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Developing Model of Fuzzy Constraints Based on Redundancy Allocation Problem by an Improved Swarm Algorithm

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ABSTRACT This study aims to develop a novel credibility-based fuzzy model for the constraints of a series-parallel redundancy allocation problem (RAP) with a mix of components under an uncertainty environment and investigates the reliability enhancement of wireless sensor networks (WSN) through a RAP for objective system reliability by subjecting it to two nonlinear constraints, i.e., cost and weight constraints, under an uncertainty environment. In this work, fuzzy numbers have been adopted to overcome the problems related to the exact values of cost and weight of components used in system that are hard to obtain due to uncertainty in the real world. Next, the credibility theory is utilized to convert the fuzzy numbers to crisp numbers. In addition, the difficulty of the RAP encountered is the need to resiliently sustain the two nonlinear constraints, while also aiming to maximize system reliability by optimizing the redundancy allocation of components in parallel of each subsystem in the WSN. Hence, the model is solved by an improved simplified swarm optimization (ISSO). To prove the effectiveness of the ISSO in solving the researching model, experimental results are tested on random benchmarks. The experimental results show a significant efficiency and effectiveness of the proposed ISSO.

INDEX TERMS Credibility theory, credibility-based fuzzy constraints, redundancy allocation problem (RAP), soft computing, simplified swarm optimization (SSO).

ACRONYM

WSN Wireless sensor networks
RAP Redundancy allocation problem
SSO Simplified swarm optimization
ISSO An improved SSO algorithm

NOTATIONS

$x_{i,j}$ The component number of the j th type component used in the i th subsystem for $i = 1, 2, \dots, n$; $j = 1, 2, \dots, k_i$
 X $X = (x_{1,j}, x_{2,j}, \dots, x_{n,j})$ is the solution vector of the redundancy allocation of components in the system
 $R(X)$ The system reliability
 R_i The subsystem reliability for $i = 1, 2, \dots, n$

$r_{i,j}$ The component reliability of the j th type component used in the i th subsystem
 $C(X)$ The system cost
 $W(X)$ The system weight
 $c_{i,j}$ The component cost of the j th type component used in the i th subsystem
 $w_{i,j}$ The component weight of the j th type component used in the i th subsystem
 C_u The predetermined upper bound of the system cost
 W_u The predetermined upper bound of the system weight
 L_i The lower bound of the number of components used in the i th subsystem for $i = 1, 2, \dots, n$
 U_i The upper bound of the number of components used in the i th subsystem for $i = 1, 2, \dots, n$
 $\tilde{c}_{i,j}$ The component cost with fuzzy number of the j th type component used in the i th subsystem

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$w_{i,j}$ The component weight with fuzzy number of the j th type component used in the i th subsystem

I. INTRODUCTION

This paper investigates the reliability enhancement of wireless sensor networks (WSN) under an uncertainty environment using the redundancy allocation problem (RAP) to devise and to optimize the redundancy allocation of components in parallel in the system, in order to attain superior system reliability [1]–[4]. Reliability plays an important role in assessing the performance of systems, especially the frequent use of high-tech systems, which are characteristically complex and expensive [4]–[19]. Therefore, many system engineers and scholars are devoted to designing and researching enhanced reliability in various systems, such as semiconductor industries [12], Internet of Things (IOT) [13], mechanical devices [14], electronic equipment [15], transportation systems [16], and so forth. In the design phase, the evaluation and analysis of system reliability is an important task to help the enhancement of reliability, which has been noted by numerous researchers [17], [18]. In several decades, RAP has attracted the attention of a number of researchers because its use and application can effectively enhance the reliability of many different systems during the design phase. Fyffe *et al.* [1] originally developed a famous series-parallel RAP to optimize system reliability. Chern [19] showed RAP is NP-hard. Nakagawa and Miyazaki [20] indicated RAP under surrogate constraints. Kuo and Prasad [21] suggested an overview of system-reliability optimization in 2000 and solved optimal RAP. Kuo and Wan [22] also provided an overview of system-reliability optimization in 2007. Onishi *et al.* [23] and Yeh [24] showed the RAP with mix of components. Hsieh and Yeh [25] suggested a penalty guided to solve the series-parallel RAP. Yun and Kim [26] indicated a multilevel RAP in series system. Yun *et al.* [27] and Yeh [28] extended the study in [26] to a multiple multilevel RAP in Series System. Yeh [29] showed the RAP in a complex system. In addition, numerous researchers such as Yeh [24] and Hsieh [30] indicated the series-parallel RAP following the famous series-parallel RAP with 14 subsystems developed by Fyffe *et al.* [1] to optimize the system reliability by the various meta-heuristic algorithms. Herein, we study the series-parallel RAP to optimize the redundancy allocation of mixed components while maximizing the system reliability by subjecting it to cost and weight constraints.

In most cases, the RAP is studied based on the assumption that the performance rate of components is an accurate value. In the real-world systems of WSN, it is difficult to obtain the significant precise value of the performance rate of components in the system with the associated uncertainty conditions, such as design, manufacture, and environment. The RAP with consideration for the uncertainty conditions has rarely been discussed in previous studies. Ravi *et al.* [31] studied RAP with respect to addressing the uncertainty state of the objective function solved by fuzzy

objective optimization function, and Mousavi *et al.* [32] presented RAP with the fuzzy objective function with respect to the performance rates and the availabilities of components in fuzzy. In the above previous RAP studies, which have introduced the uncertainty condition, both the studies focus on the uncertainty state by discussing the fuzzy objective function, and neither discuss the performance rates of the cost and weight of components under uncertainty conditions. This work considers the RAP subjected to the constraints of the cost and weight of components under an uncertainty environment.

Due to the uncertainty condition in a WSN, the imprecision of performance rates of components can always be resolved by the fuzzy number, which was first developed by Zadeh in 1965 [33]. The effectiveness of the fuzzy number in resolving the uncertainty condition has been approved by numerous studies in different fields, such as the above rare studies of RAP under the fuzzy environment [31], [32], the reliability optimization design by fuzzy uncertainties [34], and the optimal routes design in transportation under uncertainties by the fuzzy method [35]. It has been shown that fuzzy numbers play a very significant role in managing the uncertainties in different area. This work intends to explore the series-parallel RAP with mix of components subjected to the cost and weight constraints under an uncertainty environment, while the objective is to optimize the system reliability. Therefore, in this study, the cost and weight of components are evaluated in fuzzy numbers to overcome the uncertainty condition. To the best of our knowledge, the series-parallel RAP with a mix of components aimed at optimizing the redundancy allocation of the components to maximize the system reliability subjected to the cost and weight constraints in a fuzzy environment has not been addressed by the previous literatures, and this work is the first attempt to study it.

In an actual situation, many uncertainty values are absorbed as fuzzy numbers that finally need to be shifted to crisp numbers. Charnes & Cooper in 1959 [36] first developed a novel chance-constrained programming which models the constraints to be met with at least α of the probability, where α means the confidence level. Following this, Liu and Iwamura [37] applied the chance-constrained programming in a fuzzy environment which is adopted by several studies [38]–[40]. Later, in 2004, Liu [41] first proposed a credibility theory based approach to the fuzzy chance-constrained programming by applying it to topics in financial research. The credibility theory preserves the concept of the fuzzy chance-constrained programming, which means a fuzzy constraint can certainly satisfy at a confidence level, i.e., at a credibility value. The effectiveness and efficiency of the credibility theory in resolving the fuzzy constraints has been approved by several studies [42]–[44]. In this study of the series-parallel RAP subjected to the cost and weight constraints under a fuzzy environment, the cost and weight of components are evaluated in fuzzy numbers to overcome the uncertainty condition. Due to the cost and weight constraints including fuzzy numbers, this work adopts

the credibility theory to shift the fuzzy numbers to crisp numbers. Finally, the cost and weight constraints will be satisfied at a predetermined confidence level, i.e., a credibility level. Finally, this study develops a credibility-based fuzzy model for constraints solving in the series-parallel redundancy allocation problem with mix of components under an uncertainty environment.

