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Fusion Particle Filter for Nonlinear Systems Based on Segmental Gauss-Hermite Approximation

YUN LI
School of Computer and Information Engineering, Harbin University of Commerce, Harbin 150001, China
e-mail: 101090@hrbcu.edu.cn

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ABSTRACT Multisensor fusion estimators play an important role in modern information processing. Weighted measurement fusion (WMF) algorithm has widely been applied to data compression of multisensor linear systems. Due to the complexity and uncertainty of nonlinear systems, the application of WMF algorithms is limited in multisensor nonlinear systems. In this article, an approximate linear relationship is established by using the segmental Gauss-Hermite approximation method for multisensor nonlinear systems. Based on the relationship and weighted least squares (WLS) method, a WMF algorithm is presented to compress the data for multisensor nonlinear systems. By combining the WMF algorithm with Particle Filter (PF), a weighted measurement fusion Particle Filter (WMF-PF) is presented for multisensor nonlinear systems. Compared with the centralized fusion PF, the proposed WMF-PF has a fair accuracy and less computational cost. It has a potential application in navigation, GPS, target tracking, communications, big data and so on. An example is given to show the effectiveness of the proposed algorithms.

INDEX TERMS Multisensor, nonlinear system, weighted measurement fusion, segmental Gauss-Hermite approximation, particle filter.

I. INTRODUCTION

IN order to obtain the comprehensive information, the multisensor is always the irreplaceable way. Multisensor information fusion can improve the estimation accuracy and the fault tolerance. And it always play an important part in control, target tracking, navigation, fault diagnosis and so on.

For the fusion estimation, there are two main kinds of forms: the centralized structure and the distributed structure. The centralized fusion algorithm is optimal because no information is lost, but poor real-time because of the expensive computational cost. Distributed fusion algorithms make the fused estimates in the fusion center by weighting local state estimates under a certain criterion [1]–[4], such as distributed fusion information Kalman filter [5], distributed fusion federated Kalman filter [6], distributed weighted fusion estimators [7], and distributed covariance intersection fusion estimators [8]. They have good robustness and strong fault tolerance. However, they are local optimal and global suboptimal. Based on the variances of measurement noises and relationships among measurement functions, the weighted measurement fusion (WMF) algorithms compress a high-dimension measurement of centralized fusion systems to a low-dimension one [9]–[11], which can reduce computational cost. For linear systems, the WMF is numerically equivalent to the centralized fusion [9], [11]. So, they are also optimal in the sense of least mean squares. However, in nonlinear systems, time-varying, saturation, etc. often occur [12]. Due to the complexity and uncertainty of nonlinear systems, the WMF is difficult to implement. In this article, the WMF method will be studied for multisensor nonlinear systems, which is seldom mentioned in the literatures [13].

Nonlinear characteristics widely exist in various practical engineering systems [14]–[16]. Many scholars are devoted to the research of control and estimation for nonlinear systems, and have proposed many practical methods, such as linear approximation, neural networks, T-S fuzzy rules [17]–[21]. The fusion estimation for nonlinear systems has also attracted the attention of many scholars [22]–[26]. In literature [27], based on Taylor series and Unscented Kalman filter (UKF), a weighted measurement fusion Unscented Kalman filter (WMF-UKF) was presented. The WMF-UKF

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algorithm needs to calculate the coefficients of Taylor series in real time, which brings the expensive online computational cost. In literature [28], based on Gauss-Hermite approximation, a WMF algorithm is presented, which avoids the online calculation of coefficient matrices, but it could only deal with scalar cases and the Hermite polynomials are fixed.

Gauss-Hermite approximation [29]–[31] can approximate most elementary functions by Gauss functions, Hermite polynomials and some selected points. This approximation method does not need to calculate the Jacobian in real time like Taylor series approximation, and has a very good fitting effect. The main contributions of this article are listed below:

1. In this article, based on segmental Gauss-Hermite approximation, a method of measurement data compression (or weighted measurement fusion) is proposed. This method can effectively reduce the dimension of the measurement equation and reduce the computational cost.

2. Based on the fusion method and PF, a WMF-PF is presented. The proposed WMF-PF can adjust the computational complexity by Hermite polynomials according to the approximated functions, and can be used for multi-dimensional nonlinear systems.

The rest of the paper is organized as follows. Problem formulation is proposed for nonlinear systems in Section II. In Section III Gauss-Hermite approximation is proposed. Universal WMF based on Gauss-Hermite approximation is proposed in Section IV. In Section V WMF-PF algorithm based on segmental Gauss-Hermite approximation is presented. The simulation analysis is given in Section VI. The conclusions are summarized in Section VII.

II. PROBLEM FORMULATION

Consider a multisensor nonlinear dynamic system:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{f}_k(\boldsymbol{x}_k) + \boldsymbol{w}_k & (1)\\ \boldsymbol{y}_k^{(j)} &= \boldsymbol{h}_k^{(j)}(\boldsymbol{x}_k) + \boldsymbol{v}_k^{(j)},\ j = 1, 2, \cdots, L & (2)\end{aligned}$$

where $\boldsymbol{f}_k(\cdot) \in \Re^n$ and $\boldsymbol{h}_k^{(j)}(\cdot) \in \Re^{m_j}$ are the known process and measurement functions, $\boldsymbol{x}_k \in \Re^n$ is the state vector at time k, $\boldsymbol{y}_k^{(j)} \in \Re^{m_j}$ is the measurement vector of the jth sensor at time k, and $\boldsymbol{w}_k \sim p_{\omega_k}(\cdot)$ and $\boldsymbol{v}_k^{(j)} \sim p_{v_k}(\cdot)$ are the independent white process and measurement noises with zero mean and variances $\boldsymbol{Q}_w$ and $\boldsymbol{R}^{(j)}$, respectively, i.e.,

$$\mathrm{E}\left\{\begin{bmatrix} \boldsymbol{w}_t \\ \boldsymbol{v}_t^{(j)} \end{bmatrix}\begin{bmatrix} \boldsymbol{w}_k^{\mathrm{T}} & \left(\boldsymbol{v}_k^{(l)}\right)^{\mathrm{T}} \end{bmatrix}\right\} = \begin{bmatrix} \boldsymbol{Q}_w & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}^{(j)}\delta_{jl} \end{bmatrix}\delta_{tk} \tag{3}$$

where E means mathematical expectation, the superscript T means transpose, and δ_{tk} and δ_{jl} are the Kronecker delta functions.

For such systems in Equations (1) and (2), the centralized fusion algorithm is widely used.

