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Quickest Multistate Flow Networks With the Deterioration Effect

MIN-FAN HE[1,2], ZHIFENG HAO[1], WEI-CHANG YEH[3], (Senior Member, IEEE), CHIA-CHEN KUO[3], NEAL N. XIONG[4], AND YEOU-REN SHIUE[5]

[1]Schools of Mathematics and Big Data, Foshan University, Foshan 528000, China
[2]Institute of Systems Engineering, Macau University of Science and Technology, Taipa, Macau
[3]Department of Industrial Engineering and Engineering Management, National Tsing Hua University, Hsinchu City 30071, Taiwan
[4]Department of Mathematics and Computer Science, Northeastern State University, Tahlequah, OK 74464, USA
[5]Department of Management Engineering, Fujian Business University, Fuzhou 350000, China

Corresponding author: Wei-Chang Yeh (yeh@ieee.org)

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ABSTRACT In a traditional multistate quickest path problem (MQPP), the system reliability is evaluated based on a strict assumption that the net flow into and out of a system is equal to zero. However, certain networks, which are known as deteriorated networks, suffer a loss due to the deterioration effect, resulting a delivery shortage. For example, the data or goods will deteriorate or decay because the transmission distance is too long, which affects whether the delivered data or goods arrive intact. To provide a practical solution to this problem, a novel MQPP model, known as the deteriorated MQPP ($\mathrm{MQPP_{de}}$) model, is proposed in this work. The goal is to evaluate the system reliability, which is defined as the probability that the end user receives at least d units of data or goods in transmission time T in the case of a MQPP with the deterioration effect. A simple path-based algorithm based on an integer programming model of the flow conservation law is presented to generate all of the lower boundary points (d, T)-$\mathrm{MP_{de}}$s. Next, the reliability of the $\mathrm{MQPP_{de}}$ model can be calculated in terms of all of the (d, T)-$\mathrm{MP_{de}}$s.

INDEX TERMS Multistate flow network (MFN), quickest path problem (QPP), deteriorated network, reliability.

I. INTRODUCTION

The quickest path problem (QPP), which is a variation of the shortest path problem, has been proposed by Chen and Chin [1]. The general form of this problem is to find a single path along which to transmit a given amount of data or goods from a source node to a sink node in a flow network that minimizes the transmission time [1], [2]. This path is called the quickest path. The past decade has witnessed a growing number of variants of the QPP, such as the constrained QPP [3], [4], k-QPP [5]–[7], the all-pairs QPP [8]–[10] and the multistate QPP (MQPP) [11]–[16].

The MQPP is an extension of the QPP in a multistate flow network (MFN) in which the capacities of nodes and arcs may be uncertain due to failure, maintenance, etc. [11]. The MQPP is a connected network that has no self-loops, all of the nodes are completely reliable, the capacity of each arc is a non-negative integer, and the capacity of each arc is stochastic and statistically independent with a given probability distribution.

To assess the reliability of an MQPP, all of the lower boundary points, called (d, T)-MPs (MPs is the abbreviation of minimal paths), must be searched first. If the level d and transmission time T are given, then the MQPP's reliability can be calculated in terms of the (d, T)-MPs and expressed as the probability that the system can transfer at least d units of data from the source node to the sink node through one MP within a given time T. Note that a MP is a subset of order sequence arcs from the source node to the sink node with no cycle such that if any arc is removed from the set, the remaining arcs no longer form a path set [17]–[24].

A number of studies have investigated practical applications of the MQPP [11]–[16]. All of those studies evaluated the system reliability based on the strict assumption that the net flow into and out of the system is equal to zero. However, some networks, called deteriorated networks, may suffer flow losses of arcs due to the deterioration effect [25], [26] of which the flow value is decreased during transmission

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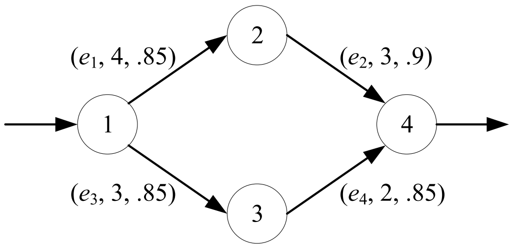

FIGURE 1. A MQP$_{Pd}$e in which each arc label represents (arc ID, $W(ei)$, δi).

through the arcs. To provide a practical solution to this problem, a novel MQPP model, known as the deteriorated MQPP (MQPP$_{\text{de}}$) model, is proposed in this work. In the proposed MQPP$_{\text{de}}$, all of the data or goods are transmitted along one single minimal path and all of the flows in each node satisfy the flow conservation law, i.e., the net flow into and out of a node is equal to zero, but the flow in each arc may decay at the deterioration rate of that arc.

