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Observer Design for Actuator Failure of a Quadrotor

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ABSTRACT This article addresses the problem of fault estimation for a quadrotor unmanned aircraft vehicle (UAV). A robust H_∞ observer is proposed to achieve fault and state estimation of a quadrotor UAV with actuator fault in the presence of external disturbances, parameter uncertainties and nonlinear terms. The observer can observe the system state and actuator fault simultaneously. The actuator fault estimation error is taken as an auxiliary state to transform the original system into an augmented generalized system, and a nonlinear robust H_∞ observer is designed. Based on the Lyapunov stability theory, stability analysis was carried out and a sufficient condition for the stability of the observer was established, which is expressed as an LMI optimization problem to satisfy H_∞ performance. Finally, the typical faults of two actuators of a quadrotor UAV are given, and the faults and states can be estimated by using the method proposed in this article. The results show that the proposed observer can accurately observe the system state and actuator fault before and after the occurrence of two typical faults.

INDEX TERMS Fault estimation, observer, quadrotor, H_∞.

I. INTRODUCTION

Quadrotor UAVs have the advantageous characteristics of a simple mechanical structure, low cost, vertical take-off and landing, and stable hovering, thereby providing a well-suited mission platform for a wide range of military and civilian applications [1]–[3]. In military applications, quadrotor UAVs carry out detection in restricted terrains, sample collection, military surveillance, and search and destroy missions. In the civilian applications, the quadrotor can be used for image recognition, environmental monitoring, reconnaissance and mapping after disasters, volcanic activity monitoring, and atmospheric sampling. With the further development of the quadrotor UAVs, they will inevitably be used to perform an increasing range of tasks [4]–[6].

The range of tasks performed by quadrotor aircraft has increased over the past three decades with the more in-depth research on quadrotor aircraft. The increasing demand for reliability, availability, safety and performance stability of a quadrotor UAV systems has encouraged the development of research on fault detection and isolation (FDI) and fault-tolerant control (FTC). A key feature of this approach is that it aims to prevent a simple fault evolving into a serious failure [7].

Fault-tolerant control systems may be grouped into two main families: passive fault-tolerant control systems and active fault-tolerant systems [8], [9].

In a passive fault-tolerant control system, deviations of the plant parameters from their true values or deviations of the actuators from their expected position may be efficiently compensated by a fixed robust feedback controller [10]. However, if these deviations become excessively large and exceed the robustness bound, some actions must be taken. Therefore, an active fault-tolerant control architecture is needed in order to achieve extended fault-tolerance capability.

In an active fault-tolerant control system, faults are detected and isolated by an FDI scheme, and the controllers are reconfigured accordingly online in real time. Accurate fault estimation is critical for designing high-performance active fault-tolerant control systems [11].

In recent years, with the development of system identification technology, the model-based FDI method has been widely investigated. In this approach, a residual signal is generated and then processed to detect the occurrence of the fault, and then to determine the type, location and severity of the fault [11]–[14].

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In [11], an active fault-tolerant flight control system for sensor/actuator failures of unmanned aerial vehicles (UAVs) is proposed. An adaptive two-stage linear Kalman filtering algorithm is used to isolate the sensor and actuator faults and to estimate the loss of control effectiveness and the severity of the stuck faults in a UAV model.

Reference [12] presents a fault detection, isolation, and accommodation algorithm for quadrotor actuator faults using nonlinear adaptive estimation techniques. The fault diagnosis architecture consists of a nonlinear fault detection estimator and an array of nonlinear adaptive fault isolation estimators designed based on the functional structures of the faults under consideration. Adaptive thresholds for fault detection and isolation are systematically designed to enhance the robustness and fault sensitivity of the diagnostic algorithm.

In [13], a robust model-based observer for actuators fault detection and diagnosis (FDD) is proposed and applied to a quadrotor unmanned aerial vehicle. The observer is designed to maximize the residual sensitivity to a fault by using the H_- index properties, and minimizing the H_∞ norm for worst case exogenous signal attenuation. Fault detection is formulated as a Linear matrix inequality (LMI) feasibility problem for minimizing a cost function based on a trade-off between fault sensitivity and robustness against disturbances.

In [14], a design method based on the H_-/L_∞ fault detection observer is proposed. The H_- index in the finite frequency domain is used to describe the minimum fault sensitivity of the residual generated by the observer, and the L_∞ norm from unknown disturbance to residual is used to describe the robust performance of the residual to disturbance.

However, this method cannot provide accurate information for fault magnitude. Accurate fault estimation is also an important basis for fault-tolerant control (FTC) tasks.

References [15] and [16] address the problem of fault estimation for quadrotor UAV. A robust fault estimation observe is proposed to achieve fault estimation of quadrotor. In both works, observers are designed based on a linear system.

References [17] and [18] propose a novel fault estimation observer for nonlinear and normal systems that are simultaneously subjected to actuator faults, sensor faults and unstructured non-parametric uncertainties. Sufficient conditions for the existence of the proposed observer with an H_∞ performance have been derived based on the Lyapunov stability theory. However, the parameter uncertainty of the system is not considered.

In [19], an estimation method for actuator failure is proposed. A parallel bank of recurrent neural network is designed. With the trained network, the severity of actuator failure can be accurately estimated. However, external disturbances have not yet been addressed in the design.

In this work, we will investigate the fault and state estimation problems of quadrotor system with parameter uncertainties and external disturbances. The actuator fault estimation error is taken as an auxiliary state to transform the original system into an augmented generalized system, and a nonlinear robust H_∞ observer is designed. Based on the Lyapunov stability theory, a sufficient condition for the stability of the observer is established, and is expressed as an LMI optimization problem to satisfy H_∞ performance. Simulation results demonstrate the effectiveness of the proposed method.

The main contributions of the paper are summarized as follows:

a. The method proposed in this article can estimate the state of the system and the fault of the actuator at the same time, and can give accurate fault information when the actuator fault occurs.

b. For the fault and state estimation of quadrotor UAV, the parameter uncertainty, nonlinear term and external disturbance are considered simultaneously.

The rest of this article is organized as follows. In Section II, the quadrotor UAV dynamics and problem formulation are addressed. In Section III, the proposed observer is presented. Simulations results are presented in section VI. Finally, conclusions are drawn in section V.

