IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received July 10, 2020, accepted July 29, 2020, date of publication August 3, 2020, date of current version August 17, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3013939

Adaptive Fuzzy PI Prescribed Performance Tracking Control for Switched Nonlinear Systems With Dead-Zone Input and External Disturbances

HONGYAO LI[1], HAIBIN SUN[1], (Member, IEEE), AND LINLIN HOU[2]
[1]School of Engineering, Qufu Normal University, Rizhao 276826, China
[2]School of Information Science and Engineering, Qufu Normal University, Rizhao 276826, China

Corresponding author: Haibin Sun (seusunhaibin@gmail.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61773236, Grant 61773235, Grant 61703233, Grant 61873331, and Grant 61803225; in part by the Taishan Scholar Project of Shandong Province under Grant TSQN20161033 and Grant TS201712040; in part by the Natural Science Foundation of Shandong Province under Grant ZR2016FQ09; in part by the Postdoctoral Science Foundation of China under Grant 2017M612236 and Grant 2019T120574; and in part by the Interdisciplinary Scientific Research Projects of Qufu Normal University under Grant xkjjc201905.

ABSTRACT This paper studies the problem of adaptive fuzzy proportional-integral (PI) tracking control for a class of switched nonlinear systems with dead-zone input, unknown nonlinear functions and external disturbances. By using backstepping method with fuzzy logic systems, a set of adaptive fuzzy PI tracking controllers are proposed, which have a simple structure and a clear physical meaning. The designed controllers don't need priori information of dead-zone input via introducing Nussbaum function scheme. To enhance the tracking performance, the prescribed performance technique is utilized to constrain the tracking error. The stability analysis of the closed-loop switched systems is established via common Lyapunov function, and the tracking error remains within a prescribed bound and all signals of the closed-loop system are bounded. Finally, the effectiveness of the scheme is verified by two examples.

INDEX TERMS Adaptive control, backstepping, dead-zone input, fuzzy logic systems, PI control, prescribed performance, switched nonlinear system, tracking control.

I. INTRODUCTION

Switched systems, as a special type of hybrid systems, contain a set of subsystems and a switching rule that arranges switching among subsystems [1]–[7]. In recent years, the analysis and synthesis of switched systems have aroused general concern because it can be employed to model many engineering systems such as robot control systems [8]–[10], and automobile engine control systems [11], etc. As a very important topic, the stability analysis and control synthesis of switched linear or nonlinear systems are widely investigated. For example, the stabilization or tracking problems are investigated for switched nonlinear systems by combining backstepping method and common Lyapunov function technique [12]–[15]. In [12], a backstepping control scheme is proposed with the existing condition on common virtual control laws. Afterwards, in [13], inequality constraint conditions are applied to the virtual control function to eliminate the assumptions in [12]. Later, the adaptive tracking control problem for switched nonlinear systems with uncertainties is dealt with by using fuzzy logic systems in [14], in which some inequality restrictions are removed and common virtual control laws are constructed. Recently, [15] aims at solving the tracking control problem of uncertain switched systems, where two control schemes, robust adaptive and direct adaptive, are proposed. In [16], the problem of adaptive fuzzy tracking control for a class of switched uncertain nonlinear systems is investigated, where a new adaptive state-dependent switching law is designed. Based on adaptive fuzzy control technique and command filter, the state and output feedback control schemes are proposed for Multi-Input and Multi-Output nonstrict-feedback switched systems [17]. By constructing a state observer independent of switching signal, an adaptive fuzzy backstepping controller is designed for a class of stochastic systems with unmodeled dynamics [18]. However, the above references do not refer to constraint problem for considered systems.

The associate editor coordinating the review of this manuscript and approving it for publication was Fanbiao Li.

System constraints, which widely exist in practical engineering systems, for example state constraints, actuator saturation, and dead zone input, and so on, have stirred much emotion [19], [20]. In [21], an adaptive output feedback control strategy is designed for switched nonlinear large systems with dead-zone input. Recently, the problem of adaptive neural tracking control for switched systems with non-lower triangular structure [22], where the controller is designed by using the prior information of dead-zone input. In [23], the problem of adaptive fuzzy tracking control is investigated for switched nonlinear systems with input saturation and dead-zone output. In addition, in order to obtain a better control performance, the prescribed performance of the system is investigated during the controller design [24], [25]. In [4], the problem of adaptive output feedback control is studied for a class of switched nonlinear system with prescribed performance. Adaptive prescribed performance control (PPC) is considered for switched nonlinear systems in a general form [26]. In [27], the problem of PPC is investigated for switched nonlinear systems via dynamic surface technique. It is not difficult to see that the PPC scheme can effectively enhance the control performance of the closed-loop systems. Although many useful results have been obtained for all types of systems, the proposed solutions still lack simplicity and an intuitive structure.

On the other hand, the structure of PI control is simple and intuitive, which has been very popular in engineering practice. The key of PI control design is to determine its gain. A self-tuning PID gain method is proposed in [28], however, this method is only suitable for linear systems. For a nonlinear system, a PI/PID controller is constructed for the first time [29]. Subsequently, the problem of PI control for nonlinear system has received more attention. In [20], a neural adaptive PI control algorithm is developed for nonlinear systems with unknown actuation characteristics. Recently, considering full state constraints, a neural adaptive PI control method is come up in [19]. Nevertheless, as far as we know, there are scarcely results studying the prescribed performance PI controller design for switched nonlinear systems with unknown functions and dead-zone input. This is an open problem and needs to be further discussed. In a practice system, the actuator always exists dead-zone feature, which may influence the control performance. To guarantee the closed-loop system have a good control performance, the PPC scheme provides a feasible solution. However, most controllers designed in the existing literature about switched nonlinear systems have complex structure and unclear physical meaning. Inspired by the above discussion that it is significant from the perspective of both practical application and theory to study the problem of adaptive prescribed performance PI tracking control for switched nonlinear systems with dead-zone input, which motivates us for this study. To solve this problem, two questions must be addressed: (*i*). how can we construct the common virtual control laws and controller with PI structure via the backstepping technique? (*ii*). how can we deal with the dead-zone input with unknown information?

In this paper, a set of adaptive fuzzy PI controllers are proposed for a class of switched nonlinear systems with dead-zone input, external disturbances and unknown nonlinear functions. The common virtual control functions are constructed by backstepping technique, and the prescribed performance technique is used to constrain the tracking error and improve the tracking performance. By using Lyapunov function theory, the stability of the closed-loop system is analyzed, and all the closed-loop signals are bounded and the tracking error remains within a prescribed bound.

