \\ &\quad + u^T(k)B_2^T(r(k))\bar{\chi}(s_i)\alpha(x(k))\theta \\ &\quad + \tilde{\theta}^T(k+1)\Gamma\tilde{\theta}(k+1)\Big] \\ &\quad - x^T(k)\chi(s_i)x(k) - \tilde{\theta}^T(k)\Gamma\tilde{\theta}(k) \\ &\quad + y^T(k)y(k) - y^T(k)y(k) \\ &\quad + \gamma\omega^T(k)\omega(k) - \gamma\omega^T(k)\omega(k). \end{aligned} \quad (20)$$

Using (6) to (15), we can obtain

$$\Delta V(k, r(k) = s_i) \le \gamma\omega^T(k)\omega(k) - y^T(k)y(k), \quad (21)$$

which implies

$$\mathrm{E}[V(k+1, r(k+1)) | r(k) = s_i] - V(k, r(k) = s_i) \le \gamma\omega^T(k)\omega(k) - y^T(k)y(k). \quad (22)$$

Because (45) is true for every s_i, we can obtain

$$\begin{aligned}&\mathrm{E}[V(k+1, r(k+1))] - V(k, r(k) = s_i) \\ &= \sum_{j=1}^{N}(\mathrm{E}[V(k+1, r(k+1)) | r(k) = s_i] \\ &\quad - V(k, r(k) = s_i))p_{s_i} \\ &\le \left(\gamma\omega^T(k)\omega(k) - y^T(k)y(k)\right)p_{s_i} \\ &= \gamma\omega^T(k)\omega(k) - Ey^T(k)y(k). \end{aligned} \quad (23)$$

Summing both sides from 0 to ∞, we have

$$\mathrm{E}(\sum_{k=0}^{\infty} y^T(k)y(k) \le V(0, r(0)) + \gamma\sum_{k=0}^{\infty}\omega^T(k)\omega(k). \quad (24)$$

Hence, the resulting closed-loop system is stochastically stable when $\omega(k)$ is 0 and achieves H_∞ performance when $\omega(k)$ is nonzero, i.e., from the disturbance input $\omega(k)$ to the output $y(k)$, it has finite L_2-gain not larger than γ.

III. APPLICATION TO GAS ENGINE AS AN ELECTRIC VEHICLE EXTENDER

The designed control law is applied to the control problem of the air-fuel ratio of a gas engine intended as an electric vehicle extender. The process of in-cylinder gas exchange of the gas engine is exhibited in Fig. 1.

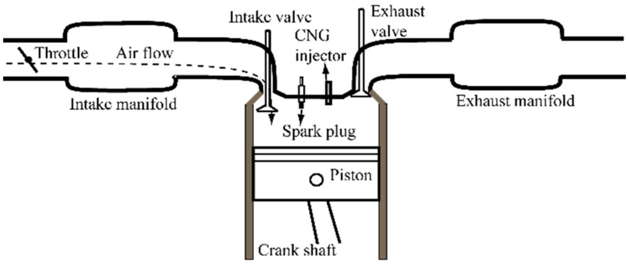

FIGURE 1. Sketch map of gas exchange process of gas engine as an electric vehicle extender.

A. DYNAMIC MODEL DESCRIPTION

The air-fuel ratio control dynamic model of the gas engine as an electric vehicle extender is same as the one for the gasoline engines shown in [32]:

$$\begin{aligned} y(k) &= M_a(k) - \lambda_d M_f(k), M_a(k+1) \\ &= \left(M_a(k) - \lambda_d M_f(k)\right)\xi(k) \\ &\quad + M_{an}(k), M_f(k+1) \\ &= M_f(k)(1-\mu)\xi(k) + M_{fn}(k), \end{aligned} \tag{25}$$

