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A Traffic Assignment Approach for Multi-Modal Transportation Networks Considering Capacity Constraints and Route Correlations

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ABSTRACT A multi-modal urban transportation network provides travelers with diversified and convenient travel options. The study of multi-modal traffic assignment encounters great challenges from the common-line problem, route correlation, and the variability of travel tools. This article starts with the construction of super-networks to solve the common-line problem of multi-modal system. From the dimensions of time, fee, comfort, and transfer penalty, a multi-modal generalized travel cost function is proposed to reflect the impact of travel mode and transfer on route choice. Based on C-logit model, considering multi-modal capacity constraint and route correlation, a nonlinear programming model equivalent to the multi-modal stochastic user equilibrium is set up. The corresponding solving algorithm is designed by combining the augmented Lagrangian multiplier method and the successive weight average algorithm. Finally, the effectiveness of the proposed model and algorithms is verified through a numerical example, and the traffic assignment approach is applied in some typical scenarios. The multi-modal transportation network equilibrium approach proposed in this article takes into account the capacity constraints of different travel modes and solves the path overlapping problem in combined modes. It provides a basis and tool to formulate the traffic management strategy for public transport and combined mode trips.

INDEX TERMS Traffic assignment, multi-modal, stochastic user equilibrium, capacity constraints, C-logit.

I. INTRODUCTION

With the continuous development of transportation, urban travel has changed from a single mode to a diversified and complex multi-modal mode. Multi-modal travel and transportation systems have become mature. In order to predict the traffic flow effectively and exactly in the complex transportation network, it is necessary to study the traffic assignment model under multi-modal conditions.

Since Wardrop put forward the principle of user equilibrium (UE) and system optimization (SO) in 1952 [1], traffic assignment has become the research focus in the field of transportation, and the principle and method of traffic assignment have been constantly improved. The Wardrop user equilibrium principle is also called deterministic user equilibrium, which assumes that travelers know exactly the network traffic status. In fact, it is impossible for travelers to know the actual network information exactly. To improve this unrealistic assumption, Daganzo and Sheffi proposed the theory of stochastic assignment in 1977 assuming a randomness bias between users' estimation of network and the reality [2].

With the in-depth study of traffic theory, scholars have discovered some further problems. The first one is that the road capacity cannot be limited adequately by the BPR (Bureau of Public Road) function, which is commonly used in the travel time estimation. The second one is the irrationality of Logit method commonly used in stochastic traffic assignment, because Logit model has the independence of irrelevant alternatives (IIA). In addition, with the transition from a single mode to a multi-modal mode, traffic assignment includes not only route choices, but also choices of mode and transfer point.

In terms of road capacity constraints, a large number of solutions have been proposed to keep the road traffic flow

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within the capacity limit. These solutions are mainly divided into two categories: advance in the BPR function and capacity restriction for the assignment model.

To improve the BPR function, Daganzo used Davidson function instead of BPR function [3]. The Davidson function is a progressive function that adopts the idea of queuing theory. When the traffic flow reaches the capacity limit of road, the travel time will be very long. Yang considered the queuing phenomenon when the capacity exceeds the limit, and found that the travel time will be infinite when the traffic flow exceeds the capacity [4]. Zudhy re-calibrated the BPR function, which improves the problem that the original BPR function has little change in the saturation state [5].

Adding a capacity constraint to the assignment model is relatively direct, but it will make the commonly used Frank-Wolfe (F-W) method invalid. In order to solve F-W failure, scholars introduced the penalty function to convert the capacity constraint into a non-capacity constraint. For example, Cheng adopted the internal penalty function method [6], and Nie adopted the external penalty function method [7]. Another method is the augmented Lagrangian multiplier method (ALM), which can solve nonlinear programming problems well. Larsson used ALM to solve the assignment problem with capacity constraint, and deduced that Lagrangian multiplier corresponds to queuing delay [8]. Bell added capacity constraints to the SUE problem, and proposed an equivalent Logit-SUE model with road capacity constraints [9].

In terms of logit-based assignment method, it is easy to understand and has obvious advantages in large networks [10]. But due to its own structure, the Logit method has IIA characteristics and its accuracy needs to be improved [11]. IIA characteristics reflect the lack of sensitivity of logit-based method to network structure, resulting in excessive traffic flow assignment to overlapping paths [11]. Therefore, many scholars have proposed various improvements to reduce the impact of IIA characteristics. The improved models mainly include: C-Logit, path size logit, route perception logit, etc. C-logit considers the impact of path overlap by adding commonality factors [12]. Path size logit reduces the impact of overlapping by adding a logarithmic correction term of the ratio of overlapping paths to overall path length [13]. Route perception logit reduces the selective probability of overlapping paths by adding correction items [14].

As for the content of traffic assignment, the multi-modal traffic assignment involves not only route choice but also mode choice at the same time, because travelers usually make integrated travel choices in the multi-modal transportation system. At the beginning of multi-modal, transportation was underdeveloped. Combined mode trips only accounted for a small proportion. Therefore, the study of multi-modal traffic assignment focused on the combined model of mode split and traffic assignment. Florian *et al.* studied the traffic equilibrium problem of two modes of bus and car [15]. They develop a trip distribution, mode split, and traffic assignment model based on Wardrop's UE principle, and finally established an equivalent minimization model to solve the problem. Lam *et al.* studied the combined model of trip distribution and traffic assignment in multi-modal and established an equivalent convex optimization model, assuming that the impedance on the road segment is caused by multiple modes [16]. Abrahamsson *et al.* studied the reverse nested logit combination model that includes three stages of trip distribution, mode split and traffic assignment [17]. CEA studied the trip distribution, mode split and traffic assignment combined model considering the impact of congestion on cars and buses, in which the demand model is a hierarchical model [18]. In order to study the traveler's choice behavior of intercity buses and trains in the multi-modal urban economic circle transportation network, Li *et al.* established a combined model of mode split and traffic assignment [19].

With the encouragement of transportation policies and the construction of transfer facilities, combined mode trips have greatly increased. There are more and more researches on multi-modal traffic assignment including combined mode trips. In multi-modal trips with combined mode, travelers face a choice between pure mode and combined mode. If the combined mode is selected, the transfer point and route need to be decided. Travel choices are usually modeled by demand model or network model. The modelling approaches to address the travel choices in multi-modal transportation network can be divided into three categories [20].

The first category considers route choice only, and takes the choices of the combined mode and the transfer point just as a part of the network route choice. It assumes that the network is subject to congestion effect and the route choice is subject to the UE principle. The cost of the combined mode route is considered to be the same as the cost of other routes. Lo proposed the use of state-augmented multi-modal network (SAM) network to solve the problem of transfer and nonlinear structure in multi-modal trip, and built the SUE model based on SAM network model [21]. Gentile [22] *et al.* simplified cordon congestion pricing and suboptimal pricing by using hypergraph for network modeling and considered multi-modal assignment as route choice. Xiao *et al.* combined the activity-time-space (ATS) network and SAM network to construct ATS-SAM network. By expressing travel choices in different sections of ATS-SAM super network, a reliability-based user equilibrium model is proposed for scheduling daily activity-travel patterns in multi-modal transit networks under uncertainty [23]. Si *et al.* studied the UE model in multi-modal networks, based on the principle of minimum generalized travel cost in the mode split stage, and the principle of shortest travel time in the route choice stage, and finally constructed an equivalent variational inequality (VI) function to solve it [24]. Wu *et al.* expressed the mode split and route choice as a nested logit model and established an equivalent VI model [25]. Liu *et al.* studied the multi-modal probit-based SUE model under elastic demand considering congestion pricing and P+R facility layout [26].

