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# Leader-Following Formation Control and Collision Avoidance of Second-Order Multi-Agent Systems With Time Delay

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**ABSTRACT** In this article, the formation control problem has been considered for second-order multi-agent system with time delay. The involved controller is divided into two parts. The first part is to design the leader-following and adaptive control strategies that are utilized to achieve the specified formation shape. Based on a potential field function, the second part is applied to realizing the collision avoidance of the agents communicating with each other. By using the Lyapunov theory, some sufficient criteria are derived to ensure the specified formation shape of all agents and collision avoidance of any pair of agents. The derived criteria are formulated in terms of algebraic conditions, in which the control gains play an important role. Finally, a numerical simulation is given to illustrate the effectiveness of the derived results.

**INDEX TERMS** Formation control, collision avoidance, multi-agent system, time delay.

## I. INTRODUCTION

As a striking way to apply dynamics of autonomous agents in practical problems, distributed formation control strategies of multi-agent systems have been considered in many related fields over the past decades [1]–[7]. The so-called formation control usually means that all of the agents cooperate with each other by utilizing local information, meanwhile approaching to some desired positions as a geometric shape. As mentioned in [8], many methods have been proposed to realize the desired formation shape, e.g., the position-based formation control scheme in [9], displacement-based scheme in [10], and distance-based scheme in [11]. Among these methods, the displacement-based formation control scheme has become an important and active topic, since the control law is easy to be designed and implemented [10], [12]–[24].

In practical application, there are many problems to be considered for the distributed formation control, e.g., collision avoidance [25], obstacle avoidance [26], [27], and connectivity assurance [28]. Collision avoidance, as a basic requirement in the design of formation control law, is attributed to some task constraints. Such constraints rely on the implementation of real-world flight vehicles and computer-based simulations offering better reliability. In view of theory analysis, such constraints present a separation control law that affects the distance of between agents in close proximity. The basic idea of this separation control law is that a potential field is utilized for characterizing a repulsive force, such that the collision avoidance among the agents can be achieved. This idea had been introduced for obstacle avoidance of manipulators and mobile robots in [29], and developed to navigate the robot systems in [30], [31]. Recently, several kinds of artificial potential fields have been considered to deal with the collision avoidance or flocking behaviors of multi-agent systems in [6], [25], [32]. For example, in [6], a general potential field function has been constructed for the event-triggered formation control problem of nonlinear uncertain second-order multi-agent. In [32], a kind of formation control

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problem has been considered in the second-order multi-agent system, where the collision avoidance is described by a sequence of potential fields with respect to estimated position states.

In addition to collision avoidance, time delay is ubiquitous in the control processes. In some cases, it is difficult for the agents with time delay to cooperate with each other, and they may even present oscillation behavior. Therefore, it is crucial to consider time delay into control problem. Recently, many kinds of time delay have been introduced in the distributed control problem of multi-agent systems in [3], [15]–[17], [33]–[39]. In these works [3], [15]–[17], [33]–[37], time delay has many presentations: a typical example is system delay. The system delay, as an inherent delay, is usually caused by the finite response speed of hardware, such that the control signals in the system may not be instantaneous responses [37]. For example, in [15], formation tracking control problem of second-order multi-agent systems has been investigated, where time delay is introduced into the tracking control law. In [17], the leader-following formation control problem has been studied for nonlinear second-order multi-agent systems with time delay. In [36], the time delay has been considered in consensus problem of the second-order multi-agent system with jointly-connected topologies. In [37], the stability problem has been examined for a type of stochastic delayed systems, which can be readily extended for application to the consensus and formation control problem of multi-agent systems.

On the other hand, the leader-following control problem is one of the most important topics in the field of multi-agent systems [10], [14], [15], [17], [27], [40]–[46]. Generally, the aim of leader-following control is to design a control law such that all of the agents can track the dynamics of the leader. However, the dynamics of followers not only cooperates with each other under the effects of network communication, but also exhibits their own motions that are acted by the leader. These properties of multi-agent systems reveal that it is possible to control a fraction of agents in order to achieve some final control objectives. This idea is the so-called pinning control strategy. Recently, the pinning control strategy has been utilized to study the leader-following control problem in [10], [47], [48]. For example, in [10], the formation control problem of second-order multi-agent system with fixed and switching topologies has been considered by using a pinning control strategy. In [47], several pinning control strategies have been examined for the synchronization problem of stochastic dynamical networks, where the selection of control nodes is optimized according to the evolutionary algorithms and the convex method.

Although it is important to consider the leader-following formation control and collision avoidance of multi-agent systems with time delay, there are still some difficulties and challenges which remain to be investigated. The primary difficulties and challenges can be listed as three aspects.

1) There are many recent works that study the formation control and collision avoidance of multi-agent systems in [3], [4], [10]. However, in some works, the potential field function has been designed by only considering the minimum radius of the avoidance region and ignoring radius of the detection region. In this case, how to design an appropriate control protocol that, not only can be utilized to realize the specified formation shape of all the agents and the collision avoidance of any pair of agents, but also considers both the radius of the avoidance region and radius of the detection region, must be determined.
2) Different from the relevant works in [2], [3], [5], [6], the main aim of this article focuses on the dynamic relationship between the system delay and the control protocol rather than some certain cases, e.g., only studying the collision avoidance or the specified formation shape. Therefore, the relationship between the time delay and the control parameters must be determined.
3) Due to the requirement of collision avoidance, the common Lyapunov function candidate may be hard to be utilized. Thus, how to apply a decentralized Lyapunov function to deal with the system delay and the potential field in the adaptive control protocol must be determined.