The RAP is an NP-hard problem whose computational demands exponentially grow with respect to the number of components in the system [19]. Consequently, various meta-heuristic algorithms, such as variable neighborhood search (VNS) algorithms [45], [46], genetic algorithms (GA) [47], [51], simplified swarm optimization (SSO) algorithms [24], [28], and a hybridization of SSO, particle swarm optimization (PSO) and simulated annealing (SA) [29], optimized memetics algorithm (OMA) [48], [49], PSO [50], are adopted to resolve the RAP in the previous studies.

The SSO algorithm, originally called a two-stage discrete particle swarm optimization algorithm, was first developed by Yeh in 2009 [28]. The SSO belongs to the swarm algorithm and evolution computing is shown to be a simple, efficient, and flexible algorithm [52]–[54]. To push the improvement of the solution quality in solving the researching model, an ISSO algorithm is proposed to resolve the credibility-based fuzzy model for constraint solving in the series-parallel RAP with a mix of components under an uncertainty environment whose objective is to maximize the system reliability subjected to cost and weight constraints under an uncertainty environment.

The main contributions and novelty of this paper are further summarized in the following 3 bullets:

1. To the best of the authors' knowledge, the model of RAP subjected to the constraints of the cost and weight of components under an uncertainty environment that has been studied in this paper has not been discussed in the literature thus far.
2. The credibility theory used in this work is first adopted to shift the constraints of cost and weight of components with triangular fuzzy numbers to crisp values in the RAP.
3. An improved simplified swarm optimization (ISSO) algorithm with an update mechanism of the novel three-stage parameter setting is developed to optimize the RAP, which is an NP-hard problem.

The remainder of this paper is organized as follows. A related work of RAP is presented in Section II. The mathematical model of a series-parallel RAP subjected to cost and weight constraints under a fuzzy environment, fuzzy set theory, and credibility theory are presented in Section III. In Section IV, a review of SSO algorithms has been introduced. The proposed ISSO algorithm is presented in Section V. The performance of the proposed ISSO algorithm is shown in the numerical example in Section VI. Conclusions and suggestions for future research are discussed in Section VII.

II. A RELATED WORK OF RAP

The common model of the series-parallel RAP is presented as the integer nonlinear programming subjected to cost and weight constraints in Eqs. (1)-(4) [30].

$$\text{Maximize } R(\mathbf{X}) \quad (1)$$

$$\text{Subject to } C(\mathbf{X}) \leq C_u \quad (2)$$

$$W(\mathbf{X}) \leq W_u \quad (3)$$

$$L_i \leq \mathbf{X} = (x_1, x_2, \dots, x_n) \leq U_i, \quad i = 1, 2, \dots, n \quad (4)$$

The objective function maximizes the system reliability $R(\mathbf{X})$, as shown in Eq. (1), by optimizing the redundancy allocation of components in parallel in each subsystem and subjecting them to the cost and weight constraints as shown in Eqs. (2)-(3), respectively. The lower bound and upper bound of the number of components used in each subsystem are shown in Eq. (4). The lower bound L_i should not be less than two, i.e., $L_i \geq 2$, due to at least one redundancy component that has been used in each subsystem.

Furthermore, this study uses a series-parallel RAP with a mix of components to allow different type of components in the system to be subjected to the cost and weight constraints. Its mathematics model is presented in Eqs. (5)-(8) [30].

$$\text{Maximize } R(\mathbf{X}) = \prod_{i=1}^n R_i = \prod_{i=1}^n \left[1 - \prod_{j=1}^{k_i} (1 - r_{i,j})^{x_{i,j}} \right] \quad (5)$$

$$\text{Subject to } C(\mathbf{X}) = \sum_{i=1}^n \sum_{j=1}^{k_i} c_{i,j} x_{i,j} \leq C_u \quad (6)$$

$$W(\mathbf{X}) = \sum_{i=1}^n \sum_{j=1}^{k_i} w_{i,j} x_{i,j} \leq W_u \quad (7)$$

$$L_i \leq \sum_{j=1}^{k_i} x_{i,j} \leq U_i, \quad i = 1, 2, \dots, n \quad (8)$$

The objective function maximizes the system reliability $R(\mathbf{X})$, as shown in Eq. (5), by optimizing the redundancy allocation of components in parallel in each subsystem by subjecting them to the cost and weight constraints as shown in Eqs. (6)-(7), respectively, where $x_{i,j}$ means the component number of the j th type component used in the i th subsystem for $i = 1, 2, \dots, n; j = 1, 2, \dots, k_i$. The lower bound and upper bound of the number of components used in each subsystem are shown in Eq. (8).

III. MATHEMATICAL MODEL

The fuzzy model, fuzzy set theory, credibility theory, the credibility fuzzy model, and a small-scale example of series-parallel RAP subjected to cost and weight constraints in a fuzzy environment are presented in Section II.

A. FUZZY MODEL

Generally, addressing the WSN using the series-parallel RAP subjected to the constraints of the cost and weight of

components used in the system in Eqs. (6)-(7) are not explicit. While it aims to maximize the system reliability, the cost and weight of the components are dragged into various uncertain conditions in the WSN. To make the formulations more elastic and useful, the WSN using the series-parallel RAP subjected to the constraints of the cost and weight of components used in the system in Eqs. (6)-(7) are remodeled as fuzzy constraints with fuzzy numbers in Eqs. (9)-(10).

$$\text{Subject to } C(\mathbf{X}) = \sum_{i=1}^n \sum_{j=1}^{k_i} c_{i,j} x_{i,j} \leq C_u \quad (9)$$

$$W(\mathbf{X}) = \sum_{i=1}^n \sum_{j=1}^{k_i} w_{i,j} x_{i,j} \leq W_u \quad (10)$$

where $c_{i,j}$ and $w_{i,j}$ are the component cost and weight employed as fuzzy numbers for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, k_i$.

B. FUZZY TECHNIQUE

Zadeh in 1965 [33] first developed the fuzzy set theory as a technique of resolving inaccurate data due to the uncertainty environment in the real world. The definition of a fuzzy set is provided in following definition 1.

Definition 1. Fuzzy set: The fuzzy set is defined as the following model in Eq. (11) that can intelligently solve the uncertainty problems in the real world.

$$\{(x, u_A(x)|x \in \mathbf{X})\} \quad (11)$$

where \underline{A} is denoted as a fuzzy set; $u_A(x)$ is the membership function of \underline{A} for $x \in \mathbf{X}$, here $\mathbf{X} \in R$ (real number), $0 \leq u_A(x) \leq 1$.

The series-parallel RAP subjected to the cost and weight constraints under a fuzzy environment has been investigated such that the cost and weight of components are evaluated in triangular fuzzy numbers to overcome the uncertainty condition. Thus, the triangular fuzzy numbers have an introduction in definition 2.

