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Novel Modeling and Filtering for Stochastic Switched Systems With Heterogeneous Behaviors

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ABSTRACT A novel heterogeneous behavior representation for linear stochastic switched system is proposed in the discrete-time domain. The switching modes of state evolution and measurement output are described by two random sequences with known probability information. The more general and flexible framework covers several classes of well-studied models as special cases, and can be served to manage different complex systems with random abrupt changes in structure and parameter, so that it has wider applicability than existing models. By introducing an equivalent auxiliary system in virtue of the mode-dependent random parameter matrices, the filter design schemes, including an optimal and a suboptimal recursive algorithms, are performed for the established model in the minimum mean square error sense to meet different application requirements. Illustrative numerical examples demonstrate the effectiveness of the proposed formulation and the corresponding filters that enjoy a promising application prospect.

INDEX TERMS State estimation, minimum mean square error, heterogenous, switching modes, random parameters matrices.

I. INTRODUCTION

Recent years have witnessed a constant research interest on stochastic switched systems in light of their extensive applications on target tracking, fault detection and identification, signal processing, networked control and so on [1]–[4]. Referred to as a class of hybrid systems, stochastic switched systems are characterized by switches among a finite number of subsystems or modes in the control of a switching signal. Such systems have the superior capacity of modeling the systems subject to some abrupt variations on the underlying structures and/or sudden environment disturbances. Owing to the high practicability, stochastic switched systems have been attached a great deal of attention and a number of results have been published. Various issues have been investigated, e.g., filter design [2], [5], stability analysis [6], [7], and model reduction [8].

Usually, the states of stochastic switched systems are not available and/or the measurement outputs are often subject to random uncertainties. Tailored for specific applications of interest, diverse modeling manners have emerged to describe

the switches in the associated systems. For example, in [1], a linear dynamic system, where the state switches among a set of unknown number modes, is presented for modeling the complex dynamical phenomena. The study of networked systems also calls for the framework of stochastic switched systems. In the networked systems, switches in the measurement output modes frequently take place whenever the status in the communication networks changes [9], [10]. A finite state Markov chain governs the switching modes of stochastic systems with random delays in [11], [12] and missing measurements in [13]. It is common to use Bernoulli binary switching sequences as an alternative approach to describe the network-induced uncertainties [14], [15]. Besides, switched systems, which consist of a finite number of subsystems and a switching mode signal that determines the active subsystem, have been extensively studied in many branches [16]–[20]. For other interesting schemes of stochastic switched systems, the references [21]–[26] may provide a boarder perspective.

While the aforementioned achievements do represent major progress, to the best of our knowledge, they are still limited in several aspects. For example, there has been little research on the heterogeneous behavior representation for

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stochastic switched systems with the individual switching characteristics of state evolution and measurement output. A common switching signal is shared between the state and measurement modes for many of the existing models (see, e.g., [27]–[29]). This manner is powerless in the cases where the state evolution and the measurement output inherently have their own switching modes. The typical example of maneuvering target tracking in a cluttered environment can be employed to illustrate the mentioned point [30]–[32]. Besides, many researches are conducted on the study for the modeled switched systems, but it appears that a systematic regime that can expose their common attributes is still lacking. As a consequence, we have to make a new and different research mechanism for each switched system. Now, there is a growing awareness of the necessity to investigate a universal formulation for describing complex random phenomena and discovering simple underlying temporal structures of stochastic switched systems.

Filtering is an important theme for stochastic systems. It is necessary to consider the underlying uncertainties, such as switching modes, unmodeled dynamics, incomplete measurement information and so on, when a switched system is investigated. Since the performance of the classical and well-known Kalman filtering can deteriorate appreciably when the system model under consideration is not exactly known, considerable effort has been devoted to develop alternative filtering techniques with specific aims to accommodate the system model uncertainties in the past decades (see, e.g., [2], [5], [20], [33]). Besides, considering the fact that inaccuracies or uncertainties do occur in filter implementation, the filtering should be designed to be insensitive against certain errors/variations with respect to its gain. For this purpose, some results on non-fragile filtering could be found in the recent references [34], [35]. In addition, it should be noted that the filtering problem for systems with random parameter matrices, due to their the powerful capacity of describing the random variations, has become a focus of research attracting an ever-increasing interest [36], [37]. For instance, linear filter and quadratic filter are offered separately in [38] and [39] for their respective concerned models where random uncertainties can be depicted by multiplicative noises in terms of random parameter matrices. For the systems with some random phenomena induced by networks, which can be further transformed into ones with random parameter matrices, the filter problems have been tackled in [40]–[44]. Besides, as for the framework simultaneously involving random state transition and measurement matrices, some filter results are prepared in [45]–[47]. More recent works on the filtering for systems with random parameter matrices are referred to [48]–[51] and the references therein. Nonetheless, the filter design of switched systems in virtue of random parameter matrices has not been fully explored, which makes the further study necessary and motivates our investigation.

Inspired by the aforementioned discussions, in this paper, we are dedicated to investigating the filtering problem for a class of linear stochastic switched systems with

heterogeneous behaviors in the discrete-time domain. The heterogeneous behaviors of stochastic switched systems result from the potential individual characteristics of state evolution and measurement output in some practical applications. As far as we are aware, the simultaneous consideration of state and measurement uncertainties, whose switching modes are heterogenous, is still in its infancy in the framework of random parameter matrices. Therefore, this situation constitutes an interesting study challenge. The main contributions of the current research are highlighted as follows:

- (1) A novel and flexible model, which allows for capturing the heterogeneity of state evolution and measurement output modes, is introduced for discrete-time linear stochastic switched systems. The proposed formulation has an excellent capacity of depicting various systems that are subject to random abrupt changes in structure and/or parameter using a unified form.
- (2) By introducing an equivalent auxiliary system in virtue of the mode-dependent random parameter matrices, the filtering schemes are conducted in the minimum mean square error (MSE) sense for cases with complete and incomplete prior mode information respectively. The algorithms designed are of Kalman-type filters and conveniently implementable in a recursive form.
- (3) As significant applications of the proposed modeling and filtering algorithms, the state estimation problems are addressed in the context of networked systems with uncertain measurements, multiple model systems, and jump Markov linear systems with uncertain measurements, respectively.

The remainder of this paper is organized as follows. Section II formulates the novel model of discrete-time linear stochastic switched system whose serviceability is revealed by two practical problems notably. The random parameter Kalman filtering (RPKF) algorithms, including an optimal and a suboptimal algorithms in the minimum MSE sense, are devised for the systems under consideration in Section III. In Section IV, some illustrative numerical examples are introduced to demonstrate the applicability and feasibility of the presented model and filter algorithms. Finally, conclusions are drawn and some future topics are discussed in Section V.

A. NOTATIONS

Throughout the paper, we use lightface letters to denote scalars and functions, boldface lowercase letters to denote vectors, and boldface uppercase letters to denote matrices. The notation \mathbb{R}^r denotes the r -dimensional Euclidean space. For a matrix \mathbf{A} , \mathbf{A}' and \mathbf{A}^\dagger represent its transpose and Moore–Penrose generalized inverse respectively. The symbols \mathbf{I} and $\mathbf{0}$ stand for the identity matrix and the matrix having all zero entries with appropriate dimension respectively. The operations $\mathbb{P}\{\cdot\}$ and $\mathbb{E}[\cdot]$ denote the probability of some random event and mathematical expectation of random variable respectively. Matrices, if their dimensions are not

explicitly specified, are assumed to be compatible for algebraic operations.

II. PROBLEM FORMULATION

Consider the following discrete-time linear stochastic switching system:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{F}_{k-1}(\theta_k)\mathbf{x}_{k-1} + \mathbf{B}_{k-1}(\theta_k)\mathbf{v}_{k-1}, \\ \mathbf{y}_k &= \mathbf{H}_k(\gamma_k)\mathbf{x}_k + \mathbf{C}_k(\gamma_k)\mathbf{w}_k, \end{aligned} \quad (1)$$

where k is the time index, $\mathbf{x}_k \in \mathbb{R}^p$ is the state to be estimated, $\mathbf{y}_k \in \mathbb{R}^N$ is the measurement output, $\mathbf{v}_k \in \mathbb{R}^q$ and $\mathbf{w}_k \in \mathbb{R}^r$ are the process noise and measurement noise respectively. The random sequences $\{\theta_k\}$ and $\{\gamma_k\}$, taking values in the finite sets $\mathcal{M} \triangleq \{1, 2, \dots, M\}$ and $\mathcal{N} \triangleq \{1, 2, \dots, N\}$, respectively, govern the switches among M state modes and N measurement modes. For $\theta_k = i \in \mathcal{M}$ and $\gamma_k = j \in \mathcal{N}$, the pairs of matrices of the i th state mode and the j th measurement output mode, respectively denoted by $(\mathbf{F}_{k-1}^i, \mathbf{B}_{k-1}^i)$ and $(\mathbf{H}_k^j, \mathbf{C}_k^j)$, are known with appropriate dimensions.

Remark 1: Two switching signals, θ_k and γ_k , are employed to indicate the state evolution and measurement output modes, respectively. The potential individual switching characteristics of state evolution and measurement output motivate the framework for stochastic switched system with heterogeneous modes (1). Different from the conventional switched models where common switching signals are often shared (see, e.g., [16]–[20], [27]–[29]), our model allows for capturing the heterogeneity of state evolution and measurement output modes, which can accommodate the dynamical complexity and environmental changes in many real applications. Only scattered results are available on the stochastic switched systems with heterogeneous modes in the literatures. It is, therefore, of particular significance to shorten such a gap by initiating a study on the corresponding formulation. The formulation (1) also covers several classes of well-studied models as special cases. Some evident examples are the systems with multiple models [28], [29], [52], the networked systems with random measurement losses [41], and the systems with state-dependent multiplicative noise [38], [39].

Similar to [22], [53], a new overall mode set can be defined by the product of the state and measurement mode sets as $\{s_k = (\theta_k, \gamma_k) : \theta_k = 1, \dots, M, \gamma_k = 1, \dots, N\}$. Specifically, if the structure (or environment) of the system considered in the paper does not change in the time interval $[k, k + 1]$, then $\theta_{k+1} = \theta_k$ and $\gamma_{k+1} = \gamma_k$; otherwise, $\theta_{k+1} \neq \theta_k$ or/and $\gamma_{k+1} \neq \gamma_k$.

Furthermore, our proposed scheme has a simple structure, offers more flexibility and is favorable to depict a broader class of uncertain stochastic systems. Two specific applications will be demonstrated in the following.

