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Testability Modeling and Test Point Optimization Method of Multi-State System

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ABSTRACT Existing testability models are difficult to describe the multi-state characteristics of the system, so it is necessary to study the testability modeling method applicable to multi-state systems. A testability model with structure and function as the object is established in this paper. In order to describe the relationship between system state and test, the calculation method of the detectable state set of the test set is introduced. In order to quantitatively describe the testability of the system state, the concept of state detection rate is proposed for the first time. A test point optimization method that comprehensively considers the system fault detection rate, fault isolation rate, and state detection rate under the constraints of test cost is proposed. A numerical example shows that the best test set obtained by this method cannot only complete the system fault detection and isolation, but also obtain more system state.

INDEX TERMS Multi-state system, testability modeling, state detection, test point optimization, fault diagnosis and isolation.

I. INTRODUCTION

Compared with a binary system, a multi-state system considers one or more degraded states between the system and components from normal to fault. It can describe the relationship between system state and component state as well as system state and system fault through quantitative transition probability. Multi-state information is important to discover the potential failure mechanism and law of the complex system, which is helpful to improve the efficiency of system fault diagnosis and prediction and reduce the cost of diagnosis [1]. The research on MSS mainly focuses on the reliability modeling and analysis [2], [3], reliability optimization design [4], [5], maintenance management strategy [6], etc. Almost all the reported studies on MSS reliability analysis and assessment are based on the critical premise that the transition intensities and the initial state of elements and systems are exactly known in advance. However, the research on multi-state reliability focuses on the calculation of state transition intensities, while ignoring the problem of acquiring the initial state. Their approach is to assume that the initial state is known, which is unreasonable in many cases. Therefore, the purpose of this paper is to study how to obtain the state of MSS through system testing.

Testability modeling is a method of model-based diagnostic reasoning, popularly used in testability design and analysis. Testability models are mainly divided into dependency model (known as the diagnostic inference model or causal effect model), information flow model, multi-signal flow model, and hybrid diagnostic model. Dependency model mainly describes the correlation between functional units and test points, but can not reflect the correlation between test and failure mode. Information flow model can describe the information flow between fault and test, but its model is quite different from the actual structure. The multi-signal flow model connects fault and test through the flow direction of functional information, thus describing the relationship between fault and test. The modeling method of the hybrid diagnosis model is similar to that of the multi-signal model. As the multi-signal model is based on the hierarchical structure of the system, the structural units are connected through the functional signal flow relationship, so it is easy to model and understand, so it has been widely used [7]–[9].

In terms of testability modeling, Azam *et al.* [10] use a cause-effect relationship model to trace the power quality-related events to particular equipment of a system under consideration. Sheppard and Simpson [11] proposed an information flow model that directly reflects the information flow of faults and tests in the form of directed graphs and can automatically calculate the correlation matrix and quickly

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calculate various testability indicators. Deb *et al.* [12] proposed a multi-signal model, which does not directly describe the relationship between fault and test but connects the relationship between fault and test with functional signals. Some applications of the multi-signal model are discussed in [13]–[16]. Gould [17] proposed a hybrid diagnostic model, which is an extension of diagnostic dependency modeling that allows the inter-relationships between a system or device's tests, functions and failure modes to be captured in a single representation. Shi and Fan [18] provided a detailed analysis and summary of the diagnosis model generation rules of Rodon. In the testability analysis of the above models, it is considered that the system has only two states: perfect and fault. Considering the uncertainty of fault propagation, Gao *et al.* [19] proposed a fuzzy probability multi-signal flow graph model by combining the fuzzy theory with multi-signal flow graph. Considering the uncertainty of the test, Yang *et al.* [20] calculates the fault detection rate and fault isolation rate through the uncertainty dependency matrix. Although testability models have been applied in many fields, because these models do not describe the degradation state between normal and fault, they only use the normal or fault of test unit to qualitatively infer the normal or fault of other units of the system. Therefore, the existing test models are more suitable for the analysis of binary system.

In terms of test optimization of the binary system, scholars have made extensive research and put forward many optimization methods, mainly including sorting methods based on information theory [21]–[23] and search algorithm based on combination optimization [24]–[27]. The former mainly uses information entropy to define the test importance of fault detection and fault isolation and takes it as the weight of test, preferentially selects the test with high weight until it meets the requirements of testability index. The latter takes the minimum test cost as the objective function, the testability index requirements as the constraints to establish the mathematical model, and then uses an intelligent optimization algorithm to solve it.

