Hyperchaotic Behavior in the Novel Memristor-Based Symmetric Circuit System

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ABSTRACT A novel five-dimensional (5D) memristor-based symmetric circuit, which consists of two symmetric capacitors, two symmetric inductors and only one memristor is presented in this article. The multivariable first order and multivariable second order polynomial functions are used for the internal state function of the memristor respectively. Theoretical and simulation analyses of the novel memristive circuit are investigated using equilibrium points, phase portraits, bifurcation diagrams and Lyapunov exponent spectra, etc. Complex chaotic behaviors are observed and analyzed through simulation results. The first order internal state function memristor-based symmetric circuit system can only exhibit chaotic behavior whereas the second order internal state function memristor-based symmetric circuit system can generate not only chaotic attractors, but also hyperchaotic attractors in proper parameters.

INDEX TERMS Chaos attractor, hyperchaotic attractor, memristor, second order internal state, symmetric circuit.

I. INTRODUCTION

Memristor, known as the fourth basic fundamental circuital element in electronic circuits, was first put forward the hypothesis by Leon Chua in 1971 [1]. Not until 2008, this postulation was proved by Hewlett-Packard (HP) laboratory. They successfully fabricated the first actual physical model of the memristor as a TiO2 nanocomponent [2]. Since then, various kinds of circuits with the memristor have been investigated and the nonlinear dynamical characteristic of these circuits have been studied. Such as memristor oscillators and memristor-based crossbar [3]–[9]. As a novel element, its ability to combine processing and memory are expected to bring various benefit in artificial intelligence, machine learning, neuromorphic computing and self-learning. For example, researchers have designed some neural networks combined with memristor. Paper [10] proposed a new circuit based on memristor and two MOSFET transistors to realize synapse. Wen *et al.* [11] proposed online least mean square algorithm for echo state network based on memristor. A simple electronic circuit composed of a LC contour and a memristor is proposed in [12] to get series of voltage pulses that mimic

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environment changes. Hu *et al.* [13] proposed a dynamic synaptic design based on memristor and experimental calibration of memristor model. Liu *et al.* [14] given a simple and effective method for designing autonomous memristor chaotic system with infinite chaotic attractor.

Memristor-based circuits can exhibit complex dynamics and easily generate chaotic signals due to the nonlinearity induced by memristors. Many memristor-based circuits have been reported to construct chaotic systems in the last few years. Sun *et al.* [15] designed the autonomous memristor chaotic systems with infinite chaotic attractors. Fan *et al.* [16] proposed a simplified neural network based on fractional-order memristor with discontinuous memductance function. Paper [17] studied a class of chaotic systems based on piecewise linear memristor. Zhang *et al.* [18] designed the Josephson Junction circuit employing memristor, and investigated the chaos encryption as well. Paper [19] derived a novel four-wing memristive chaotic system by bringing a flux-controlled memristor with quadratic nonlinearity into the Liu-Chen system as a feedback term.

Hyperchaos was first published by Rössler in 1979 [20] and has attracted more and more attention from various scientific and engineering communities. Hyperchaotic systems with multiple positive Lyapunov exponents usually have
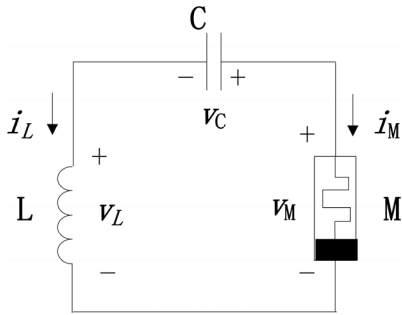
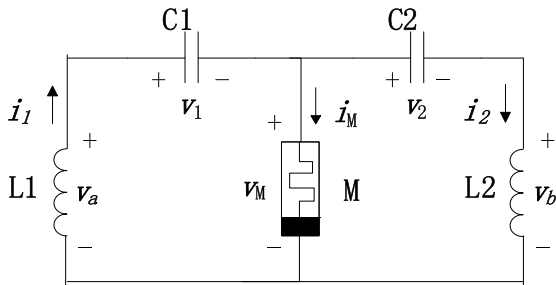**FIGURE 1.** Circuit structure of the simplest chaotic circuit.

FIGURE 2. Circuit structure of the memristor-based symmetric chaotic circuit.

more complex and richer dynamic behaviors than chaotic systems which can enhance the randomness and higher unpredictability of the corresponding system. Generating hyperchaotic attractors is a very attractive work in theory, but it is quite challenging in technology. Some researchers have investigated the hyperchaotic attractor in multi-dimensional circuits systems and memristor-based dynamical circuits systems. Paper [21] studied the nonlinear dynamics of the TCMNL hyperchaotic oscillator with gyrators based on a smooth mathematical model of the system. Sambas *et al.* [22] reported a new five-dimensional four-wing hyperchaotic system with hidden attractor. Wei *et al.* [23] proposed and investigated a 5D hyperchaotic generalization of a 3D model for a self-exciting homopolar disc dynamo without unstable equilibria but with three positive Lyapunov exponents. Paper [24] reported the finding of a new hyperchaotic temperature fluctuations model and described its modelling. Li *et al.* [25] studied a four-dimensional (4D) memristive system modified from the 3D chaotic system proposed by Lü and Chen. A novel memristive hyperchaotic system is presented in paper [26] by introducing a memristor to instead a coupling resistor in the realization of three-dimensional chaotic circuit system. Bao *et al.* [27] utilized a memristor to substitute a linear resistor to expand the active band pass filter-based memristive circuit, and presented a novel fifth-order two memristor-based Chua's hyperchaotic circuit. The good characteristics of chaotic systems, such as pseudo-random behavior, sensitive on initial conditions etc., can be applied to cryptography and confidential communication [28]–[30]. And hyperhcaotic systems with more complex and richer dynamic behaviors can improve the security of the encryption schemes.
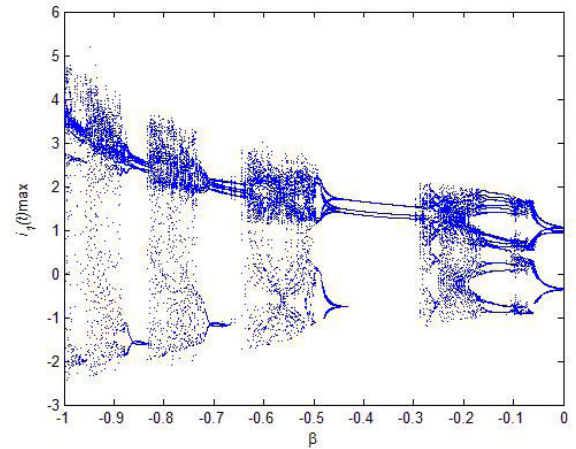
(a) Bifurcation diagram for $i_{L1}(t)$ against β
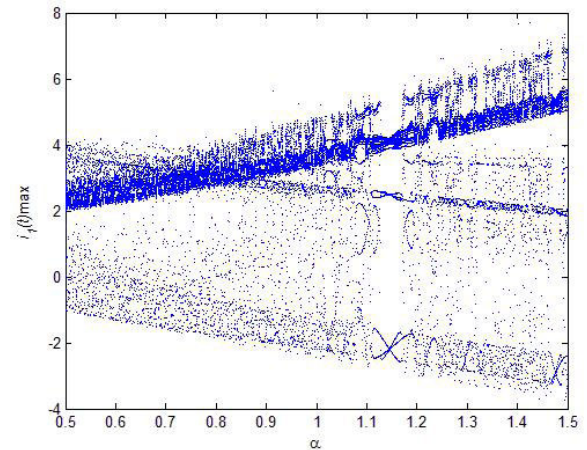
(b) Bifurcation diagram for $i_{L1}(t)$ against α

FIGURE 3. Bifurcation diagrams of the first order memristor-based symmetric system.