Lemma 1 [27]: For the systems in Equations (1) and (2), the measurement equation of the optimal centralized fusion system can be written as:

$$\boldsymbol{y}_k^{(\mathrm{C})} = \boldsymbol{h}_k^{(\mathrm{C})}(\boldsymbol{x}_k) + \boldsymbol{v}_k^{(\mathrm{C})} \tag{4}$$

where

$$\boldsymbol{y}_k^{(\mathrm{C})} = \left[\left(\boldsymbol{y}_k^{(1)}\right)^{\mathrm{T}}, \left(\boldsymbol{y}_k^{(2)}\right)^{\mathrm{T}}, \cdots, \left(\boldsymbol{y}_k^{(L)}\right)^{\mathrm{T}}\right]^{\mathrm{T}} \tag{5}$$

$$\boldsymbol{h}_k^{(\mathrm{C})}(\boldsymbol{x}_k) = \left[\left(\boldsymbol{h}_k^{(1)}(\boldsymbol{x}_k)\right)^{\mathrm{T}}, \left(\boldsymbol{h}_k^{(2)}(\boldsymbol{x}_k)\right)^{\mathrm{T}}, \cdots, \left(\boldsymbol{h}_k^{(L)}(\boldsymbol{x}_k)\right)^{\mathrm{T}}\right]^{\mathrm{T}} \tag{6}$$

$$\boldsymbol{v}_k^{(\mathrm{C})} = \left[\left(\boldsymbol{v}_k^{(1)}\right)^{\mathrm{T}}, \left(\boldsymbol{v}_k^{(2)}\right)^{\mathrm{T}}, \cdots, \left(\boldsymbol{v}_k^{(L)}\right)^{\mathrm{T}}\right]^{\mathrm{T}} \tag{7}$$

and its covariance is:

$$\boldsymbol{R}^{(\mathrm{C})} = \mathrm{diag}(\boldsymbol{R}^{(1)}, \boldsymbol{R}^{(2)}, \cdots, \boldsymbol{R}^{(L)}) \tag{8}$$

where the $'\mathrm{diag}(\boldsymbol{.})'$ means a diagonal matrix.

Using the nonlinear filtering algorithms (e.g. Extended Kalman Filter (EKF), UKF, PF [32], [33], Cubature Kalman Filter (CKF) [25], [34]–[37]), the optimal centralized fusion filtering algorithms can be obtained for the centralized fusion systems in Equations (1) and (4). Because no information is lost, centralized fusion is considered to be optimal. But the fusion measurement function in Equation (4) has a high dimension, which brings expensive computational cost. So it is significant to find the equivalent or approximate ways to reduce the computational cost. However, in many cases, the measurement functions are different because of the different types of sensors, locations and so on. Next, we will introduce a Lemma, which enables the WMF to be applied to the systems with different measurement functions.

Lemma 2 [27]: For the systems in Equations (1) and (2), if there exists a intermediary function $\boldsymbol{\phi}_k(\boldsymbol{x}_k) \in \Re^n$ satisfying $\boldsymbol{h}_k^{(j)}(\boldsymbol{x}_k) = \boldsymbol{H}_k^{(j)}\boldsymbol{\phi}_k(\boldsymbol{x}_k)$, where $\boldsymbol{H}^{(j)} \in \Re^{m_j \times p}(j = 1, 2, \cdots, L)$, the measurement function of the optimal WMF system is given as:

$$\boldsymbol{y}_k^{(\mathrm{I})} = \boldsymbol{H}^{(\mathrm{I})}\boldsymbol{\phi}_k(\boldsymbol{x}_k) + \boldsymbol{v}_k^{(\mathrm{I})} \tag{9}$$

where

$$\boldsymbol{y}_k^{(\mathrm{I})} = \left(\boldsymbol{M}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\boldsymbol{M}\right)^{-1}\boldsymbol{M}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\boldsymbol{y}_k^{(\mathrm{C})} \tag{10}$$

$$\boldsymbol{v}_k^{(\mathrm{I})} = \left(\boldsymbol{M}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\boldsymbol{M}\right)^{-1}\boldsymbol{M}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\boldsymbol{v}_k^{(\mathrm{C})} \tag{11}$$

and the covariance matrix of $\boldsymbol{v}_k^{(\mathrm{I})}$ is computed as:

$$\boldsymbol{R}^{(\mathrm{I})} = \left(\boldsymbol{M}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\boldsymbol{M}\right)^{-1} \tag{12}$$

where $\boldsymbol{M}$ (full-column rank) and $\boldsymbol{H}^{(\mathrm{I})}$ (full-row rank) are the full rank decomposition matrices of the matrix $\boldsymbol{H}^{(\mathrm{C})} = \left[(\boldsymbol{H}^{(1)})^{\mathrm{T}}, \cdots, (\boldsymbol{H}^{(L)})^{\mathrm{T}}\right]^{\mathrm{T}}$, i.e.,

$$\boldsymbol{H}^{(\mathrm{C})} = \boldsymbol{M}\boldsymbol{H}^{(\mathrm{I})} \tag{13}$$

which can be computed by Hermite canonical.

Remark 1: Lemma 1 gives an effective data compression method. This method can compress the centralized measurement $\boldsymbol{y}_k^{(\mathrm{C})}$ into $\boldsymbol{y}_k^{(\mathrm{I})}$ through the $\boldsymbol{M}$ matrix. This method is widely used in linear systems, but due to the complexity and uncertainty of nonlinear systems, it is difficult to obtain the intermediary function, which makes it difficult to apply for nonlinear systems.

Next, a method of using function approximation to obtain the intermediary function will be proposed. This method attempts to approximately express the general functions as unified Gauss-Hermite functions, and the unified Gauss-Hermite functions are the intermediate functions we are looking for.

III. GAUSS-HERMITE APPROXIMATION

In this section, the Gauss-Hermite approximation will be introduced which can approximate elementary functions by Gauss functions and Hermite polynomials. Based on the approximation method, the measurement functions can be converted into $\boldsymbol{h}_k^{(j)}(\boldsymbol{x}_k) \approx \bar{\boldsymbol{h}}_k^{(j)}(\boldsymbol{x}_k) = \bar{\boldsymbol{H}}_k^{(j)}\bar{\boldsymbol{\phi}}_k(\boldsymbol{x}_k)$, where $\bar{\boldsymbol{h}}_k^{(j)}(\boldsymbol{x}_k)$ is the approximation function of $\boldsymbol{h}_k^{(j)}(\boldsymbol{x}_k)$, $\bar{\boldsymbol{\phi}}_k(\boldsymbol{x}_k)$ formed by Gauss functions and Hermite polynomials is the approximation intermediary function, and $\bar{\boldsymbol{H}}_k^{(j)}$ is the corresponding constant coefficient matrix. In this way, Lemma 2 can be implemented.