The main contributions of this paper is illustrated as follows.

- An adaptive fuzzy PI tracking controller with prescribed performance is constructed for a class of switched nonlinear systems with dead-zone input, external disturbances. The proposed controller has a simple structure and clear physical meaning and guarantees the tracking error satisfy prescribed performance.
- In order to deal with the unknown parameters of dead-zone input, the Nussbaum function technique is utilized to design controllers.

Notations: Define the following notations, which will not be covered in this article. For any $i = 1, \cdots, n$, $g_{i,min} = min\{g_{i,k}, k \in M\}$, $\hat{\theta}_i$ denotes the estimation of $\theta_i = \|W_{i,max}\|^2$, $W_{i,max} = max\{W_{i,k}, k \in M\}$, $\hat{\delta}_i$ represents the estimation of δ_i, where M is a positive integer set and $W = [w_1, \cdots, w_N]^T$ is a vector composed of points of fuzzy membership function will appear later.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. PROBLEM FORMULATION

Consider the following switched nonlinear systems:

$$\begin{aligned} \dot{x}_1 &= g_{1,\sigma(t)}x_2 + f_{1,\sigma(t)}(x_1) + d_1(t), \\ \dot{x}_2 &= g_{2,\sigma(t)}x_3 + f_{2,\sigma(t)}(x_1, x_2) + d_2(t), \\ &\vdots \\ \dot{x}_n &= g_{n,\sigma(t)}u(D_{\sigma(t)}(t)) + f_{n,\sigma(t)}(x_1, x_2, \cdots, x_n) \\ &\quad + d_n(t), \\ y &= x_1, \end{aligned} \tag{1}$$

where $\sigma(t) : [0, +\infty) \rightarrow M = \{1, \cdots, m\}$ denotes the switching signal, $x = [x_1, x_2, \cdots, x_n]^T$ is system state, $g_{i,k}$ are positive constants with $i = 1, \cdots, n$ and $k \in M$, $f_{i,k}$ mean smooth unknown nonlinear functions, $y = x_1$ and $u(D_{\sigma(t)}(t))$ denote the system output and control input, respectively; $d_i(t)$ are external disturbances.

$u(D_{\sigma(t)}(t))$ is written as

$$u(D_{\sigma(t)}(t)) = \begin{cases} \iota(D_{\sigma(t)}(t) - D_1), & D_{\sigma(t)}(t) \geq D_1 \\ 0, & -D_2 < D_{\sigma(t)}(t) < D_1 \\ \iota(D_{\sigma(t)}(t) + D_2), & D_{\sigma(t)}(t) \leq -D_2 \end{cases}$$

where D_1, D_2 are positive constants and the ι represents slope of the dead-zone.

Then, $u(D_{\sigma(t)}(t))$ can be written as

$$u(D_{\sigma(t)}(t)) = \iota D_{\sigma(t)}(t) + b(t), \tag{2}$$

where

$$b(t) = \begin{cases} -\iota D_1, & D_{\sigma(t)}(t) \geq D_1, \\ -\iota D_{\sigma(t)}(t), & -D_2 < D_{\sigma(t)}(t) < D_1, \\ \iota D_2, & D_{\sigma(t)}(t) \leq -D_2, \end{cases} \tag{3}$$

where $w \leq b(t) \leq \bar{b}$, $w = \min(\iota D_1, \iota D_2)$, $\bar{b} = \max(\iota D_1, \iota D_2)$. Substituting (2) into (1) yields

$$\begin{aligned} \dot{x}_1 &= g_{1,\sigma(t)}x_2 + f_{1,\sigma(t)}(x_1) + d_1(t), \\ \dot{x}_2 &= g_{2,\sigma(t)}x_3 + f_{2,\sigma(t)}(x_1, x_2) + d_2(t), \\ &\vdots \\ \dot{x}_n &= g_{n,\sigma(t)}(\iota D_{\sigma(t)}(t) + b(t)) + f_{n,\sigma(t)}(x_1, x_2, \cdots, x_n) \\ &\quad + d_n(t), \\ y &= x_1. \end{aligned} \tag{4}$$

Adaptive PI tracking control Problem: A group of adaptive fuzzy PI controllers will be designed for switched nonlinear system (4), such that the signals of closed-loop system are bounded and the tracking error $\varepsilon_1 = y - y_d$ remains within a prescribed bound, where y_d is a reference signal.

To acquire the main results, some Definition, Lemmas and Assumptions are reviewed.

Assumption 1 [15]: $y_d, \dot{y}_d, \cdots, y_d^{(n)}$ are smooth and bounded.

Remark 1: In order to guarantee the boundedness of other signals, the above assumption is necessary.

Definition 1 [30]: A continuous function is called as Nussbaum-type function if the following conditions satisfy

$$\lim_{t\to\infty} \sup \frac{1}{t}\int_0^t N(\tau)d\tau = +\infty,$$
$$\lim_{t\to\infty} \inf \frac{1}{t}\int_0^t N(\tau)d\tau = -\infty.$$

In fact, many functions can be called as Nussbaum-type function, for example $N(\tau) = \tau^2 \cos\tau$.

Lemma 1 [31]: Consider the following system:

$$\dot{\hat{\theta}}(t) = -\kappa\hat{\theta}(t) + \nu\iota(t),$$

with $\nu > 0$, $\kappa > 0$, $\iota(t) > 0$ and initial condition $\hat{\theta}(t_0) \geq 0$. Then we have $\hat{\theta}(t) \geq 0$.

Lemma 2 [32]: If $V(t)$, $\tau(t)$ are smooth functions defined on $[0, t_f)$ and for $\forall t \in [0, t_f)$, the following inequality holds

$$0 \leq V(t) \leq b_0 + e^{-h_0 t}\int_0^t (\iota(\Im(\varsigma))N(\tau) + 1)\dot{\tau}e^{h_0\varsigma}d\varsigma, \tag{5}$$

where h_0, b_0 are positive constants, $\iota(\Im(\varsigma))$ is a time-varying parameter. Then for any $t \in [0, t_f)$, $V(t)$, $\Im(\varsigma)$, and $\int_0^t \iota(\Im(\varsigma))N(\tau)\dot{\tau}d\tau$ are bounded.