Example 1 (Distributed networked systems with uncertain measurements): Consider a class of discrete-time linear systems with missing measurements coming from L sensors, which is modeled as the following state and measurement

equations in [11]:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{v}_{k-1}, \\ \mathbf{y}_k^l &= \gamma_k^l \mathbf{H}_k^l \mathbf{x}_k + \mathbf{C}_k^l \mathbf{w}_k, \quad l = 1, \dots, L, \end{aligned} \quad (2)$$

where \mathbf{x}_k is the state and \mathbf{y}_k^l is the measurement provided by the l -th sensor at time k , \mathbf{F}_k , \mathbf{B}_k , \mathbf{H}_k^l and \mathbf{C}_k^l are known time-varying matrices with compatible dimensions. For each $l = 1, \dots, L$, $\{\gamma_k^l : k \geq 0\}$ is a sequence of independent Bernoulli random variables with known probabilities such that $\gamma_k^l = 1$ indicates that the signal is present in the measurement from the l -th sensor at time k , whereas $\gamma_k^l = 0$ means that this measurement is only noise. At time k , let us construct an entirety by stacking the measurement vectors in equation (2) as

$$\begin{bmatrix} \mathbf{y}_k^1 \\ \vdots \\ \mathbf{y}_k^L \end{bmatrix} = \begin{bmatrix} \gamma_k^1 \mathbf{H}_k^1 \\ \vdots \\ \gamma_k^L \mathbf{H}_k^L \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \mathbf{C}_k^1 \\ \vdots \\ \mathbf{C}_k^L \end{bmatrix} \mathbf{w}_k.$$

Consequently, the system model (2) can be casted within the proposed framework (1) just by setting $N = 2^L$,

$$\begin{aligned} \mathbf{y}_k &= [(\mathbf{y}_k^1)', \dots, (\mathbf{y}_k^L)']', \\ \mathbf{H}_k(\gamma_k) &= \begin{cases} [(\mathbf{H}_k^1)', \mathbf{0}, \dots, \mathbf{0}]', & \gamma_k = 1, \\ [\mathbf{0}, (\mathbf{H}_k^2)', \dots, \mathbf{0}]', & \gamma_k = 2, \\ \dots \\ [\mathbf{0}, \mathbf{0}, \dots, \mathbf{0}]', & \gamma_k = N, \end{cases} \\ \mathbf{C}_k(\gamma_k) &= [(\mathbf{C}_k^1)', \dots, (\mathbf{C}_k^L)']', \quad \gamma_k = 1, \dots, N. \end{aligned}$$

This mechanism allows us to deal with the focused problem for single and multi-sensor systems in a unified way.

Example 2 (Tracking multiple targets in a cluttered environment): Assume that there are T targets and D measurements but no knowledge on each measurement being noise or associating with which target. In [51], the multiple individual target tracking system is modeled as

$$\begin{aligned} \mathbf{x}_k^t &= \mathbf{F}_{k-1}^t \mathbf{x}_{k-1}^t + \mathbf{B}_{k-1}^t \mathbf{v}_{k-1}, \quad t = 1, \dots, T, \\ \mathbf{y}_{k,d} &= \begin{cases} \mathbf{H}_k \mathbf{x}_k^1 + \mathbf{C}_{k,d} \mathbf{w}_k, & \text{with probability } q_{k,d}^1, \\ \dots \\ \mathbf{H}_k \mathbf{x}_k^T + \mathbf{C}_{k,d} \mathbf{w}_k, & \text{with probability } q_{k,d}^T, \\ \mathbf{C}_{k,d} \mathbf{w}_k, & \text{with probability } q_{k,d}^0, \end{cases} \end{aligned} \quad (3)$$

where $d = 1, \dots, D$, \mathbf{x}_k^t is the system state of the t -th target, $\mathbf{y}_{k,d}$ is the d -th measurement at time k , $q_{k,d}^t$ denotes the probability that the measurement $\mathbf{y}_{k,d}$ is associated with the t -th target at time k and $q_{k,d}^0$ denotes the probability that the measurement $\mathbf{y}_{k,d}$ belongs to the set of false measurements at time k . Representing the above multiple individual target tracking system (3) in a compact form by an augmented state vector, we have

$$\begin{aligned} \mathbf{x}_k &= \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{v}_{k-1}, \\ \begin{bmatrix} \mathbf{y}_{k,1} \\ \vdots \\ \mathbf{y}_{k,D} \end{bmatrix} &= \begin{bmatrix} \mathbf{H}_{k,1} \\ \vdots \\ \mathbf{H}_{k,D} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \mathbf{C}_{k,1} \\ \vdots \\ \mathbf{C}_{k,D} \end{bmatrix} \mathbf{w}_k, \end{aligned} \quad (4)$$

where,

$$\begin{aligned} \mathbf{x}_k &= [(\mathbf{x}_k^1)', \dots, (\mathbf{x}_k^T)']', \\ \mathbf{F}_k &= \text{diag}(\mathbf{F}_k^1, \dots, \mathbf{F}_k^T), \\ \mathbf{B}_k &= [(\mathbf{B}_k^1)', \dots, (\mathbf{B}_k^T)']', \\ \mathbf{H}_{k,d} &= \begin{cases} [\mathbf{H}_k, \mathbf{0}, \dots, \mathbf{0}], & \text{with probability } q_{k,d}^1, \\ \dots \\ [\mathbf{0}, \dots, \mathbf{0}, \mathbf{H}_k], & \text{with probability } q_{k,d}^T, \\ \mathbf{0}, & \text{with probability } q_{k,d}^0. \end{cases} \end{aligned}$$

With the feasible development implemented in Example 1, the measurement equation (4) has the same form as that in (1) where $\gamma_k \in \{1, \dots, (T + 1)^D\}$. As a result, we integrate the multiple individual target tracking system (3) into the proposed model (1).

Since we have elaborated the serviceability of the established model, the main objective is to design filter algorithms for system (1). Some prerequisites should be claimed before performing any operation. So, assume that the dynamical system (1) satisfies the following statistical properties:

- 1) The initial state \mathbf{x}_0 is a random vector with known mean $\bar{\mathbf{x}}_0$ and covariance matrix \mathbf{P}_0 .
- 2) The noise sequences $\{\mathbf{v}_k\}$ and $\{\mathbf{w}_k\}$ are independent and mutually independent with zero means and known covariance matrices

$$\mathbb{E}[\mathbf{v}_k \mathbf{v}_k'] = \mathbf{R}_{\mathbf{v}_k}, \quad \mathbb{E}[\mathbf{w}_k \mathbf{w}_k'] = \mathbf{R}_{\mathbf{w}_k}.$$

- 3) The mode switching sequence $\{(\theta_k, \gamma_k)\}$ is independent of the noise sequences $\{\mathbf{v}_k\}$ and $\{\mathbf{w}_k\}$, and all of them are independent of the initial state \mathbf{x}_0 .

III. RANDOM PARAMETER KALMAN FILTERING

In this section, the RPKF algorithms designed for the system (1) in the minimum MSE sense will be presented. To be more specific, depending on what information is available, the RPKF algorithms are composed of two parts: an optimal and a suboptimal algorithms.

A. OPTIMAL LINEAR RECURSIVE FILTERING

The optimal linear recursive filter algorithm will be obtained under the assumption that the evolution modes of the focused system are unknown and only information about their mode probabilities is available. This message is specified in the following assumption.

Assumption 1: The random sequences $\{\theta_k\}$ and $\{\gamma_k\}$ are independent and mutually independent with known probabilities

$$\begin{aligned} p_k^i &= \mathbb{P}\{\theta_k = i\}, \quad i = 1, \dots, M, \\ q_k^j &= \mathbb{P}\{\gamma_k = j\}, \quad j = 1, \dots, N. \end{aligned} \quad (5)$$

The following is naturally satisfied

$$\sum_{i=1}^M p_k^i = 1, \quad \sum_{j=1}^N q_k^j = 1.$$

Remark 2: Assumption 1 is made to account for the statistical properties of system modes, which is applicable for some of the most investigated stochastic models, such as the systems with uncertain measurements, random delays or packet dropouts [41], [42]. The derivation of the algorithms does not require the knowledge of the exact system modes, but only the probabilities of modes involved. We make use of the known mode probability information at each time instant for the desired filter so as to possess the optimality.

Defining the random matrices $\mathbf{F}_k, \mathbf{H}_k, \mathbf{B}_k$ and \mathbf{C}_k that are satisfied by

$$\begin{aligned} \mathbb{P}\{\mathbf{F}_{k-1} = \mathbf{F}_{k-1}^i\} &= \mathbb{P}\{\mathbf{B}_{k-1} = \mathbf{B}_{k-1}^i\} \\ &= p_k^i, \quad i = 1, \dots, M, \\ \mathbb{P}\{\mathbf{H}_k = \mathbf{H}_k^j\} &= \mathbb{P}\{\mathbf{C}_k = \mathbf{C}_k^j\} \\ &= q_k^j, \quad j = 1, \dots, N, \end{aligned} \quad (6)$$

the compact form is obtained from (1) as follows:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{F}_{k-1} \mathbf{x}_{k-1} + \mathbf{B}_{k-1} \mathbf{v}_{k-1}, \\ \mathbf{y}_k &= \mathbf{H}_k \mathbf{x}_k + \mathbf{C}_k \mathbf{w}_k. \end{aligned} \quad (7)$$

Therefore, the problem is reformulated as that of attaining the minimum MSE estimator of \mathbf{x}_k , based on the measurements $\mathbf{y}_1, \dots, \mathbf{y}_k$ for the system (7).

Remark 3: The filter design in this paper is for the switched system with the simultaneous uncertainties of state and measurement modes. The different modes correspond to the changing dynamics as well as the uncertain measurements. The main difficulty is how to deal with these random uncertainties reflected by the behaviors of two switching signals θ_k and γ_k . A reasonable way of modeling such random effects is to formulate the system parameters as random matrices [36]–[44]. With the definition of random matrices $\mathbf{F}_k, \mathbf{H}_k, \mathbf{B}_k$ and \mathbf{C}_k , which are associated with the mode probabilities (5), we construct an auxiliary system (7) as an equivalent version of the original model (1). The represented form (7) is concise and convenient for us to devise the recursive filter algorithms.

Dividing a random matrix into the deterministic and random parts, we can get

$$\mathbf{F}_{k-1} = \bar{\mathbf{F}}_{k-1} + \tilde{\mathbf{F}}_{k-1}, \quad \mathbf{H}_k = \bar{\mathbf{H}}_k + \tilde{\mathbf{H}}_k, \quad (8)$$

where

$$\begin{aligned} \bar{\mathbf{F}}_{k-1} &= \mathbb{E}[\mathbf{F}_{k-1}] = \sum_{m=1}^M p_k^m \mathbf{F}_{k-1}^m, \\ \tilde{\mathbf{F}}_{k-1} &= \mathbf{F}_{k-1}^i - \sum_{m=1}^M p_k^m \mathbf{F}_{k-1}^m \quad \text{with probability } p_k^i, \end{aligned}$$

and

$$\begin{aligned} \bar{\mathbf{H}}_k &= \mathbb{E}[\mathbf{H}_k] = \sum_{n=1}^N q_k^n \mathbf{H}_k^n, \\ \tilde{\mathbf{H}}_k &= \mathbf{H}_k^j - \sum_{n=1}^N q_k^n \mathbf{H}_k^n \quad \text{with probability } q_k^j, \end{aligned}$$

then $\mathbb{E}[\tilde{\mathbf{F}}_k] = \mathbf{0}$ and $\mathbb{E}[\tilde{\mathbf{H}}_k] = \mathbf{0}$. Substituting (8) into (7), it is obvious that the system (7) can be rewritten as

$$\begin{aligned} \mathbf{x}_k &= \bar{\mathbf{F}}_{k-1}\mathbf{x}_{k-1} + \tilde{\mathbf{v}}_{k-1}, \\ \mathbf{y}_k &= \bar{\mathbf{H}}_k\mathbf{x}_k + \tilde{\mathbf{w}}_k, \end{aligned} \quad (9)$$

where

$$\begin{aligned} \tilde{\mathbf{v}}_{k-1} &= \tilde{\mathbf{F}}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{v}_{k-1}, \\ \tilde{\mathbf{w}}_k &= \tilde{\mathbf{H}}_k\mathbf{x}_k + \mathbf{C}_k\mathbf{w}_k. \end{aligned} \quad (10)$$

To facilitate the subsequent developments, the following lemma is introduced.