Most memristive hyperchaotic systems are generated by introducing or substituting memristor into the classical circuit. In this article, we extended the simplest chaotic circuit [31] which only consists three elements and one of them is already memristor to build a hyperchaotic system. This simplest chaotic circuit is extended to build a five-dimensional (5D) memristor-based symmetric circuit which consists of two symmetric linear passive inductors, two symmetric linear passive capacitors and only one non-linear active memristor. The multivariable first order and multivariable second order polynomial functions are used for the internal state function of the memristor respectively. We use the second order internal state of memristor in order to make the chaotic attractors more complexity. The chaotic system can be hyperchaotic when its parameters are taken appropriately. Chaotic behaviors are illustrated using equilibrium points, bifurcation diagrams, Lyapunov exponent spectra and phase portraits, etc. Simulation results show that the first order internal state function memristor-based symmetric circuit system can only generate chaotic attractors whereas the second order internal state function memristor-based symmetric circuit system can generate not only chaotic
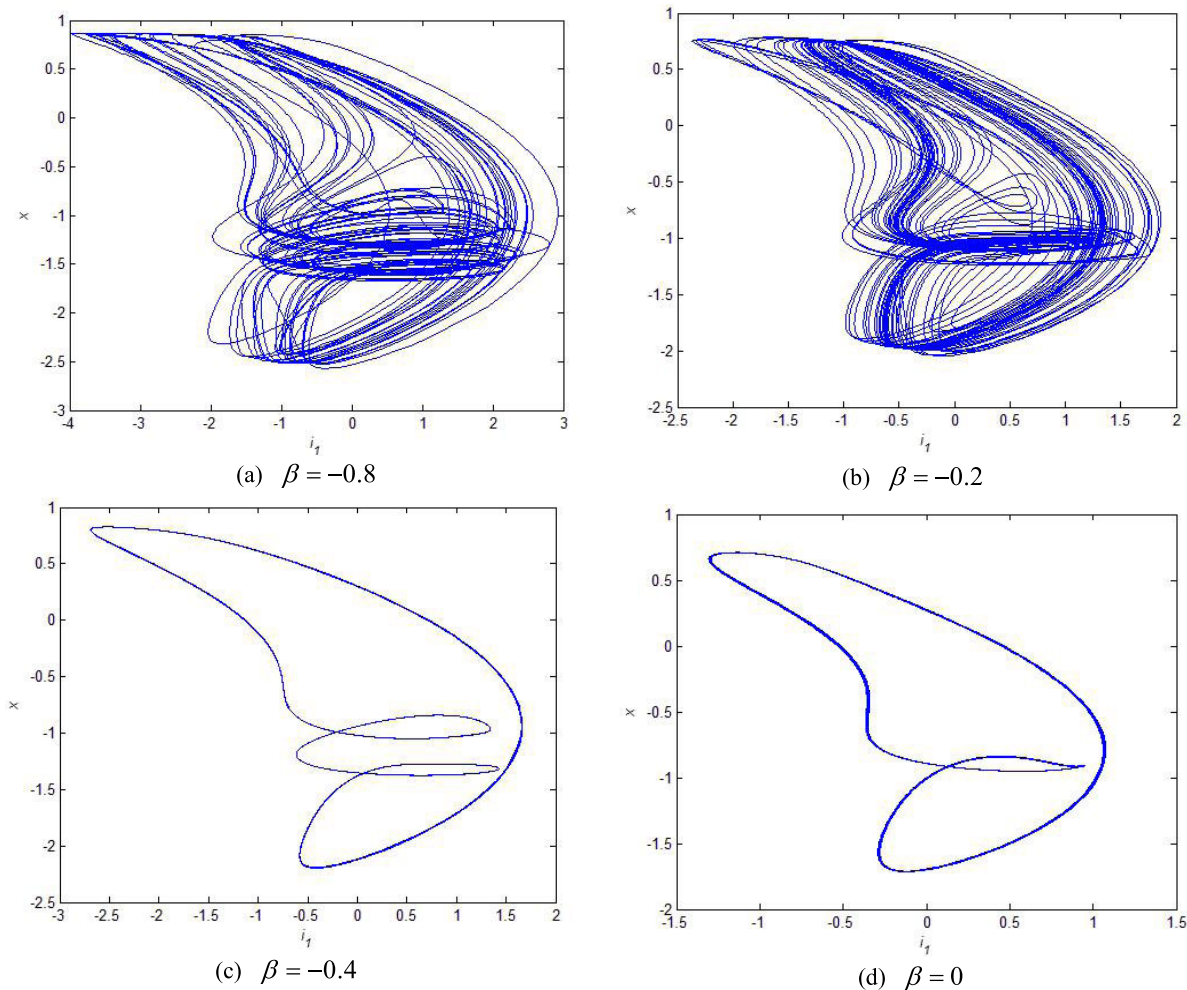
(a) $\beta = -0.8$

(b) $\beta = -0.2$

(c) $\beta = -0.4$

(d) $\beta = 0$

FIGURE 4. The $i_{L1}(t)$ versus $x(t)$ phase plots of the first order memristor-based symmetric circuit system.

attractors, but also hyperchaotic attractors in proper parameters.

This article is organized as follows. In section 2, a novel memristor-based symmetric chaotic system circuit is presented. In section 3, the first order of the memristor internal state case is analyzed. In section 4, the second order of the memristor internal state case is analyzed, the bifurcation diagram and the corresponding Lyapunov exponent spectrum are plotted with the system parameters changed, which reveals the dynamic behavior of hyperchaos. Section 5 concludes the paper.

II. MEMRISTOR-BASED SYMMETRIC CHAOTIC SYSTEM

Paper [31] presented the simplest memristor-based circuit which can exhibited chaotic behavior. This circuit only consists of one linear passive inductor, one linear passive capacitor and one non-linear active memristor in series shown in Figure 1.