Because the degradation state between normal and fault is considered in a MSS, not only the fault diagnosis of the system but also the degradation state diagnosis of the system should be considered in testability analysis. The test optimization problem of MSS is to consider the cost of tests and the acquisition of system state information when the testability index meets the requirements. The description of the relationship between system test and system state and the selection of test points are the main research contents of multi-state system test optimization. Therefore, to describe the system state and test, the testability model of multi-state system is established based on the characteristics of multi-state system. Then on the basis of the model, the relationship between system state and test is analyzed. Finally, a selection method of test points is proposed, which considers fault detection rate (FDR), fault isolation rate (FIR), and state detection rate (SDR) under the constraints of test resources.

The remaining sections of this paper are arranged as follows. In section II, the generic model of MSS is described and a testability model with structure and function as the object is established. In section III, the correlation between the system state and the test is analyzed, and the concept of state detection rate is proposed. Then a method for test points selecting that comprehensively considers the system fault detection rate, fault isolation rate, and state detection rate under the constraints of test cost is proposed. In section IV, a numerical example shows the effectiveness of the method in this paper and compared it with the test of the binary system. Finally, section V concludes the paper with a discussion of future research extensions.

II. TESTABILITY MODELING OF MSS

A. DESCRIPTION OF MSS

Consider a multi-state system composed of n elements, and any system element m may have $h_m + 1$ different states. The state-space of the element is described as $s_m = \{0, 1, \cdots, h_m\}$ in the order from low-performance level to perfect performance, where 0 is complete failure state and h_m is perfect performance state. The corresponding performance rates are represented by the set $g_m = \left\{g_{m0}, \cdots g_{mh_m}\right\}$, where g_{mk} is the performance rate if element m in state k. The set $p_m = \left\{p_{m0}, \cdots, p_{mh_m}\right\}$ is the probability of element m in different states, where p_{mk} is the probability if element m in state k. The performance rate $G_m(t)$ of system element m at any instant t is a random variable that takes its values $g_m : G_m(t) \in g_m$. Let $L^n = \left\{g_{10}, \cdots g_{1h_1}\right\} \times \cdots \times \left\{g_{n0}, \cdots g_{nh_n}\right\}$ be the space of possible combinations of performance rates for all of the MSS elements, and $M = \{v_1, \cdots, v_n\}$ be the space of performance rate for the entire system. The mapping $\phi(G_1(t), \cdots, G_n(t)) : L^n \to M$ represents the transformation relationship between elements performance rates space and system performance rates space, then we get the performance rate of the entire MSS at any instant t.

$$V(t) = \varphi(G_1(t), \cdots, G_n(t)) \tag{1}$$

where $\varphi(\bullet)$ is the structure-function of MSS, which represents the transfer relationship between element performance and system performance, and determined by the structural relationship of the system and the element performance attribute.

B. SYSTEM TEST MODELING

In the aspect of binary system testing, there are mature quantitative indicators for the evaluation of system fault testing, such as fault detection rate and fault isolation rate. For testability modeling of a multi-state system, there are three difficulties: 1. Describing the relationship between functional state and testing; 2. Establishing quantitative indicators of state testing that can be used for evaluation; 3. How to integrate fault test index and state test index, and optimize system test.

In the testability modeling of the binary system, the multi-signal model can obtain the correlation information between the system fault and test, and then searches for the best test with the lowest cost by satisfying the specified fault detection rate and fault isolation rate. For multi-state system testing, it is not only required to be able to diagnose and isolate system faults but also to obtain system state information. Therefore, in the testability design of MSS, it is necessary to analyze the state transfer process of MSS and obtain the correlation information between system state and test which also including the information between system fault and test. Because the multi-signal model is modeling of system structure, it cannot get the state information of system function, so it is not suitable for the test modeling analysis of MSS. In this paper, a multi-state system testability model is proposed, which takes advantage of hierarchical structure modeling of the multi-signal model, takes the function as a direct modeling object, describes the relationship between each element functions, and allows to add fault mode and test information, so as to realize the description of the correlation between system state and test.

The test model of MSS can be described as a directed graph $G = \{C, CF, FM, E, T, D\}$, where $C = \{c_1, \cdots, c_n\}$ is the set of n elements of the system; $CF = \{cf_1, \cdots, cf_r\}$ is the set of r functions of elements; $FM = \{fm_1, \cdots, fm_l\}$ is the set of fault mode, and each fault mode depends on the corresponding function; $E = \left\{e_{ij}\right\}$ is the set of directed edges from function i to function j, and describes the flow direction relationship of system functions; $T = \{t_1, \cdots t_k\}$ is the set of k available test points in the system; $D = \left[d_{ij}\right]$ is the dependency matrix of elementary functions and test, which take the function as row and test as the column. If the output signal from function i can reach test j through the directed edge, then $d_{ij} = 1$, otherwise $d_{ij} = 0$. When considering the uncertainty of fault propagation and test, if the probability that the output signal of function cf_i can reach test cf_j through the directed side is p_{ij} and the reliability of test t_i is σ_{t_i}, then $d_{ij} = p_{ij}\sigma_{t_j}$.