Lemma 2: If the random matrix \mathbf{F} and random vector \mathbf{x} are independent, then

$$\mathbb{E}[\mathbf{F}\mathbf{x}\mathbf{x}'\mathbf{F}'] = \mathbb{E}[\mathbf{F}]\mathbb{E}[\mathbf{x}\mathbf{x}']\mathbb{E}[\mathbf{F}'].$$

Theorem 3: Under Assumption 1, the dynamical system (9) has the following properties:

- 1) The pseudo-noises $\tilde{\mathbf{v}}_k$ and $\tilde{\mathbf{w}}_k$ are uncorrelated zero-mean white sequences, and have respective covariance matrices

$$\begin{aligned} \mathbf{R}_{\tilde{\mathbf{v}}_{k-1}} &= \sum_{i=1}^M p_k^i ((\mathbf{F}_{k-1}^i - \bar{\mathbf{F}}_{k-1})\mathbb{E}[\mathbf{x}_{k-1}\mathbf{x}'_{k-1}] \\ &\quad \cdot (\mathbf{F}_{k-1}^i - \bar{\mathbf{F}}_{k-1})' + \mathbf{B}_{k-1}^i \mathbf{R}_{\mathbf{v}_{k-1}} (\mathbf{B}_{k-1}^i)'), \\ \mathbf{R}_{\tilde{\mathbf{w}}_k} &= \sum_{j=1}^N q_k^j ((\mathbf{H}_k^j - \bar{\mathbf{H}}_k)\mathbb{E}[\mathbf{x}_k\mathbf{x}'_k](\mathbf{H}_k^j - \bar{\mathbf{H}}_k)' \\ &\quad + \mathbf{C}_k^j \mathbf{R}_{\mathbf{w}_k} (\mathbf{C}_k^j)'), \end{aligned}$$

- 2) The initial state \mathbf{x}_0 is uncorrelated with $\tilde{\mathbf{v}}_k$ and $\tilde{\mathbf{w}}_k$.

Proof: It can be seen from Equation (7) that the state \mathbf{x}_l linearly depends on $\mathbf{F}_{l-1}\mathbf{F}_{l-2}\cdots\mathbf{F}_1\mathbf{F}_0\mathbf{x}_0$, $\mathbf{B}_{l-1}\mathbf{v}_{l-1}$ and $\mathbf{F}_{l-1}\cdots\mathbf{F}_{l-i+1}\mathbf{B}_{l-i}\mathbf{v}_{l-i}$ for $i = 2, \dots, l$. With Assumption 1 and the independence of \mathbf{x}_0 , $\{\mathbf{v}_k\}$, $\{\mathbf{w}_k\}$, $\{\theta_k\}$ and $\{\gamma_k\}$, we have that \mathbf{x}_l is independent of \mathbf{F}_t , \mathbf{H}_t , \mathbf{v}_t and \mathbf{w}_t for any $l \leq t$. Therefore,

$$\begin{aligned} \mathbb{E}[\tilde{\mathbf{v}}_{k-1}] &= \mathbb{E}[\tilde{\mathbf{F}}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{v}_{k-1}] \\ &= \mathbb{E}[\tilde{\mathbf{F}}_{k-1}]\mathbb{E}[\mathbf{x}_{k-1}] + \mathbb{E}[\mathbf{B}_{k-1}]\mathbb{E}[\mathbf{v}_{k-1}] \\ &= \mathbf{0}, \\ \mathbb{E}[\tilde{\mathbf{w}}_k] &= \mathbf{0}, \end{aligned}$$

and then

$$\begin{aligned} \mathbf{R}_{\tilde{\mathbf{v}}_{k-1}} &= \mathbb{E}[\tilde{\mathbf{v}}_{k-1}\tilde{\mathbf{v}}_{k-1}'] \\ &= \mathbb{E}[\tilde{\mathbf{F}}_{k-1}\mathbb{E}[\mathbf{x}_{k-1}\mathbf{x}'_{k-1}]\tilde{\mathbf{F}}_{k-1}' \\ &\quad + \mathbf{B}_{k-1}\mathbb{E}[\mathbf{v}_{k-1}\mathbf{v}'_{k-1}]\mathbf{B}_{k-1}'] \\ &= \sum_{i=1}^M p_k^i ((\mathbf{F}_{k-1}^i - \bar{\mathbf{F}}_{k-1})\mathbb{E}[\mathbf{x}_{k-1}\mathbf{x}'_{k-1}] \\ &\quad \cdot (\mathbf{F}_{k-1}^i - \bar{\mathbf{F}}_{k-1})' + \mathbf{B}_{k-1}^i \mathbf{R}_{\mathbf{v}_{k-1}} (\mathbf{B}_{k-1}^i)'), \\ \mathbf{R}_{\tilde{\mathbf{w}}_k} &= \mathbb{E}[\tilde{\mathbf{w}}_k\tilde{\mathbf{w}}_k'] \\ &= \mathbb{E}[\tilde{\mathbf{H}}_k\mathbb{E}[\mathbf{x}_k\mathbf{x}'_k]\tilde{\mathbf{H}}_k' + \mathbf{C}_k\mathbb{E}[\mathbf{w}_k\mathbf{w}'_k]\mathbf{C}_k'] \end{aligned}$$

$$\begin{aligned} &= \sum_{j=1}^N q_k^j ((\mathbf{H}_k^j - \bar{\mathbf{H}}_k)\mathbb{E}[\mathbf{x}_k\mathbf{x}'_k](\mathbf{H}_k^j - \bar{\mathbf{H}}_k)' \\ &\quad + \mathbf{C}_k^j \mathbf{R}_{\mathbf{w}_k} (\mathbf{C}_k^j)'). \end{aligned}$$

Without loss of generality, assume $k > l$. Noting (10) and using the independence of random variables aforementioned, we arrive at

$$\begin{aligned} \text{Cov}(\tilde{\mathbf{v}}_k, \tilde{\mathbf{v}}_l) &= \mathbb{E}[\tilde{\mathbf{v}}_k\tilde{\mathbf{v}}_l'] \\ &= \mathbb{E}[\tilde{\mathbf{F}}_k\mathbf{x}_k\mathbf{x}'_l\tilde{\mathbf{F}}_l'] + \mathbb{E}[\tilde{\mathbf{F}}_k\mathbf{x}_k\mathbf{v}'_l\mathbf{B}'_l] \\ &\quad + \mathbb{E}[\mathbf{B}_k\mathbf{v}_k\mathbf{x}'_l\tilde{\mathbf{F}}_l'] + \mathbb{E}[\mathbf{B}_k\mathbf{v}_k\mathbf{v}'_l\mathbf{B}'_l] \\ &= \mathbf{0}, \\ \text{Cov}(\tilde{\mathbf{w}}_k, \tilde{\mathbf{w}}_l) &= \mathbb{E}[\tilde{\mathbf{w}}_k\tilde{\mathbf{w}}_l'] \\ &= \mathbb{E}[\tilde{\mathbf{H}}_k\mathbf{x}_k\mathbf{x}'_l\tilde{\mathbf{H}}_l'] + \mathbb{E}[\tilde{\mathbf{H}}_k\mathbf{x}_k\mathbf{w}'_l\mathbf{C}'_l] \\ &\quad + \mathbb{E}[\mathbf{C}_k\mathbf{w}_k\mathbf{x}'_l\tilde{\mathbf{H}}_l'] + \mathbb{E}[\mathbf{C}_k\mathbf{w}_k\mathbf{w}'_l\mathbf{C}'_l] \\ &= \mathbf{0}, \end{aligned}$$

and

$$\begin{aligned} \text{Cov}(\tilde{\mathbf{v}}_k, \tilde{\mathbf{w}}_l) &= \mathbb{E}[\tilde{\mathbf{v}}_k\tilde{\mathbf{w}}_l'] \\ &= \mathbb{E}[\tilde{\mathbf{F}}_k\mathbf{x}_k\mathbf{x}'_l\tilde{\mathbf{H}}_l'] + \mathbb{E}[\tilde{\mathbf{F}}_k\mathbf{x}_k\mathbf{w}'_l\mathbf{C}'_l] \\ &\quad + \mathbb{E}[\mathbf{B}_k\mathbf{v}_k\mathbf{x}'_l\tilde{\mathbf{H}}_l'] + \mathbb{E}[\mathbf{B}_k\mathbf{v}_k\mathbf{w}'_l\mathbf{C}'_l] \\ &= \mathbf{0}. \end{aligned}$$

Similarly, it is clear that

$$\begin{aligned} \text{Cov}(\mathbf{x}_0, \tilde{\mathbf{v}}_k) &= \mathbb{E}[\mathbf{x}_0\tilde{\mathbf{v}}_k'] \\ &= \mathbb{E}[\mathbf{x}_0\mathbf{x}'_k\tilde{\mathbf{F}}_k'] + \mathbb{E}[\mathbf{x}_0\mathbf{v}'_k\mathbf{B}'_k] \\ &= \mathbf{0}, \\ \text{Cov}(\mathbf{x}_0, \tilde{\mathbf{w}}_k) &= \mathbb{E}[\mathbf{x}_0\tilde{\mathbf{w}}_k'] \\ &= \mathbb{E}[\mathbf{x}_0\mathbf{x}'_k\tilde{\mathbf{H}}_k'] + \mathbb{E}[\mathbf{x}_0\mathbf{w}'_k\mathbf{C}'_k] \\ &= \mathbf{0}. \end{aligned}$$

This theorem thus holds. \square

So far, we have provided the statistical properties for the quantities in system (9) besides $\mathbb{E}[\mathbf{x}_k\mathbf{x}'_k]$ whose recursive expression also needs to be figured out.

