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Optimal Second Order Integral Sliding Mode Based Composite Nonlinear Feedback Approach for an Electrostatic Micromirror

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ABSTRACT In this article, an optimal second order integral sliding mode based composite nonlinear feedback (SOISM-CNF) controller is presented for an electrostatic micromirror. The proposed controller is able to improve the transient performance and guarantee robustness against uncertainties simultaneously. A tabu search and particle swarm optimization (TS-PSO) algorithm is used to solve the parameter tuning problem. The input saturation issue is considered in the SOISM-CNF controller design. The stability of closed-loop system is proved by Lyapunov stability theory. The simulation results demonstrate the effectiveness of SOISM-CNF controller with guaranteed transient performance under the external disturbances.

INDEX TERMS Composite nonlinear feedback, second order sliding mode control, particle swarm optimization, electrostatic micromirror.

I. INTRODUCTION

With the development of Micro-electro-mechanical system (MEMS), electrostatic micromirrors become the key components for large-port-count high capacity optical cross-connects (OXC) [1], biomedical imaging [2] and high resolution displays [3]. Accurate positioning, fast responses and large operation range are essential for micromirror-based applications, which motivated the majority of the studies on the control problem of MEMS micromirror. PID controller has been reported to improve the positioning performance of a MEMS micromirror [4]. A nonlinear controller based on feedback linearization is reported for an electrostatic micromirror [5]. The developed control system achieves stable operation beyond the pull-in voltage for set-point and scanning controls. The controlled micromirrors are inevitable affected by external disturbance in practical systems. The aforementioned PID and nonlinear control schemes are not able to eliminate the effect of lumped disturbances, which comes from the micromirrors' uncertainties and external disturbances.

The robust control for micromirrors has gained significant attention in recent years, and various research works have been reported. H_∞ robust control has been proposed for a large-piston MEMS micromirror-based compact fourier transform spectrometer systems [6]. The closed-loop system maintains good stability and robustness under various driving conditions. A second order twisting sliding mode controller is proposed to provide enhanced transient response and positioning performance for an electrostatic micromirror-based laster scanning system [7]. An adaptive self-tuning controller is reported to on-line compensate parameter variations and improve positioning performance of an electrostatic micromirror [8]. Although these aforementioned algorithms have demonstrated benefits of enhancing the positioning performance and system robustness for electrostatic micromirrors, the transient response is not directly considered in previous works.

In practice, MEMS systems are commonly subject to constraints leading to a rigorous consideration of the transient performance of controlled system. For example, position constraints of micromirror should be imposed to prevent collisions with electrodes. In the micromirror-based OXC system, the angle of micromirrors receiving the command

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signal is required to achieve the desired position within 5 to 10 microseconds in order to reduce the insertion loss [1].

Inspired by the work of Lin *et al.* [9], the composite nonlinear feedback (CNF) control is proposed for linear systems [10]. The CNF controller consists of a linear feedback part and a nonlinear feedback part. The linear feedback law is responsible for yielding a closed-loop system with small damping ratio to achieve short settling time. The nonlinear feedback law is introduced to increase the damping ratio as the controlled output approaches the tracking target in order to reduce the overshoot. The composite nonlinear feedback (CNF) technique is one effective approach to deal with the constraints related to the transient performance issue of linear systems. The result has been extended to the partially linear composite system with input saturation [11]. A CNF control law is designed for a hard-disk-drive (HDD) servo system [12]. The parameter tuning problem of CNF is formulated as an optimization problem and solved by numerical methods. A CNF controller by using standard backstepping technique has been reported to solve the tracking control problem of strict-feedback nonlinear systems with input saturation [13]. The conventional CNF control is designed based on assumption that no disturbance is acting on the system. However, the system disturbances are inevitable which intensively interfere with the system stability and transient performance. Among the exiting control techniques, sliding mode control (SMC) is an efficient robust control approach [14] which is widely used in many field such as spacecraft [15], linear motor [16] and MEMS gyroscope [17].