Let $G(V, E, W, \Delta)$ be an MQPP$_{\text{de}}$ with node set $V = \{1, 2, \ldots, n\}$, arc set $E = \{e_1, e_2, \ldots, e_m\}$, $W = (W(e_1), W(e_2), \ldots, W(e_m))$, $\Delta = (\delta_1, \delta_2, \ldots, \delta_m)$, the source node 1, and the sink node n, where $W(e_i)$ is the maximum capacity of e_i without and with the deterioration effect and δ_i is the deterioration rate of e_i. For example, an MQPP$_{\text{de}}$ with $V = \{1, 2, 3, 4\}$, $E = \{e_1, e_2, e_3, e_4\}$, $W = (4, 3, 3, 2)$, and $\Delta = (0.85, 0.9, 0.85, 0.85)$ is shown in Fig. 1.

As shown in figure 1, a deteriorated network $G(V, E, W, \Delta)$ consists of four arcs and nodes with two MPs, $P_1 = \{e_1, e_2\}$ and $P_2 = \{e_3, e_4\}$. Let I_i and O_i be the amount of input flow and output flow through the ith MP P_i in G, respectively. A flow into the system, along P_1, and to the end user with a value $I_1 = 2$ will decay from 2 to $2 \times 0.85 = 1.7$ on e_1, from 1.7 to $1.7 \times 0.9 = 1.5$ on e_2, and exit the system along P_1 with a value $O_1 = 1.5 < I_1 = 2$. That is, the net flow into and out of the system is not equal to zero in a deteriorated network.

From the system management viewpoint, it is important to guarantee that the quantity of data or goods transmitted meets the needs of customers or end users. In a deteriorated network, the system may fail to satisfy this requirement due to losses in the flow. In addition, the reliability of a deteriorated network may be over–estimated when the traditional MQPP is considered and the deterioration effect is neglected.

Therefore, this study proposes a novel MQPP model called the deteriorated MQPP (MQPP$_{\text{de}}$) model to address this real and practical problem. In addition, a simple and efficient algorithm for evaluating the reliability of the MQPP$_{\text{de}}$ model is proposed. First, we find all of the lower boundary points for level (d, T) with the deterioration effect included. These points are called (d, T)-MP$_{\text{de}}$s. Then, the system reliability is calculated in terms of them.

This work is organized as follows. In Section II, the problem of a deteriorated network is defined and the MQPP$_{\text{de}}$ model is formulated. A simple algorithm for generating all of the (d, T)-MP$_{\text{de}}$s is presented in Section III. Section IV describes a method for assessing the MQPP$_{\text{de}}$ reliability using the proposed algorithm with a numerical example. Section V presents conclusions and a discussion of future work.

II. MODEL FORMULATION

A. ASSUMPTIONS

Suppose that an MQPP network, $G(V, E, W)$, which does not include the deterioration effect, has p MPs from the source node to the sink node. Let $M(P)$ be the maximum capacity of MP P and $X = (x_1, x_2, \ldots, x_m)$ be a system-state vector with $X(e_i) = x_i$ without considering the deterioration effect. According to reference [11], the transmission time, denoted by $\Psi(d, X, P_j)$, in which the system can send d units of data along P_j under the system-state vector X, is:

$$\text{the lead time of } P_j + \left\lceil \frac{d}{M(P_j)} \right\rceil = \sum_{i=1}^{m} l_i + \left\lceil \frac{d}{\text{Min}\{W(e_i)\}} \right\rceil, \quad \text{for each } e_i \in P_j \tag{1}$$

where $\lceil x \rceil$ is the smallest integer that is greater than or equal to x and l_i is the lead time of the ith arc of the jth MP.

The minimal capacity v_j of the jth MP plays a key role generating all of the lower boundary points, (d, T)-MP, and needs to be found first. It must be such that the system-state vector X_j has minimal capacity and the system can send d units of data along the jth MP within time T under X_j. Therefore, the minimal capacity v_j is the smallest integer such that:

$$\sum_{i=1}^{m} l_i + \left\lceil \frac{d}{v_j} \right\rceil \le T, \quad \text{for each } e_i \in P_j \tag{2}$$

If $v_j \le M(P_j)$, then (d, T)-MP X_j can be generated using the following equations.

$$\begin{cases} x_i = \text{the minimal capacity } S(e_i), \text{ such that } S(e_i) \ge v_j, \\ \quad \text{if } e_i \in P_j, \\ x_i = 0, \\ \quad \text{if } e_i \notin P_j. \end{cases} \tag{3}$$

If $v_j > M(P_j)$, then X_j does not exist.

Let $Y \ge X$ if and only if $y_i \ge x_i$ for each i and $Y > X$ if and only if $y_i > x_i$ for at least one i. Let $\Lambda(d, X)$ denote the minimum time in which the system can transmit d units of data from the source node to the sink node subject to the system-state vector X, and then, we have the following lemma from [11]:

Lemma 1: If X is a (d, T)-MP, then $\Lambda(d, Y) \le T$ for any $Y \ge X$.