Inspired by the above considerations, the aim of this article is to explore a mathematical framework to describe the relationship among the pinning control strategy, system delay and collision avoidance in the formation control problem. First, the dynamics of each agent is modeled by a nonlinear function with time delay acting on the position and velocity states. Then, a mixed control law is designed, where the pinning and adaptive control strategies are utilized to achieve the specified formation shape. Meanwhile, a typical potential field function is considered for ensuring collision avoidance. Based on the Lyapunov theory, two sufficient criteria are derived to ensure leader-following formation control and collision avoidance of second-order multi-agent systems with time delay. Finally, an example is given to show the effectiveness of results. The contributions of this article are summarized as follows:

1) Compared with the usual formation control problems in [3], [4], [10], a mathematical framework of formation control and collision avoidance in second-order multi-agent systems with time delay is constructed, in which both the radius of the avoidance region and the detection region are considered for ensuring collision avoidance and connectivity preservation in a unify potential field function.
2) Different from the formation control problems with collision avoidance [6], [25], [32], time delay, along with pinning and adaptive control strategies are simultaneously considered. Such strategies provide a mixed

control approach that, on one hand, the specified formation shape and collision avoidance can be achieved, and on the other hand, the relationship among control parameters can be revealed.

The rest of this article is organized as follows. In Section II, some basic concepts and the control problem are formulated, where the potential field function, as well as pinning and adaptive control strategies are introduced, respectively. In Section III, the main results are presented. In Section IV, an example is given to show the effectiveness of results. In Section V, the conclusion is drawn.

Moreover, throughout this article, $\mathbb{R}^{n\times m}$ indicates the set of the $n\times m$ real matrix, and $\mathbb{R}^n$ is the $n$-dimensional Euclidean space. For matrices $X\in\mathbb{R}^{q\times p}$ and $Y\in\mathbb{R}^{n\times m}$, $X\otimes Y$ stands for their Kronecker product. For a matrix $A\in\mathbb{R}^{n\times m}$, the 2-norm of $A$ is $\|A\|=\sqrt{\lambda_{\max}(A^TA)}$, where the superscript "T" means the transpose of matrix $A$ and $\lambda_{\max}(\cdot)$ is the largest eigenvalue. For a vector $x\in\mathbb{R}^n$, $\|x\|=\sqrt{x^Tx}$ denotes the Euclidean vector norm.

## II. MODEL AND CONTROL PROBLEM FORMULATION

Denote an undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ as the communication structure among the agents, where $\mathcal{V}=\{1,\ldots,N\}$ stands for the agent set and $\mathcal{E}=\{(i,j)\}$ means the edge set. Let $\mathcal{L}=[\ell_{ij}]\in\mathbb{R}^{N\times N}$ be the Laplacian matrix of graph $\mathcal{G}$, and the elements of the Laplacian matrix $\mathcal{L}$ be defined as: for $i\neq j$, $\ell_{ij}=\ell_{ji}=-1$ holds if $(i,j)\in\mathcal{E}$; otherwise $\ell_{ij}=\ell_{ji}=0$ holds if $(i,j)\notin\mathcal{E}$, and for $i=j$, $\ell_{ii}=\sum_{j\in\mathcal{N}_i}^{N}\ell_{ij}$, where $\mathcal{N}_i=\{j\in\{(i,j)\}\}$ is the neighboring set of agent $i$. In this article, the graph $\mathcal{G}$ is assumed to be undirected, connected and simple (i.e., without multiple edges and self-loops). As shown in [47], if the graph $\mathcal{G}$ is undirected and connected, all eigenvalues of the Laplacian matrix $\mathcal{L}$ can be reordered by $0=\lambda_1(\mathcal{L})<\lambda_2(\mathcal{L})\leq\ldots\leq\lambda_N(\mathcal{L})$, where $\lambda_i(\mathcal{L})$ means the $i$-smallest eigenvalue of the Laplacian matrix $\mathcal{L}$.

Considering a nonlinear second-order multi-agent system, the dynamics of the $i$-th agent is expressed by

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=f(x_i(t-\tau),v_i(t-\tau))+u_i(t),\end{cases}\tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ means the position state, $v_i(t)\in\mathbb{R}^n$ is the velocity state, $\tau\geq 0$ is the bounded system delay, $f(x_i(t),v_i(t))$ is a nonlinear function, and $u_i(t)\in\mathbb{R}^n$ is the formation control law. Moreover, for $t\in[\tau,0]$, $x_i(t)=\varrho_i(t)$ and $v_i(t)=\rho_i(t)$ are the initial functions, where the initial functions $\varrho_i(t)$ and $\rho_i(t)$ are continuous for all $t\in[\tau,0]$ and $i\in\mathcal{V}$.

The dynamics of the virtual leader is given by

$$\begin{cases}\dot{x}_0(t)=v_0(t),\\ \dot{v}_0(t)=f(x_0(t-\tau),v_0(t-\tau)),\end{cases}\tag{2}$$

where $x_0(t)\in\mathbb{R}^n$ means the position state of the virtual leader, and $v_0(t)\in\mathbb{R}^n$ is the velocity state of the virtual leader.

Then, denote $\mathcal{P}=\{1,2,\ldots,M\}$ $(M\leq N)$ as a fixed pinning set, and therefore, $\mathcal{P}\subseteq\mathcal{V}$. For $i\in\mathcal{P}$, the formation control law is designed by

$$\begin{aligned}u_i(t)=&f(x_0(t-\tau),v_0(t-\tau))-f(x_0(t-\tau)\\&+d_i,v_0(t-\tau))+\alpha_i(t)\sum_{j\in\mathcal{N}_i}(x_j(t)-x_i(t)\\&+d_i-d_j)+\alpha_i(t)\sum_{j\in\mathcal{N}_i}(v_j(t)-v_i(t))\\&+u_i^o(t)+u_i^p(t),\end{aligned}\tag{3}$$

and for $i\notin\mathcal{P}$, the formation control law is given by

$$\begin{aligned}u_i(t)=&f(x_0(t-\tau),v_0(t-\tau))-f(x_0(t-\tau)\\&+d_i,v_0(t-\tau))+\alpha_i(t)\sum_{j\in\mathcal{N}_i}(x_j(t)-x_i(t)\\&+d_i-d_j)+\alpha_i(t)\sum_{j\in\mathcal{N}_i}(v_j(t)-v_i(t))\\&+u_i^o(t),\end{aligned}\tag{4}$$

where $\alpha_i(t)$ is an adaptive control gain given later, $d_i\in\mathbb{R}^n$ is an absolute desired position of agent $i$, $u_i^p(t)\in\mathbb{R}^n$ is a pinning control law, and $u_i^o(t)\in\mathbb{R}^n$ is a collision avoidance control law.

