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# Risk-Based Multi-Attribute Decision-Making for Normal Cloud Model Considering Pre-Evaluation Information

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**ABSTRACT** An uncertain multi-attribute decision-making (MADM) problem is studied based on cloud models. Cloud models, referring to fuzziness and randomness, are utilized to depict evaluation and pre-evaluation information which can reflect the future development performance of alternatives. Because of bounded rationality, decision maker's (DM) risk attitudes should be considered when facing uncertainty. Thus, a behavioral MADM (BMADM) method is proposed by considering DM's risk attitudes and pre-evaluation. First, a distance measure for normal cloud models is developed with consideration of both DM's risk preferences and random distribution, aiming at making full use of information. Second, as a basis of applying prospect theory, positive ideal reference point is set by considering both evaluation and pre-evaluation information from three aspects: risk-averse, risk-neutral, and risk-seeking preference coefficients, in which the sign of distance is not necessary to determine. The third element is the establishment of an optimization model for handling incomplete attribute weights, following which is to obtain the ranking of alternatives. The final phase is the application of the proposed method to one case, along with sensitivity and comparison analyses, as a means of illustrating the applicability and feasibility of the new method.

**INDEX TERMS** Multi-attribute decision making, normal cloud model, risk attitudes, pre-evaluation information, prospect theory.

## I. INTRODUCTION

Multi-attribute decision making (MADM) is a process of ranking alternatives or selecting the best alternative from several alternatives with respect to a set of attributes [1]. In decision-making practice, decision-makers (DMs) prefer to use linguistic terms to express their preferences due to the fact that this expression is not only intuitive and easy to express but can well align human source [2]–[4]. However, most of existing methods based on linguistic terms rely on the conversion between linguistic information and exact numbers, and thus ignore the randomness and fuzziness which are the two key aspects of various uncertainties involved in the linguistic expression [5]. Cloud model, initially proposed by Li based on probability theory and fuzzy set theory [6],

is a very good tool to describe the qualitative concept by using the quantitative method. This kind of expression can not only allow a stochastic disturbance of the membership degree encircling a central value rather than a crisp number, but also simultaneously reflect fuzziness (the vague boundary of the extension) and randomness (frequency of occurrence) [7]. Three numerical characteristics are utilized to describe one cloud model [6], [8]: expectation, entropy, and hyper entropy, which can explicitly disclose the randomness and the fuzziness of qualitative concepts. Up to now, cloud model is considered one of the most effective tools for handling linguistic expressions, and normal cloud model is mainly utilized for handling various linguistic expressions [9]–[12]. Moreover, normal cloud model can be also used to depict quantitative things with fuzziness and randomness being included, such as interval numbers [13]. Besides the theoretical researches of normal cloud model, it has been also used for solving many

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practical MADM problems, such as selection and evaluation of groundwater management schemes [7], sustainable supplier selection [13], and risk evaluation [14]. These existing studies have played an important role in the development of rational decision theory with uncertainty.

However, some behavioral experiments have been conducted to prove that DMs are not completely rational in numerous actual cases [15], [16]. Based on this consideration, some behavioral decision theories were developed for solving problems with bounded rationality being included, such as prospect theory (PT) [17], regret theory (RT) [18], [19], fairness theory [20], and disappointment theory [21]. These studies successfully take into account loss aversion and other psychological behaviors of DMs into the decision-making process. Compared to other behavioral decision theories which only consider DM's risk-averse preference, PT, which is proposed by Kahneman and Tversky, takes into account both risk-averse and risk-seeking preferences towards losses and gains [17]. In PT, the value function is assumed to be a S-shape curve, denoting the risk aversion of DMs in face of gains and the risk propensity in case of losses [17]. Thus, PT is more applicable in real-world applications [22], [23]. When applying PT in decision-makings, distance measure for normal cloud models is quite significant for disclosing the relevance between alternatives and reference points, and has been investigated in some studies, such as maximum-minimum-value based distance measurement [4], [7], [24], Hamming distance [25], arithmetic square root-based distance measurement [14], relative entropy distance measurement [23], cosine similarity measure [26], and fuzzy distance measure [27]. The measures reported in [4], [7], [24], [25] that are slightly different with each other consider that the expectation plays a leading role when measuring the distance between clouds, but the limitation is that they ignore the impact of random distribution whereas different distributions might lead to various distances. The measures reported in [14], [23], [26], [27] assume that expectation, entropy and hyper entropy have equal importance, and thus failed to highlight the leading role of expectation which is the most representative value for one alternative. Also, some of the measures [14], [23], [26], [27] do not reflect the intrinsic property of the random distribution. Moreover, in the above reported studies, in spite of its essential role in enhancing the distance measures of cloud models from various perspectives, no consideration was given to the influence of DM's risk attitudes either during the distance measure or throughout the decision-making process. For individuals, reality is a completely personal phenomenon with consideration of their needs, experience, personality traits, and subjective judgments [28]. Risk preferences of different DMs might differ from each other according to what they perceive to be reality. Thus, a role of DM's attitudinal characters towards risk, including risk aversion, risk seeking and risk neutrality, should be taken into account during decision-making procedures [29]. Especially for a normal cloud model involving in both fuzziness and randomness, DMs of different risk

attitudes are likely to have different preferences when facing the same cloud model. It is therefore taken into account DM's risk preferences when calculating cloud distance.

However, the above studies commonly depended on the existed performance (i.e., past and present performance) of alternatives to make an evaluation or selection, and did not consider their future potential development status which is quite important for collecting the information comprehensively and forming an accurate value judgement. Normally, comprehensive information is not merely the evaluation information for the past and present performance of one alternative, but also the pre-evaluation information for its relevant future performance [30]. Different from traditional prediction approaches which are only based on the past data of one alternative, pre-evaluation refers to as an advance evaluation for the future performance of one alternative through identifying and analyzing potential favorable and negative factors of one alternative's development according to some relevant basic materials collected and sorted [30]. It is one requirement of successful decision-makings, that is because: (a) it offers more helpful and explicit information for DMs to make an evaluation and selection; (b) it is the embodiment of grasping the development law and essence of things, and helps DMs have a definite object in view; (c) the pre-evaluation information might provide DMs a reference for choosing a long-term cooperative partner, and thus helps to reduce the follow-up selection cost. Things such as the planning of tactical policy and the establishment of strategic thinking illustrate the importance of pre-evaluation. A few researches have been directed at pre-evaluation-based decision-making methodologies, such as project bidding [30], environmental management partner selection [31], and environmental technologies for sustainable revitalization [32]. Therefore, pre-evaluation is fairly important for cloud model-based BMADM problems.

Regarding the previous researches, three challenges need to be addressed with respect to normal cloud model -based BMADM approaches (NC-BMADM). (a) Many distance measures have been investigated for cloud models from various aspects, some limitations still exist that should be compensated for by combing random distribution and DM's risk attitudes, thus, a novel cloud distance measure with wider application is still a challenging problem in actual applications. (b) The primary element of PT is the setting of reference points. DMs who hold different risk attitudes have different views on uncertainty and thus are inclined to choose different reference points. So, setting reference points from risk preference perspective, which is less taken into account in existing researches, still requires further investigation for cloud models. (c) Few of the aforementioned studies focus on the combination impact of evaluation and pre-evaluation information which often exists in real-world cases. How to aggregate evaluation and pre-evaluation information is therefore another important issue worthy of study.

Motivated by the aforementioned challenges, a NC-BMADM method is proposed by considering DM's

risk preferences and pre-evaluation information. To highlight characteristics of both random distribution and risk preferences, a novel distance measure between normal cloud models is investigated based on distribution function, risk preferences, and three numerical characteristics. To satisfy different requirements of DMs of different risk preferences, reference points are set by considering three risk preferences: risk neutrality, risk aversion, and risk seeking. Then, to increase the differentiation between alternatives, an optimization model is established for solving incomplete attribute weights based on maximum deviation theory. Those are also considered the main innovations of the proposed method.

The remainder of the paper is structured as follows. Section II introduces some relevant basic concepts, including prospect theory and cloud models. Section III presents the proposed method in detail. Section IV mainly discusses the case study in which the comparison and sensitivity analyses are also presented. Section V concludes the main contributions and shortcomings of the proposed method, and identifies possible areas for future research.

**II. PRELIMINARIES**

This section introduces some basic concepts related to prospect theory and cloud models, and describes the problem addressed in this article.

**A. PROSPECT THEORY**

Prospect theory (PT) [17] is a decision-making model of the descriptive paradigm. In PT, a value function can reflect the DM's attitudes towards risk and subjective preferences when faced with gains or losses. In the function, the difference between one attribute value and its corresponding reference point is used as a basis for decision-making instead of the absolute value, because the former is in line with the DM's mindset and the decision-making scenario. The function can be expressed as follows:

$$v(x) = \begin{cases} x^\alpha, & x \geq 0 \\ -\rho(-x)^\beta, & x < 0 \end{cases} \quad (1)$$

where  $x$  indicates the difference between the performance value and the reference point,  $\alpha$  and  $\beta$  indicates the difference between the performance value and the reference point, respectively. If  $0 < \alpha, \beta \leq 1$ , the greater  $\alpha$  or  $\beta$  value indicates the DM is more inclined to risk. Moreover,  $\rho$  denotes the loss-averse coefficient, and  $\rho > 1$  means that the DMs are more sensitive to the losses when losses are the same as gains, the greater  $\rho$  value means the more sensitive the DM is to the losses. Many researches have been carried out using the values  $\alpha, \beta$ , and  $\rho$ , and Tversky and Kahneman [33] found that when  $\alpha = \beta = 0.88$  and  $\rho = 2.25$ , the empirical results are more consistent with each other.