The circuit dynamics are described by:

$$\begin{cases} \dot{x} = \dfrac{y}{C} \\ \dot{y} = \dfrac{-1}{L}\left[x + \beta(z^2 - 1)y\right] \\ \dot{z} = -y - \alpha z + yz \end{cases}$$

In this article, we extended this circuit to build an autonomous memristor-based symmetric circuit which consists of two symmetric linear passive inductors, two symmetric linear passive capacitors and one non-linear active memristor. Memristors can be classified as flux-controlled and charge-controlled memristors. A charge-controlled memristor is used in this article, and its circuit structure is presented in Figure 2.

This memristor-based symmetric circuit consists five elements, so the state variables for this circuit can defined as

$$z(t) = \begin{pmatrix} v_{C1}(t) \\ i_{L1}(t) \\ x(t) \\ v_{C2}(t) \\ i_{L2}(t) \end{pmatrix} \tag{1}$$

where $v_{C1}(t)$ and $v_{C2}(t)$ denotes the voltage across the terminals of capacitors $C1$ and $C2$ respectively, $i_{L1}(t)$ and $i_{L2}(t)$ represents the currents flowing through inductor $L1$ and $L2$ respectively, and the internal state variable of the memristor is represented by $x(t)$.
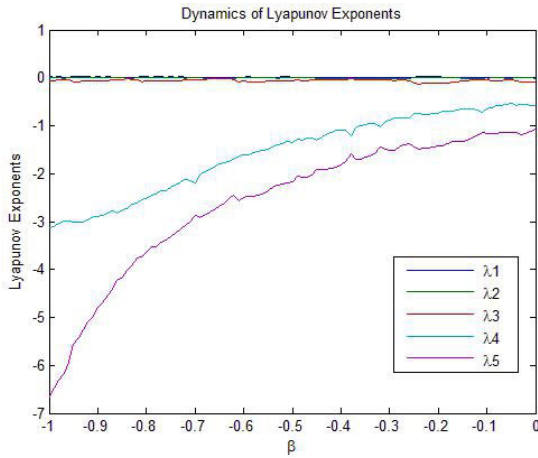
(a) Lyapunov exponents spectrum
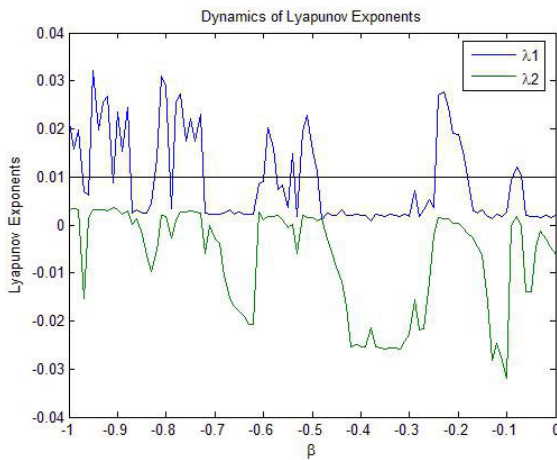
(b) Two maximum Lyapunov exponents λ_1 and λ_2

FIGURE 5. Lyapunov exponents spectrum versus β of the first order memristor-based symmetric circuit system.

The first two state equations to construct the dynamical system are the current-voltage relations of two capacitors.

$$C1 \frac{dv_{C1}(t)}{dt} = i_{L1}(t) \tag{2}$$

$$C2 \frac{dv_{C2}(t)}{dt} = i_{L2}(t) \tag{3}$$

There are two loops in this memristor-based symmetric circuit, so the other two state equations can be obtained by applying Kirchhoff's voltage law around the loops.

$$L1 \frac{di_{L1}(t)}{dt} = - \{v_{C1}(t) + R(x(t)) \times (i_1(t) - i_2(t))\} \tag{4}$$

$$L2 \frac{di_{L2}(t)}{dt} = -v_2(t) + R(x(t)) \times (i_1(t) - i_2(t)) \tag{5}$$

The memristance function of M is $R(x(t))$ and is defined as $R(x(t)) = 2x(t)^2 - 1 + \beta$ in this article.

The multivariable first order and multivariable second order polynomial functions are used as the internal state memristor function respectively. We analysis the chaotic behaviors in these two cases and find that the exponent of the internal state chosen second order can make the chaotic

TABLE 1. Lyapunov exponents of the first order memristor-based symmetric circuit system.

Figure	Parameter	Lyapunov exponents	Property
4(a)	$\beta = -0.8$	{0.0292, 0.0017, -0.075, -2.51, -3.632}	Chaos
4(b)	$\beta = -0.2$	{0.0172, 0, -0.1133, -0.7533}	Chaos
4(c)	$\beta = -0.4$	{0.0019, -0.025, -0.031, -1.093, -1.822}	Periodicity
4(d)	$\beta = 0$	{0.0019, -0.006, -0.082, -0.59, -1.078 }	Periodicity

attractor and orbit more complex, the chaotic system can be hyperchaotic when its parameters are taken appropriately.

The first order internal state of the memristor defined as

$$\frac{dx(t)}{dt} = -(i_1(t) - i_2(t)) - \alpha x(t) + (i_1(t) - i_2(t)) x(t) \tag{6}$$

The second order internal state of the memristor defined as

$$\frac{dx(t)}{dt} = -(i_1(t) - i_2(t)) - \alpha x(t) + (i_1(t) - i_2(t))^2 x(t) \tag{7}$$

III. FIRST ORDER OF THE MEMRISTOR INTERNAL STATE CASE

The first order of the memristor internal state case is analyzed. The five-dimensional symmetric memristive chaotic system with first order of the memristor internal state can be constructed using (2)-(6):

$$\begin{cases} \dfrac{dv_1(t)}{dt} = \dfrac{i_1(t)}{C_1} \\ \dfrac{di_1(t)}{dt} = -\dfrac{1}{L_1}(v_1(t) + R(x(t)) \times (i_1(t) - i_2(t))) \\ \dfrac{dx(t)}{dt} = -(i_1(t) - i_2(t)) - \alpha x(t) + (i_1(t) - i_2(t)) x(t) \\ \dfrac{dv_2(t)}{dt} = \dfrac{i_2(t)}{C_2} \\ \dfrac{di_2(t)}{dt} = \dfrac{1}{L_2}(-v_2(t) + R(x(t)) \times (i_1(t) - i_2(t))) \end{cases} \tag{8}$$

where the circuit parameters are $C_1 = 3$, $L_1 = 1$, $C_2 = 1$, $L_2 = 1$.

A. ANALYSIS OF BIFURCATION DIAGRAMS AND PHASE PORTRAITS

Fixing system parameter as $\alpha = 1$ and the setting the five initial condition of state variables as $v_{C1}(0) = 0.1$, $i_{L1}(0) = 0$, $x(0) = 0.1$, $v_{C2}(0) = 0.1$ and $i_{L2}(0) = 0$, the bifurcation diagram for control parameter β over the range $-1 \le \beta \le 0$ is generated in Figure 3(a). From Figure 3(a), the system goes through a complex alternation of numerous chaotic orbit and periodic states. Take $\beta = -0.8$ and $\beta = -0.2$ as examples, the system exhibits the chaotic behavior which are shown in Figure 4(a) and Figure 4(b). The system exhibits the periodic behavior as shown in Figure 4(c) and Figure 4(d) when $\beta = -0.4$ and $\beta = 0$.