The adaptive control gain $\alpha_i(t)$ is updated according to the following form

$$\begin{aligned}\dot{\alpha}_i(t)=&\alpha\sum_{j\in\mathcal{N}_i}(x_j(t)-x_i(t)+d_i-d_j+v_j(t)-v_i(t))^T\\&\times(x_j(t)-x_i(t)+d_i-d_j+v_j(t)-v_i(t)),\end{aligned}\tag{5}$$

where $\alpha>0$.

The pinning control law $u_i^p(t)$ is written by

$$u_i^p(t)=b_i(x_i(t)-x_0(t)-d_i+v_i(t)-v_0(t)),\tag{6}$$

where $b_i$ is a control gain.

The collision avoidance control law $u_i^o(t)$ has the following form

$$u_i^o(t)=-\sum_{j=1}^{N}\frac{\partial W_{ij}^T(x_i(t),x_j(t))}{\partial x_i(t)}.\tag{7}$$

The collision avoidance control law $u_i^o(t)$ in (3) is composed of a potential field function that is utilized to realize the collision avoidance between any pair of agents. Given two positive parameters $r$ and $R$, $r$ expresses a minimum radius of the avoidance region, and $R$ means a radius of the detection region, where $r\leq\min_{i,j\in\mathcal{V}}\{\|d_i-d_j\|\}$ and $R\geq r$. The potential field function $W_{ij}(x_i(t),x_j(t))$ is described by

$$W_{ij}(x_i(t),x_j(t))=\left(\min\left\{\frac{\|x_i(t)-x_j(t)\|^2-R^2}{\|x_i(t)-x_j(t)\|^2-r^2},0\right\}\right)^2.$$

Taking the partial differential of the potential field function $W_{ij}(x_i(t),x_j(t))$ with respect to $x_i(t)$, for the case of

$\|x_i(t) - x_j(t)\| \in [r, R]$, one has

$$\frac{\partial W_{ij}^T(x_i(t), x_j(t))}{\partial x_i(t)} = \frac{4(R^2 - r^2)(\|x_i(t) - x_j(t)\|^2 - R^2)}{(\|x_i(t) - x_j(t)\|^2 - r^2)^3} \times (x_i(t) - x_j(t))^T,$$

and for the case of $\|x_i(t) - x_j(t)\| \in (R, \infty]$, one gets

$$\frac{\partial W_{ij}^T(x_i(t), x_j(t))}{\partial x_i(t)} = 0.$$

Based on the above discussions, the potential field function $W_{ij}(x_i(t), x_j(t))$ has the following properties:

- If the relative distance of any pair of agents $i, j$ is less than $r$, $W_{ij}(x_i(t), x_j(t))$ is not equal to 0.
- If the relative distance of any pair of agents $i, j$ tends to $r$, $W_{ij}(x_i(t), x_j(t))$ increases.
- If the relative distance of any pair of agents $i, j$ is larger than $r$ and less than $R$, $W_{ij}(x_i(t), x_j(t))$ increases.

In this case, if the relative distance of any pair of agents $i, j$ belongs to $[r, R]$, the potential field function $W_{ij}(x_i(t), x_j(t))$ can be regarded as an extra control input such that the collision avoidance is realized.

Let $e_i(t) = x_i(t) - d_i - x_0(t)$ and $\nu_i(t) = v_i(t) - v_0(t)$, then the system in (1)-(6) is transformed into the following form

$$\begin{cases} \dot{e}_i(t) = \nu_i(t), \\ \dot{\nu}_i(t) = \widetilde{f}(e_i(t-\tau), \nu_i(t-\tau)) - \alpha_i(t) \\ \qquad \times \sum_{j=1}^{N} \ell_{ij}(e_j(t) + \nu_j(t)) - b_i(e_i(t) + \nu_i(t)) \\ \qquad - \sum_{j=1}^{N} \frac{\partial W_{ij}^T(x_i(t), x_j(t))}{\partial x_i(t)}, \end{cases} \quad (8)$$

where $\widetilde{f}(e_i(t-\tau), \nu_i(t-\tau)) = f(x_i(t-\tau), v_i(t-\tau)) - f(x_0(t-\tau) + d_i, v_0(t-\tau))$, and for $i \in \mathcal{P}$, $b_i \neq 0$, otherwise, $b_i = 0$.

For the sake of obtaining the main results, the following assumption, lemma and definition are necessary.

*Assumption 1:* For any $x(t), y(t), v(t), u(t) \in \mathbb{R}^n$, there exist two nonnegative parameters $\varphi$ and $\phi$, such that the nonlinear function $f(\cdot, \cdot)$ satisfies the following condition,

$$\|f(x(t), v(t)) - f(y(t), u(t))\| \leq \varphi\|x(t) - y(t)\| + \phi\|v(t) - u(t)\|.$$

*Lemma 1:* For any vectors $X, Y \in \mathbb{R}^n$, the following inequality holds

$$2X^T Y \leq X^T X + Y^T Y.$$

*Definition 1:* The leader-following formation control of second-order multi-agent systems with time delay in (8) is said to be ensured, if the following conditions hold

$$\lim_{t\to\infty} \|x_i(t) - d_i - x_0(t)\| = 0,$$
$$\lim_{t\to\infty} \|v_i(t) - v_0(t)\| = 0.$$

*Remark 1:* In this article, the formation control problem has been considered for a second-order multi-agent system with time delay in (8). Different from the usual formation control problems in [3], [4], [10], the collision avoidance control law in (7) is constructed by a typical potential field function. This potential field function can be regarded as a repulsive force, such that the collision avoidance among any pair of agents can be achieved. Compared with the formation control problems with collision avoidance [6], [25], [32], a mixed control approach is considered. The controller is designed by using this mixed control approach that not only involves the leader-following and adaptive control strategies to ensure the specified formation shape, but also depends on the potential field function in order to guarantee the collision avoidance.