Lemma 3 [29]: Assuming $\{\boldsymbol{X}_i' \in \Re^n\}$ $(i = 1,\ 2,\ \cdots,\ \Upsilon)$ is an selected ensemble of Υ points, to each selected point $\boldsymbol{X}_i' = [x_{i_1}', x_{i_2}', \cdots, x_{i_n}']$, $(a \le x_{i_\lambda}' \le x_{i+1_\lambda}' \le b,\ \lambda = 1, \cdots, n)$, there exists a point $\boldsymbol{Z}_i'(x_{i_1}', x_{i_2}', \cdots, x_{i_n}') = [z_{i_1}, z_{i_2}, \cdots, z_{i_p}]$ satisfies $\boldsymbol{Z}_i' = \boldsymbol{Z}(\boldsymbol{X}_i')$ where $\boldsymbol{Z}(\cdot)$ is a determined multidimensional function. Then, the approximate function $\bar{\boldsymbol{Z}}(x_1, x_2, \cdots, x_n)$ of $\boldsymbol{Z}(x_1, x_2, \cdots, x_n)$ by Gauss-Hermite folding reads:

$$\begin{aligned}\bar{\boldsymbol{Z}}(x_1, x_2, \cdots, x_n) \approx \sum_{i_1=1}^{\Upsilon} \Delta x_{i_1} \sum_{i_2=1}^{\Upsilon} \Delta x_{i_2} \cdots \sum_{i_n=1}^{\Upsilon} \Delta x_{i_n} \\ \cdot \boldsymbol{Z}'(x_{i_1}', x_{i_2}', \cdots, x_{i_n}') \prod_{\lambda=1}^{n} \frac{1}{\mu_\lambda \sqrt{\pi}} \\ \cdot \exp\left\{-\left(\frac{x_\lambda - x_{i_\lambda}'}{\mu_\lambda}\right)^2\right\} f_\hbar\left(\frac{x_\lambda - x_{i_\lambda}'}{\mu_\lambda}\right) \end{aligned} \tag{14}$$

where $\Delta x_{i_\lambda} = \frac{1}{2}(x_{i_\lambda+1} - x_{i_\lambda-1})$, $\mu_\lambda(\lambda = 1, \cdots, n)$ is a coefficient, and $f_\hbar(\varpi)$ is a correction polynomial which can be decomposed into a series of Hermite polynomials:

$$f_\hbar(\varpi) = \sum_{\lambda\!\!\!-=0}^{\hbar} C_{\lambda\!\!\!-} H_{\lambda\!\!\!-}(\varpi) \tag{15}$$

$$C_{\lambda\!\!\!-} = \frac{1}{2^{\lambda\!\!\!-}\lambda\!\!\!-!} H_{\lambda\!\!\!-}(0) \tag{16}$$

where $H_{\lambda\!\!\!-}(\varpi) = (-1)^{\lambda\!\!\!-} e^{\varpi^2}(e^{-\varpi^2})^{(\lambda\!\!\!-)}$ is a Hermite polynomial [38], $H_{\lambda\!\!\!-}(0)$ is:

$$H_{\lambda\!\!\!-}(0) = \begin{cases} 1 & \lambda\!\!\!- = 0 \\ 2^\rho(-1)^\rho(2\rho-1)!! & \lambda\!\!\!- = 2\rho \\ 0 & \lambda\!\!\!- = 2\rho+1 \end{cases}, \quad \rho = 0, 1 \cdots \tag{17}$$

and $C_{\lambda\!\!\!-}$ is:

$$C_{\lambda\!\!\!-} = \begin{cases} 1 & \lambda\!\!\!- = 0 \\ (-1)^\rho \dfrac{(2\rho-1)!!}{2^\rho(2\rho)!} & \lambda\!\!\!- = 2\rho \qquad \rho = 0, 1 \cdots \\ 0 & \lambda\!\!\!- = 2\rho+1, \end{cases} \tag{18}$$

See the detailed proof in the literature [2], [29], [31].

Remark 2: $\mu_\lambda(\lambda = 1, \cdots, n)$ are arbitrary coefficients related to Δx_{i_λ} $(i = 1, \cdots, \Upsilon)$ and should be used in conjunction with $\hbar$ [29]. According to the known approximated function, we can get the suitable coefficients in advance by experiment.

Remark 3: From Equations (15)-(17), we have the specific forms:

$$\begin{aligned} &C_0 = 1, \quad H_0 = 1 \\ &C_2 = -\frac{1}{4}, \quad H_2(\varpi) = 4\varpi^2 - 2 \\ &C_4 = \frac{1}{32}, \quad H_4(\varpi) = 16\varpi^4 - 48\varpi^2 + 12 \end{aligned} \tag{19}$$

Letting $\psi(\varepsilon) = \exp(-\varepsilon^2) f_\hbar(\varepsilon)$, Equation (14) can be simplified to:

$$\begin{aligned}\bar{\boldsymbol{Z}}(x_1, x_2, \cdots, x_n) = \sum_{i_1=1}^{\Upsilon} \Delta x_{i_1} \sum_{i_2=1}^{\Upsilon} \Delta x_{i_2} \cdots \sum_{i_n=1}^{\Upsilon} \Delta x_{i_n} \\ \cdot \boldsymbol{Z}'(x_{i_1}', x_{i_2}', \cdots, x_{i_n}') \prod_{\lambda=1}^{n} \frac{1}{\mu_\lambda\sqrt{\pi}} \psi\left(\frac{x_\lambda - x_{i_\lambda}'}{\mu_\lambda}\right) \end{aligned} \tag{20}$$

IV. UNIVERSAL WMF BASED ON GAUSS-HERMITE APPROXIMATION

In section III, the nonlinear function is uniformly expressed in the form of Gauss-Hermite function. From Equation (14), if the function $\sum_{i_1=1}^{\Upsilon} \Delta x_{i_1} \sum_{i_2=1}^{\Upsilon} \Delta x_{i_2} \cdots \sum_{i_n=1}^{\Upsilon} \Delta x_{i_n} \prod_{\lambda=1}^{n} \frac{1}{\mu_\lambda\sqrt{\pi}} \cdot \psi\left(\frac{x_\lambda - x'_{i_\lambda}}{\mu_\lambda}\right)$ $(i = 1, \cdots, \Upsilon;\ \lambda = 1, \cdots, n)$ is seen as the intermediary function $\boldsymbol{\phi}_k(\boldsymbol{x}_k)$ and $\boldsymbol{Z}'(x_{i_1}', x_{i_2}', \cdots, x_{i_n}')$ is seen as $\boldsymbol{H}_k^{(j)}$ in Lemma 2, the relationships which are needed in Lemma 2 can be established and the Lemma 2 can be implemented.