The second category regards the combined mode as an independent mode. Travelers can choose combined mode trips in the mode split model. Once they have selected the mode, they will choose the route in the pure mode sub-network or the combined mode network. This kind of models allows the user perceived cost of the combined mode to be different from that of the pure mode, but the perceived costs for all combined mode routes are equal, regardless of the chosen transfer nodes. Hamdouch *et al.* used the binomial logit function solution method to divide in a multi-modal network, and studied the traffic assignment methods for UE and SO principle [27]. Li *et al.* [19] and Song *et al.* [28] applied the logit-based model to solve the mode split, assuming that each mode internally satisfies the UE principle, and finally constructed an equivalent VI model. Zhang *et al.* [29] studied the multi-modal network design problem, and constructed the VI model as the equivalent multi-modal user equilibrium nonlinear programming problem (MUE-NLP) model in the lower layer model. In order to maintain the continuity of mode split and route choice, Wu and Lam [30] chose a logit-SUE-based path selection model and constructed an equivalent VI model. Kitthamkesorn *et al.* [31] used nested-logit model for mode split, and cross-nested-logit model for traffic assignment. In addition, Ryu *et al.* [32] transformed the mode split into an elastic demand problem by making appropriate modifications to the network, and then designed a route-based projection gradient algorithm.

The third category is an extension of the second category. In this kind, the combined mode trip contains a detailed transfer point choice in demand sub-model. Lam *et al.* [33] divided travel choices into mode split, transfer point choice, and route choice in the car and P+R dual-mode network. Logit-based model was used in the three parts. Li *et al.* [34] studied the multi-modal equilibrium model under elastic demand, and used logit model to solve the mode split. The choice of transfer point and route conforms to the UE principle.

Existing research has achieved many results in multi-modal traffic assignment. However, it is found that the following three points can be improved.

1) For the multi-modal traffic assignment containing combined mode trips, dividing travel choice into multiple stages is too complex. This kind of division is not appropriate for large-scale road network, and will weaken the relationships among the stages in travel choice. Besides, there are still some drawbacks in the route choice model, such as the multi-modal common-line problem, the complexity of multi-modal perceived cost, and so on.

2) Logit model is mainly used to split the travel flow, but the premise of logit model is that the selection branches are independent. However, most of the time there will be correlation between routes because there are many overlap links, and when logit model is still used for assignment, too much flow will be allocated to those overlap links. When there is a combined mode, the travel modes are not independent from each other, and the overlapping effect between routes is more obvious. The logit model needs to be improved in multi-modal traffic assignment.

3) Capacity constraint is a defect in the traditional traffic assignment model, and affects the accuracy of assignment results. The impact of capacity constraint needs to be further considered in multi-modal traffic assignment. All kinds of travel tools have capacity constraints, and no one is consistent. Therefore, in the multi-modal traffic assignment, it's not the right way to solve all the capacity constraints by a single method.

In the face of these problems, this article provides a set of solutions. First, a multi-modal transfer network model is proposed to solve the multi-modal common-line problem, and lays the foundation for a multi-modal SUE model based on route choice. Then, the generalized travel costs of different modes are quantified from the dimensions of time, fee, comfort, and transfer penalty, to reflect the traveler's perception of different travel modes and routes. Moreover, the C-logit model is adopted to overcome the IIA characteristics of logit model. Finally, after analyzing the capacity constraints of different modes, a multi-modal SUE model considering capacity constraints and route correlations is established.

The structure of the remainder of the article is as follows. Section 2 studies the multi-modal transfer network model. Section 3 studies the multi-modal generalized travel cost function. Based on the consideration of capacity constraints and route correlations, a multi-modal SUE model and corresponding solution algorithm are proposed in Section 4. Section 5 demonstrates the effectiveness of the traffic assignment approach through a numerical example, and the proposed approach is applied in typical scenarios.

II. MULTI-MODAL TRANSPORTATION NETWORK MODEL

The urban transportation network model is the basis for the intuitive display, scientific calculation and data analysis of urban traffic system. Multi-modal transportation network includes multiple modes of transportation and inter-mode transfer. The network structure can fully express not only single mode trip but also combined mode trip. According to the structure requirement, we can use super-network to describe the multi-modal transportation network. Multi-modal super-network is composed of different sub-networks and transfer links between sub-networks.

The multi-modal super-network is represented as $G = (N, L)$, N is the set of nodes and L is the set of links. The node set includes car node, bus node, subway node and P+R node. The line set can be divided into travel link and transfer link. Travel link includes car link, bus link, and subway link. Transfer link includes the transfer link within the public transportation network and the transfer link among multi-modals.

An example is given to illustrate a typical multi-modal super network. Among a group of O-D pairs, there are three sub-networks, namely, car network, bus network and subway network, as well as transfer arcs and transfer facilities (as show in Fig. 1). The car network consists of 9 nodes and 12 edges. The bus network consists of 6 nodes and 3 bus lines. The subway network consists of 2 nodes

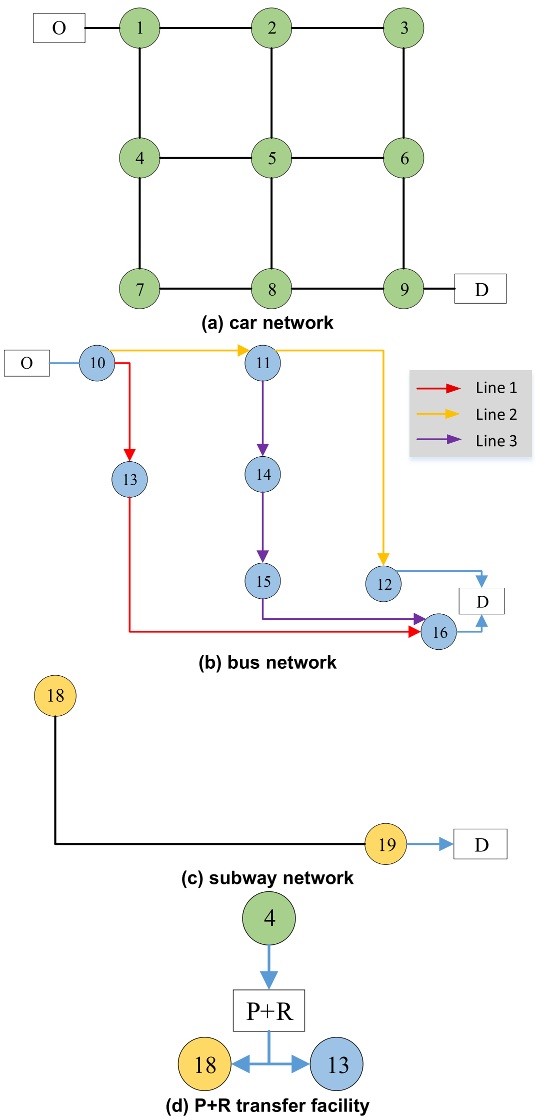

FIGURE 1. Networks of different traffic modes.

and 1 line. In addition, the P+R transfer facilities and transfer links between different modes are also included. It is obvious that different subnetworks have distinct network structures. The network structure of Fig 1 can be described as super-network shown in Fig 2. The multi-modal super-network contains all modes and inter-mode transfers, and can describe any travel choice through the hyper-path.