II. MATHEMATICAL MODEL OF UAV DYNAMICS

The dynamic characteristics of an aircraft must be known in order to estimate the state and fault of the system. The equations governing the motion of a quadrotor are derived in this section [20]–[22].

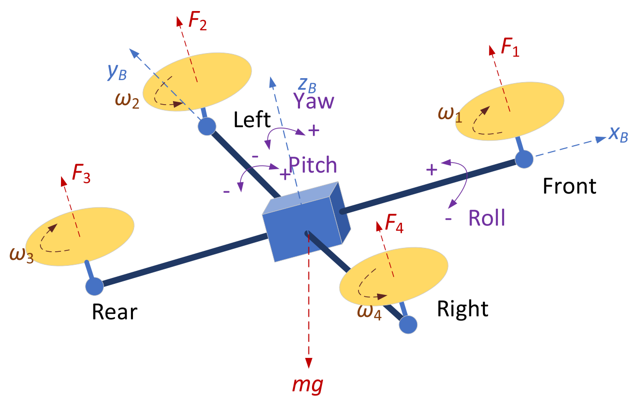

FIGURE 1. Quadrotor aircraft.

The derivation of nonlinear dynamics is carried out in the ground coordinate system $\{x_E, y_E, z_E\}$ and the body coordinate system $\{x_B, y_B, z_B\}$ as defined in Fig. 1. The Euler angles of the body axes are $\mathbf{\Phi} = [\varphi, \theta, \psi]^T$ with respect to the x_E, y_E and z_E axes, respectively, and are referred to as roll, pitch and yaw. Five forces act on the vehicle: the weight mg, and the four propeller forces of magnitude F_i that act in the body fixed direction $z_B = (0, 0, 1)$ as defined in the figure. ($i = 1$, 2, 3, 4 to represent four propellers respectively). Boldface symbols are used throughout this section to denote vectors and matrices, while non-boldface symbols such as m will generally be used for scalars, unless specified otherwise.

Six rigid body equations consisting of three force and three moment equations are obtained for the UAV. The following assumptions are made [11]:

- The aircraft is a rigid body;
- The mass of the aircraft remains constant for a relatively short duration; and
- The xz plane of the aircraft is the plane of symmetry.

Based on the above assumptions, according to the Newton-Euler formula, the dynamic model of a quadrotor UAV is established as follows

$$\dot{\boldsymbol{P}} = \boldsymbol{v} \tag{1}$$

$$m\dot{\boldsymbol{v}} = \boldsymbol{R}z_B \sum\nolimits_{i=1}^{4} F_i - m\boldsymbol{g} + \boldsymbol{F}_d \tag{2}$$

$$\dot{\boldsymbol{\Phi}} = \boldsymbol{T\omega} \tag{3}$$

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}\times\boldsymbol{J\omega} + \boldsymbol{M}_f + \boldsymbol{M}_g + \boldsymbol{M}_d \tag{4}$$

where m is the mass of the aircraft, $\boldsymbol{P} = [x, y, z]^T$ is the position in the ground coordinates, $\boldsymbol{v}= [u, v, w]^T$ is the velocity vector of the quadrotor in the ground coordinate system, $\boldsymbol{g} = (0, 0, g)^T$ is the acceleration due to gravity, $\boldsymbol{\omega} = (p, q, r)^T$ is the angular velocity in the body-coordinate system, $J = \operatorname{diag}\left(I_x, I_y, I_z\right)$ is the moment of inertia matrix; The transformation matrix $\boldsymbol{R}$ from the body coordinate system to the ground coordinate system is given by

$$\boldsymbol{R} = \begin{bmatrix} c_\theta c_\psi & s_\varphi s_\theta c_\psi - c_\varphi s_\psi & C_\varphi s_\theta c_\psi + s_\varphi s_\psi \\ c_\theta s_\psi & s_\varphi s_\theta s_\psi + c_\varphi c_\psi & c_\varphi s_\theta s_\psi - s_\varphi c_\psi \\ -s_\theta & s_\varphi c_\theta & c_\varphi c_\theta \end{bmatrix} \tag{5}$$

The transformation matrix from the triaxial angular velocity to the Euler angular velocity is given by

$$\boldsymbol{T} = \begin{bmatrix} 1 & s_\varphi t_\theta & c_\varphi t_\theta \\ 0 & c_\varphi & -s_\varphi \\ 0 & s_\varphi / c_\theta & c_\varphi / c_\theta \end{bmatrix} \tag{6}$$

where $s_{(\cdot)} \triangleq \sin(\cdot)$, $c_{(\cdot)} \triangleq \cos(\cdot)$, $t_{(\cdot)} \triangleq \tan(\cdot)$.

The aircraft is generally in a small angle and low speed flight state, so the relationship between the attitude angle and angular velocity can be simplified as follows:

$$\dot{\boldsymbol{\Phi}} = \boldsymbol{\omega} \tag{7}$$

The control torque generated by the four rotors $\boldsymbol{M_f}$ is given by

$$\boldsymbol{M}_f = \begin{bmatrix} L\left(F_4 - F_2\right) \\ L\left(F_3 - F_1\right) \\ k_g\left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right) \end{bmatrix} \tag{8}$$

where $F_i = k_t\omega_i^2$; k_t and k_g are the lift and drag coefficients, respectively, and L is the distance between the aircraft axis and the rotor axis.

The vector $\boldsymbol{M}_g$ contains the gyroscopic torques due to the combination of the rotation of the airframe and the four rotors, and is given by

$$\boldsymbol{M}_g = \sum\nolimits_{i=1}^{4} I_r\left(\boldsymbol{\omega}\times\boldsymbol{e}_z\right)(-1)^{i+1}\omega_i \tag{9}$$

where $\boldsymbol{e}_z = (0, 0, 1)^T$ is the unit vector in the ground coordinate system, and I_r represents the moment of inertia of the rotor;

$$\boldsymbol{M}_g = I_r\left(\omega_1 - \omega_2 + \omega_3 - \omega_4\right)\begin{bmatrix} q \\ -p \\ 0 \end{bmatrix} \tag{10}$$

$-\boldsymbol{\omega}\times\boldsymbol{J\omega}$ is the gyro moment that can be expressed as

$$-\boldsymbol{\omega}\times\boldsymbol{J\omega} = \begin{bmatrix} \left(I_y - I_z\right)qr \\ \left(I_z - I_x\right)pr \\ \left(I_x - I_y\right)pq \end{bmatrix} \tag{11}$$