Theorem 1: For the systems in Equations (1) and (2), the approximate centralized measurement fusion equation is given as:

$$\bar{\boldsymbol{y}}_k^{(\mathrm{C})} = \bar{\boldsymbol{H}}^{(\mathrm{C})}\bar{\boldsymbol{\phi}}_k(\boldsymbol{x}_k) + \bar{\boldsymbol{v}}_k^{(\mathrm{C})} \tag{21}$$

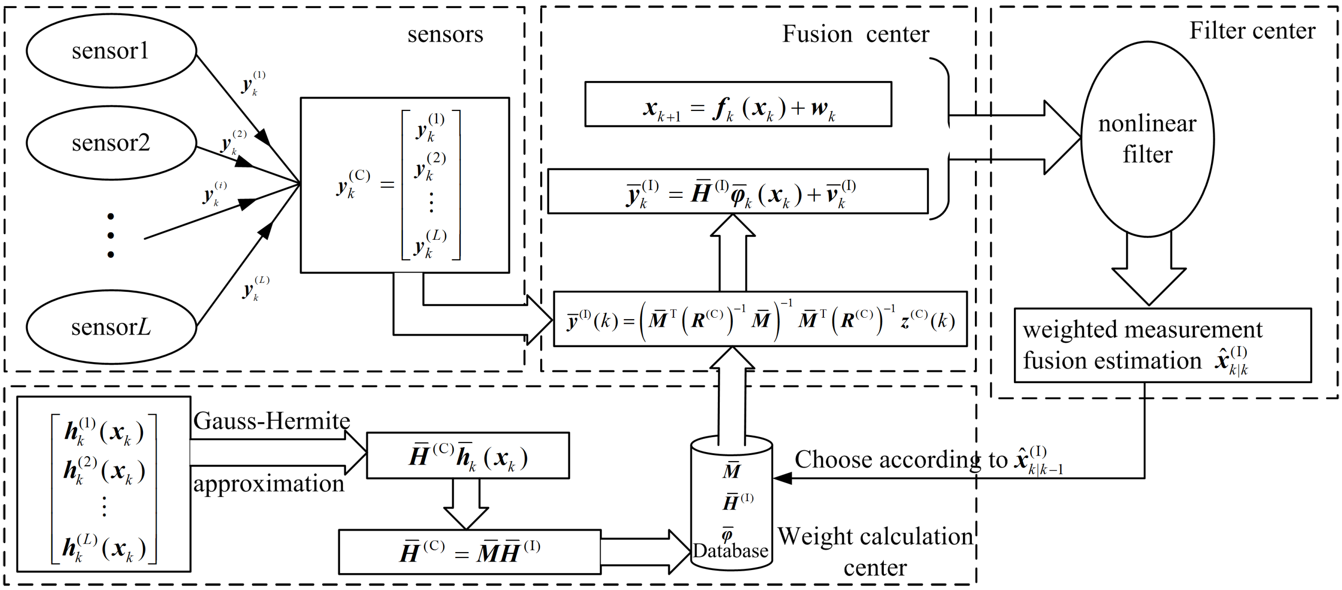

FIGURE 1. Implementing process of WMF filter.

where $\bar{\boldsymbol{\phi}}_k(\boldsymbol{x}_k)$ is shown as Equation (26), as shown at the bottom of the next page, $x_\lambda(\lambda = 1, \cdots, n)$ is the λth state component, x'_{i_λ} $(i = 1, \cdots, \Upsilon)$ is the ith selected point of the λth state component, and Υ and μ_λ are defined as Lemma 3. $\bar{\boldsymbol{H}}^{(\mathrm{C})}$ is shown as Equation (27), as shown at the bottom of the next page, where $\boldsymbol{h}^{(j)}(\cdot)$ $(j = 1, \cdots, L)$ are the corresponding function value of the jth sensor function for x_λ. Using Lemma 2, the approximate measurement equation of the WMF system is given as:

$$\bar{\boldsymbol{y}}_k^{(\mathrm{I})} = \bar{\boldsymbol{H}}^{(\mathrm{I})}\bar{\boldsymbol{\phi}}_k(\boldsymbol{x}_k) + \bar{\boldsymbol{v}}_k^{(\mathrm{I})} \tag{22}$$

where

$$\bar{\boldsymbol{y}}_k^{(\mathrm{I})} = \left(\bar{\boldsymbol{M}}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\bar{\boldsymbol{M}}\right)^{-1}\bar{\boldsymbol{M}}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\boldsymbol{y}_k^{(\mathrm{C})} \tag{23}$$

$\bar{\boldsymbol{M}}$ and $\bar{\boldsymbol{H}}^{(\mathrm{I})}$ are the full rank decomposition matrices of $\bar{\boldsymbol{H}}^{(\mathrm{C})}$, and $\bar{\boldsymbol{M}} \in \boldsymbol{R}^{\sum_{i=1}^{L} m_i \times r}$ is full-column rank and $\bar{\boldsymbol{H}}^{(\mathrm{I})} \in \boldsymbol{R}^{r \times S^n}(r \leq \sum_{i=1}^{L} m_i)$ is full-row rank.

$$\bar{\boldsymbol{v}}_k^{(\mathrm{I})} = \left(\bar{\boldsymbol{M}}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\bar{\boldsymbol{M}}\right)^{-1}\bar{\boldsymbol{M}}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\boldsymbol{v}_k^{(\mathrm{C})} \tag{24}$$

and its covariance matrix is:

$$\bar{\boldsymbol{R}}^{(\mathrm{I})} = \left(\bar{\boldsymbol{M}}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\bar{\boldsymbol{M}}\right)^{-1} \tag{25}$$

Proof: From Lemma 3, the measurement functions $\boldsymbol{h}_k^{(j)}(\boldsymbol{x}_k)(j = 1, \cdots, L)$ can be approximated as:

$$\begin{aligned}
&\boldsymbol{h}_k^{(j)}(\boldsymbol{x}_k)\\
&\approx \bar{\boldsymbol{h}}^{(j)}(x_1, x_2, \cdots, x_n)\\
&= \sum_{i_1=1}^{\Upsilon}\Delta x_{i_1}\sum_{i_2=1}^{\Upsilon}\Delta x_{i_2}\cdots\sum_{i_n=1}^{\Upsilon}\Delta x_{i_n}\boldsymbol{h}^{(j)}(x'_{i_1}, x'_{i_2}, \cdots, x'_{i_n})\\
&\quad\cdot\prod_{\lambda=1}^{n}\frac{1}{\mu_\lambda\sqrt{\pi}}\psi\left(\frac{x_\lambda - x'_{i_\lambda}}{\mu_\lambda}\right),\ j = 1, \cdots, L
\end{aligned} \tag{28}$$

Taking Equation (28) into the centralized fusion measurement Equation (4), Equations (21)-(27) can be yielded. From Lemma 2, the Equation (21) can be compressed into Equation (22). Proof is completed.