Multi-modal super-network integrates all modes of network, transfers within public transit mode and inter-mode transfers. If the effectiveness of path is not limited, some unreasonable paths in the multi-modal network will be found in the path search, while actually the selective probability of these paths is close to zero.

Effective multi-modal paths should observe the following rules:

(1) There are no circle and repeated link in the effective path.

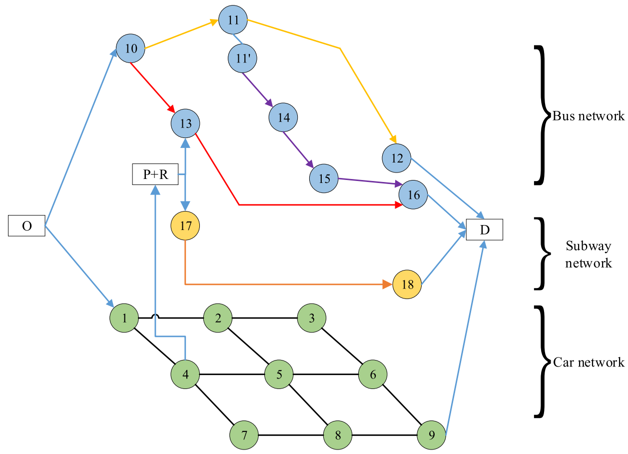

FIGURE 2. Multi-modal transportation super-network.

(2) The links in an effective path can only come from two modes at most. The car links can only appear at the beginning or the end, and the subway links can not appear intermittently.

(3) There are limits on the number of transfer links in the effective path, and there are no consecutive transfer links.

III. MULTI-MODAL GENERALIZED TRAVEL COST FUNCTION

A. PARAMETER AND VARIABLE DEFINITIONS

t_a	travel time on link a (min);
t_a^0	free-flow travel time on link a (min);
x_a	flow on link a (pcu/h);
C_a	capacity on link a (pcu/h);
α_1, β_1	parameters of bpr function, $\alpha_1 = 0.15$, $\beta_1 = 4$;
x_a^c	flow of cars on link a (pcu/h);
x_a^b	flow of buses on link a (pcu/h);
ψ	conversion coefficient between car and bus;
p_a	fuel fee on link a (yuan);
l_a	length of link a (km);
ρ	fuel fee per unit mile (yuan/km);
T_d	waiting time before boarding (min);
f_l	bus service frequency of line l (veh/h);
v_a	passenger volume waiting on the station (person/h);
v_d	passenger volume remaining in the bus (person/h);
U_l	bus operational capability of line l(person/h);
α_2, β_2, o	calibration parameters of waiting time function;
T_z	perceived travel time in the crowded bus (min);
E	standing area in the transit carriage (m^2);
x_b	passenger volume in the bus on link b(person);
s	seat capacity in the carriage (person)
α_3, β_3	calibration parameters of comfort loss;
K^{rs}	set of routes between r and s;
R, S	sets of origins and destinations, respectively;
f_k^{rs}	flow on route k between r and s (pcu/h);
q^{rs}	travel demand between r and s (pcu/h);
p_k^{rs}	route choice probability of route k between r and s;

θ	positive calibrated parameter which is used to measure the cost sensitivity on route choices
c_k^{rs}	generalized travel cost on route k between r and s (Yuan);
CF_k^{rs}	commonality factor on route k between r and s;
$G_a(x_a)$	generalized travel cost function of link a (Yuan);
$\delta_{a,k}^{rs}$	route and road incidence variable, $\delta_{a,k}^{rs} = 1$ if link a is on route k between r and s, otherwise $\delta_{a,k}^{rs} = 0$;
φ	correction parameters of commonality factor;
L_{kl}	length of the shared links between routes k and l (km);
L_k, L_l	length of route k and l, respectively (km);
A	set of links;
D	set of links with capacity constraints;
G_k^{car}	generalized travel cost by car on route k (Yuan);
G_k^{bus}	generalized travel cost by bus on route k (Yuan);
G_k^{subway}	generalized travel cost by subway on route k (Yuan);
α_t	value of travel time (Yuan/min);
α_d	value of waiting time (Yuan/min);
α_z	value of comfort loss (Yuan/min);
α_c	value of transfer penalty (Yuan/min).

B. MULTI-MODAL GENERALIZED TRAVEL COST FUNCTION

When travelers make travel choices, they always choose the route with the lowest travel cost. In the pure mode trip, people only need to make route choice, and most of the travel cost merely includes travel time. In the multi-modal transportation system, travelers consider the travel costs of different travel modes comprehensively. In this article, generalized travel costs are defined to reflect the travel costs of different traffic modes.

Different travel modes bring different feelings to travelers, and their distinctions are mainly reflected through four dimensions: travel time, travel fee, comfort loss and transfer penalty. Travel time and travel fee can be objectively measured, while comfort loss and transfer penalty bring subjective feelings and are difficult to count. The comfort level is brought by different traffic modes. The car has comfortable seats and cannot be disturbed by others on the travel, so the comfort level is the highest. However, passengers in bus or subway are vulnerable to interference from others, they have to reduce their own space or even lose their seats due to overcrowding. Transfer behaviors include the transfer between public transit lines, and the transfer between different traffic modes. Transfer may increase the walking distance and reduce the probability of finding a seat, so most travelers have an aversion to it.

The definition of generalized travel cost enables the route impedance of different modes to be compared, and can be used as the basis of mode choice and route choice for travelers.

1) GENERALIZED TRAVEL COST IN THE CAR NETWORK

The car's travel cost consists of travel time, fuel fee and parking fee. The generalized travel cost by car on route k, which is measured in terms of equivalent money units [35], can be expressed as:

$$G_k^{car} = \alpha_t \sum_{a\in A} t_a \delta_{a,k} + \sum_{a\in A} p_a \delta_{a,k} + I_p \tag{1}$$

a: TRAVEL TIME

When there are excess vehicles on the road, it will cause road congestion and make the car's driving time longer. The travel time function of car on the road is usually expressed by the BPR function [10].

$$t_a = t_a^0 [1 + \alpha_1 (\frac{x_a}{C_a})^{\beta_1}] \tag{2}$$

Cars and buses will interfere with each other on the same road at the same time. To simplify the study, we convert the buses to cars.

$$x_a = x_a^c + \psi x_a^b \tag{3}$$

b: FUEL FEE

Travelers have to pay for fuel, assuming that fuel fee is related merely to mileage.

$$p_a = l_a \cdot \rho \tag{4}$$

c: PARKING FEE

When choosing a car to travel, travelers need to take the parking problem into account, which involves parking fee. we assume the parking fee as a constant I_p.