where $y(k)$ denotes the output of the system, $M_a(k)$ denotes the whole in-cylinder air mass, $M_f(k)$ denotes the whole in-cylinder fuel mass, the ideal air-fuel ratio λ_d is set to 17.4, $\mu \in [0, 1]$ denotes the efficiency of combustion, the mass of intake air $M_{an}(k)$ is viewed as an unknown constant intake air mass M_{an0} that adds a disturbance $\Delta M_{an}(k)$ belonging to $L_2[0, \infty]$, and the fresh fuel mass $M_{fn}(k)$ is the input. The residual gas fraction $\xi(k)$ is treated as a finite homogeneous Markov chain, its state space is $S = \{\xi_1, \cdots, \xi_n\}$ and the one-step transition probability is

$$p_{ij} = P\left(r(k+1) = \xi_j \mid r(k) = \xi_i\right). \tag{26}$$

System (25) is rearranged as follows:

$$\begin{aligned} y(k+1) &= \xi(k)y(k) - \lambda_d M_{fn}(k) + M_{an}(k) \\ &= \xi(k)y(k) - \lambda_d M_{fn}(k) + M_{an0} \\ &\quad + \Delta M_{an}(k). \end{aligned} \tag{27}$$

It is clear that system (27) meets the form of system (1).

B. CONTROLLER DESIGN

The unknown constant mass of intake air M_{an0} is calculated using the adaptive law [20]:

$$\widehat{M}_{an0}(k+1) = \widehat{M}_{an0}(k) + \left(y(k+1) - \hat{y}(k+1)\right), \tag{28}$$

where $\widehat{M}_{an0}(k)$ denotes the estimation of M_{an0} of the k th cycle, and the estimated value of $y(k)$ of the $k+1$ th cycle is given as:

$$\widehat{y}(k+1) = \xi(k)y(k) - \lambda_d M_{fn}(k) + \widehat{M}_{an0}(k). \tag{29}$$

The design process of the adaptive disturbance attenuation air-fuel ratio controller is given in the following:

Theorem 2: For system (27), after setting a constant $\gamma > 0$, if a class of $\chi(\xi_i) > 0$, $\xi_i \in \Omega$ exists, and satisfies

$$\zeta > \bar{\eta}(\xi_i) + \mu, \tag{30}$$

$$\phi(\xi_i) < 0, \tag{31}$$

where

$$\bar{\eta}(\xi_i) = \sum_{j=1}^{N} \eta(\xi_j) p_{ij}, \tag{32}$$

$$\begin{aligned} R(\xi_i) &= \bar{\eta}(\xi_i)\xi_i^2 - \eta(\xi_i) + 1 + \epsilon_1^{-1}\bar{\eta}^2(\xi_i)\xi_i^2\Omega_3^{-1}(\xi_i) \\ &\quad + \epsilon_4^{-1}\bar{\eta}^2(\xi_i)\xi_i^2\Omega_4^{-1}(\xi_i) \\ &\quad - \bar{\eta}^2(\xi_i)\xi_i^2\Big(\bar{\eta}(\xi_i) + \epsilon_2^{-1}\bar{\eta}^2(\xi_i)\Omega_3^{-1}(\xi_i) \\ &\quad + \epsilon_5^{-1}\bar{\eta}^2(\xi_i)\Omega_4^{-1}(\xi_i)\Big)^{-1}, \end{aligned} \tag{33}$$

where

$$\Omega_3(\xi_i) = \zeta - \bar{\eta}(\xi_i) - \mu, \tag{34}$$

$$\Omega_4(\xi_i) = \mu - \bar{\eta}(\xi_i) - \epsilon_3^{-1}\bar{\eta}^2(\xi_i)\Omega_3^{-1}(\xi_i). \tag{35}$$

and $\mu > 0, \epsilon_1 > 0, \epsilon_2 > 0, \epsilon_3 > 0, \epsilon_4 > 0, \epsilon_5 > 0$ are design parameters satisfying:

$$\begin{aligned} \Omega_4(\xi_i) &> 0, \\ \epsilon_1 + \epsilon_2 + \epsilon_3 &= 1, \\ \epsilon_4 + \epsilon_5 &= 1. \end{aligned} \tag{36}$$