An approximate intermediary function $\bar{\boldsymbol{\phi}}_k(\boldsymbol{x}_k)$ was constructed in Theorem 1 by Gauss-Hermite approximation, which makes the local measurement equations have the relationships needed in Lemma 2, then lemma 2 can be carried out. We can use nonlinear filtering algorithms (e. g. EKF, UKF, PF, CKF etc.) and Theorem 1 to obtain the WMF filtering algorithms for the approximate WMF systems in Equations (1) and (22). It is worth noting that the measurement Equation (22) has a reduced dimension compared with the centralized measurement fusion Equation (4). So, based on the approximate WMF systems in Equations (1) and (22), the designed filtering algorithms will have reduced computational cost compared with the centralized fusion filter based on Equations (1) and (4).

If the range of the state value is too large, the sample points will increase sharply, which would increase the computational cost. So the method of segmentation will be used here. For example, for the systems with 1-dimensional state, the range of the state can be divide into a number of line segments, and for the systems with 2-dimensional state, the range of the state can be divide into a number of small areas. For simplicity, all the $\bar{\boldsymbol{H}}^{(\mathrm{I})}$ and $\bar{\boldsymbol{M}}$ for every segmentation can be calculate off-line in advance, and the suitable $\bar{\boldsymbol{H}}^{(\mathrm{I})}$ and $\bar{\boldsymbol{M}}$ can be chosen directly in the database according to the predictor $\hat{\boldsymbol{x}}_{k|k-1}^{(\mathrm{I})}$. The implementing process of WMF filtering algorithms is shown as Figure 1.

Compared with the WMF algorithm in literature [27], the proposed WMF algorithm does not need to calculate the fusion matrices online in real time, so its computational complexity will be lower. Compared with the WMF algorithm in literature [28], the proposed WMF algorithm can be used for multi-dimensional systems. And we can adjust the coefficients μ_λ, Δx_{i_λ} and Hermite polynomials according to the simulation results of approximation, so the proposed WMF algorithm is more flexible and has a wider range of applications.

V. WMF-PF ALGORITHM BASED ON SEGMENTAL GAUSSÍCHERMITE APPROXIMATION

In fact, the universal WMF algorithm in Theorem 1 can be combined with various nonlinear fusion algorithms (EKF, UKF, PF, CKF, etc.) to form the nonlinear multisensor WMF filtering algorithms. A nonlinear multisensor WMF Particle Filter (WMF-PF) algorithm will be presented for example. From Algorithm 1, the differences between the centralized fusion PF (CF-PF) and the WMF-PF are in step 4), 5) and 6). The CF-PF needs to deal with the function with $\sum_{j=1}^{L} m_j-$dimension, so its time complexity is $O\left(\left(\sum_{j=1}^{L} m_j\right)^2\right)$. But the WMF-PF needs to deal with the function with $r-$dimension so its time complexity is $O\left(r^2\right)$. From Theorem 1, it is evident that $\sum_{j=1}^{L} m_j \geq r$, so the time complexity of WMF-PF is smaller than CF-PF.

$$\bar{\boldsymbol{\phi}}_k(\boldsymbol{x}_k) = \pi^{-\frac{n}{2}} \begin{bmatrix} \prod_{\lambda=1}^{n} \Delta x_{1_\lambda}\mu_\lambda^{-n}\psi\left(\frac{x_\lambda - x'_{i_\lambda}}{\mu_\lambda}\right) \\ \prod_{\lambda=1}^{n-1} \Delta x_{1_\lambda}\mu_\lambda^{-1}\psi\left(\frac{x_\lambda - x'_{1_\lambda}}{\mu_\lambda}\right)\cdot \Delta x_{2_n}\mu_n^{-1}\psi\left(\frac{x_n - x'_{2_n}}{\mu_n}\right) \\ \vdots \\ \prod_{\lambda=1}^{n-1} \Delta x_{1_\lambda}\mu_\lambda^{-1}\psi\left(\frac{x_\lambda - x'_{1_\lambda}}{\mu_\lambda}\right)\cdot \Delta x_{\Upsilon_n}\mu_n^{-1}\psi\left(\frac{x_n - x'_{\Upsilon_n}}{\mu_n}\right) \\ \prod_{\lambda=1}^{n-2} \Delta x_{1_\lambda}\mu_\lambda^{-1}\psi\left(\frac{x_\lambda - x'_{1_\lambda}}{\mu_\lambda}\right)\cdot \Delta x_{2_{n-1}}\mu_{n-1}^{-1}\psi\left(\frac{x_{n-1} - x'_{2_{n-1}}}{\mu_{n-1}}\right)\cdot \Delta x_{1_n}\mu_n^{-1}\psi\left(\frac{x_n - x'_{1_n}}{\mu_n}\right) \\ \vdots \\ \prod_{\lambda=1}^{n-2} \Delta x_{1_\lambda}\mu_\lambda^{-1}\psi\left(\frac{x_\lambda - x'_{1_\lambda}}{\mu_\lambda}\right)\cdot \Delta x_{2_{n-1}}\gamma_{n-1}^{-1}\psi\left(\frac{x_{n-1} - x'_{2_{n-1}}}{\mu_{n-1}}\right)\cdot \Delta x_{\Upsilon_n}\gamma_n^{-1}\psi\left(\frac{x_n - x'_{\Upsilon_n}}{\mu_n}\right) \\ \vdots \\ \prod_{\lambda=1}^{n-1} \Delta x_{S_\lambda}\mu_\lambda^{-1}\psi\left(\frac{x_\lambda - x'_{\Upsilon_\lambda}}{\mu_\lambda}\right)\cdot \Delta x_{1_n}\mu_n^{-1}\psi\left(\frac{x_n - x'_{1_n}}{\mu_n}\right) \\ \vdots \\ \prod_{\lambda=1}^{n} \Delta x_{\Upsilon_\lambda}\mu_\lambda^{-1}\psi\left(\frac{x_\lambda - x'_{\Upsilon_\lambda}}{\mu_\lambda}\right) \end{bmatrix}_{\Upsilon^n\times 1} \tag{26}$$

$$\bar{\boldsymbol{H}}^{(\mathrm{C})} = \left[\begin{matrix} \boldsymbol{h}^{(1)}(x'_{1_1},x'_{1_2},\cdots,x'_{1_n}) & \boldsymbol{h}^{(1)}(x'_{1_1},x'_{1_2},\cdots,x'_{2_n}) & \cdots & \boldsymbol{h}^{(1)}(x'_{1_1},x'_{1_2},\cdots,x'_{\Upsilon_n}) \\ \boldsymbol{h}^{(2)}(x'_{1_1},x'_{1_2},\cdots,x'_{1_n}) & \boldsymbol{h}^{(2)}(x'_{1_1},x'_{1_2},\cdots,x'_{2_n}) & \cdots & \boldsymbol{h}^{(2)}(x'_{1_1},x'_{1_2},\cdots,x'_{\Upsilon_n}) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{h}^{(L)}(x'_{1_1},x'_{1_2},\cdots,x'_{1_n}) & \boldsymbol{h}^{(L)}(x'_{1_1},x'_{1_2},\cdots,x'_{2_n}) & \cdots & \boldsymbol{h}^{(L)}(x'_{1_1},x'_{1_2},\cdots,x'_{\Upsilon_n}) \end{matrix}\right.$$

