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Robust Adaptive Control for Uncertain Input Delay MIMO Nonlinear Non-Minimum Phase System: A Fuzzy Approach

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ABSTRACT In this paper, the robust controller design problem of uncertain multi-input multi-output nonlinear non-minimum phase system is discussed. The nonlinear system is suffering from both uncertainty and input delay, so the controller design is difficult. The traditional stable inversion controller is utilized and extended to uncertain case. An integral of past control input is constructed and fuzzy logical system is utilized for approaching the unknown state matrix and input matrix. Then a robust adaptive control strategy is presented. Finally, a numerical simulation on vertical takeoff and landing aircraft is given to show the effectiveness of the proposed method.

INDEX TERMS Nonlinear non-minimum phase, fuzzy logical system (FLS), parameter uncertainty, unmodelled dynamics, input delay.

I. INTRODUCTION

The dynamic characteristic of a practical servo system is often viewed as a linear function, while the real dynamic characteristic is really complex. From receiving the control order to providing a regular control input, the running time of the servo system, which is usually called input delay, is not considered in most situation. For an actual system, the processing speed is limited, so the input is usually time delay, besides, the nonlinear dynamics of the servo is complex, this can also cause time delay in input. Time delay of input may affect the control performance or even lead to unstability of a real system, so we must take it seriously in controller design [2], [3], [8].

Control for input delay system has been widely discussed for linear continuous system [37] and discrete-time system [29]. But unfortunately, most of the real system are nonlinear, so designing a controller for input delay nonlinear system is more significative. T-S model based fuzzy controller is an efficient way for this problem, and has been widely researched [9], [15], [33]. Obviously, the more complex the built T-S fuzzy model is, the better the approximation result is. But when the T-S model is complex, a feasible solution for the T-S fuzzy model is hard to be get [4]. To reduce the conservative of the designed controller, diffeomorphism coordinate transformation (DCT) based controller design methods are proposed for input delay nonlinear system. Through choosing appropriate DCT, the original nonlinear system is simplified, and then, lots of controller design strategies, like back-stepping method [23], [34], [35], sliding mode control [32] and adaptive neural network method [18] are proposed for the input delay nonlinear system. It is noteworthy that, the methods for input delay nonlinear system directly are all based on an assumption that, the nonlinear model can be completely linearization, or its internal dynamics are stable. When the internal dynamics is unstable, the above methods will lose effectiveness [4].

The internal dynamics of a lot of practical systems are unstable [5]–[7], so the controller design of such a kind of system is worthy to be studied. The minimum nonlinear system

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can be converted as an inverted triangle form, and then, some powerful nonlinear control approach can be utilized directly [4]. But if it is non-minimum phase, the inverted triangle form cannot be got, then the traditional power nonlinear control strategy will lose efficacy. In this case, the control design becomes complex and challenging, and some novel control design strategy is needed.

The output tracking control of non-minimum phase system has been widely studied, and ideal internal dynamics (IID) based controller method is a widely used [10]. By choosing appropriate DCT, the partially linearized dynamics and the internal dynamics can be constructed. After computing IID, a state tracking control problem can be constructed, then a nonlinear stable inversion controller can be constructed [4]. The IID based control strategy can track a given command exactly, and guarantee the closed-loop stabilization meanwhile. But the IID based controller is based on an exact model, and the parameters are all supposed to be time-invariant. When the parameters are uncertainty, or the nonlinear model is not exact known, IID based method is unsuitable. For a practical system, the exact nonlinear model is difficult to be got, and uncertainty parameters and unmodelled dynamics are always in existence, so the designed controller for it must be robust.

As an efficient and frequently utilized way, fuzzy logic system (FLS) can approximate a smooth nonlinear function accurately [25], [27], so it has been widely used to identify and control unknown nonlinear systems, such as time delay nonlinear system [16], [28], output control of nonlinear system [26], nonlinear hyperbolic PDE systems [20], input nonlinearity nonlinear system [17] and fuzzy fault-tolerant control for stochastic system [22], [36]. Considering the benefits of it, FLS is adopted in this paper in order to deal with the unknown uncertainty and unmodelled disturbance.

Motivated by the reasoning above, control design of uncertain input delay MIMO nonlinear non-minimum phase system is considered here, and a FLS based control design method is discussed. The original nonlinear dynamic is linearized firstly, and IID of the nonlinear system is computed based on the chosen diffeomorphism coordinate transformation. Then a state tracking model is constructed, in which the system uncertainty and unmodelled disturbance is included. The constructed internal dynamics model adopted here is not a standard form [13], instead, is a common form. The input in a standard form is disappeared in the nonlinear expression, so the controller design is simple. While the common form is more complex, and the controller design is also difficult. Unfortunately, standard form is hard to be get, so a controller for nonlinear system with common form ID is more important.

The model of vertical takeoff and landing aircraft (VTOL) is non-minimum phase and the control design has been studied and a lot of results can be get in literature, such as the robust control of VTOL [24], [30], [31], fault tolerant control of VTOL [1] and so on. But the robust output tracking control for VTOL with both uncertainties and unmodelled disturbance has not been completely solved. In this case, an adaptive robust controller design method is considered in this paper, and a numerical simulation is listed to confirm its' effectiveness.

The main feature and contribution of this paper is:

(1) A robust controller is designed for input delay MIMO nonlinear non-minimum phase system.

(2) Not only parameter uncertainty and unmodelled dynamic, but also input delay are considered in this paper.

(3) The stable inversion based nonlinear controller for exact nonlinear non-minimum phase system is extended to input delay and uncertain case.

A nonlinear model which is suffering from input delay is listed in section 2, together with the control objective of the paper. The controller design method is proposed in Section 3. Numerical simulation is developed in section 4, and we summarize this paper in Section 5.

II. PROBLEM AND OBJECTIVE

A. MODEL DESCRIPTION

For an input delay nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u(t-\tau) \\ y &= h(x) = [y_1, y_2, \cdots, y_m]^T \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t-\tau) \in \mathbb{R}^m$, τ is time delay which is already known and $u(t-\tau) = 0$ if $t < \tau$. $y \in \mathbb{R}^m$ is system output. Assuming that x_0 $(x = 0, u = 0)$ is a balance point of (1), and its' relative degree is r, $r < n$. Then (1) can be input/output linearized, and:

$$\begin{aligned} y_1^{(r_1)} &= F_1(x) + G_1(x)u(t-\tau) \\ &\vdots \\ y_m^{(r_m)} &= F_m(x) + G_m(x)u(t-\tau) \\ \dot{\eta} &= s(\varsigma, \eta, u(t-\tau)) \end{aligned} \tag{2}$$