Thus, the adaptive disturbance attenuation air-fuel ratio controller exists:

$$\begin{aligned} &M_{fn}(k) \\ &= \lambda_d^{-1}\Big(\hat{M}_{an0}(k) + \bar{\eta}(\xi_i)\xi_i\Big(\bar{\eta}(\xi_i) + \epsilon_2^{-1}\bar{\eta}^2(\xi_i)\Omega_3^{-1}(\xi_i) \\ &\quad + \epsilon_5^{-1}\bar{\eta}^2(\xi_i)\Omega_4^{-1}(\xi_i)\Big)^{-1} y(k)\Big). \end{aligned} \tag{37}$$

Therefore, the system (27) is stochastically stable when $\Delta M_{an}(k)$ is 0 and achieves H_∞ performance when $\Delta M_{an}(k)$ is nonzero, i.e., from $\Delta M_{an}(k)$ to the output $y(k)$, it has finite L_2-gain not larger than ζ:

$$\mathrm{E}\sum_{k=0}^{\infty} y^2(k) < \zeta \sum_{k=0}^{\infty} \Delta M_{an}^2(k). \tag{38}$$

Proof: Choose a stochastic Lyapunov function as follows

$$V(k, \xi(k) = \xi_i) = \eta(\xi_i)y^2(k) + \mu\tilde{M}_{an}^2(k), \tag{39}$$

where $\tilde{M}_{an0}(k) = M_{an0} - \hat{M}_{an0}(k)$. By differential calculus, we have

$$\begin{aligned} \Delta V(k, \xi(k) = \xi_i) &= \mathrm{E}\left[V(k+1, \xi(k+1)) \mid \xi(k) = \xi_i\right] \\ &\quad - V(k, \xi(k) = \xi_i) \\ &= E\Big[\eta(\xi(k+1))y^2(k+1) \\ &\quad + \mu\tilde{M}_{an}^2(k+1)\Big] - \eta(\xi_i)y^2(k) \\ &\quad - \mu\tilde{M}_{an}^2(k). \end{aligned} \tag{40}$$

Substituting (27) into (40), we obtain

$$\begin{aligned}\Delta V(k, \xi(k) = \xi_i) &= \mathrm{E}\Big[\eta(\xi(k+1))\big(\xi_i y(k) - \lambda_d M_{fn}(k) \\ &\quad + M_{an0} + \Delta M_{an}(k)\big)^2 + \mu \tilde{M}_{an}^2(k+1)\Big] \\ &\quad - \eta(\xi_i) y^2(k) - \mu \tilde{M}_{an}^2(k),\end{aligned} \tag{41}$$

and based on (41), we can obtain

$$\begin{aligned}\Delta V(k, \xi(k) = \xi_i) &= \mathrm{E}\Big[\eta(\xi(k+1))\big(\xi_i y(k) - \lambda_d M_{fn}(k) \\ &\quad + M_{an0} + \Delta M_{an}(k)\big)^2 + \mu \tilde{M}_{an}^2(k+1)\Big] \\ &\quad - \eta(\xi_i) y^2(k) - \mu \tilde{M}_{an}^2(k) + y^2(k) \\ &\quad - y^2(k) + \zeta \Delta M_{an}^2(k) - \zeta \Delta M_{an}^2(k).\end{aligned} \tag{42}$$