$$\begin{matrix} \boldsymbol{h}^{(1)}(x'_{1_1},x'_{1_2},\cdots,x'_{2_{n-1}},x'_{1_n}) & \cdots & \boldsymbol{h}^{(1)}(x'_{1_1},x'_{1_2},\cdots,x'_{2_{n-1}},x'_{\Upsilon_n}) & \cdots \\ \boldsymbol{h}^{(2)}(x'_{1_1},x'_{1_2},\cdots,x'_{2_{n-1}},x'_{1_n}) & \cdots & \boldsymbol{h}^{(2)}(x'_{1_1},x'_{1_2},\cdots,x'_{2_{n-1}},x'_{\Upsilon_n}) & \cdots \\ \vdots & \cdots & \vdots & \cdots \\ \boldsymbol{h}^{(L)}(x'_{1_1},x'_{1_2},\cdots,x'_{2_{n-1}},x'_{1_n}) & \cdots & \boldsymbol{h}^{(L)}(x'_{1_1},x'_{1_2},\cdots,x'_{2_{n-1}},x'_{\Upsilon_n}) & \cdots \end{matrix}$$

$$\left.\begin{matrix} \boldsymbol{h}^{(1)}(x'_{\Upsilon_1},x'_{\Upsilon_2},\cdots,x'_{\Upsilon_{n-1}},x'_{1_n}) & \cdots & \boldsymbol{h}^{(1)}(x'_{\Upsilon_1},x'_{\Upsilon_2},\cdots,x'_{\Upsilon_n}) \\ \boldsymbol{h}^{(2)}(x'_{\Upsilon_1},x'_{\Upsilon_2},\cdots,x'_{\Upsilon_{n-1}},x'_{1_n}) & \cdots & \boldsymbol{h}^{(2)}(x'_{\Upsilon_1},x'_{\Upsilon_2},\cdots,x'_{\Upsilon_n}) \\ \vdots & \cdots & \vdots \\ \boldsymbol{h}^{(L)}(x'_{\Upsilon_1},x'_{\Upsilon_2},\cdots,x'_{\Upsilon_{n-1}},x'_{1_n}) & \cdots & \boldsymbol{h}^{(L)}(x'_{\Upsilon_1},x'_{\Upsilon_2},\cdots,x'_{\Upsilon_n}) \end{matrix}\right]_{\sum_{i=1}^{L} m_i\times\Upsilon^n} \tag{27}$$

Algorithm 1 WMF-PF Algorithm

1)Initialization:
Initial particles:$\hat{\boldsymbol{x}}_{0|0}^{(\mathrm{I})(i)} \sim p_{x_0}(\boldsymbol{x}_0)$ $(i = 1, \cdots, N_s)$ and initial prediction: $\hat{\boldsymbol{x}}_{1|0}^{(\mathrm{I})}$;

2)State prediction particles:

$$\hat{\boldsymbol{x}}_{k|k-1}^{(\mathrm{I})(i)} = \boldsymbol{f}_{k-1}(\hat{\boldsymbol{x}}_{k-1|k-1}^{(\mathrm{I})(i)}) + \boldsymbol{\zeta}_{k-1}^{(\mathrm{I})(i)}$$

($\boldsymbol{\zeta}_{k-1}^{(\mathrm{I})(i)}$ is random number with the same distribution of the process noise $\boldsymbol{w}_{k-1}$);

3) According to the prediction $\hat{\boldsymbol{x}}_{k|k-1}^{(\mathrm{I})}$, choose the segments and the corresponding $\bar{\boldsymbol{H}}^{(\mathrm{I})}$, $\bar{\boldsymbol{M}}$ and $\bar{\boldsymbol{\phi}}_k(\boldsymbol{x}_k)$ in weighted calculation center;

4) Fusion measurement:

$$\bar{\boldsymbol{y}}_k^{(\mathrm{I})} = \left(\bar{\boldsymbol{M}}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\bar{\boldsymbol{M}}\right)^{-1}\bar{\boldsymbol{M}}^{\mathrm{T}}\left(\boldsymbol{R}^{(\mathrm{C})}\right)^{-1}\boldsymbol{y}_k^{(\mathrm{C})}$$

5) Measurement prediction particles:

$$\hat{\boldsymbol{y}}_{k|k-1}^{(\mathrm{I})(i)} = \bar{\boldsymbol{H}}^{(I)}\bar{\boldsymbol{h}}_k(\hat{\boldsymbol{x}}_{k|k-1}^{(\mathrm{I})(i)})$$

6) The importance weight:

$$\omega_k^{(\mathrm{I})(i)} = \frac{1}{N_s}p_{v_k^{(\mathrm{I})}}(\boldsymbol{y}_k^{(\mathrm{I})} - \hat{\boldsymbol{y}}_{k|k-1}^{(\mathrm{I})(i)})$$

$$\bar{\omega}_k^{(\mathrm{I})(i)} = \frac{\omega_k^{(\mathrm{I})(i)}}{\sum_{i=1}^{N}\omega_k^{(\mathrm{I})(i)}}$$

7) Filtering:

$$\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{I})} = \sum_{i=1}^{N_s}\bar{\omega}_k^{(\mathrm{I})(i)}\hat{\boldsymbol{x}}_{k|k-1}^{(\mathrm{I})(i)}$$

$$\boldsymbol{P}_{k|k}^{(\mathrm{I})} \approx \sum_{i=1}^{N_s}\bar{\omega}_k^{(\mathrm{I})(i)}(\hat{\boldsymbol{x}}_{k|k-1}^{(\mathrm{I})(i)} - \hat{\boldsymbol{x}}_{k|k}^{(\mathrm{I})(i)})(\hat{\boldsymbol{x}}_{k|k-1}^{(\mathrm{I})(i)} - \hat{\boldsymbol{x}}_{k|k}^{(\mathrm{I})(i)})^{\mathrm{T}}$$

8) Prediction:

$$\hat{\boldsymbol{x}}_{k+1|k}^{(\mathrm{I})} = \boldsymbol{f}_k(\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{I})})$$

9) Resampling:
$\varpi_i = \dfrac{(i-1)+r}{N}$($r \sim U[0, 1]$, $i = 1, \cdots, N_s$), if $\sum_{j=1}^{m-1}\bar{\omega}_k^{(\mathrm{I})(j)} < \varpi_i \leq \sum_{j=1}^{m}\bar{\omega}_k^{(\mathrm{I})(j)}$, we copy m particles as resampling particles $\hat{\boldsymbol{x}}_{k|k}^{(\mathrm{I})(i)}$ directly;
Turn to step 2) and reiterated.