Lemma 4: Under Assumption 1, the state of system (9) satisfies the following recursion:

$$\begin{aligned} \mathbb{E}[\mathbf{x}_k\mathbf{x}'_k] &= \bar{\mathbf{F}}_{k-1}\mathbb{E}[\mathbf{x}_{k-1}\mathbf{x}'_{k-1}]\bar{\mathbf{F}}_{k-1}' \\ &\quad + \sum_{i=1}^M p_k^i ((\mathbf{F}_{k-1}^i - \bar{\mathbf{F}}_{k-1})\mathbb{E}[\mathbf{x}_{k-1}\mathbf{x}'_{k-1}] \\ &\quad \cdot (\mathbf{F}_{k-1}^i - \bar{\mathbf{F}}_{k-1})' + \mathbf{B}_{k-1}^i \mathbf{R}_{\mathbf{v}_{k-1}} (\mathbf{B}_{k-1}^i)'), \end{aligned} \quad (11)$$

with the initial value

$$\mathbb{E}[\mathbf{x}_0\mathbf{x}'_0] = \bar{\mathbf{x}}_0\bar{\mathbf{x}}_0' + \mathbf{P}_0.$$

Proof: From Theorem 3, we obtain

$$\mathbb{E}[\mathbf{x}_{k-1}\tilde{\mathbf{v}}_{k-1}] = \mathbf{0}, \quad \mathbb{E}[\tilde{\mathbf{v}}_{k-1}\mathbf{x}'_{k-1}] = \mathbf{0}.$$

Therefore, for the system (9), we have

$$\mathbb{E}[\mathbf{x}_k\mathbf{x}'_k] = \bar{\mathbf{F}}_{k-1}\mathbb{E}[\mathbf{x}_{k-1}\mathbf{x}'_{k-1}]\bar{\mathbf{F}}_{k-1}' + \bar{\mathbf{F}}_{k-1}\mathbb{E}[\mathbf{x}_{k-1}\tilde{\mathbf{v}}_{k-1}']$$

$$+ \mathbb{E}[\tilde{\mathbf{v}}_{k-1} \mathbf{x}'_{k-1}] \bar{\mathbf{F}}'_{k-1} + \mathbb{E}[\tilde{\mathbf{v}}_{k-1} \tilde{\mathbf{v}}'_{k-1}],$$

and then (11) can be obtained by simple derivation. \square

Up to now, all the preparations for deriving the optimal recursive estimator for the dynamical system (1) have been completed. We are in the position to present the following theorem.

Theorem 5: For the dynamic system (1) with Assumption 1, the minimum MSE estimate of the state \mathbf{x}_k can be computed recursively as follows:

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{K}_k(\mathbf{y}_k - \bar{\mathbf{H}}_k \mathbf{x}_{k|k-1}), \quad (12)$$

$$\mathbf{x}_{k|k-1} = \bar{\mathbf{F}}_{k-1} \mathbf{x}_{k-1|k-1}, \quad (13)$$

$$\mathbf{P}_{k|k-1} = \bar{\mathbf{F}}_{k-1} \mathbf{P}_{k-1|k-1} \bar{\mathbf{F}}'_{k-1} + \mathbf{R}_{\tilde{\mathbf{v}}_{k-1}}, \quad (14)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \bar{\mathbf{H}}'_k (\bar{\mathbf{H}}_k \mathbf{P}_{k|k-1} \bar{\mathbf{H}}'_k + \mathbf{R}_{\tilde{\mathbf{w}}_k})^{-1}, \quad (15)$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \bar{\mathbf{H}}_k) \mathbf{P}_{k|k-1}, \quad (16)$$

where

$$\bar{\mathbf{F}}_{k-1} = \sum_{i=1}^M p_k^i \mathbf{F}_{k-1}^i,$$

$$\bar{\mathbf{H}}_k = \sum_{j=1}^N q_k^j \mathbf{H}_k^j,$$

$$\mathbf{R}_{\tilde{\mathbf{v}}_{k-1}} = \sum_{i=1}^M p_k^i ((\mathbf{F}_{k-1}^i - \bar{\mathbf{F}}_{k-1}) \mathbb{E}[\mathbf{x}_{k-1} \mathbf{x}'_{k-1}] \cdot (\mathbf{F}_{k-1}^i - \bar{\mathbf{F}}_{k-1})' + \mathbf{B}_{k-1}^i \mathbf{R}_{\mathbf{v}_{k-1}} (\mathbf{B}_{k-1}^i)'),$$

$$\mathbf{R}_{\tilde{\mathbf{w}}_k} = \sum_{j=1}^N q_k^j ((\mathbf{H}_k^j - \bar{\mathbf{H}}_k) \mathbb{E}[\mathbf{x}_k \mathbf{x}'_k] (\mathbf{H}_k^j - \bar{\mathbf{H}}_k)' + \mathbf{C}_k^j \mathbf{R}_{\mathbf{w}_k} (\mathbf{C}_k^j)'),$$

with $\mathbb{E}[\mathbf{x}_k \mathbf{x}'_k]$ recursively given by (11) and the initial values

$$\mathbf{x}_{0|0} = \bar{\mathbf{x}}_0, \quad \mathbf{P}_{0|0} = \mathbf{P}_0, \quad \mathbb{E}[\mathbf{x}_0 \mathbf{x}'_0] = \bar{\mathbf{x}}_0 \bar{\mathbf{x}}'_0 + \mathbf{P}_0.$$

Remark 4: The way to straightforwardly derive Theorem 5 is based on the existing results of Kalman filtering (see, e.g., [54]) since Theorem 3 guarantees the optimality of proposed algorithm in the sense of linear minimum MSE. Such a derivation seems simple without requiring redundant deductions, because Kalman filtering can be served as a tool for obtaining the optimal estimator once some preconditions verified in Theorem 3 have been met, and the optimality of proposed algorithm is thereby in the sense of linear minimum MSE as Kalman filtering.

Remark 5: In Theorem 5, all the system parameters as well as the statistical information of considered model are included. To be specific, the terms $\bar{\mathbf{F}}_k$ and $\bar{\mathbf{H}}_k$ capture the system parameter matrices. The covariance matrices of pseudo-noises $\mathbf{R}_{\tilde{\mathbf{v}}_k}$ and $\mathbf{R}_{\tilde{\mathbf{w}}_k}$ capture the characteristics of noises and random disturbances. Remarkably, the notations in Theorem 5 and in standard Kalman filtering represent very distinct meanings respectively, so we prefer to regard Theorem 5 as a Kalman-type filter. Moreover, compared with the

standard Kalman filtering, the optimal linear recursive state estimation formulae for the system (1) given by Theorem 5 has one more recursion of $\mathbb{E}[\mathbf{x}_k \mathbf{x}'_k]$ as (11).

B. SUBOPTIMAL LINEAR FILTERING

If the real-world random phenomena are not always in agreement with the Assumption 1, it would be difficult to pursue the optimality of the filter. A common workaround is the addition of limited information on the switching modes. Among them, Markov switching behavior has attracted a great deal of attention since it is very appropriate to model random variations verified by numerous report research findings. A suboptimal algorithm for the system (1) will be presented in this section.

Assumption 6: The sequences $\{\theta_k\}$ and $\{\gamma_k\}$ are mutually independent finite state first order Markov chains according to the transition probability matrices $\Phi = [\phi_{im}]_{M \times M}$ with $\phi_{im} = \mathbb{P}\{\theta_k = m | \theta_{k-1} = i\}$ and $\Psi = [\psi_{jn}]_{N \times N}$ with $\psi_{jn} = \mathbb{P}\{\gamma_k = n | \gamma_{k-1} = j\}$ respectively.

Recall that Theorem 5 states the optimal recursive state estimate under the minimum MSE sense with the condition of known mode probabilities (5). For the case that the mode probabilities are unknown, it is natural to replace them with posterior mode probabilities. In the following, we will give the calculation of the posterior mode probabilities under Assumption 6 and the relevant state estimation formulae for the system (1).

Assume that the initial prior mode probabilities are known as

$$p_0^i = \mathbb{P}\{\theta_0 = i\}, \quad i = 1, \dots, M, \\ q_0^j = \mathbb{P}\{\gamma_0 = j\}, \quad j = 1, \dots, N.$$

Given the cumulative set of measurements up to time $k - 1$, $\mathbf{y}_{1:k-1} \triangleq \{\mathbf{y}_1, \dots, \mathbf{y}_{k-1}\}$, the posterior probability distributions of the state mode and measurement mode at time $k - 1$ are

$$\hat{p}_{k-1}^i \triangleq \mathbb{P}\{\theta_{k-1} = i | \mathbf{y}_{1:k-1}\}, \quad i = 1, \dots, M, \\ \hat{q}_{k-1}^j \triangleq \mathbb{P}\{\gamma_{k-1} = j | \mathbf{y}_{1:k-1}\}, \quad j = 1, \dots, N.$$

Under the assumption that the random variables \mathbf{x}_0 , \mathbf{v}_k and \mathbf{w}_k are of Gaussian distribution, for $i = 1, \dots, M$ and $j = 1, \dots, N$, the mode likelihood is

$$\lambda_k^{(i,j)} \triangleq f(\mathbf{y}_k | \theta_k = i, \gamma_k = j, \mathbf{y}_{1:k-1}), \quad (17)$$

where f is the probability density function of Gaussian distribution $\mathcal{N}(\boldsymbol{\mu}_k^{(i,j)}, \mathbf{S}_k^{(i,j)})$ with

$$\boldsymbol{\mu}_k^{(i,j)} = \mathbf{H}_k^j \mathbf{F}_{k-1}^i \mathbf{x}_{k-1|k-1}, \\ \mathbf{S}_k^{(i,j)} = \mathbf{H}_k^j (\mathbf{F}_{k-1}^i \mathbf{P}_{k-1|k-1} (\mathbf{F}_{k-1}^i)' + \mathbf{B}_{k-1}^i \mathbf{R}_{\mathbf{v}_{k-1}} (\mathbf{B}_{k-1}^i)' \cdot (\mathbf{H}_k^j)' + \mathbf{C}_k^j \mathbf{R}_{\mathbf{w}_k} (\mathbf{C}_k^j)'),$$

and the process and measurement mode likelihoods respectively are

$$\lambda_k^{(i,:)} \triangleq f(\mathbf{y}_k | \theta_k = i, \mathbf{y}_{1:k-1}) = \sum_{j=1}^N \lambda_k^{(i,j)} \hat{q}_{k|k-1}^j,$$

$$\lambda_k^{(:,j)} \triangleq f(\mathbf{y}_k | \gamma_k = j, \mathbf{y}_{1:k-1}) = \sum_{i=1}^M \lambda_k^{(i,j)} \hat{p}_{k|k-1}^i,$$

where

$$\hat{p}_{k|k-1}^i \triangleq \mathbb{P}\{\theta_k = i | \mathbf{y}_{1:k-1}\} = \sum_{m=1}^M \phi_{mi} \hat{p}_{k-1}^m,$$

$$\hat{q}_{k|k-1}^j \triangleq \mathbb{P}\{\gamma_k = j | \mathbf{y}_{1:k-1}\} = \sum_{n=1}^N \psi_{nj} \hat{q}_{k-1}^n.$$

According to the law of total probability and Bayes formula, the posterior mode probabilities at time k can be obtained as

$$\hat{p}_k^i = \frac{\hat{p}_{k|k-1}^i \lambda_k^{(i,:)}}{\sum_{m=1}^M \hat{p}_{k|k-1}^m \lambda_k^{(m,:)}}, \quad i = 1, \dots, M,$$

$$\hat{q}_k^j = \frac{\hat{q}_{k|k-1}^j \lambda_k^{(:,j)}}{\sum_{n=1}^N \hat{q}_{k|k-1}^n \lambda_k^{(:,n)}}, \quad j = 1, \dots, N. \quad (18)$$

An analogous derivation procedure as in Section III-A leads us, after some manipulations, to the following theorem.