B. FUZZY LOGIC SYSTEMS (FLSs)

Assume that $U_1^j, U_2^j, \cdots, U_n^j$ and V^j are fuzzy sets defined according to the following if-then rule:

The *jth* rule is: if x_1 is U_1^j, x_2 is $U_2^j, \cdots, x_n$ is U_n^j, then y is V^j. $x = [x_1, \cdots, x_n]^T$ and y represent FLSs input and output, respectively. So the FLSs can be represented as

$$y = W^T S(x),$$

where $W = [w_1, \cdots, w_N]^T$ is a vector composed of fuzzy membership function, $S(x) = [s_1, \cdots, s_N]^T$ is basis function vector, and $s_i = \frac{\prod_{l=1}^n \mu_{U_l^i}(x_l)}{\sum_{j=1}^N [\prod_{l=1}^n \mu_{U_l^i}(x_l)]}$.

Lemma 3 [33]: There exists FLSs $y = W^T S(x)$ such that $sup_{x\in\Omega}|f(x) - W^T S(x)| \leq \epsilon$, where $f(x)$ is a continuous function defined on a compact.

Remark 2: From Lemma 3, FLSs can approximate the unknown function $f_{i,k}(x)$: $f_{i,k}(x) = W_{i,k}^T S_{i,k}(x) + \eta_{i,k}(\epsilon)$, and $S_{i,k}(x)$ satisfies: $0 < S_{i,k}^T S_{i,k} \leq 1$.

Assumption 2: There exists a positive constant δ_i such that $|\eta_{i,k} + d_i(t)| \leq \delta_i$.

C. PERFORMANCE BOUNDARY SPECIFICATION

To enhance the tracking performance, the tracking error restricts within prescribed region

$$-\gamma_2\zeta(t) < \varepsilon_1(t) < \gamma_1\zeta(t), \tag{6}$$

where $0 < \gamma_1, \gamma_2 \leq 1$ are stated parameters, $\zeta(t)$ is a smooth and decreasing function with the character $\lim_{t\to\infty}\zeta(t) = \zeta_1 > 0$. In this paper, $\zeta(t)$ is chosen as

$$\zeta(t) = (\zeta_2 - \zeta_1)e^{-bt} + \zeta_1, \tag{7}$$

where ζ_2, ζ_1, b are positive constants and $\zeta_2 > \zeta_1$.

D. TRANSFORMED ERRORS SYSTEM

In this part, to transform the constrained condition (6), a smooth function $T(s)$ is introduced and satisfies the following properties [24]:

(I) $-\gamma_2 < T(s) < \gamma_1$;
(II) $\lim_{s\to\infty} T(s) = \gamma_1$, $\lim_{s\to-\infty} T(s) = \gamma_2$;
(III) $\lim_{s\to 0}\frac{T(s)}{s} = q$, where q is positive constant;
(IV) $T(0) = 0$;
(V) $T(s)$ is a strictly increasing function.

According to the above properties, we choose

$$T(s) = \frac{\gamma_1\gamma_2 e^s - \gamma_1\gamma_2 e^{-s}}{\gamma_2 e^s + \gamma_1 e^{-s}} \tag{8}$$

Therefore one has

$$\varepsilon_1(t) = \zeta(t)T(s), \tag{9}$$

where $\varepsilon_1(t) = x_1(t) - y_d(t)$. By using property (V), the following equation is set up

$$s = T^{-1}(\frac{\varepsilon_1(t)}{\zeta}), \tag{10}$$

where s is a transformed error. The derivative of s is

$$\dot{s} = \xi(\dot{\varepsilon}_1 - \frac{\varepsilon_1 \dot{\zeta}}{\zeta}), \tag{11}$$

where $\xi = \frac{\partial T^{-1}(s)}{\zeta \partial s} > 0$.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

The adaptive PI tracking controllers are designed as follows.

Define the virtual errors

$$\varepsilon_2 = x_2 - \alpha_1,$$
$$\vdots$$
$$\varepsilon_n = x_n - \alpha_{n-1},$$

where $\alpha_i (i = 1, 2, \cdots, n-1)$ is a virtual control law, which will appear in the later.

Step 1: Define the generalized tracking error as $z_1 = s + \mu_1 \int_0^t s d\tau$ with $\mu_1 > 0$. The derivative of z_1 is

$$\begin{aligned} \dot{z}_1 &= \dot{s} + \mu_1 s \\ &= \xi(\dot{\varepsilon}_1 - \frac{\varepsilon_1 \dot{\zeta}}{\zeta}) + \mu_1 s \\ &= \xi(g_{1,k} x_2 + f_{1,k} + d_1 - \dot{y}_d - \frac{\varepsilon_1 \dot{\zeta}}{\zeta}) + \mu_1 s. \end{aligned} \tag{12}$$

Define the unknown nonlinear function as

$$\Psi_{1,k} = f_{1,k} - \dot{y}_d + \frac{\mu_1 s}{\xi} - \frac{\varepsilon_1 \dot{\zeta}}{\zeta} - g_{1,k}\mu_2 \int_0^t \varepsilon_2 d\tau, \tag{13}$$

where $\varepsilon_2 = x_2 - \alpha_1$, μ_2 is a positive constant. By using *Lemma 3*, $\Psi_{1,k}$ can be approximated as

$$\Psi_{1,k} = W_{1,k}^T S_{1,k} + \eta_{1,k}. \tag{14}$$

Choose the Lyapunov function as

$$V_1 = \frac{1}{2} z_1^2. \tag{15}$$

Applying Assumption 2, Young's inequality and differentiating V_1 result in

$$\begin{aligned} \dot{V}_1 &= z_1[\xi(g_{1,k} x_2 + f_{1,k} + d_1 - \dot{y}_d - \frac{\varepsilon_1 \dot{\zeta}_1}{\zeta_1}) + \mu_1 s] \\ &= z_1 \xi(\Psi_{1,k} + d_1 + g_{1,k} x_2 + g_{1,k}\mu_2 \int_0^t \varepsilon_2 d\tau) \\ &= z_1 \xi(W_{1,k}^T S_{1,k} + \eta_{1,k} + d_1 + g_{1,k} x_2 \\ &\quad + g_{1,k}\mu_2 \int_0^t \varepsilon_2 d\tau) \\ &\le \frac{z_1^2 \xi^2 \|W_{1,k}\|^2}{2a_1^2} + \frac{a_1^2}{2} + |z_1 \xi|\delta_1 + g_{1,k} z_1 \xi(z_2 + \alpha_1) \\ &= \frac{z_1^2 \xi^2 \theta_1}{2a_1^2} + \frac{a_1^2}{2} + |z_1 \xi|\delta_1 + g_{1,k} z_1 \xi \alpha_1 \\ &\quad + g_{1,k} \xi z_1 z_2, \end{aligned} \tag{16}$$

where $\theta_1 = \|W_{1,k}\|^2$, $z_2 = \varepsilon_2 + \mu_2 \int_0^t \varepsilon_2 d\tau$. The virtual control law can be designed as