**B. NORMAL CLOUD MODELS**

Cloud model, proposed by Li *et al.* [6], is an uncertain transformation model between a qualitative concept expressed

by linguistic values and its quantitative representation. The model is utilized to reflect the uncertainty of concepts in natural language, i.e., fuzziness and randomness.

*Definition 1 [6]:* Let  $U$  be the quantitative universe of discourse, and  $C$  is a qualitative concept on the universe. If  $x \in U$ , and  $x$  is a random generation to  $C$ , its corresponding membership, denoted by  $\mu(x)$  ( $\mu(x) \in [0, 1]$ ), is a random number with stable tendency. Then  $\forall x \in U$  and  $x \rightarrow \mu(x)$ , the distribution of  $x$  on the domain is defined as a cloud, and one  $x$  denotes a droplet in the cloud.

Three numerical characteristics are utilized to represent one cloud model: (a) Expectation ( $Ex$ ) denotes the value in the universe corresponding to the centroid of the area covered by one cloud, it is the most representative value of one qualitative concept. (b) Entropy ( $En$ ) is a measurement for uncertainty of one qualitative concept. It can reflect the randomness of the qualitative concept in terms of dispersion degree and the degree of fuzziness which is used to depict the range of cloud droplets. (c) Hyper entropy ( $He$ ), a measure for entropy uncertainty, is determined by the randomness and fuzziness of the entropy.

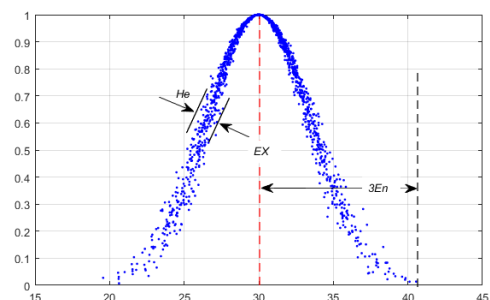
Normal cloud model is an important and common cloud model, which can be defined as follows.

*Definition 2 [6]:* Suppose that  $U$  is the quantitative universe of discourse, and  $C$  denotes a qualitative concept on  $U$ . If  $x \in U$  and  $x$  is a random generation to  $C$  that satisfies  $x \sim N(Ex, En^2)$  and  $En' \sim N(En, He^2)$ , then the degree of membership of  $x$  corresponding to  $C$  is expressed by

$$f(x) = e^{-\frac{(x-Ex)^2}{2En^2}},$$

and the distribution of  $x$  on  $U$  is defined as a normal cloud model.

According to 3En rule for clouds, 99.7% of the cloud droplets fall into the range of  $[Ex - 3En, Ex + 3En]$ , and the droplets falling outside of this range can be ignored [13]. Then based on forward and backward cloud generator algorithms [34], a qualitative concept with three numerical characters ( $Ex, En, He$ ) and cloud droplets can be transformed into each other. For example, supposing that  $C = (30, 3.5, 0.29)$ , the droplets of the cloud can be generated as in Fig.1.



**FIGURE 1.** Normal cloud generation of  $C = (30, 3.5, 0.29)$ .

If the information is provided by using linguistic terms, they can be converted into normal cloud models by using the golden ratio-based method or normal distribution-based method presented in [25]. When facing information in terms of values, some cloud drops (sample data) can be transformed into three numerical characters by using backward cloud generator algorithms [7] based on the following equations:

$$Ex = \frac{1}{n} \sum_{i=1}^n x_i, \tag{2}$$

$$En = \sqrt{\frac{\pi}{2}} \times \frac{1}{n} \sum_{i=1}^n |x_i - Ex|, \tag{3}$$

$$He = \sqrt{\left| \frac{1}{n-1} \sum_{i=1}^n (x_i - Ex)^2 - \frac{\pi}{2} \times \left( \frac{1}{n} \sum_{i=1}^n |x_i - Ex| \right)^2 \right|}. \tag{4}$$

**C. PROBLEM DESCRIPTION**

A MADM problem often involves multiple attributes and alternatives. Suppose that  $A = \{a_1, a_2, \dots, a_m\}$  ( $a_i \in A, i = 1, \dots, m$ ) denotes a set of alternatives. Each alternative is assessed with respect to a predefined attribute set  $C = \{c_1, c_2, \dots, c_n\}$  ( $c_j \in C, j = 1, \dots, n$ ), and the weight vector of attributes is assumed to be  $W = \{w_1, w_2, \dots, w_n\}$ , where  $w_j \in [0, 1]$ , and  $\sum_{j=1}^n w_j = 1$ . There are two kinds of initial information that are considered in the paper: evaluation and pre-evaluation information. Let  $C^E = (c_{ij}^E)_{mn}$  be the evaluation matrix related to alternative  $a_i$  with respect to attribute  $c_j$ , where  $(c_{ij}^E)_{mn} = (Ex_{ij}^E, En_{ij}^E, He_{ij}^E)$ . It is notable that if the information is provided in terms of linguistic terms or values, they can be transformed into the forms of normal cloud models by using the approach described in Section II-B, we do not discuss it in detail in the paper in order to highlight the key work of the proposed method.

Using the MADM framework as the basis, pre-evaluation and DM's risk attitude are further considered and a new NC-BMADM method is thereby generated. Suppose that  $C^P = (c_{ij}^P)_{mn}$  indicates pre-evaluation matrix associated with alternative  $a_i$  with respect to attribute  $c_j$ , where  $(c_{ij}^P)_{mn} = (Ex_{ij}^P, En_{ij}^P, He_{ij}^P)$ . In the NC-BMADM framework, evaluation information is a concentrated reflection of past and present performance of one alternative, and pre-evaluation information can indicate the future development trend of alternatives, which is a significant feature of the proposed method. In addition, DM's risk preference coefficient, indicated by  $\lambda$ , is also taken into account, which forms another important feature in the study.

**III. RISK-BASED PRE-EVALUATION AND EVALUATION INTEGRATION FOR NORMAL CLOUD MODELS**

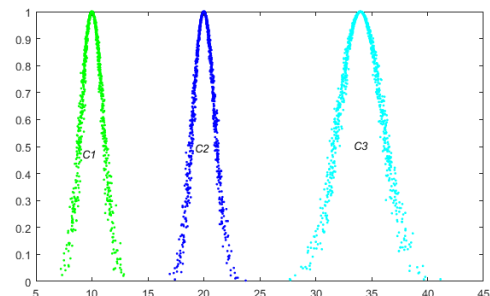
This section introduces the components of the proposed method in detail: risk-based distance measurement for cloud models, risk-based reference point setting for cloud models, prospect theory-based integration of evaluation and

pre-evaluation information, and an optimization model for incomplete attribute weights.

**A. RISK-BASED DISTANCE MEASUREMENT FOR CLOUD MODELS**

Information similarity or distance is an essential element of MADM problems, especially for behavioral decision-makings, because it might involve a variety of reference points, and similarity or distance measurement between alternatives and these reference points is quite useful for making an effective comparison and selection. An additional reason for measuring distance is to provide a reference for clustering when a MADM problem encompasses an extensive variety of data resources. Investigating an accurate method of distance measurement is thus a critically important component of solving BMADM problems. In contrast to the distance measurement associated with cloud models, distance not only includes expectation, entropy and hyper entropy, but also reflects the characteristic of randomness and DM's attitude to uncertainty.

A review of studies of cloud distance measures [7], [23], [26] reveals that some studies ignored the impact of randomness on distance. For example, through using Ren's method [7], distance that is obtained when cloud droplets obey normal distribution is the same as the one when the droplets obey other distributions, and thus reducing the decision-making accuracy to some extent. Wang's method [36] assumed that similarity between two clouds was zero when the intersection of droplets in two clouds was empty, but this is inappropriate in many real-world decision makings. For another example, supposing that two cloud models  $C_1 = (10, 1, 0.115)$  and  $C_2 = (20, 1, 0.1)$ , their corresponding reference point is  $C_3 = (34, 2, 0.195)$  (see Fig.2). The similarity between  $C_1$  and  $C_3$  is the same as the one between  $C_2$  and  $C_3$ , and both are equal to zero by using Wang's method [26]. Based on this result,  $C_1$  and  $C_2$  are hard to identify, but according to Fig.2, it is obvious that they have different distances with  $C_3$ , and thus can be differentiated. Additionally, the above methods omit consideration of DM' risk attitudes towards uncertainty. DM's behavior under risk, referring to as three kinds of attitudes: risk aversion, risk neutrality, and risk seeking, has great impact on decision



**FIGURE 2.** An example of comparing three normal cloud models.

results in many real-world cases, and thus should be taken into account in the distance measurement.