However, in the MQPP$_{\text{de}}$ model, the flow along each MP may decay because of the deterioration rates of its arcs. This may result in a flow out of the MP that is less than the input flow, i.e., $O_j < I_j$. For example, suppose that a deteriorated network, as shown in Fig. 1, need to transmit a demand $d = 8$ within a transmission time $T = 9$ to an end user along P_1. Because $v_1 = 2 \le M(P_1) = 3$ can be derived using Eq. (1-2), (8, 9)-MP $X_1 = \{2, 2, 0, 0\}$ can be generated using

Eq. (3). This implies the input flow value and the output flow value along P_1 are both equal to 2 under the conservation law, i.e., $O_1 = I_1 = 2$. However, the flow will decrease from 2 to $\lfloor 2 \times \delta_1 \rfloor = \lfloor 2 \times 0.85 \rfloor = 1$ on e_1 and to $\lfloor 1 \times 0.9 \rfloor = 0$ on e_2, where $\lfloor x \rfloor$ is the largest integer that is less than or equal to x.

Therefore, when the deterioration effect is considered, the flow out of the system along P_1 is $0 < v_j = 2$, which fails to meet the requirements of the end user within the transmission time T.

Definition 1: A (d, T)-MP$_{de}$ is a (d, T)-MP that includes the deterioration effect to satisfy level (d, T) along a single MP in an MQPP$_{de}$.

From the above discussion, all of the (d, T)-MP$_{\mathrm{de}}$s cannot be found using the smallest integer v_j directly. The remainder of this section describes how to generate all of the (d, T)-MP$_{\mathrm{de}}$s.

Let e^i be the ith arc in the MP, x'_i be the current capacity of the ith arc for some X with the deterioration effect respectively. Without loss of generality, the order of arcs in a MP is denoted by a superscript, i.e., $P_i = \{e^1_j, e^2_k, \ldots, e^u_q\}$, with j, k and $q \in [1, m]$ and $j \neq k \neq q$. For example, $P_1 = \{e^1_1, e^2_2\}$ and $P_2 = \{e^1_3, e^2_4\}$ in Fig. 1. A modified model of the MQPP$_{\mathrm{de}}$ reliability problem that is based on an integer programming model of the flow conservation law [27], called the F-IP$_{\mathrm{de}}$, is presented in the following equations, where $X(e^i) = x^i$ and each $e^i \in P_j$ and $W'(e_i)$ is the maximum capacity of the ith arc with the deterioration effect, i.e., $W'(e_i) = \left\lfloor \frac{W(e_i)}{\delta_i} \right\rfloor$:

$$I_j = x^1, \tag{4}$$
$$x'^i = x^{i+1}, \quad i = 1, 2, \ldots, u-1, \tag{5}$$
$$x'^u = O_j, \tag{6}$$
$$x_k = 0, \textit{for each } e_k \notin P_j \textit{ and } k = 1,2,\ldots,m, \tag{7}$$
$$x^i \in \{0, 1, \ldots, W(x^i)\} \text{ and } x'^i \in \{0, 1, \ldots, W'(e^i)\} \tag{8}$$

According to Eq. (2-3), the minimal capacity v_j is a key to determining (d, T)-MP for a system-state vector X_j in the MQPP. Therefore, we let $O_j = v_j$; then, the F-IP$_{\mathrm{de}}$ can be solved in inverted sequence using Eq. (9). In addition, the system-state vector X_j, which has the minimum capacity that allows the end user to receive d units of data along P_j within time T when the deterioration effect is included, can be generated as follows:

$$\begin{cases} x'^u = S'(e^u), & \text{with } S'(e^u) \geq v_j \\ x^i = S(e^i), & \text{with } S(e^i) \geq \left\lceil \dfrac{x'^i}{\delta^i} \right\rceil, \quad i = 1, 2, \ldots, u \\ x_k = 0, & \text{if } e_k \notin P_j, \end{cases} \tag{9}$$

where $S(e_i)$ and $S'(e_i)$ are the minimal capacities of the ith arc without and with the deterioration effect, respectively, and δ^i is the deterioration rate of the ith arc in some MP, respectively.

For example, Fig. 1 can be modeled as follows:

$$P_1 : I_1 = x_1, x'_1 = x_2, \ x'_2 = O_1, x_3 \text{ and } x_4 = 0 \tag{10}$$
$$P_2 : I_2 = x_3, x'_3 = x_4, \ x'_4 = O_2, x_1 \text{ and } x_2 = 0 \tag{11}$$
$$0 \leq x_1 \leq 4, \tag{12}$$
$$0 \leq x_2 \leq 3, \tag{13}$$
$$0 \leq x_3 \leq 3, \tag{14}$$
$$0 \leq x_4 \leq 2, \tag{15}$$
$$0 \leq x'_1 \leq 3, \tag{16}$$
$$0 \leq x'_2 \leq 2, \tag{17}$$
$$0 \leq x'_3 \leq 2, \tag{18}$$
$$0 \leq x'_4 \leq 1, \tag{19}$$