*Remark 2:* Recently, the adaptive control strategies have been introduced into many problems of multi-agent system [5], [35], [46]. However, time delay is inevitable due to the finite response speed of hardware. In this article, the adaptive control strategy is utilized to design the formation control protocol in (3) and (4), and the updated law in (5). On the other hand, the pinning control method has been proven to be an effective method which only acts on a small fraction of agents, rather than the whole system in the control process. In this case, this article develops the adaptive control and pinning control methods for application to the formation control problem of second-order multi-agent system with time delay in (8).

*Remark 3:* In this article, the time delay $\tau$ has been considered for the second-order multi-agent system in (1). The time delay $\tau$ is molded by a kind of system delay, which is caused by the finite response speed of hardware and is regarded as a signal for an exact amount of time in (1). For this reason, the time delay $\tau$ is assumed to be a same constant in all the leader and follower agents. Although, an ideal time delay in networked systems would be heterogeneous, it may result in some complex mathematical analyses. For the case that the time delay is heterogeneous, we will try to explore these in the following work.

## III. FORMATION CONTROL AND COLLISION AVOIDANCE OF SECOND-ORDER MULTI-AGENT SYSTEM

In this section, the formation control problem of the second-order multi-agent system in (8) is studied. The derived results are divided into two cases. The first case considers the second-order multi-agent system in (8) without time delay, and the second case examines the second-order multi-agent system with time delay in (8).

*Theorem 1:* Suppose that Assumption 1 and $\tau = 0$ hold. The formation control and collision avoidance of the second-order multi-agent system in (8) are achieved, if there is a positive parameter $\widetilde{\alpha}$, such that the following condition holds,

$$\widetilde{\varphi} - \widetilde{\alpha}\lambda_1(\mathcal{L} + B) < 0, \quad (9)$$

where $\widetilde{\varphi} = \max\{1 + \varphi + 2\phi, 3 + \phi + 2\varphi\}/2$, the matrix $B = \mathrm{diag}\{b_1, \ldots, b_N\}$, and $\lambda_1(\mathcal{L}+B)$ is the smallest eigenvalue of matrix $\mathcal{L} + B$.

*Proof:* Denote $W_{ij}(x_i(t), x_j(t))$ as $W_{ij}(t)$ for brevity. Similarly to [38], [39], consider the following Lyapunov function for the system second-order multi-agent system in (7),

$$V(t) = \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\widetilde{\alpha}e_{ji}^T(t)e_{ji}(t) + \sum_{i\in\mathcal{V}}e_i^T(t)\nu_i(t) + \frac{1}{2}\sum_{i\in\mathcal{V}}(e_i^T(t)e_i(t) + \nu_i^T(t)\nu_i(t)) + \frac{1}{2\alpha}\sum_{i\in\mathcal{V}}(\alpha_i(t)-\widetilde{\alpha})^2 + \frac{1}{2}\sum_{i,j\in\mathcal{V}}W_{ij}, \quad (10)$$

where $e_{ji}(t) = e_j(t) - e_i(t)$. In view of Schur complement theory [49], the Lyapunov function $V(t)$ can be rewritten by

$$V(t) = \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\widetilde{\alpha}e_{ji}^T(t)e_{ji}(t) + \frac{1}{2}\sum_{i\in\mathcal{V}}\begin{bmatrix} e_i(t) & \nu_i(t)\end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 1\end{bmatrix}\begin{bmatrix} e_i(t) \\ \nu_i(t)\end{bmatrix} + \frac{1}{2\alpha}\sum_{i\in\mathcal{V}}(\alpha_i(t)-\widetilde{\alpha})^2 + \frac{1}{2}\sum_{i,j\in\mathcal{V}}W_{ij}, \quad (11)$$

which means that the Lyapunov function $V(t)$ is positive definite from (5)-(7).

For $i \neq j$, the derivative of $V(t)$ can be obtained as follows

$$\begin{aligned}\dot{V}(t) &= 2\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\widetilde{\alpha}e_{ij}^T(t)\dot{e}_{ij}(t) + \sum_{i\in\mathcal{V}}\Big(\dot{e}_i^T(t)\nu_i(t) + e_i^T(t)\dot{\nu}_i(t)\Big) + \sum_{i\in\mathcal{V}}\Big(e_i^T(t)\dot{e}_i(t) + \nu_i^T(t)\dot{\nu}_i(t)\Big) \\ &\quad + \frac{1}{\alpha}\sum_{i\in\mathcal{V}}(\alpha_i(t)-\widetilde{\alpha})\dot{\alpha}_i(t) + \frac{1}{2}\sum_{i,j\in\mathcal{V}}\Big(\frac{\partial W_{ij}^T(t)}{\partial x_i(t)} \times \dot{x}_i(t) + \frac{\partial W_{ij}^T(t)}{\partial x_j(t)}\dot{x}_j(t)\Big) \\ &\leq 2\sum_{i=1}^N\sum_{i=1}^N\widetilde{\alpha}e_{ij}^T(t)\nu_{ij}(t) + \frac{1}{2}\Big(\sum_{i=1}^N e_i^T(t)e(t) + 3\nu_i^T(t)\nu_i(t)\Big) + \sum_{i=1}^N\Big(e_i(t)+\nu_i(t)\Big)^T \\ &\quad \times\Big(\widetilde{f}(e_i(t),\nu_i(t)) - \alpha_i(t)\sum_{j=1}^N\ell_{ij}(e_j(t)+\nu_j(t)) - b_i(e_i(t)+\nu_i(t)) - \frac{\partial W_{ij}^T(t)}{\partial x_i(t)}\Big) + \sum_{i=1}^N(\alpha_i(t)-\widetilde{\alpha}) \\ &\quad \times\sum_{j=1}^N\Big(e_{ji}(t)+\nu_{ji}(t)\Big)^T\Big(e_{ji}(t)+\nu_{ji}(t)\Big) + \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N\Big(\frac{\partial W_{ij}^T(t)}{\partial x_i(t)}\dot{x}_i(t) + \frac{\partial W_{ij}^T(t)}{\partial x_j(t)}\dot{x}_j(t)\Big), \quad (12)\end{aligned}$$

where $\nu_{ji}(t) = \nu_j(t) - \nu_i(t)$.