It is assumed that the model satisfies the following conditions:

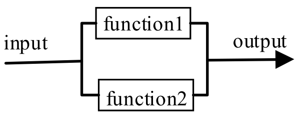
1) The fault identification standard of the whole system is consistent with that of the element functions. That is to say, when the element functions normally, the system functions normally.
2) The functional state division of the system and its subsystem is credible. That is to say when the functional state of the system element is determined, the system state can be determined by the system structure-function.

III. TEST POINT SELECTION METHOD OF MSS

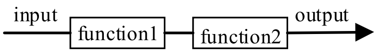
A. STATE TESTING CONSIDERATIONS

Whether the output signal of the system function conforms to the specified performance value is the basis for judging whether the system is at fault. Therefore, by testing the output signal of the function, we can judge whether the system has any fault according to whether the test result meets the specified performance index. However, although the test results can diagnose whether the system is in fault, it cannot determine whether the element function is at fault. To judge the functional state of the element, not only the output signal of the function but also the corresponding input signal should be obtained. If we want to obtain all of the input and output of elementary functions in the system, the test is expensive. In addition, in many cases (limited to space, weight, or technical conditions occupied by the test), the function output signal or input signal of some elements cannot be obtained. Therefore, we need to consider two aspects of the MSS test: 1. How to establish the relationship between the system test signal and element functional state. 2. How to choose the test point that can meet the system testability requirements.

For a MSS with n components, there may be $|L^n| = \prod_{i=0}^{n} h_i$ states. However, due to the structural relationship between the elements and the system, the number of system performance levels that can be test is not necessarily equal to $|L^n|$. For example, for the oil supply system shown in Figure 1, it is assumed that each oil supply pump may have one function and the function have three states, i.e. $s_1 = s_2 = \{0, 1, 2\}$. Corresponding to the fault, degradation performance and perfect performance of the oil supply ability is $g_1 = g_2 = \{0, 0.5, 1\}$. Then there may be 9 different states of the system shown in Fig. 1 (a) and (b). It is assumed that functions 1 and 2 in the parallel structure shown in Fig. 1 (a) are work-sharing relationships, i.e. performance level $V_a(t) = G_1(t) + G_2(t)$. $M_a = \{0, 0.5, 1\} + \{0, 0.5, 1\} = \{0, 0.5, 1, 1.5, 2\}$ is the performance level space of $V_a(t)$, which contains 5 states. We can infer that $G_1(t) = G_2(t) = 0$ when $M_a = 0$, and $G_1(t) = 1, G_2(t) = 1$ when $M_a = 2$. But cannot infer other states only by output. For the series system as shown in Figure 1 (b) as $V_b(t) = \min\{G_1(t), G_2(t)\}$, the system state can be completely inferred only when $M_b = 1$, then we can infer that $G_1(t) = 1, G_2(t) = 1$. The state of the system can only be obtained by fuzzy inference with a certain probability at other performance levels.



(a) parallel structure



(b) series structure

FIGURE 1. Multi-state series and parallel systems.

If $g_1 = \{0, 0.5, 1\}$ is the performance levels of function 1 and $g_2 = \{10, 15, 20\}$ of element 2, we get $M_a = \{0, 0.5, 1\} + \{10, 15, 20\} = \{10, 10.5, 11, 15, 15.5, 16, 20, 20.5, 21\}$, which contains 9 states. At this time, the state of function 1 and 2 can be inferred by any system state. In this case, the system is homogeneous, i.e. all functions and the whole system have the same distinguishable state.

According to the above analysis, we know that the state of entire system functions cannot be determined only by output test except for the homogeneous. In order to determine the state of the system, it is necessary to analyze the relationship between each state of the system and the test. Moreover, it is necessary to consider adding a reasonable test to comprehensively analyze the system state from multiple test results.