Definition 2. Triangular Fuzzy number: Let fuzzy set \underline{A} be a triangular fuzzy number as $\underline{A} = (a_l, a, a_u)$, where $a_l < a < a_u$ and a_l, a , and a_u are real numbers. The membership function $u_{\underline{A}}(x)$ of triangular fuzzy number $\underline{A} = (a_l, a, a_u)$ is defined in Eq. (12) and demonstrated in Fig. 1.

$$u_{\underline{A}}(x) = \begin{cases} 0, & \text{if } x < a_l \\ (x - a_l)/(a - a_l), & \text{if } a_l \leq x < a \\ 1, & \text{if } x = a \\ (a_u - x)/(a_u - a), & \text{if } a < x < a_u \\ 0, & \text{if } a_u < x \end{cases} \quad (12)$$

In this work, the WSN uses the series-parallel RAP subjected to the constraints of the cost and weight of components with fuzzy numbers as shown in above Eqs. (9)-(10). According to the above definitions of a fuzzy set and triangular fuzzy number, the constraints of the cost and weight of

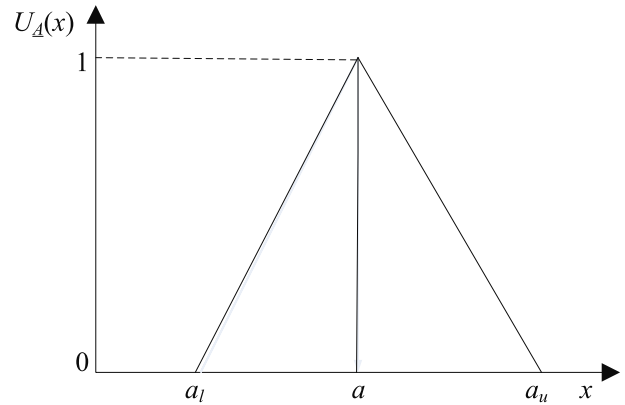


FIGURE 1. The triangular fuzzy number.

components with triangular fuzzy number can be well defined as follows.

The component cost with a triangular fuzzy number:

$c_{i,j} = (c_{i,j,l}, c_{i,j}, c_{i,j,u})$, where $c_{i,j,l} < c_{i,j} < c_{i,j,u}$ and $c_{i,j,l}, c_{i,j}$, and $c_{i,j,u}$ are real numbers.

The component weight with a triangular fuzzy number:

$w_{i,j} = (w_{i,j,l}, w_{i,j}, w_{i,j,u})$, where $w_{i,j,l} < w_{i,j} < w_{i,j,u}$ and $w_{i,j,l}, w_{i,j}$, and $w_{i,j,u}$ are real numbers.

C. CREDIBILITY THEORY

The following is an overview of the credibility theory that is used in this work to shift the constraints of cost and weight of components with triangular fuzzy numbers to crisp values.

Charnes & Cooper in 1959 [36] first developed the chance-constrained programming which ensures that the constraints will be met with at least α of the probability, where α means the confidence level. Following, Liu and Iwamura [37] applied the chance-constrained programming in a fuzzy environment, which is adopted by several studies [38]–[40]. The fuzzy chance-constrained programming proposed by Liu and Iwamura [37] has an introduction as follows in definition 3.

Definition 3. Fuzzy chance-constraints: Suppose $g(x, \underline{A})$ is the constraint with respect to x and \underline{A} , which are the variable number and the parameter with fuzzy number, respectively. In addition, *limit* is the upper bound of the constraint. Accordingly, the fuzzy chance-constrained programming can be modeled as following in Eq. (13). That means, in a fuzzy environment, the constraint has achieved at confidence level α , and the chance is demonstrated by the possibility of the constraint is fulfilling.

$$\begin{aligned} &\text{Maximize } f(x) \\ &\text{Subject to } \text{Pos}\{\underline{A}|g(x, \underline{A}) \leq \text{limit}\} \geq \alpha \end{aligned} \quad (13)$$

where the object is to maximize the objective function $f(x)$ subjected to the fuzzy chance-constraints, α is a confidence level which is predetermined, and $\text{Pos}\{\bullet\}$ is the possibility of an event $\{\bullet\}$.

Liu in 2004 [41] first proposed credibility theory based on the fuzzy chance-constrained programming, and numerous studies have shown its effectiveness on resolving the constraints in fuzzy environments [42]–[44]. This work adopts the credibility theory to solve the series-parallel RAP subjected to the constraints of the cost and weight of components with triangular fuzzy numbers. The credibility theory is defined as follows in definition 4.

Definition 4. Credibility theory: Due to the constraints including fuzzy numbers, the researchers tend to solve the constraints being fulfilled at a high credibility level, i.e., confidence level. Accordingly, the credibility theory is developed to present the fuzzy number on the left side of the constraints. The credibility theory is developed based on the fuzzy chance-constrained programming. The possibility concept of the fuzzy chance-constrained programming in Eq. (13) can be transformed into the credibility constraints as follows in Eq. (14). That means, in a fuzzy environment, the constraint has achieved at confidence level α .

$$\text{Subject to } Cr\{A|g(x, A) \leq \text{limit}\} \geq \alpha \quad (14)$$

where α is a confidence level which is predetermined, and $Cr\{\bullet\}$ is the credibility of an event $\{\bullet\}$.

In Eq. (14), suppose the parameter with fuzzy number A is taken as a triangular fuzzy number $\underline{a}_{i,j} = (a_{i,j,l}, a_{i,j}, a_{i,j,u})$ where $a_{i,j,l} < a_{i,j} < a_{i,j,u}$ and $a_{i,j,l}$, $a_{i,j}$, and $a_{i,j,u}$ are real numbers so that the constraint $g(x, A) = g(x, \underline{a}_{i,j})$ for $i = 1, 2, \dots, n$ subsystems and $j = 1, 2, \dots, k_i$ type components used in the i th subsystem. Thus, suppose the constraint $g(x, \underline{a}_{i,j})$ equals $\sum_{i=1}^n \sum_{j=1}^{k_i} a_{i,j} x_{i,j}$. According to the credibility theory, Eq. (14) can be transformed to the credibility constraints as follows in Eq. (15).

$$(2\alpha - 1) \sum_{i=1}^n \sum_{j=1}^{k_i} a_{i,j,u} x_{i,j} + 2(1 - \alpha) \sum_{i=1}^n \sum_{j=1}^{k_i} a_{i,j} x_{i,j} \leq \text{limit} \quad (15)$$

By the credibility constraints in Eq. (15), the fuzzy numbers have been successfully converted to the crisp numbers because the values of $a_{i,j,u}$ and $a_{i,j}$ are crisp values.

D. THE CREDIBILITY-BASED FUZZY MODEL

According to the constraint defined in Eq. (14) by the credibility theory, the model of the series-parallel RAP with a mix of components subjected to the constraints of the cost and weight of components in fuzzy number in Eqs. (9)-(10) can be converted to the following Eqs. (16)-(17).

$$\text{Subject to } Cr\{C(\mathbf{X}) = \sum_{i=1}^n \sum_{j=1}^{k_i} c_{i,j} x_{i,j} \leq C_u\} \geq \alpha \quad (16)$$

$$Cr\{W(\mathbf{X}) = \sum_{i=1}^n \sum_{j=1}^{k_i} w_{i,j} x_{i,j} \leq W_u\} \geq \beta \quad (17)$$

where α and β are the confidence levels, which are predetermined, and $Cr\{\bullet\}$ is the credibility of an event $\{\bullet\}$.

Therefore, in a fuzzy environment, Eqs. (16)-(17) show that the constraints of the cost and weight of components used in the system have achieved confidence levels α and β , respectively. In other words, the objective function maximizes the system reliability $R(\mathbf{X})$ as shown in Eq. (5) by optimizing the redundancy allocation of components in parallel in each subsystem subjected to the constraints of the cost and weight of components with fuzzy numbers that can be achieved at confidence level α and β , as shown in Eqs. (16)-(17), respectively.

As mentioned in section II.B, the triangular fuzzy number is used in this work, and therefore the cost and weight of the components with triangular fuzzy numbers are defined as follows.

$\underline{c}_{i,j} = (c_{i,j,l}, c_{i,j}, c_{i,j,u})$ is the component cost with triangular fuzzy number where $c_{i,j,l} < c_{i,j} < c_{i,j,u}$ and $c_{i,j,l}$, $c_{i,j}$, and $c_{i,j,u}$ are real numbers.