$$\alpha_1 = -(k_{P_1} + \Delta k_{P_1}(\cdot))s - (k_{I_1} + \Delta k_{I_1}(\cdot)) \int_0^t s d\tau, \tag{17}$$

where $k_{P_1} = c_1$, $\Delta k_{P_1}(\cdot) = \frac{\xi \hat{\theta}_1}{2a_1^2 g_{1,min}} + \frac{\xi \hat{\delta}_1}{g_{1,min}(|z_1 \xi| + \varpi_1)}$, $k_{I_1} = \mu_1 k_{P_1}$, $\Delta k_{I_1}(\cdot) = \mu_1 \Delta k_{P_1}(\cdot)$, c_1, μ_1, μ_2, $\hat{\theta}_1$ and $\hat{\delta}_1$ are the estimation of θ_1 and δ_1, respectively, a_1 and ϖ_1 are positive constants, $g_{1,min} = \min\{g_{1,k}\}$.

Substituting the virtual control law (17) into (16) yields

$$\begin{aligned} \dot{V}_1 \le& (\frac{\theta_1}{2a_1^2} - \frac{g_{1,k}\hat{\theta}_1}{2a_1^2 g_{1,min}})\xi^2 z_1^2 + |z_1 \xi|\delta_1 + \frac{a_1^2}{2} \\ &- \frac{g_{1,k}\xi^2 \hat{\delta}_1 z_1^2}{g_{1,min}(|z_1 \xi| + \varpi_1)} + g_{1,k}\xi z_1 z_2 \\ &- c_1 g_{1,k} \xi z_1^2. \end{aligned} \tag{18}$$

Step 2: Calculating the derivative of z_2 leads to

$$\begin{aligned} \dot{z}_2 &= \dot{\varepsilon}_2 + \mu_2 \varepsilon_2 \\ &= \dot{x}_2 - \dot{\alpha}_1 + \mu_2(x_2 - \alpha_1) \\ &= g_{2,k} x_3 + f_{2,k} + d_2 - \dot{\alpha}_1 + \mu_2(x_2 - \alpha_1). \end{aligned} \tag{19}$$

The unknown function $\Psi_{2,k}$ is defined as

$$\begin{aligned} \Psi_{2,k} = f_{2,k} - \dot{\alpha}_1 + \mu_2(x_2 - \alpha_1) - g_{2,k}\mu_3 \int_0^t \varepsilon_3 d\tau \\ + g_{1,k} \xi z_1, \end{aligned} \tag{20}$$

where $\mu_3 > 0$. The Lyapunov function V_2 is selected as

$$V_2 = \frac{1}{2} z_2^2. \tag{21}$$

Its derivative is

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2(g_{2,k} x_3 + f_{2,k} + d_2 - \dot{\alpha}_1 + \mu_2(x_2 - \alpha_1)) \\ &= \dot{V}_1 + z_2(\Psi_{2,k} + d_2 + g_{2,k} x_3 + g_{2,k}\mu_3 \int_0^t \varepsilon_3 d\tau) \\ &\le (\frac{\theta_1}{2a_1^2} - \frac{g_{1,k}\hat{\theta}_1}{2a_1^2 g_{1,min}})\xi^2 z_1^2 + |z_1 \xi|\delta_1 + \frac{a_1^2}{2} \\ &\quad - \frac{g_{1,k}\xi^2 \hat{\delta}_1 z_1^2}{g_{1,min}(|z_1 \xi| + \varpi_1)} - c_1 g_{1,k} \xi z_1^2 + \frac{z_2^2 \theta_2}{2a_2^2} \\ &\quad + \frac{a_2^2}{2} + |z_2|\delta_2 + g_{2,k} z_2 \alpha_2 + g_{2,k} z_2 z_3, \end{aligned} \tag{22}$$

where $\theta_2 = \|W_{2,k}\|^2$. α_2 is designed as

$$\alpha_2 = -(k_{P_2} + \Delta k_{P_2}(\cdot))\varepsilon_2 - (k_{I_2} + \Delta k_{I_2}(\cdot)) \int_0^t \varepsilon_2 d\tau, \tag{23}$$

where $k_{P_2} = c_2$, $\Delta k_{P_2}(\cdot) = \frac{\hat{\theta}_2}{2a_2^2 g_{2,min}} + \frac{\hat{\delta}_2}{g_{2,min}(|z_2| + \varpi_2)}$, $k_{I_2} = \mu_2 k_{P_2}$, $\Delta k_{I_2}(\cdot) = \mu_2 \Delta k_{P_2}(\cdot)$, $\hat{\theta}_2$ and $\hat{\delta}_2$ are the estimation of θ_2 and δ_2, respectively, c_2, μ_3, a_2 and ϖ_2 are positive constants, $g_{2,min} = \min\{g_{2,k}\}$.

Substituting (23) into (22) yields

$$\begin{aligned} \dot{V}_2 \le& (\frac{\theta_1}{2a_1^2} - \frac{g_{1,k}\hat{\theta}_1}{2a_1^2 g_{1,min}})\xi^2 z_1^2 + |z_1 \xi|\delta_1 + \frac{a_1^2}{2} \\ &- \frac{g_{1,k}\xi^2 \hat{\delta}_1 z_1^2}{g_{1,min}(|z_1 \xi| + \varpi_1)} - c_1 g_{1,k} \xi z_1^2 + |z_2|\delta_2 \end{aligned}$$

$$+(\frac{\theta_2}{2a_2^2} - \frac{g_{2,k}\hat{\theta}_2}{2a_2^2 g_{2,min}})z_2^2 + \frac{a_2^2}{2} - c_2 g_{2,k} z_2^2 - \frac{g_{2,k}\hat{\delta}_2 z_2^2}{g_{2,min}(|z_2| + \varpi_2)} + g_{2,k} z_2 z_3. \quad (24)$$