Let $e(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$ and $\nu(t) = [\nu_1^T(t), \ldots, \nu_N^T(t)]^T$. It follows from the definitions of $e(t)$, $\nu(t)$ and $\mathcal{L}$ that

$$\sum_{i=1}^N\sum_{j=1}^N\widetilde{\alpha}e_{ij}^T(t)\nu_{ij}(t) = \widetilde{\alpha}e^T(t)(\mathcal{L}\otimes I_n)\nu(t),$$

$$\sum_{i=1}^N\Big(e_i(t)+\nu_i(t)\Big)^T\Big(\alpha_i(t)\sum_{j=1}^N\ell_{ij}e_j(t) + b_ie_i(t)\Big) = (e(t)+\nu(t))^T((\alpha_i(t)\mathcal{L}+B)\otimes I_n)e(t),$$

and

$$\sum_{i=1}^N\Big(e_i(t)+\nu_i(t)\Big)^T\Big(\alpha_i(t)\sum_{j=1}^N\ell_{ij}\nu_j(t) + b_i\nu_i(t)\Big) = (e(t)+\nu(t))^T((\alpha_i(t)\mathcal{L}+B)\otimes I_n)\nu(t).$$

Using Assumption 1 and Lemma 1, one has the following

$$\begin{aligned}&\sum_{i=1}^N\Big(e_i(t)+\nu_i(t)\Big)^T\widetilde{f}(e_i(t),\nu_i(t)) \\ &\leq \sum_{i=1}^N(\|e_i(t)\| + \|\nu_i(t)\|)(\varphi\|e_i(t)\| + \phi\|\nu_i(t)\|) \\ &\leq \sum_{i=1}^N(\frac{2\varphi+\phi}{2}\|e_i(t)\|^2 + \frac{\varphi+2\phi}{2}\|\nu_i(t)\|^2). \quad (13)\end{aligned}$$

From the definition of $\partial W_{ij}(t)$, it is easy to verify that

$$\frac{\partial W_{ij}(t)}{\partial x_i(t)} = -\frac{\partial W_{ij}(t)}{\partial x_j(t)} = \frac{\partial W_{ji}(t)}{\partial x_i(t)} = -\frac{\partial W_{ji}(t)}{\partial x_j(t)},$$

which implies that

$$\begin{aligned}&\sum_{i=1}^N\nu_i^T(t)\sum_{j=1}^N\frac{\partial W_{ij}^T(t)}{\partial x_i(t)} \\ &= \frac{1}{2}\sum_{i=1}^N\nu_i^T(t)\sum_{j=1}^N\Big(\frac{\partial W_{ij}^T(t)}{\partial x_i(t)} + \frac{\partial W_{ij}^T(t)}{\partial x_i(t)}\Big) \\ &= \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N\Big(\frac{\partial W_{ij}^T(t)}{\partial x_i(t)}\nu_i(t) + \frac{\partial W_{ij}^T(t)}{\partial x_j(t)}\nu_j(t)\Big). \quad (14)\end{aligned}$$

Based on the matrix decomposition theory, there is a unitary matrix $H$ such that $H^T\mathcal{L}H = \Lambda$ holds, where $\Lambda = \mathrm{diag}\{0, \lambda_2(\mathcal{L}), \ldots, \lambda_N(\mathcal{L})\}$, $H = [H_1, \ldots, H_N]$ and $H_1 = 1_N^T/\sqrt{N}$. Let $z(t) = (H^T\otimes I_n)e(t)$ and $y(t) = (H^T\otimes I_n)\nu(t)$, where $z(t) = [z_1^T(t), \ldots, z_N^T(t)]^T$ and $y(t) = [y_1^T(t), \ldots, y_N^T(t)]^T$. Thus, it follows from the equations in (9)-(14) that

$$\begin{aligned}\dot{V}(t) &\leq \sum_{i=1}^N(\frac{2\varphi+\phi+1}{2}\|e_i(t)\|^2 + \frac{\varphi+2\phi+3}{2}\|\nu_i(t)\|^2) \\ &\quad - \widetilde{\alpha}e^T(t)((\mathcal{L}+B)\otimes I_n)e(t) - \widetilde{\alpha}\nu^T(t) \times((\mathcal{L}+B)\otimes I_n)\nu(t) \\ &= \sum_{i=1}^N\Big[\Big(\frac{2\varphi+\phi+1}{2} - \widetilde{\alpha}\lambda_1(\mathcal{L}+B)\Big)\|z_i(t)\|^2 + \Big(\frac{\varphi+2\phi+3}{2} - \widetilde{\alpha}\lambda_1(\mathcal{L}+B)\Big)\|y_i(t)\|^2\Big] \\ &< 0. \quad (15)\end{aligned}$$

In addition, for $i \neq j$, it is easy to deduce that

$$\lim_{\|x_i(t)-x_j(t)\|\to r^+} W_{ij}(t) = \infty,$$

$$\lim_{\|x_i(t)-x_j(t)\|\to r^+} \frac{\partial W_{ij}(t)}{\partial x_i(t)} = \infty.$$

That is, the collision avoidance is ensured. This completes the proof. ■

Theorem 1 studies the formation control problem of the second-order multi-agent system without time delay in (8). Note that the matrix $\mathcal{L}+B$ is a positive definite matrix, since the Laplacian matrix $\mathcal{L}$ is the positive semi-definite matrix and the matrix $B$ is a positive definite diagonal matrix. Now, time delay is considered in the following theorem.