r_i is the differential order of y_i, and $r_1 + r_2 + \cdots + r_m = r$. $\eta \in \mathbb{R}^{n-r}$ is the internal state, $\varsigma = \left[y_1, y_1', \cdots, y_1^{(r_1-1)}, y_2, \cdots, y_m^{(r_m-1)}\right]^T$ represents the external state, $s(\varsigma, \eta, u(t-\tau))$ represents the nonlinear expression of internal dynamics. Then the partially linearized system (2) is

$$\begin{aligned} \dot{\varsigma} &= A_\varsigma \varsigma + B_\varsigma v = A_\varsigma \varsigma + B_\varsigma \left[F(x) + G(x)u(t-\tau)\right] \\ \dot{\eta} &= s(\varsigma, \eta, u(t-\tau)) \\ &= s(\varsigma, \eta, v) \\ &= s(\varsigma, \eta, (F(x) + G(x)u(t-\tau))), \end{aligned} \tag{3}$$

where

$$F(x) = \left[F_1(x), F_2(x), \cdots, F_m(x)\right]^T,$$

$$G(x) = \left[G_1(x), G_2(x), \cdots, G_m(x)\right]^T,$$

$$A_\varsigma = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_m \end{bmatrix}_{|r\times r},$$

$$B_\varsigma = \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_m \end{bmatrix}_{|r\times m},$$

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & 0 & 0 & \ddots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{|r_i\times r_i},$$

$$B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{|r_i\times 1} \qquad (i = 1, 2, \cdots, m).$$

B. CONTROL OBJECTIVE

For a real system, the given reference trajectory y^d is smooth and bounded, then the IID of the original system can be computed according to output regulation [14], stable system center [21], and noncausal stable inversion [12] methods. Based on IID, a state tracking control problem is built:

$$\dot{e} = Ae + B\left[F(x) + G(x)u(t-\tau) - y^{d(r)} + d\right], \qquad (4)$$

where $e = [e_\varsigma^T, e_\eta^T]^T$, $e_\varsigma = \varsigma - \varsigma^d$, $e_\eta = \eta - \eta^d$, $\varsigma^d = \left[y_1^d, y_1^{d(1)}, \cdots, y_1^{d(r_1-1)}, y_2^d, \cdots, y_m^{d(r_m-1)}\right]^T$ is the reference trajectory and it's $(r-1)$ order differential, η^d represents IID of (1), and $y^{d(r)} = \left[y_1^{d(r_1)}, y_2^{d(r_2)}, \cdots, y_m^{(r_m)}\right]^T$ represents r order differential of the given command, d is the linearization error,

$$A = \begin{bmatrix} A_\varsigma & 0 \\ A_{\eta 1} & A_{\eta 2} \end{bmatrix}, \quad B = \begin{bmatrix} B_\varsigma \\ B_\eta \end{bmatrix},$$

$$A_{\eta 1} = \frac{\partial s(\varsigma, \eta, v)}{\partial \varsigma}\Big|_{\varsigma=0,\eta=0,v=0},$$

$$A_{\eta 2} = \frac{\partial s(\varsigma, \eta, v)}{\partial \eta}\Big|_{\varsigma=0,\eta=0,v=0},$$

$$B_\eta = \frac{\partial s(\varsigma, \eta, v)}{\partial v}\Big|_{\varsigma=0,\eta=0,v=0},$$

where $v = F(x) + G(x)u(t-\tau)$. Since $F(x)$ and $G(x)$ are uncertain, the expression of (4) should be improved as:

$$\dot{e} = (A + \Delta A)\, e + (B + \Delta B) \times \left(v - y^{d(r)} + d\right), \qquad (5)$$

where ΔA is uncertain part of matrix A, and ΔB is uncertain part of matrix B. Because of the existence of ΔA and ΔB, the control design method proposed in [12] is unsuitable, let alone the unmodelled dynamics and input delay τ. In this case, the control goal of (5) is: Find a robust controller which can guarantee the stability of (5) under unmodelled dynamics, unknown matrices ΔA and ΔB, and input delay τ.

III. MAIN RESULT

Because of the existence of input delay τ, controller design cannot be carried out directly for (5), so a transformation is needed. Defining

$$e_p = \int_{t-\tau}^{t} u(z)dz \qquad (6)$$

then a new error defined as

$$e_s = e + (B + \Delta B)\, G(x) e_p \qquad (7)$$

is constructed.

Assumption 1: Through out the running time, e_p is always bounded, and can be described by

$$\|e_p\| \le \rho_p e.$$

where ρ_p is a known positive scalars.

Remark 1: For the designed controller, $u(t)$ can be viewed as the function of system states. If the controller is designed appropriately, it should be bounded. In this case, as the finite time integral of $u(t)$, e_p is bounded.

From (4) and (7), the derivative of e_s is

$$\dot{e}_s = \dot{e} + (B + \Delta B)\left[\frac{\partial G(x)}{\partial x} e_p + G(x)\dot{e}_p\right]$$

Considering (4) we can get

$$\dot{e}_s = (A + \Delta A)\, e + (B + \Delta B) \times \left[F(x) + G(x)u(t) + N(x) - y^{d(r)} + d\right] \qquad (8)$$

where

$$N(x) = \frac{\partial G(x)}{\partial x} e_p = \begin{bmatrix} \frac{\partial G_{11}(x)}{\partial x^T}\dot{x} & \cdots & \frac{\partial G_{1m}(x)}{\partial x^T}\dot{x} \\ \vdots & \ddots & \vdots \\ \frac{\partial G_{m1}(x)}{\partial x^T}\dot{x} & \cdots & \frac{\partial G_{mm}(x)}{\partial x^T}\dot{x} \end{bmatrix} e_p \qquad (9)$$

Remark 2: From (9) we can see that, $N(x)$ is determined by $G(x)$. Since $G(x)$ is unknown, $N(x)$ should be also viewed as an unknown function of x.

Since parameter uncertainties and disturbances always exist, the controller of such a system should be, not only a robust controller for ΔA and ΔB, but also an identified method for unmodelled dynamics in $F(x)$, $G(x)$ and $N(x)$. As an universal approximation, fuzzy logic system (FLS) has good approximation effect, so it is utilized here. The FLS adopted in this paper has the same form of [11], then the output of FLS can be wrote as

$$\hat{y}_j = \theta_j^T \xi(\hat{x}),$$

where $\theta_j^T = \left[\theta_j^1\ \theta_j^2 \cdots \theta_j^M\right]^T$ are adaptive parameters, $\xi_i\left(\hat{x}\right)$ is basis function. By training θ_j^T and $\xi(\hat{x})$, $F(x)$ $G(x)$ and $N(x)$ can be replaced by fuzzy sets:

$$F(x|\theta_f) = \begin{bmatrix} F_1(x|\theta_f) \\ F_2(x|\theta_f) \\ \vdots \\ F_r(x|\theta_f) \end{bmatrix} = \xi_f(x)\theta_f,$$