Suppose that this system is required to transmit a demand $d = 8$ within a transmission time $T = 9$ to an end user along P_1, with $v_1 = 2$, as calculated previously. Let $O_1 = v_1 = 2$ and $x'_2 = S'(e_2) = 2$, so that $S'(e_2) = 2 \geq O_1 = 2$. Let $x'_1 = x_2 = S(e_2) = 3$, so that $S(e_2) \geq \left\lceil \frac{x'_2}{\delta_2} \right\rceil = \left\lceil \frac{2}{0.9} \right\rceil = 3$. Let $x_1 = S(e_1) = 4$, so that $S(e_1) \geq \left\lceil \frac{x'_1}{\delta_1} \right\rceil = \left\lceil \frac{3}{0.85} \right\rceil = 4$. Finally, a system-state vector $X_1 = \{4, 3, 0, 0\}$ is obtained, which implies that the system needs to send a flow value of at least $X(e_1, e_2) = (4, 3)$ along P_1 to satisfy the demand $d = 8$ successfully within the time $T = 9$.

Lemma 2: The set of $X_1, X_2, \ldots, X_m$ contains only lower boundary points, *(d, T)-MP$_{de}$*s, if and only if

1) $v_j \leq M'(P_j)$, and
2) X_j satisfies Eq. (9)-(18) with $O_j = v_j$.

Proof: For each X_j, we have $v_j \leq M'(P_j)$ and $O_j = v_j$. It is trivial that the transmission time T satisfies $\Psi'(d, X, P_j) \leq T$ and $\Lambda'(d, X_j) \leq T$, according to Lemma 1. Moreover, if any system-state vector $Y = (y_1, y_2, \ldots, y_n) < X_j = (x_1, x_2, \ldots, x_n)$, where $Y(y_i) < X_j(x_i)$ and $e_i \in P_j$ is such that $O(Y) < O(X_j) = v_j$, then, $\Psi'(d, X, P_j) > T$ and $\Lambda'(d, X_j) > T$. Conversely, if $Y > X$, where $Y(y_i) > X_j(x_i)$ and $e_i \in P_j$, and $O(Y) \geq O(X_j) = v_j$, then $\Psi'(d, X, P_j) \leq T$ and $\Lambda'(d, X_j) \leq T$. Therefore, we can conclude that each X_j obtained from the F-IP$_{de}$ is a *(d, T)-MP$_{de}$*.

III. SOLUTION PROCEDURE

A simple and efficient algorithm for searching all of the (d, T)-MP$_{\mathrm{de}}$s for evaluating the reliability of the MQPP$_{\mathrm{de}}$ is described by the following steps:

Input: all of the MPs $P_1, P_2, \ldots, P_p$ in the MQPP$_{\mathrm{de}}$ $G(V, E, W, \Delta)$ with level (d, T).
Output: All of the (d, T)-MP$_{\mathrm{de}}$s.
Step 0: Let $\Omega = \emptyset$ and $j = 1$.
Step 1: Find the smallest integer v_j for P_j using equation (2).
Step 2: If $v_j \leq M'(P_j)$, go to Step 3. Otherwise, set $X_j = \emptyset$ and go to Step 4.
Step 3: For $P_j = \{e^1_i, e^2_k, \ldots, e^u_q\}$, find the minimal capacity vector $X_j = (x_1, x_2, \ldots, x_m)$ such that the required value of d can be received along path P_j within time T despite the deterioration effect.
Step 3.1: Construct the F-IP$_{\mathrm{de}}$ of P_j and let $O_j = v_j$.
Step 3.2: Solve the F-IP$_{\mathrm{de}}$ in inverted sequence using Eq. (9). If any X_j does not exist, let $X_j = \emptyset$.

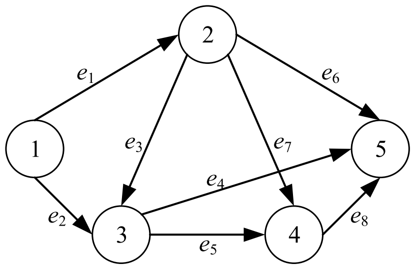

FIGURE 2. A benchmark MFN.

Step 4: Let $\Omega = \Omega \cap X_j$, increment j if $j < p$, and go to Step 1. Otherwise, halt; Ω is the complete set of (d, T)-MP$_{\text{de}}$s.

After all of the (d, T)-MP$_{\text{de}}$s have been found by the proposed algorithm, they can be used to assess the reliability of the MQPP$_{\text{de}}$ to achieve level (d, T). The system reliability can be calculated by disjoint subset methods [28]–[30], the state-space decomposition method [31], [32], and the inclusion-exclusion method [4], [25], [31], [33], [34]. This study uses the inclusion-exclusion method to calculate the system reliability because it is a simple and fundamental tool for evaluating MFN reliability.