Recently, CNF control has been proposed to be combined with SMC to cope with disturbances [18], [19]. A nonlinear controller based on composite nonlinear feedback and integral sliding mode (ISM-CNF) has been reported to improve the transient performance of matched and unmatched uncertain linear systems [20]. A CNF controller based nonlinear integral sliding surface is proposed for matched uncertain linear systems with multiple state-delays [21]. A ISM-CNF tracking controller is proposed for linear MIMO systems with multiple time-delays and external disturbances, and the control parameters are optimally tuned using a modified random search algorithm [22]. In [23], the result is further extended for the uncertain switched systems. The traditional CNF controller is more applicable for the constant references. A modified CNF technique in path following control for autonomous ground vehicle (AGV) is proposed in [24], where the linear state feedback law of CNF is designed by considering the time varying multiple references. Integral sliding mode based composite nonlinear feedback control is reported for descriptor system [25], [26].

Significant progress has been made in design and application of the CNF and SMC technique. However, high-frequency switching in SMC results in chattering at the control input. The second order sliding mode control techniques are considered as an effective solution for alleviating the chattering problem [27]. Twisting algorithm is a widely used second order sliding mode controller [28]. However, the control signal is discontinuous. An integral sliding mode controller is reported for linear time-invariant implicit systems [29]. A continuous modified twisting controller is proposed for position control of stewart platform [30]. A continuous twisting algorithm has been reported [31] and applied for third-order systems [32]. To the best of the authors' knowledge, little efforts have been made in the study of second order sliding mode based CNF control. The combination problem of SOISM and CNF control for the fast and accurate robust control of uncertain micromirror system with input saturation is still open in the literature, which has motivated the present study. Furthermore, the selection of a general nonlinear gain remains the key issue and must be further investigated, especially in the context of controlled system with external disturbance.

To tackle this dilemma, one effective solution is to convert the parameter tuning problem into a minimization problem with some performance criteria [12]. Various computational intelligence-based techniques have been reported to the optimization problem under different conditions. The HL-RF iterative algorithm is used to evaluate reliability index in reliability-based design optimization, where the formulation of the Lyapunov exponents is derived for the non-convergence problems [33]. A novel relaxed exponential nominal value method (RENVM) is proposed to evaluate the corresponding nonprobabilistic reliability index [34]. Meng *et al.* review the relevant optimization algorithms including particle swarm optimization, artificial bee colony, ant lion optimizer and multi-verse optimizer in the recent work [35]. Compared with the other similar optimization techniques, PSO is not largely affected by the size and nonlinearity of the problem, and therefore, it is easy to implement. PSO has also been reported to solve the parameters tuning problem of CNF controller [36]. To further increase the convergence speed, a tabu search and particle swarm optimization (TS-PSO) algorithm has been reported to solve job shop scheduling optimization problem [37].

The main contributions of this work are: (1) Different from the existing scheme [18]–[24], an optimal second order integral sliding mode (SOISM) based composite nonlinear feedback (CNF) controller is presented to solve the control problem of an electrostatic MEMS micromirror. The output could track the reference signal as fast as possible with the improved transient performance in presence of disturbances. (2) A tabu search and particle swarm optimization (TS-PSO) algorithm is utilized to solve multi-parameters adjustment problem. (3) The stability of the closed-loop system with input saturation and disturbance is proved by Lyapunov method.

II. MATHEMATICAL MODEL OF ELECTROSTATIC MEMS MICROMIRROR

As shown in Figure 1, an electrostatic actuated MEMS micromirror contains mirror plate, spring beams and actuating electrodes. The dynamic equation of the studied

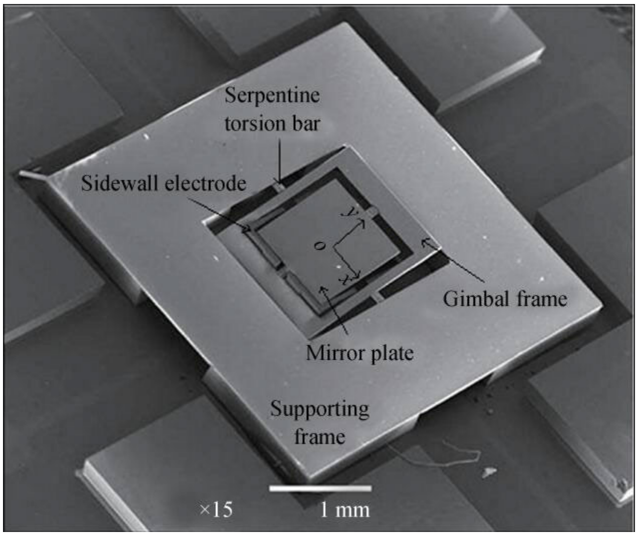

FIGURE 1. The electrostatic torsional MEMS micromirror.