Rearranging (42), we have

$$\begin{aligned}\Delta V&(k, \xi(k) = \xi_i) \\ &= \bar{\eta}(\xi_i)\xi_i^2 y^2(k) + \bar{\eta}(\xi_i)\lambda_d^2 M_{fn}^2(k) \\ &\quad + \bar{\eta}(\xi_i) M_{an0}^2 + \bar{\eta}(\xi_i)\Delta M_{an}^2(k) \\ &\quad - 2\bar{\eta}(\xi_i)\xi_i\lambda_d y(k) M_{fn}(k) + 2\bar{\eta}(\xi_i)\xi_i y(k) M_{an0} \\ &\quad + 2\bar{\eta}(\xi_i)\xi_i y(k)\Delta M_{an}(k) \\ &\quad - 2\bar{\eta}(\xi_i)\lambda_d M_{fn}(k) M_{an0} \\ &\quad - 2\bar{\eta}(\xi_i)\lambda_d M_{fn}(k)\Delta M_{an}(k) \\ &\quad + 2\bar{\eta}(\xi_i) M_{an0}\Delta M_{an}(k) + \mu \tilde{M}_{an}^2(k+1) \\ &\quad - \eta(\xi_i) y^2(k) - \mu \tilde{M}_{an}^2(k) + y^2(k) \\ &\quad - y^2(k) + \zeta \Delta M_{an}^2(k) - \zeta \Delta M_{an}^2(k).\end{aligned} \tag{43}$$

By (28) to (37), we can obtain

$$\Delta V(k, \xi(k) = \xi_i) \leq \zeta \Delta M_{an}^2(k) - y^2(k), \tag{44}$$

which implies

$$\begin{aligned}\mathrm{E}[V(k+1, \xi(k+1)) | \xi(k) = \xi_i] - V(k, \xi(k) = \xi_i) \\ \leq \zeta \Delta M_{an}^2(k) - y^2(k).\end{aligned} \tag{45}$$

Because (45) is true for every ξ_i, we can obtain

$$\begin{aligned}\mathrm{E}&[V(k+1, \xi(k+1))] - V(k, \xi(k) = \xi_i) \\ &= \sum_{j=1}^{N} \big(\mathrm{E}[V(k+1, \xi(k+1)) | \xi(k) = \xi_i] \\ &\quad - V(k, \xi(k) = \xi_i)\big) p_{\xi_i} \\ &\leq \left(\zeta \Delta M_{an}^2(k) - y^2(k)\right) p_{\xi_i} \\ &= \zeta \Delta M_{an}^2(k) - E y^2(k).\end{aligned} \tag{46}$$

Summing both sides from 0 to ∞, we have

$$\mathrm{E}\sum_{k=0}^{\infty} y^2(k) \leq V(0, \xi(0)) + \zeta \sum_{k=0}^{\infty} \Delta M_{an}^2(k). \tag{47}$$

Hence, the resulting closed-loop system is stochastically stable when $\Delta M_{an}(k)$ is 0 and achieves H_∞ performance when $\Delta M_{an}(k)$ is nonzero, i.e., from the disturbance input $\Delta M_{an}(k)$ to the output $y(k)$, it has finite L_2-gain not larger thanζ.

Remark 1: The computation process of the adaptive disturbance attenuation air-fuel ratio controller (37) is the same as the one of the robust adaptive control algorithm (13), i.e., (37) can be calculated by replacing the multi-dimensional variables in system (13) by the corresponding one-dimension variables of the gas engine intended as an electric vehicle extender because system (27) is the typical form of system (1).

Remark 2: The main results of this paper contain the design of a robust adaptive control algorithm (13) for stochastic discrete-time nonlinear systems with Markovian jumping parameters and unknown system parameters (1), the methodological systems of which are dissimilar to those of the corresponding continuous-time systems. The design of the adaptive disturbance attenuation air-fuel ratio controller (37) for the gas engine intended as an electric vehicle extender is an application of the proposed control algorithm, that overcomes the influence from the intake air and residual gas on the control accuracy of the air-fuel ratio.