2) GENERALIZED TRAVEL COST IN THE BUS NETWORK

The bus's travel cost consists of ticket fee, waiting time, travel time, comfort loss and transfer penalty. The generalized travel cost by bus on route k can be formulated as:

$$G_k^{bus} = \alpha_t \sum_{a\in A} t_a \delta_{a,k} + \alpha_d T_d + \alpha_z T_z + \alpha_c T_c + I_l \tag{5}$$

a: TICKET FEE

Let the fee of the bus line l be I_l. It shall be paid every time when travelers get on the bus.

b: WAITING TIME

As the demand for bus increases, the queue length at the station will be lengthened, resulting in a continuous extends of the passengers' average waiting time at the station. It is assumed that in addition to the service frequency, the number of waiting passengers at the station and the number of remaining passengers in the bus will both affect the waiting time [36].

$$T_{\mathrm{d}} = \frac{1}{f_l} + \alpha_2 [(v_a + \beta_2 v_d)/U_l]^o \tag{6}$$

c: TRAVEL TIME
Because the bus and the car are mixed on the road, the travel time function in-vehicle link is calculated according to the time function in the car network.

d: COMFORT LOSS
As more and more passengers get into the bus, travelers will feel that the time inside the bus is longer due to the increasing crowding, and the comfort loss will increase the travelers' perceived in-vehicle time [37].

$$T_z = t_a[\alpha_3(\frac{\max(0, x_b - s)}{E})^{\beta_3}] \tag{7}$$

e: TRANSFER PENALTY
Transfer penalty T_c refers to the quantification of the travelers' additional psychological burden caused by the transfer between different traffic modes or different lines.

3) GENERALIZED TRAVEL COST IN THE SUBWAY NETWORK
The travel cost of subway is similar to that of bus. It also contains the ticket fee, the waiting time before boarding, the in-vehicle travel time, the comfort loss and the transfer penalty. The differences between subway and bus are as follows.

1. In the subway system, the ticket fee is mainly determined by the distance between the starting and ending points, without considering the number of transfers. The ticket fee I_l is fixed when the O-D points are determined.

2. The travel time of subway is relatively fixed and in accordance with the schedule. The travel time of subway includes: travel time t_s on the link s, and waiting time t_{s_0} at any station. The congestion of subway is taken into account in terms of the comfort loss and waiting time before boarding.

The generalized travel cost by subway on route k can be formulated as

$$G_k^{subway} = \alpha_t \sum_{s \in A} (t_s + t_{s_0})\delta_{s,k} + \alpha_d T_d + \alpha_z T_z + \alpha_c T_c + I_l \tag{8}$$

4) GENERALIZED TRAVEL COST FUNCTION OF COMBINED MODE
1. Generalized Travel Cost Function of Combined Public Transit Mode

The combined public transit mode refers to the transfer mode from the bus network to the subway network or from the subway to the bus. When travelers adopt this travel mode, the generalized travel cost can be divided into three parts according to the travel process: the cost of bus, the cost of subway, and the transfer cost between the two modes. Here only the transfer cost between the two modes required to be studied on account of the generalized cost functions of two pure modes being analyzed in the previous section. The transfer cost mainly includes two parts, one is the walking time on transfer links, and the other is the psychological penalty brought by transfer. These costs do not change with the traffic volume, and are defined as constants in this article. For simplicity, the two costs are collectively named as public transit transfer cost T_g.

2) Generalized Travel Cost Function of Combined Mode of Car and Public Transit

The combined mode of car and public transit refers to the transfer mode between car and public transit at the P+R facility. Similar to the combined public transit mode, its cost can be divided into three parts: the cost of car, the cost of bus or subway, and the transfer cost between car and public transit. Here we only need to analyze the transfer cost between car and public transit. The transfer cost includes two parts: the travel time on transfer links and the psychological penalty, which are regarded as constants. These two parts of costs are collectively referred to as P+R transfer cost T_r.

IV. THE MULTI-MODAL TRAFFIC ASSIGNMENT MODEL CONSIDERING CAPACITY CONSTRAINTS AND ROUTE CORRELATIONS
Since the SUE is more realistic than UE and Logit assignment technology has its advantages in large networks, the multi-modal traffic assignment is studied based on the Logit-based SUE model in this article. But the Logit-based stochastic user equilibrium model still has two drawbacks to be concerned.

1) In the assignment model, the BPR function is constantly used for the travel time of vehicle on the road. Whereas BPR is a polynomial function fitted with a large amount of measured data, it performs a weak ability to limit the capacity constraints.

2) Logit-based assignment technology is insensitive to network structure, and has IIA characteristics. In the multi-modal transportation network, there are same links in several routes, and the real time of these routes cannot be irrelevant. This will lead the logit-based model to allocate too much traffic flow on the common lines.

In view of these two problems, we improve the multi-modal traffic assignment model from the analysis of the characteristics of capacity constraints and route correlations in the multi-modal transportation system.

A. CAPACITY CONSTRAINTS AND ROUTE CORRELATIONS ANALYSIS
1) CAPACITY LIMITATION ANALYSIS
Each mode has the capacity constraint in the multi-modal transportation system. If the capacity constraint is not considered, the outcomes of traffic assignment may be quite far from the reality, especially in the case of a congested network.

For a car network, if we do not consider the capacity constraint, the traffic volume allocated to critical roads is likely to exceed the road capacity, and even reach two or three times over the limit [38], while the other roads may not be allocated with traffic volume. In fact, it is impossible for the traffic volume to exceed the capacity of the road. Once the traffic demand exceeds the capacity, the queue will occur on the adjacent upstream road.

For public transit, the congestion effect caused by capacity constraint is mainly reflected in two aspects:

①Although the desired bus is coming, some passengers are unable to board the bus due to capacity constraint, which will result in an increase in passengers' waiting time.

②The congestion degree of the bus carriage is beyond the passengers' tolerance of comfort. In particular, whether you can find a seat or a comfortable standing space has a great influence on the passenger's willingness to take the bus.

2) ROUTE CORRELATIONS ANALYSIS

In order to solve the adverse impact of route correlation, scholars have proposed many improvement methods, including C-Logit, path size logit, route perception logit, etc. Among them, C-logit model identifies the correlation degree of correlative route through commonality factors. It maintains the same single-layer pattern as polynomial logit. It conforms to the real route choice behavior of travelers. However, all the models are used to solve the problem of route correlations in pure-mode.

In a multi-modal transport system, traffic assignment includes not only route choice but also mode choice. When every mode is independent, you can directly use logit to divide. But when there is a combined mode, the route in the combined mode is not independent, and is related to the two single modes of transfer.