$$G(x|\theta_g) = \begin{bmatrix} G_{11}(x|\theta_g) & \cdots & G_{1m}(x|\theta_g) \\ \vdots & \ddots & \vdots \\ G_{m1}(x|\theta_g) & \cdots & G_{mm}(x|\theta_g) \end{bmatrix} = \xi_g(x)\theta_g,$$

$$N(x|\theta_n) = \begin{bmatrix} N_1(x|\theta_g) \\ \vdots \\ N_m(x|\theta_g) \end{bmatrix} = \xi_n(x)\theta_n,$$

where

$$\theta_f = \begin{bmatrix} \theta_{f_1} \\ \theta_{f_2} \\ \vdots \\ \theta_{f_m} \end{bmatrix},$$

$$\xi_f(x) = \begin{bmatrix} \xi_{f_1}(x) & 0 & \cdots & 0 \\ 0 & \xi_{f_2}(x) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \xi_{f_n}(x) \end{bmatrix},$$

$$\theta_g = \begin{bmatrix} \theta_{g11} & \theta_{g12} & \cdots & \theta_{g1m} \\ \theta_{g21} & \theta_{g22} & \cdots & \theta_{g2m} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{gm1} & \theta_{gm2} & \cdots & \theta_{gmm} \end{bmatrix},$$

$$\xi_g(x) = \begin{bmatrix} \xi_{g_1}(x) & 0 & \cdots & 0 \\ 0 & \xi_{g_2}(x) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \xi_{g_m}(x) \end{bmatrix},$$

$$\theta_n = \begin{bmatrix} \theta_{n_1} \\ \theta_{n_2} \\ \vdots \\ \theta_{n_m} \end{bmatrix},$$

$$\xi_n(x) = \begin{bmatrix} \xi_{n_1}(x) & 0 & \cdots & 0 \\ 0 & \xi_{n_2}(x) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \xi_{n_m}(x) \end{bmatrix}.$$

Based on the FLS expression, (8) can be replaced by

$$\begin{aligned} \dot{e}_s = (A + \Delta A)\, e + (B + \Delta B) \big(& F(x|\theta_f) + G(x|\theta_g)u \\ & + N(x|\theta_m) - y^{d(r)} + F(x) - F(x|\theta_f) \\ & + N(x) - N(x|\theta_m) + G(x)u - G(x|\theta_g)u + d \big) \end{aligned}$$

For the adaptive training parameters, assuming the optimal value of them are

$$\begin{aligned} \theta_f^* &= \arg\min \left| F(x) - F(x|\theta_f^*) \right| \\ \theta_g^* &= \arg\min \left| G(x) - G(x|\theta_g^*) \right| \\ \theta_n^* &= \arg\min \left| N(x) - N(x|\theta_n^*) \right| \end{aligned}$$

Then a FLS based robust controller is constructed:

$$u = \frac{1}{G(x|\theta_g)} \left(-F(x|\theta_f) - N(x|\theta_n) + y^{d(r)} + Ke + u_h + u_s \right) \tag{10}$$

where K and u_h, u_s will be discussed in the following part. By substituting (10) into (8), we can get a new tracking expression:

$$\begin{aligned} \dot{e}_s = (A + \Delta A)\, e + (B + \Delta B) \Big[& Ke + \xi_f(x)\tilde{\theta}_f \\ & + \xi_g(x)\tilde{\theta}_g u + \xi_n(x)\tilde{\theta}_n + u_h + u_s \\ & + \Delta F + \Delta M + \Delta G u + d \Big] \end{aligned} \tag{11}$$

where $\Delta F = F(x) - F(x|\theta_f^*)$, $\Delta G = G(x) - G(x|\theta_g^*)$, $\Delta N = N(x) - N(x|\theta_n^*)$, $\tilde{\theta}_f = \theta_f^* - \theta_f$, $\tilde{\theta}_g = \theta_g^* - \theta_g$, $\tilde{\theta}_n = \theta_n^* - \theta_n$.

The following assumptions are listed for ΔA, ΔB and $\Delta G G^{-1}(x|\theta_g)$:

Assumption 2: $G(x|\theta_g)$ is invertible.

Assumption 3: $\left\| \Delta G G^{-1}(x|\theta_g) \right\| \le \kappa_G$, κ_G is already known scalar and $\kappa_G < 1$.

Assumption 4: The uncertain matrices ΔA and ΔB are bounded and $\Delta B = B\Delta H$, $\|\Delta H\| \le \kappa_B < 1$, where κ_B is a known scalar.

Assumption 5: Define $M_h(x) = e_s^T P (B + \Delta B) \cdot \left(\Delta F + \Delta G + \Delta N + \Delta G G^{-1}(x|\theta_g) \right) \left(-F(x|\theta_f) + y^{d(r)} + Ke \right) - e_s^T PBR^{-1}B^T P (B + \Delta B)\, G(x|\theta_g) e_p$
$-e_s^T P \left[(A + \Delta A) + (B + \Delta B) K \right] (B + \Delta B)\, G(x|\theta_g) e_p$. $M_h(x)$ is unknown and bounded, which means that, $\|M_h(x)\| \le (\rho_0 + \rho_1 \|e\|) \left\| B^T P e \right\|$, where ρ_0 and ρ_1 are unknown scalars.

Remark 3: For a real system, $G(x)$ is invertible. As a fuzzy approximation of $G(x)$, $G(x|\theta_g)$ should also be invertible. So Assumption 2 is reasonable.

Remark 4: When the value of $G(x|\theta_g)$ is close to zero, the computing of $\frac{1}{G(x|\theta_g)}$ becomes an *ill-conditioned inverse* problem. This problem should be taken seriously, but a lot of results, such as Newton algorithm [38], iterative refinement method [39], can be used and referenced.

Remark 5: $\Delta G = G(x) - G(x|\theta_g^*)$, so ΔG is bounded. From Assumption 2, $G(x|\theta_g)$ is also bounded. Then $G^{-1}(x|\theta_g)$ is bounded and the product of ΔG and $G^{-1}(x|\theta_g)$ is also bounded. So Assumption 3 is reasonable.

Remark 6: Since the parameter uncertainty and unmodelled dynamics in practice is bounded, the derived uncertainty matrices ΔA and ΔB are bounded. The input matrix B is full column rank, so $\Delta H = \left(B^T B \right)^{-1} B^T \Delta B$, and ΔH is bounded. So Assumption 4 is reasonable.

Remark 7: From the expression of $M_h(x)$ we can see that, ΔF, ΔG and ΔM are approximation errors which can be viewed as a small amount, $G(x|\theta_g)$ is bounded, and $F(x|\theta_f)$ is Lipschitz, e_p and Ke are all functions of tracking error e, so Assumption 5 is reasonable.