Theorem 7: Replacing the mode probabilities p_k^i and q_k^j in Theorem 5 with \hat{p}_k^i and \hat{q}_k^j respectively, the recursive state estimate of the system (1) can be modified as the same form as Theorem 5.

Remark 6: For the suboptimal filter algorithm given in Theorem 7, we can make a comparative analysis with the result in [22]. A robust state estimation algorithm for jump Markov linear systems with missing measurements, where the behavior of missing measurements is described by a two-state Markov chain, is presented in [22]. The considered model is casted into the framework of the interacting multiple model (IMM), and several sub-filters operate in parallel and cooperate with each other through an interacting strategy. Since we construct an auxiliary system (7) as the equivalent version of the original model (1), only a single filter is carried out at each time instant. This reorganization makes the provided algorithms consider all information of the mode variations without increasing the number of parallel filters, and thus our scheme is more appealing during execution. More importantly, the obtained results are not simple generalization of the existing ones. Some attempts would be made in Section IV to demonstrate that the RPKF algorithms may be promising in applications.

Remark 7: In contrast with Theorem 5, the mode probabilities in Theorem 7 is determined by (18), which is a compromise to the ones in (5). This major difference results in the sub-optimality of Theorem 7, although two theorems have the same recursive form. Comparing with the existing results, our designed filter algorithms in Theorems 5 and 7 exhibit the following distinct features: 1) Some random matrices are introduced in the filter design to cover the effects from the random switching behaviors of state and measurement modes in the established model; 2) All the information on the system parameters, the switching laws and the mode probabilities

has been fully considered and reflected in the algorithms; 3) Only a single filter is carried out at each time instant; 4) The designed filters are of a simple structure and easy to be implemented.

IV. EXAMPLES

In this section, three example scenarios are introduced to evaluate the proposed model and RPKF algorithms. Firstly, in Section IV-A, an example of networked system with uncertain measurements is utilized to examine the effectiveness of the proposed optimal algorithm. Secondly, in Section IV-B, a maneuvering target tracking is adopted to test the performance of the proposed suboptimal algorithm which is also compared with IMM [27], the information theoretic IMM (ITIMM) [29] and the scalar-weight IMM (SIMM) [28] algorithms in terms of error performance. The significance of our work is embodied not only by casting the classical problems within our framework but also by investigating new serviceability. Therefore, thirdly, a promising application of the proposed algorithms compared with the H_∞ filter IMM (HFIMM) [22] algorithm is illustrated in Section IV-C.

To compare the performance of the algorithms, the root mean square error (RMSE) of the state estimate is evaluated by averaging Monte Carlo simulation results as

$$\text{RMSE}_k = \sqrt{\frac{1}{R} \sum_{i=1}^R \|\mathbf{x}_{k|k}^i - \mathbf{x}_k^i\|_2^2},$$

where R is the Monte Carlo runs, and $\mathbf{x}_{k|k}^i$ and \mathbf{x}_k^i denote the estimated and true states at the i -th Monte Carlo run at time k respectively. In next, we set $R = 100$ for all simulations.

A. NETWORKED SYSTEMS WITH UNCERTAIN MEASUREMENTS

Consider a discrete-time linear stochastic system with the same parameters in [41]:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{F}_{k-1} \mathbf{x}_{k-1} + \mathbf{B}_{k-1} v_{k-1}, \\ y_k &= \gamma_k \mathbf{H}_k \mathbf{x}_k + c_k w_k, \end{aligned} \quad (19)$$

where

$$\mathbf{F}_k = \begin{bmatrix} 1.7240 & -0.7788 \\ 1 & 0 \end{bmatrix},$$

$$\mathbf{B}_k = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$\mathbf{H}_k = [0.0286, 0.0264],$$

$$c_k = 0.2,$$

and v_k and w_k are mutually independent white noises with zero means and unit variances. The random parameter γ_k characters the uncertainty in measurement and is modeled as a Bernoulli distributed white sequence taking values 0 and 1 with $\mathbb{P}\{\gamma_k = 1\} = \alpha$. Set the initial values $\mathbf{x}_0 = [2, -2]'$ and $\mathbf{P}_0 = 0.1\mathbf{I}$.

According to Theorem 5, the optimal linear filter of system (19) can be solved by means of the designed filter

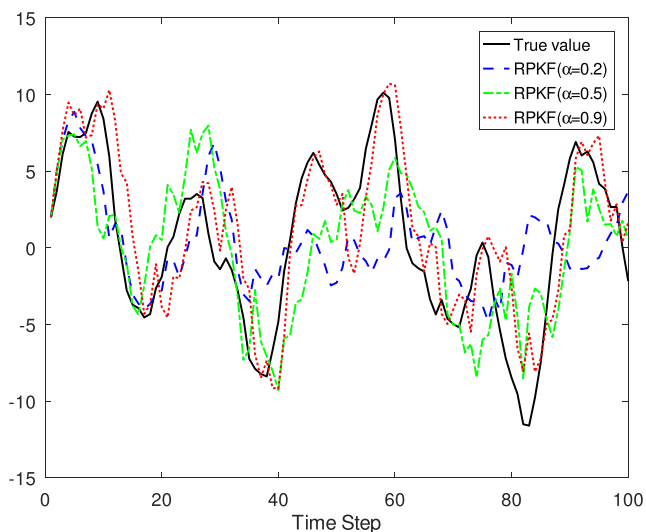


FIGURE 1. The optimal state estimates of first component for $\alpha = 0.2, 0.5$ and 0.9 .

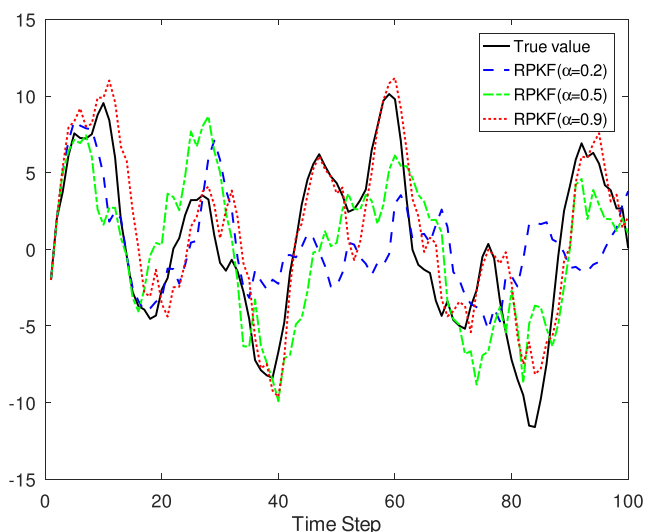


FIGURE 2. The optimal state estimates of second component for $\alpha = 0.2, 0.5$ and 0.9 .

structure (12)–(16). Figures 1 and 2 respectively show the true state components and their estimates for $\alpha = 0.2, 0.5$ and 0.9 . The simulation results illustrate that the presented filter scheme can perform well to estimate the system state. Beyond that, we can find that the estimation performance for $\alpha = 0.9$ is better than that for $\alpha = 0.5$ and $\alpha = 0.2$, and this discovery can be further verified from Figure 3 since the RMSEs of the estimators become smaller as α increases. Furthermore, the estimator reduces to Kalman estimator for the system with deterministic parameter matrices when $\alpha = 1$.

To further investigate the relationship between the performance of the proposed algorithm and the probability of measurement occurrence α , we assess the estimation performance by the average RMSE (ARMSE) over the whole simulation run time $K = 100$:

$$\text{ARMSE} = \frac{1}{K} \sum_{k=1}^K \text{RMSE}_k.$$

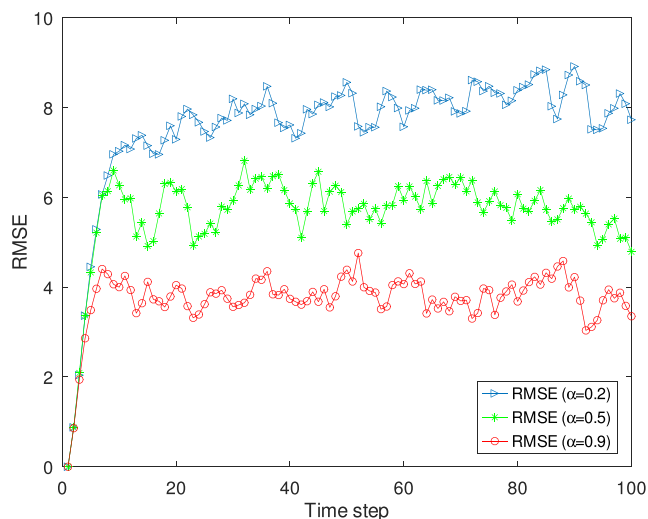


FIGURE 3. The RMSEs for $\alpha = 0.2, 0.5$ and 0.9 .

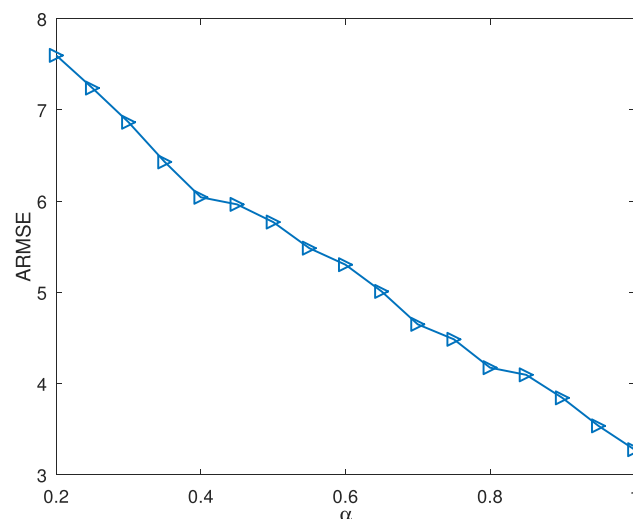


FIGURE 4. The ARMSEs versus the probability α .

Figure 4 depicts the ARMSE for different α . It can be seen that, as α increasing, the ARMSE of the proposed algorithm becomes smaller which implies the performance improvement.

By comparing the state estimates with the true state components, we can see that the proposed formulation (1) and the RPKF algorithm are valid for networked systems with uncertain measurements. This is due to the fact that we have made specific efforts to investigate a more feasible framework of stochastic switched systems and design an efficient novel filtering algorithm for the proposed. It is without doubt that the state estimation problem for networked systems with uncertain measurements belongs to the focused scope.