Based on the  $3En$  rule,  $En'$  belongs to the range of  $[En - 3He, En + 3He]$ . For DMs with different risk attitudes, the ranges of  $En'$  are quite different [29]. According to the idea reported by Chu *et al.* [29], risk-seeking DMs are inclined to accept a larger  $En'$  within  $[En - 3He, En + 3He]$  compared with risk-averse DMs, and the greater risk preference indicates a larger value of  $En'$ , whereas it is opposite for risk-averse DMs. With this idea in mind, a risk-based entropy is defined by considering hyper entropy as follows

$$En' = En - 3\lambda \times He, \tag{5}$$

where  $\lambda$  ( $-1 \leq \lambda \leq 1$ ) is the risk preference coefficient,  $-1 \leq \lambda < 0$  denotes risk-seeking situation,  $\lambda = 0$  indicates risk-neutral situation, and  $0 < \lambda \leq 1$  expresses risk-averse situation. When collecting the information on  $\lambda$  values, an easy and direct way is to incorporate into surveys completed by DMs questions that ask them to subjectively indicate their attitude towards risks. This type of operation has also been utilized in the study reported in [29]. Then, according to the meaning of entropy  $En'$ , there should be  $En' \geq 0$ , i.e.,  $En - 3\lambda \times He \geq 0$ . To satisfy this requirement,  $\lambda$  is limited to the range of  $[-1, \min\{1, En/(3He)\}]$ , and then we have  $En - 3 \min\{1, En/(3He)\} \times He \leq En' \leq En - 3He$  according to (5). However, there might exist  $En' = 0$  when  $0 < \lambda \leq \min\{1, En/(3He)\}$ , the randomness is therefore eliminated, then we take  $E(C) = Ex$ . When  $En' = En - 3\lambda \times He > 0$ , there exists randomness and based on Definition 2, we have

$$f(x) = e^{-\frac{(x-Ex)^2}{2En'^2}}, \tag{6}$$

where  $f^{\min} = e^{-\frac{(x-Ex)^2}{2[En-3 \min\{1, En/(3He)\} \times He]^2}}$  and  $f^{\max} = e^{-\frac{(x-Ex)^2}{2(En+3He)^2}}$  are the minimum and maximum expectation curves, respectively. When  $\lambda = 0$ ,  $f(x) = f^N = e^{-\frac{(x-Ex)^2}{2En^2}}$  indicates the risk-neutral expectation curve; when  $-1 \leq \lambda < 0$ ,  $f(x) \in (f^N, f^{\max})$  indicates the risk-seeking expectation curve; and when  $0 < \lambda \leq \min\{1, En/(3He)\}$ ,  $f(x) \in (f^{\min}, f^N)$  indicates the risk-averse expectation curve. For example, supposing that  $C_1 = (15, 1, 0.15)$ , then expectation curves of  $C_1$  are obtained with consideration of DM's risk attitudes, see Fig.3, where EC indicates the expectation curve, RA means risk aversion, RN represents risk neutrality, RS denotes risk seeking.

Then based on the above consideration, risk-based comprehensive expectation of one cloud model can be defined as

$$E(C) = \int_{-\infty}^{+\infty} xf(x)dx = \int_{-\infty}^{+\infty} (xe^{-\frac{(x-Ex)^2}{2En'^2}})dx, \tag{7}$$

where (7) is integrable which can be proved by the following Lemma 1.

*Lemma 1:*  $E(C)$  is integrable and equal to  $\sqrt{2\pi} \times Ex \times (En - 3\lambda \times He)$ .

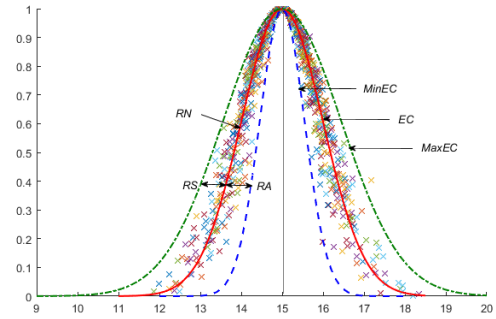


FIGURE 3. Normal cloud model with consideration of risk preference coefficient.

*Proof:* The proof of Lemma 1 is seen in Section A of Appendix.

Equation (7) is the embodiment of the randomness, DM's risk attitude, and expectation of one cloud, which can achieve the effective use of information. Based on the equation, suppose that there are  $N$  ( $N \geq 2$ ) clouds involved in a comparison, and that two clouds are denoted by  $C_i = (Ex_i, En_i, He_i)$  and  $C_k = (Ex_k, En_k, He_k)$ , where  $i \in N, k \in N$  and  $i \neq k$ , then the risk-based comprehensive expectation distance between two clouds is defined as

$$D^E(C_i \| C_k) = \begin{cases} 0, & \max_{i \in N} E(C_i) = \min_{i \in N} E(C_i) \\ \frac{|E(C_i) - E(C_k)|}{\max_{i \in N} E(C_i) - \min_{i \in N} E(C_i)}, & \max_{i \in N} E(C_i) > \min_{i \in N} E(C_i) \end{cases} \tag{8}$$

where (8) determines the distance from the relative perspective, which is relevant to the number of cloud models involved in the comparison, aiming at ensuring that  $0 \leq D^E(C_i \| C_k) \leq 1$ . That is because for different groups of cloud models, the maximum and minimum comprehensive expectations might be different. In order to better deal with various actual cases, the distance obtained in (8) is applicable for all cases in real-world applications due to its range of  $[0, 1]$ , but also is distinguishable when facing different  $N$  values. In addition, the distance has three properties which are discussed in Lemma 2, this also conforms to the idea reported by Xu and Xia [35].

*Lemma 2:* (1)  $0 \leq D^E(C_i \| C_k) \leq 1$ , and if and only if  $E(C_i) = E(C_k)$ ,  $D^E(C_i \| C_k) = 0$ ; (2)  $D^E(C_i \| C_k) = D^E(C_k \| C_i)$ ; (3)  $D^E(C_i \| C_i) = 0$ .

*Proof:* The proof of Lemma 2 is seen in Section B of Appendix.

The distance in (8) is insufficient to differentiate two clouds. For instance, supposing that two clouds  $C_1 = (15, 4.15, 0.1)$  and  $C_2 = (15, 4.3, 0.2)$ , and that  $\lambda = 0.5$ , then according to (8),  $E(C_1) = E(C_2)$ , but it is obvious that two clouds are different due to the different shapes. Thus, the shape difference of two clouds should also be taken into consideration when making a comparison. With this idea in

mind, the shape distance is defined as follows:

$$D^S(C_i \parallel C_k) = \frac{1}{2} \left[ \frac{|En_i - En_k|}{\max_{i \in N} En_i - \min_{i \in N} En_i} + \frac{|He_i - He_k|}{\max_{i \in N} He_i - \min_{i \in N} He_i} \right]. \quad (9)$$

where in (9), if  $\max_{i \in N} En_i = \min_{i \in N} En_i$  or  $\max_{i \in N} He_i = \min_{i \in N} He_i$ , it is assumed to be  $\frac{|En_i - En_k|}{\max_{i \in N} En_i - \min_{i \in N} En_i} = 0$  or  $\frac{|He_i - He_k|}{\max_{i \in N} He_i - \min_{i \in N} He_i} = 0$ . Furthermore, the same as the idea illustrated in (8), (9) is also the relative distance which has three properties.

**Lemma 3:** (1)  $0 \leq D^S(C_i \parallel C_k) \leq 1$ , and if and only if  $En_i = En_k$  and  $He_i = He_k$ ,  $D^S(C_i \parallel C_k) = 0$ ; (2)  $D^S(C_i \parallel C_k) = D^S(C_k \parallel C_i)$ ; (3)  $D^S(C_i \parallel C_i) = 0$ .

*Proof:* The proof of Lemma 3 is seen in Section C of Appendix.

With consideration of two kinds of distances in (8) and (9), we define risk-based distance between two clouds as follows:

$$D(C_i \parallel C_k) = \frac{1}{N} D^S(C_i \parallel C_k) + (1 - \frac{1}{N}) D^E(C_i \parallel C_k), \quad (10)$$

where  $1/N$  denotes the importance degree of shape distance, which is determined by the number of clouds.  $N$  takes an important role in determining the importance degree, and the shape distance will account for the smaller proportion with the increase of  $N$ . That conforms to real operations. Expectation, the most representative value of one alternative, is the first reference that DMs pay attention to when evaluating alternatives [7]. If expectation of one alternative has a huge gap with others, DMs are not inclined to choosing it even when its distance is quite close to others. Shape distance is essentially used to increase the discrimination among alternatives. Moreover, fewer cloud models could lead to larger expectation distance between two clouds even when their expectation values are quite close, then shape distance, occupying greater proportion than that with more cloud models, can balance the differences when two cloud models have a similar shape. For example, if there are two clouds which are denoted by  $C_1 = (15, 4.15, 0.1)$  and  $C_2 = (14, 4.15, 0.1)$ , then  $D^E(C_1 \parallel C_2) = 1$  and  $D(C_1 \parallel C_2) = 1/2$ ; if suppose that three clouds are involved, e.g.,  $C_1 = (15, 4.15, 0.1)$ ,  $C_2 = (14, 4.15, 0.1)$ , and  $C_3 = (15, 4.15, 0.1)$ , then  $D^E(C_1 \parallel C_2) = 1$  and  $D(C_1 \parallel C_2) = 2/3$ . It is obvious to obtain that  $C_1$  and  $C_2$  have greater differentiation with the increase of the number of clouds, making the decision-making more effective. In addition,  $D(C_i \parallel C_k)$  also has three properties which can obviously differentiate two clouds, see Lemma 4.