micromirror are described as

$$\begin{aligned} (J_1+J_2)\ddot{\alpha}+D_1\dot{\alpha}+K_1\alpha &= T_\alpha \\ J_1\ddot{\beta}+D_2\dot{\beta}+K_2\beta &= T_\beta \end{aligned} \tag{1}$$

where J_1, J_2 are mass moment of inertias, D_1, D_2 are damping coefficients, K_1, K_2 are stiffness coefficients. The mirror plate is actuated to rotate at two axes by applying voltages to the actuating electrodes. The tilt angles of x-axis and y-axis are α and β respectively. T_α and T_β are electrostatic torques for x-axis and y-axis respectively.

Introducing the normalized parameters $\tau = \sqrt{\frac{K_2}{J_1}}t$ and letting $x_1 = \alpha, x_2 = \frac{d\alpha}{d\tau}, x_3 = \beta, x_4 = \frac{d\beta}{d\tau}$, we can rewrite the equation (1) as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -R_1x_2 - \lambda_{\alpha\beta}x_1 + G_1T_\alpha \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -R_2x_4 - x_3 + G_2T_\beta \end{aligned} \tag{2}$$

where the parameters are $\lambda_{\alpha\beta} = 0.2251$, $R_1 = 0.16$, $G_1 = 3.0827 \times 10^6$, $R_2 = 0.15$ and $G_2 = 1.7894 \times 10^7$.

The parameter variation and external disturbance in the micromirror system are inevitable. Taking the input saturation, uncertainty and disturbance into consideration, the above equation can be depicted as

$$\begin{aligned} \dot{x} &= Ax + Bsat(u) + Bd \\ y &= Cx \end{aligned} \tag{3}$$

where A, B, C are constant matrices, $x \in R^n$ is the system state, $y \in R$ is the output and $u \in R$ is the control input. The maximum of control input is u_{max} and $sat(u) = sgn(u)min\{u_{max}, |u|\}$. $d \in R$ denotes the bounded system uncertainties and disturbances, $|d| \le d_{max}$, $|\dot{d}| \le D_{max}$. The following conditions [12] are satisfied: (A, B) is stabilizable, (A, C) is detectable and (A, B, C) is invertible with no zeros at $s = 0$.

III. OPTIMAL SECOND ORDER INTEGRAL SLIDING MODE BASED COMPOSITE NONLINEAR FEEDBACK CONTROLLER

In this section, an optimal second order integral sliding mode based composite nonlinear feedback (SOISM-CNF) controller is proposed for the MEMS micromirror system (3). The control objective is to develop a control scheme such that the angle of MEMS micromirror can track the step command input r with fast response and small overshoot in the presence of bounded disturbances. For this purpose, the optimal composite nonlinear feedback control law is designed to improve the transient performance. The second order sliding mode control law with an integral sliding surface is incorporated such that the system has strong robustness. The TS-PSO algorithm is utilized to optimize the control parameters.

A. NOMINAL CONTROL DESIGN USING OPTIMAL COMPOSITE NONLINEAR FEEDBACK

The design procedure of optimal CNF is carried out in several steps. First, a linear feedback control item is designed to ensure a fast rise time of the controlled system. Then, a nonlinear feedback control item is proposed to reduce the overshoot. In the third step, the CNF control is formed by integrating the linear and nonlinear control laws. Finally, an optimization algorithm is introduced to solve the parameter tuning problem.

Step 1: Design a linear feedback control part u_L as [10]

$$u_L = Fx + Gr \tag{4}$$

where the state feedback gain F is chosen such that the matrix $(A + BF)$ is asymptotically stable whilst the closed-loop system $C(sI - A - BF)^{-1}$ has desired properties. To further achieve fast response, the gain F of linear control (4) should be designed such that $(A + BF)$ has a low damping ratio [10]. In (4), G is the feedforward gain of the step command input r, which is chosen as

$$G = -[C(A + BF)^{-1}B]^{-1} \tag{5}$$

Step 2: Design a nonlinear feedback control u_N as [10]