Assume that $X_1, X_2, \ldots, X_p$ are all (d, T)-MPs and $\Pr(X_i)$ be the probability of event X_i. Then, the system reliability calculated using the inclusion-exclusion method is given by:

$$\sum_{i=1}^{p} \Pr(X_i) - \sum_{j=2}^{p}\sum_{i=1}^{j-1} \Pr(X_i \cap X_j) + \sum_{j=3}^{p}\sum_{i=2}^{j-1}\sum_{k=1}^{i-1} \Pr(X_i \cap X_j \cap X_k) + \ldots + (-1)^{p+1} \Pr(X_1 \cap X_2 \cap \ldots \cap X_p)$$

where

$$\Pr(X_j) = \Pr\{X | X \geq X_j\} = \prod_{i=1}^{m} \Pr\{X(e_i) \geq X_j(e_i)\}. \tag{20}$$

IV. EXAMPLE ILLUSTRATION

An illustration of the proposed algorithm using the benchmark network in Fig. 2 is introduced in this section. We generate all of the (d, T)-MP$_{\text{de}}$s using the proposed algorithm in Stage 1. Then, the reliability of this MQPP$_{\text{de}}$ can be calculated in terms of all of the (d, T)-MP$_{\text{de}}$s obtained in Stage 1, as shown in Stage 2.

Stage 1. The capacity, lead time and deterioration rate of each arc are listed in Table 1. This benchmark has six MPs: $P_1 = \{e_1, e_6\}$, $P_2 = \{e_1, e_7, e_8\}$, $P_3 = \{e_1, e_3, e_4\}$, $P_4 = \{e_1, e_3, e_5, e_8\}$, $P_5 = \{e_2, e_4\}$, and $P_6 = \{e_2, e_5, e_8\}$. Given that level $(d, T) = (6, 10)$, all of the (6, 10)-MP$_{\text{de}}$s can be generated as follows:

Solve:

Step 0: Let $\Omega = \emptyset$ and $i = 1$.

TABLE 1. The arc data for the example of network in Fig. 2.

Arc	Capacity	*Pr.*	l_i	δ_i
1	5	0.60	2	0.90
	4	0.20		
	3	0.05		
	2	0.05		
	1	0.05		
	0	0.05		
2	3	0.80	1	0.85
	2	0.10		
	1	0.05		
	0	0.05		
3	4	0.75	3	0.90
	3	0.10		
	2	0.05		
	1	0.05		
	0	0.05		
4	3	0.80	3	0.85
	2	0.10		
	1	0.05		
	0	0.05		
5	3	0.80	1	0.85
	2	0.10		
	1	0.05		
	0	0.05		
6	4	0.60	2	0.85
	3	0.20		
	2	0.10		
	1	0.05		
	0	0.05		
7	5	0.55	2	0.85
	4	0.10		
	3	0.10		
	2	0.10		
	1	0.10		
	0	0.05		
8	3	0.70	1	0.8
	2	0.20		
	1	0.05		
	0	0.05		

Step 1: The lead time along P_1 is $l_1 + l_6 = 2 + 2 = 4$. Therefore, $v_1 = 1$ is the smallest integer such that $4 + \left\lceil \frac{6}{v_1} \right\rceil \leq 10$.

Step 2: $v_1 \leq M'(P_1) = 3$; therefore, go to Step 3.1.

Step 3.1: Construct the F-IP$_{\text{de}}$ for $P_1 = \{e_1, e_6\}$ and let $O_1 = v_1 = 1$.
Then, the F-IP$_{\text{de}}$ for P_1 has $I_1 = x_1, x_1' = x_6$, and $x_6' = O_1 = 1$.

Step 3.2.1: Let $x_6' = S'(e_6) = 1$, so that $S'(e_6) \geq O_1 = 1$.

Step 3.2.2: Let $x_1' = x_6 = S(e_6) = 2$, so that $S(e_6) \geq \left\lceil \frac{x_6'}{\delta_6} \right\rceil = \left\lceil \frac{1}{0.85} \right\rceil = 2$. Because $x_6 = 2 \leq W(e_6)$ and $x_1' = 2 \leq W'(e_1) = 4$, go to Step 3.2.3.

Step 3.2.3: Let $x_1 = S(e_1) = 3$, so that $S(e_1) \geq \left\lceil \frac{x_1'}{\delta_1} \right\rceil = \left\lceil \frac{2}{0.9} \right\rceil = 3$. Because $x_1 = 3 \leq W(e_1) = 5$, we obtain $X_1 = (3, 0, 0, 0, 0, 2, 0, 0)$.

Step 4: Let $\Omega = \Omega \cap X_1$. Because $i = 1 < p = 6$, let $i = 2$ and go to Step 1.