**Lemma 4:** (1)  $0 \leq D(C_i \parallel C_k) \leq 1$ , and if and only if  $Ex_i = Ex_k$ ,  $En_i = En_k$  and  $He_i = He_k$ ,  $D(C_i \parallel C_k) = 0$ ; (2)  $D(C_i \parallel C_k) = D(C_k \parallel C_i)$ ; (3)  $D(C_i \parallel C_i) = 0$ .

*Proof:* The proof of Lemma 4 is seen in Section D of Appendix.

**B. PROSPECT THEORY -BASED INTEGRATION OF EVALUATION AND PRE-EVALUATION INFORMATION**

Prospect theory (PT) is a very useful tool for addressing uncertain decision-making problems, as it takes into account

both risk-seeking and risk-averse risk attitudes when facing gains and losses. Moreover, it is also appropriate for the proposed method in which DM's risk preference coefficient is taken into consideration. In line with the framework of uncertain BMADM problems, reference points should be considered as a part of decision-making processes, because absolute values cannot be utilized simply for expressing the advantages and disadvantages of alternatives without comparisons. Through setting the reference points, DMs can acquire better knowledge of the detailed status of alternatives. In general, positive ideal points are helpful for reflecting the gap between one alternative and other external competitors, and thus for illustrating the competitive advantages of the outward attributes [22]. Moreover, the key part of applying PT is to determine the positive or negative distance between alternatives and reference points. However, based on the distance in (10), it is difficult to directly detect the positive or negative direction of the distance. This can be solved by setting positive ideal reference which can be considered the alternative of best performance, and accordingly the distance between each alternative and the positive ideal reference is considered to be negative. For these reasons, the positive ideal point is considered to be the criterion for alternatives' comparisons.

When setting positive ideal reference point for cloud models which contain two kinds of uncertainties: entropy and hyper entropy, the attitudes of DMs should be considered in the face of uncertainty. Risk-seeking DMs are inclined to accept a wider range of uncertainty as they are unwilling to miss any chance for gains, and thus choose a higher entropy and hyper entropy as the reference, whereas risk-averse DMs hold opposite opinions [29]. For risk-neutral DMs, they intend to pay much attention on the average level of uncertainty. Additionally, whether for DMs of any type of risk, they are inclined to choose the alternative of better performance, i.e., larger expectation associated with benefit attribute, lower expectation value with respect to cost attributes. Thus, based on the above considerations, we consider the setting of positive ideal points for both evaluation and pre-evaluation information from the following aspects:

(a) For risk-neutral DMs ( $\lambda = 0$ ), the positive ideal point is defined as:

$$\begin{aligned} R_{jB}^N &= (Ex_{jB}^{RN}, En_{jB}^{RN}, He_{jB}^{RN}) = (\max\{\max_i Ex_{ijB}^E, \max_i Ex_{ijB}^P\}, \\ &\quad \frac{1}{2m} \sum_{i=1}^m (En_{ijB}^E + En_{ijB}^P), \\ &\quad \frac{1}{2m} \sum_{i=1}^m (He_{ijB}^E + He_{ijB}^P)), \\ R_{jC}^N &= (Ex_{jC}^{RN}, En_{jC}^{RN}, He_{jC}^{RN}) = (\min\{\min_i Ex_{ijC}^E, \min_i Ex_{ijC}^P\}, \\ &\quad \frac{1}{2m} \sum_{i=1}^m (En_{ijC}^E + En_{ijC}^P), \\ &\quad \frac{1}{2m} \sum_{i=1}^m (He_{ijC}^E + He_{ijC}^P)). \end{aligned} \quad (11)$$

where  $C^B$  ( $j_B \in C^B$ ) and  $C^C$  ( $j_C \in C^C$ ) denote the benefit attribute set and the cost attribute set, respectively, and the reference setting for entropy and hyper entropy is developed from the average perspective.

(b) For risk-averse DMs ( $0 < \lambda \leq 1$ ), the positive ideal point is defined as:

$$\begin{aligned}
 R_{j_B}^A &= (Ex_{ij_B}^{RA}, En_{ij_B}^{RA}, He_{ij_B}^{RA}) = (\max\{\max_i Ex_{ij_B}^E, \max_i Ex_{ij_B}^P\}, \\
 &\quad \lambda \min\{\min_i En_{ij_B}^E, \min_i En_{ij_B}^P\} + (1 - \lambda) \\
 &\quad En_{ij_B}^{RN}, \lambda \min\{\min_i He_{ij_B}^E, \min_i He_{ij_B}^P\} + (1 - \lambda)He_{ij_B}^{RN}), \\
 R_{j_C}^A &= (Ex_{ij_C}^{RA}, En_{ij_C}^{RA}, He_{ij_C}^{RA}) = (\min\{\min_i Ex_{ij_C}^E, \min_i Ex_{ij_C}^P\}, \\
 &\quad \lambda \min\{\min_i En_{ij_C}^E, \min_i En_{ij_C}^P\} + (1 - \lambda) \\
 &\quad En_{ij_C}^{RN}, \lambda \min\{\min_i He_{ij_C}^E, \min_i He_{ij_C}^P\} + (1 - \lambda)He_{ij_C}^{RN}).
 \end{aligned} \tag{12}$$

where reference entropy and hyper entropy are determined by risk preference coefficient  $\lambda$ . They become increasingly smaller with the increase of  $\lambda$ . That means when  $\lambda$  is closer to 0, reference entropy and hyper entropy become closer to the average level, and complete risk averters choose the lowest uncertainty. This conforms to the cognition in real-world operations.

(c) For risk-seeking DMs ( $-1 \leq \lambda < 0$ ), the positive ideal point is defined as:

$$\begin{aligned}
 R_{j_B}^S &= (Ex_{ij_B}^{RS}, En_{ij_B}^{RS}, He_{ij_B}^{RS}) = (\max\{\max_i Ex_{ij_B}^E, \\
 &\quad \max_i Ex_{ij_B}^P\}, -\lambda \max\{\max_i En_{ij_B}^E, \max_i En_{ij_B}^P\} + (1 + \lambda) \\
 &\quad En_{ij_B}^{RN}, -\lambda \max\{\max_i He_{ij_B}^E, \max_i He_{ij_B}^P\} \\
 &\quad + (1 + \lambda)He_{ij_B}^{RN}), \\
 R_{j_C}^S &= (Ex_{ij_C}^{RS}, En_{ij_C}^{RS}, He_{ij_C}^{RS}) = (\min\{\min_i Ex_{ij_C}^E, \\
 &\quad \min_i Ex_{ij_C}^P\}, -\lambda \max\{\max_i En_{ij_C}^E, \max_i En_{ij_C}^P\} + (1 + \lambda) \\
 &\quad En_{ij_C}^{RN}, -\lambda \max\{\max_i He_{ij_C}^E, \max_i He_{ij_C}^P\} \\
 &\quad + (1 + \lambda)He_{ij_C}^{RN}).
 \end{aligned} \tag{13}$$

where the setting method for reference entropy and hyper entropy is opposite to the one for risk-averse DMs. It is notable that the reference points are set by considering both evaluation and pre-evaluation information from a dynamic perspective rather than a separate perspective. Because it can reflect the development status of alternatives from the past to the future, and thus be helpful for making a vertical comparison. Thus, the positive ideal points setting above takes into account both external competition and interval development comparison. Suppose that three cloud models with respect to a benefit attribute are denoted by  $C_1 = (15, 1, 0.15)$ ,  $C_2 = (20, 2, 0.3)$ , and  $C_3 = (25, 1.5, 0.2)$ , then  $R = (25, 1.5, 0.217)$  when  $\lambda = 0$ ,  $R = (25, 1, 0.15)$  when  $\lambda = 1$ , and  $R = (25, 2, 0.3)$  when  $\lambda = -1$ , see Fig.4.

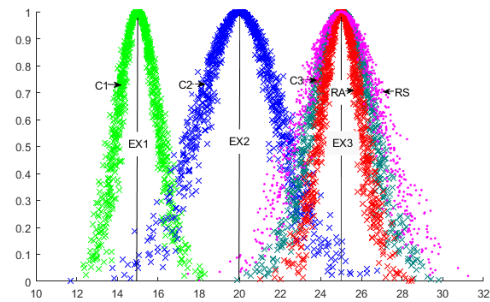


FIGURE 4. Reference setting for cloud models with respect to benefit attributes.