B. MANEUVERING TARGET TRACKING

Consider the scenario of tracking a maneuvering target in two-dimensional plane described in [29], [55]. The target starts a slow 90° turn with acceleration inputs

$\ddot{x}_k^1 = \ddot{x}_k^2 = 0.075\text{m/s}^2$, occurring for $40 \leq k \leq 60$. The second maneuver, a faster 90° turn, starts at $k = 61$ with acceleration inputs $\ddot{x}_k^1 = -0.3\text{m/s}^2$ and $\ddot{x}_k^2 = 0.3\text{m/s}^2$ and completes at $k = 66$. Before and after the turn, the target moves in plane with constant velocity (CV) until $k = 100$. The target position is sampled every $T = 10\text{s}$ and the initial position and velocity are $[2000\text{m}, 10000\text{m}]$ and $[0\text{m/s}, -15\text{m/s}]$ respectively.

The target state vector is denoted by

$$\mathbf{x}_k \triangleq [x_k^1, \dot{x}_k^1, \ddot{x}_k^1, x_k^2, \dot{x}_k^2, \ddot{x}_k^2]'$$

and the system parameter matrices are given by

$$\mathbf{F}_k^1 = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{F}_k^2 = \begin{bmatrix} 1 & T & T^2/2 & 0 & 0 & 0 \\ 0 & 1 & T & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & T^2/2 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{H}_k^1 = \mathbf{H}_k^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}_k^i \mathbf{R}_{\mathbf{v}_k} (\mathbf{B}_k^i)' = b_k^i \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix},$$

$$\mathbf{T} = \begin{bmatrix} T^5/20 & T^4/8 & T^3/6 \\ T^4/8 & T^3/3 & T^2/2 \\ T^3/6 & T^2/2 & T \end{bmatrix},$$

$$\mathbf{C}_k^1 \mathbf{R}_{\mathbf{w}_k} (\mathbf{C}_k^1)' = \mathbf{C}_k^2 \mathbf{R}_{\mathbf{w}_k} (\mathbf{C}_k^2)' = \begin{bmatrix} 100^2 & 0 \\ 0 & 100^2 \end{bmatrix},$$

where $i = 1, 2$, and $b_k^1 = 0.01$ and $b_k^2 = 0.25$.

The initial state estimate is set as a sum of the actual state and a bias $[100\text{m}, 15\text{m/s}, 0\text{m/s}^2, 100\text{m}, 15\text{m/s}, 0\text{m/s}^2]'$. The initial mode probabilities for two modes are identical with $p_0^1 = p_0^2 = 0.5$. The transition probability matrix between two state modes is taken as

$$\Phi = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}.$$

The performance is compared with respect to the RMSEs in position and velocity, illustrated in Figures 5 and 6 respectively. It can be seen that all algorithms achieve higher tracking accuracy when the target moves with CV, and perform a bit of worse when the target is maneuvering. The proposed RPKF algorithm does not manifest the superiority when the target moves with CV compared with the IMM and ITIMM algorithms, but it seems better for the maneuver motion. To illustrate this point, two other tracking scenarios are conducted:

- One is the scenario where the target always moves with accelerations $\ddot{x}_k^1 = \ddot{x}_k^2 = 0.075\text{m/s}^2$ from $k = 1$ to

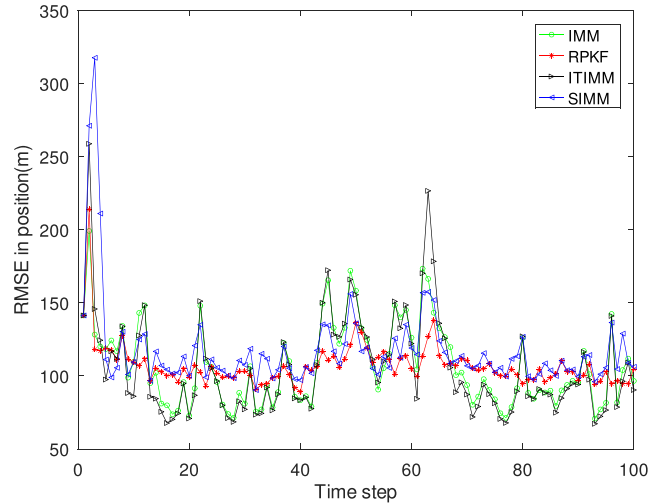


FIGURE 5. Example in Section IV-B: RMSE of position versus time.

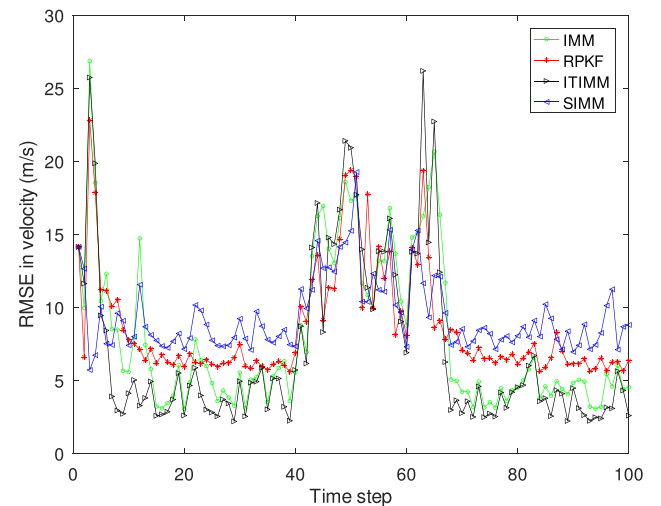


FIGURE 6. Example in Section IV-B: RMSE of velocity versus time.

$k = 100$. The RMSEs in position and velocity are presented in Figures 7 and 8 respectively. The result indicates that the proposed RPKF algorithm outperforms the IMM, the ITIMM and the SIMM algorithms when the target moves with maneuver;

- The other scenario is where the target always moves with CV all the time, and the performance are shown by the Figures 9 and 10. When the target moves with CV all the time, the RPKF algorithm does not show much superiority, although it has a higher accuracy compared with the SIMM algorithm.

All these facts evidence that the proposed RPKF algorithm is favorable for the tracking scenarios with maneuver modes.

To further verify the conclusions drawn above, Tables 1 and 2 give the ARMSEs in position and velocity of the algorithms in the three scenarios. By numerical comparison, it is not hard to see that the proposed RPKF algorithm has better performance than the others both in position and velocity when the target is maneuvering. As shown in [29],

TABLE 1. The ARMSEs in position.

Algorithm	The first scenario	The second scenario	The third scenario
RPKF	107.02	132.76	103.61
IMM	103.44	136.12	93.79
ITIMM	101.67	140.69	91.95
SIMM	116.58	178.24	113.14

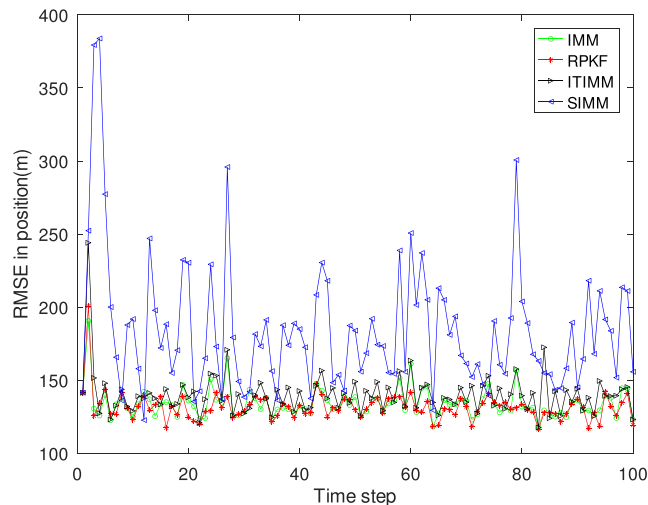


FIGURE 7. Example in Section IV-B having maneuver all the time: RMSE of position versus time.

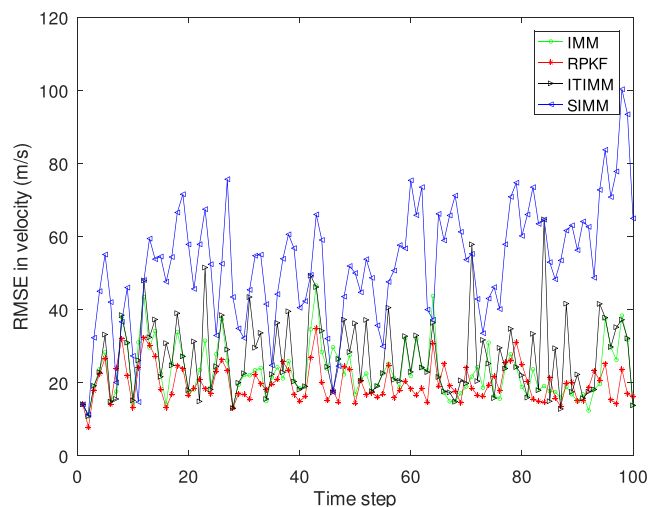


FIGURE 8. Example in Section IV-B having maneuver all the time: RMSE of velocity versus time.

the ITIMM algorithm performs good when the target moves with CV, but has worse performance than the RPKF and the IMM algorithms when the target is maneuvering. Since the target in the first scenario spends 70% of the simulation time moving with CV, the ARMSE of the ITIMM algorithm is the lowest. Besides, compared with the others, the SIMM algorithm has the worst performance no matter what the target motion mode is.

The elapsed time of the four algorithms versus sample size from 100 to 1000 is illustrated in Figure 11. The RPKF algorithm is computationally more costly than the SIMM algorithm in which the scalar weights are determined by the

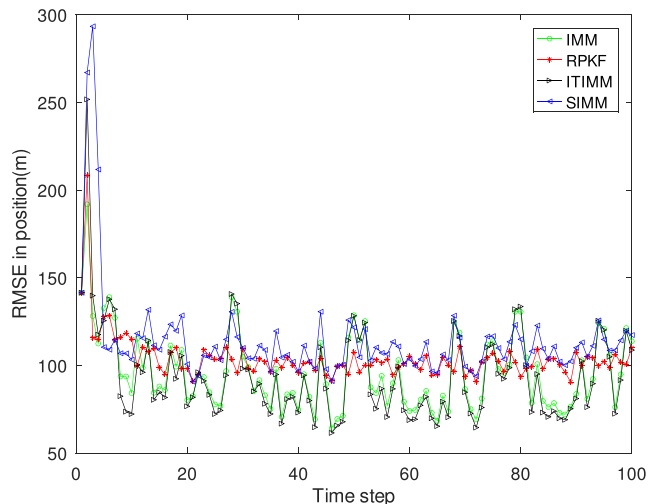


FIGURE 9. Example in Section IV-B having CV all the time: RMSE of position versus time.