Considering the external disturbance force (moment) and system modeling uncertainty of a quadrotor UAV system, the sum of disturbances $\boldsymbol{F}_d$ and $\boldsymbol{M}_d$ is considered:

$$\boldsymbol{F}_d = \left[d_x, d_y, d_z\right]^T \tag{12}$$

$$\boldsymbol{M}_d = \left[d_p, d_q, d_r\right]^T \tag{13}$$

Based on the above analysis, the dynamic equations of a quadrotor UAV are established as follows:

$$\begin{cases} \dot{u} = \dfrac{U_1}{m}\left(C_\varphi S_\theta C_\psi + S_\varphi S_\psi\right) + d_x \\ \dot{v} = \dfrac{U_1}{m}\left(C_\varphi S_\theta S_\psi - S_\varphi C_\psi\right) + d_y \\ \dot{w} = \dfrac{U_1}{m}C_\varphi C_\theta - g + d_z \\ \dot{p} = \dfrac{1}{I_x}\left[LU_2 + \left(I_y - I_z\right)qr\right] - \dfrac{I_r}{I_x}q\cdot\gamma + d_\varphi \\ \dot{q} = \dfrac{1}{I_y}\left[LU_3 + \left(I_z - I_x\right)pr\right] + \dfrac{I_r}{I_y}p\cdot\gamma + d_\theta \\ \dot{r} = \dfrac{1}{I_z}\left[U_4 + \left(I_x - I_y\right)pq\right] + d_\psi \end{cases} \tag{14}$$

where $\gamma = -\omega_1 + \omega_2 - \omega_3 + \omega_4$, $\boldsymbol{u} = [U_1, U_2, U_3, U_4]^T$ is the control vector.

$$\boldsymbol{u} = \boldsymbol{D\tau} \tag{15}$$

$$\boldsymbol{D} = \begin{bmatrix} k_t & k_t & k_t & k_t \\ 0 & -k_t & 0 & k_t \\ -k_t & 0 & k_t & 0 \\ -k_g & k_g & -k_g & k_g \end{bmatrix} \tag{16}$$

where k_t and k_g are the lift and drag coefficients, respectively, and $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3, \tau_4]^T$ can be expressed as

$$\begin{bmatrix} \tau_1 & \tau_2 & \tau_3 & \tau_4 \end{bmatrix}^T = \begin{bmatrix} \omega_1^2 & \omega_2^2 & \omega_3^2 & \omega_4^2 \end{bmatrix}^T \tag{17}$$

III. DESIGN OF FAULT OBSERVER

The quadrotor is a typical underactuated system [23]. The system states x and y are indirectly driven by θ and φ, and the system states θ, φ, ψ, z are full drive subsystems. The control structure of the aircraft is shown in Fig. 2 where (x_d, y_d, z_d) is the reference trajectory of the aircraft.

In this work, the problem of fault tolerance is studied by taking the attitude control of aircraft as an example, so that $\boldsymbol{x} = [\varphi, \theta, \psi, p, q, r]^T$, and the dynamic model (14) of

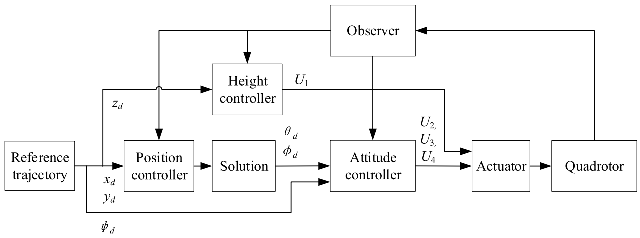

FIGURE 2. Structure of control system.

the quadrotor is written in the form of matrix that can be expressed as:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{u} + \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{E}\boldsymbol{d} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \end{cases} \tag{18}$$

where $\boldsymbol{Ax}$ is a linear term and $\boldsymbol{A}$ can be expressed as

$$A = \begin{bmatrix} \boldsymbol{O}_{3\times3} & \boldsymbol{I}_{3\times3} \\ \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} \end{bmatrix}. \tag{19}$$

$\boldsymbol{B}_1$ can be expressed as

$$\boldsymbol{B}_1 = \begin{bmatrix} & \boldsymbol{O}_{3\times4} & & \\ 0 & \dfrac{L}{I_x} & 0 & 0 \\ 0 & 0 & \dfrac{L}{I_y} & 0 \\ 0 & 0 & 0 & \dfrac{1}{I_z} \end{bmatrix}. \tag{20}$$

$f(\boldsymbol{x})$ is the nonlinear term that can be expressed as

$$f(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{O}_{3\times1} \\ \dfrac{1}{I_x}\left[\left(I_y - I_z\right)qr\right] - \dfrac{I_r}{I_x}q\cdot\gamma \\ \dfrac{1}{I_y}\left[\left(I_z - I_x\right)pr\right] + \dfrac{I_r}{I_y}p\cdot\gamma \\ \dfrac{1}{I_z}\left[\left(I_x - I_y\right)pq\right] \end{bmatrix}. \tag{21}$$

$\boldsymbol{E} \in \boldsymbol{R}^{6\times6}$ is the uncertainty matrix, $\boldsymbol{d} \in \boldsymbol{R}^6$ is the uncertainty vector of the system, $\boldsymbol{y} \in \boldsymbol{R}^6$ is the measurement output vector, and $\boldsymbol{C} \in \boldsymbol{R}^{6\times6}$ is the output matrix.

The aircraft model with actuator failure is represented as follows:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{B}\left(\boldsymbol{\tau} + \boldsymbol{f}_a\right) + \boldsymbol{E}\boldsymbol{d} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \end{cases} \tag{22}$$

where $\boldsymbol{B} = \boldsymbol{B}_1\boldsymbol{D}, \boldsymbol{f}_a \in \boldsymbol{R}^4$ denote the unknown actuator fault.

The fault observer of the system is designed based on the following assumptions:

Assumption 1: the nonlinear term $f(x)$ is assumed to be known and Lipschitz about $\boldsymbol{x}$ uniformly [24], [25], i.e., $\forall \boldsymbol{x}, \hat{x} \in \boldsymbol{R}^6$,

$$\left\| f(\boldsymbol{x}) - f(\hat{\boldsymbol{x}}) \right\| \le L_f \left\| \boldsymbol{x} - \hat{\boldsymbol{x}} \right\| \tag{23}$$

where L_f is referred to as the Lipschitz constant and is independent of $\boldsymbol{x}$ and t.