B. MULTI-MODAL STOCHASTIC USER EQUILIBRIUM PRINCIPLE ANALYSIS

Based on the Wardrop's first principle, we propose the SUE principle in multi-modal assignment: all travelers will make mode and route choice with the lowest generalized travel cost. Every traveler's choice of mode and route will affect the choices of other travelers. When the travel cost of a mode or route changes, the number of travelers who choose this mode or route will change accordingly. When the multi-modal network reaches an equilibrium states, no traveler can unilaterally reduce the minimum generalized travel cost by changing mode and route.

According to the multi-modal assignment principle and the characteristics analyzed in section 4.1, the multi-modal SUE principle considering capacity constraints and route correlations is set up in Formulas (9) - (13).

$$f_k^{rs} = q^{rs} p_k^{rs}, \quad \forall k \in K^{rs}, r \in R, s \in S \tag{9}$$

$$p_k^{rs} = \frac{\exp\left(-\theta\left(c_k^{rs} + \mathrm{CF}_k^{rs}\right)\right)}{\sum\limits_{l \in K^{rs}} \exp\left(-\theta\left(c_l^{rs} + \mathrm{CF}_l^{rs}\right)\right)}, \quad \forall k \in K^{rs}, r \in R, s \in S \tag{10}$$

$$c_k^{rs} = \sum_a G_a(x_a)\delta_{a,k}^{rs}, \quad \forall a \in A,\ \forall r \in R,\ \forall s \in S, \forall k \in K^{rs} \tag{11}$$

$$CF_k^{rs} = \varphi \ln \sum_{l \in K^{rs}} \left(\frac{L_{kl}}{\sqrt{L_k}\sqrt{L_l}}\right), \quad \forall k \in K^{rs}, r \in R, s \in S \tag{12}$$

$$x_a = \sum_{r,s}\sum_k f_k^{rs}\delta_{a,k}^{rs} \le C_a, \quad \forall a \in D,\ \forall r \in R,\ \forall s \in S,\ \forall k \in K_{rs} \tag{13}$$

Formula (9) is the calculation formula of path flow with the SUE principle. Formula (10) refers to the route choice probability of C-logit model, in which adds a commonality factor reflecting the degree of route correlation. Formula (11) is the calculation formula of route generalized travel cost. Formula (12) is the calculation formula of the commonality factor. Formula (13) is the limitation of capacity constraint.

The model proposed in this article directly makes the route choice in the multi-modal network based on the SUE principle, and let the choice of the combined mode and the transfer point be a part of the network route choice model. The traditional study of multi-modal traffic assignment focused on the combined model of mode split and traffic assignment. When the traditional combined model conducts mode split and traffic assignment according to the SUE principle, the entire routes of road network reach equilibrium ultimately. It is consistent with the established equilibrium state. However, the combined model is difficult to discuss the route correlation problem when there are combined mode trips in the multi-modal assignment. Our multi-modal traffic assignment model analyzes the correlation degree of routes and allocates the volumes directly, so it solves the correlation problem of combined mode in the mode split.

C. NONLINEAR PROGRAM FORMULATION

An equivalent nonlinear program formulation for the C-logit SUE traffic assignment model considering capacity constraints and route correlations can be expressed as follows:

$$\min z(f) = \sum_{a \in A}\int_0^{x_a} G_a(\omega)\mathrm{d}\omega + \frac{1}{\theta}\sum_{r,s}\sum_k f_k^{rs}\ln f_k^{rs} + \sum_{r,s}\sum_k f_k^{rs} CF_k^{rs} \tag{14a}$$

$$\sum_k f_k^{rs} = q^{rs}, r \in R, s \in S \tag{14b}$$

$$f_k^{rs} \ge 0, \quad \forall k, r, s \tag{14c}$$

$$x_a = \sum_{r,s}\sum_k f_k^{rs}\delta_{a,k}^{rs} \le C_a, \forall a \in D, \forall r \in R, \forall s \in S, \forall k \in K_{rs} \tag{14d}$$

Proposition 1: The solution of the C-logit SUE model considering capacity constraints and route correlations satisfies the C-logit route choice probability.

Proof: Construct Lagrangian functions for (14a), (14b), (14c), and (14d).

$$L = z(f) + \sum_{r,s}\mu^{rs}\left(q^{rs} - \sum_k f_k^{rs}\right) + \sum_a d_a(x_a - C_a) + \lambda\sum_{r,s}\sum_k f_k^{rs} \tag{15}$$

where:

$$L_0 = z(f) + \sum_{r,s} \mu^{rs}(q^{rs} - \sum_k f_k^{rs}) + \sum_a d_a(x_a - C_a) \quad (16)$$

$\mu^{rs}(r \in R, s \in S), \lambda$ and $d_a(a \in D)$ are Lagrangian multipliers to limitation (14b), (14c) and (14d), respectively.

According to the Karush–Kuhn–Tucker (KKT) conditions in nonlinear programming theory, the Lagrangian function must meet the following criteria at the extreme point:

$$\begin{cases} \nabla_f L_0 = 0 & if\ f_k^{rs} > 0 \\ \nabla_f L = 0 & if\ f_k^{rs} = 0 \end{cases} \quad (17a)$$

$$d_a(x_a - C_a) = 0, a \in D \quad (17b)$$

$$d_a \geq 0, a \in D \quad (17c)$$

$$\lambda \leq 0 \quad (17d)$$

where:

$$\nabla_f L_0 = \sum_{r,s} \sum_k (\sum_a G_a(x_a)\,\delta_{ak}^{rs} + \frac{1}{\theta}\left(\ln f_k^{rs} + 1\right) + \mathrm{CF}_k^{rs} - \mu^{rs} + \sum_a d_a \delta_{ak}^{rs}) = 0 \quad (18)$$

$$\nabla_f L = \sum_{r,s} \sum_k (\sum_a G_a(x_a)\,\delta_{ak}^{rs} + \frac{1}{\theta}\left(\ln f_k^{rs} + 1\right) + \mathrm{CF}_k^{rs} - \mu^{rs} + \sum_a d_a \delta_{ak}^{rs} + \lambda) = 0 \quad (19)$$

Let $\tilde{c}_k^{rs} = \sum_a \tilde{G}_a(x_a)\delta_{rs}^{ak}, \tilde{G}_a(x_a) = G_a(x_a) + d_a$, where $\tilde{c}_k^{rs}$ represents a more generalized route travel cost. Formulas (17b) and (17c) ensure that when $x_a = C_a$, $d_a \geq 0$. Lagrangian multiplier d_a denotes the equilibrium waiting time in the queue.