B. CORRELATION ANALYSIS OF STATE AND TEST

In order to establish the relationship between the test and functional state, set the function set related to the test point t_i as $RCF_{t_i} = \left\{ cf_j \left| d_{ji} = 1, cf_j \in CF \right. \right\}$. Define the test value space of system test point t_i as

$$H(t_i) = \varphi_i (g_1, \cdots g_n) \tag{2}$$

where $\varphi_i(\bullet)$ named as the test structure-function, indicating the mapping relationship between the performance level of each function and the test value. It is determined by the structure relationship and performance attribute from each function to the test point. If the test value space is no intersection when the function cf_m in its state s_{mk} and cf_m is not in the state, then this state can be detected. The state set that the test t_i can detect is described as

$$Hs(t_i) = \left\{ s_{mk} \left| \begin{array}{l} H(t_i | g_{mk}) \cap H(t_i | g_{mp}) = \phi, \\ \forall cf_m \in RCF_i, g_{mk}, g_{mp} \in g_m, k \neq p \end{array} \right. \right\} \tag{3}$$

where $H(t_i | g_{mk})$ is the value space of the test t_i when the function cf_m is in State s_{mk}.

When test $t_i = r \in H(t_i)$, its corresponding combination space of function state can be described as

$$L_{t_i}^r = \left\{ s_{im}, \cdots s_{jn} \left| \varphi_i (g_{im}, \cdots g_{jn}) = r \right. \right\} \tag{4}$$

If the elements in the combined space of function state contain some common states, these states can be determined as a whole by test t_i. Then, the combination function state set that can be determined is expressed as

$$Hcs(t_i) = \left\{ L_{t_i}^r(1) \cap \cdots \cap L_{t_i}^r(q) \left| \forall r \in H(t_i) \right. \right\} \tag{5}$$

where $q = \left| L_{t_i}^r \right|$ represents the number of combination states in $L_{t_i}^r$.

We can prove that $Hs(t_i) \subseteq Hcs(t_i)$. Set $s_{mk} \in Hs(t_i)$ and $\varphi_i(g_{im}, \cdots, g_{mk}, g_{jn}) = r$. If $\varphi_i(g_{ik}, \cdots, g_{mx}, g_{jp}) = r$, there must be $x = k$. According to the definition of $Hs(t_i)$, if $x \neq k$, then $H(t_i | g_{mk}) \cap H(t_i | g_{mx}) = \phi$. It conflicts with $\varphi_i(g_{ik}, \cdots, g_{mx}, g_{jp}) = r$. Therefore we can obtain $x = k$, and $s_{mk} \in [s_{ik}, \cdots, s_{mk}, s_{jp}] \cap [s_{ik}, \cdots, s_{mx}, s_{jp}]$. From this reasoning, we can get $Hs(t_i) \subseteq Hcs(t_i)$.

To describe the relationship between the test and other tests, set $RCF_{t_j} \subset RCF_{t_i}$, it can be seen that the function tested by t_j is the sub-function tested by t_i. At this condition, the test value space $H(t_i)$ can be calculated through $H(t_j)$, that is

$$H(t_i) = \varphi_{i-j}\left(g_{cf_k}, \cdots g_{cf_p}, H(t_j)\right) \tag{6}$$

where $cf_k, \cdots cf_p \in RCF_i - RCF_j, \varphi_{i-j}(\bullet)$ is the structure-function from t_j to t_i. It's easy to know that if $H(t_i)$ takes a certain value and can determine the value of $H(t_j)$, then $H(t_i)$ can determine the combination state determined by $H(t_j)$; if $H(t_i)$ takes any value and cannot determine the value of $H(t_j)$, then $H(t_i)$ cannot determine the combination state determined by $H(t_j)$.

Consider that $t_j = b \in H(t_j)$, and $t_i = a \in H(t_i)$ can determine some states in $cf_k, \cdots cf_p \in RCF_i - RCF_j$. Then in the condition that test t_j is known, the combined function state determined by the test t_i can be expressed as

$$Hucs(t_i, t_j) = \left\{ \begin{array}{l} L_{t_i,t_j}^{a,b}(1) \cap \cdots \cap L_{t_i,t_j}^{a,b}(l) \left| \forall a \in H(t_i), \right. \\ b \in H(t_j) \end{array} \right\} \tag{7}$$

where $L_{t_i,t_j}^{a,b} = \left\{ s_{km}, \cdots s_{pn} \left| H(t_i) = a, H(t_j) = b \right. \right\}$ is combined state space when $t_i = a \in H(t_i)$ and $t_j = b \in H(t_j)$. And $l = \left| L_{t_i,t_j}^{a,b} \right|$ is the number of state combinations of $L_{t_i,t_j}^{a,b}$.

The combination state set that test set T_s can determine includes the combination state that can be determined by a single test and the combination state that can be determined by the test together. In fact, for the test set T_s, the combined state determined by the test t_i is only related to the next subtest, because the subtest value is the result of its subtest value and the related function state. Therefore, the combination state set determined by the test set T_s can be expressed as

$$Hcs(T_s) = \bigcup_{t_i \in T_s} Hcs(t_i) \bigcup_{t_i, t_j \in T_s} Hucs(t_i, t_j) \tag{8}$$

Test structure-function describes the relationship between the system test and function state, and also between the high-level function test and low-level function test. It can ensure that the number of functional states is convergent in the process of transferring from the low level to the high level through the structural function, so as to avoid the number explosion problem of system state space.