$\underline{w}_{i,j} = (w_{i,j,l}, w_{i,j}, w_{i,j,u})$ is the component weight with triangular fuzzy number where $w_{i,j,l} < w_{i,j} < w_{i,j,u}$ and $w_{i,j,l}$, $w_{i,j}$, and $w_{i,j,u}$ are real numbers.

Thus, by the credibility constraints in Eq. (15), the above constraints in Eqs. (16)-(17) can be transformed to the credibility constraints as follows in Eqs. (18)-(19).

Subject to

$$(2\alpha - 1) \sum_{i=1}^n \sum_{j=1}^{k_i} c_{i,j,u} x_{i,j} + 2(1 - \alpha) \sum_{i=1}^n \sum_{j=1}^{k_i} c_{i,j} x_{i,j} \leq C_u \quad (18)$$

$$(2\beta - 1) \sum_{i=1}^n \sum_{j=1}^{k_i} w_{i,j,u} x_{i,j} + 2(1 - \beta) \sum_{i=1}^n \sum_{j=1}^{k_i} w_{i,j} x_{i,j} \leq W_u \quad (19)$$

By the credibility constraints in Eqs. (18)-(19), the fuzzy numbers has been successfully converted to the crisp numbers because the values of $c_{i,j,u}$, $c_{i,j}$, $w_{i,j,u}$ and $w_{i,j}$ are crisp values.

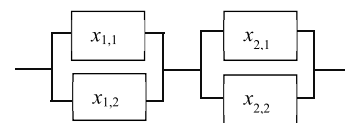


FIGURE 2. An example of series-parallel system.

E. A SMALL-SCALE EXAMPLE OF THE CREDIBILITY-BASED FUZZY MODEL

A small-scale example of the credibility fuzzy model is presented as follows.

Example 1: As shown in the following Fig. 2, a series-parallel RAP with two subsystems ($i = 1, 2$) and two type components ($j = 1, 2$) can be used in each subsystem subjected to the constraints of the cost and weight of the components with a triangular fuzzy number. This example has been presented to demonstrate

the effectiveness of the proposed credibility theory to resolve the constraints in fuzzy environment.

Suppose the lower bound and upper bound of the number of components used in each subsystem are $L_i = 2$ and $U_i = 5$, respectively. As shown in Fig. 2, the series-parallel RAP with a mix of components, which aims to maximize the system reliability, subjected to the constraints of the cost and weight of the components in fuzzy environment taken as triangular fuzzy numbers can be modeled as follows in Eqs. (20)-(23), according to the models in Eq. (5), Eqs. (18)-(19), and Eq. (8).

$$\text{Maximize } R(\mathbf{X}) = \prod_{i=1}^2 R_i = \prod_{i=1}^2 [1 - \prod_{j=1}^2 (1 - r_{i,j})^{x_{i,j}}] \quad (20)$$

Subject to

$$(2\alpha - 1) \sum_{i=1}^2 \sum_{j=1}^2 c_{i,j,u} x_{i,j} + 2(1 - \alpha) \sum_{i=1}^2 \sum_{j=1}^2 c_{i,j} x_{i,j} \leq C_u \quad (21)$$

$$(2\beta - 1) \sum_{i=1}^2 \sum_{j=1}^2 w_{i,j,u} x_{i,j} + 2(1 - \beta) \sum_{i=1}^2 \sum_{j=1}^2 w_{i,j} x_{i,j} \leq W_u \quad (22)$$

$$2 \leq \sum_{j=1}^2 x_{i,j} \leq 5, \quad i = 1, 2 \quad (23)$$

Assume a set of solutions is $\mathbf{X} = (x_{1,j}, x_{2,j}) = (x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}) = (1, 2, 2, 3)$, i.e., a total 3 components used in the first subsystem with one first type component and two second type components ($x_{1,1} + x_{1,2} = 1 + 2 = 3$), and total 5 components used in the second subsystem with two first type components and three second type components ($x_{2,1} + x_{2,2} = 2 + 3 = 5$). In addition, the reliability of components is $(r_{1,1}, r_{1,2}, r_{2,1}, r_{2,2}) = (0.92, 0.95, 0.89, 0.99)$, and the data for the cost and weight of components with a triangular fuzzy number are presented in Table 1.

TABLE 1. The cost and weight of components with a triangular fuzzy number.

		<i>i</i>	
		<i>j</i>	
		1	2
Cost	1	(1.1, 1.68289, 2.1)	(0.5, 1.04729, 1.5)
	2	(2.1, 2.68842, 3.1)	(1.1, 1.67305, 2.1)
Weight	1	(8.4, 8.98818, 9.4)	(9.7, 10.27220, 10.7)
	2	(22.5, 23.08210, 23.5)	(25.8, 26.37954, 26.8)

Suppose the upper bound of the system cost and system weight are $C_u = 20$ and $W_u = 165$, respectively. To compare the solutions with one lower credibility level and two higher credibility levels, three different credibility levels for the fuzzy cost and fuzzy weight constraints are predetermined as 0.85, 0.95, and 0.99, i.e., $\alpha = \beta = 0.85$, $\alpha = \beta = 0.95$, and $\alpha = \beta = 0.99$, respectively.

The system reliability of the series-parallel RAP in Fig. 2 is calculated in the following Eq. (24). and the system cost and weight in a fuzzy environment are shown in Eqs. (25)-(26), with the credibility level $\alpha = \beta = 0.85$ as a computational example according to the formulations in Eqs. (20)-(22).

$$R(\mathbf{X}) = [1 - (1 - r_{1,1})(1 - r_{1,2})^2][1 - (1 - r_{2,1})^2(1 - r_{2,2})^3] = 0.999799988 \quad (24)$$

$$C(\mathbf{X}) = (2\alpha - 1) \sum_{i=1}^2 \sum_{j=1}^2 c_{i,j,u} x_{i,j} + 2(1 - \alpha) \sum_{i=1}^2 \sum_{j=1}^2 c_{i,j} x_{i,j} = 16.572038000 \quad (25)$$

$$W(\mathbf{X}) = (2\beta - 1) \sum_{i=1}^2 \sum_{j=1}^2 w_{i,j,u} x_{i,j} + 2(1 - \beta) \sum_{i=1}^2 \sum_{j=1}^2 w_{i,j} x_{i,j} = 163.814506600 \quad (26)$$

Further, the whole computation result of the series-parallel RAP in Fig. 2 can be obtained and is shown in Table 2.

TABLE 2. The computation results of the series-parallel RAP in Fig. 2.

	$\alpha = \beta = 0.85$	$\alpha = \beta = 0.95$	$\alpha = \beta = 0.99$
<i>R</i>	0.999799988	0.999799988	0.999799988
System Cost	16.572038000	17.257346000	17.531469200
System Weight	163.814506600	164.551502200	164.846300440

As mentioned above, the upper bound of the system cost and system weight are $C_u = 20$ and $W_u = 165$, respectively. Hence, the system reliability is equal to 0.999799988 for the solution of $\mathbf{X} = (x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}) = (1, 2, 2, 3)$ subjected to the system cost and weight in a fuzzy environment, which is obtained by the credibility theory at three different credibility levels, i.e., $\alpha = \beta = 0.85$, $\alpha = \beta = 0.95$, and $\alpha = \beta = 0.99$ and are all satisfied for the limitation of the upper bound.

IV. SIMPLIFIED SWARM OPTIMIZATION (SSO)

This study shows that a credibility-based fuzzy model for constraints solving the series-parallel RAP with a mix of components under an uncertainty environment, in addition to an objective function maximizing the system reliability subjected to the cost and weight constraints under a fuzzy environment, is resolved by an improved simplified swarm optimization (ISSO) algorithm. This section shows that the RAP, which is an NP-hard problem, can be efficiently resolved by the proposed ISSO algorithm based on the SSO algorithm that belongs to the swarm intelligence algorithm and evolution computing.