Assumption 2: when the actuator fails, the failure can be identified, and $\dot{f}_a \in L_2(0, \infty)$

For system (22), the following state observer and fault observer are designed based on assumptions 1 and 2:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}\left(\boldsymbol{\tau} + \hat{\boldsymbol{f}}_a\right) + \boldsymbol{L}\left(\boldsymbol{y} - \hat{\boldsymbol{y}}\right) + \boldsymbol{f}(\hat{\boldsymbol{x}}) \\ \hat{\boldsymbol{y}} = \boldsymbol{C}\hat{\boldsymbol{x}} \\ \dot{\hat{\boldsymbol{f}}}_a = \boldsymbol{KCe} \end{cases} \tag{24}$$

where $\hat{\boldsymbol{x}} \in \boldsymbol{R}^6$ is the state estimation vector, $\hat{\boldsymbol{f}}_a \in \boldsymbol{R}^4$ is the actuator fault deviation estimate, and $\boldsymbol{L} \in \boldsymbol{R}^{6\times6}$ and $\boldsymbol{K} \in \boldsymbol{R}^{4\times6}$ are the observer proportional gain matrix to be designed.

From (22) and (24), the estimation error equation can be expressed as follows:

$$\begin{cases} \dot{\boldsymbol{e}} = (\boldsymbol{A} - \boldsymbol{LC})\boldsymbol{e} + \boldsymbol{B}\boldsymbol{e}_f + \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{E}\boldsymbol{d} \\ \dot{\boldsymbol{e}}_f = -\boldsymbol{KCe} + \dot{\boldsymbol{f}}_a \end{cases} \tag{25}$$

where $\boldsymbol{e}_f = \boldsymbol{f}_a - \hat{\boldsymbol{f}}_a$ is the fault estimation error. $\boldsymbol{e} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ is the state estimation error.

Equation (25) can be represented in a more compact form as

$$\dot{\tilde{\boldsymbol{e}}} = \tilde{\boldsymbol{A}}\tilde{\boldsymbol{e}} + \tilde{\boldsymbol{f}}(\boldsymbol{x}) + \tilde{\boldsymbol{E}}\tilde{\boldsymbol{d}} \tag{26}$$

where

$$\tilde{\boldsymbol{e}} = \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{e}_f \end{bmatrix}^T \tag{27}$$

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A} - \boldsymbol{LC} & \boldsymbol{B} \\ -\boldsymbol{KC} & \boldsymbol{O}_{4\times4} \end{bmatrix} \tag{28}$$

$$\tilde{\boldsymbol{f}}(\boldsymbol{x}) = \begin{bmatrix} f(\boldsymbol{x}) - f(\hat{\boldsymbol{x}}) \\ \boldsymbol{O}_{4\times4} \end{bmatrix}^T \tag{29}$$

$$\tilde{\boldsymbol{d}} = \begin{bmatrix} \boldsymbol{d} \\ \dot{\boldsymbol{f}}_a \end{bmatrix}^T \tag{30}$$

$$\tilde{\boldsymbol{E}} = \begin{bmatrix} \boldsymbol{E} & \boldsymbol{O}_{6\times4} \\ \boldsymbol{O}_{4\times6} & \boldsymbol{I}_{4\times4} \end{bmatrix}. \tag{31}$$

The following theorem gives the existence condition of the proposed observer satisfying the following: the observer error dynamics (26) are asymptotically stable when $\tilde{\boldsymbol{d}} = 0$, namely, there is no system uncertainty and fault, or the actuator fault is time-invariant.

Theorem 1: Consider system (22) satisfying Assumptions 1 and 2, when $\tilde{\boldsymbol{d}} = 0$. If there exist positive definite matrices, $\boldsymbol{P}, \boldsymbol{Q}$ and matrices $\boldsymbol{Y}_1, \boldsymbol{Y}_2$ such that

$$\begin{bmatrix} \boldsymbol{\Omega} & \boldsymbol{PB} - \boldsymbol{C}^T\boldsymbol{Y}_2^T & \boldsymbol{P} & 0 \\ \boldsymbol{B}^T\boldsymbol{P} - \boldsymbol{Y}_2\boldsymbol{C} & \boldsymbol{L}_f^2\boldsymbol{I} & 0 & \boldsymbol{Q} \\ \boldsymbol{P} & 0 & -\boldsymbol{I} & 0 \\ 0 & \boldsymbol{Q} & 0 & -\boldsymbol{I} \end{bmatrix} < 0 \tag{32}$$

where $\boldsymbol{\Omega} = \boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{PA} - \boldsymbol{C}^T\boldsymbol{Y}_1^T - \boldsymbol{Y}_1\boldsymbol{C} + \boldsymbol{L}_f^2\boldsymbol{I}$, then the observer error dynamics (26) are asymptotically stable.

Proof: Consider the Lyapunov function as

$$V(\tilde{e}) = \tilde{e}^T\tilde{\boldsymbol{P}}\tilde{e} \tag{33}$$

where $\widetilde{P} = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}$, $P \in R^{6\times 6}$ and $Q \in R^{4\times 4}$ are symmetric positive definite matrices.

Then, the time derivative of $V(\widetilde{e})$ can be shown to be

$$\begin{aligned} \dot{V}(\widetilde{e}) &= \dot{\widetilde{e}}^T \widetilde{P}\widetilde{e} + \widetilde{e}^T \widetilde{P}\dot{\widetilde{e}} \\ &= \left(\widetilde{e}^T \widetilde{A}^T + \widetilde{f}^T + \widetilde{d}^T \widetilde{E}^T\right) \widetilde{P}\widetilde{e} + \widetilde{e}^T \widetilde{P}\left(\widetilde{A}\widetilde{e} + \widetilde{f} + \widetilde{E}\widetilde{d}\right) \end{aligned} \tag{34}$$

when $\widetilde{d} = 0$, $\dot{V}(\widetilde{e})$ can be expressed as

$$\begin{aligned} \dot{V}(\widetilde{e}) &= \widetilde{e}^T \left(\widetilde{A}^T \widetilde{P} + \widetilde{P}\widetilde{A}\right)\widetilde{e} + \widetilde{f}^T \widetilde{P}\widetilde{e} + \widetilde{e}^T \widetilde{P}\widetilde{f} \\ &\le \widetilde{e}^T \left(\widetilde{A}^T \widetilde{P} + \widetilde{P}\widetilde{A}\right)\widetilde{e} + \widetilde{f}^T \widetilde{f} + \widetilde{e}^T \widetilde{P}\widetilde{P}\widetilde{e} \end{aligned} \tag{35}$$