Theorem 1: If Assumption 1-4 is hold, and there are matrices P, K, and scalars ε_A, ε_B, satisfying

$$\begin{aligned} & P > 0,\ \varepsilon_A > 0,\ \varepsilon_B > 0 \\ & \quad P(A + BK) + (A + BK)^T P + Q \\ & \quad + \lambda_{\max}^2 (\Delta A)\, \varepsilon_A I + \lambda_{\max}^2 (\Delta B)\, \varepsilon_B K^T K \\ & \quad + \left(\varepsilon_A^{-1} + \varepsilon_B^{-1} \right) PP + PB \left(\rho^{-2} I + R^{-1} \right) B^T P < 0, \end{aligned} \tag{12}$$

where $Q > 0$ is a given matrix, R is a given control gain and ρ is prescribed attenuation index, then (11) is stable under the following robust controller

$$u_h = \frac{1}{2\left(1+\kappa_G\right)\left(1+\kappa_B\right)} R^{-1} B^T Pe \tag{13}$$

$$u_s = -\frac{\hat{\rho}_0 + \hat{\rho}_1 \left\|e\right\|}{(1+\kappa_G)\Theta} \mathtt{sgn}(B^T Pe) \tag{14}$$

$\hat{\rho}_0$ and $\hat{\rho}_1$ are estimates of ρ_0 and ρ_1, and

$$\dot{\theta}_f = \gamma_f \Theta \xi_f^T(x) B^T Pe \tag{15}$$

$$\dot{\theta}_{g_{ij}} = \gamma_g \Theta u_j(t) (e^T PB)_i \xi_{g_i}^T(x) \tag{16}$$

$$\dot{\theta}_n^T = \gamma_n \Theta \xi_n^T(x) B^T Pe \tag{17}$$

$$\dot{\hat{\rho}}_0 = q_0 \left\|B^T Pe\right\| \tag{18}$$

$$\dot{\hat{\rho}}_1 = q_1 \left\|e\right\| \left\|B^T Pe\right\| \tag{19}$$

where $\Theta = (1 + \lambda_{\max}^2\left(B\right)\left(1+\kappa_B\right)\rho_p\rho_G)$.

Proof: Choosing a Lyapunov function for system (11) as

$$\begin{aligned} V &= \frac{1}{2} e_s^T P e_s + \frac{1}{2\gamma_f} \tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_g} tr\left(\tilde{\theta}_g^T \tilde{\theta}_g\right) \\ &\quad + \frac{1}{2\gamma_n} \tilde{\theta}_n^T \tilde{\theta}_n + \frac{1}{2q_0} \tilde{\rho}_0^2(t) + \frac{1}{2q_1} \tilde{\rho}_1^2(t) \end{aligned}$$

where γ_f, γ_g, γ_n, q_0 and q_1 are given scalars. Since $\dot{\tilde{\theta}}_f = -\dot{\theta}_f$, $\dot{\tilde{\theta}}_g = -\dot{\theta}_g$, $\dot{\tilde{\theta}}_n = -\dot{\theta}_n$, $\dot{\tilde{\rho}}_0 = -\dot{\hat{\rho}}_0$ and $\dot{\tilde{\rho}}_1 = -\dot{\hat{\rho}}_1$, differentiating (11) we have

$$\begin{aligned} \dot{V} &= \frac{1}{2} \dot{e}_s^T P e_s + \frac{1}{2} e_s^T P \dot{e}_s - \frac{1}{\gamma_f} \dot{\theta}_f^T \tilde{\theta}_f - \frac{1}{\gamma_g} tr\left(\dot{\theta}_g^T \tilde{\theta}_g\right) \\ &\quad - \frac{1}{\gamma_n} \dot{\theta}_n^T \tilde{\theta}_n - \frac{1}{q_0} \tilde{\rho}_0^T \dot{\hat{\rho}}_0 - \frac{1}{q_1} \tilde{\rho}_1^T \dot{\hat{\rho}}_1 \\ &= e_s^T P \left[\left(A + \Delta A\right) + \left(B + \Delta B\right) K\right]^T e_s \\ &\quad - e_s^T P \left[\left(A + \Delta A\right) + \left(B + \Delta B\right) K\right] \left(B + \Delta B\right) G(x|\theta_g) e_p \\ &\quad + e_s^T P \left(B + \Delta B\right) \left(I + \Delta G G^{-1}(x|\theta_g)\right) u_h \\ &\quad + e_s^T P \left(B + \Delta B\right) \left(I + \Delta G G^{-1}(x|\theta_g)\right) u_s \\ &\quad + e_s^T P \left(B + \Delta B\right) \left(\Delta F + \Delta N + \Delta G G^{-1}(x|\theta_g)\right) \\ &\quad \times (-F(x|\theta_f) - N(x|\theta_n) + y^{d(r)} + Ke) \\ &\quad - \frac{1}{q_0} \tilde{\rho}_0^T \dot{\hat{\rho}}_0 - \frac{1}{q_1} \tilde{\rho}_1^T \dot{\hat{\rho}}_1 \\ &\quad + e_s^T P \left(B + \Delta B\right) \xi_f(x) \tilde{\theta}_f - \frac{1}{\gamma_f} \dot{\theta}_f^T \tilde{\theta}_f \\ &\quad + e_s^T P \left(B + \Delta B\right) \xi_g(x) \tilde{\theta}_g u(t) - \frac{1}{\gamma_g} tr\left(\dot{\theta}_g^T \tilde{\theta}_g\right) \\ &\quad + e_s^T P \left(B + \Delta B\right) \xi_n(x) \tilde{\theta}_n - \frac{1}{\gamma_n} \dot{\theta}_n^T \tilde{\theta}_n \\ &\quad + e_s^T P \left(B + \Delta B\right) d \end{aligned}$$

Since

$$\begin{aligned} \Delta A^T P + P \Delta A &\le \varepsilon_A \Delta A^T \Delta A + \varepsilon_A^{-1} PP \\ &\le \lambda_{\max}^2\left(\Delta A\right) \varepsilon_A I + \varepsilon_A^{-1} PP \end{aligned}$$

$$\begin{aligned} P \Delta B K + K^T \Delta B^T P &\le \varepsilon_B K^T \Delta B^T \Delta B K + \varepsilon_B^{-1} PP \\ &\le \lambda_{\max}^2\left(\Delta B\right) \varepsilon_B K^T K + \varepsilon_B^{-1} PP, \end{aligned}$$

then

$$\begin{aligned} & P \left[\left(A + \Delta A\right) + \left(B + \Delta B\right) K\right] \\ &\quad + \left[\left(A + \Delta A\right) + \left(B + \Delta B\right) K\right]^T P \\ &\le P(A + BK) + (A + BK)^T P \\ &\quad + \lambda_{\max}^2\left(\Delta A\right) \varepsilon_A I + \lambda_{\max}^2\left(\Delta B\right) \varepsilon_B K^T K \\ &\quad + \left(\varepsilon_A^{-1} + \varepsilon_B^{-1}\right) PP \end{aligned}$$