Step i($3 \leq i < n$)*:* Select the Lyapunov function at *Step i-1* as

$$V_{i-1} = \frac{1}{2}\sum_{j=1}^{i-1} z_j^2. \quad (25)$$

$\dot{V}_{i-1}$ satisfies

$$\begin{aligned}\dot{V}_{i-1} =& (\frac{\theta_1}{2a_1^2} - \frac{g_{1,k}\hat{\theta}_1}{2a_1^2 g_{1,min}})\xi^2 z_1^2 + |z_1\xi|\delta_1 + \frac{a_1^2}{2}\\ &- \frac{g_{1,k}\xi^2\hat{\delta}_1 z_1^2}{g_{1,min}(|z_1\xi| + \varpi_1)} - c_1 g_{1,k}\xi z_1^2\\ &+ \sum_{j=2}^{i-1}\{(\frac{\theta_j}{2a_j^2} - \frac{g_{j,k}\hat{\theta}_j}{2a_j^2 g_{j,min}})z_j^2 + |z_j|\delta_j\\ &- \frac{g_{j,k}\hat{\delta}_j z_j^2}{g_{j,min}(|z_j| + \varpi_j)} + \frac{a_j^2}{2} - c_j g_{j,k} z_j^2\}\\ &+ g_{i-1,k} z_{i-1} z_i, \end{aligned} \quad (26)$$

where $\theta_j = \|W_{j,k}\|^2$, $g_{j,min} = \min\{g_{j,k}\}$. The generalized error is defined as $z_i = \varepsilon_i + \mu_i \int_0^t \varepsilon_i d\tau$ with $\mu_i > 0$, then

$$\begin{aligned}\dot{z}_i &= \dot{\varepsilon}_i + \mu_i\varepsilon_i\\ &= \dot{x}_i - \dot{\alpha}_{i-1} + \mu_i(x_i - \alpha_{i-1})\\ &= g_{i,k}x_{i+1} + f_{i,k} + d_i + \mu_i(x_i - \alpha_{i-1})\\ &\quad -\dot{\alpha}_{i-1}. \end{aligned} \quad (27)$$

Define unknown nonlinear functions $\Psi_{i,k}$ as

$$\Psi_{i,k} = f_{i,k} - \dot{\alpha}_{i-1} + \mu_i(x_i - \alpha_{i-1}) + g_{i-1,k}z_{i-1} - g_{i,k}\mu_{i+1}\int_0^t \varepsilon_{i+1} d\tau, \quad (28)$$

where $\mu_{i+1} > 0$. Construct Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2. \quad (29)$$

Computing the derivative V_i gives

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + z_i\dot{z}_i\\ &= \dot{V}_{i-1} + z_i[g_{i,k}x_{i+1} + f_{i,k} + d_i + \mu_i(x_i - \alpha_{i-1})\\ &\quad -\dot{\alpha}_{i-1}]\\ &= \dot{V}_{i-1} + z_i(\Psi_{i,k} + d_i + g_{i,k}x_{i+1} - g_{i-1,k}z_{i-1}\\ &\quad + g_{i,k}\mu_{i+1}\int_0^t \varepsilon_{i+1} d\tau)\\ &\leq (\frac{\theta_1}{2a_1^2} - \frac{g_{1,k}\hat{\theta}_1}{2a_1^2 g_{1,min}})\xi^2 z_1^2 + |z_1\xi|\delta_1 + \frac{a_1^2}{2}\\ &\quad - \frac{g_{1,k}\xi^2\hat{\delta}_1 z_1^2}{g_{1,min}(|z_1\xi| + \varpi_1)} - c_1 g_{1,k}\xi z_1^2 + \frac{a_i^2}{2}\\ &\quad + \sum_{j=2}^{i-1}\{(\frac{\theta_j}{2a_j^2} - \frac{g_{j,k}\hat{\theta}_j}{2a_j^2 g_{j,min}})z_j^2 + |z_j|\delta_j\\ &\quad - \frac{g_{j,k}\hat{\delta}_j z_j^2}{g_{j,min}(|z_j| + \varpi_j)} + \frac{a_j^2}{2} - c_j g_{j,k} z_j^2\} + |z_i|\delta_i\\ &\quad + \frac{z_i^2\|W_{i,k}\|^2}{2a_i^2} + g_{i,k}z_i\alpha_i + g_{i,k}z_i z_{i+1}. \end{aligned} \quad (30)$$

α_i is designed as

$$\alpha_i = -(k_{P_i} + \Delta k_{P_i}(\cdot))\varepsilon_i - (k_{I_i} + \Delta k_{I_i}(\cdot))\int_0^t \varepsilon_i d\tau, \quad (31)$$

where $k_{P_i} = c_i$, $k_{I_i} = \mu_i k_{P_i}$, $\Delta k_{P_i}(\cdot) = \frac{\hat{\theta}_i}{2a_i^2 g_{i,min}} + \frac{\hat{\delta}_i}{g_{i,min}(|z_i| + \varpi_i)}$, $\Delta k_{I_i}(\cdot) = \mu_i \Delta k_{P_i}(\cdot)$, c_i, μ_i, a_i and ϖ_i are positive parameters, $\hat{\theta}_i$ and $\hat{\delta}_i$ are the estimation of θ_i and δ_i, respectively. By combining the *Lemma 1*, Young's inequality, (31) and (30), one has

$$\begin{aligned}\dot{V}_i =& \dot{V}_{i-1} + z_i(W_{i,k}^T S_{i,k} + \eta_{i,k} + d_i + g_{i,k}x_{i+1}\\ &+ g_{i,k}\mu_{i+1}\int_0^t \varepsilon_{i+1} d\tau - g_{i-1,k}z_{i-1})\\ \leq& (\frac{\theta_1}{2a_1^2} - \frac{g_{1,k}\hat{\theta}_1}{2a_1^2 g_{1,min}})\xi^2 z_1^2 + |z_1\xi|\delta_1 + \frac{a_1^2}{2}\\ &- c_1 g_{1,k}\xi z_1^2 - \frac{g_{1,k}\xi^2\hat{\delta}_1 z_1^2}{g_{1,min}(|z_1\xi| + \varpi_1)}\\ &+ \sum_{j=2}^{i}\{(\frac{\theta_j}{2a_j^2} - \frac{g_{j,k}\hat{\theta}_j}{2a_j^2 g_{j,min}})z_j^2 + |z_j|\delta_j\\ &- \frac{g_{j,k}\hat{\delta}_j z_j^2}{g_{j,min}(|z_j| + \varpi_j)} + \frac{a_j^2}{2} - c_j g_{j,k} z_j^2\}\\ &+ g_{i,k}z_i z_{i+1}. \end{aligned} \quad (32)$$

Step n: Define $z_n = \varepsilon_n + \mu_n \int_0^t \varepsilon_n d\tau$ with $\mu_n > 0$. Its derivative is expressed as

$$\begin{aligned}\dot{z}_n &= \dot{\varepsilon}_n + \mu_n\varepsilon_n\\ &= g_{n,k}(\iota D_k(t) + b(t)) + f_{n,k} + d_n - \dot{\alpha}_{n-1}\\ &\quad + \mu_n(x_n - \alpha_{n-1}). \end{aligned} \quad (33)$$