In this article, we agree that the vector norm is $\|X\| = \sqrt{X^T X}$

$$\begin{aligned} \widetilde{f}^T \widetilde{f} &= \|\widetilde{f}\|^2 = \|f(x) - f(\hat{x})\|^2 \\ &\le L_f^2 \|x - \hat{x}\|^2 = L_f^2 \|\widetilde{e}\|^2 = L_f^2 \widetilde{e}^T \widetilde{e} \end{aligned} \tag{36}$$

$$\begin{aligned} \dot{V}(\widetilde{e}) &\le \widetilde{e}^T \left(\widetilde{A}^T \widetilde{P} + \widetilde{P}\widetilde{A}\right)\widetilde{e} + L_f^2 \widetilde{e}^T \widetilde{e} + \widetilde{e}^T \widetilde{P}\widetilde{P}\widetilde{e} \\ &\le \widetilde{e}^T (\widetilde{A}^T \widetilde{P} + \widetilde{P}\widetilde{A} + L_f^2 I + \widetilde{P}\widetilde{P})\widetilde{e} \end{aligned} \tag{37}$$

Substituting $\widetilde{A} = \begin{bmatrix} A - LC & B \\ -KC & 0 \end{bmatrix}$, $\widetilde{P} = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}$ into (37), we obtain

$$\dot{V}(\widetilde{e}) \le \widetilde{e}^T \begin{bmatrix} \Omega + PP & PB - C^T K^T Q \\ B^T P - QKC & L_f^2 I\ C\ QQ \end{bmatrix} \widetilde{e} \tag{38}$$

where $\Omega = A^T P + PA - C^T L^T P - PLC + L_f^2 I$

Let $Y_1 = PL$, $Y_2 = QK$, then (38) can be expressed as

$$\dot{V}(\widetilde{e}) \le \widetilde{e}^T \begin{bmatrix} \Omega + PP & PB - C^T Y_2^T \\ B^T P - Y_2 C & L_f^2 I + QQ \end{bmatrix} \widetilde{e} \tag{39}$$

where $\Omega = A^T P + PA - C^T Y_1^T - Y_1 C + L_f^2 I$

Using the Schur complement, condition (32) is equivalent to

$$\begin{bmatrix} \Omega + PP & PB - C^T Y_2^T \\ B^T P - Y_2 C & L_f^2 I + QQ \end{bmatrix} < 0 \tag{40}$$

substituting (40) into (39), we obtain

$$\dot{V}(\widetilde{e}) < 0. \tag{41}$$

Thus, $\widetilde{e} \to 0$ as $t \to 0$. This completes the proof.

The following theorem gives the existence condition of the proposed observer when the external disturbance and the actuator fault exist.

Theorem 2: Consider system (22) satisfying Assumptions 1 and 2. Given a positive scalar, if there exist positive definite matrices P, Q and matrices Y_1, Y_2 such that

$$\begin{bmatrix} \Omega + I & PB - C^T Y_2^T & P & 0 & PE & 0 \\ B^T P - Y_2 C & \left(L_f^2 + 1\right) I & 0 & Q & 0 & Q \\ P & 0 & -I & 0 & 0 & 0 \\ 0 & Q & 0 & -I & 0 & 0 \\ E^T P & 0 & 0 & 0 & -\gamma^2 I & 0 \\ 0 & Q & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{42}$$

then the observer error dynamics (26) are asymptotically stable and satisfy H_∞ performance $\|\widetilde{e}\|_\infty \le \gamma \|\widetilde{d}\|_\infty$,

Proof. By using (34) and (39), $\dot{V}(\widetilde{e})$ can be derived as

$$\begin{aligned} \dot{V}(\widetilde{e}) &\le \widetilde{e}^T \begin{bmatrix} \Omega + PP & PB - C^T Y_2^T \\ B^T P - Y_2 C & L_f^2 I + QQ \end{bmatrix} \widetilde{e} \\ &\quad + \widetilde{e}^T \widetilde{P}\widetilde{E}\widetilde{d} + \widetilde{d}^T \widetilde{E}^T \widetilde{P}\widetilde{e} \end{aligned} \tag{43}$$

Let

$$H = \dot{V}(\widetilde{e}) + \widetilde{e}^T \widetilde{e} - \gamma^2 \widetilde{d}^T \widetilde{d} \tag{44}$$

Then, we have

$$\begin{aligned} H &\le \widetilde{e}^T \begin{bmatrix} \Omega + PP + I & PB - C^T Y_2^T \\ B^T P - Y_2 C & L_f^2 I + QQ + I \end{bmatrix} \widetilde{e} \\ &\quad + \widetilde{e}^T \widetilde{P}\widetilde{E}\widetilde{d} + \widetilde{d}^T \widetilde{E}^T \widetilde{P}\widetilde{e} - \gamma^2 \widetilde{d}^T \widetilde{d} \\ &= \begin{bmatrix} \widetilde{e}^T & \widetilde{d}^T \end{bmatrix} \begin{bmatrix} \Omega_2 & \widetilde{P}\widetilde{E} \\ \widetilde{E}^T \widetilde{P} & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \widetilde{e} \\ \widetilde{d} \end{bmatrix} \end{aligned} \tag{45}$$

where

$$\Omega_2 = \begin{bmatrix} \Omega + PP + I & PB - C^T Y_2^T \\ B^T P - Y_2 C & L_f^2 I + QQ + I \end{bmatrix} \tag{46}$$

Write condition (42) as

$$\begin{bmatrix} \Omega_1 & \widetilde{P} & \widetilde{P}\widetilde{E} \\ \widetilde{P} & -I & 0 \\ \widetilde{e}^T \widetilde{P} & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{47}$$

where

$$\Omega_1 = \begin{bmatrix} \Omega + I & PB - C^T Y_2^T \\ B^T P - Y_2 C & \left(L_f^2 + 1\right) I \end{bmatrix} \tag{48}$$

Using the Schur complement, (47) is equivalent to

$$\Omega_1 + \widetilde{P}\widetilde{P} + \frac{1}{\gamma^2} \widetilde{P}\widetilde{E}\widetilde{E}^T \widetilde{P} < 0 \tag{49}$$

Because $\Omega_2 = \Omega_1 + \widetilde{P}\widetilde{P}$, (49) can be written as

$$\Omega_2 + \frac{1}{\gamma^2} \widetilde{P}\widetilde{E}\widetilde{E}^T \widetilde{P} < 0 \tag{50}$$

Using the Schur complement, condition (50) is equivalent to

$$\begin{bmatrix} \Omega_2 & \widetilde{P}\widetilde{E} \\ \widetilde{E}^T \widetilde{P} & -\gamma^2 I \end{bmatrix} < 0 \tag{51}$$

(51) indicates that H in (45) is negative definite.