(a) The plot of $v_1(t)$ versus $i_1(t)$

(b) The plot of $v_2(t)$ versus $i_2(t)$

(c) The plot of $v_1(t)$ versus $x(t)$

(d) The plot of $v_2(t)$ versus $x(t)$

(d) The plot of $v_2(t)$ versus $x(t)$

(f) The plot of $i_2(t)$ versus $x(t)$

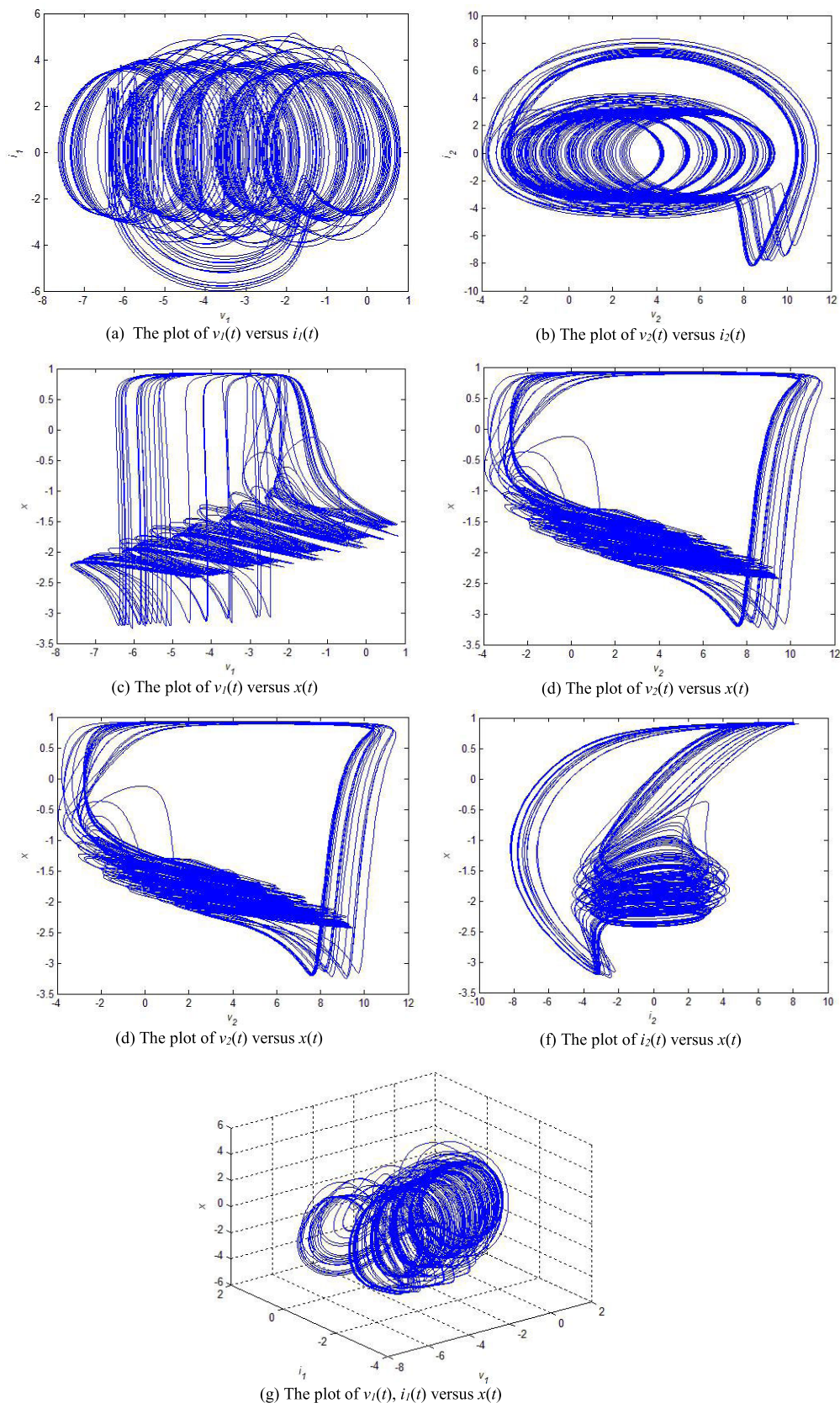(g) The plot of $v_1(t)$, $i_1(t)$ versus $x(t)$

FIGURE 6. Chaotic attractors of the first order internal state function symmetric system.

Fixing system parameter as $\beta = -1$, the bifurcation diagram for control parameter α over the range $0.5 \leq \alpha \leq 1.5$ is generated in Figure 3(b). It can be acquired that the system can generate chaotic attractors all through the range.
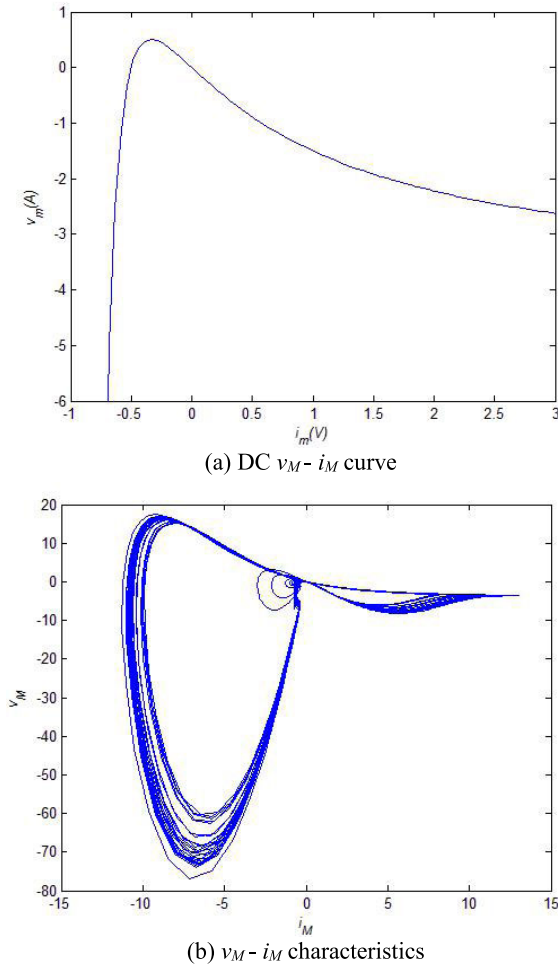
(a) DC v_M - i_M curve

(b) v_M - i_M characteristics

FIGURE 7. Memristor characteristics of the first order internal state function symmetric system.