In 2009, Yeh [28] originally developed the SSO algorithm that is a popular population-based swarm intelligence algorithm. The simplification, efficiency, and flexibility of the SSO has been approved by numerous studies [52]–[56]. The update mechanism (UM) of the SSO algorithm that is simple and efficient in optimizing the NP-hard problems is presented

in Eq. (27).

$$x_{i,j}^t = \begin{cases} p_{gBest,j} & \text{if } \rho_{[0,1]} \in [0, C_g = c_g) \\ p_{i,j} & \text{if } \rho_{[0,1]} \in [C_g, C_p = C_g + c_p) \\ x_{i,j}^{t-1} & \text{if } \rho_{[0,1]} \in [C_p, C_w = C_p + c_w) \\ x & \text{if } \rho_{[0,1]} \in [C_w, 1] \end{cases} \quad (27)$$

where c_g , c_p , c_w and c_r are the predetermined probability values satisfying $c_g + c_p + c_w + c_r = 1$, and the number of $\rho_{[0,1]}$ is randomly generated from $[0, 1]$.

The update process of the UM in Eq. (27) is the solution of $x_{i,j}^t$ of the i th solution at the t th generation for the j th variable equals the global best $p_{gBest,j}$ if $\rho_{[0,1]}$ falls into $[0, C_g)$; or equals the local best $p_{i,j}$ if $\rho_{[0,1]}$ falls into $[C_g, C_p)$; or equals the solution of the last generation $x_{i,j}^{t-1}$ if $\rho_{[0,1]}$ falls into $[C_p, C_w)$; otherwise a new solution x is randomly generated from the uniform distribution $[lower_j, upper_j]$ if $\rho_{[0,1]}$ falls into $[C_w, 1]$, where $lower_j$ and $upper_j$ are the predetermined values of lower bound and upper bound for each variable number.

The steps of SSO are presented as follows:

- Step 1.** Initialize the solution of the i th solution at the 0^{th} generation: $x_i^0 = p_i$, for $i = 1, 2, \dots, n$.
- Step 2.** Evaluate the fitness function of x_i^0 to obtain p_{gBest} .
- Step 3.** Let the generation $t = 1$.
- Step 4.** Let $i = 1$.
- Step 5.** The solution of the i^{th} solution at the t^{th} generation: x_i^t is generated by the UM of SSO in Eq. (27).
- Step 6.** Evaluate the fitness function of x_i^t .
- Step 7.** Let $p_i = x_i^t$ if the fitness function of x_i^t is better than the fitness function of p_i . Otherwise, move to step 9.
- Step 8.** Let $gBest = i$ if the fitness function of p_i is better than the fitness function of p_{gBest} .
- Step 9.** Let $i = i + 1$ if $i < n$, return to step 4.
- Step 10.** Halt or let $t = t + 1$ and return to step 1.

V. THE PROPOSED ISSO ALGORITHM

This section discusses the fitness function, the novel initiation, the novel three-stage parameter setting, the new update mechanism, and the novel local search without calculating the fitness for the proposed ISSO algorithm.

A. THE FITNESS FUNCTION

A penalty mechanism of the fitness function as shown in Eq. (28) is used here to lead the solution to obtain the feasible solution [53], [57]–[59].

$$F(\mathbf{X}) = \begin{cases} R(\mathbf{X}) & \\ \text{if } C(\mathbf{X}) \leq C_u \text{ and } W(\mathbf{X}) \leq W_u & \\ R(\mathbf{X}) \text{Min} \left\{ \frac{C_u}{C(\mathbf{X})}, \left[\frac{W_u}{W(\mathbf{X})} \right]^3 \right\} & \\ \text{otherwise} & \end{cases} \quad (28)$$

where the exponent value in Eq. (28) is the penalty mechanism of the fitness function.

An example as following presents the calculation of the penalty mechanism of the fitness function using the solution \mathbf{X} given in Example 1.

Example 2: Assume a set of solutions is provided as given in Example 1, i.e., $\mathbf{X} = (x_{1,j}, x_{2,j}) = (x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}) = (1, 2, 2, 3)$ and let $\alpha = \beta = 0.99$, $C_u = 15$ and $W_u = 160$. Hence,

$$\begin{aligned} F(\mathbf{X}) &= R(\mathbf{X}) \text{Min} \left\{ \frac{C_u}{C(\mathbf{X})}, \left[\frac{W_u}{W(\mathbf{X})} \right]^3 \right\} \\ &= 0.999799988 \cdot \text{Min} \left\{ \frac{15}{17.5314692}, \left[\frac{160}{164.84630044} \right]^3 \right\} \\ &= 0.855433144 \end{aligned} \quad (29)$$

B. THE NOVEL INITIATION

The novel initiation is obtained as shown in the following Lemma 1.

Lemma 1: The series system reliability is zero if and only if the reliability of any of its subsystems is zero.

C. THE NOVEL THREE-STAGE PARAMETER SETTING

How to tune the parameters is always an important and difficult problem in soft computing. During the update process, the solution quality is increased quickly with the increase of the generation number in the beginning and converges slowly from generation to generation to a solution, which may be a local optimum or a global optimum.

To push the improvement of the solution quality, there are three different setting in the proposed algorithm. In the 100 generations, the proposed algorithm is more focused on the $gBest$ and $pBest$, in the next 100 generations, i.e., generation 101 to 200, the values of C_g , C_p , and C_w are all decreased to increase the value of cr to have more momentum to escape the possible local trap. In the last stage, i.e., the generation from 300 to the end, these values of c_g , c_p , and c_w are reduced to 10% to give more chance for cr . The details of the three-stage parameter setting are listed below.

$$\begin{cases} C_g = 0.20 \\ C_p = 0.40 \\ C_w = 0.50 \end{cases} \quad \text{if } g < 100$$

$$\begin{cases} C_g = 0.15 \\ C_p = 0.35 \\ C_w = 0.45 \end{cases} \quad \text{if } g < 200$$

$$\begin{cases} C_g = 0.10 \\ C_p = 0.20 \\ C_w = 0.30 \end{cases} \quad \text{if } g < 300$$

D. THE NEW UPDATE MECHANISM

In the traditional discrete SSO, there is no need to add additional terms for the update value, e.g., the new value of the

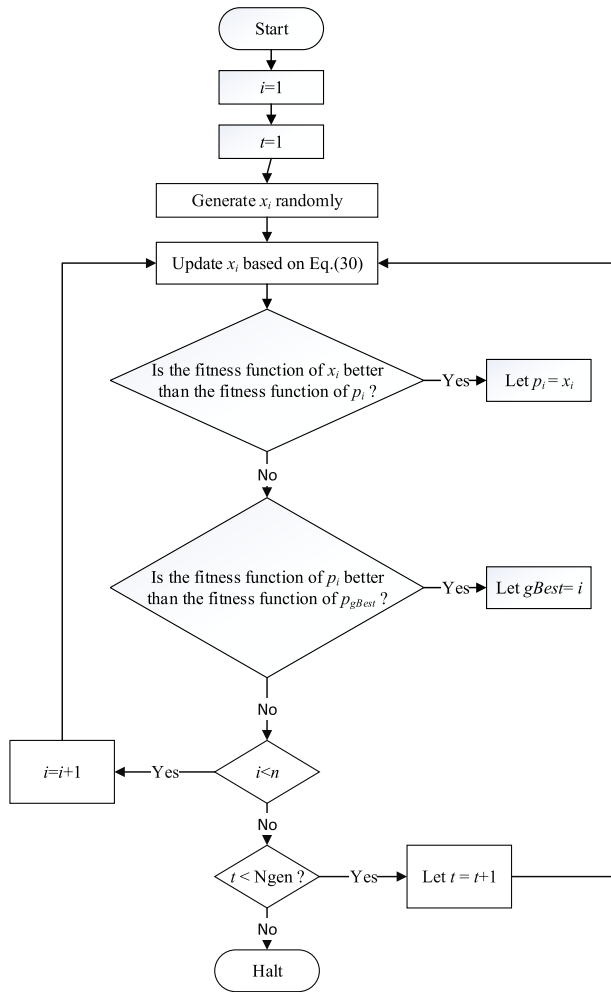


FIGURE 3. The flowchart of the proposed ISSO.