Since $\left\|\Delta G G^{-1}(x|\theta_g)\right\| \le \kappa_G < 1$, $\left\|\Delta H\right\| \le \kappa_B < 1$, and $u_h = \frac{1}{2(1+\kappa_G)(1+\kappa_B)} R^{-1} B^T Pe$,

$$\begin{aligned} & u_h^T \left(I + \Delta G G^{-1}(x|\theta_g)\right)^T \left(B + \Delta B\right)^T P e_s \\ &\quad + e_s^T P \left(B + \Delta B\right) \left(I + \Delta G G^{-1}(x|\theta_g)\right) u_h \\ &\le e_s^T P B R^{-1} B^T P e_s \\ &\quad - e_s^T P B R^{-1} B^T P \left(B + \Delta B\right) G(x|\theta_g) e_p \end{aligned}$$

From Assumption 4, $M_h(x)$ is bounded, $u_s = -\frac{\hat{\rho}_0 + \hat{\rho}_1 \|e\|}{(1+\kappa_G)(1+\lambda_{\max}^2(B)(1+\kappa_B)\rho_G\rho_p)} sgn(B^T Pe)$, and

$$\begin{aligned} \left\|e_s\right\| &= \left\|e + \left(B + \Delta B\right) G(x|\theta_g) e_p\right\| \\ &\le \left(1 + \lambda_{\max}^2\left(B\right)\left(1 + \kappa_B\right)\rho_G\rho_p\right) \left\|e\right\| \\ \dot{\hat{\rho}}_0 &= q_0 \left\|B^T Pe\right\| \\ \dot{\hat{\rho}}_1 &= q_1 \left\|e\right\| \left\|B^T Pe\right\| \end{aligned}$$

so

$$\begin{aligned} & u_s^T \left(I + \Delta G G^{-1}(x|\theta_g)\right)^T \left(B + \Delta B\right)^T P e_s \\ &\quad + M_h(x) - \frac{1}{q_0} \tilde{\rho}_0^T \dot{\rho}_0 - \frac{1}{q_1} \tilde{\rho}_1^T \dot{\rho}_1 \\ &\le \left(\rho_0 - \hat{\rho}_0\right) \left\|B^T Pe\right\| - \frac{1}{q_0} \dot{\hat{\rho}}_0 \tilde{\rho}_0 \\ &\quad + \left(\rho_1 \left\|e\right\| - \hat{\rho}_1 \left\|e\right\|\right) \left\|B^T Pe\right\| - \frac{1}{q_1} \dot{\hat{\rho}}_1 \tilde{\rho}_1 \\ &\le 0 \end{aligned}$$

Also form Lemma 1 of [19],

$$\begin{aligned} & \left[d^T \left(I + \Delta H\right)^T B^T P e_s + e_s^T P B \left(I + \Delta H\right) d\right] \\ &\quad \le \rho^2 d^T \left(I + \Delta H\right)^T \left(I + \Delta H\right) d + \rho^{-2} e_s^T P B B^T P e_s \\ &\quad \le \left(1 + \kappa_B\right)^2 \rho^2 d^T d + \rho^{-2} e_s^T P B B^T P e_s \end{aligned}$$

So

$$\begin{aligned} \dot{V} &\le \frac{1}{2} e_s^T \Big[P(A + BK) + (A + BK)^T P + \lambda_{\max}^2\left(\Delta A\right) \varepsilon_A I \\ &\quad + \lambda_{\max}^2\left(\Delta B\right) \varepsilon_B K^T K + \left(\varepsilon_A^{-1} + \varepsilon_B^{-1}\right) PP \\ &\quad + P B R^{-1} B^T P + \rho^{-2} P B B^T P\Big] e_s \\ &\quad + \frac{1}{2} (1 + \kappa_B)^2 \rho^2 \omega^T \omega \\ &\quad + \frac{1}{2} \left[\tilde{\theta}_f^T \xi_f^T(x) \left(I + \Delta H\right)^T B^T P e_s\right. \end{aligned}$$

$$+e_s^T PB(I+\Delta H)\xi_f(x)\tilde{\theta}_f\Big] - \frac{1}{\gamma_f}\dot{\theta}_f^T\tilde{\theta}_f$$
$$+\frac{1}{2}\Big[u^T(t)\tilde{\theta}_g^T\xi_g^T(x)(I+\Delta H)^T B^T Pe_s$$
$$+e_s^T PB(I+\Delta H)\xi_g(x)\tilde{\theta}_g u(t)\Big] - \frac{1}{\gamma_g}tr\left(\dot{\theta}_g^T\tilde{\theta}_g\right)$$

From (15), $\dot{\theta}_f^T = \gamma_f(1+\lambda_{\max}^2(B)(1+\kappa_B)\rho_G\rho_p)(1+\kappa_B)e^T PB\xi_f(x)$, and

$$e_s^T PB(I+\Delta H)\xi_f(x) \le (1+\lambda_{\max}^2(B)(1+\kappa_B)\rho_G\rho_p)(1+\kappa_B)e^T PB\xi_f(x)$$

so

$$e_s^T PB(I+\Delta H)\xi_f(x) - \frac{1}{\gamma_f}\dot{\theta}_f^T\tilde{\theta}_f \le 0$$

Similarly,

$$e_s^T P(B+\Delta B)\xi_g(x)\tilde{\theta}_g u(t) - \frac{1}{\gamma_g}tr\left(\dot{\theta}_g^T\tilde{\theta}_g\right) \le 0$$
$$e_s^T P(B+\Delta B)\xi_m(x)\tilde{\theta}_m - \frac{1}{\gamma_m}\dot{\theta}_m^T\tilde{\theta}_m \le 0$$

from (12) we have

$$\dot{V} \le -\frac{1}{2}e_s^T Qe_s + \frac{1}{2}(1+\kappa_B)^2\rho^2 d^T d$$

Since d is bounded, for e_s, if

$$\|e_s\| \ge \frac{\rho\sqrt{(1+\kappa_B)d^T d}}{\sqrt{\lambda_{\min}(Q)}}$$

$\dot{V} \le 0$ is always hold. Then for the H_∞ performance, we have:

$$\int_0^{t_f} e_s^T Qe_s dt \le e_s^T(0)Pe_s(0) + \frac{1}{\gamma_f}\tilde{\theta}_f^T(0)\tilde{\theta}_f(0)$$
$$+\frac{1}{\gamma_g}tr\left(\tilde{\theta}_g^T(0)\tilde{\theta}_g(0)\right)$$
$$+\frac{1}{\gamma_m}\tilde{\theta}_m^T(0)\tilde{\theta}_m(0) + \rho^2\int_0^{t_f} d^T d dt.$$

The proof is completed. ■

Remark 8: Theorem 1 gives the designing method for input delay non-minimum phase system, but it seems a little complex. Actually, inequation (12) can be easily solved by LMI toolbox of MATLAB, while (15), (16) and (17) can be easily approached by FLS toolbox of MATLAB. So relying on existing MATLAB toolbox, Theorem 1 is easily to be implemented.