(a) Bifurcation diagram for $i_{LI}(t)$ against β

(b) Bifurcation diagram for $i_{LI}(t)$ against α

FIGURE 8. Bifurcation diagrams of the second order memristor function symmetric system.

B. LYAPUNOV EXPONENT SPECTRA

The evidence of chaos can be provided by Lyapunov exponent spectra [32], [33]. For the five-dimensional system, assume the five Lyapunov exponents are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$, where $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4 > \lambda_5$ and $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 < 0$. The relations between the Lyapunov exponents and the orbits of the system are described as follows:

(1) When the system is in periodic orbits, the Lyapunov exponents of the system are one zero and four negatives. That is if the Lyapunov spectrum is $\lambda_1 = 0$ and $\lambda_2, \lambda_3, \lambda_4, \lambda_5 < 0$, system produces a stable limit cycle. (2) When the system is in quasi-periodic orbits, the Lyapunov exponents of the system are two zero and three negatives. That is if the Lyapunov spectrum is $\lambda_1 = 0, \lambda_2 = 0$ and $\lambda_3, \lambda_4, \lambda_5 < 0$, system produces a two-dimensional closed torus. (3) When the system is in chaotic orbits, the Lyapunov exponents of the system are one positive, one zero and three negatives. That is if the Lyapunov spectrum is $\lambda_1 > 0, \lambda_2 = 0$ and $\lambda_3, \lambda_4, \lambda_5 < 0$, system is in chaotic state. (4) When the system is in hyperchaotic orbits, the Lyapunov exponents of the system are two positives, one zero and two negatives. That is if the Lyapunov spectrum is $\lambda_1 > 0, \lambda_2 > 0, \lambda_3 = 0$ and $\lambda_4, \lambda_5 < 0$, system is in hyperchaotic state.
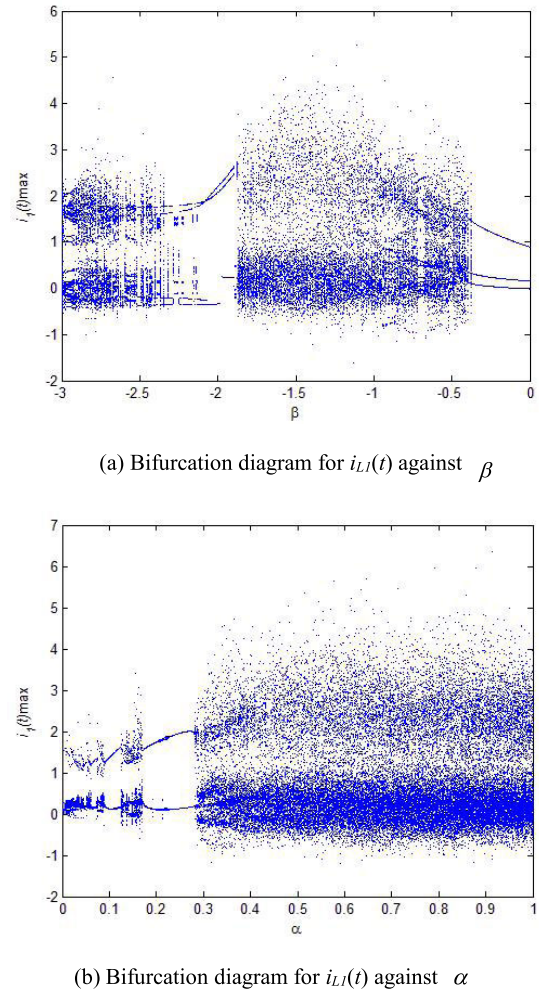
The Lyapunov exponents are calculated by wolf method [25]. Due to the numerical error in the calculation process by computer, that $|\lambda_i| < 0.01$ can be considered equivalent to $\lambda_i = 0$. Fixing $\alpha = 1$ and varying β, the Lyapunov exponents spectrum is shown in Figure 5.

Figure 5(b) shows the maximum Lyapunov exponent spectrum and the second largest Lyapunov exponent spectrum. From Figure 5(b) and the bifurcation diagram Figure 3(a), it can be seen that the spectrum of Lyapunov exponents and the bifurcation diagram are one to one correspondence. There are only one positive Lyapunov exponent during some intervals, that means the first order memristor-based symmetric circuit system can only generate chaotic orbits but cannot generate hyperchaotic attractors.

The phase portraits of Figure 4 and the corresponding Lyapunov exponents results are listed in Table 1. From data shown in Table 1, we can see that the systems with parameter β selected in Figure 4(a) and Figure 4(b) display the chaotic state, while the systems parameter corresponding to Figure 4(c) and Figure 4(d) produce a limit cycle and in periodic state.
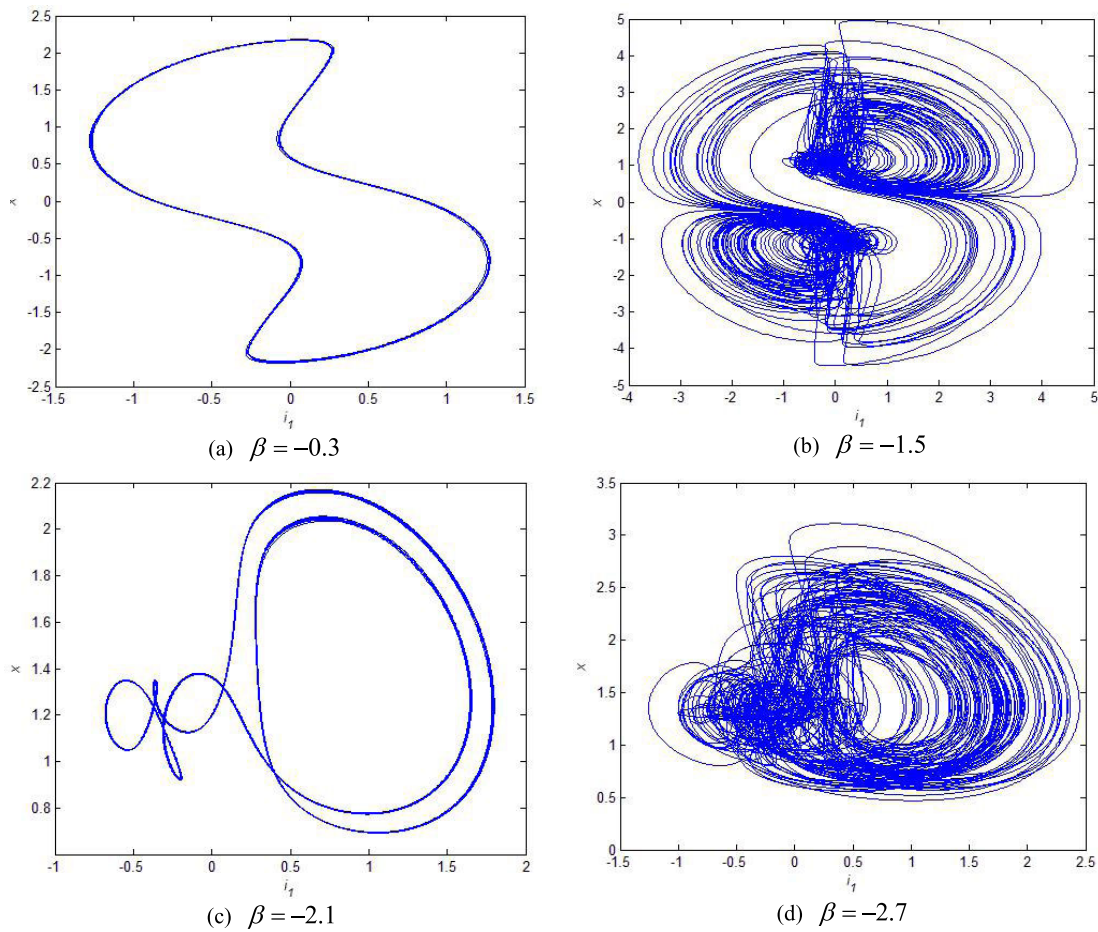
(a) $\beta = -0.3$