IV. NUMERICAL SIMULATION

Validation of the given control algorithm (37) is demonstrated using the numerical simulation, which is same as the one in [32]:

$$\begin{aligned} y(k) &= M_a(k) - M_f(k)\lambda_d, \\ M_a(k+1) &= \big(M_a(k) - \lambda_d \mu M_f(k)\big) r(k) + M_{an}(k), \\ M_f(k+1) &= M_f(k)(1-\mu) r(k) + M_{fn}(k), \\ \dot{M}_{an} &= \frac{\rho_a V_d \eta_v}{4\pi P_a}\omega_e P_m, \\ T_e &= \frac{H_u V_d \eta_i \eta_v P_m}{4\pi R T_m \lambda}, \\ J\dot{\omega}_e &= T_e - T_l, \\ \dot{P}_m &= \frac{R T_m}{V_m}\big(\dot{M}_i - \dot{M}_{an}\big), \\ \dot{M}_i &= s_0(1 - \cos\phi)\frac{P_a}{\sqrt{R T_a}}\psi\left(\frac{P_a}{P_m}\right), \end{aligned} \tag{48}$$

where $\dot{M}_{an}$ denotes the flow rate of the air mass leaving the manifold, ρ_a denotes the atmospheric density, V_d denotes the cylinder displacement, η_v denotes the volumetric efficiency, ω_e denotes the engine revolution, P_m denotes the manifold pressure, P_a denotes the atmospheric pressure, T_e denotes the mean indicated torque, H_u denotes the fuel low heating value, η_i denotes the indicated efficiency, R denotes the constant of gas, T_m denotes the manifold temperature, J denotes the rotational inertia, T_l denotes the external load, $\dot{M}_i$ denotes the mass flow rate of the air pass throttle, s_0 denotes the area of the throttle, ϕ denotes the opening of the throttle, and

$$\psi(s) = \begin{cases} s^{\frac{2}{k}}\left(\dfrac{2k}{k-1}(1-s)\right)^{\frac{k-1}{k}} & \text{if } s \geq \left(\dfrac{2}{k+1}\right)^{\frac{k}{k-1}} \\ k\left(\dfrac{2}{k+1}\right)^{\frac{k+1}{k-1}} & \text{if otherwise.} \end{cases} \tag{49}$$

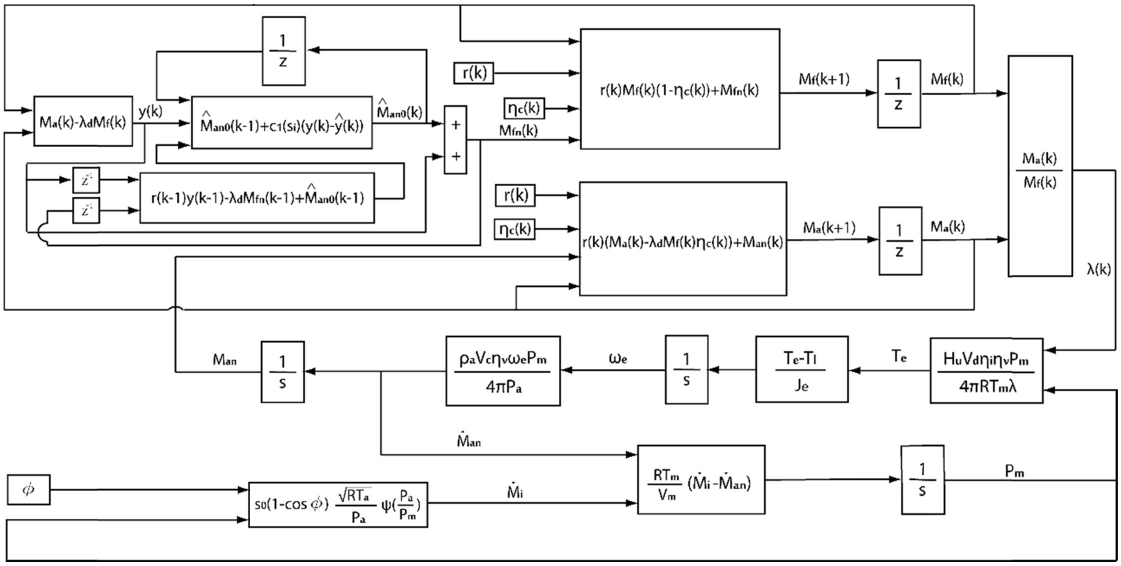

FIGURE 2. Sketch map of simulation.