As a result, under zero initial conditions, we have

$$\begin{aligned} J &= \int_0^\infty \left(\widetilde{e}^T \widetilde{e} - \gamma^2 \widetilde{d}^T \widetilde{d}\right) \mathrm{d}t \\ &= \int_0^\infty \left(\widetilde{e}^T \widetilde{e} - \gamma^2 \widetilde{d}^T \widetilde{d} + \dot{V}\right) \mathrm{d}t - \int_0^\infty \dot{V} \mathrm{d}t \\ &= \int_0^\infty \left(\widetilde{e}^T \widetilde{e} - \gamma^2 \widetilde{d}^T \widetilde{d} + \dot{V}\right) \mathrm{d}t - (V(\infty) - V(0)) \end{aligned}$$

$$\le \int_0^{\infty} \left(\tilde{e}^T \tilde{e} - \gamma^2 \tilde{d}^T \tilde{d} + \dot{V} \right) \mathrm{d}t$$
$$= \int_0^{\infty} H \mathrm{d}t < 0 \tag{52}$$

which means

$$\int_0^{\infty} \left(\tilde{e}^T \tilde{e} \right) \mathrm{d}t \le \gamma^2 \int_0^{\infty} \left(\tilde{d}^T \tilde{d} \right) \mathrm{d}t \tag{53}$$

Therefore, the H_∞ performance $\|\tilde{e}\|_\infty \le \gamma \left\| \tilde{d} \right\|_\infty$ has been established and the proof is complete.

The following theorem gives the conditions for the existence of the observer when the system has parameter uncertainty.

Lemma 1: Let $\boldsymbol{U}, \boldsymbol{W}, \boldsymbol{V}$ be any given real matrixes that have the corresponding dimensions. For a given symmetric matrix $\boldsymbol{M}$, the following statements are equivalent.

(a) For any given matrix V that meets $\boldsymbol{V^T V} \le \boldsymbol{I}, \boldsymbol{U}, \boldsymbol{W}, \boldsymbol{V}$ and $\boldsymbol{M}$ satisfy

$$\boldsymbol{M} + \boldsymbol{UVW} + \boldsymbol{W^T V^T U^T} < 0 \tag{54}$$

(b) There exists a real number $\varepsilon > 0$ that makes

$$\boldsymbol{M} + \varepsilon^{-1} \boldsymbol{U U^T} + \varepsilon \boldsymbol{W^T W} < 0 \tag{55}$$

Theorem 3: Consider system (22) satisfying Assumptions 1 and 2. The parameter uncertainty satisfies the conditions

$$\begin{pmatrix} \Delta A & \Delta \boldsymbol{B} \end{pmatrix} = \boldsymbol{HF}\begin{pmatrix} \boldsymbol{E_1} & \boldsymbol{E_2} \end{pmatrix} \tag{56}$$

where $\boldsymbol{H}, \boldsymbol{E}_1, \boldsymbol{E}_2$ are constant matrices with the corresponding dimensions, and $\boldsymbol{F}$ is an unknown matrix and satisfies $\boldsymbol{F^T F} \le \boldsymbol{I}$, Given a positive scalar γ, if there exist positive definite matrices $\boldsymbol{P}, \boldsymbol{Q}$, matrices $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$ and positive scalar ε, such that (57) then the observer error dynamics (26) are asymptotically stable and satisfy H_∞ performance $\|\tilde{e}\|_\infty \le \gamma \left\| \tilde{d} \right\|_\infty$. Furthermore, the proportional gains $\boldsymbol{L}$ and $\boldsymbol{K}$ can be computed respectively as $\boldsymbol{L} = \boldsymbol{P^{-1} Y}_1$, $\boldsymbol{K} = \boldsymbol{Q^{-1} Y}_2$

$$\begin{bmatrix} \boldsymbol{\Omega}+\boldsymbol{I} & \boldsymbol{PB}-\boldsymbol{C^T Y_2^T} & \boldsymbol{P} & 0 & \boldsymbol{PE} & 0 & \boldsymbol{E}_1^T & \boldsymbol{PH} \\ \boldsymbol{B^T P}-\boldsymbol{Y_2 C} & \left(\boldsymbol{L_f^2}+1\right)\boldsymbol{I} & 0 & \boldsymbol{Q} & 0 & \boldsymbol{Q} & \boldsymbol{E}_2^T & 0 \\ \boldsymbol{P} & 0 & -\boldsymbol{I} & 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{Q} & 0 & -\boldsymbol{I} & 0 & 0 & 0 & 0 \\ \boldsymbol{E^T P} & 0 & 0 & 0 & -\gamma^2 \boldsymbol{I} & 0 & 0 & 0 \\ 0 & \boldsymbol{Q} & 0 & 0 & 0 & -\gamma^2 \boldsymbol{I} & 0 & 0 \\ \boldsymbol{E}_1 & \boldsymbol{E}_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \boldsymbol{H^T P} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0 \tag{57}$$

Proof: Let

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{\Omega}+\boldsymbol{I} & \boldsymbol{PB}-\boldsymbol{C^T Y_2^T} & \boldsymbol{P} & 0 & \boldsymbol{PE} & 0 \\ \boldsymbol{B^T P}-\boldsymbol{Y_2 C} & \left(\boldsymbol{L_f^2}+1\right)\boldsymbol{I} & 0 & \boldsymbol{Q} & 0 & \boldsymbol{Q} \\ \boldsymbol{P} & 0 & -\boldsymbol{I} & 0 & 0 & 0 \\ 0 & \boldsymbol{Q} & 0 & -\boldsymbol{I} & 0 & 0 \\ \boldsymbol{E^T P} & 0 & 0 & 0 & -\gamma^2 \boldsymbol{I} & 0 \\ 0 & \boldsymbol{Q} & 0 & 0 & 0 & -\gamma^2 \boldsymbol{I} \end{bmatrix} \tag{58}$$