*Theorem 2:* Suppose that Assumption 1 and $\tau \neq 0$ hold. The formation control and collision avoidance of the second-order multi-agent system in (8) are achieved, if there is a positive parameter $\widetilde{\alpha}$, such that the following condition holds,

$$\widehat{\varphi} - \widetilde{\alpha}\lambda_1(\mathcal{L}+B) < 0, \tag{16}$$

where the matrix $B = \text{diag}\{b_1, \ldots, b_N\}$, $\widehat{\varphi} = \phi + \varphi + \tau/2 + 3/2$, and $\lambda_1(\mathcal{L}+B)$ is the smallest eigenvalue of matrix $\mathcal{L}+B$.

*Proof:* Consider the following Lyapunov function for the system second-order multi-agent system in (7),

$$\begin{aligned} V(t) &= \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} \widetilde{\alpha} e_{ji}^T(t)e_{ji}(t) + \sum_{i\in\mathcal{V}} e_i^T(t)\nu_i(t) \\ &\quad + \frac{1}{2}\sum_{i\in\mathcal{V}}(e_i^T(t)e_i(t) + \nu_i^T(t)\nu_i(t)) \\ &\quad + \frac{1}{2\alpha}\sum_{i\in\mathcal{V}}(\alpha_i(t)-\widetilde{\alpha})^2 + \frac{1}{2}\sum_{i,j\in\mathcal{V}} W_{ij}(t) \\ &\quad + \sum_{i\in\mathcal{V}}\frac{\varphi+\phi}{2}\int_{t-\tau}^{t}(e_i^T(\theta)e_i(\theta) + \nu_i^T(\theta)\nu_i(\theta))d\theta \\ &\quad + \sum_{i\in\mathcal{V}}\int_0^{\tau}\int_{t-\theta}^{t}(e_i^T(\vartheta)e_i(\vartheta) + \nu_i^T(\vartheta)\nu_i(\vartheta))d\vartheta d\theta. \end{aligned}$$

Based on the similar discussions of (11), the above Lyapunov function is positive. Then, for $i \neq j$, the derivative of $V(t)$ can be obtained as follows

$$\begin{aligned} \dot{V}(t) &\leq 2\sum_{i=1}^{N}\sum_{i=1}^{N}\widetilde{\alpha} e_{ij}^T(t)\nu_{ij}(t) + \frac{1}{2}\sum_{i=1}^{N}\Big(e_i^T(t)e_i(t) \\ &\quad + 3\nu_i^T(t)\nu_i(t)\Big) + \sum_{i=1}^{N}\Big(e_i(t)+\nu_i(t)\Big)^T \\ &\quad \times \Big(\widetilde{f}(e_i(t-\tau), \nu_i(t-\tau)) - \alpha_i(t)\sum_{j=1}^{N}\ell_{ij}(e_j(t) \\ &\quad + \nu_j(t)) - b_i(e_i(t)+\nu_i(t)) - \frac{\partial W_{ij}^T(t)}{\partial x_i(t)}\Big) \\ &\quad + \sum_{i=1}^{N}(\alpha_i(t)-\widetilde{\alpha})\sum_{j=1}^{N}\Big(e_{ji}(t)+\nu_{ji}(t)\Big)^T\Big(e_{ji}(t) \\ &\quad + \nu_{ji}(t)\Big) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\Big(\frac{\partial W_{ij}^T(t)}{\partial x_i(t)}\dot{x}_i(t) \\ &\quad + \frac{\partial W_{ij}^T(t)}{\partial x_j(t)}\dot{x}_j(t)\Big) + \frac{\varphi+\phi}{2}\sum_{i=1}^{N}\Big(e_i^T(t)e_i(t) \\ &\quad - e_i^T(t-\tau)e_i(t-\tau) + \nu_i^T(t)\nu_i(t) \\ &\quad - \nu_i^T(t-\tau)\nu_i(t-\tau)\Big) + \sum_{i=1}^{N}\Big(\tau(e_i^T(t)e_i(t) \\ &\quad + \nu_i^T(t)\nu_i(t)) - \int_{t-\tau}^{t}(e_i^T(\theta)e_i(\theta) + \nu_i^T(\theta) \\ &\quad \times \nu_i(\theta))d\theta. \end{aligned} \tag{17}$$

Using Assumption 1 and Lemma 1, one has the following

$$\begin{aligned} &\sum_{i=1}^{N}\Big(e_i(t)+\nu_i(t)\Big)^T\widetilde{f}(e_i(t-\tau), \nu_i(t-\tau)) \\ &\leq \sum_{i=1}^{N}(\|e_i(t)\| + \|\nu_i(t)\|)(\varphi\|e_i(t-\tau)\| \\ &\quad + \phi\|\nu_i(t-\tau)\|) \\ &\leq \sum_{i=1}^{N}\frac{\varphi+\phi}{2}(\|e_i(t)\|^2 + \|e_i(t-\tau)\|^2 \\ &\quad + \|\nu_i(t)\|^2 + \|\nu_i(t-\tau)\|^2). \end{aligned} \tag{18}$$

Similar to (15), based on the matrix decomposition theory, there is a unitary matrix $H$ such that $H^T\mathcal{L}H = \Lambda$ holds, where $\Lambda = \text{diag}\{0, \lambda_2(\mathcal{L}), \ldots, \lambda_N(\mathcal{L})\}$, $H = [H_1, \ldots, H_N]$ and $H_1 = 1_N^T/\sqrt{N}$. Let $z(t) = (H^T \otimes I_n)e(t)$ and $y(t) = (H^T \otimes I_n)\nu(t)$, where $z(t) = [z_1^T(t), \ldots, z_N^T(t)]^T$ and $y(t) = [y_1^T(t), \ldots, y_N^T(t)]^T$. Moreover, it is easy to check that the last term in satisfies $-\int_{t-\tau}^{t}(e_i^T(\theta)e_i(\theta) + \nu_i^T(\theta)\nu_i(\theta))d\theta \leq 0$. In this case, it is not hard to obtain that