$$u_N = \rho(y, r)B^TP(x - x_e) \tag{6}$$

The nonlinear control u_N is designed to increase the damping ratio and attenuate overshoot. The nonlinear function $\rho(y, r)$ is non-positive and used to change the damping ratio of the closed-loop system as the output y approaches the reference r. The nonlinear function has different forms [23] which is generally chosen as

$$\rho(y, r) = -\bar{\beta}e^{-m(y-r)^2} \tag{7}$$

where $\bar{\beta} > 0$ and $m > 0$ are tuning parameters. In (6), P is the positive definite solution of

$$(A + BF)^TP + P(A + BF) = -W \tag{8}$$

for some positive definite matrix W. The desired state x_e is defined as

$$x_e = -(A + BF)^{-1}BGr \tag{9}$$

Step 3: The linear and nonlinear feedback control are integrated to form the CNF controller as

$$u_{CNF} = u_L + u_N = Fx + Gr + \rho(y, r)B^T P(x - x_e) \quad (10)$$

Theorem 1: Consider system (3) under the CNF control scheme (10), for any $\delta \in (0, 1)$, $c_\delta > 0$ is selected to be the largest positive scalar satisfying the following conditions [18]

$$|Fx| \leq (1 - \delta)u_{max}, \forall x \in X_\delta := \{x|x^T Px \leq c_\delta\} \quad (11)$$

Let the initial state x_0 and reference r satisfy

$$x_0 - x_e \in X_\delta, |Hr| \leq \delta_1 u_{max} \quad (12)$$

where $0 \leq \delta_1 < \delta$, $|(\delta - \delta_1)u_{max}| = d_{max}$ and H is defined as

$$H = [I - F(A + BF)^{-1}B]G \quad (13)$$

For any non-positive function $\rho(y, r)$, the control law (10) forces the system output y to track the reference r asymptotically in the presence of uncertainty.

Step 4: Adjust the parameter by TS-PSO algorithm. We utilize the Integrated Time and Absolute Error (ITAE) as the adaptation value to select F, m, $\bar{\beta}$ and W. Such a multi-parameter adjustment problem is transformed into the following minimization problem

$$\min_{F,m,\bar{\beta},W} = \int_0^\infty t|y - r|dt \quad (14)$$

Let

$$W = 10^\theta \cdot \hat{E} \quad (15)$$

where $\hat{E}$ is an identity matrix, θ is a tunable parameter which is set to be $\theta \in [-5, 5]$ in this section. The minimization problem (14) is solved by a hybrid optimization algorithm based on TS-PSO as follows [37].

1. Initialize TS-PSO parameters including inertia weight $\omega_1 = 0.9$, acceleration constants $c_1 = 1.2$, $c_2 = 1.2$, tabu length $L_{max} = 8$. Set tabu table empty.

2. Generate an initial population randomly and use PSO algorithm to find the global optimal solution. The following evolution equation (16) is utilized to calculate the position and velocity of each particle

$$\begin{aligned} v_{ij}(t+1) &= \omega_1 v_{ij}(t) + c_1 r_1(p_{ij}(t) - x_{ij}(t)) \\ &\quad + c_2 r_2(p_{gj}(t) - x_{ij}(t)) \\ x_{ij}(t+1) &= x_{ij}(t) + v_{ij}(t+1) \end{aligned} \quad (16)$$

where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, D$ with n and D are the population size and particle dimension, respectively. $x_{ij}(t)$ and $v_{ij}(t)$ are the ith particle's position and velocity respectively. r_1, r_2 are two random numbers with uniform distribution in the range of [0, 1]. t represents the iteration time step. p_{ij} is the individual best position of ith particle, and p_{gj} is the global best position.

3. Take the current solution optimized by PSO algorithm as the initial solution of tabu search (TS). Produce a number of neighborhood solutions from the current solution, and find the optimal neighborhood solution as candidate solution. Judge whether the candidate solution fits the amnesty criterion, if the candidate solution fits the amnesty criterion, replace current solution with the candidate solution and update the tabu table. Otherwise, judge whether the candidate solution is in the tabu table. Choose the non-tabu candidate solution as the current solution and update the tabu table. Judge whether the termination condition is satisfied. If the termination condition is met, terminate the search TS. Otherwise, perform the next iteration.