Substituting $\boldsymbol{A}$ and $\boldsymbol{B}$ in (42) with $\boldsymbol{A} + \Delta \boldsymbol{A}$ and $\boldsymbol{B} + \Delta \boldsymbol{B}$, and at the same time substituting (56) into (42) as follows,

$$\boldsymbol{Z} + \boldsymbol{M_1 F M_2} + \boldsymbol{M_2^T F^T M_1^T} < 0 \tag{59}$$

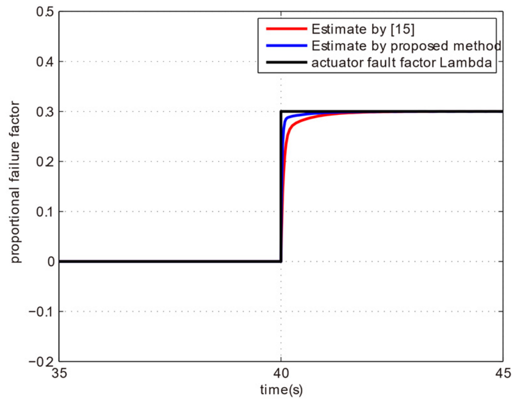

FIGURE 3. Fault estimation of the actuator for a proportional fault.

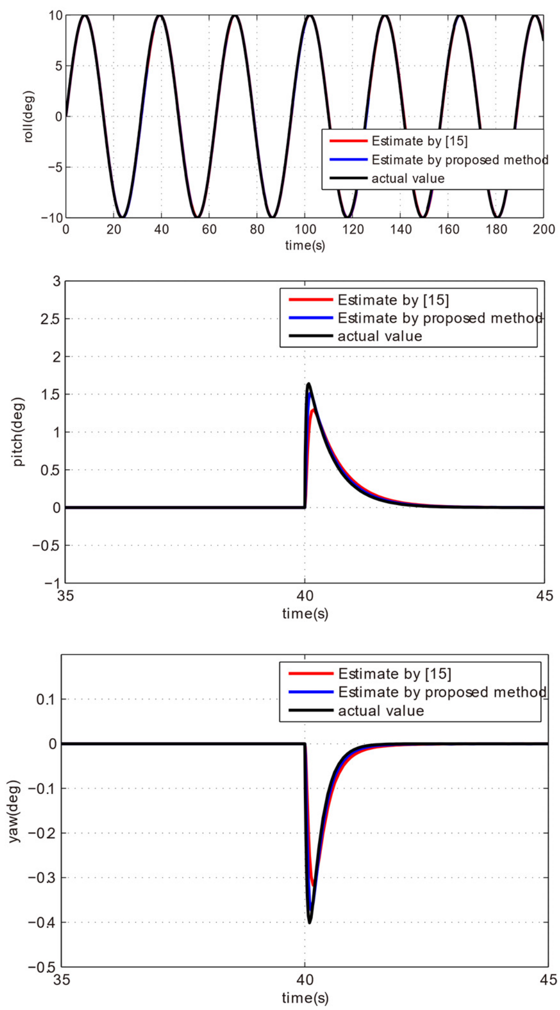

FIGURE 4. Attitude estimation of actuator for a proportional failure.

where $\boldsymbol{M_1} = \begin{bmatrix} \boldsymbol{PH} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\boldsymbol{M_2} = \begin{bmatrix} \boldsymbol{E}_1 & \boldsymbol{E}_2 & 0 & 0 & 0 & 0 \end{bmatrix}$

TABLE 1. Main parameters of the Quadrotor UAV.

Symbol	Quantity	Value
g	gravity acceleration	$9.81 m/s^2$
m	mass of the quadrotor	$0.65\ kg$
l	distance from each rotor to the center of mass	$0.225\ m$
I_x	inertia of x-axis	$0.025 kgm^2$
I_y	inertia of y-axis	$0.025 kgm^2$
I_z	inertia of z-axis	$0.043 kgm^2$
I_r	rotor inertia	$4.6 \times 10^{-5} kgm^2$
k_t	lift coefficient	$1.9 \times 10^{-6} Ns^2$
k_g	drag coefficient	$7.0 \times 10^{-8} Nms^2$

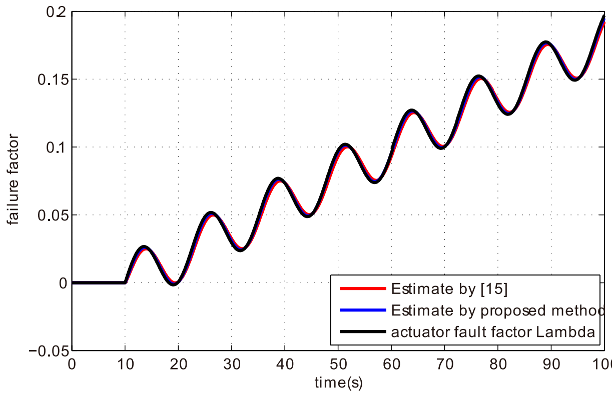

FIGURE 5. Failure estimation of the actuator for a time varying fault.

According to lemma 1, (59) is equivalent to

$$Z + \varepsilon M_1 M_1^T + \varepsilon^{-1} M_2^T M_2 < 0 \tag{60}$$

Using the Schur complement, (60) is equivalent to

$$\begin{bmatrix} Z & M_2^T & M_1 \\ M_2 & -\varepsilon I & 0 \\ M_1^T & 0 & -\varepsilon^{-1} I \end{bmatrix} < 0 \tag{61}$$

Multiply (61) both on the left-hand and right-hand sides by $\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \varepsilon I \end{bmatrix}$

The proof is completed.

The variables in the linear matrix (57) are P, Q, ε, Y_1, Y_2, so that the controller design finds the feasible solution of the linear matrix inequality.

To obtain a robust fault estimation observer for the system (22), using the LMI toolbox in MATLAB, the convex optimization problem with LMI constraints is solved.