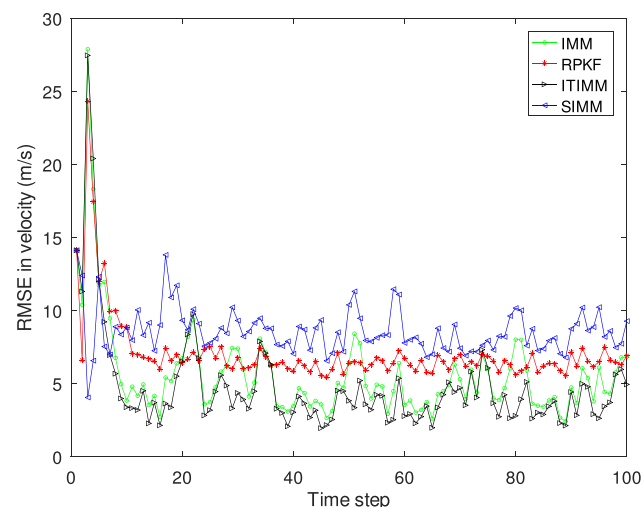


FIGURE 10. Example in Section IV-B having CV all the time: RMSE of velocity versus time.

TABLE 2. The ARMSEs in velocity.

Algorithm	The first scenario	The second scenario	The third scenario
RPKF	8.46	20.99	7.08
IMM	8.15	23.73	5.64
ITIMM	7.30	26.08	4.90
SIMM	9.61	46.46	8.54

multiple model fusion criteria. Due to the computation of the matrix inversion in the weight calculations, the ITIMM algorithm is the most expensive one. More details, please refer to [29] and [28]. The simulation results suggest that the RPKF algorithm makes a tradeoff between the estimation accuracy and computation complexity and has superior performance for maneuver motion.

C. JUMP MARKOV LINEAR SYSTEMS WITH UNCERTAIN MEASUREMENTS

Consider a similar maneuvering target tracking example with five modes used in [56].

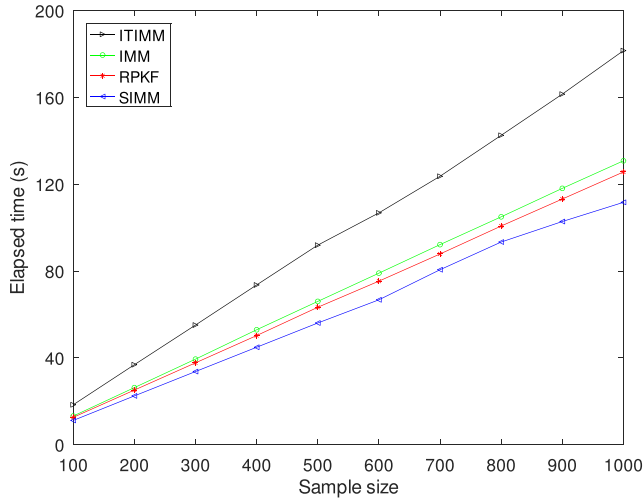


FIGURE 11. The elapsed time versus sample size.

Target motion model: The target dynamics are modeled by the following jump Markov linear system

$$x_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_{k-1} + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} a_0 \theta_k + \begin{bmatrix} 10 \\ 2 \end{bmatrix} v_{k-1},$$

where $x_k = [x_k, \dot{x}_k]'$ denotes the target state, a_0 is a quantity of acceleration, T is the sampling time and θ_k is a five-state Markov chain taking values in the set $\{-2, -1, 0, 1, 2\}$.

Adopt the simulation parameters in [56] as follows. Take the initial target state $x_0 = [500m, 10m/s]'$, the initial state covariance $P_0 = \text{diag}(1, 0.1)$, $a_0 = 1$ and $T = 1s$. The total simulation time is 200s. For $1 \leq k \leq 50$, $81 \leq k \leq 145$ and $161 \leq k \leq 200$, the target moves with CV; for $51 \leq k \leq 80$, the target accelerates with $1m/s^2$; for $146 \leq k \leq 160$, the target decelerates with $-1m/s^2$. The initial target state estimate $x_{0|0}$ is combination of the target state and a bias vector $[20m, 3m/s]'$. The initial state mode probabilities for five modes are identical with $p_0^1 = p_0^2 = p_0^3 = p_0^4 = p_0^5 = 0.2$. The transition probability matrix of the five state modes is taken as

$$\Phi = \begin{bmatrix} 0.6 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.6 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.6 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.6 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.6 \end{bmatrix}.$$

Target measurement model: The available measurement is only the x -coordinate location of the target with possibility of missing data [22]

$$y_k = \gamma_k [1 \quad 0] x_k + v_k,$$

where γ_k is a two-state Markov chain taking values in the set $\mathcal{N} = \{0, 1\}$ with transition probability matrix

$$\Psi = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}.$$

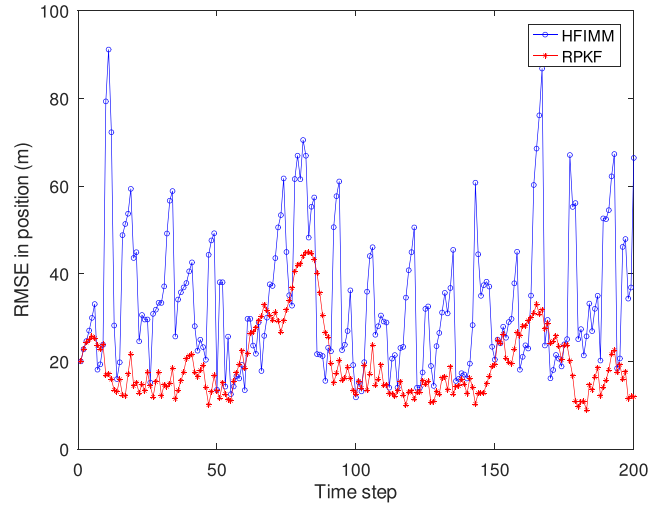


FIGURE 12. Example in Section IV-C: RMSE of position versus time.

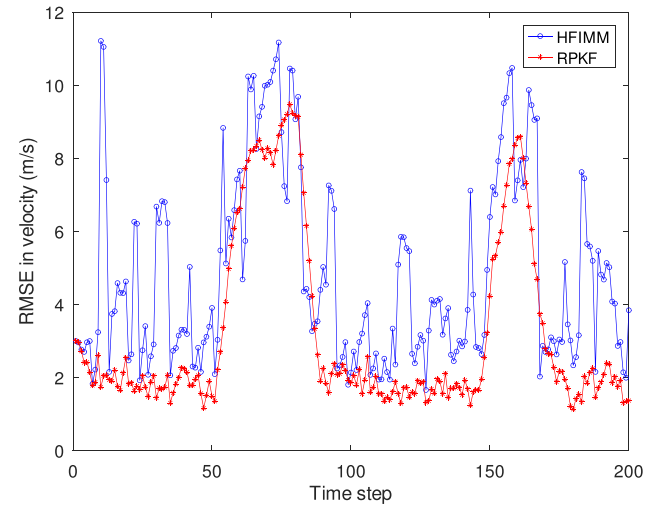


FIGURE 13. Example in Section IV-C: RMSE of velocity versus time.

The initial distribution of γ_k is $\mathbb{P}\{\gamma_k = 0\} = \mathbb{P}\{\gamma_k = 1\} = 0.5$, and the measurement noise v_k follows a zero-mean Gaussian random variable with standard deviation 50m.

The estimation performance with respect to RMSEs in position and velocity is shown in Figures 12 and 13. The results reveal that the target state can be accurately estimated in the presence of motion uncertainty and measurement uncertainty, which implies that the designed RPKF algorithm is robust with respect to the model uncertainties. Compared with the HFIMM algorithm, our proposed algorithm shows the better performance, thus it is more appealing in practical applications.

V. CONCLUSION

The paper presents a general formulation for discrete-time linear stochastic switched systems, where the switching characteristics of the state transition and measurement output are reflected by two random sequences respectively. Such modeling reveals a more flexible feature in describing stochastic switched systems and can be served to illustrate several

practical applications, such as distributed networked systems with uncertain measurements, multiple individual target tracking systems. An optimal and a suboptimal recursive filtering algorithms in the sense of minimum MSE are devised to solve the state estimation problem of the studied systems. The applicability of the novel modeling and the developed filtering algorithms are illustrated by several examples. Valuable directions for future work include developing efficient algorithms for the nonlinear systems with correlated heterogeneous modes and designing controller for unmanned aerial vehicles having complex maneuvering behaviors.