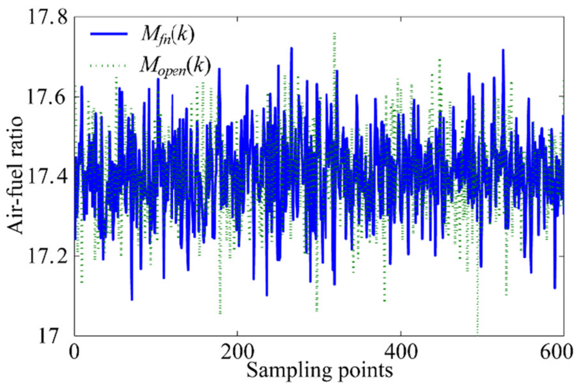

FIGURE 3. Air-fuel ratio of A.

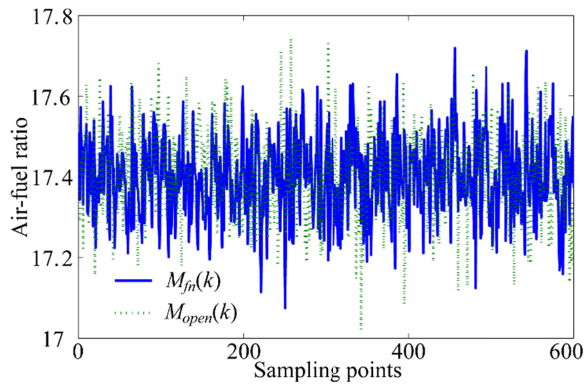

FIGURE 4. Air-fuel ratio of B.

Fig. 2 shows a sketch map of the simulation. The simulations were conducted under working conditions denoted by A, B and C. For A, the engine revolution is 800 rpm, the external load is 60 Nm, and $\hat{M}_{an0}(0)$ is 0.13 g. For B, the engine revolution is 1200 rpm, the external load is 60 Nm, and $\hat{M}_{an0}(0)$ is 0.14 g. For C, the engine revolution is 1200 rpm, the external load is 90 Nm, and $\hat{M}_{an0}(0)$ is 0.15 g. The control parameters γ, Γ, ξ_i, ϵ_1, ϵ_2, ϵ_3, ϵ_4, and ϵ_5 are chosen as 4.9, 2.5, 1.2, 0.4, 0.3, 0.3, 0.5, and 0.5, respectively.

The performance of the proposed control algorithm is shown in Figs. 3-8, where $M_{open}(k)$ denotes the open-loop controller as follows:

$$M_{open}(k) = \frac{M_{an}(k)}{\lambda_d}, \tag{50}$$

where $M_{an}(k)$ can be estimated by the air mass flow rate sensor, and $M_{fn}(k)$ denotes the adaptive disturbance attenuation air-fuel ratio controller (37). From the signals of the air-fuel ratio shown in Figs. 3-5, we observe that the air-fuel ratio can be controlled in a small range of the ideal air-fuel ratio by $M_{fn}(k)$ and $M_{open}(k)$ under all of the working conditions. The signals given in Figs. 6-8 show that the adaptive law can be tuned to a steady value by $M_{fn}(k)$ under each working condition.

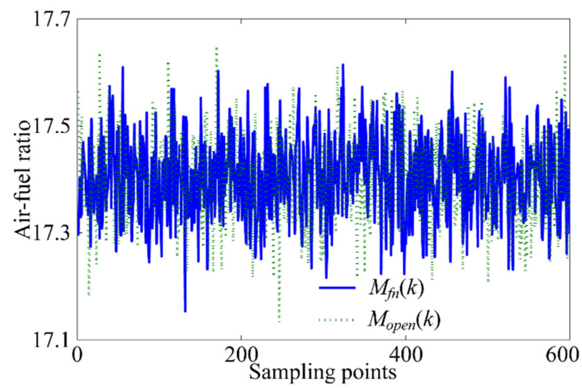

FIGURE 5. Air-fuel ratio of C.