Step 1: The lead time for P_2 is $l_1 + l_7 + l_8 = 2 + 2 + 1 = 5$. Therefore, $v_2 = 2$ is the smallest integer such that $5 + \left\lceil \frac{6}{v_2} \right\rceil \leq 10$.

Step 2: Because $v_2 \leq M'(P_2) = 2$, go to Step 3.1.

Step 3.1: Construct the F-IP$_{\text{de}}$ for $P_2 = \{e_1, e_7, e_8\}$ and let $O_2 = v_2 = 2$.
The F-IP$_{\text{de}}$ of P_2 has: $I_2 = x_1, x_1' = x_7, x_7' = x_8, x_8' = O_2 = 2$.

Step 3.2.1: Let $x_8' = S'(e_8) = 2$, so that $S'(e_8) \geq O_2 = 2$.

Step 3.2.2: Let $x_7' = x_8 = S(e_8) = 3$, so that $S(e_8) \geq \left\lceil \frac{x_8'}{\delta_8} \right\rceil = \left\lceil \frac{2}{0.8} \right\rceil = 3$. Because $x_8 = 3 \leq W(e_8) = 3$ and $x_7' = 3 \leq W'(e_7) = 4$, go to Step 3.2.3.

Step 3.2.3: Let $x_1' = x_7 = S(e_7) = 4$, so that $S(e_7) \geq \left\lceil \frac{x_7'}{\delta_7} \right\rceil = \left\lceil \frac{3}{0.85} \right\rceil = 4$. Because $x_7 = 4 \leq W(e_7) = 5$ and $x_1' = 4 \leq W'(e_1) = 4$, go to Step 3.2.4.

Step 3.2.4: Let $x_1 = S(e_1) = 5$, so that $S(e_1) \geq \left\lceil \frac{x_1'}{\delta_1} \right\rceil = \left\lceil \frac{4}{0.9} \right\rceil = 5$. Because $x_1 = 5 \leq W(e_1) = 5$, we obtain $X_2 = (5, 0, 0, 0, 0, 0, 4, 3)$.

Step 4: Let $\Omega = \Omega \cap X_2$. Because $i = 2 < m = 6$, let $i = 3$ and go to Step 1.

Step 1: The lead time for P_3 is $l_1 + l_3 + l_4 = 2 + 3 + 3 = 8$. Therefore, $v_3 = 3$ is the smallest integer such that $8 + \left\lceil \frac{6}{v_3} \right\rceil \leq 10$.

Step 2: Because $v_3 > M'(P_3) = 2$, X_3 does not exist; go to Step 4.

Step 4: Because $i = 3 < m = 6$, let $i = 4$ and go to Step 1.

Step 1: The lead time for P_4 is $L_4 = l_1 + l_3 + l_5 + l_8 = 2 + 3 + 1 + 1 = 7$. Therefore, $v_4 = 2$ is the smallest integer such that $7 + \left\lceil \frac{6}{v_4} \right\rceil \leq 10$.

Step 2: Because $v_4 \leq M'(P_4) = 2$, go to Step 3.1.

Step 3.1: Construct the F-IP$_{\text{de}}$ for $P_2 = \{e_1, e_3, e_5, e_8\}$ and let $O_4 = v_4 = 2$.
The F-IP$_{\text{de}}$ of P_4 has: $I_4 = x_1, x_1' = x_3, x_3' = x_5, x_5' = x_8, x_8' = O_4 = 2$.

Step 3.2.1: Let $x_8' = S'(e_8) = 2$, so that $S'(e_8) \geq O_4 = 2$.

Step 3.2.2: Let $x_5' = x_8 = S(e_8) = 3$, so that $S(e_8) \geq \left\lceil \frac{x_8'}{\delta_8} \right\rceil = \left\lceil \frac{2}{0.8} \right\rceil = 3$. Because $x_5' = 3 > M'(e_5) = 2$, X_4 does not exist; go to Step 4.

Step 4: Because $i = 4 < m = 6$, let $i = 5$ and go to Step 1.

Step 1: The lead time for P_5 is $L_5 = l_2 + l_4 = 1 + 3 = 4$. Therefore, $v_5 = 1$ is the smallest integer such that $4 + \left\lceil \frac{6}{v_5} \right\rceil \leq 10$.

Step 2: Because $v_1 \leq M'(P_5) = 2$, go to Step 3.1.

Step 3.1: Construct the F-IP$_{\text{de}}$ for $P_1 = \{e_1, e_6\}$ and let $O_5 = v_5$.
The F-IP$_{\text{de}}$ of P_5 has: $I_5 = x_2, x_2' = x_4, x_4' = O_5 = 1$.

Step 3.2.1: Let $x_4' = S'(e_4) = 1$, so that $S'(e_4) \geq O_5 = 1$.