C. TEST POINT SELECTION METHOD

The multi-state system with n element functions has a state combination $|L^n|$, and $S_l = \left\{ s_{1i}, \cdots s_{nj} \right\}$ is s state combination of the system. In this combination, the state set that can be determined by the test set T_s is $S_l D_{T_s}$. Without considering the importance of functions, the state detection rate of S_l is defined as the proportion of the number of states in $S_l D_{T_s}$ to the number of states in S_l, namely $S_l DR_{T_s} = \frac{|S_l D_{T_s}|}{n}$. Then the state detection rate of $T_s$for the entire system is defined as

$$SDR_{T_s} = \sum_{l=1}^{|L^n|} p(S_l) \cdot S_l DR_{T_s} \tag{9}$$

where $p(S_l)$ is the probability of the system in state combination S_l.

For a test set T_s, the function set related to the test set is $F_{DT_s} = \bigcup_{t_i \in T_s} RCF_i$, which is also the function fault set that T_s can detect. Then the system fault detection rate of test set T_s

can be expressed as

$$FDR_{T_s} = \frac{\sum_{cf_m \in FD_{T_s}} p\left(s_{m0}\right)}{\sum_{i=1}^{r} p\left(s_{i0}\right)} \tag{10}$$

Set the test set related to function cf_i in the system as $RT_i = \left\{t_j \left| d_{ji} = 1, t_j \in T\right.\right\}$, the function fault set that T_s can isolate is $F_{IT_s} = \left\{cf_i \left| cf_i \in F_{DT_s}, RT_i \oplus RT_j \neq 0, \forall cf_j \in F_{DT_s}\right.\right\}$. Then the system fault isolate rate of test set T_s can be expressed as

$$FIR_{T_s} = \frac{\sum_{cf_m \in FD_{T_s}} p\left(s_{m0}\right)}{\sum_{cf_m \in FI_{T_s}} p\left(s_{m0}\right)} \tag{11}$$

The test optimization of the binary system is to find the best test based on meeting the testability requirements (FIR, FDR), so as to minimize the test cost. When we design the test point of MSS, we hope that the test can get as much system state information as possible. So it is necessary to make the SDR maximum and the cost minimum based on meeting the requirements of the testability index. Generally speaking, the optimal test can obtain the higher testability index and state detection rate with the allowed test cost increases. Therefore, we can choose the optimal test by adjusting the test cost. Set the test cost of t_i is c_{t_i} and the test cost of T_s is $C_{T_s} = \sum_{t_i \in T_s} c_{t_i}$. Under the limitation of test cost C^*, the optimal test set can be expressed as

$$\begin{cases} T_s = \arg\max\left(r_{F_D} + r_{F_I} + SD_{T_s}\right) \\ s.t.\ C_{T_s} \leq C^* \end{cases} \tag{12}$$

It should be noted that the test cost mentioned here is a comprehensive consideration of the space, weight, price and time occupied by the test. In different engineering situations, the factors to be considered are also different. Here, the test cost is a unified description of the factors.

D. TEST OPTIMIZATION ALGORITHM

When a MSS contains a large number of elements, more test points are generally needed to meet the test requirements of the system. Assuming there are N available test points in the system, the system has 2^N test combination according to whether each test point is selected or not. Therefore, when N is a large value, the application of enumeration method to analyze each test combination will become a difficult NP complete problem. So it is necessary to use a heuristic search algorithm to find test combinations that meet the requirements. Because it is suitable for test point selection coding, we choose genetic algorithm (GA) to search the optimal test set. The algorithm flow is as Figure 2.

The search process of the optimal test set based on the genetic algorithm can be divided into the following steps:

1. Code each test set. For a system with n available test points, set the n-bit binary array. When the corresponding bit value is 1, the test point is selected; when the value is 0, the test point is not selected.

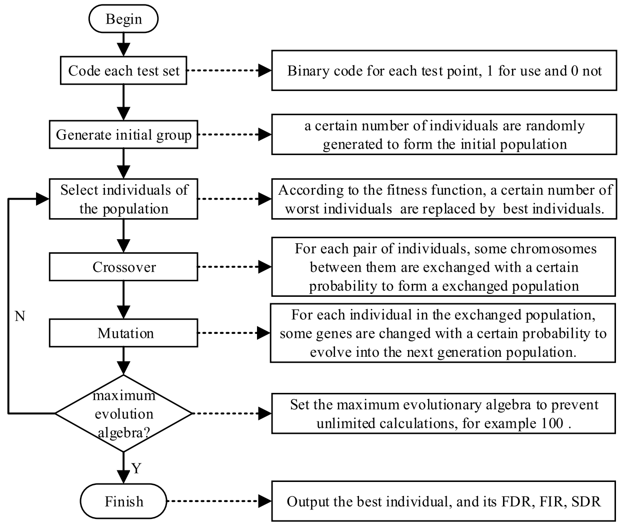


FIGURE 2. Algorithm flow of test optimization by GA.