B. DESIGN OF MODIFIED INTEGRAL SLIDING MODE CONTROL

The system performance is affected by external disturbances in practical systems. It can be noted that the aforementioned nominal controller u_{CNF} does not consider the rejection of external disturbances. To guarantee the robustness for the closed-loop system, a modified integral sliding mode controller u_s is added with u_{CNF}. The control input for the micromirror system (3) is proposed as

$$u = u_{nominal} + u_s \quad (17)$$

where $u_{nominal}$ is used to provide the desired system performance under the assumption that there exist no disturbances acting on the system. u_s is designed to eliminate the disturbance. An integral sliding surface is designed as [14]

$$s = B^+(x - x_0 - \int_o^t (Ax + Bu_{nominal})d\tau) \quad (18)$$

where s is the sliding variable, B^+ is selected as the pseudo-inverse of B. The term $\int_o^t Ax(\tau) + B(u_{nominal})(\tau)d\tau)$ can be considered as the trajectory of system with the nominal control $u_{nominal}$ in the absence of disturbances. Consider the input saturation, the integral sliding surface (18) is modified as [24]

$$s = B^+(x - x_0 - \int_o^t (Ax + B(sat(u) - u_s))d\tau) \quad (19)$$

where u_s is designed as a modified twisting controller which is given as [30]

$$u_s = \int_0^t u_T d\tau \quad (20)$$

$$u_T = -k_1 sgn(s) - k_2 sgn(\dot{s}) \quad (21)$$

where k_1 and k_2 are the tunable parameters of twisting control u_T, $k_2 > D_{max}$ and $k_1 > k_2 + D_{max}$. It can be noted that the knowledge of $\dot{s}$ is necessary and unmeasurable, which is obtained by using a differentiator [30].

The control law (17) can be rewritten as

$$u = u_{CNF} + \int_0^t u_T d\tau \quad (22)$$

where u_{CNF} is selected as the nominal control.

Finally, the proposed SOISM-CNF controller can be derived from equation (10)and (20) as

$$u = Fx + Gr - \bar{\beta} e^{-m(y-r)^2} B^T P(x - x_e) + \int_0^t (-k_1 sgn(s) - k_2 sgn(\dot{s}))d\tau \quad (23)$$

C. STABILITY

The closed-loop system composed of the system (3) and the control law (22) can be expressed as

$$\dot{x} = Ax + Bsat(Fx + Gr + u_N + u_s) + Bd \quad (24)$$

Letting $\tilde{x} = x - x_e$, we have

$$\dot{\tilde{x}} = (A + BF)\tilde{x} + B(sat(F\tilde{x} + Hr + u_N + u_s) - F\tilde{x} - Hr + d) \quad (25)$$

From equation (19), the derivative of sliding surface can be obtained as

$$\begin{aligned} \dot{s} &= B^+(\dot{x} - (Ax + Bsat(u) - Bu_s)) \\ &= B^+(Bd + Bu_s) \\ &= d + u_s \end{aligned} \quad (26)$$

Taking the derivative of $\dot{s}$ and using the equation (21), we obtain

$$\begin{aligned} \ddot{s} &= u_T + \dot{d} \\ &= -k_1 sgn(s) - k_2 sgn(\dot{s}) + \dot{d} \end{aligned} \quad (27)$$

Since the disturbance as well as its derivation are bounded $|\dot{d}| \leq D_{max}$, choosing $k_2 > D_{max}$ and $k_1 > k_2 + D_{max}$ leads to the conclusion that s and $\dot{s}$ would converge to zero in finite time [28], [30]. When system reaches the sliding mode, $\dot{s} = d + u_s = 0$. It can be noted that the disturbance is canceled out by the u_s, we have $u_s = -d$.

From Theorem 1, it can be obtained that

$$|F\tilde{x} + Hr + u_s| \leq |F\tilde{x}| + |Hr| + |u_s| \leq u_{max} \quad (28)$$

To prove the stability, a Lyapunov function is defined as

$$V = \tilde{x}^T P\tilde{x} \quad (29)$$

Taking the time derivative of V

$$\begin{aligned} \dot{V} &= \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} \\ &= \tilde{x}^T (A + BF)^T P\tilde{x} + \tilde{x}^T (A + BF)P\tilde{x} \\ &\quad + 2\tilde{x}^T PB\psi \\ &= -\tilde{x}^T W\tilde{x} + 2\tilde{x}^T PB\psi \end{aligned} \quad (30)$$

where $\psi = sat(F\tilde{x} + Hr + u_N + u_s) - F\tilde{x} - Hr + d$.