Suppose that  $C_{ij}^E = (Ex_{ij}^E, En_{ij}^E, He_{ij}^E)$  and  $C_{ij}^P = (Ex_{ij}^P, En_{ij}^P, He_{ij}^P)$  indicate the evaluation and pre-evaluation of alternative  $a_i$  with respect to attribute  $c_j$ , respectively. Through using (7),  $E(C_{ij}^E)$ ,  $E(C_{ij}^P)$ , and  $E(R_j)$  can be obtained. Then, according to (8)-(10), the risk-based distance between each kind of information and reference point is calculated as:

$$\begin{aligned}
 D(C_{ij}^E \parallel R_j) &= \frac{1}{2m + 1} D^S(C_{ij}^E \parallel R_j) \\
 &\quad + (1 - \frac{1}{2m + 1}) D^E(C_{ij}^E \parallel R_j),
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 D(C_{ij}^P \parallel R_j) &= \frac{1}{2m + 1} D^S(C_{ij}^P \parallel R_j) \\
 &\quad + (1 - \frac{1}{2m + 1}) D^E(C_{ij}^P \parallel R_j),
 \end{aligned} \tag{15}$$

where  $N = 2m + 1$  contains  $m$  evaluation cloud models,  $m$  pre-evaluation cloud models and one reference point, that means the maximum and minimum values involved in (14) and (15) are obtained with consideration of  $C_{ij}^E$ ,  $C_{ij}^P$  and  $R_j$ . For example, the maximum expectation is expressed as  $\max\{\max_i E(C_{ij}^E), \max_i E(C_{ij}^P), E(R_j)\}$ . Then the prospect values with respect to evaluation and pre-evaluation information can be calculated by using (1), respectively.

$$v_{ij}^E = -\rho(D(C_{ij}^E \parallel R_j))^\beta, \tag{16}$$

$$v_{ij}^P = -\rho(D(C_{ij}^P \parallel R_j))^\beta. \tag{17}$$

It is notable that (16) and (17) only refer to one situation because the reference point is the positive ideal point which is considered the optimum value.

Considering the above two kinds of values  $v_{ij}^E$  and  $v_{ij}^P$ , the comprehensive value of alternative  $a_i$  with respect to attribute  $c_j$  can be calculated as

$$v_{ij} = \tau v_{ij}^E + (1 - \tau) v_{ij}^P, \tag{18}$$

where  $\tau$  represents the preference coefficient for evaluation value, and  $\tau \in [0, 1]$ . When  $\tau > 0.5$ , the importance that the evaluation information accounts for is higher than pre-evaluation information for DMs, so the result is more in line with evaluated performance of alternatives; when  $\tau < 0.5$ , the pre-evaluation information is more emphasized; when there is no special preference difference,  $\tau = 0.5$ , denoting that they are considered to have equal importance.

**C. AN OPTIMIZATION MODEL FOR INCOMPLETE ATTRIBUTE WEIGHT**

In a MADM problem, attribute weight is crucial to aggregate the information of decision-makings, which should not only reflect the DM’s subjective judgments but also adequately represent the information involved in the decision-making. In terms of determining the incomplete attribute weights, various methods have been developed, such as trapezoidal fuzzy neutrosophic entropy-based [37] and similarity degrees-based [38] weight determination approaches for completely unknown attribute weights, relative closeness-based [39] and group satisfaction-based [40] linear programming models, Best-Worst Method [2], and maximum deviation method [36] for incompletely known attribute weights. Here, we only discuss decision-makings with incompletely known attribute weights. Compared with other methods [2], [39], [40], an optimization model with the maximum deviation [36], aiming at sorting the alternatives by the weight of each attribute, is more effective for sorting alternatives with certain distinction degree. The greater contribution of one attribute on the summation of weighted values’ deviations of the alternatives indicates the greater importance of the attribute, and thus a higher weight value is assigned, and vice versa [22]. For that reason, the attribute-weighting model in this study has been built by using the maximum deviation method.

Suppose that a set of attribute weights is  $W = \{w_1, w_2, \dots, w_n\}$ , then the weighted comprehensive prospect value of each alternative is expressed as:

$$v_i = \sum_{j=1}^n w_j v_{ij}, \tag{19}$$

where  $v_{ij}$  is obtained by using (18).

Then, with the idea of the maximum deviation method [36] as a basis, a model aimed at optimizing the weights of attributes and maximizing the deviation values of  $v_i$  is established by combining the prior subjective information with objective decision-making information, the objective function and constraints can be summarized as follows:

$$\begin{aligned} & \max \left[ \frac{1}{m(m-1)} \sum_{i=1}^m \sum_{k=1, k \neq i}^m |v_i - v_k| \right] \\ & = \max \left[ \frac{1}{m(m-1)} \sum_{i=1}^m \sum_{k=1, k \neq i}^m \left| \sum_{j=1}^n w_j (v_{ij} - v_{kj}) \right| \right] \\ & \text{s.t. } \begin{cases} w_j^L \leq w_j \leq w_j^U, \\ \sum_{j=1}^n w_j = 1. \end{cases} \end{aligned} \tag{M1}$$

where  $[w_j^L, w_j^U]$  represents prior information regarding the attribute weight,  $\sum_{j=1}^n w_j^L \leq 1$ ,  $\sum_{j=1}^n w_j^U \geq 1$ , and  $0 \leq w_j^L \leq w_j \leq w_j^U \leq 1$ . In order to effectively apply the local search approach to achieve an optimum solution, the objective function in M1 should be firstly transformed into the following

one to eliminate the absolute value sign:

$$\begin{aligned} & \max \sum_{i=1}^m \sum_{k=1, k \neq i}^m \left[ \sum_{j=1}^n w_j (v_{ij} - v_{kj}) \right]^2 \\ & \text{s.t. } \begin{cases} w_j^L \leq w_j \leq w_j^U, \\ \sum_{j=1}^n w_j = 1. \end{cases} \end{aligned} \tag{M2}$$

*Theorem 1:* An optimal solution exists in M2.

*Proof:* Let the feasible region of M2 be denoted as  $\Omega = \{w_j = \{w_1, \dots, w_n\} | w_j^L \leq w_j \leq w_j^U, \sum_{j=1}^n w_j = 1\}$ . it is not difficult to find that the constraints in  $\Omega$  are bounded and non-empty.  $\Omega$  is thus a non-empty and bounded closed region. Furthermore, it is not difficult to find that the objective function of M2 is a continuous function on region  $\Omega$ . Thus, the objective function must attain a maximum according to the extreme value theorem of multivariate functions [37]. Therefore, the optimal solution exists in M2.

Through solving M2, the weight of each attribute is obtained. Then through using (19), the weighted comprehensive value of alternative  $a_i$  is calculated as  $v_i$ . Accordingly, the ranking of alternatives can be obtained by comparing  $v_i$  values.

The detailed steps of the proposed method can be summarized as follows:

**Step 1:** Prepare the information required for the proposed method, such as  $c_{ij}^E = (Ex_{ij}^E, En_{ij}^E, He_{ij}^E)$ ,  $c_{ij}^P = (Ex_{ij}^P, En_{ij}^P, He_{ij}^P)$ ,  $w_j^L, w_j^U, \lambda$ , and  $\tau$ .

**Step 2:** Determine the distance between the alternatives’ values and reference points. According to the  $\lambda$  value, one formula should be detected from (11), (12) and (13) to determine the positive ideal reference points. Then by using (14) and (15), the risk-based distance between the evaluation or pre-evaluation information and the reference point can be obtained as a basis of applying PT.

**Step 3:** Generate the comprehensive value of alternatives associated with each attribute based on both evaluation and pre-evaluation information. Through utilizing (16) and (17), prospect values associated with both two kinds of information are obtained, following which is the calculation of the comprehensive value of each alternative by combining the prospect values of evaluation and pre-evaluation information, which can be obtained by using (18).

**Step 4:** Optimize the attribute weights. According to model M1 and the prior information of the attribute weights, the optimized weights of attributes can be determined as a part of making a final decision.

**Step 5:** Make a selection among alternatives. Based on (19), the weighted comprehensive value of alternative  $a_i$  is calculated as  $v_i$ . For two alternatives  $a_i$  and  $a_k$ , if  $v_i > v_k$ ,  $a_i > a_k$ ; if  $v_i = v_k$ ,  $a_i \equiv a_k$ ; if  $v_i < v_k$ ,  $a_i < a_k$ . The ranked order of alternatives can be therefore obtained to make a final selection.



#### IV. CASE STUDY

In this section, we present a case study conducted for the purposes of demonstrating the feasibility and rationality of the proposed method. To highlight its advantages, the proposed approach is also compared with two existing methods.

##### A. BACKGROUND DESCRIPTION

Catalytic converters are a kind of device installed in the exhaust system of a motor vehicle. It can reduce the emission of harmful gases while reducing the power loss of the engine, so as to achieve the goal of energy conservation and environmental protection. A good catalytic converter should have low ignition temperature, high conversion efficiency, and wide three-way air-fuel ratio work window. In this regard, the construction project of catalyst industrialization of new vehicle environmental protection is quite necessary because it can promote the sustainable development of the national economy and conform to the basic national policy of protecting the environment. In a case, a motor vehicle institute intends to invest one construction project of catalysts which is required to be selected from four candidates (indicated by  $a_1, a_2, a_3,$  and  $a_4$ ).