VI. SIMULATION EXAMPLES

Let us consider a 2-dimensions target tracking system with eight sensors. In 2-dimension Cartesian coordinate, the state equation is:

$$\boldsymbol{x}_{k+1} = \boldsymbol{\Phi}\boldsymbol{x}_k + \boldsymbol{\Gamma}w_k \tag{29}$$

where $\boldsymbol{x}_k = \begin{pmatrix} x_k & \dot{x}_k & y_k & \dot{y}_k \end{pmatrix}^{\mathrm{T}}$ is the state vector,

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} 0.5T^2 & 0 \\ T & 0 \\ 0 & 0.5T^2 \\ 0 & T \end{bmatrix}, \boldsymbol{w}(k)$$

is Gaussian noise with variance $\boldsymbol{Q}_w = \mathrm{diag}(0.1^2,\ 0.1^2)$.

Assume that there are eight angle sensors $\theta^{(j)}(x^{(j)},\ y^{(j)})$, $(j = 1, \cdots, 8)$ which is located at four points, i.e., $\theta^{(1)}(5,\ 5)$, $\theta^{(2)}(5,\ 5)$, $\theta^{(3)}(-5,\ 5)$, $\theta^{(4)}(-5,\ 5)$, $\theta^{(5)}(-5,\ -5)$, $\theta^{(6)}(-5,\ -5)$, $\theta^{(7)}(5,\ -5)$, $\theta^{(8)}(5,\ -5)$. Then, the measurement functions of the eight sensors can be written as:

$$\begin{aligned} z_k^{(j)} &= h_k^{(j)} + v_k^{(j)} \\ &= \arctan\left((y_k - y^{(j)})\Big/(x_k - x^{(j)})\right) + v_k^{(j)}, \\ &\qquad j = 1, \cdots, 8 \end{aligned} \tag{30}$$

where $v_k^{(i)}, v_k^{(j)}, i \neq j$ are uncorrelated and the variances are $\sigma_{v1}^2 = 0.021^2$, $\sigma_{v2}^2 = 0.022^2$, $\sigma_{v3}^2 = 0.023^2$, $\sigma_{v4}^2 = 0.024^2$, $\sigma_{v5}^2 = 0.025^2$, $\sigma_{v6}^2 = 0.026^2$, $\sigma_{v6}^2 = 0.026^2$ and $\sigma_{v8}^2 = 0.028^2$, respectively. In simulation, we set the sample period is $T = 200$ms and the initial state is $\boldsymbol{x}(0) = 0$.

$$\bar{\boldsymbol{\phi}}_k(\boldsymbol{x}_k) = \frac{1}{\pi\,(0.9)^2}\begin{bmatrix} \exp\left\{-\left(\frac{x-x_1'}{0.9^2}\right)^2\left(\frac{y-y_1'}{0.9^2}\right)^2\right\} \\ \exp\left\{-\left(\frac{x-x_1'}{0.9^2}\right)^2\left(\frac{y-y_2'}{0.9^2}\right)^2\right\} \\ \vdots \\ \exp\left\{-\left(\frac{x-x_1'}{0.9^2}\right)^2\left(\frac{y-y_S'}{0.9^2}\right)^2\right\} \\ \exp\left\{-\left(\frac{x-x_2'}{0.9^2}\right)^2\left(\frac{y-y_1'}{0.9^2}\right)^2\right\} \\ \vdots \\ \exp\left\{-\left(\frac{x-x_2'}{0.9^2}\right)^2\left(\frac{y-y_S'}{0.9^2}\right)^2\right\} \\ \vdots \\ \exp\left\{-\left(\frac{x-x_S'}{0.9^2}\right)^2\left(\frac{y-y_1'}{0.9^2}\right)^2\right\} \\ \vdots \\ \exp\left\{-\left(\frac{x-x_S'}{0.9^2}\right)^2\left(\frac{y-y_S'}{0.9^2}\right)^2\right\} \end{bmatrix}_{4^2\times 1} \tag{31}$$

From the experiment of approximation offline, we use $\mu_i = 0.9$, $\Delta x_{i_\lambda} = 1$ $(i = 1, \cdots, \Upsilon)$ and $\hbar = 0$ in the simulation. From Equations (15)-(19), we have Equation (31).

In order to reduce the computational cost, the target area is divided into 1 square kilometers as Figure 2 (a). Take No.7 piece for example, its distribution of the sample points is shown as Figure 2 (b). According to the sample

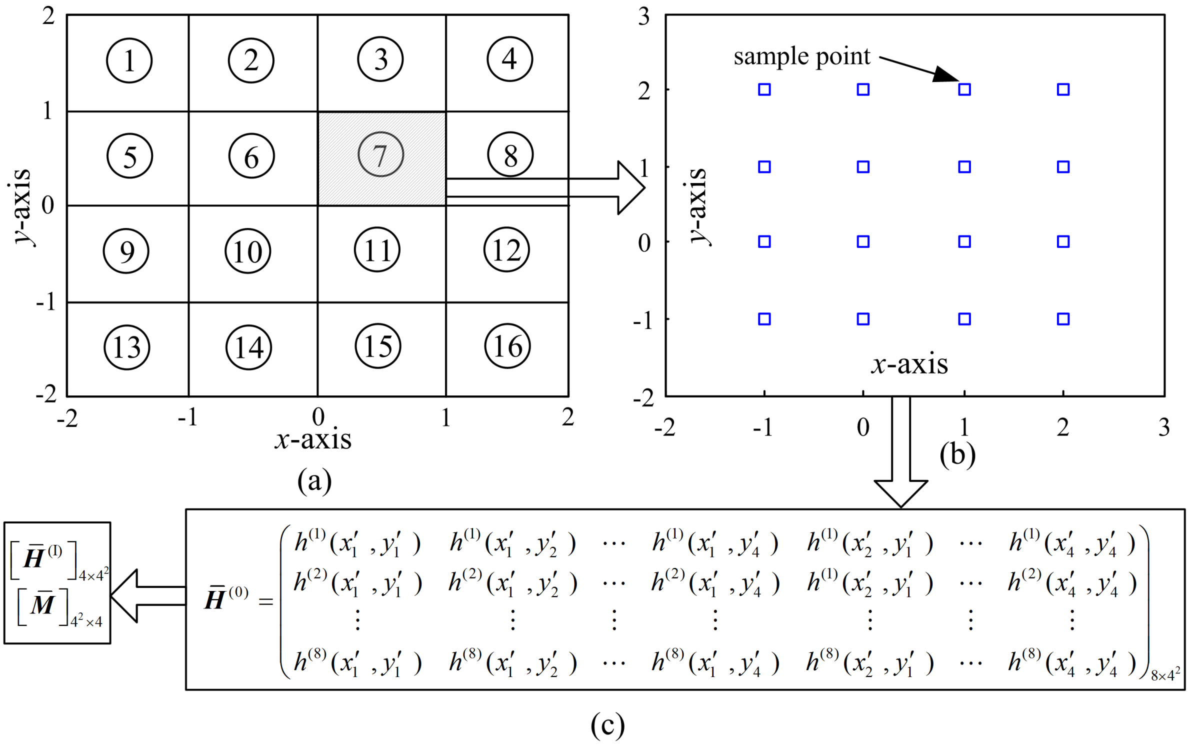

FIGURE 2. Fusion matrix $\bar{H}^{(\mathrm{I})}$ and $\bar{M}$.