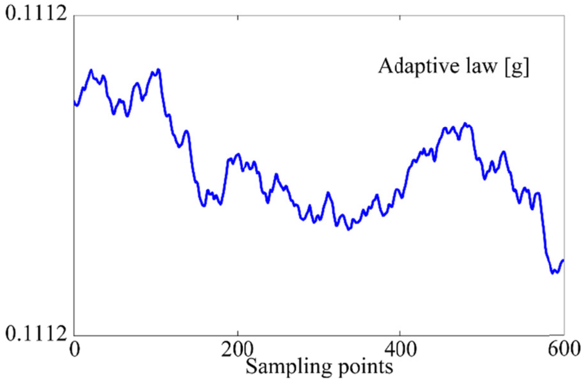

FIGURE 6. Adaptive law of A.

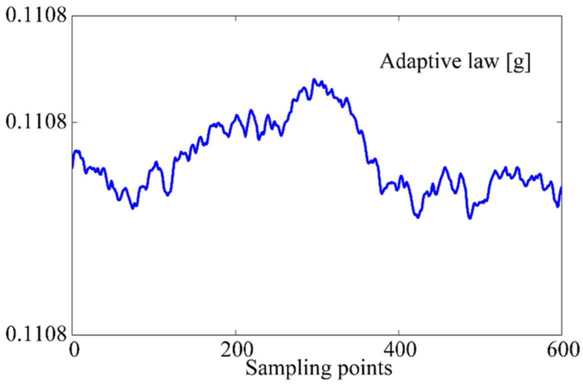

FIGURE 7. Adaptive law of B.

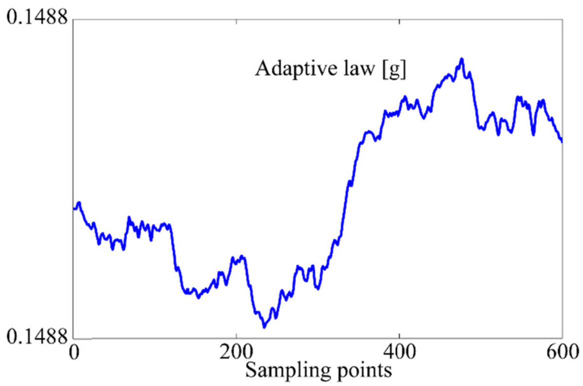

FIGURE 8. Adaptive law of C.

The control performance indices over 140000 sampling points of $M_{fn}(k)$ and $M_{open}(k)$ under the three working conditions are exhibited in TABLE I, where

$$J(N) = \sum_{k=1}^{N} (\lambda(k) - \lambda_d)^2 . \quad (51)$$

From TABLE I we find that the dispersion of the air-fuel ratio of $M_{fn}(k)$ is smaller than that of $M_{open}(k)$, which means that $M_{fn}(k)$ achieves better control performance under all of the working conditions.

TABLE 1. Control performance indices of A, B and C.

	A	B	C
$J_{M_{fn}(k)}(140000)$	1.0795E-02	1.0954E-02	6.0929E-03
$J_{M_{open}(k)}(140000)$	1.0804E-02	1.0963E-02	6.0983E-03

V. CONCLUSION

The paper addressed the adaptive control problem and disturbance attenuation for stochastic systems with nonlinear characteristics and Markovian jumping parameters. The robust adaptive controller is given, and the closed-loop system achieves stochastic stability without disturbance and achieves H_∞ performance over $[0, \infty]$ with nonzero disturbance. The proposed control law is validated through the robust adaptive air-fuel ratio control problem of the gas engine intended as an electric vehicle extender. The disadvantage of the proposed control algorithm is that the structure is complex because many influencing factors are considered, such as unknown parameters and external disturbance, which results in a complicated design and theoretical analysis process for the proposed control algorithm. For use of alternative fuels, the control problem of gas engines fueled by biogas will be the subject of our future work.

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