In general, indexes are the premise of making an evaluation or selection. With consideration of intra-industry evaluation criteria, the selection indexes mainly involve four aspects: construction scale, process technology, device configuration, and construction condition, which are denoted by  $c_1, c_2, c_3,$  and  $c_4,$  respectively. Construction scale mainly considers the construction area and fixed investment of one project. In order to easily quantify, we utilize cost to represent the construction scale. Process technology reflects the comprehensive technology level of one product, including in-engine purification and out-engine purification technologies. On the premise of meeting the production scale and ensuring the product quality, device configuration can calculate and equip the main process equipment by combining the characteristics such as advancement, practicability, and economical efficiency. Construction condition reflects the integrated environment of one project, involving the geographic position, meteorological condition, traffic condition, and environmental protection condition, and the smooth progress of one project must depend on a favorable construction condition. Thus, four projects above are evaluated based on these four aspects. Moreover,  $c_1$  is a quantitative and cost attribute, whereas  $c_2, c_3$  and  $c_4$  are qualitative and benefit attributes. Accordingly,  $c_1$  can be depicted by using a numerical number,  $c_2, c_3$  and  $c_4$  are quantified in the range of 0 – 10. The industry allocates attribute weights as follows:  $c_1$  generally accounts for 25% – 35%,  $c_2$  accounts for 25% – 30%,  $c_3$  accounts for 15% – 20%, and  $c_4$  generally accounts for 20% – 25%.

In this case, DMs belong to the risk-averse type, then we take  $\lambda > 0$ . Moreover, they do not express their preference coefficient explicitly, so we consider that they have a moderate risk-averse preference, i.e.,  $\lambda = 0.5$ . In order to decrease the risk, the project belongs to the strategic

planning. So, they also need to know future adjustment and development of alternatives with respect to each attribute. To solve this issue, they ask the industry professional to pre-evaluate the possible future development of projects. Accordingly, the decision-making considers both evaluation and pre-evaluation information in order to make a comprehensive acquaintance of each alternative. The evaluation information is an appraisal for the current performance of projects with respect to attributes, and pre-evaluation is an appraisal for the possible future development trend of projects for which some adjustments might be made. Two kinds of information both involve in uncertainty and randomness, and thus can be transformed by using normal cloud models, making the information expressions more flexible and accurate. Thus, the information in the case is expressed in forms of normal cloud models after being transferred. The detailed information of alternatives with respect to attributes is presented in Table 1, where EI denotes the evaluation information and PEI indicates the pre-evaluation information.

TABLE 1. Initial information of the case.

	$c_1$ (million)	$c_2$	$c_3$	$c_4$
$a_1$ EI	(99.71,5.208,0.298)	(6.2,0.985,0.121)	(7.3,0.977,0.111)	(7.8,1.153,0.142)
$a_1$ PEI	(95.43,5.314,0.357)	(8.7,0.995,0.152)	(8.9,0.998,0.212)	(8.9,1.211,0.152)
$a_2$ EI	(101.95,5.211,0.298)	(7.6,0.883,0.119)	(7.8,0.961,0.124)	(8.1,1.174,0.137)
$a_2$ PEI	(97.34,5.718,0.377)	(6.3,0.999,0.158)	(8.3,1.031,0.211)	(8.5,1.223,0.168)
$a_3$ EI	(105.92,6.425,0.398)	(2.3,0.919,0.133)	(5.1,0.988,0.211)	(5.9,1.444,0.222)
$a_3$ PEI	(102.37,7.613,0.423)	(3.4,1.013,0.188)	(6.7,1.017,0.241)	(6.5,1.515,0.243)
$a_4$ EI	(102.96,5.712,0.289)	(6.2,0.991,0.121)	(7.1,0.988,0.119)	(7.2,1.145,0.133)
$a_4$ PEI	(99.37,6.003,0.357)	(7.5,0.995,0.152)	(7.9,1.032,0.199)	(7.9,1.199,0.221)

##### B. COMPUTATION PROCESS AND ANALYSIS OF THE RESULTS

Based on the proposed method, we solved this case by applying the following five steps. Some solutions were obtained using the MATLAB optimization toolbox.

**Step 1:** In the case, the evaluation information and pre-evaluation information are provided in Table 1. Prior information on weights of attributes is denoted by  $w_1 \in [0.25, 0.35], w_2 \in [0.25, 0.3], w_3 \in [0.15, 0.2],$  and  $w_4 \in [0.2, 0.25],$  and  $\lambda = 0.5$ .

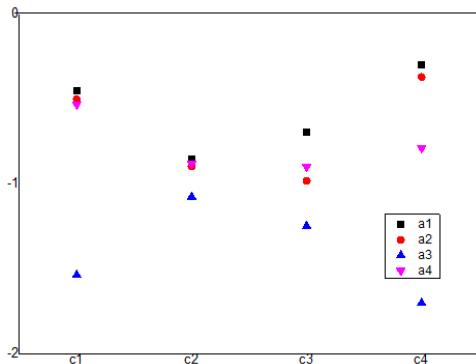
**Step 2:** Equation (12) is utilized to determine the ideal positive reference points because of  $\lambda = 0.5,$  i.e.,  $R_1^A = (95.43, 5.554, 0.319), R_2^A = (8.7, 0.928, 0.131), R_3^A = (8.9, 0.98, 0.145),$  and  $R_4^A = (8.9, 1.202, 0.155).$  Then by using (14) and (15), the risk-based distance can be obtained, see Table 2.

It can be seen from Table 2 that positive ideal points setting can reveal the development status of alternatives and thus is useful for making vertical and horizontal comparisons. Take alternatives  $a_1$  and  $a_2$  with respect to attribute  $c_1$  as an example, from the vertical perspective,  $a_1$  has a poor development trend because of  $0.139 < 0.182,$  that is because higher uncertainty is involved in the PEI of  $a_1$  associated with  $c_1;$  based on the horizontal comparison,  $a_1$  has a better development prospect because of  $0.182 < 0.229.$

**TABLE 2.** Results of risk-based distance between the information and reference points.

		c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>
a <sub>1</sub>	EI	0.139	0.291	0.163	0.177
	PEI	0.182	0.372	0.367	0.03
a <sub>2</sub>	EI	0.135	0.253	0.214	0.146
	PEI	0.229	0.452	0.574	0.111
a <sub>3</sub>	EI	0.479	0.147	0.388	0.666
	PEI	0.823	0.744	0.64	0.79
a <sub>4</sub>	EI	0.155	0.31	0.173	0.249
	PEI	0.233	0.379	0.544	0.359

**Step 3:** Based on the distances obtained in the above step, the prospect values of alternatives with respect to attributes are obtained for the evaluation and pre-evaluation information by utilizing (16) and (17). In the case, there is no special preference for both two kinds of information, we have  $\tau = 0.5$ , following which, the comprehensive values of alternatives with respect to each attribute are obtained, see Fig.5, where X-axis represents attributes, and Y-axis indicates their corresponding prospect values. It is evident from the figure that there are two features of the prospect values associated with alternatives with respect to each attribute: (a) the prospect values are all negative, this is because the reference points are set by using positive idea points, its merit lies that the direction of distance is not required to be firstly detected, which eliminates the bias brought by subjective judgements; (b) the rankings of alternatives are different under different attributes, illustrating that an appropriate approach is required for aggregating these values, and thus the following step is used to determine the attribute weights.



**FIGURE 5.** Comprehensive values of alternatives with respect to each attribute.

**Step 4:** According to M1, the weights of attributes are obtained as  $w_1 = 0.35$ ,  $w_2 = 0.25$ ,  $w_3 = 0.15$ , and  $w_4 = 0.25$ . That illustrates that cost plays the most important role in evaluating one alternative, whereas device configuration accounts for the minimum importance. Thus, if alternatives want to take place in the relevant market, more attention should be paid to reduce the cost under the condition of limited resources.

**Step 5:** Through using (19), the weighted comprehensive values of alternatives are obtained in Table 3. The ranking of alternatives is  $a_1 > a_2 > a_4 > a_3$ , and the best alternative is  $a_1$  which should be selected.

**TABLE 3.** Weighted comprehensive values of alternatives.

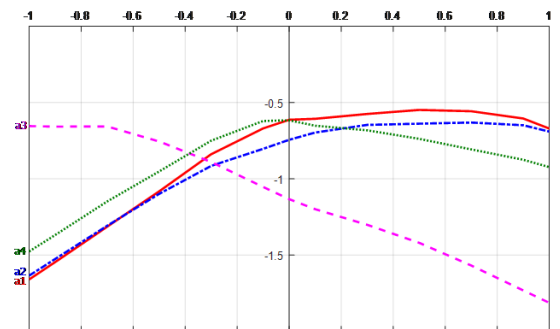
	a <sub>1</sub>	a <sub>2</sub>	a <sub>3</sub>	a <sub>4</sub>	SD	Ranking
EI	-0.519	-0.493	-1.056	-0.591	0.229	$a_2 > a_1 > a_4 > a_3$
PEI	-0.577	-0.784	-1.782	-0.884	0.461	$a_1 > a_2 > a_4 > a_3$
EI+PEI	-0.548	-0.638	-1.419	-0.737	0.344	$a_1 > a_2 > a_4 > a_3$

SD=Standard deviation

Moreover, the ranking of alternatives is  $a_2 > a_1 > a_4 > a_3$  if not considering the pre-evaluation information, which has neglected the future development trend and the adjustments made for alternatives, and thus might result in decision failure. For example, the weather in last week was clear, it does not mean tomorrow will be fine. Thus, pre-evaluation information is an important reference which should be rendered when reaching a decision. In addition, it is obvious that the consideration of both EI and PEI can balance the standard deviation of alternatives and thus reduce the bias that might be brought by utilizing a single information.