Formula (18) is simplified as

$$\frac{1}{\theta}(\ln f_k^{rs} + 1) + \tilde{c}_k^{rs} + CF_k^{rs} - \mu_{rs} = 0;$$

Formula (19) is simplified as

$$\frac{1}{\theta}(\ln f_k^{rs} + 1) + \tilde{c}_k^{rs} + CF_k^{rs} - \mu_{rs} = -\lambda.$$

From formula (17d) we can obtain:

$$\frac{1}{\theta}(\ln f_k^{rs} + 1) + \tilde{c}_k^{rs} + CF_k^{rs} - \mu_{rs} \geq 0.$$

Formula (17a) is equivalent to:

$$\begin{cases} \frac{1}{\theta}(\ln f_k^{rs} + 1) + \tilde{c}_k^{rs} + CF_k^{rs} - \mu_{rs} = 0 & if\ f_k^{rs} > 0 \\ \frac{1}{\theta}(\ln f_k^{rs} + 1) + \tilde{c}_k^{rs} + CF_k^{rs} - \mu_{rs} \geq 0 & if\ f_k^{rs} = 0 \end{cases} \quad (20)$$

From formula (20), it can be obtained that when $f_k^{rs} > 0$, $\frac{1}{\theta}(\ln f_k^{rs} + 1) + \tilde{c}_k^{rs} + CF_k^{rs} = \mu_{rs}$, that is $f_k^{rs} = \exp(\theta\mu_{rs} - 1)\exp(-\theta(\tilde{c}_k^{rs} + CF_k^{rs}))$. Put these positive route flows into formula (14b), $\exp(\theta\mu_{rs} - 1) = q^{rs} / \sum_{k \in EK^{rs}} \exp(-\theta(\tilde{c}_k^{rs} + CF_k^{rs}))$. EK^{rs} is a set of routes with positive route flow, EK^{rs} can be replaced by K^{rs} if all route flow is positive. Therefore, C-logit route choice probability can be obtained:

$$f_k^{rs} = q^{rs} \frac{\exp(-\theta(\tilde{c}_k^{rs} + CF_k^{rs}))}{\sum_{l \in EK^{rs}} \exp(-\theta(\tilde{c}_l^{rs} + CF_l^{rs}))} \quad (21)$$

This completes the proof.

Proposition 2: The solution of the C-logit SUE model considering capacity constraints and route correlations is unique.

Proof: If the objective function and the constraint set are convex, the optimal programming problem is strictly convex optimization. For convex optimization problem, the local minimum point is the global minimum point. In addition, the first derivative necessary condition of minimum point is the sufficient condition of convex optimization problem.

Therefore, it is just required to prove that the programming problem (14a)-(14d) is a convex optimization problem.

The constraints are linear according to the analysis of Formulas (14a)-(14d). Linear functions are both convex and concave functions, so the model conforms to the requirement that 'constraint set' is a convex set.

Now it is necessary to prove the objective function

$$\min z(f) = \sum_{a \in A} \int_0^{x_a} G_a(\omega)\mathrm{d}\omega + \frac{1}{\theta}\sum_{r,s}\sum_k f_k^{rs} \ln f_k^{rs} + \sum_{r,s}\sum_k f_k^{rs} CF_k^{rs}$$

is a convex function.

According to the nonlinear programming theory, we can establish the Hessian matrix of the objective function and prove the positive definiteness of Hessian matrix to demonstrate that the objective function is a convex function.

First, the objective function takes the partial derivative of variable f:

$$\frac{\partial Z}{\partial f_k^{rs}} = \sum_{r,s}\sum_k (\sum_a G_a(x_a)\delta_{rs}^{ak} + \frac{1}{\theta}(\ln f_k^{rs} + 1) + CF_k^{rs})$$

The generalized travel cost of link is only related to its own traffic flow, so $\frac{\partial G_a(x_a)}{\partial x_b} = 0$.

Then

$$\frac{\partial G_a(x_a)}{\partial f_l^{mn}} = \frac{\partial G_a(x_a)}{\partial x_b}\frac{\partial x_b}{\partial f_l^{mn}} = \frac{\partial G_a(x_a)}{\partial x_b} \cdot \delta_{rs}^{bl} = 0$$

$$\frac{\partial f_k^{rs}}{\partial f_l^{mn}} = \begin{cases} 1 & if\ m = r, n = s, l = k \\ 0 & otherwise \end{cases}$$

so

$$\frac{\partial^2 Z}{\partial f_k^{rs}\partial f_l^{mn}} = \begin{cases} \frac{\mathrm{d}G_a}{\mathrm{d}x_b} + \frac{1}{\theta}\frac{1}{f_l^{mn}} & if\ m = r, n = s, l = k, a = b \\ 0 & otherwise \end{cases}$$

Therefore, the Hessian matrix of the objective function Z is a diagonal matrix. Since $\frac{1}{f_l^{mn}} > 0$, and the generalized travel cost function increases monotonically with the flow, that is, the Hessian matrix of the objective function Z is positive definite, and the objective function is strictly convex. Thus, the solution of the model is unique. This completes the proof.

V. SOLUTION ALGORITHM

The augmented Lagrangian multiplier (ALM) is adopted to solve the traffic assignment problem with capacity constraints, which can be viewed as an extension to the penalty method.

The ALM algorithm consists of four main steps:

Step 1. Define the augmented Lagrangian function and initialize the Lagrangian multiplier vector and the penalty parameter;

Step 2. Solve the Lagrangian multiplier and the unconstrained problem with penalty parameter;

Step 3. Determine whether the stopping criterion has been met;

Step 4. Update Lagrangian multiplier vector and penalty parameter repeatedly.

Since the augmented Lagrangian functions are strictly convex for each Lagrangian multiplier and penalty parameter, all existing algorithms for solving classical traffic assignment problems can be used in Step 2. In Step 4, if the dual constraint is not reduced to a sufficiently unfeasible level, the value of the penalty parameter will increase [10]. Readers who are interested in the proofs of the convergence of the ALM algorithm may refer to Larsson and Partriksson [8].

In Step 2, the MSWA (method of successive weight average) algorithm is adopted, which is an improved algorithm of MSA (method of successive average). MSA algorithm is an iterative algorithm, which obtains the final solution by the weighted average of several iterations. In solving the assignment problem, a set of auxiliary traffic volume of every link are obtained in each iteration. Then calculate the weighted average of the current traffic volume and auxiliary traffic volume of each link for the next cycle. This method is simple to use and has strong applicability, but the convergence speed is slow. MSWA changes the step size setting to give greater weight to the auxiliary travel volume from the latter. Liu *et al.* [39] and Meng *et al.* [40] prove that MSWA converges faster than MSA in dealing with SUE problems. In addition, when facing with large-scale systems, we can use the second-order cone program to meet accuracy and efficiency requirements simultaneously [41].

To sum up, the ALM algorithm is used as surrounding loop to solve the problem of link capacity constraints, and MSWA algorithm is used to solve the traffic assignment of C-logit model.

The augmented Lagrangian function of Formulas (14a)-(14d) is shown in Formula (22).

$$L_\rho(f,\mu) = z(f)+\sum_a \frac{1}{2\rho}\left\{\max\left[0,\ \mu_a+\rho(x_a-C_a)\right]-\mu_a^2\right\} \tag{22}$$

μ is a Lagrangian multiplier vector with dual constraints.

The specific steps of ALM algorithm are as follows:

Step 1: Initialization. Select the initial value of μ, then set $\rho^1(\rho^1>0)$ and iteration $n=1$.

Use the depth-first search algorithm to search out all paths, and then use the rules of effective paths to filter out the path set. And calculate the commonality factor of each path.

Step 2: Solving subproblem——C-logit model. Let the link cost function be defined as $G_a(x_a)+\max\{0,\ \mu_a^1+\rho(x_a-C_a)\}, a\in D$, solve the traffic assignment problem with unconstraint, and get the solution of link flow.