For facilitating the application of the proposed method, an overall block diagram is given in Figure 1.

In order to ensure the adaptive parameters θ_f, θ_g and θ_m to be bounded, the projection algorithm is utilized here to amend the adaptive law (15), (16) and (17). Assume the constraint sets Ω_f, Ω_g and Ω_m are specified as $\Omega_f \triangleq \{\theta_f \,|\, \|\theta_f\| \le \sigma_f\}$, $\Omega_g \triangleq \{\theta_{gij} \,|\, \|\theta_{gij}\| \le \sigma_{gij}\}$ and $\Omega_m \triangleq \{\theta_m \,|\, \|\theta_m\| \le \sigma_m\}$, $(i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, m)$, respectively, where σ_f, σ_{gij} and σ_m are all positive constants and can be arbitrarily specified. Thus, the adaptive law (15), (16) and (17) can be modified as

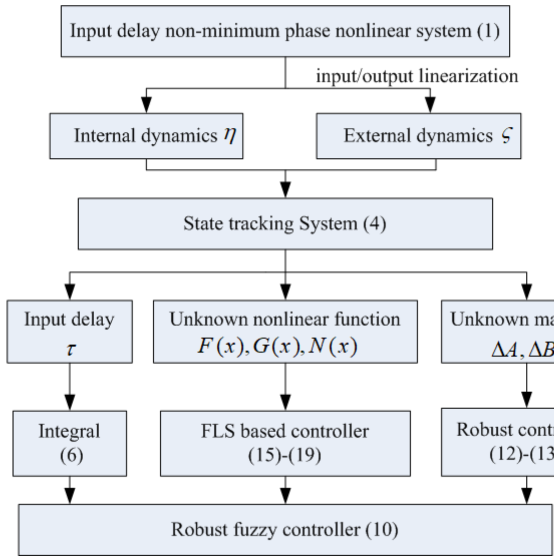

FIGURE 1. Implementation process of the proposed controller.

$$\dot{\theta}_f = \begin{cases} \gamma_f(1+\kappa_B)\left(\xi_f^T(x)B^T Pe - \dfrac{\|\theta_f\|^2\xi_f^T(x)B^T Pe}{\|\theta_f\|^2}\right) \\ \quad \text{If } \|\theta_f\|^2 = \sigma_f \\ \text{and } \xi_f^T(x)B^T Pe > 0 \\ \gamma_f(1+\kappa_B)\xi_f^T(x)B^T Pe \\ \quad \text{otherwise} \end{cases}$$

$$\dot{\theta}_{gij} = \begin{cases} \gamma_g(1+\kappa_B)\Big(\left(e^T PB\right)_i u_j(t)\xi_{gi}^T(x) \\ \quad - \dfrac{\|\theta_{gij}\|^2\left(e^T PB\right)u_j(t)\xi_{gi}^T(x)}{\|\theta_{gij}\|^2}\Big) \\ \quad \text{If } \|\theta_{gij}\|^2 = \sigma_{gij} \\ \text{and } \left(e^T PB\right)u_j(t)\xi_{gi}^T(x) > 0 \\ \gamma_g(1+\kappa_B)\left(e^T PB\right)_i u_j(t)\xi_{gi}^T(x) \\ \quad \text{otherwise} \end{cases}$$

$$\dot{\theta}_m = \begin{cases} \gamma_m(1+\kappa_B)\left(\xi_m^T(x)B^T Pe - \dfrac{\|\theta_m\|^2\xi_m^T(x)B^T Pe}{\|\theta_m\|^2}\right) \\ \quad \text{If } \|\theta_m\|^2 = \sigma_m \\ \text{and } \xi_m^T(x)B^T Pe > 0 \\ \gamma_m(1+\kappa_B)\xi_m^T(x)B^T Pe \\ \quad \text{otherwise} \end{cases}$$

IV. SIMULATION RESULTS

A numerical simulation example on VTOL is considered, and the nonlinear expression of VTOL is adopted here and a sketch of VTOL is given in Figure 2. The nonlinear expressions are listed here:

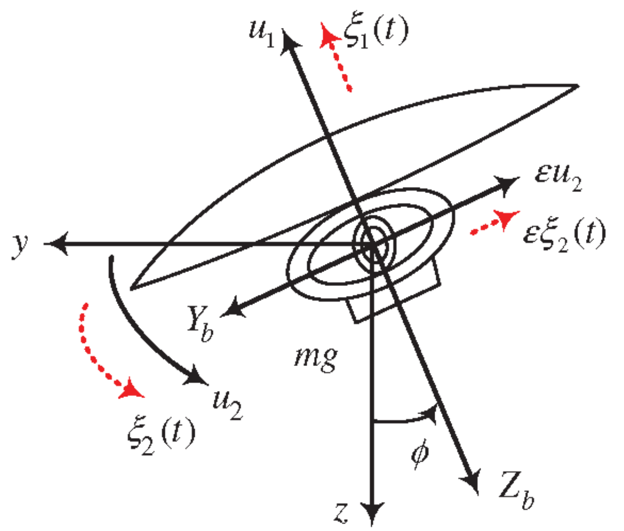

FIGURE 2. **Reference frame of VTOL.**

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} = f(x) + g(x)\left(u(t-\tau) + \zeta\right) \tag{20}$$

where

$$f(x) = \begin{pmatrix} x_2 \\ 0 \\ x_4 \\ -1 \\ x_6 \\ 0 \end{pmatrix}, \quad g(x) = \begin{pmatrix} 0 & 0 \\ -\sin x_5 & \varepsilon(t)\cos x_5 \\ 0 & 0 \\ \cos x_5 & \varepsilon(t)\sin x_5 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \zeta = \begin{bmatrix} \zeta_1 & \zeta_2 \end{bmatrix}^T.$$

The physical means of the above nonlinear expression can be founded in [10], and $\varepsilon(t) = 0.5 + 0.1\sin x_5$. ζ_1 and ζ_2 are additional unknown dynamics. Then (20) can be replaced by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} = \begin{pmatrix} x_2 \\ \varepsilon(t)\cos x_5 \zeta_2 - \sin x_5 \zeta_1 \\ x_4 \\ -1 + \cos x_5 \zeta_1 + \varepsilon(t)\sin x_5 \zeta_2 \\ x_6 \\ \zeta_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\sin x_5 & \varepsilon(t)\cos x_5 \\ 0 & 0 \\ \cos x_5 & \varepsilon(t)\sin x_5 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1(t-\tau) \\ u_2(t-\tau) \end{pmatrix} \tag{21}$$