2. Generate the initial group. An n-bit binary code represents an individual, then a certain number of individuals are randomly generated to form the initial population as the first generation of genetic evolution.
3. Select individuals of the population. Evaluating the advantages and disadvantages of individuals according to the fitness function. A certain number of worst evaluated individuals in the population are replaced by the same number of best individuals.
4. Crossover and mutation. For each pair of individuals in the selected population, some chromosomes between them are exchanged with a certain probability, and for each individual in the exchanged population, some genes are changed with a certain probability to evolve into the next generation population.
5. Determine whether the optimal test set meets the requirements or reaches the maximum evolution algebra. If it is, find the optimal individual and end, otherwise, repeat steps 3 and 4.

In different cases, designers may have different biases on failure detection rate, failure isolation rate, state detection rate. The fitness function can be given by the following formula:

$$f\left(T_s\right) = \lambda_1 FDR_{T_s} + \lambda_2 FIR_{T_s} + \lambda_3 SDR_{T_s} \tag{13}$$

By adjusting the values of λ_1, λ_2, and λ_3, we can realize the bias of different test indexes. In particular, it should be noted that when the value of λ_3 is 0, the test optimization is not related to SDR, but only considers the FDR and FIR, which is the same as the test optimization of a binary system.

IV. NUMERICAL EXAMPLE ANALYSIS

This method is suitable for the case where the state data of multistate systems are known. For systems lacking multi-state data, there are application limitations. In many engineering systems, when the statistical results of multi-state data are unknown, this paper makes a theoretical example analysis.

TABLE 1. Performance of each function in different states.

State	Function														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	50	50	50	50	50	100	40	50	40	40	40	40	50	40	180
2	100	100	100	100	100	200	80	100	80	80	80	80	100	80	360

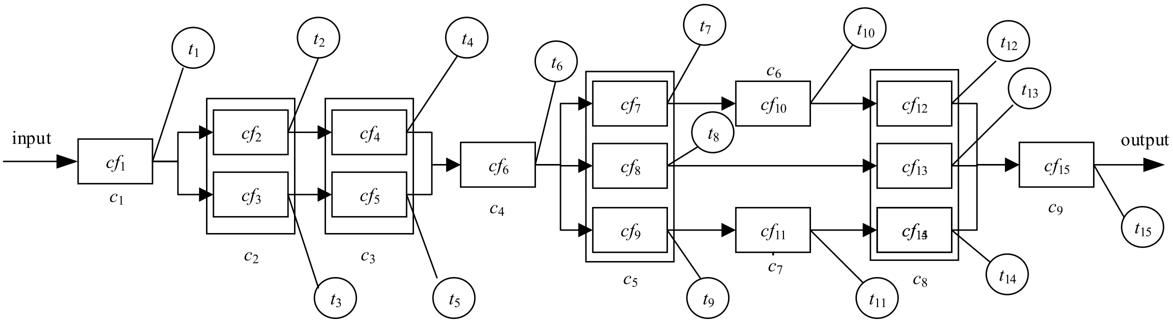


FIGURE 3. Testability model of a data processing system.

A. EXAMPLE DESCRIPTION

Figure 3 shows a data processing system, which consists of 9 elements with 15 functions and 15 available test points. Each function has a failure mode, i.e. the data transmission volume is 0, so the failure mode is not modeled in this example. Each function is assumed to contain three states, namely $s_i = \{0, 1, 2\}$, and the corresponding state probability is $p_i = \{0.1, 0.3, 0.6\}, i \in [1, 15]$.