Step 2.1: Initialization. Set the iteration number $m=0$ and let each link flow $x_a^{(0)}=0$;

Step 2.2: Calculate the generalized cost of each path $\tilde{G}_l^{(\mathrm{m})}=0$ according to the link generalized cost function; calculate the generalized cost of all paths in the path set, and get the generalized cost of the shortest path $\min\{\tilde{c}_k^{rs}+CF_k^{rs}\}$;

Step 2.3: Filter out the valid route from the path set. The decision condition is the route generalized cost $\tilde{c}_k^{rs}+CF_k^{rs}\le(1+\sigma)\min\{\tilde{c}_k^{rs}+CF_k^{rs}\}, \sigma>0$;

Step 2.4: The C-logit model is used to allocate the traffic flow on the valid path, and then obtain the auxiliary link flow $y_a^{(m)}$;

Step 2.5: Calculate the current traffic volume $x_a^{(m+1)}$ of each link by MSWA algorithm.

$$x_a^{(m+1)} = x_a^{(m)}+\chi^{(m)}(y_a^{(m)}-x_a^{(m)})$$
$$\chi^{(m)} = \frac{n}{\sum_{i=1}^{n} i}$$

Step 2.6: Stop criteria. If satisfied

$$\varphi=\sqrt{\sum_{a\in A}(x_a^{(m+1)}-x_a^{(m)})^2}\Bigg/\left(\sum_{a\in A}x_a^{(m)}\right)\le\varepsilon,$$

(where ε is the convergence criteria), then $x_a^{(m+1)}$ is the solution, continue to Step 3; otherwise, set m=m+1, and return to Step 2.2.

Step 3: Update multiplier vector and penalty parameter.

$$\mu_a^{n+1} = \max\left\{0,\ \mu_a^n+\rho\left[x_a(\rho,\mu)-C_a\right]\right\}, a\in D$$
$$\rho^{n+1} = \begin{cases} k\rho^n, & \sqrt{\sum_a \max{}^2\{-\mu_a^n/\rho^n, x_a^n-C_a\}} \\ & \quad >\gamma\sqrt{\sum_a \max^2\{-\mu_a^{n-1}/\rho^{n-1}, x_a^{n-1}-C_a\}} \\ \rho^n, & Otherwise \end{cases}$$

Step 4: If $\sqrt{\sum_{a\in A|x_a>C_a}(x_a-C_a)^2}\le\varepsilon$, stop the iteration and (x^n,μ^n) is the solution. Otherwise, turn to Step 2.

VI. CASE STUDY

A. A TESTING ROAD NETWORK

Take the road network shown in Fig. 2 as an example and assume an OD pair with a demand of 2400 person/h, including three modes of transportation: car, bus and subway. In Fig. 2, the links between nodes 1 to 9 represent the car network, the links between nodes 10 to 16 represent the bus network, and nodes 17 to 18 represent the subway network.

Bus network consists of three lines, L1:10-13-16, L2:10-11-12, L3:11-14-15-16, and subway network has only one line, L4:17-18.

The values of the parameters in the multi-modal generalized cost function and the traffic assignment model are shown in Table 1.

TABLE 1. Parameter values.

Parameter	value	Parameter	value
α_1	0.15	β_1	4
α_2	1	β_2	0.2
α_3	0.02	β_3	1.8
ψ	2	θ	0.5
o	2	ρ	0.4(Yuan/km)
α_t	0.2(Yuan/min)	α_d	0.4(Yuan/min)
α_z	0.2(Yuan/min)	α_c	0.4(Yuan/min)
T_g	15(min)	T_c	10(min)
T_r	20(min)	--	--

The detailed information of public transit is shown in Table 2.

TABLE 2. Transit line information.

Bus line	Departure interval f_l (min)	Seat capacity	Standing area (m^2)	Ticket price (Yuan)
L1	12	200	30	1
L2	12	200	30	1
L3	12	200	30	1
L4	15	300	50	2

Details of the road network are shown in Table 3.

TABLE 3. Road network information.

Link	Length (m)	Free-flow time (min)	Capacity (pcu/h)
1-2	1000	5	400
1-4	1200	6	400
2-3	1000	5	300
2-5	1500	8	400
4-5	1500	8	400
4-7	1000	5	300
3-6	1000	5	300
7-8	1200	6	300
5-6	1500	8	400
5-8	1500	8	400
6-9	1200	6	400
8-9	1200	6	400

B. RESULT ANALYSIS

The depth-first algorithm is used to search all the paths and then filter according to the definition of the effective paths. The road network contains 6 pure mode effective paths for cars and 3 pure mode effective paths for buses, there are 2 effective paths of car and public transit combined mode, and 1 effective path of public transit combined mode. Details of the paths are shown in Table 4.

TABLE 4. Effective paths information.

Mode	index	Path	Commonality factor
Car mode	1	0-1, 1-2, 2-3, 3-6, 6-9, 9-D	1.0382
	2	0-1, 1-2, 2-5, 5-6, 6-9,	1.1625
	3	0-1, 1-2, 2-5, 5-8, 8-9, 9-D	1.1601
	4	0-1, 1-4, 4-5, 5-6, 6-9, 9-D	1.2777
	5	0-1, 1-4, 4-5, 5-8, 8-9, 9-D	1.2756
	6	0-1, 1-4, 4-7, 7-8, 8-9, 9-D	1.1678
combined car and public transportation mode	7	0-1, 1-4, 4-P+R, P+R-17, 17 -18, 18-D	1.0916
	8	0-1,1-4, 4-P+R, P+R-13, 13-16, 16-D	1.1270
public transit mode	9	0-10, 10-11, 11-12, 12-D	0.3401
	10	0-10, 10-11, 11-11', 11'-14, 14-15, 15-16, 16-D	0.4621
	11	0-10, 10-13, 13-16, 16-D	0.7841
combined public transportation mode	12	0-10, 10-13, 13-17, 17-18, 18-D	0.7272

For this road network, MATLAB is used to design the MSWA algorithm program for SUE model under non-capacity constraint, and the ALM algorithm embedded MSWA algorithm program for the SUE model under capacity constraints with different φ

The comparative results of the solution are shown in Table 5.

According to the analysis of the Table 5, we can get the following information.

1. The capacity constraints can ensure that the traffic flow on the road does not exceed its capacity. The change of φ will influence the result of flow distribution, and the bigger the value is, the greater the impact on the result will be. It indicates that the model can effectively adjust the impact of route correlation on the flow distribution. Therefore, the model and algorithm can effectively reflect the capacity constraint and route correlation on the impact of traffic flow.

2. In any case, the flow of P+R-13 is always zero, and people who transfer from car to public transit always choose subway at the P+R node. It shows that the subway has a great advantage over the public transport at the same starting point and ending point.

3. Under the condition of capacity constraint, the maximum flow of road network is determined by capacity, and it does not change with the variation of commonality factors. It indicates that commonality factors cannot influence the traffic flow of critical sections in the crowded environment.