(b) $\beta = -1.5$

(c) $\beta = -2.1$

(d) $\beta = -2.7$

FIGURE 9. The plot of $i_{L1}(t)$ versus $x(t)$ with the second order memristor function symmetric system.

C. ANALYSIS OF CHAOTIC ATTRACTORS

Set the parameters as $\beta = -1$ and $\alpha = 1$, chose the initial conditions as $v_{C1}(0) = 0.1$, $i_{L1}(0) = 0$, $x(0) = 0.1$, $v_{C2}(0) = 0.1$ and $i_{L2}(0) = 0$ to simulate the first order memristor function symmetric system. The chaotic attractors can be obtained as shown in Figure 6. We can see that the system exhibits multiple-scroll attractor.

When the first order memristor function symmetric system exhibits the chaotic state, the memristor DCv_M- i_M curve (9) and the plot of v_M- i_M memristor characteristics are shown in Figure 7.

$$\begin{cases} x_{DC} = \dfrac{i_M}{\alpha + i_M} \\ v_M = R(x_{DC})i_M \end{cases} \tag{9}$$

in this equation, set the i_M to 10A amplitude sine-wave and the frequency to 0.5Hz.

IV. SECOND ORDER OF THE MEMRISTOR INTERNAL STATE CASE

Hyperchaotic systems which have multiple positive Lyapunov exponents usually exhibit more complex and richer dynamic behaviors than chaotic systems which only have one positive Lyapunov exponents. Because the first order memristor-based symmetric circuit system can only generate chaotic attractor, the multivariable second order polynomial function is adopted as the internal state of the memristor to increase the complexity of the chaotic attractor and attempt to get the hyperchaos. The five-dimensional symmetric memristive chaotic system with second order of the memristor internal state can be constructed using (2)-(5) and (7):

$$\begin{cases} \dfrac{dv_1(t)}{dt} = \dfrac{i_1(t)}{C_1} \\ \dfrac{di_1(t)}{dt} = -\dfrac{1}{L_1}\left(v_1(t) + R(x(t)) \times (i_1(t) - i_2(t))\right) \\ \dfrac{dx(t)}{dt} = -(i_1(t) - i_2(t)) - \alpha x(t) + (i_1(t) - i_2(t))^2 x(t) \\ \dfrac{dv_2(t)}{dt} = \dfrac{i_2(t)}{C_2} \\ \dfrac{di_2(t)}{dt} = \dfrac{1}{L_2}\left(-v_2(t) + R(x(t)) \times (i_1(t) - i_2(t))\right) \end{cases} \tag{10}$$

where the circuit parameters are $C_1 = 3$, $L_1 = 1$, $C_2 = 1$, $L_2 = 1$.

A. STABILITY ABALYSIS

In order to calculate the equilibrium point of the memristive chaotic system with second-order memristor internal state, the left side of (10) is set to zero. Obviously, $P_0 = (0, 0, 0, 0, 0)$
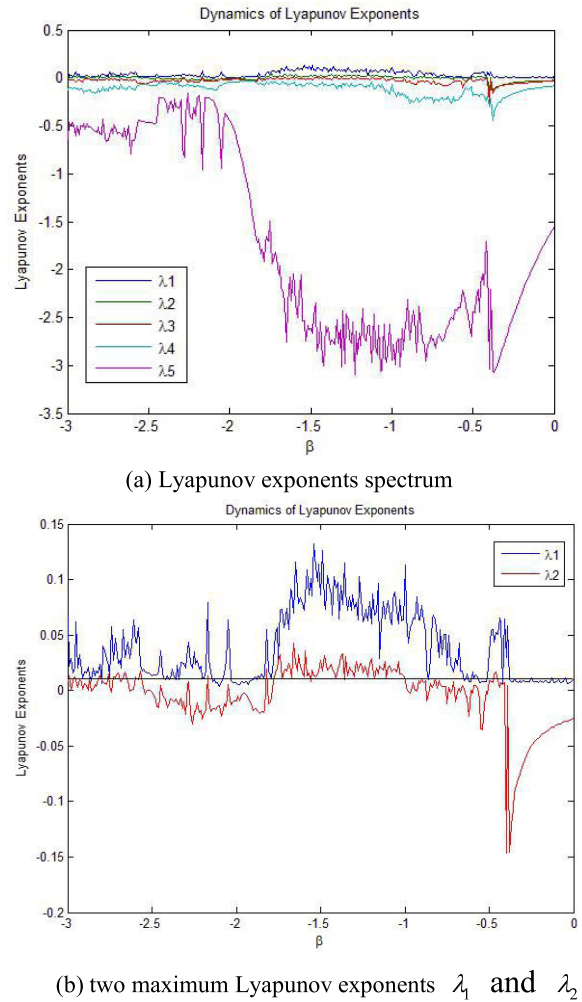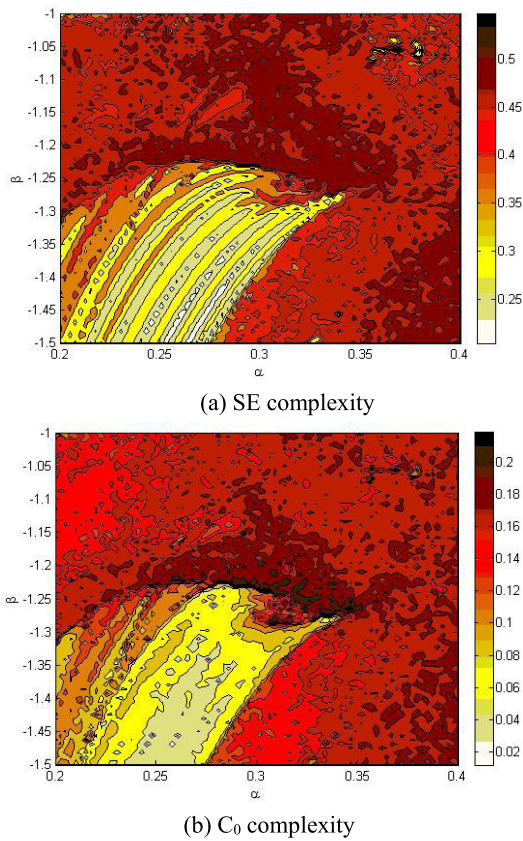
(a) SE complexity

(b) C_0 complexity

FIGURE 10. The dynamical analysis.

is the only one equilibrium set of this system. The corresponding Jacobian matrix at equilibrium P_0 can be derived in (11), as shown at the bottom of the page.