REFERENCES

- [1] E. Fox, E. B. Sudderth, M. I. Jordan, and A. S. Willsky, "Bayesian non-parametric inference of switching dynamic linear models," *IEEE Trans. Signal Process.*, vol. 59, no. 4, pp. 1569–1585, Apr. 2011.
- [2] M. Zhang, C. Shen, Z.-G. Wu, and D. Zhang, "Dissipative filtering for switched fuzzy systems with missing measurements," *IEEE Trans. Cybern.*, vol. 50, no. 5, pp. 1931–1940, May 2020.
- [3] Q. Su, Z. Fan, T. Lu, Y. Long, and J. Li, "Fault detection for switched systems with all modes unstable based on interval observer," *Inf. Sci.*, vol. 517, pp. 167–182, May 2020.
- [4] A. Cetinkaya, H. Ishii, and T. Hayakawa, "Analysis of stochastic switched systems with application to networked control under jamming attacks," *IEEE Trans. Autom. Control*, vol. 64, no. 5, pp. 2013–2028, May 2019.
- [5] Y. Chen, Z. Wang, Y. Yuan, and P. Date, "Distributed H_∞ filtering for switched stochastic delayed systems over sensor networks with fading measurements," *IEEE Trans. Cybern.*, vol. 50, no. 1, pp. 2–14, Jan. 2020.
- [6] A. R. Teel, A. Subbaraman, and A. Sferlazza, "Stability analysis for stochastic hybrid systems: A survey," *Automatica*, vol. 50, no. 10, pp. 2435–2456, Oct. 2014.
- [7] L. Zhang, Y. Leng, and P. Colaneri, "Stability and stabilization of discrete-time semi-Markov jump linear systems via semi-Markov kernel approach," *IEEE Trans. Autom. Control*, vol. 61, no. 2, pp. 503–508, Feb. 2016.
- [8] L. Wu and W. X. Zheng, "Weighted H_∞ model reduction for linear switched systems with time-varying delay," *Automatica*, vol. 45, no. 1, pp. 186–193, Jan. 2009.
- [9] M. Moayedi, Y. K. Foo, and Y. C. Soh, "Adaptive Kalman filtering in networked systems with random sensor delays, multiple packet dropouts and missing measurements," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1577–1588, Mar. 2010.
- [10] J. Hu, Z. Wang, D. Chen, and F. E. Alsaadi, "Estimation, filtering and fusion for networked systems with network-induced phenomena: New progress and prospects," *Inf. Fusion*, vol. 31, pp. 65–75, Sep. 2016.
- [11] M. J. García-Ligero, A. Hermoso-Carazo, and J. Linares-Pérez, "Distributed fusion estimation in networked systems with uncertain observations and Markovian random delays," *Signal Process.*, vol. 106, pp. 114–122, Jan. 2015.
- [12] C. Han, H. Zhang, and M. Fu, "Optimal filtering for networked systems with Markovian communication delays," *Automatica*, vol. 49, no. 10, pp. 3097–3104, Oct. 2013.
- [13] S. C. Smith and P. Seiler, "Estimation with lossy measurements: Jump estimators for jump systems," *IEEE Trans. Autom. Control*, vol. 48, no. 12, pp. 2163–2171, Dec. 2003.
- [14] Z. Wang and W. Zhou, "Recursive estimation for sensor systems with one-step randomly delayed and censored measurements," *IEEE Access*, vol. 8, pp. 11710–11721, 2020.
- [15] H. Rezaei, R. M. Esfanjani, and M. H. Sedaaghi, "Improved robust finite-horizon Kalman filtering for uncertain networked time-varying systems," *Inf. Sci.*, vol. 293, pp. 263–274, Feb. 2015.
- [16] A. Hocine, M. Chadli, and H. R. Karimi, "A structured filter for Markovian switching systems," *Int. J. Syst. Sci.*, vol. 45, no. 7, pp. 1518–1527, Jul. 2014.
- [17] Z. Wang, Q. Wang, and C. Dong, "Asynchronous H_∞ control for unmanned aerial vehicles: Switched polytopic system approach," *IEEE/CAA J. Automatica Sinica*, vol. 2, no. 2, pp. 207–216, Apr. 2015.
- [18] P. Shi and F. Li, "A survey on Markovian jump systems: Modeling and design," *Int. J. Control, Autom. Syst.*, vol. 13, no. 1, pp. 1–16, Feb. 2015.
- [19] B. Cai, L. Zhang, and Y. Shi, "Observed-Mode-Dependent state estimation of hidden semi-Markov jump linear systems," *IEEE Trans. Autom. Control*, vol. 65, no. 1, pp. 442–449, Jan. 2020.
- [20] X. Xiao, J. H. Park, and L. Zhou, "Event-triggered H_∞ filtering of discrete-time switched linear systems," *ISA Trans.*, vol. 77, pp. 112–121, Jun. 2018.
- [21] Z.-G. Wu, Y. Shen, H. Su, R. Lu, and T. Huang, " H_2 performance analysis and applications of 2-D hidden Bernoulli jump system," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 10, pp. 2097–2107, Oct. 2019.
- [22] W. Li, Y. Jia, J. Du, and J. Zhang, "Robust state estimation for jump Markov linear systems with missing measurements," *J. Franklin Inst.*, vol. 350, no. 6, pp. 1476–1487, Aug. 2013.
- [23] W. Xu, Z. G. Feng, G.-H. Lin, K. F. C. Yiu, and L. Yu, "Optimal switching of switched systems with time delay in discrete time," *Automatica*, vol. 112, Feb. 2020, Art. no. 108696.
- [24] D. Zhang, Z. Xu, H. R. Karimi, and Q.-G. Wang, "Distributed filtering for switched linear systems with sensor networks in presence of packet dropouts and quantization," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 64, no. 10, pp. 2783–2796, Oct. 2017.
- [25] J. Wang, L. Shen, J. Xia, Z. Wang, and X. Chen, "Asynchronous dissipative filtering for nonlinear jumping systems subject to fading channels," *J. Franklin Inst.*, vol. 357, no. 1, pp. 589–605, Jan. 2020.
- [26] H. Shen, Y. Men, J. Cao, and J. H. Park, " H_∞ filtering for fuzzy jumping genetic regulatory networks with round-robin protocol: A hidden-Markov-model-based approach," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 1, pp. 112–121, Jan. 2020.
- [27] H. A. P. Blom and Y. Bar-Shalom, "The interacting multiple model algorithm for systems with Markovian switching coefficients," *IEEE Trans. Autom. Control*, vol. 33, no. 8, pp. 780–783, Aug. 1988.
- [28] X. Fu, Y. Jia, F. Yu, and J. Du, "New interacting multiple model algorithms for the tracking of the manoeuvring target," *IET Control Theory Appl.*, vol. 4, no. 10, pp. 2184–2194, Oct. 2010.
- [29] W. Li and Y. Jia, "An information theoretic approach to interacting multiple model estimation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 3, pp. 1811–1825, Jul. 2015.
- [30] X. Rong Li and V. P. Jilkov, "Survey of maneuvering target tracking. Part V. Multiple-model methods," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, no. 4, pp. 1255–1321, Oct. 2005.
- [31] S.-S. Jan and Y.-C. Kao, "Radar tracking with an interacting multiple model and probabilistic data association filter for civil aviation applications," *Sensors*, vol. 13, no. 5, pp. 6636–6650, May 2013.
- [32] J. F. P. Kooij, G. Englebienne, and D. M. Gavrilu, "Mixture of switching linear dynamics to discover behavior patterns in object tracks," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 38, no. 2, pp. 322–334, Feb. 2016.
- [33] G. Wang, X. Wang, and Y. Zhang, "Variational Bayesian IMM-filter for JMSs with unknown noise covariances," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 56, no. 2, pp. 1652–1661, Apr. 2020.
- [34] W.-W. Che, H. Su, P. Shi, and Z.-G. Wu, "Nonfragile and nonsynchronous synthesis of reachable set for Bernoulli switched systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 2, pp. 726–731, Feb. 2020.
- [35] R. Rakkiyappan, K. Maheswari, K. Sivaranjani, and Y. H. Joo, "Non-fragile finite-time l_2 - l_∞ state estimation for discrete-time neural networks with semi-Markovian switching and random sensor delays based on Abel lemma approach," *Nonlinear Anal., Hybrid Syst.*, vol. 29, pp. 283–302, Aug. 2018.
- [36] J. Hu, Z. Wang, and H. Gao, "Recursive filtering with random parameter matrices, multiple fading measurements and correlated noises," *Automatica*, vol. 49, no. 11, pp. 3440–3448, Nov. 2013.
- [37] R. Caballero-Águila, A. Hermoso-Carazo, and J. Linares-Pérez, "Networked fusion estimation with multiple uncertainties and time-correlated channel noise," *Inf. Fusion*, vol. 54, pp. 161–171, Feb. 2020.
- [38] Y. Yang, Y. Qin, Q. Pan, Y. Yang, and Z. Li, "Recursive linear optimal filter for Markovian jump linear systems with multi-step correlated noises and multiplicative random parameters," *Int. J. Syst. Sci.*, vol. 50, no. 4, pp. 749–763, Mar. 2019.
- [39] Q. Liu, Z. Wang, Q.-L. Han, and C. Jiang, "Quadratic estimation for discrete time-varying non-Gaussian systems with multiplicative noises and quantization effects," *Automatica*, vol. 113, Mar. 2020, Art. no. 108714.
- [40] N. Pang, Y. Luo, and Y. Zhu, "A novel model for linear dynamic system with random delays," *Automatica*, vol. 99, pp. 346–351, Jan. 2019.
- [41] M. Sahebsara, T. Chen, and S. L. Shah, "Optimal H_2 filtering with random sensor delay, multiple packet dropout and uncertain observations," *Int. J. Control*, vol. 80, no. 2, pp. 292–301, Feb. 2007.

- [42] J. Ma and S. Sun, "Optimal linear estimators for systems with random sensor delays, multiple packet dropouts and uncertain observations," *IEEE Trans. Signal Process.*, vol. 59, no. 11, pp. 5181–5192, Nov. 2011.
- [43] S. Wang, H. Fang, and X. Tian, "Recursive estimation for nonlinear stochastic systems with multi-step transmission delays, multiple packet dropouts and correlated noises," *Signal Process.*, vol. 115, pp. 164–175, Oct. 2015.
- [44] S. Sun, L. Xie, W. Xiao, and Y. C. Soh, "Optimal linear estimation for systems with multiple packet dropouts," *Automatica*, vol. 44, no. 5, pp. 1333–1342, May 2008.
- [45] F. Han, H. Dong, Z. Wang, G. Li, and F. E. Alsaadi, "Improved tobit Kalman filtering for systems with random parameters via conditional expectation," *Signal Process.*, vol. 147, pp. 35–45, Jun. 2018.
- [46] W. Wang and J. Zhou, "Optimal linear filtering design for discrete-time systems with cross-correlated stochastic parameter matrices and noises," *IET Control Theory Appl.*, vol. 11, no. 18, pp. 3353–3362, Dec. 2017.
- [47] J. Linares-Pérez, R. Caballero-Águila, and I. García-Garrido, "Optimal linear filter design for systems with correlation in the measurement matrices and noises: Recursive algorithm and applications," *Int. J. Syst. Sci.*, vol. 45, no. 7, pp. 1548–1562, Jul. 2014.
- [48] L. Wang, Z. Wang, Q.-L. Han, and G. Wei, "Event-based variance-constrained H_∞ filtering for stochastic parameter systems over sensor networks with successive missing measurements," *IEEE Trans. Cybern.*, vol. 48, no. 3, pp. 1007–1017, Mar. 2018.
- [49] J. Feng, R. Yang, H. Liu, and B. Xu, "Robust recursive estimation for uncertain systems with delayed measurements and noises," *IEEE Access*, vol. 8, pp. 14386–14400, 2020.
- [50] S. Sun, T. Tian, and L. Honglei, "State estimators for systems with random parameter matrices, stochastic nonlinearities, fading measurements and correlated noises," *Inf. Sci.*, vols. 397–398, pp. 118–136, Aug. 2017.
- [51] Y. Luo, Y. Zhu, X. Shen, and E. Song, "Novel data association algorithm based on integrated random coefficient matrices Kalman filtering," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 1, pp. 144–158, Jan. 2012.
- [52] Y. Ma, S. Zhao, and B. Huang, "Multiple-model state estimation based on variational Bayesian inference," *IEEE Trans. Autom. Control*, vol. 64, no. 4, pp. 1679–1685, Apr. 2019.
- [53] Y. Niu, W. Dong, and Y. Ji, "State estimation for networked systems with a Markov plant in the presence of missing and quantised measurements," *IET Control Theory Appl.*, vol. 10, no. 5, pp. 599–606, Mar. 2016.
- [54] D. Simon, *Optimal State Estimation: Kalman, H_∞ , and Nonlinear Approaches*. Hoboken, NJ, USA: Wiley, 2006.
- [55] L. A. Johnston and V. Krishnamurthy, "An improvement to the interacting multiple model (IMM) algorithm," *IEEE Trans. Signal Process.*, vol. 49, no. 12, pp. 2909–2923, Dec. 2001.
- [56] M. Hernandez, B. Ristic, A. Farina, T. Sathyan, and T. Kirubarajan, "Performance measure for Markovian switching systems using best-fitting Gaussian distributions," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 2, pp. 724–747, Apr. 2008.



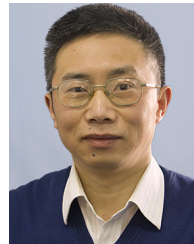
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