IV. SIMULATION RESULT

To verify the correctness and effectiveness of the proposed robust fault estimation observer design in quadrotor aircraft, the method proposed in this article is compared to the results of reference [15], and the following two kinds of actuator faults are simulated and analyzed. The failure is expressed as $\tau_i(t) = (1 - \lambda_i)\tau_{di}(t), t \geq t_i, i = 1, 2, 3, 4, \tau_i(t), \tau_{di}(t)$ represents the actual input and design input of the ith rotor of the system, and t_i represents the time of failure. λ_i represents the severity of the ith rotor fault, and the larger λ_i, the more serious the fault. Table 1 lists the parameters of the quadrotor model. The desired attitude trajectories in the following simulations are chosen as $\varphi_d = 10\sin(0.2t)\ deg, \theta_d = 0, \psi_d = 0$. Considering the imbalance of the body counterweight [20], the constant disturbance added to the attitude loop is $\left[d_\varphi, d_\theta, d_\psi\right] = [0.5, -0.5, 0.5]$. The uncertain parameter can be expressed as $H = [1, 1, 1, 1, 1, 1]$,

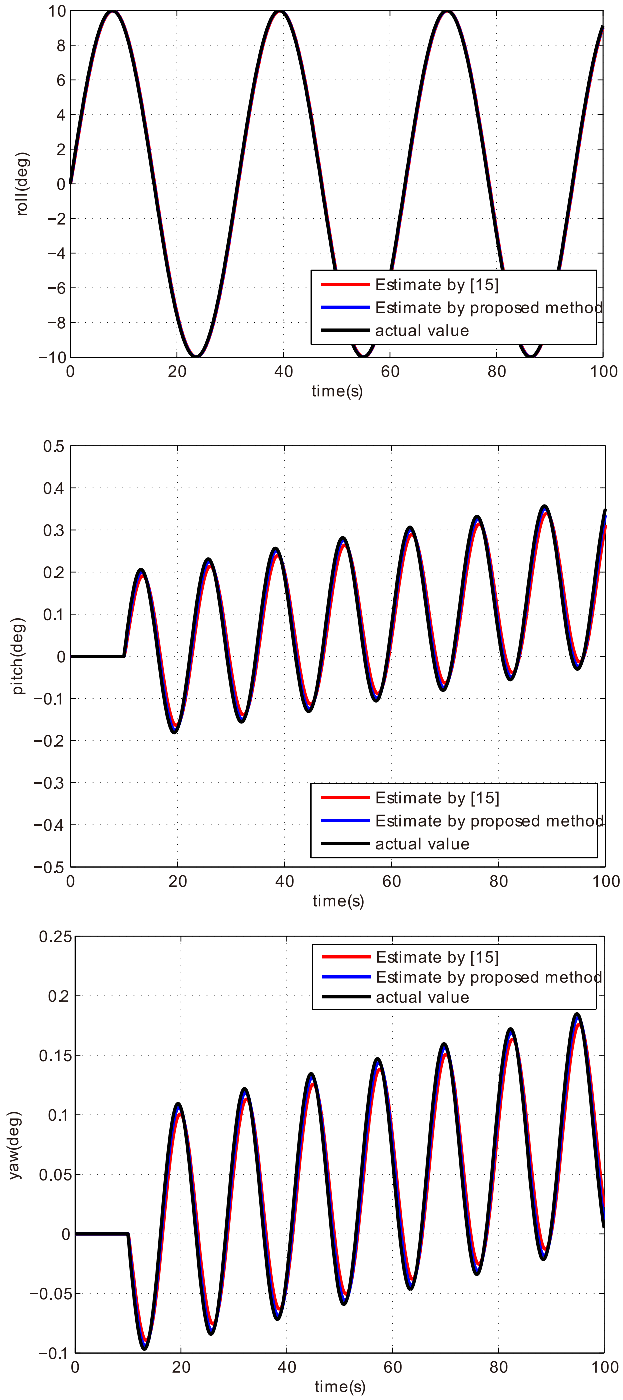

FIGURE 6. Attitude estimation for a time varying fault.

$E_1 = [0.1, 0.1, 0.1, 0.1, 0.1, 0.1]^T$, $F = \sin(t)$, $E_2 = 10^{-5} \times [0.3, 0.3, 0.3, 0.3, 0.3, 0.3]$, $L_f = 0.3$

A. PROPORTIONAL FAULT SIMULATION

Proportional fault means that the lift and torque moment of the propeller decrease at the same rotation speed, but the lift and the square of speed, the torque moment torque and the square of speed still meet the linear relationship. Proportional fault means $\lambda_i =$ constant. The loss of propeller blades is an example of such a fault [26].

For example, regarding the first rotor of the aircraft, it is assumed that there is a proportional failure with a failure factor of 0.3. The fault occurred at 40 s. The method of fault estimation is used to estimate the fault. The results of fault estimation are illustrated in Figs. 3 and 4.

These figures show that when the actuator has a proportional fault, the proposed observer can accurately estimate the state before and after the actuator failure, and can successfully estimate the actuator fault. Compared to the method in reference [15], the proposed observer has a higher estimation speed and higher estimation accuracy.

B. TIME VARYING FAULT SIMULATION

For a time-varying failure such as rotor speed instability, the failure loss ratio changes with time. The time varying fault is expressed as $\lambda_i = 0.02 \sin(0.5(t-10)) + 0.002\,(t-10)$. The fault occurred at 10 s.

Fig. 5 shows the estimation results of a time-varying fault. Clearly, the proposed method is also valid for time-varying faults. Fig. 6 shows the attitude estimation under a time varying-fault. It is observed that the proposed method has better estimation performance than the method of reference [15].

V. CONCLUSION

In this article, the problem of fault estimation of an actuator of a quadrotor UAV with parameter uncertainty and external disturbance is studied. A new method for the simultaneous estimation of the system state and actuator fault is proposed. Specifically, the actuator fault estimation error is taken as an auxiliary state to transform the original system into an augmented generalized system, and nonlinear robust H_∞ observer is designed. Based on the Lyapunov stability theory, a sufficient condition for the observer stability is established, and is expressed as an LMI optimization problem to satisfy H_∞ performance. Comparison of the simulations under different actuator fault scenarios demonstrate that the proposed observer can effectively estimate the system state and actuator fault simultaneously. In the future, we will make further improvements and verify the effectiveness of the proposed observer through real-time flight experiments on a quadrotor UAV.

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