The characteristic polynomial of (11) with the parameters set above is

$$(\lambda + \alpha)(\lambda^4 + 2\lambda^3(\beta - 1) + \frac{4}{3}\lambda^2 + \frac{4}{3}(\beta - 1)\lambda + \frac{1}{3}) = 0 \tag{12}$$

where α is a positive number, denote

$$\lambda^4 + 2\lambda^3(\beta - 1) + \frac{4}{3}\lambda^2 + \frac{4}{3}(\beta - 1)\lambda + \frac{1}{3} = 0 \tag{13}$$

According to the Routh-Hurwitz stability conditions, not all the real parts of the root λ of (13) are negative. Therefore, not all roots λ of (12) have negative real parts. This means that the equilibrium point P_0 is unstable. The necessary condition for the existence of chaotic attractor is the eigenvalue λ in the unstable region. Therefore, the five-dimensional symmetric memristive chaotic system with second order of the

(a) Lyapunov exponents spectrum

(b) two maximum Lyapunov exponents λ_1 and λ_2

FIGURE 11. Lyapunov exponents spectrum versus β of the second order memristor function symmetric system.
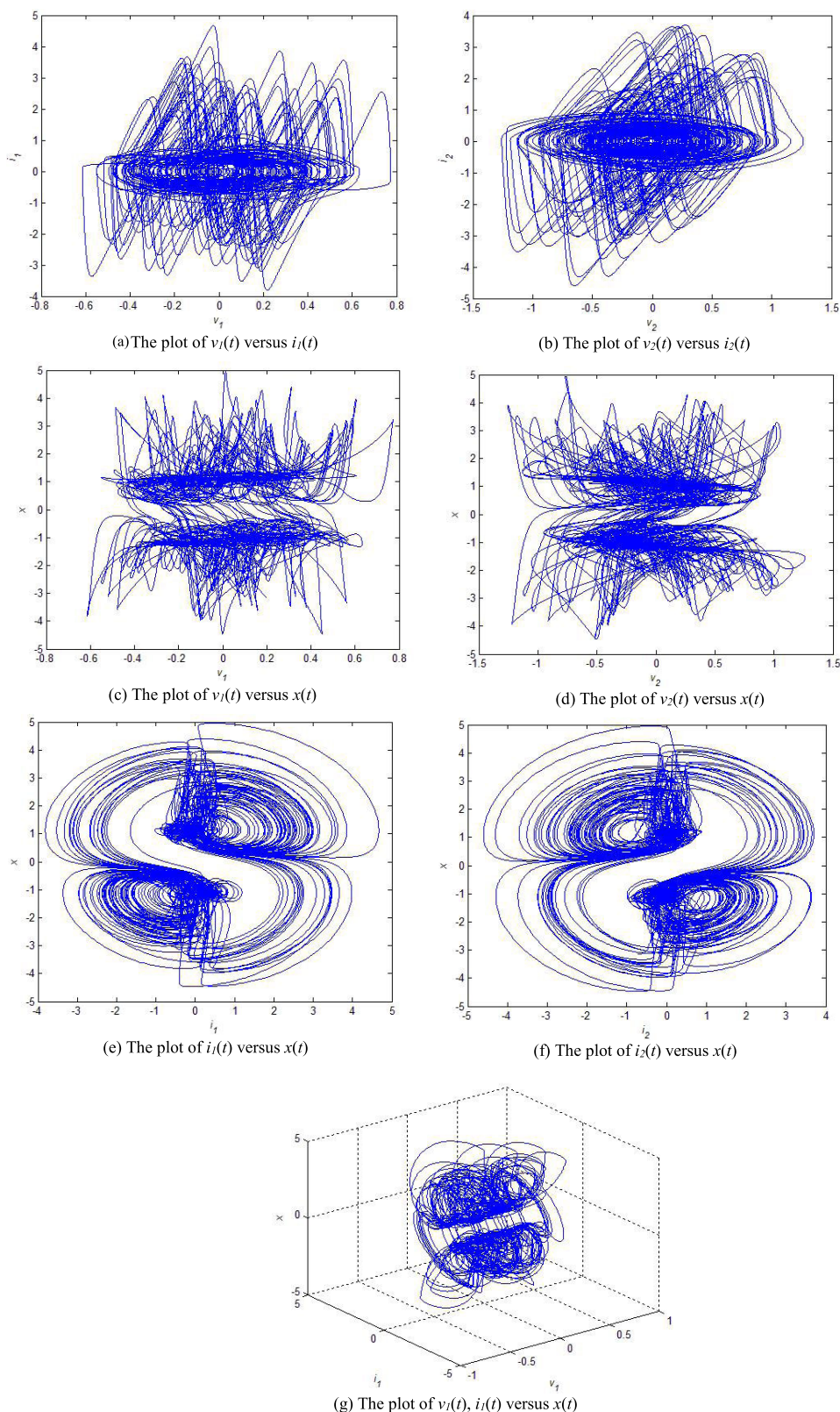
TABLE 2. Lyapunov exponents of the SECOND order memristor-based symmetric circuit system.

Figure	Parameter	Lyapunov exponents	Property
9(a)	$\beta = -0.3$	{0, -0.065, -0.066, -0.205, -2.763}	Periodicity
9(b)	$\beta = -1.5$	{0.084, 0.010, -0.002, -0.074, -2.59}	Hyperchaos
9(c)	$\beta = -2.1$	{0, -0.028, -0.03, -0.125, -0.138}	Periodicity
9(d)	$\beta = -2.7$	{0.029, 0.001, -0.02, -0.107, -0.643}	Chaos

memristor internal state is able to produce self-excited chaotic attractors or self-excited hyperchaotic attractors.

$$J(p_0) = \begin{bmatrix} 0 & 1/C_1 & 0 & 0 & 0 \\ -1/L_1 & -1/L_1 \cdot (\beta - 1) & 0 & 0 & 1/L_1 \cdot (\beta - 1) \\ 0 & -1 & -\alpha & 0 & 1 \\ 0 & 0 & 0 & 0 & 1/C_2 \\ 0 & 1/L_2 \cdot (\beta - 1) & 0 & -1/L_2 & -1/L_2 \cdot (\beta - 1) \end{bmatrix} \tag{11}$$

(a) The plot of $v_1(t)$ versus $i_1(t)$

(b) The plot of $v_2(t)$ versus $i_2(t)$

(c) The plot of $v_1(t)$ versus $x(t)$

(d) The plot of $v_2(t)$ versus $x(t)$

(e) The plot of $i_1(t)$ versus $x(t)$

(f) The plot of $i_2(t)$ versus $x(t)$

(g) The plot of $v_1(t)$, $i_1(t)$ versus $x(t)$

FIGURE 12. Hyperchaotic attractors of the second order internal state function symmetric system.