variable is 5 if it is copied from the $gBest$ and the value of $gBest$ is 5. The above update is simple and fast, but it may also lose a chance to exploit the neighborhood of the $gBest$, $pBest$, and itself. In addition, in the traditional SSO, no matter if it is discrete or continuous, the value of the current variable is always replaced with a new feasible value. However, such simple and fast replacement will result in some of the constraints being violated seriously. To overcome the above two problems in an SSO, a new update mechanism is shown below Eq. (30), and a flowchart of the proposed ISSO is shown in Fig. 3.

$$x_i = \begin{cases} g_i + \Delta_g & \text{if } \rho_{[0,1]} \in [0, C_g) \\ p_i + \Delta_p & \text{if } \rho_{[0,1]} \in [C_g, C_p) \\ x_i + \Delta_w & \text{if } \rho_{[0,1]} \in [C_p, C_w) \\ x & \text{otherwise} \end{cases} \quad (30)$$

where

$$\Delta_g = \begin{cases} 0 & \text{if } \rho_{[0,1]} \in [0, 1/3) \text{ and } g_i = 0 \\ g_i - 1 & \text{if } \rho_{[0,1]} \in [0, 1/3) \text{ and } g_i > 0 \\ g_i & \text{if } \rho_{[0,1]} \in [1/3, 2/3) \\ g_i + 1 & \text{otherwise} \end{cases}$$

$$\Delta_p = \begin{cases} 0 & \text{if } \rho_{[0,1]} \in [0, 1/3) \text{ and } p_i = 0 \\ p_i - 1 & \text{if } \rho_{[0,1]} \in [0, 1/3) \text{ and } p_i > 0 \\ p_i & \text{if } \rho_{[0,1]} \in [1/3, 2/3) \\ p_i + 1 & \text{otherwise} \end{cases}$$

$$\Delta_w = \begin{cases} 0 & \text{if } \rho_{[0,1]} \in [0, 1/3) \text{ and } x_i = 0 \\ x_i - 1 & \text{if } \rho_{[0,1]} \in [0, 1/3) \text{ and } x_i > 0 \\ x_i & \text{if } \rho_{[0,1]} \in [1/3, 2/3) \\ x_i + 1 & \text{otherwise} \end{cases}$$

The computational cost of the proposed approach, which has been further discussed by the time and space complexities of the proposed algorithm, has been calculated below:

The time complexities: $Ngen * Nsol * Nvar$

The space complexities: $Nsol * Nvar * 2$

where $Ngen$, $Nsol$, and $Nvar$ represent the number of generations, solutions, and variables, respectively.

For the new update mechanism of the proposed ISSO as shown in Figure 3, the new UM is improved to overcome the problems of the SSO algorithm. Therefore, Eq. (27) in the UM of the SSO is replaced by Eq. (30) in the UM of the proposed ISSO.

E. THE NOVEL LOCAL SEARCH WITHOUT CALCULATING THE FITNESS

Finally, the novel local search without calculating the fitness is obtained as shown in the following Lemma 2 and Lemma 3.

Lemma 2: The system reliability is at most r if and only if the smallest reliability of its any subsystem is r .

Lemma 3: The system reliability is increased if any of its subsystems is increased r and the other subsystem are unchanged without considering the constraints.

VI. NUMERICAL EXAMPLE

A novel credibility-based fuzzy model for constraint solving in WSN using the series-parallel RAP with a mix of components under an uncertainty environment, whose objective is to maximize the system reliability subjected to the cost and weight constraints with fuzzy numbers to address the uncertainty circumstances, is received by an ISSO algorithm.

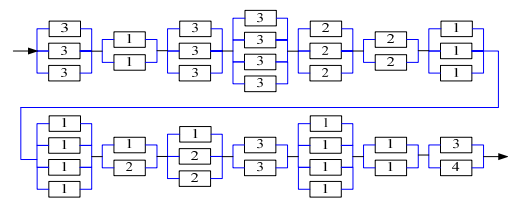


FIGURE 4. The series-parallel system proposed by Fyffe et al. [1].

To demonstrate the effectiveness of the proposed ISSO algorithm in this researching model, the experimental results are tested on a famous series-parallel RAP with a mix of components having 14 subsystems in the system, as shown in Fig. 4, which was originally develop by Fyffe et al. [1]. In addition, in order to illustrate the efficiency of the proposed

ISSO approach, it is compared to other state-of-the-art methods, including particle swarm optimization (PSO) and genetic algorithm (GA). Experimental results demonstrate the high performance of the proposed ISSO algorithm in terms of the statistical results of system reliability and the running time (seconds).

In this study, the RAP optimizes the redundancy allocation of components in parallel to design a superior system reliability subjected to the cost and weight constraints under a fuzzy environment. Thus, the data of cost and weight is adopted as the triangular fuzzy number of cost and weight, as shown in Table 3.

TABLE 3. The triangular fuzzy number of cost and weight for the Fyffe’s RAP in Fig. 3.

(a) The triangular fuzzy number of cost					
		$C_{i,j}$			
$j \backslash i$		1	2	3	4
1		(0.5,1,1.5)	(0.5,1,1.5)	(1.5,2,2.5)	(1.5,2,2.5)
2		(1.5,2,2.5)	(0.5,1,1.5)	(0.5,1,1.5)	
3		(1.5,2,2.5)	(2.5,3,3.5)	(0.5,1,1.5)	(3.5,4,4.5)
4		(2.5,3,3.5)	(3.5,4,4.5)	(4.5,5,5.5)	
5		(1.5,2,2.5)	(1.5,2,2.5)	(2.5,3,3.5)	
6		(2.5,3,3.5)	(2.5,3,3.5)	(1.5,2,2.5)	(1.5,2,2.5)
7		(3.5,4,4.5)	(3.5,4,4.5)	(4.5,5,5.5)	
8		(2.5,3,3.5)	(4.5,5,5.5)	(5.5,6,6.5)	
9		(1.5,2,2.5)	(2.5,3,3.5)	(3.5,4,4.5)	(2.5,3,3.5)
10		(3.5,4,4.5)	(3.5,4,4.5)	(4.5,5,5.5)	
11		(2.5,3,3.5)	(3.5,4,4.5)	(4.5,5,5.5)	
12		(1.5,2,2.5)	(2.5,3,3.5)	(3.5,4,4.5)	(4.5,5,5.5)
13		(1.5,2,2.5)	(2.5,3,3.5)	(1.5,2,2.5)	
14		(3.5,4,4.5)	(3.5,4,4.5)	(4.5,5,5.5)	(5.5,6,6.5)