Case 1: $|F\tilde{x} + Hr + u_N + u_s| \leq u_{max}$. Then

$$\begin{aligned} \psi &= F\tilde{x} + Hr + u_N + u_s - F\tilde{x} - Hr + d \\ &= u_N \\ &= \rho(y, r)B^T P\tilde{x} \end{aligned} \quad (31)$$

where $\rho(y, r)$ is a non-positive function. Then, (30) becomes $\dot{V} \leq -\tilde{x}^T W\tilde{x}$.

Case 2: $(F\tilde{x} + Hr + u_N + u_s) > u_{max}$. Then

$$\psi = u_{max} - (F\tilde{x} + Hr + u_s) + (u_s + d) \geq 0. \quad (32)$$

In this case, we can get

$$u_N = \rho(y, r)B^T P\tilde{x} \geq u_{max} - (F\tilde{x} + Hr + u_s) \geq 0 \quad (33)$$

Since $\rho(y, r)$ is a non-positive function, we have $\tilde{x}^T PB \leq 0$. Hence $\dot{V} \leq -\tilde{x}^T W\tilde{x}$.

Case 3: $(F\tilde{x} + Hr + u_N + u_s) < -u_{max}$. Then

$$\psi = -u_{max} - (F\tilde{x} + Hr + u_s) + (u_s + d) \leq 0. \quad (34)$$

Here

$$u_N = \rho(y, r)B^T P\tilde{x} \leq -u_{max} - F\tilde{x} - Hr - u_s \leq 0 \quad (35)$$

Since $\rho(y, r)$ is a non-positive function, we have $\tilde{x}^T PB \geq 0$. Thus $\dot{V} \leq -\tilde{x}^T W\tilde{x}$.

Finally, we can conclude that $\dot{V} \leq -\tilde{x}^T W\tilde{x} < 0$, which means that the close-loop system is asymptotically stable.

IV. SIMULATION RESULTS

To demonstrate the performance of the proposed algorithm, the closed-loop micromirror system is tested by simulations. The comparisons under CNF, ISM-CNF and the proposed optimal SOISM-CNF controller are conducted by using MATLAB software. The electrostatic torques T_α and T_β of equation (2) are generated by driving voltages u_α and u_β respectively. Let $u_\alpha = 10^6 T_\alpha$ and $u_\beta = 10^7 T_\beta$. The system equation (2) can be simplified as two independent linear systems as

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -0.2251 & -0.16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 3.0827 \end{bmatrix} \times (sat(u_\alpha) + d_\alpha) \\ y_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \quad (36)$$

$$\begin{aligned} \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -1 & -0.15 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1.7894 \end{bmatrix} (sat(u_\beta) + d_\beta) \\ y_2 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \end{aligned} \quad (37)$$

where d_α and d_β are bounded external disturbances, and the control inputs u_α and u_β are restricted with the maximum values as $u_{\alpha max} = u_{\beta max} = 10$.

The control parameters for (36) are chosen as $k_{1\alpha} = 8$, $k_{2\alpha} = 4$, and the CNF control parameters are tuned by TS-PSO algorithm as $F_\alpha = [-43.5675 \quad -7.0154]$, $m_\alpha = 56.1354$, $\bar{\beta}_\alpha = 76.5631$ and $\theta_\alpha = -0.1634$. Then, $G_\alpha = 43.6405$, $W_\alpha = [\begin{smallmatrix} 0.6864 & 0 \\ 0 & 0.6864 \end{smallmatrix}]$, $P_\alpha = [\begin{smallmatrix} 2.1907 & 0.0026 \\ 0.0026 & 0.0159 \end{smallmatrix}]$.

The control parameters for (37) are chosen as $k_{1\beta} = 10$, $k_{2\beta} = 5$, and the CNF control parameters are tuned by TS-PSO algorithm as $F_\beta = [-73.4816 \quad -12.1508]$, $m_\beta = 149.1508$, $\bar{\beta}_\beta = 190.4299$ and $\theta_\beta = -4.2524$. Then, $G_\beta = 74.0404$, $W_\beta = 10^{-4} \cdot [\begin{smallmatrix} 0.5592 & 0 \\ 0 & 0.5592 \end{smallmatrix}]$, $P_\beta = 10^{-3} \cdot [\begin{smallmatrix} 0.1751 & 0.0002 \\ 0.0002 & 0.1751 \end{smallmatrix}]$.