**C. SENSITIVITY ANALYSIS**

To find the effect of different risk preference coefficients on the ranking of alternatives, let  $\lambda$  take different values within the range of  $[-1, 1]$ , then the weighted comprehensive values ( $v_i$ ) of alternatives can be obtained under different  $\lambda$  values, see Fig.6, where X-axis represents  $\lambda$  values, and Y-axis indicates  $v_i$  of alternatives. Larger  $v_i$  means better performance of alternative  $a_i$ .



**FIGURE 6.** Relationship between risk preference coefficient and  $v_i$  of alternatives.

It is obvious from Fig.6 that there are two characteristics of the proposed method: (a) the rankings of alternatives differ greatly with different  $\lambda$  values. For example, when  $\lambda = -1$ , the ranked order of alternatives is  $a_3 > a_4 > a_2 > a_1$ , whereas it is  $a_1 > a_2 > a_4 > a_3$  when  $\lambda = 1$ . That is because when facing the uncertainty, DM's attitudes towards uncertainty are different and thus result in different decision-making results. (b)  $\lambda$  values produce different effects on alternatives of different uncertainties

(i.e.,  $En$  or  $He$ ). It can be seen from Fig.6 that the volatility of the alternative  $a_3$  is the largest with different  $\lambda$  values and the one of alternative  $a_4$  is the smallest, because the  $En$  and  $He$  of  $a_3$  are generally higher than those of  $a_4$ . This is applicable to many cases. Suppose that one cloud model is denoted by  $C_1 = (Ex, 0, 0)$ , and thus its reference point can be set as  $C_2 = (Ex, 0, 0)$ , then the distance between two cloud models with different risk preferences is constant. With the increase of  $En$  and  $He$  which can be supposed to be  $C_1 = (Ex, x, y)$ , then its reference point is varying from  $C_2 = (Ex, 0, 0)$  with complete risk-averse preference to  $C_2 = (Ex, x, y)$  with complete risk-seeking preference, accordingly, the distance between the cloud model and its preference point is varying from 0 to 1. Therefore, the risk preference coefficient has great impact on decision-makings, and thus should be rendered in accordance with actual situations to make an effective decision. In addition, it is obvious that  $\tau$  also has great impact on rankings of alternatives and thus is not discussed in detail.

**D. METHOD COMPARISONS**

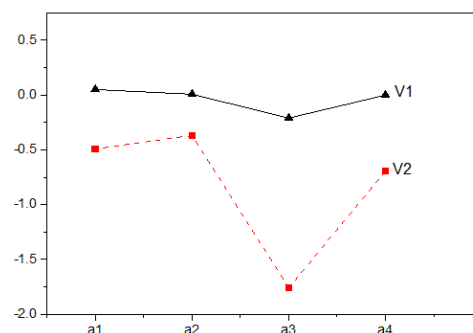
To further illustrate the feasibility of the proposed NC-BMADM method, some comparisons between the proposed method and two existing methods [7], [23] are made from different perspectives. The details are discussed as follows.

(1) Song’s method

Song’s method [23] investigated a distance-based decision-making method for cloud models by using prospect theory. In the method, distance between two clouds was determined based on the relative entropy in which expectation, hyper entropy and entropy have equal importance, and the average level of alternatives is considered one reference point, those are different from the proposed method.

In order to demonstrate the rationality of the proposed distance measure and the reference point setting approach, we compared the results produced by the proposed method with those acquired using the corresponding approaches presented in [23], and the other calculation procedures are the same as the proposed method to eliminate the influence of other factors. Accordingly, based on the case in Section IV-A, the result obtained using Song’s method is  $a_2 > a_4 > a_1 > a_3$ , but it failed to take into account the attribute types: cost attributes and benefit ones, which might lead to an ineffective decision. When considering the attribute types in Song’s method, it is  $a_1 > a_2 > a_4 > a_3$  which is based on the evaluation information. However, through using the same information, it is  $a_2 > a_1 > a_4 > a_3$  by using the proposed method. Two results obtained are different due to the fact that: (a) in Song’s method, DM’s risk preference coefficient is not taken into consideration when defining the reference point, and the sign of distance is judged only based on expectation, this might induce the bias for the decision. For example, when expectation of one alternative is the same as the one in the reference point, then through using Song’s method, the prospect value of the alternative is positive whether the

entropy and hyper entropy are more or less than the ones in the reference point. Thus, the definition of reference point in Song’s method is irrational. However, the positive idea reference point defined in the proposed method can solve this issue because the distance between one alternative and the point is considered to be negative, see Fig.7, where  $V_1$  denotes the result obtained by using Song’s method,  $V_2$  indicates the result obtained by using the proposed method without consideration of pre-evaluation information, X-axis represents alternatives, and Y-axis indicates  $v_i$  of alternatives. (b) Randomness, the inherent nature of cloud model, is not considered when measuring the distance in Song’s method, and thus might affect the accuracy of decision making. This is also compensated for by the proposed method. It is also evident from Fig.7 that alternatives have greater differentiation by using the proposed method than Song’s method. The ranking produced by our method is therefore reasonable and convincing. Additionally, there is another advantage of the proposed method by comparing with Song’s method: pre-evaluation information is considered in the proposed method to well understand the future development of alternatives and thus to help make an accurate decision.



**FIGURE 7. Ranking comparison between the proposed method and Song’s method.**

(2) Ren’s method

Ren’s method [7] mainly studied a cloud model-based multi-attribute decision making approach. The distance measure proposed by the method takes into account expectation, entropy and hyper entropy, where expectation plays a leading role. Also, because the method did not consider the situation when attribute weights are incomplete, we use the attribute weights obtained by using the proposed method to compute. Then the ranking of alternatives is obtained as  $a_1 \equiv a_2 > a_4 > a_3$ , which is a little different from  $a_2 > a_1 > a_4 > a_3$  obtained by using the proposed method without considering PEI. It illustrates that the proposed method is helpful to greatly differentiate the alternatives and thus makes the decision more efficient. Moreover, from the perspective of the future development,  $a_1$  has a better development trend than  $a_2$ . So, the PEI should be also considered to make a comprehensive evaluation.

Furthermore, in Ren’s method, the distance measure neglected the impacts of randomness and DMs’ risk attitudes

which can greatly affect the real-world decision operations, because DMs of different risk preferences have different attitudes towards uncertainty and thus adopt different measures to address the uncertainty. From the ranked results obtained by using the two methods, the best alternatives are different. That demonstrates the significance of the consideration of randomness and risk preference coefficient. In addition, the distance measure is invalid if the expectation of one cloud model is equal to zero in Ren's method as it is considered the denominator in the distance calculation.

Built on the aforementioned analysis for the case and the comparisons, the paper has three advantages. (a) Pre-evaluation information is considered in order to reflect the future development trends of alternatives, aiming at making an accurate decision. (b) Distance measure for cloud method is proposed involving in DM's risk preference coefficient, randomness, and three numerical numbers, making the method more applicable. (c) Positive ideal points are defined as the reference points by considering different types of risk preferences for enhancing the rationality and the application range of the proposed method. Undoubtedly, the considerations of the pre-evaluation information and decision-makers' risk attitudes guarantee the validity of results. Pre-evaluation information is helpful to make an accurate judgement for the following investment or cooperation, and DMs' risk attitudes can affect the reference setting and the distance measure. Thus, ranked results obtained by using the proposed method are helpful to obtain a more applicable and convincing decision.

## V. CONCLUSION

For MADM problems with some uncertainty and risk being included, we propose a NC-BMADM method by considering DM's risk attitudes and pre-evaluation information. For making a comprehensive evaluation for alternatives including their development trend, pre-evaluation information is taken into consideration by using normal cloud models. To well combine DM's risk attitudes and uncertainty involved in cloud models, a new distance measure is developed from perspectives of risk preferences and randomness by considering three numerical characteristics of cloud models. Then, PT is applied for addressing normal cloud models in which positive idea reference points are set from three perspectives: risk neutrality, risk aversion, and risk seeking, enhancing the application of the proposed method. Next, an optimization model is designed to obtain attribute weights and thus to increase the differentiation of alternatives. The results of a case study conducted to provide a sensitivity analysis as well as a comparison of two existing methods reveal the proposed method to be more applicable and scientifically sound. The primary contributions of the work presented in this article are outlined as follows:

(a) Pre-evaluation information is considered, which is helpful to disclose the future development trend of alternatives and thus to make an accurate decision.

(b) A means of distance measurement is proposed by considering both randomness and risk preferences, which has two advantages: uncertainty involved in cloud models is taken into account that conforms to the real application; the intrinsic properties of three numerical values of cloud models are integrated into the measurement.

(c) A setting approach of reference points has been developed from three perspectives: risk aversion, risk neutrality, and risk seeking, increasing the scope of application in real-world cases.

However, there are still two problems that have not been solved in the proposed method: (1) risk preference coefficient and the preference coefficient in (18) are subjectively provided which might introduce certain bias for decision-makings; (2) the solution obtained in M2 might be the locally optimal solution rather than globally optimal solution. Thus, further research could be directed at determining a method for measuring the pre-evaluation accuracy, developing approaches for eliminating the bias through the subjective detection of DM's risk attitude, and improving M1 for obtaining unique optimal solution. The proposed method can be also extended to include addressing uncertain group MADM problems. These topics all constitute appropriate areas for future investigation.