The output of the plant is

$$y_1 = x_1, \quad y_2 = x_3$$

and through input/output linearization, (21) can be simplified as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} x_2 \\ \varepsilon(t)\cos x_5 \zeta_2 - \sin x_5 \zeta_1 \\ x_4 \\ -1 + \cos x_5 \zeta_1 + \varepsilon(t)\sin x_5 \zeta_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\sin x_5 & \varepsilon(t)\cos x_5 \\ 0 & 0 \\ \cos x_5 & \varepsilon(t)\sin x_5 \end{pmatrix} \begin{pmatrix} u_1(t-\tau) \\ u_2(t-\tau) \end{pmatrix}$$

What is with mentioning, (20) is a common form, and the internal state is

$$\eta = \begin{bmatrix} x_5 \\ x_6 \end{bmatrix}$$

and

$$F(x) = \begin{bmatrix} \varepsilon(t)\cos x_5 \zeta_2 - \sin x_5 \zeta_1 \\ -1 + \cos x_5 \zeta_1 + \varepsilon(t)\sin x_5 \zeta_2 \end{bmatrix},$$

$$G(x) = \begin{bmatrix} -\sin x_5 & \varepsilon(t)\cos x_5 \\ \cos x_5 & \varepsilon(t)\sin x_5 \end{bmatrix}$$

$$s(\varsigma, \eta, (F(x) + G(x)u)) = \begin{bmatrix} x_6 \\ -\zeta_2 + \dfrac{\sin x_5}{\varepsilon(t)} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{\cos x_5}{\varepsilon(t)} & \dfrac{\sin x_5}{\varepsilon(t)} \end{bmatrix} (F(x) + G(x)u(t-\tau)) \tag{22}$$

Based on the analysis of [31], (21) is non-minimum phase. For the construction of IID, here we utilize the method presented in [21], and in calculation, ε is assumed to be constant and chosen as $\varepsilon = 0.5$. Then IID $\eta^d = (\eta_1^d, \eta_2^d)^T \triangleq (x_5^d, x_6^d)^T$ can be constructed by solving the equation below:

$$\ddot{\eta}^d + c_1\dot{\eta}^d + c_0\eta^d = -\left(P_1\dot{\varkappa}^d + P_0\varkappa^d\right)$$

where $c_0 = 1$, $c_1 = 2$, $\varkappa^d = (0, 1)^T(\ddot{y}_1^d \cos\eta_1^d + (1 + \ddot{y}_2^d)\sin\eta_1^d - \eta_1^d)$, P_0 and P_1 are gain matrices, and the solving details can be found in [31].

Remark 9: For an uncertain input delay nonlinear system, IID is also computed by the exact model (20) with a constant coupling coefficient ε and $\tau = 0$, $\zeta_1 = 0$, and $\zeta_2 = 0$, since IID is a ideal state for internal dynamic which we want.

Reference trajectories $y_1^d = R\cos(\omega t)$, $y_2^d = R\sin(\omega t)$ are chosen for simulation here, and $R = 1$, $\omega = 0.1$. The given trajectories and IID are shown in Figure (3).

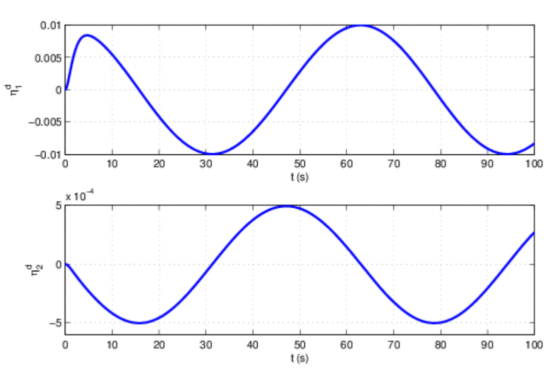

FIGURE 3. **IID of VTOL.**

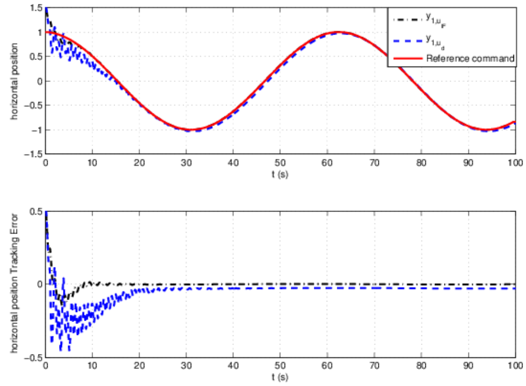

FIGURE 4. **Tracking performance of y_1.**

Defining states error as

$$\begin{aligned} e_1 &= y_1 - y_1^d = x_1 - x_1^d \\ e_2 &= \dot{y}_1 - y_2^d = x_2 - \dot{x}_1^d \\ e_3 &= y_2 - y_3^d = x_3 - x_2^d \\ e_4 &= \dot{y}_2 - y_4^d = x_4 - \dot{x}_2^d \\ e_5 &= \eta_1 - \eta_1^d = x_5 - x_5^d \\ e_6 &= \eta_2 - \eta_2^d = x_6 - x_6^d \end{aligned}$$

then

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \\ \dot{e}_5 \\ \dot{e}_6 \end{pmatrix} = (A + \Delta A) \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{pmatrix} + (B + \Delta B) \times \left(F(x) + G(x)u(t-\tau) - \left[y_1^{d(2)} \; y_2^{d(2)} \right]^T \right) \quad (23)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{\cos x_5}{\varepsilon} & 0 \end{bmatrix},$$

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\partial \zeta_2}{\partial x_5} & -\frac{\partial \zeta_2}{\partial x_6} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ \frac{\cos x_5}{0.5} & \frac{\sin x_5}{0.5} \end{bmatrix},$$

$$\Delta B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ \frac{\cos x_5}{\varepsilon} - \frac{\cos x_5}{0.5} & \frac{\sin x_5}{\varepsilon} - \frac{\sin x_5}{0.5} \end{bmatrix}.$$

The initial states $x(0) = \left[1.5,\ 0,\ -0.5,\ 0.2,\ 0.28,\ 0\right]^T$, $\varepsilon(t) = 0.5 + 0.2\sin x_5$, the disturbance caused by wind ζ_1 and ζ_2 are chosen as $\zeta_1 = 0.2\cos x_5$, $\zeta_2 = 0.2\sin x_5$, the input delay τ is already known and $\tau = 0.15$. The initial value for θ_f, θ_g and θ_m are all set to be "1".