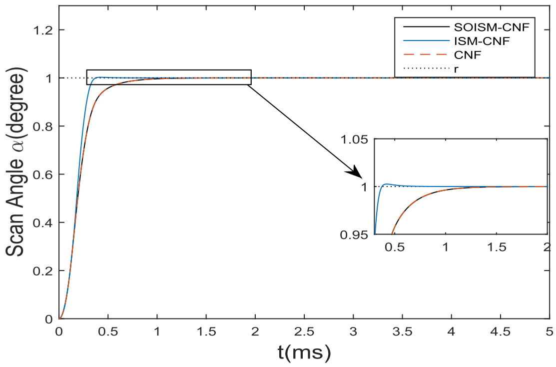

FIGURE 2. Set-point regulation of α when $r = 1$, $d_\alpha = 0$.

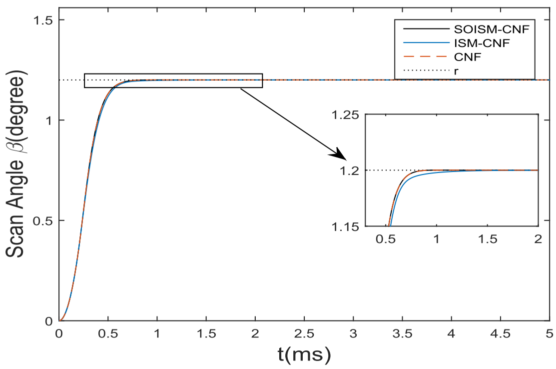

FIGURE 3. Set-point regulation of β when $r = 1.2$, $d_\beta = 0$.

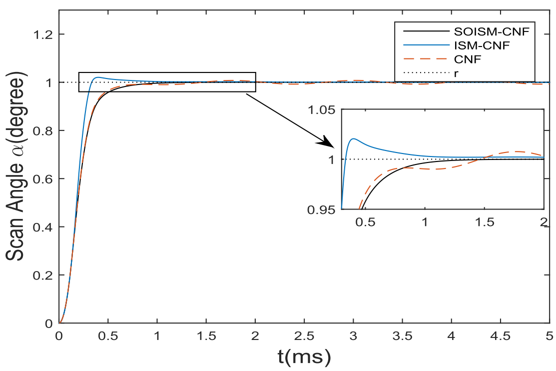

FIGURE 4. Set-point regulation of α when $r = 1$, $d_\alpha = 0.5sin(5t)$.

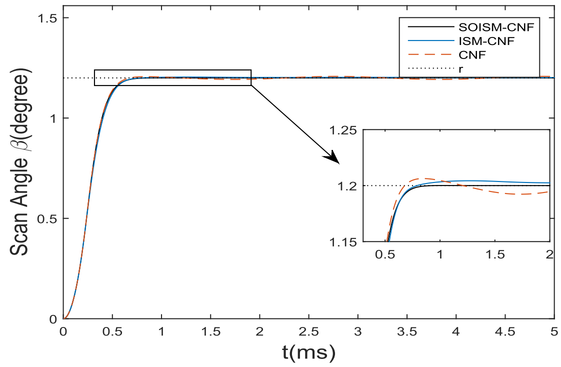

FIGURE 5. Set-point regulation of β when $r = 1.2$, $d_\beta = 0.6sin(3t)$.

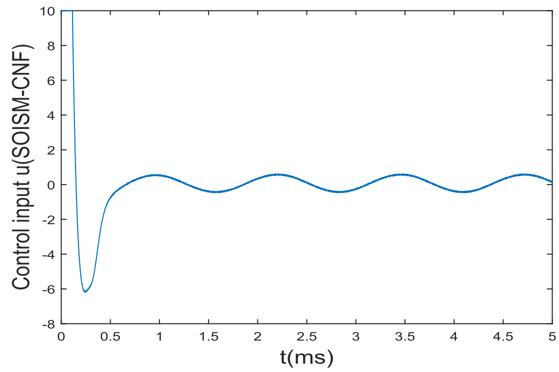

FIGURE 6. Control input u_α of SOISM-CNF when $d_\alpha = 0.5sin(5t)$.