## APPENDIX A

### THE PROOFS OF LEMMAS

#### A. THE PROOF OF LEMMA 1

The proof of Lemma 1 is provided as follows.

*Proof:* Suppose that  $(x - Ex)/(\sqrt{2}En') = \mu$ , then we have  $x = Ex + \sqrt{2}En' \times \mu$ ,  $dx = \sqrt{2}En' d\mu$ , and  $\mu \in (-\infty, +\infty)$ , (7) can be transformed into  $E(C) = \int_{-\infty}^{+\infty} [(Ex + \sqrt{2}En' \times \mu) \times e^{-\mu^2} \times \sqrt{2}En'] d\mu = \sqrt{2}En' \times (\int_{-\infty}^{+\infty} Ex \times e^{-\mu^2} d\mu + \int_{-\infty}^{+\infty} \sqrt{2}En' \times \mu \times e^{-\mu^2} d\mu) = \sqrt{2\pi} \times En' \times Ex = \sqrt{2\pi} \times Ex \times (En - 3\lambda He)$ .

Lemma 1 is therefore proved.

#### B. THE PROOF OF LEMMA 2

The proof of Lemma 2 is provided as follows.

*Proof:* (a) When  $\max_{i \in N} E(C_i) = \min_{i \in N} E(C_i)$ , we have  $E(C_i) = E(C_k) = \max_{i \in N} E(C_i) = \min_{i \in N} E(C_i)$ , so  $D^E(C_i \parallel C_k) = 0$ .

When  $\max_{i \in N} E(C_i) > \min_{i \in N} E(C_i)$ ,  $\min_{i \in N} E(C_i) - \max_{i \in N} E(C_i) = \min_{i \in N} E(C_i) - \max_{k \in N} E(C_k) \leq E(C_i) - E(C_k) \leq \max_{i \in N} E(C_i) - \min_{k \in N} E(C_k) = \max_{i \in N} E(C_i) - \min_{i \in N} E(C_i)$ . Thus,  $0 \leq D^E(C_i \parallel C_k) = \frac{|E(C_i) - E(C_k)|}{\max_{i \in N} E(C_i) - \min_{i \in N} E(C_i)} \leq 1$ .

If  $E(C_i) \neq E(C_k)$ , that means  $\max_{i \in N} E(C_i) > \min_{i \in N} E(C_i)$ , and  $|E(C_i) - E(C_k)| > 0$ , then  $D^E(C_i \parallel C_k) > 0$ . It is obvious to prove that  $D^E(C_i \parallel C_k) = 0$  if  $E(C_i) = E(C_k)$ . Hence, if and only if  $E(C_i) = E(C_k)$ ,  $D^E(C_i \parallel C_k) = 0$ .

(b) When  $\max_{i \in N} E(C_i) = \min_{i \in N} E(C_i)$ ,  $D^E(C_i \parallel C_k) = D^E(C_k \parallel C_i) = 0$ . When  $\max_{i \in N} E(C_i) > \min_{i \in N} E(C_i)$ ,

$$D^E(C_i \parallel C_k) = \frac{|E(C_i) - E(C_k)|}{\max_{i \in N} E(C_i) - \min_{i \in N} E(C_i)} = \frac{|E(C_k) - E(C_i)|}{\max_{i \in N} E(C_i) - \min_{i \in N} E(C_i)} = D^E(C_k \parallel C_i).$$

(c) Based on Lemma 1, we have  $E(C_i) = E(C_i)$ , then  $D^E(C_i \parallel C_i) = \frac{|E(C_i) - E(C_i)|}{\max_{i \in N} E(C_i) - \min_{i \in N} E(C_i)} = 0$ .

Lemma 2 is therefore proved.

### C. THE PROOF OF LEMMA 3

The proof of Lemma 3 is provided as follows.

*Proof:* (a) When  $\max_{i \in N} En_i = \min_{i \in N} En_i$ ,  $\frac{|En_i - En_k|}{\max_{i \in N} En_i - \min_{i \in N} En_i} = 0$ . When  $\max_{i \in N} En_i > \min_{i \in N} En_i$ , similar to the proof in Lemma 2,  $0 \leq \frac{|En_i - En_k|}{\max_{i \in N} En_i - \min_{i \in N} En_i} \leq 1$ . Then,  $\frac{|En_i - En_k|}{\max_{i \in N} En_i - \min_{i \in N} En_i} = 0 \Leftrightarrow |En_i - En_k| = 0 \Leftrightarrow En_i = En_k$ .

By the same token, we have  $0 \leq \frac{|He_i - He_k|}{\max_{i \in N} He_i - \min_{i \in N} He_i} \leq 1$ , and  $\frac{|He_i - He_k|}{\max_{i \in N} He_i - \min_{i \in N} He_i} = 0 \Leftrightarrow He_i = He_k$ . Then, we have  $0 = \frac{1}{2}(0 + 0) \leq D^S(C_i \parallel C_k) \leq \frac{1}{2}(1 + 1) = 1$ , and  $D^S(C_i \parallel C_k) = 0 \Leftrightarrow \frac{|En_i - En_k|}{\max_{i \in N} En_i - \min_{i \in N} En_i} = 0$  and  $\frac{|He_i - He_k|}{\max_{i \in N} He_i - \min_{i \in N} He_i} = 0 \Leftrightarrow En_i = En_k$  and  $He_i = He_k$ .

Hence, if and only if  $En_i = En_k$  and  $He_i = He_k$ ,  $D^S(C_i \parallel C_k) = 0$ .

(b)  $D^S(C_i \parallel C_k) = \frac{1}{2} \left( \frac{|En_i - En_k|}{\max_{i \in N} En_i - \min_{i \in N} En_i} + \frac{|He_i - He_k|}{\max_{i \in N} He_i - \min_{i \in N} He_i} \right) = \frac{1}{2} \left( \frac{|En_k - En_i|}{\max_{i \in N} En_i - \min_{i \in N} En_i} + \frac{|He_k - He_i|}{\max_{i \in N} He_i - \min_{i \in N} He_i} \right) = D^S(C_k \parallel C_i)$ .

(c)  $D^S(C_i \parallel C_i) = \frac{1}{2} \left( \frac{|En_i - En_i|}{\max_{i \in N} En_i - \min_{i \in N} En_i} + \frac{|He_i - He_i|}{\max_{i \in N} He_i - \min_{i \in N} He_i} \right) = 0$ .

Lemma 3 is therefore proved.

### D. THE PROOF OF LEMMA 4

The proof of Lemma 4 is provided as follows.

*Proof:* (a) Because of  $N \geq 2$ , then  $0 < \frac{1}{N} < 1$ , and thus  $0 < 1 - \frac{1}{N} < 1$ . Based on Lemmas 2 and 3, we have  $0 \leq D(C_i \parallel C_k) = \frac{1}{N} D^S(C_i \parallel C_k) + (1 - \frac{1}{N}) D^E(C_i \parallel C_k) \leq 1$ .

Because of  $0 \leq D^S(C_i \parallel C_k) \leq 1$ ,  $0 \leq D^E(C_i \parallel C_k) \leq 1$ ,  $0 < \frac{1}{N} < 1$ , and  $0 < 1 - \frac{1}{N} < 1$ , we have  $D(C_i \parallel C_k) = 0 \Leftrightarrow D^S(C_i \parallel C_k) = 0$  and  $D^E(C_i \parallel C_k) = 0 \Leftrightarrow E(C_i) = E(C_k)$ ,  $En_i = En_k$ , and  $He_i = He_k$ .

When  $E(C_i) = E(C_k)$ , based on Lemma 1, we have  $\sqrt{2\pi} \times Ex_i \times (En_i - 3\lambda He_i) = \sqrt{2\pi} \times Ex_k \times (En_k - 3\lambda He_k)$ . As  $En_i = En_k$ , and  $He_i = He_k$ , we have  $En_i - 3\lambda He_i = En_k - 3\lambda He_k$ . If  $En_i - 3\lambda He_i = En_k - 3\lambda He_k = 0$ ,  $E(C_i) = Ex_i$  and  $E(C_k) = Ex_k$ , then,  $E(C_i) = E(C_k) \Leftrightarrow Ex_i = Ex_k$ . If  $En_i - 3\lambda He_i = En_k - 3\lambda He_k \neq 0$ ,  $Ex_i = Ex_k$  is also obtained.

Thus, if and only if  $Ex_i = Ex_k$ ,  $En_i = En_k$ , and  $He_i = He_k$ ,  $D(C_i \parallel C_k) = 0$ .

(b) Based on Lemmas 2 and 3, we have  $D(C_i \parallel C_k) = \frac{1}{N} D^S(C_i \parallel C_k) + (1 - \frac{1}{N}) D^E(C_i \parallel C_k) = \frac{1}{N} D^S(C_k \parallel C_i) + (1 - \frac{1}{N}) D^E(C_k \parallel C_i) = D(C_k \parallel C_i)$ .

(c)  $D(C_i \parallel C_i) = \frac{1}{N} D^S(C_i \parallel C_i) + (1 - \frac{1}{N}) D^E(C_i \parallel C_i) = 0$ .

Lemma 4 is therefore proved here.

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