Taking the data processing capacity of each function as its performance index, the performance of each function in each state is listed in Table 1

From the data transmission characteristics of the system, we can obtain the system structure function from (1) that

$$V(t) = \min \left\{ \begin{array}{l} G_1(t), \min[G_2(t), G_4(t)] + \\ \min[G_3(t), G_5(t)], G_6(t), \\ \min[G_7(t), G_{10}(t), G_{12}(t)] + \\ \min[G_8(t), G_{13}(t)] + \\ \min[G_9(t), G_{11}(t), G_{14}(t)], G_{15}(t) \end{array} \right\} \tag{14}$$

According to the definition of the dependency matrix between function and test. We can get

$$D = \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 0&1&0&1&0&1&1&1&1&1&1&1&1&1&1 \\ 0&0&1&0&1&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&1&0&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&1&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&0&1&0&0&1&0&1&1&0&1 \\ 0&0&0&0&0&0&0&1&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&0&0&1&0&1&0&0&1&1 \\ 0&0&0&0&0&0&0&0&0&1&0&1&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \tag{15}$$

TABLE 2. Value spaces of each test.

t_i	Value Space	t_i	Value Space	t_i	Value Space
1	0,50,100	6	0,50,100,150,200	11	0,40,50,80
2	0,50,100	7	0,40,50,80	12	0,40,50,80
3	0,50,100	8	0,50,100	13	0,50,100
4	0,50,100	9	0,40,50,80	14	0,40,50,80
5	0,50,100	10	0,40,50,80	15	0,40,50,80,90,100, 130,140,150 ,180,190, 200,210,220,,230,260

According to formula (2), the test value spaces are shown in Figure 3.

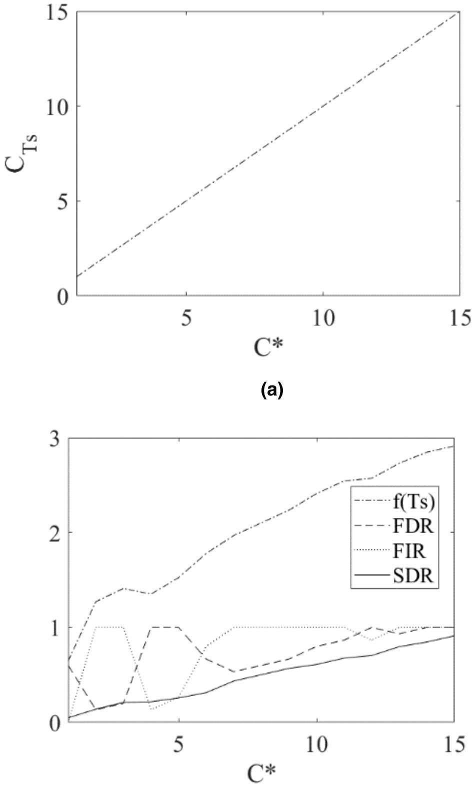
According to the value space of each test and formulas (4) and (5), the detectable state sets and corresponding test values of each test are shown in Table 3.

B. STATIONARY DISTRIBUTION OF SYSTEM STATES

Set $\lambda_1 = \lambda_2 = \lambda_3 = 1$, the test cost of each test point is $c_{t_i} = c = 1$. Then, for the test set $T_s \subset T$, the SDR, FDR, FIR can be calculated respectively according to equations (9), (10) and (11). According to the fitness function (13), the fitness of the corresponding test set is calculated. Figure 4 shows the test cost of the best test set under different test cost constraints, and the corresponding fitness function value, FDR and FIR, SDR. It can be seen that with the increase of test cost constraints, the test cost of the optimal test set is increasing, and the SDR is increasing. The final FDR and FIR are both 1, while the SDR is not 1, which indicates that every fault of the system can be detected and isolated, but the system states

TABLE 3. Detectable state sets and corresponding values of each test.

Test	detectable state sets	corresponding test values	Test	detectable state sets	corresponding test values
1	h_{10},h_{11},h_{12}	0,50,100	9	h_{91},h_{92}	40,80
2	(h_{12},h_{22})	100	10	(h_{72},h_{102})	80
3	(h_{12},h_{32})	100	11	(h_{92},h_{112})	80
4	(h_{12},h_{22},h_{42})	100	12	(h_{72},h_{102},h_{122})	80
5	(h_{12},h_{32},h_{52})	100	13	(h_{82},h_{132})	100
6	$\begin{pmatrix} h_{12},h_{22}, \\ h_{32},h_{42}, \\ h_{52},h_{62} \end{pmatrix}$	200	14	(h_{92},h_{112},h_{142})	80
7	h_{71},h_{72}	40,80	15	$\begin{pmatrix} h_{72},h_{92},h_{102},h_{112}, \\ h_{122},h_{142},h_{152} \end{pmatrix},$ $\begin{pmatrix} h_{72},h_{82},h_{92},h_{102},h_{112}, \\ h_{122},h_{132},h_{142},h_{152} \end{pmatrix}$	210, 260
8	h_{82}	100			