$$\begin{aligned} \dot{V}(t) &\leq \frac{1}{2}\sum_{i=1}^{N}\Big((1+\tau+2\varphi+2\phi)\|e_i(t)\|^2 + (3+\tau+2\varphi \\ &\quad + 2\phi)\|\nu_i(t)\|^2\Big) - \widetilde{\alpha}e^T(t)((\mathcal{L}+B)\otimes I_n)e(t) \\ &\quad - \widetilde{\alpha}\nu^T(t)((\mathcal{L}+B)\otimes I_n)\nu(t) \\ &= \frac{1}{2}\sum_{i=1}^{N}\Big[\Big((1+\tau+2\varphi+2\phi) - \widetilde{\alpha}\lambda_1(\mathcal{L}+B)\Big) \\ &\quad \times \|z_i(t)\|^2 + \Big((3+\tau+2\varphi+2\phi) - \widetilde{\alpha}\lambda_1(\mathcal{L}+B)\Big) \\ &\quad \times \|y_i(t)\|^2\Big] \\ &< 0. \end{aligned} \tag{19}$$

In addition, for $i \neq j$, it is easy to know that

$$\lim_{\|x_i(t)-x_j(t)\|\to r^+} W_{ij(t)} = \infty,$$

$$\lim_{\|x_i(t)-x_j(t)\|\to r^+} \frac{\partial W_{ij}(t)}{\partial x_i(t)} = \infty.$$

Thus, the proof is completed. ■

*Remark 4:* Note that the sufficient criteria in Theorems 1 and 2 are presented by the algebraic conditions instead of the terms of linear matrix inequalities. This means that the sufficient criteria in Theorems 1 and 2 are easier for calculation and simulation. Particularly, it can be anticipated that the calculation of the obtained criteria only depends on the size of matrix $\mathcal{L}+B$ if applying large-scale system. On the other hand, in many works of pinning control, the number of pinning nodes can be solved or optimized. However, the best solution of pinning nodes cannot be solved in this article. In our future works, we will attempt to investigate this problem.

*Remark 5:* In views of Theorems 1 and 2, it has been proved that for $i \in \mathcal{V}$, all the states $x_i(t)$ and $v_i(t)$ approach to $x_0(t)+d_i$ and $v_0(t)$, respectively. Thus, the control gain $\alpha_i(t)$ in (5) is convergent for all $i \in \mathcal{V}$. Actually, it can be anticipated that there exist two positive constants $T_0$ and $\hat{\alpha}$, such that $\int_t^{t+T_0} \sum_{i\in\mathcal{V}} \chi_i^T(\tau)\chi_i(\tau)d\tau \geq \hat{\alpha}I_{2N}$ holds if all conditions in Theorems 1 and 2 are satisfied, where $\chi_i(t) = [e_i^T(t), v_i^T(t)]^T$ and $\hat{\alpha}$ is a constant depended on $\widetilde{\alpha}$ in Theorems 1 and 2. At this stage, $\widetilde{\alpha} > 0$ can be regarded as a special case of persistent excitation (PE) condition which has been investigated in many adaptive problems. For more details of PE condition, the reference [50] is recommended.

*Remark 6:* Note that the Lyapunov method plays an important role in the proof of Theorems 1 and 2. When constructing these Lyapunov functions, the primary aim is to consider the dynamic relationship among the parameters of multi-agent system in (8). In this case, the first challenge is how to apply a Lyapunov function that can reflect the system delay $\tau$, the control gains $\alpha_i(t)$ and $b_i$, and the topological structure. Inspired by [3], [5], [33], [36], the Lyapunov functions in the proof of Theorems 1 and 2 have been constructed. Then, the second challenge is how to design an appropriate potential field function of the avoidance control (7) into the Lyapunov functions in the proof of Theorems 1 and 2. It is worth highlighting that the derivative of the potential field function $W_{ij}(t)$ between any pair of agents $i, j$ will approach to zero as time evolves. It means that the derivative of these Lyapunov functions is always negative definite.

*Remark 7:* Recently, there are many leader-following control methods that have been considered for multi-agent systems [41]–[46]. Compared with these works, in this article, the leader-following control method is designed for formation control problem with collision avoidance and time delay. In this control method, on one hand, the pinning and adaptive control strategies have considered to achieve the specified formation shape, and on the other hand, the relationship among control parameters is shown. Moreover, a typical potential field function has designed in this leader-following control method. Although, this potential field function, as a collision avoidance control law, is irrelevant to the leader, it can be utilized for ensuring collision avoidance.

*Remark 8:* This article studies the formation control and collision avoidance problem of second-order multi-agent systems with time delay, where the time delay is modeled by system delay. The main idea of this article is to construct an appropriate Lyapunov function. This Lyapunov function in Theorem 2 contains some relevant terms, where the integral items $\sum_{i\in\mathcal{V}} \frac{\varphi+\phi}{2} \int_{t-\tau}^{t} (e_i^T(\theta)e_i(\theta) + v_i^T(\theta)v_i(\theta))d\theta$ and $\sum_{i\in\mathcal{V}} \int_0^{\tau} \int_{t-\theta}^{t} (e_i^T(\vartheta)e_i(\vartheta)+v_i^T(\vartheta)v_i(\vartheta))d\vartheta d\theta$ are utilized to deal with the time delay. In this case, this method focuses more on the mathematical techniques, but may not reveal the relationship among the parameters of system.

## IV. EXAMPLE

*Example 1:* An example is given to show the formation control and collision avoidance problem of a second-order multi-agent system with four agents. The communication graph is a globally coupled network, which is drawn in Fig.1.