The unknown function $\Psi_{n,k}$ is defined as

$$\Psi_{n,k} = f_{n,k} - \dot{\alpha}_{n-1} + \mu_n(x_n - \alpha_{n-1}) + g_{n-1,k}z_{n-1} + \frac{g_{n,k}^2 w^2 z_n}{2a_n^2}. \quad (34)$$

Choose Lyapunov function V_n as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \sum_{j=1}^{n}\{\frac{1}{2\hbar_j}\tilde{\theta}_j^2 + \frac{1}{2p_j}\tilde{\delta}_j^2\}, \quad (35)$$

where $\tilde{\theta}_j = \theta_j - \hat{\theta}_j$, $\tilde{\delta}_j = \delta_j - \hat{\delta}_j$, $\hbar_j > 0$, and $p_j > 0$. The derivative of V_n is

$$\dot{V}_n = \dot{V}_{n-1} + z_n\dot{z}_n - \sum_{j=1}^{n}\{\frac{1}{\hbar_j}\tilde{\theta}_j\dot{\hat{\theta}}_j + \frac{1}{p_j}\tilde{\delta}_j\dot{\hat{\delta}}_j\}$$

$$\begin{aligned}
&\leq \dot{V}_{n-1} + z_n[g_{n,k}(\iota D_k(t) + b(t)) + f_{n,k} + d_n \\
&\quad -\dot{\alpha}_{n-1} + \mu_n(x_n - \alpha_{n-1})] - \sum_{j=1}^{n}\{\frac{1}{\hbar_j}\tilde{\theta}_j\dot{\hat{\theta}}_j \\
&\quad +\frac{1}{p_j}\tilde{\delta}_j\dot{\hat{\delta}}_j\} \\
&\leq \dot{V}_{n-1} + \frac{z_n^2\theta_n}{2a_n^2} + a_n^2 + z_n g_{n,k}\iota D_k(t) \\
&\quad +|z_n|\delta_n - g_{n-1,k}z_{n-1}z_n - \sum_{j=1}^{n}\{\frac{1}{\hbar_j}\tilde{\theta}_j\dot{\hat{\theta}}_j \\
&\quad +\frac{1}{p_j}\tilde{\delta}_j\dot{\hat{\delta}}_j\} \\
&\leq (\frac{\theta_1}{2a_1^2} - \frac{g_{1,k}\hat{\theta}_1}{2a_1^2 g_{1,min}})\xi^2 z_1^2 + |z_1\xi|\delta_1 + \frac{a_1^2}{2} \\
&\quad -\frac{g_{1,k}\xi^2\hat{\delta}_1 z_1^2}{g_{1,min}(|z_1\xi| + \varpi_1)} - c_1 g_{1,k}\xi z_1^2 + a_n^2 \\
&\quad +\sum_{j=2}^{i-1}\{(\frac{\theta_j}{2a_j^2} - \frac{g_{j,k}\hat{\theta}_j}{2a_j^2 g_{j,min}})z_j^2 + |z_j|\delta_j \\
&\quad -\frac{g_{j,k}\hat{\delta}_j z_j^2}{g_{j,min}(|z_j| + \varpi_j)} + \frac{a_j^2}{2} - c_j g_{j,k}z_j^2\} + |z_n|\delta_n \\
&\quad +\frac{z_n^2\theta_n}{2a_n^2} + z_n g_{n,k}\iota D_k(t) - z_n g_{n,k}\bar{D}(t) \\
&\quad +z_n g_{n,k}\bar{D}(t) - \sum_{j=1}^{n}\{\frac{1}{\hbar_j}\tilde{\theta}_j\dot{\hat{\theta}}_j + \frac{1}{p_j}\tilde{\delta}_j\dot{\hat{\delta}}_j\}, \qquad (36)
\end{aligned}$$

where $\theta_n = \|W_{n,k}\|^2$. $D_{\sigma(t)}(t)$ and adaptive laws are designed as

$$D_k(t) = N(\tau)\bar{D}(t), \qquad (37)$$

$$\begin{aligned}
\dot{\hat{\theta}}_1 &= \frac{\hbar_1}{2a_1^2}\xi^2 z_1^2 - \beta_1\hat{\theta}_1, \\
\dot{\hat{\delta}}_1 &= \frac{p_1}{|z_1\xi| + \varpi_1}\xi^2 z_1^2 - \rho_1\hat{\delta}_1, \\
\dot{\hat{\theta}}_i &= \frac{\hbar_i}{2a_i^2}z_i^2 - \beta_i\hat{\theta}_i, \quad (i = 2, \cdots, n) \\
\dot{\hat{\delta}}_i &= \frac{p_i}{|z_i| + \varpi_i}z_i^2 - \rho_i\hat{\delta}_i \quad (i = 2, \cdots, n), \qquad (38)
\end{aligned}$$

for any $j = 1, 2, \cdots, n$. $\hbar_j$, a_j, β_j, p_j, ϖ_j and ρ_j are positive design parameters. $N(\tau)$ is a Nussbaum-type function, and $\dot{\tau} = z_n g_{n,k}\bar{D}(t)$, $\bar{D}(t)$ has a PI structure:

$$\bar{D}(t) = -(k_{P_n} + \Delta k_{P_n}(\cdot))\varepsilon_n - (k_{I_n} + \Delta k_{I_n}(\cdot))\int_0^t \varepsilon_n d\tau, \qquad (39)$$

where k_{P_n}, k_{I_n}, $\Delta k_{P_n}(\cdot)$, $\Delta k_{I_n}(\cdot)$ are designed as:

$$\begin{aligned}
k_{P_n} &= c_n,\ \Delta k_{P_n}(\cdot) = \frac{\hat{\theta}_n}{2a_n^2 g_{n,min}} + \frac{\hat{\delta}_n}{g_{n,min}(|z_n| + \varpi_n)}, \\
k_{I_n} &= \mu_n k_{P_n},\ \Delta k_{I_n}(\cdot) = \mu_n\Delta k_{P_n}(\cdot),
\end{aligned}$$

where c_n, μ_n, a_n and ϖ_n are positive constants.

Then we have

$$\begin{aligned}
z_n g_{n,k}\iota D_{\sigma(t)}(t) - z_n g_{n,k}\bar{D}(t) &= z_n g_{n,k}\iota N(\tau)\bar{D}(t) - z_n g_{n,k}\bar{D}(t) \\
&= \iota N(\tau)\dot{\tau} - \dot{\tau} \\
&= (\iota N(\tau) - 1)\dot{\tau}. \qquad (40)
\end{aligned}$$

Theorem 1: Consider system (4) with the Assumptions 1-2. If the actual control input and adaptive laws are designed as (37)-(38), then the tracking error and all the closed-loop signals are bounded.