4. In the calculation example, the commonality factor of the route obviously affects the number of people who transfer from car to public transit. Because the commonality factor of the combined mode route is smaller than that of single car

TABLE 5. Comparison of results with various constraints and φ.

Capacity Constraint	Uncon straint	Constraint					
φ	0	0	2.5	5	7.5	10	12.5
0-1	1365	800	800	800	800	800	800
1-2	699	400	400	400	400	400	400
1-4	666	400	400	400	400	400	400
2-3	385	215	225	235	245	254	263
2-5	314	185	175	165	156	147	138
4-5	246	194	183	172	161	151	140
4-7	298	191	199	206	214	220	227
3-6	385	215	225	235	245	254	263
5-6	286	185	176	166	156	147	138
5-8	274	194	182	171	161	151	140
7-8	298	191	199	206	214	220	227
6-9	671	400	401	401	401	401	401
8-9	572	385	381	377	375	371	367
9-D	1243	785	782	778	776	772	768
18-D	310	439	440	442	444	446	449
0-10	1035	1600	1600	1600	1600	1600	1600
10-11	512	765	768	771	774	778	782
10-13	523	835	832	829	826	822	818
4-P+R	122	15	18	22	25	29	33
P+R-17	122	15	18	22	25	29	33
P+R-13	0	0	0	0	0	0	0
11-12	234	390	394	396	398	400	401
11'-14	278	375	374	375	376	378	381
13-16	335	411	410	409	407	405	402
13-17	188	424	422	420	419	417	416
17-18	310	439	440	442	444	446	449
12-D	234	390	394	396	398	400	401
14-15	278	375	374	375	376	378	381
15-16	278	375	374	375	376	378	381
16-D	613	786	784	784	783	783	783

mode, when the value of φ is bigger, the combined mode route diverts more traffic flow from car mode.

C. TYPICAL SCENARIO APPLICATION

Since public transit and P+R modes are better than cars in terms of carbon emissions and overall capacity, the general idea for improving the operation of urban transport is to increase the proportion of public transit and P+R trips, and limit car mode trips. The complexity of P +R mode is that it has route correlation with other modes such as car and public transit. It is necessary to observe the proportion change of car and public transit under different measures.

Increasing the parking rate is usually taken as a way to reduce the proportion of car mode trip. Increasing the service frequency of public transit is often used to reduce the waiting time caused by capacity constraint and comfort loss. In order to verify the guidance of our model for multi-modal traffic management, the parking rate of cars and the service frequency of public transportation are selected as variables for sensitivity analysis.

To reflect the influence of various traffic management measures on the travelers' choice of mode, the sensitivity analysis of the parameters is carried out when $\varphi = 5$.

1) PARKING RATE

Based on the previous example, assuming an average parking time of 5 hours per vehicle, the car trips, P+R trips, and public transit trips are shown in Table 6 when parking rate is increased for all parking lots.

TABLE 6. The change of mode choice with increasing parking rate.

Parking rate	0~5	6	7	8
Car	782	740	676	607
P+R	18	12	6	4
Public Transport	1600	1648	1718	1789

As can be seen from Table 6, there is no effect on the flow of cars on the road network when parking rate less than 5 Yuan/h, since the road resources are in short supply and the demand is still greater than the capacity constraint of the road network. In order to improve the transportation network operation state and the speed of road operation, parking rate should be set above 6 Yuan/h. The sensitivity analysis of parking rate can also verify that the traffic assignment model considering capacity limitation is more effective for traffic management.

With the increase of parking rate, the traffic volume of P+R mode is also decreasing. In order to encourage travelers to select P+R mode, the parking rate for car trips is raised to 8 Yuan/h, and the impact of P+R reduction policies on mode choice in different degrees is shown in Table 7.

TABLE 7. The change of mode choice with the decrease of P + R parking rate.

P+R Parking rate	8	7	6	5	4	3	2~0
Car	607	577	485	365	311	301	301
P+R	4	38	155	310	365	376	378
Public Transport	1789	1785	1760	1725	1724	1723	1721

As can be seen from Table 7, when the P+R parking rate is reduced under the high parking rate of cars, there will be a large number of traffic diversion from cars. When the P+R parking rate is reduced to 3 Yuan/hour, the travel volume of the P+R mode reaches 376, accounting for 15.8% of the total travel volume. Therefore, reasonable car parking rate and P+R parking rate can have a great impact on traffic mode choice and network operation.

TABLE 8. The changes of mode choice with increasing service frequency of public transit.

Service frequency	4	5	6	7	8	9	10
Car	797	782	756	723	604	492	408
P+R	3	18	44	52	36	30	24
Public Transit	1600	1600	1600	1625	1760	1878	1968

2) SERVICE FREQUENCY OF PUBLIC TRANSIT

Based on the previous cases, with the increase of public transit service frequency (the same frequency of bus and subway), the flow changes of the mode choice are shown in Table 8.

From Table 8, we can find that with the increase of the service frequency of public transit, the volume of car trips decreases continuously. As the service frequency increases, car travelers will be attracted to the P+R mode. But as the service frequency continues to increase, the P+R travelers will be attracted to the bus mode. The increase of the service frequency of public transit can effectively reduce car trips and has a positive impact on the operation of the road network, so public transit should be reasonably promoted.

VII. CONCLUSION

The multi-modal traffic assignment problem with combined mode trips is a complex problem, which cannot be solved by the simple combination of mode split and traffic assignment. At present, Logit model is mainly used in multi-modal equilibrium problem, but the IIA characteristic is more obvious in mode choice because of the combined mode. Furthermore, in a multi-modal system, the capacity of each mode should be considered separately.

In this article, a set of multi-modal SUE assignment method is proposed. To solve the multi-modal common-line problem, we use super-network to describe the multi-modal network. In order to maintain the consistency of mode split and route correlations, a path-based assignment model is chosen. Meanwhile, the generalized travel cost function is defined in four dimensions to describe travelers' perceived costs and enhance the impact of travel modes and transfer on the route model. Based on the analysis of multi-modal capacity constraint and route correlation, a multi-modal SUE model is established, an equivalent nonlinear programming model is proposed to solve the equilibrium problem, and a solution algorithm combining ALM algorithm and MSWA algorithm is designed.

The multi-modal traffic assignment model can be widely used. In this study, the model is applied in some typical scenarios, and the following conclusions can be drawn. 1) Increasing parking rate can effectively reduce the proportion of car trips, while setting P+R parking rate discount can significantly increase the proportion of P+R trips. 2) Increasing the service frequency of public transit can improve the proportion of public transit, but the effect is limited. In practice, reasonable service frequency of public transit should be set in combination with operating costs. Furthermore, the model can also be applied to public transit network planning, P+R facility location, congestion toll design and other travel management issues.

There are still some ideal assumptions worth improving. The subsequent research can incorporate travel demand changes in real time with traffic operating conditions. In addition, every traveler has different attributes, such as travelers' perception decision-making mechanism [42], and we should consider the multiclass user in future.

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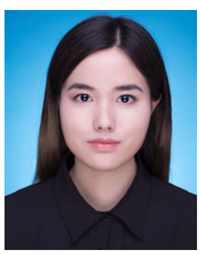

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