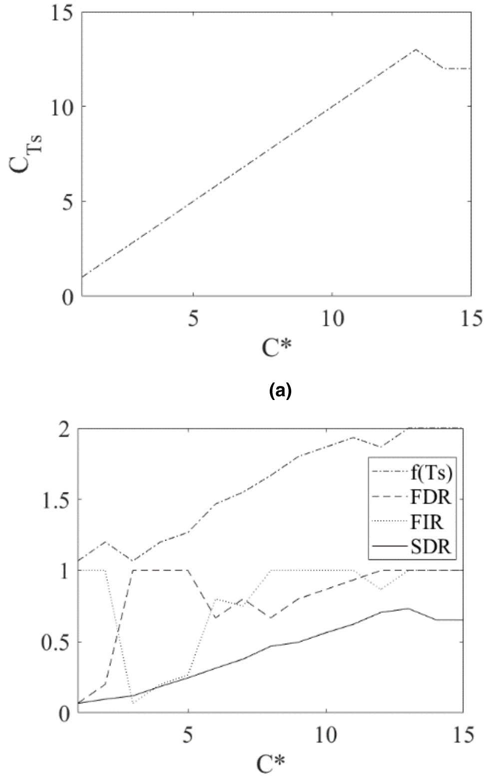
(b)

FIGURE 4. Change of parameters of MSS with test cost.

can not be fully detected even if all available test points are selected.

In order to compare the optimal test set for MSS consideration with that for binary system consideration. Set $\lambda_1 = \lambda_2 = 1$, $\lambda_3 = 0$, the test optimization is not related to SDR, but only considers the FDR and FIR, which is the same as the test optimization of multi-signal flow model for the binary system. The optimal test set parameters under different test cost constraints are shown in Figure 5. It can be seen that when the test cost constraint is 14 or 15, the test cost of the optimal test set is 12, which means that the system only needs 12 test points to complete the detection and isolation of all faults. In addition, due to the decrease of SDR when the test cost constraint is 14 compared with 13, the state information obtained by the optimal test will also be reduced.

Table 4 shows the optimal test set and corresponding parameter values obtained by the GA algorithm when considering MSS under different test cost constraints. Table 5 shows



(b)

FIGURE 5. Change of parameters of binary system with test cost.

TABLE 4. Optimal test set and parameter value of MSS.

C^*	4	6	8	10	12	∞
Optimal T_s	7,8,13,15	2,3,4,5,6,9	1,2,3,4,5,7,8,9	2,3,4,5,8,9,10,11,13,14	1,2,3,4,5,8,9,10,11,12,13,14	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15
FDR	1	0.47	0.6	0.87	0.93	1
FIR	0.2	1	1	1	1	1
SDR	0.26	0.34	0.50	0.65	0.74	0.91

TABLE 5. Optimal test set and parameter value of binary system.

C^*	4	6	8	10	12	∞
Optimal T_S	7,8,10,15	2,3,4,8,10,13	2,3,4,5,7,9,11,14	2,3,4,5,8,9,10,11,13,14	1,2,3,5,8,9,10,11,12,13,14,15	2,3,4,5,8,9,10,11,12,13,14,15
FDR	1	0.67	0.67	0.87	1	1
FIR	0.2	0.8	1	1	0.87	1
SDR	0.18	0.31	0.47	0.56	0.7	0.65

the corresponding results when considering the binary system.

By comparing Table 4 and Table 5, it can be found that when the cost constraint value is 4, the best test set of MSS and binary system have the same FDR and FIR, but the optimal test set of MSS can obtain higher SDR value. Under other test cost constraints, the sum of FDR and FIR of MSS and the binary system is only slightly different, but the optimal test set of MSS can still get higher SDR value. It is shown that the optimal test set obtained by our method can detect more system states while ensuring FDR and FIR indexes.

V. CONCLUSION

In this paper, the testability modeling and test point optimization method of the multi-state system are studied. Because the test of multi-state system requires not only the detection and isolation of system fault but also the detection of system state. Therefore, a multi-state system test model based on structure and function is established. Based on this model, the relationship between element functional state and test is analyzed and the concept of state detection rate is proposed for the first time. A test optimization method is proposed, which combines the system fault detection rate, fault isolation rate, and state detection rate. Because the search of the optimal test set is an NP-complete problem when there are many test points in a complex system, a heuristic search method of the optimal test set based on the GA algorithm is proposed. A numerical example is used to describe the test point optimization process of the multi-state system. The results show that the optimal test set obtained by this method can not only detect and isolate system faults but also obtain more system states. However, there are still some deficiencies in this paper. For example, in order to accurately calculate the state detection rate, each state of the system needs to be weighted, which may lead to NP problems when it is applied to large complex systems. Therefore, how to simplify the calculation process of state detection rate or reasonably estimate it is the focus of the next step of this paper.

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