Step 1: For the approximating of unmodelled dynamics, membership functions in FLS are chosen as x_5 and $\varepsilon(t)$, and:

$$\begin{aligned} \mu_{\hat{F}_{x_5}^1} &= \exp\left[-0.003\,(x_5 - 0.01)^2\right] \\ \mu_{\hat{F}_{x_5}^2} &= \exp\left[-0.003\,(x_5)^2\right] \\ \mu_{\hat{F}_{x_5}^3} &= \exp\left[-0.003\,(x_5 + 0.01)^2\right] \\ \mu_{\hat{F}_{\varepsilon}^1} &= \exp\left[-0.003\,(\varepsilon(t) - 0.2)^2\right] \\ \mu_{\hat{F}_{\varepsilon}^2} &= \exp\left[-0.003\,(\varepsilon(t) - 0.5)^2\right] \\ \mu_{\hat{F}_{\varepsilon}^3} &= \exp\left[-0.003\,(\varepsilon(t) - 0.7)^2\right] \end{aligned}$$

and the corresponding fuzzy rules are:

$R_{ij}^{(l)}$: If x_5 is $\hat{F}_{x_5}^i$ and $\varepsilon(t)$ is $\hat{F}_{\varepsilon}^j$, Then y is F_{ij}^l, where $i = 1, 2, 3; j = 1, 2, 3; l = 1, 2, \cdots, 9$. Then $F(x)$, $G(x)$ and $N(x)$ can be replaced by $\xi_f(x)\theta_f$, $\xi_g(x)\theta_g$, $\xi_n(x)\theta_n$.

Step 2: From (23), $\lambda_{\max}(\Delta A) = 0.036$, $\lambda_{\max}(\Delta B) = 0.71$, $\kappa_B = 0.5$, $\kappa_G = 0.3$, $\rho_G = 0.29$, $\rho_G = 0.79$. Choosing $\Omega = 1 \times 10^{-2} I$, $R = diag(20, 20)$, $\varepsilon_A = 0.2$, $\varepsilon_B = 0.2$, $\rho = 0.1$, the control matrix K can be constructed and

$$K = \begin{bmatrix} 2.11 & 7.42 & 0.92 & 2.75 & -12.45 & -8.96 \\ 0.92 & 2.75 & -0.82 & -1.35 & -3.58 & -2.57 \end{bmatrix}.$$

Step 3: The disturbance $d(t)$ are chosen the same as [10], and the parameters in theorem 1 are chosen as $\gamma_f = 1000$, $\gamma_g = 10$, $\gamma_m = 10$, $q_0 = 100$, and $q_1 = 100$.

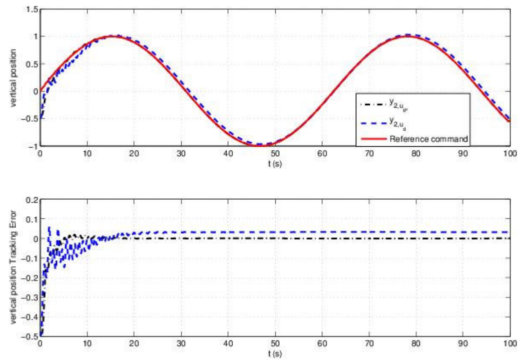

FIGURE 5. Tracking performance of y_2.

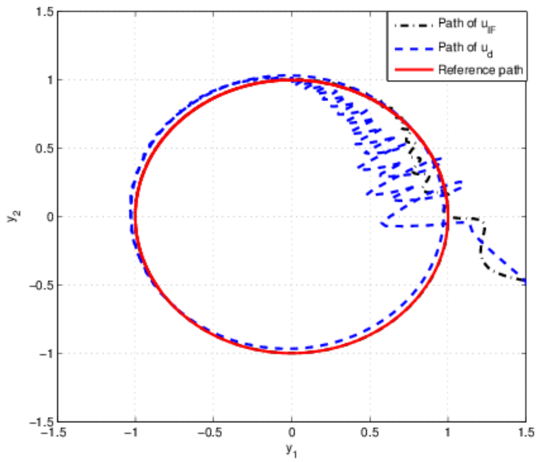

FIGURE 6. Path of VTOL.

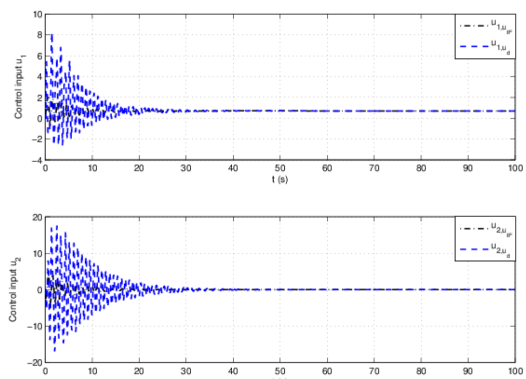

FIGURE 7. Input of the plant.

The proposed controller is marked as u_{IF}, and is carried on VTOL suffering from input delay, uncertainties and unmodelled dynamics together with a classical stable inversion controller u_d proposed in [4]. Simulation results are given in Figures (4-7), the given path and the controller response path are given in Figure (6). See the tracking performance of u_d in Figure (4), (5) and (6), a tracking error appears, while the tracking error of u_{IF} always keep remarkably small, then we can say that, the control effect of u_{IF} is much better.

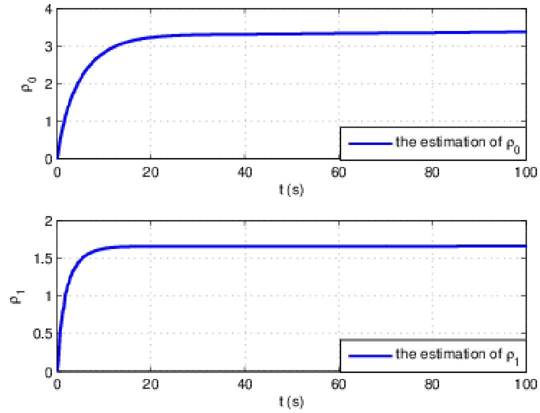

FIGURE 8. The estimation of ρ_0 and ρ_1.

Figure (7) is the input of u_{IF} and u_d. The estimation of ρ_0 and ρ_1 are also given in Figure (8).

V. CONCLUSION

The controller design of input delay uncertain MIMO nonlinear non-minimum phase system is discussed in this paper, and a FLS based controller is proposed. By input/output linearization, the expression of the nonlinear system is simplified, and then IID is constructed. Based IID, a state tracking problem is built. By defining a integral of past input, the input delay is transformed into a simple one, and then FLS is adopted in this paper to approach the uncertainty and unmodelled disturbance. Then the FLS adaptive fuzzy controller is constructed. Finally, a numerical simulation on VTOL is carried to test the good performance of the proposed strategy.

The uncertainty adopted in this paper is a norm bound uncertainty, and the priori information of them are assumed to be known. This is certainly not suitable for all real situations and will cause conservatism to the designed controller.

Further work is needed to design an online estimation method for the parameter uncertainty and unmodelled dynamics, and get a better robust controller. To testify the validity of the proposed method thoroughly, a real flight test of VTOL with the proposed control strategy is also need in the future.

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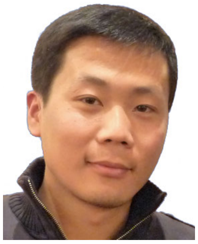

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