(b) The triangular fuzzy number of weight					
		$W_{i,j}$			
$j \backslash i$		1	2	3	4
1		(2.5,3,3.5)	(3.5,4,4.5)	(1.5,2,2.5)	(4.5,5,5.5)
2		(7.5,8,8.5)	(9.5,10,10.5)	(8.5,9,9.5)	
3		(6.5,7,7.5)	(4.5,5,5.5)	(5.5,6,6.5)	(3.5,4,4.5)
4		(4.5,5,5.5)	(5.5,6,6.5)	(3.5,4,4.5)	
5		(3.5,4,4.5)	(2.5,3,3.5)	(4.5,5,5.5)	
6		(4.5,5,5.5)	(3.5,4,4.5)	(4.5,5,5.5)	(3.5,4,4.5)
7		(6.5,7,7.5)	(7.5,8,8.5)	(8.5,9,9.5)	
8		(3.5,4,4.5)	(6.5,7,7.5)	(5.5,6,6.5)	
9		(7.5,8,8.5)	(8.5,9,9.5)	(6.5,7,7.5)	(7.5,8,8.5)
10		(5.5,6,6.5)	(4.5,5,5.5)	(5.5,6,6.5)	
11		(4.5,5,5.5)	(5.5,6,6.5)	(5.5,6,6.5)	
12		(3.5,4,4.5)	(4.5,5,5.5)	(5.5,6,6.5)	(6.5,7,7.5)
13		(4.5,5,5.5)	(4.5,5,5.5)	(5.5,6,6.5)	
14		(5.5,6,6.5)	(6.5,7,7.5)	(5.5,6,6.5)	(8.5,9,9.5)

For the fair comparison of the proposed ISSO with PSO and GA, the conditions are the same for all three algorithms. First, the credibility theory that is used in this work shifts the constraints of the cost and weight of components with triangular fuzzy numbers to crisp values, and thus, the credibility levels for the fuzzy cost and fuzzy weight constraints are predetermined as 0.80, i.e., $\alpha = \beta = 0.80$. In the UM of the ISSO, the corresponding variable of the previous generation will be updated to a random number x to ensure sufficient variability so the algorithm can fully search the solution space, meaning that the solution will not fall in the local optimal solution when $\rho [0,1]$ falls into the range of $[Cw, 1]$.

The final optimization result of each run is guaranteed to converge to the global optimal solution after reaching the convergence condition or enough generations. Therefore, second, all three algorithms were run for 50 generations ($N_{gen} = 50$), with 33 solutions. In addition, the proposed ISSO, PSO and GA are coded using DEV C++ with 64-bit Windows 10, implemented on an Intel Core i7-6650U CPU @ 2.20 GHz 2.21-GHz notebook with 64 GB of memory.

The computational cost of the proposed approach, which has been further discussed by the time and space complexities of the proposed algorithm, has been calculated below:

The time complexities: $N_{gen} * N_{sol} * N_{var} = 50 * 33 * 2 = 3300$

The space complexities: $N_{sol} * N_{var} * 2 = 33 * 2 * 2 = 132$

Moreover, for ISSO algorithm or other heuristic search methods with randomness mechanisms, optimization result every time may differ from each other. Therefore, descriptive statistics or related statistic methods are used to describe the results with the randomness property, i.e., average, standard deviation, maximum, or minimum. The average, standard deviation, maximum, or minimum of optimization results are usually chosen as the final solution. Therefore, the following statistical results of the minimum, maximum, average, and standard deviation for the system reliability and for the running time (seconds) are recorded for the proposed ISSO, PSO, and GA, and the best solutions compared among three algorithms are in bold.

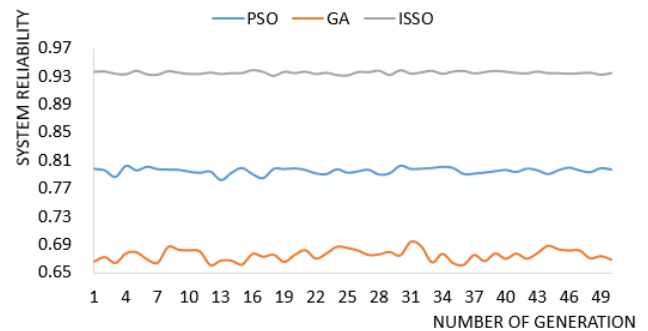


FIGURE 5. The average of the system reliability for the 50 generations.

First, the average of the system reliability for the 50 generations obtained by the proposed ISSO are compared with those found by PSO and GA, as shown in Table 4 and in Figure 5 and as concluded below.

1. The best solution obtained by ISSO is **0.938054062** at **the 16th generation**, which is better and quicker than those found by PSO and GA, which are 0.80207458 at the 30th generation and 0.69419976 at the 31st generation, respectively, in terms of the average of the system reliability and the number of generations that obtains the best solution.
2. The overall of the average of the system reliability for the 50 generations obtained by ISSO is **0.934207830**, which is superior to those found by PSO and GA, which are 0.79468481 and 0.67501724, respectively.

TABLE 4. The average of the system reliability for the 50 generations.

N_{gen}	PSO	GA	ISSO
1	0.79754978	0.66611152	0.935650179
2	0.79518056	0.67231004	0.936081664
⋮	⋮	⋮	⋮
16	0.78988745	0.67699201	0.938054062
⋮	⋮	⋮	⋮
30	0.80207458	0.67470892	0.937939507
31	0.79730710	0.69419976	0.932998527
⋮	⋮	⋮	⋮
50	0.79651923	0.66875053	0.933825420
Overall	0.79468481	0.67501724	0.934207830

TABLE 5. The overall of the minimum, maximum and standard deviation of the system reliability for the 50 generations.

Statistical result	PSO	GA	ISSO
Minimum	0.73216209	0.58422580	0.876498909
Maximum	0.85519718	0.78670806	0.967023220
Std	0.03211108	0.05140538	0.024102150

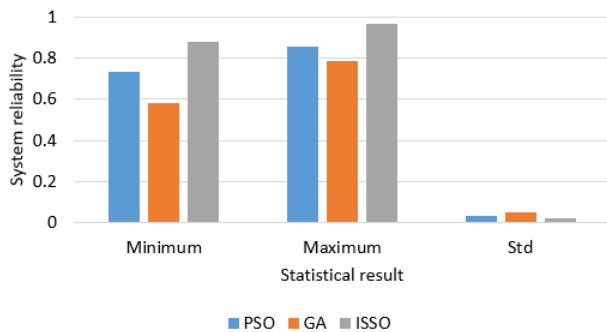


FIGURE 6. The overall of the minimum, maximum and standard deviation of the system reliability for the 50 generations.

Second, the statistical results of the overall of the minimum, maximum, and standard deviation (Std) of the system reliability for the 50 generations obtained by the proposed ISSO are also compared with those found by PSO and GA, shown as following in Table 5 and Figure 6 and concluded as below.

1. The overall of the minimum, maximum, and standard deviation of the system reliability for the 50 generations obtained by ISSO are **0.876498909**, **0.967023220**, and **0.024102150**, which that are all superior to those found by PSO and GA.

Third, the analyses of the average of the system reliability and of the overall of the minimum, maximum, and standard deviation of the system reliability above are based on the mean values of all solutions for all 50 generations. To handle the problem of premature convergence about the average of the system reliability for the 50 generations in the proposed ISSO algorithm, the generation that obtains the best solution in terms of the minimum, maximum, and standard deviation of the system reliability in all solutions of 50 generations and the value of the best solution in terms of the minimum, maximum, and standard deviation of the system reliability in

TABLE 6. The generation that obtains the best solution and the best solution in terms of the minimum, maximum, and standard deviation of the system reliability in all solutions of 50 generations for the proposed ISSO.

	Minimum	Maximum	Std
Solution	0.893255046	0.974701196	0.020777056
Generation	33	27	26

all solutions of 50 generations have been further confirmed and analyzed for the proposed ISSO, as shown in Table 6 and concluded as below.

The generation that obtains the best average, 0.938054062, in terms of the average of the system reliability for the 50 generations is at the 16th generation for the proposed ISSO, according to the results in Table 4. However, the generation that obtains the best solution, i.e., 0.893255046, 0.974701196, and 0.020777056, in terms of the minimum, maximum, and standard deviation of the system reliability in all solutions of 50 generations are at the 33rd, the 27th, and the 26th generation, respectively. There is no problem of premature convergence about the best solution in terms of the minimum, maximum, and standard deviation of the system reliability for the 50 generations in the proposed ISSO algorithm. In this way, the number of generations in the experiment can be reduced to 35 generations. Furthermore, to overcome the premature convergence, it can be enhanced in the future to monitor the population diversity during iteration and add perturbations to maintain the diversity when the diversity is insufficient that kindly see the work presented in [60].