Proof: Substituting (37), (38), (39), (40) into (36) produces

$$\begin{aligned}
\dot{V}_n &\leq (\frac{\theta_1}{2a_1^2} - \frac{g_{1,k}\hat{\theta}_1}{2a_1^2 g_{1,min}} - \frac{\tilde{\theta}_1}{2a_1^2})\xi_1^2 z_1^2 + |z_1\xi_1|\delta_1 \\
&\quad -\frac{g_{1,k}\xi_1^2\hat{\delta}_1 z_1^2}{g_{1,min}(|z_1\xi_1| + \varpi_1)} - \frac{\xi_1^2\tilde{\delta}_1 z_1^2}{|z_1\xi_1| + \varpi_1} + \frac{a_1^2}{2} \\
&\quad -c_1 g_{1,k}\xi_1 z_1^2 + \frac{\beta_1}{\hbar_1}\tilde{\theta}_1\hat{\theta}_1 + \frac{\rho_1}{p_1}\tilde{\delta}_1\hat{\delta}_j + \frac{a_n^2}{2} \\
&\quad +\sum_{j=2}^{n}\{(\frac{\theta_j}{2a_j^2} - \frac{g_{j,k}\hat{\theta}_j}{2a_j^2 g_{j,min}} - \frac{\tilde{\theta}_j}{2a_j^2})z_j^2 \\
&\quad +|z_j|\delta_j - \frac{g_{j,k}\hat{\delta}_j z_j^2}{g_{j,min}(|z_j| + \varpi_j)} - \frac{\tilde{\delta}_j z_j^2}{|z_j| + \varpi_j} + \frac{a_j^2}{2} \\
&\quad -c_j g_{j,k}z_j^2 + \frac{\beta_j}{\hbar_j}\tilde{\theta}_j\hat{\theta}_j + \frac{\rho_j}{p_j}\tilde{\delta}_j\hat{\delta}_j\} \\
&\quad +(\iota N(\tau) - 1)\dot{\tau}. \qquad (41)
\end{aligned}$$

By *Lemma 1*, if we choose $\hat{\theta}_j(0) \geq 0$ and $\hat{\delta}_j(0) \geq 0$, so $\hat{\theta}_j(t) \geq 0$ and $\hat{\delta}_j(t) \geq 0$ and for any $x > 0$, $y > 0$, one has $0 < \frac{xy}{x+y} < y$. So that

$$\begin{aligned}
&|z_1\xi_1|\delta_1 - \frac{g_{1,k}\xi_1^2\hat{\delta}_1 z_1^2}{g_{1,min}(|z_1\xi_1| + \varpi_1)} - \frac{\xi_1^2\tilde{\delta}_1 z_1^2}{|z_1\xi_1| + \varpi_1} \\
&= |z_1\xi_1|\delta_1 - \frac{g_{1,k}\xi_1^2\hat{\delta}_1 z_1^2}{g_{1,min}(|z_1\xi_1| + \varpi_1)} - \frac{\xi_1^2\delta_1 z_1^2}{|z_1\xi_1| + \varpi_1} \\
&\quad +\frac{g_{1,min}\xi_1^2\hat{\delta}_1 z_1^2}{g_{1,min}(|z_1\xi_1| + \varpi_1)} \\
&= |z_1\xi_1|\delta_1 - \frac{\xi_1^2\delta_1 z_1^2}{|z_1\xi_1| + \varpi_1} - \frac{(g_{1,k} - g_{1,min})\xi_1^2\hat{\delta}_1 z_1^2}{g_{1,min}(|z_1\xi_1| + \varpi_1)} \\
&\leq |z_1\xi_1|\delta_1 - \frac{\xi_1^2\delta_1 z_1^2}{|z_1\xi_1| + \varpi_1} \\
&= \frac{|z_1\xi_1|^2\delta_1 + |z_1\xi_1|\delta_1\varpi_1 - |z_1\xi_1|^2\delta_1}{|z_1\xi_1| + \varpi_1} \\
&= \frac{|z_1\xi_1|\delta_1\varpi_1}{|z_1\xi_1| + \varpi_1} \leq \delta_1\varpi_1. \qquad (42)
\end{aligned}$$

In the same way, we have

$$|z_j|\delta_j - \frac{g_{j,k}\hat{\delta}_j z_j^2}{g_{j,min}(|z_j| + \varpi_j)} - \frac{\tilde{\delta}_j z_j^2}{|z_j| + \varpi_j} \leq \delta_j\varpi_j. \qquad (43)$$

Substituting (42) and (43) into (41) gives rise to

$$\dot{V}_n \leq \sum_{j=1}^{n}\{\delta_j\varpi_j + \frac{a_j^2}{2} + \frac{\beta_j}{\hbar_j}\tilde{\theta}_j\hat{\theta}_j + \frac{\rho_j}{p_j}\tilde{\delta}_j\hat{\delta}_j\}$$

$$-c_1 g_{1,k}\xi_1 z_1^2 - \sum_{l=2}^{n}\{c_l g_{l,k} z_l^2\} + \frac{a_n^2}{2}$$
$$+(\iota N(\tau)-1)\dot{\tau}. \quad (44)$$

Notice that

$$\tilde{\theta}_j\hat{\theta}_j = \tilde{\theta}(\theta_j - \tilde{\theta}_j) \le -\frac{1}{2}\tilde{\theta}_j^2 + \frac{1}{2}\theta_j^2,$$
$$\tilde{\delta}_j\hat{\delta}_j = \tilde{\delta}(\delta_j - \tilde{\delta}_j) \le -\frac{1}{2}\tilde{\delta}_j^2 + \frac{1}{2}\delta_j^2. \quad (45)$$

Combining (44) and (45) leads to

$$\dot{V}_n \le \sum_{j=1}^{n}\{-\frac{\beta_j}{2\hbar_j}\tilde{\theta}_j^2 - \frac{\rho_j}{2p_j}\tilde{\delta}_j^2\} - \sum_{l=2}^{n}\{c_l g_{l,min} z_l^2\}$$
$$+\sum_{j=1}^{n}\{\frac{a_j^2}{2} + \delta_j\varpi_j + \frac{\beta_j}{2\hbar_j}\theta_j^2 + \frac{\rho_j}{2p_j}\delta_j^2\}$$
$$+\frac{a_n^2}{2} - c_1 g_{1,min}\xi_1 z_1^2 + (\iota N(\tau)-1)\dot{\tau}. \quad (46)$$