Step 3.2.2: Let $x_2' = x_4 = S(e_4) = 2$, so that $S(e_4) \geq \left\lceil \frac{x_4'}{\delta_4} \right\rceil = \left\lceil \frac{1}{0.85} \right\rceil = 2$. Because $x_4 = 2 \leq W(e_4) = 3$ and $x_2' = 2 \leq W'(e_2) = 2$, go to Step 3.2.3.

Step 3.2.3: Let $x_2 = S(e_2) = 3$, so that $S(e_2) \geq \left\lceil \frac{x_2'}{\delta_2} \right\rceil = \left\lceil \frac{2}{0.85} \right\rceil = 3$. Because $x_2 = 3 \leq W(e_2) = 3$, we obtain $X_5 = (0, 3, 0, 2, 0, 0, 0, 0)$.

Step 4: Let $\Omega = \Omega \cap X_5$. Because $i = 5 < m = 6$, let $i = 6$ and go to Step 1.

Step 1: The lead time for P_6 is $l_2 + l_5 + l_8 = 1 + 1 + 1 = 3$. Therefore, $v_6 = 1$ is the smallest integer such that $3 + \left\lceil \frac{6}{v_6} \right\rceil \leq 10$.

Step 2: Because $v_6 \leq M'(P_6) = 2$, go to Step 3.1.

Step 3.1: Construct the F-IP$_{\text{de}}$ for $P_2 = \{e_2, e_5, e_8\}$ and let $O_6 = v_6 = 1$.
The F-IP$_{\text{de}}$ of P_6 has: $I_6 = x_2, x_2' = x_5, x_5' = x_8, x_8' = O_6$.

Step 3.2.1: Let $x_8' = S'(e_8) = 1$, so that $S'(e_8) \geq O_6 = 1$.

Step 3.2.2: Let $x_5' = x_8 = S(e_8) = 2$, so that $S(e_8) \geq \left\lceil \frac{x_8'}{\delta_8} \right\rceil = \left\lceil \frac{1}{0.8} \right\rceil = 2$. Because $x_8 = 2 \leq W(e_8) = 2$ and $x_5' = 2 \leq W'(e_5) = 2$, go to Step 3.2.3.

Step 3.2.3: Let $x_2' = x_5 = S(e_5) = 3$, so that $S(e_5) \geq \left\lceil \frac{x_5'}{\delta_5} \right\rceil = \left\lceil \frac{2}{0.85} \right\rceil = 3$. Because $x_2' = 3 > W'(e_2) = 2$, X_6 does not exist; go to Step 4.

Step 4: Because $i = p = 6$, halt. Ω is the complete set of (6, 10)-MP$_{\text{de}}$s.

Stage 2. Three lower boundary points, $X_1 = (3, 0, 0, 0, 0, 2, 0, 0)$, $X_2 = (5, 0, 0, 0, 0, 0, 4, 3)$ and $X_5 = (0, 3, 0, 2, 0, 0, 0, 0)$, for level $(d, T) = (6, 10)$ are generated using the proposed algorithm in Stage 1. The MQPP$_{\text{de}}$ reliability of the system shown in Fig. 2 can be calculated by applying the inclusion-exclusion method as follows:

Solve:

$$\begin{aligned}
&[\Pr\{X_1\} + \Pr\{X_2\} + \Pr\{X_5\}] \\
&\quad - [\Pr\{X_1 \cap X_2\} + \Pr\{X_1 \cap X_5\} + \Pr\{X_2 \cap X_5\}] \\
&\quad + [\Pr\{X_1 \cap X_2 \cap X_5\}] \\
&= (0.765 + 0.273 + 0.72) \\
&\quad - (0.2457 + 0.5508 + 0.1966) + 0.1769 \\
&= 0.9418.
\end{aligned}$$

where

$$\begin{aligned}
\Pr\{X_1\} &= \Pr\{X \geq (3, 0, 0, 0, 0, 2, 0, 0)\} \\
&= \Pr\{x_1 \geq 3\} \times \Pr\{x_2 \geq 0\} \times \Pr\{x_3 \geq 0\} \\
&\quad \times \Pr\{x_4 \geq 0\} \times \Pr\{x_5 \geq 0\} \times \Pr\{x_6 \geq 2\} \\
&\quad \times \Pr\{x_7 \geq 0\} \times \Pr\{x_8 \geq 0\} \\
&= 0.85 \times 1 \times 1 \times 1 \times 1 \times 0.9 \times 1 \times 1 \\
&= 0.765, \\
\Pr\{X_1 \cap X_2\} &= \Pr\{(X \geq (3, 0, 0, 0, 0, 2, 0, 0)) \\
&\quad \cap (X \geq (5, 0, 0, 0, 0, 0, 4, 3))\}
\end{aligned}$$

$$
\begin{aligned}
&= \Pr\{X \geq (5, 0, 0, 0, 0, 2, 4, 3)\}\\
&= 0.2457,\\
\Pr\{X_1 \cap X_2 \cap X_5\} &= \Pr\{(X \geq (3, 0, 0, 0, 0, 2, 0, 0))\\
&\quad \cap (X \geq (5, 0, 0, 0, 0, 0, 4, 3))\\
&\quad \cap (X \geq (0, 3, 0, 2, 0, 0, 0, 0))\}\\
&= \Pr\{X \geq (5, 3, 0, 2, 0, 2, 4, 3)\}\\
&= 0.1769.
\end{aligned}
$$