TABLE 7. The average of the running time for the 50 generations.

N_{gen}	PSO	GA	ISSO
1	0.259	0.118	0.230
2	0.253	0.118	0.225
⋮	⋮	⋮	⋮
16	0.278	0.118	0.225
⋮	⋮	⋮	⋮
30	0.252	0.119	0.224
31	0.252	0.119	0.225
⋮	⋮	⋮	⋮
50	0.252	0.118	0.225
Overall	0.259	0.119	0.225

Fourth, the average running time (seconds) for the 50 generations obtained by the proposed ISSO are compared with those found by PSO and GA and are shown in Table 7 and Figure 7 and concluded as below.

1. The average of the running time for all methods including the proposed ISSO, PSO, and GA have good convergence at the generation that obtained the best solution, i.e., at the 16th generation, at the 30th generation and at the 31st generation, respectively.
2. The overall average of the running time for the 50 generations obtained by GA is **0.119**, which is superior to those obtained by ISSO and PSO, which are 0.225 and 0.259, respectively.

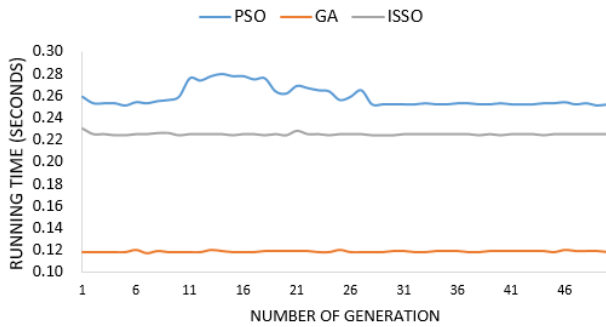


FIGURE 7. The average of the running time for the 50 generations.

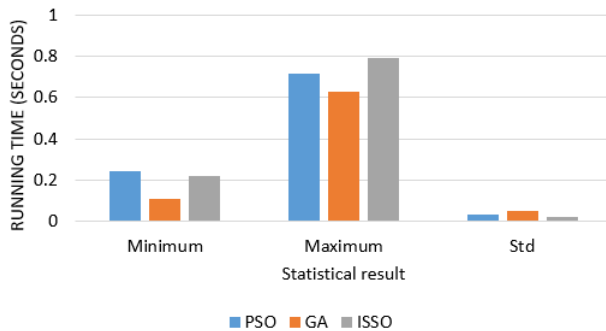


FIGURE 8. The overall of the minimum, maximum and standard deviation of the running time for the 50 generations.

TABLE 8. The overall of the minimum, maximum, and standard deviation of the running time for the 50 generations.

Statistical result	PSO	GA	ISSO
Minimum	0.243	0.108	0.218
Maximum	0.716	0.628	0.792
Std	0.031	0.050	0.024

Fifth, the statistical results of the overall of the minimum, maximum, and standard deviation (Std) of the running time for the 50 generations obtained by the proposed ISSO are also compared with those found by PSO and GA and are shown in Table 8 and Figure 8 and concluded as below.

1. The overall of the minimum and maximum of the running time for the 50 generations obtained by GA are **0.108 and 0.628**, which are superior to those found by ISSO and PSO.
2. The overall of the standard deviation of the running time for the 50 generations obtained by ISSO is **0.024**, which is superior to those found by PSO and GA.

To make a more advanced comparison of the performance among the proposed ISSO with PSO and GA, this paper used a nonparametric Kruskal-Wallis test, which is free from the restrictions of data distribution, for the system reliability and for the running time (seconds). For fair comparison, the above statistical test is performed on the solutions of the generation number of the average optimal solution for the three algorithms, i.e., the solutions at the 16th generation, at the 30th generation and at the 31st generation for ISSO, PSO and GA, respectively.

TABLE 9. Nonparametric Kruskal-Wallis test of the system reliability for ISSO, PSO, and GA.

Dependent Variable	Chi-Square	df	P-value
System reliability	84.938	2	0.000

TABLE 10. Nonparametric Kruskal-Wallis test of the running time for ISSO, PSO, and GA.

Dependent Variable	Chi-Square	df	P-value
Running time	90.339	2	0.000

The statistical results of the nonparametric Kruskal-Wallis test of the system reliability and of the running time for the proposed ISSO, PSO and GA are shown as following in Table 9 and Table 10, respectively, and concluded as below.

1. According to Table 9, the performance of the system reliability of the proposed ISSO significantly outperforms that of the PSO and GA because the P-value obtained by nonparametric Kruskal-Wallis test of the system reliability for ISSO, PSO and GA is equal 0.000, which is less than the significant level 0.05.
2. According to Table 10, the performance of the running time of GA is significantly better than ISSO and PSO because the P-value obtained by the nonparametric Kruskal-Wallis test of the system reliability for ISSO, PSO and GA is equal to 0.000 and that is less than the significant level 0.05.

The above experiment gives the results of WSN using a RAP considering uncertainty environment for optimum system reliability using the proposed ISSO compared with PSO and GA. The performance of the proposed ISSO is concluded below.

1. The novel credibility-based fuzzy model for constraints solving in WSN using the series-parallel RAP with a mix of components under an uncertainty environment, whose objective is to maximize the system reliability subjected to cost and weight constraints with fuzzy numbers to describe the uncertainty circumstances, is workable and that has been proven by the proposed ISSO, PSO, and GA.
2. The proposed ISSO outperforms PSO and GA in terms of the average, minimum, maximum, and nonparametric Kruskal-Wallis test of system reliability.
3. The proposed ISSO is more stable than either PSO and GA in terms of the standard deviation of the system reliability.
4. The proposed ISSO is quicker than both PSO and GA in terms of the number of generations needed to obtain the best solution of system reliability.
5. GA outperforms both the ISSO and PSO in terms of the average, minimum, maximum, and nonparametric Kruskal-Wallis test of running time.
6. The proposed ISSO is more stable than both PSO and GA in terms of the standard deviation of running time.

VII. CONCLUSION

In the real-world systems of WSN, it is difficult to obtain the significant precise value of the performance rate of components in the system with associated uncertainty conditions such as design, manufacture, and environment. Therefore, a novel credibility-based fuzzy model for constraints solving in WSN using the series-parallel RAP with a mix of components under an uncertainty environment, whose objective is to maximize the system reliability subjected to cost and weight constraints with fuzzy numbers to describe the uncertainty circumstances, is developed in this study and is solved by the proposed ISSO algorithm. As shown in the experimental study, the proposed ISSO algorithm possesses the ability to solve the RAP with mix of components under an uncertainty environment and is outstanding and superior to state-of-art techniques, including PSO and GA, in the performance of fitness values for optimizing system reliability of a WSN using the series-parallel RAP subjected to the constraints of the cost and weight of components under an uncertainty environment.

In future work, the presented approaches including the credibility theory and the proposed ISSO algorithm may apply in new research topics by extending the RAP to a more practical environment such as Internet of Things (IOT), smart manufacturing, and transportation systems or to the reliability redundancy allocation problem (RRAP). In addition, to the best of the authors' knowledge, the model of a RAP subjected to the constraints of the cost and weight of components under an uncertainty environment that has been studied in this paper had not been discussed in the literature so far. Therefore, the experimental results obtained by the proposed ISSO algorithm are compared with the results of state-of-the-art methods, including PSO and GA. The experimental results may be compared with the results of other more state-of-the-art methods in future work.

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