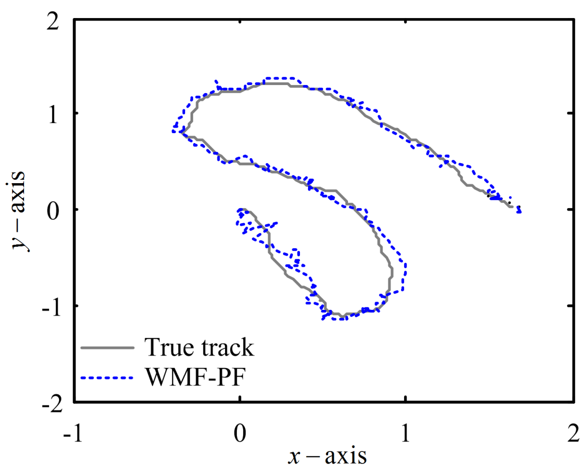

FIGURE 3. True track and the estimated tracks using WMF-PF.

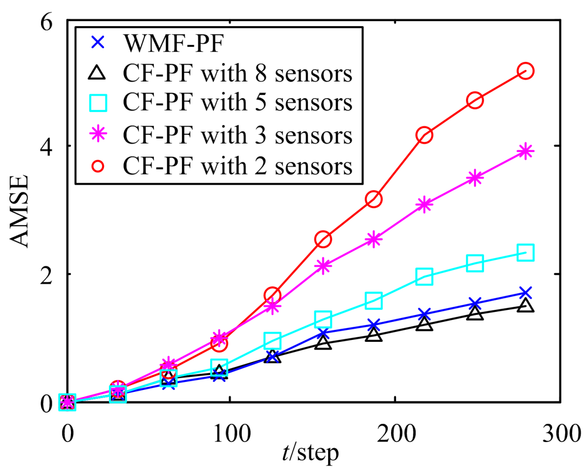

FIGURE 4. AMSEs of the distances between the actual and estimated positions.

points, we get the matrices $\bar{\boldsymbol{H}}^{(\mathrm{C})}$, $\bar{\boldsymbol{H}}^{(\mathrm{I})}$ and $\bar{\boldsymbol{M}}$ shown as Figure 2(c). In the simulation, because there are two different sensors in the same place, the $\bar{\boldsymbol{H}}^{(\mathrm{I})} \in \Re^{4\times 16}$ is a matrix with 4-rows.

In order to compare the impact of the number of sensor network nodes on system accuracy and computational cost, the sensor networks with different numbers of sensors are compared. To compare with CF-PFs with different sensors, the selection of these sensors is based on the dispersion principle, i.e., we chose sensors 1 and 3 to show CF-PF with 2 sensors, sensors 1, 3 and 5 to show CF-PF with 3 sensors, and sensors 1, 3, 5, 7 and 8 to show CF-PF with 5 sensors. The true track and the estimated track using WMF-PF are shown in Figure 3.

The estimation performance is accumulated mean square error (AMSE) in position at time k [37], [39], [40]:

$$\mathrm{AMSE}(k) = \sum_{\tau=0}^{k} \frac{1}{N} \sum_{\chi=1}^{N} \Big((x^{\chi}(\tau) - \hat{x}^{\chi}(\tau|\tau))^2 + (y^{\chi}(\tau) - \hat{y}^{\chi}(\tau|\tau))^2 \Big) \tag{32}$$

where $(x^{\tau}(t),\ y^{\tau}(t))$ and $(\hat{x}^{\tau}(t|t),\ \hat{y}^{\tau}(t|t))$ are the true and estimated positions of the ith Monte Carlo experiment at time t.

The AMSE curves of WMF-PF, CF-PFs with 8 sensors, 5 sensors, 3 sensors and 2 sensors are shown in Figure 4 with 20 Monte Carlo experiments. From Figure 4 we can see that the accuracy from high to low followed by CF-PF with 8 sensors, WMF-PF, CF-PF with 5 sensors, CF-PF with 3 sensors and CF-PF with 2 sensors. This shows that the number of sensors has an important influence on the accuracy of the estimation systems. But the computational cost from high to low followed by CF-PF with 8 sensors, CF-PF with 5 sensors, WMF-PF, CF-PF with 3 sensors and CF-PF with 2 sensors, because the $\bar{\boldsymbol{H}}^{(\mathrm{I})} \in \Re^{4\times 4^2}$ is a matrix with 4-rows. As the number of sensors increases, the dimension of the WMF measurement equation will not change, but the dimension of the centralized fusion system will greatly increase. So, the advantage of the proposed WMF-PF in computational cost will be more obvious for massive sensor systems.

VII. CONCLUSION AND FUTURE WORKS

In this article, we firstly use the segmental Gauss-Hermite approximation method combined with the WLS method to present a universal nonlinear WMF algorithm. Then, based on the WMF algorithm and PF, a nonlinear WMF-PF is presented. The proposed WMF-PF can deal with the fusion estimation problems for multidimensional nonlinear multi-sensor systems. Compared with centralized fusion PF, it has an approximate accuracy and less computational cost. The advantage of the proposed WMF filtering algorithm in computational cost will be more obvious for massive sensors systems. In simulation, we analyzed the WMF-PF, CF-PFs with 8 sensors, 5 sensors, 3 sensors and 2 sensors in accuracy and computational cost. Experimental results verify the effectiveness of the proposed WMF-PF algorithm.

Although the approximation effect of the Gauss-Hermite approximation method is better, just like other approximation functions (such as polynomial interpolation), its convergence has not been proven, that is it will not asymptotically converge as the Hermite polynomial increases. In the next work, we will study the convergence of the method. In addition, other methods that can be used to approximate nonlinear functions (such as neural networks) will also become one of our research contents.

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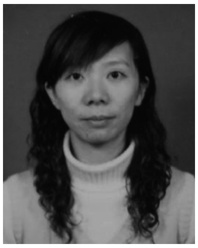

YUN LI was born in Qiqihar, Heilongjiang, China, in 1978. She received the B.S. and M.E. degrees in automation and the Ph.D. degree in control science and engineering from Heilongjiang University, Harbin, in 2005 and 2018.

Since 2005, she has been an Assistant Professor with the Computer and Information Engineering Department, Harbin University of Commerce. She is the author of seven books, more than 30 articles, and three inventions. Her research interests include information fusion and state estimation.