V. DISCUSSION AND CONCLUSION

There are p and m variables for all of the MPs and arcs in G, respectively. The time complexities of finding the minimal capacity v_j and comparing it to $M'(P_j)$ Step 1 and Step 2 are both $O(m)$ in the worst case. Therefore, both Step 1 and Step 2 take $O(pm)$ time. The F-IP$_{de}$ in Step 3 requires $O(pm)$ time to generate all of the (d, T)-MP$_{de}$s in the worst case.

In a MFN system, a delivery shortage may occur because of the deterioration effect. Therefore, a novel MQPP$_{de}$ model is presented that evaluates the reliability of an MQPP in which the deterioration effect is included to solve this real-life problem. The MQPP$_{de}$ reliability is introduced and expressed as the probability that a deteriorated network can send at least d units of data or goods from the source node to the sink node within T units of time along a single MP. In addition, a simple path-based algorithm that generates all of the lower boundary points, which are called (d, T)-MP$_{de}$s, by solving the F-IP$_{de}$ is proposed. Then, the MQPP$_{de}$ can be calculated in terms of those points.

The study of calculating the reliability of deteriorated networks is still in its infancy. Future research could analyze the effects of different deterioration rates on the system reliability, determine the most critical nodes or arcs to system reliability, and extend the deterioration problem to a large-scale network system or an MFN system that delivers the data or goods along all of the feasible MPs simultaneously and develop a model to calculate the reliability of such a system.

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MIN-FAN HE received the B.Sc. degree from Foshan University, Foshan, China, in 2003, and the M.Sc. degree from the South China University of Technology, Guangzhou, in 2006. He is currently pursuing the Ph.D. degree with the Macau University of Science and Technology. He is also an Associate Professor with the School of Mathematics and Big Date, Foshan University. His research interests include machine learning, data mining, bioinformatics, and evolutionary algorithms.

ZHIFENG HAO received the B.Sc. degree in mathematics from Sun Yat-sen University, Guangzhou, China, in 1990, and the Ph.D. degree in mathematics from Nanjing University, Nanjing, China, in 1995. He is currently a Professor with the School of Computer, Guangdong University of Technology, Guangzhou, and with the School of Mathematics and Big Date, Foshan University, Foshan, China. His current research interests include various aspects of algebra, machine learning, data mining, and evolutionary algorithms.

WEI-CHANG YEH (Senior Member, IEEE) received the M.S. and Ph.D. degrees from the Department of Industrial Engineering, The University of Texas at Arlington. He is currently a Distinguished Professor with the Department of Industrial Engineering and Engineering Management, National Tsing Hua University, Taiwan. Most of his research is focused around algorithms, including exact solution methods and soft computing. He has published more than 250 research articles in highly ranked journals and conference papers and has been awarded the Outstanding Research Award twice, the Distinguished Scholars Research Project once, and an Overseas Research Fellowship twice by the Ministry of Science and Technology in Taiwan. He proposed a novel soft computing algorithm called the simplified swarm optimization (SSO) and demonstrated the simplicity, effectiveness, and efficiency of his SSO for solving NP-hard problems. He has been granted 50 patents and earns an International Fellow, the Guoguang Invention Medal as well as the titles of Outstanding Inventor of Taiwan and Doctor of Erudition, by the Chinese Innovation and Invention Society. He has been invited to serve as an Associate Editor of the two top reliability related journals, namely, the IEEE TRANSACTIONS ON RELIABILITY and *Reliability Engineering & System Safety*.

CHIA-CHEN KUO, photograph and biography not available at the time of publication.

NEAL N. XIONG received the Ph.D. degree in software engineering from Wuhan University and the Ph.D. degree in dependable networks from the Japan Advanced Institute of Science and Technology. He is currently an Associate Professor with the Department of Mathematics and Computer Science, Northeastern State University. Before, he attended Northeastern State University, he worked at the Wentworth Technology Institution, Georgia State University, for many years. His research interests include cloud computing, security and dependability, parallel and distributed computing, networks, and optimization theory.

YEOU-REN SHIUE received the Ph.D. degree from the Department of Industrial and Management Engineering, National Chiao Tung University, Hsinchu City, Taiwan, in 2002. From 2004 to 2018, he was a Professor with the Department of Information Management, Huafan University, New Taipei City, Taiwan. He is currently a Professor with Artificial Intelligence/Machine Learning scientific research team academic leader, Fujian Business University, Fuzhou, China. His research interests include intelligent manufacturing, machine learning, and artificial intelligence and its applications.