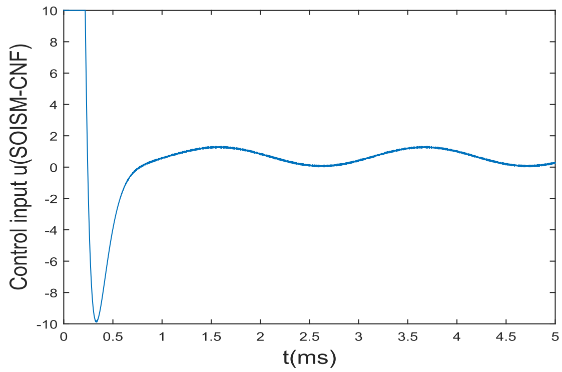

FIGURE 7. Control input u_β of SOISM-CNF when $d_\beta = 0.6sin(3t)$.

First, we compare the control performance of the closed-loop system without disturbances. The proposed optimal SOISM-CNF controller, the CNF controller and the conventional first order ISM-CNF controller are employed. The simulation results of set-point regulation are demonstrated in Figure 2 and Figure 3. It can be seen that the proposed optimal SOISM-CNF gives better performance including short settling time, small overshoot and precise positioning performance. The transient performance with the proposed controller is improved since the control parameters are properly tuned by using TS-PSO algorithm.

To test the disturbance rejection ability of the closed-loop control system, the external disturbances are set as $d_\alpha = 0.5sin(5t)$ and $d_\beta = 0.6sin(3t)$. From Figure 4 and Figure 5, it is observed that the proposed optimized SOISM-CNF controller shows the better performance in the presence of disturbances. The utilization of second order sliding mode scheme assures the robustness. However, the CNF controller exhibits worst performance because the conventional CNF control is not able to deal with external disturbances. The control inputs of optimal SOISM-CNF are shown in Figure 6 and Figure 7. It can be seen that the chattering phenomenon is reduced by utilizing the second order sliding mode technique. Figure 8 and Figure 9 further depict the tracking performance of the closed-loop system under the above three control schemes. It can be seen that the system output under the proposed SOISM-CNF controller achieve precise tracking of the command input r.

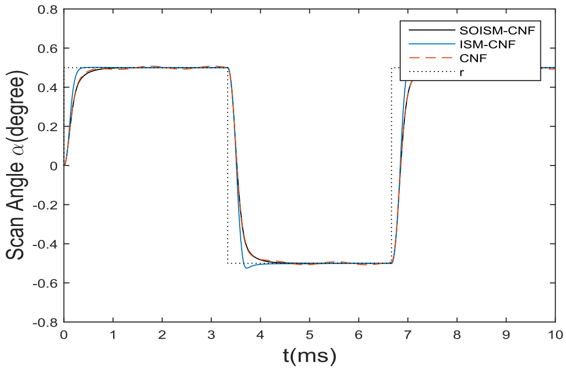

FIGURE 8. Position tracking of α when $r = 0.5sgn(sin(0.3\pi t))$.

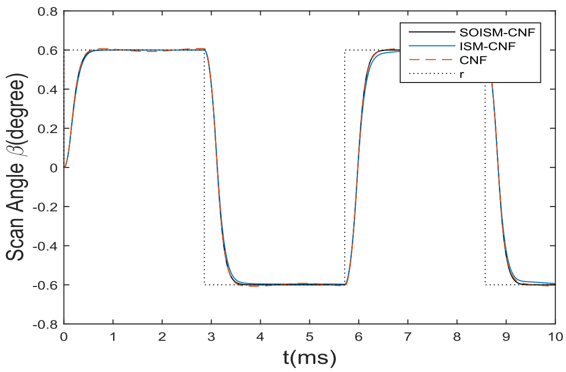

FIGURE 9. Position tracking of β when $r = 0.6sgn(sin(0.35\pi t))$.

V. CONCLUSION

In this article, an optimal second order sliding mode based composite nonlinear feedback controller is developed for micromirror system with bounded disturbances and input saturation. The stability of the closed-loop system under the proposed SOISM-CNF controller is proved by the Lyapunov theory. The effectiveness of the proposed controller is illustrated by the simulation results. Compared with the CNF and ISM-CNF control, the simulation results demonstrated that the proposed controller provides superior transient performance in the presence of disturbances.

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