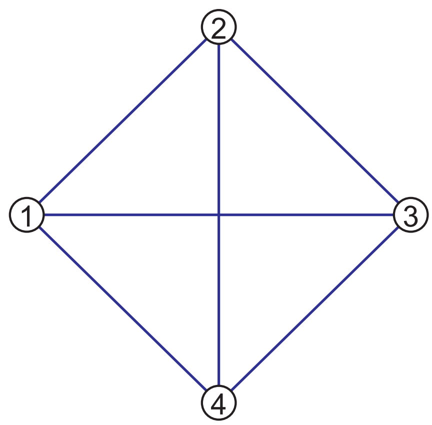


**FIGURE 1. The communication structure with 4 agents.**

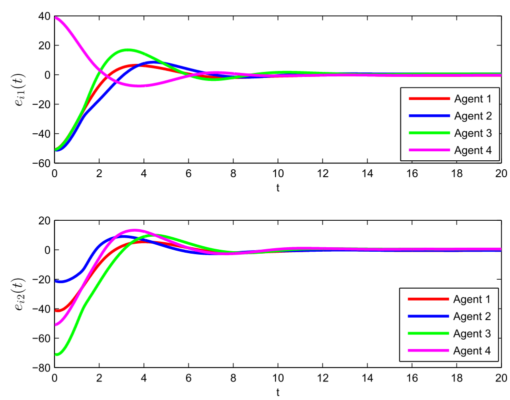


**FIGURE 2. The relative positions $x_{i1}(t) - x_0(t) - d_{i1}$ and $x_{i2}(t) - x_0(t) - d_{i2}$ of formation control and collision avoidance in second-order multi-agent system.**

The control gains are $\alpha = 0.81$ and $b_i = 1$ for all $i \in \mathcal{V}$. The nonlinear function is $f(x_i(t-\tau), v_i(t-\tau)) = 0.2\sin(x_i(t-2)) + 0.1v_i(t-2)$, which means that $\varphi = 0.2$ and $\phi = 0.1$. Now, denote the desired position as a square with side length 20, the minimum radius $r$ of the avoidance region is 3, and the radius $R$ of the detection region is 70.

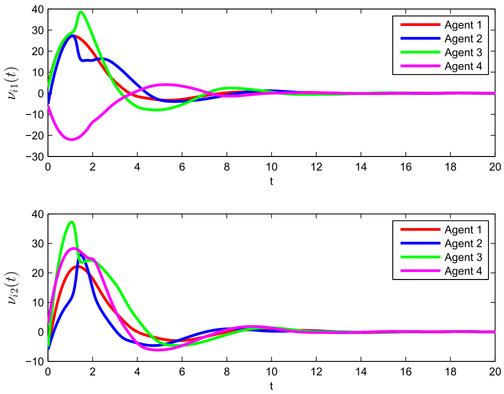


**FIGURE 3.** The relative velocities $v_{i1}(t) - v_0(t)$ and $v_{i2}(t) - v_0(t)$ of formation control and collision avoidance in second-order multi-agent system.

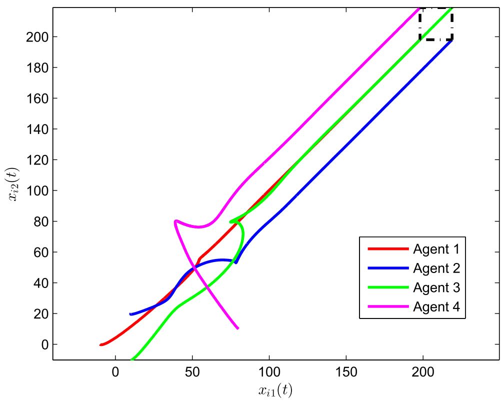


**FIGURE 4.** The trajectories of all the agents under the formation control law with collision avoidance.

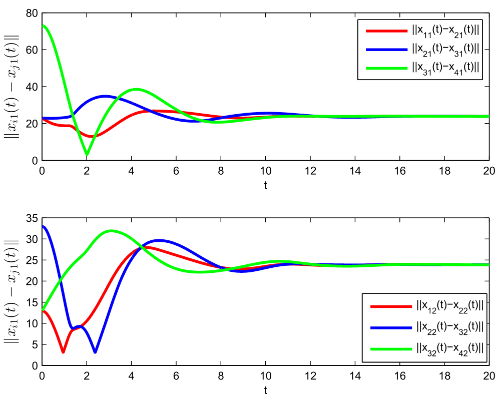


**FIGURE 5.** The relative distances $\|x_{i1}(t) - x_{j1}(t)\|$ and $\|x_{i2}(t) - x_{j2}(t)\|$ of formation control and collision avoidance in second-order multi-agent system.

In this case, it is easy to check that $\lambda_i(\mathcal{L} + B) = 1$, which means that all of the conditions in Theorem 1 are satisfied if choosing $\widetilde{\alpha} > 3$. Therefore, the formation control of second-order multi-agent system with four agents is ensured, and the collision avoidance can be avoided. The relative positions $x_{i1}(t) - x_0(t) - d_{i1}$ of all agents approach to zero in Fig. 2 as well as the relative positions $x_{i2}(t) - x_0(t) - d_{i2}$ of all agents. In Fig. 3, the relative velocities $v_{i1}(t) - v_0(t)$ and $v_{i2}(t) - v_0(t)$ converge to zero. In Fig. 4, the position trajectories of all the agents are drawn, it can be observed that all of the agents approach to the desired position of the square. The relative distances $\|x_{i1}(t) - x_{j1}(t)\|$ and $\|x_{i2}(t) - x_{j2}(t)\|$ are shown in Fig. 5.

## V. CONCLUSION

This article studies the formation control and collision avoidance problem for a second-order multi-agent system with time delay. To achieve the specified formation shape, the leader-following control method utilizing the distributed adaptive control law is considered. Then, a kind of potential field function is applied to avoid the collision avoidance. Based on the Lyapunov theory, two sufficient criteria are obtained in terms of algebraic conditions such that the formation control and collision avoidance of the second-order multi-agent system are ensured. Subsequently, a numerical simulation is given to illustrate the effectiveness of the obtained results. Our further works including formation control protocol with collision and obstacle avoidance will be carried out.

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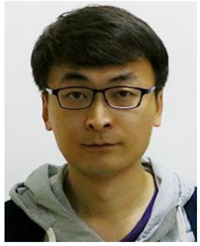

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