B. ANALYSIS OF BIFURCATION DIAGRAMS AND PHASE PORTRAITS

Fixing system parameter as $\alpha = 0.5$ and the setting the five initial condition of state variables as $v_{C1}(0) = 0.1$, $i_{L1}(0) = 0$, $x(0) = 0.1$, $v_{C2}(0) = 0.1$ and $i_{L2}(0) = 0$, the bifurcation diagram for control parameter β over the range $-3 \leq \beta \leq 0$ is generated in Figure 8(a). Fixing system parameter as $\beta = -1.5$, the bifurcation diagram for control
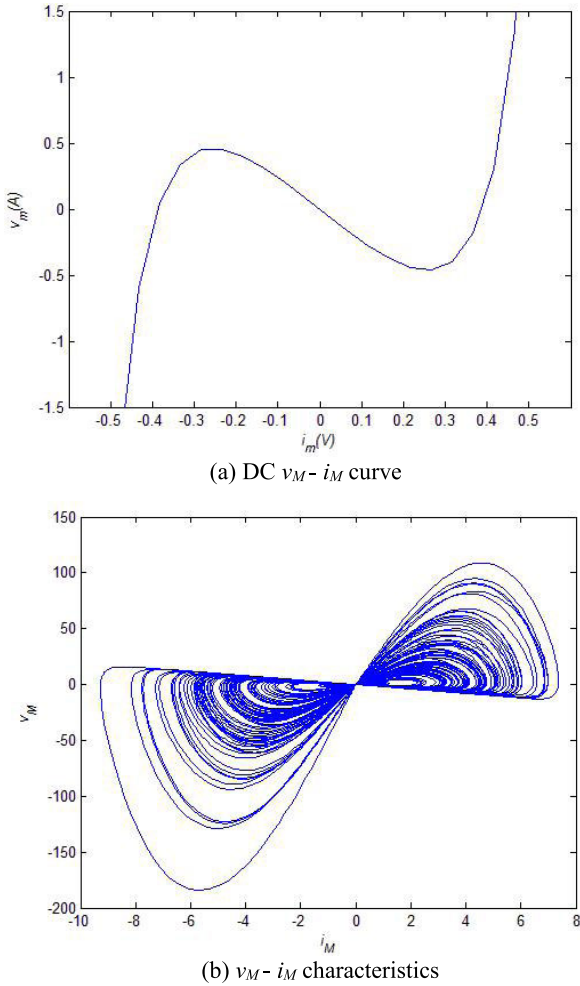
(a) DC v_M- i_M curve

(b) v_M- i_M characteristics

FIGURE 13. Memristor characteristics of the second order internal state function symmetric system.

parameter α over the range $0 \leq \alpha \leq 1$ is generated in Figure 8(b).

As can be seen from Figure 8(a), the periodic behavior is exhibited when $-0.37 < \beta \leq 0$ while the corresponding phase portrait is shown in Figure 9(a). When in range $-1.8 < \beta < -0.38$ the system shows the chaotic attractor, which is plotted in Figure 9(b). When in range $-1.39 < \beta < -2.4$ the system exhibits the limit cycle as shown in Figure 9(c). Then in the range $-3 \leq \beta < -2.4$, the system reveals the chaotic attractor again and shown in Figure 9(d). As can be seen from Figure 8(b), when over the approximate range $0.3 < \alpha < 1$, the system is in chaotic state.

To investigate the complex dynamic chaotic behaviors of the five-dimensional symmetric memristive system with second order internal state more deeply, SE and C_0 complexity varying with two system parameters are shown in Figure 10. The darker color areas are the intervals with bigger complexity. So, when utilize the symmetric memristive system, the parameters generate lighter color areas should be avoided.

C. LYAPUNOV EXPONENT SPECTRA

Fixing $\alpha = 0.5$ and varying β, the Lyapunov exponents spectrum is shown in Figure 11. Figure 11(b) shows the

maximum Lyapunov exponent spectrum and the second largest Lyapunov exponent spectrum. From Figure 11(b) and the bifurcation diagram Figure 8(a), we can see that when in range $-1.77 < \beta < -1$ the system has two positive Lyapunov exponents and that means the second order memristor-based symmetric circuit system is in hyperchaotic state.

The phase diagrams of Figure 9 and the corresponding Lyapunov exponents results are shown in Table 2. From data listed in Table 2, we can see that the systems with parameter selected in Figure 9(b) exhibit the hyperchaotic state, the system simulated for Figure 9(d) shows the chaotic state, while the systems parameter corresponding to Figure 9(a) and Figure 9(c) produce a limit cycle and in periodic state.

D. ANALYSIS OF CHAOTIC ATTRACTORS

Set the parameters as $\beta = -1$ and $\alpha = 0.5$, chose the initial conditions as $v_{C1}(0) = 0.1$, $i_{L1}(0) = 0$, $x(0) = 0.1$, $v_{C2}(0) = 0.1$ and $i_{L2}(0) = 0$ to simulate the second order memristor function symmetric system. The hyperchaotic attractors can be obtained as shown in Figure 12. We can see that the system exhibits multiple-scroll attractor.

When the second order memristor function symmetric system exhibits the hyperchaotic state, the memristor DCv_M- i_M curve (14) and the plot of v_M- i_M memristor characteristics are shown in Figure 13.

$$\begin{cases} x_{DC} = \dfrac{i_M}{\alpha - i_M^2} \\ v_M = R(x_{DC})i_M \end{cases} \quad (14)$$

in this equation, set the i_M to 10A amplitude sine-wave and the frequency to 0.5Hz.

V. CONCLUSION

This article proposed a novel fifth-order memristor-based symmetric circuit system. The chaotic behavior of this article is studied. In order to obtain the hyperchaotic state, we use multivariable first order and multivariable second order polynomial functions for the memristor internal state function respectively. The chaotic system can be hyperchaotic when its parameters are taken appropriately. The stability analysis, bifurcation diagrams, portraits and Lyapunov exponents spectrum are performed for theoretical analysis. Simulation results show that the first order internal state function memristor-based symmetric circuit system can only generate chaotic attractors whereas the second order internal state function memristor-based symmetric circuit system can generate not only chaotic attractors, but also hyperchaotic attractors in proper parameters. As future work, the physically implement and the application of this hyperchaos system in image encryption algorithms can be investigated.

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