For all $1 \le j \le n$, let $\Lambda = min\{2c_j g_{j,min}, 2c_1 g_{1,min}\xi_1, \beta_j, \rho_j\}$, $\Upsilon = \sum_{j=1}^{n}\{\frac{a_j^2}{2} + \delta_j\varpi_j + \frac{\beta_j}{2\hbar_j}\theta_j^2 + \frac{\rho_j}{2p_j}\delta_j^2\} + \frac{a_n^2}{2}$, we obtain

$$\dot{V}_n \le -\Lambda V_n + \Upsilon + (\iota N(\tau)-1)\dot{\tau}. \quad (47)$$

By multiplying by $e^{\Lambda t}$ and integrating both sides of inequality (47), gives

$$V_n(t) \le V_n(0) + \frac{\Upsilon}{\Lambda} + e^{-\Lambda t}\int_0^t (\iota N(\tau)-1)\dot{\tau}e^{\Lambda\chi}d\chi. \quad (48)$$

According to *Lemma 2*, when $t \to \infty$, $V_n(t)$ is bounded, therefore $z_1, \cdots, z_n$, $\tilde{\theta}_1, \cdots, \tilde{\theta}_n$, $\tilde{\delta}_1, \cdots, \tilde{\delta}_n$ are bounded. By combining (6) and (7), we have $\varepsilon_1 \le max\{\gamma_1, \gamma_2\}\zeta_2$. Hence all signals in the closed-loop system and the tracking error are bounded.

Remark 3: The detailed implementation of our control scheme is shown as follows.

(1) Choose positive constants $\zeta_1, \zeta_2, \gamma_1$, and γ_2 to regulate the bounds of the steady errors.

(2) Select the smooth function $T(s)$ which satisfies properties (I) to (V).

(3) Choose parameters $\mu_i > 0$ for the generalized errors z_i. $(i = 1, \cdots, n)$

(4) Select appropriate designed parameters $a_i > 0$, $\varpi_i > 0$, $\hbar_i$, β_i, p_i and ρ_i for the adaptive PI controllers.

(5) Determine the virtual control laws, the adaptive laws, and the controllers.

IV. SIMULATION RESULTS

In this section, two examples are shown to illustrate the effectiveness of the scheme.

Example 1: Consider the following switched nonlinear systems:

$$\dot{x}_1 = g_{1,\sigma(t)}x_2 + f_{1,\sigma(t)}(x_1) + d_1(t),$$
$$\dot{x}_2 = g_{2,\sigma(t)}x_3 + f_{2,\sigma(t)}(x_1, x_2) + d_2(t),$$

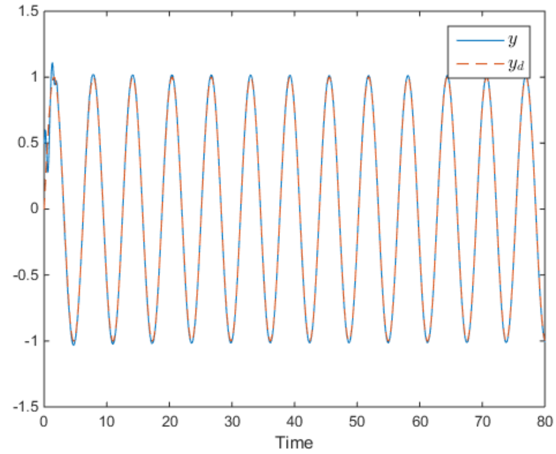


FIGURE 1. Curves of y and y_d for scheme 1 in example 1.

$$\dot{x}_3 = g_{3,\sigma(t)}u(D_{\sigma(t)}(t)) + f_{3,\sigma(t)}(x_1, x_2, x_3) + d_3(t),$$
$$y = x_1, \quad (49)$$

where $g_{1,1} = 2, g_{1,2} = 2, g_{2,1} = 2, g_{2,2} = 1, g_{3,1} = 3, g_{3,2} = 4, f_{1,1} = 0.2x_1, f_{1,2} = 0.2\sin x_1, f_{2,1} = 0.2x_1x_2, f_{2,2} = 0.2x_1x_2^2, f_{3,1} = 0.2x_1x_2x_3, f_{3,2} = 0.2x_1x_2x_3^2, d_1 = \sin t, d_2 = 2\cos t, d_3 = \sin t\cos t$. We choose the tracking signal as $y_d = \sin t$. The dead-zone parameters are selected as $\iota = 1$, $D_1 = 0.1, D_2 = 0.2$. We design $\zeta(t) = 0.65e^{-t}+0.5$ with $\gamma_1 = \gamma_2 = 1$. We choose the initial value as $x_1(0) = 0.5, x_2(0) = 0.2, x_3(0) = -0.5$, $\hat{\theta}_1(0) = \hat{\theta}_2(0) = \hat{\theta}_3(0) = 0$, $\hat{\delta}_1(0) = \hat{\delta}_2(0) = \hat{\delta}_3(0) = 0$. The controller parameters are set as $\hbar_1 = \hbar_2 = \hbar_3 = 2.8$, $\varpi_1 = \varpi_2 = 5$, $\varpi_3 = 7$, $\mu_1 = \mu_2 = \mu_3 = 2$, $a_1 = 0.5$, $a_2 = 0.5$, $a_3 = 4$, $\beta_1 = \beta_2 = 0.001$, $\beta_3 = 0.005$, $p_1 = p_2 = 2.5$, $p_3 = 1.2$, $\rho_1 = \rho_2 = 0.001$, $\rho_3 = 0.03$, $c_1 = 1$, $c_2 = 2$, $c_3 = 13$. By applying the proposed algorithm in this paper, the simulation results are presented as in Figures 1-6. Figure 1 shows curves of the system output y and track signal y_d. Figure 2 depicts tracking errors with prescribed performance. From Figures 1-2, we can see that the proposed control scheme guarantees the closed-loop system have a good control performance. In Figures 3-4, curves of adaptive parameters $\hat{\theta}_i$ and $\hat{\delta}_i$ are presented. Figure 5 depicts the control input $u_{\sigma(t)}$ and $\bar{D}$. The switching signal is shown in Figure 6. In order to further prove the superiority of the developed scheme in this paper, we compare the simulation results with the control scheme proposed in [22]. As a matter of convenience, the design scheme in this paper is set as scheme 1, and the scheme in [22] is set as scheme 2. Curves of system output, reference signal, tracking error, and controller inputs for scheme 2 are presented in Figures 7-9, respectively. Figures 10-11 show the performance comparison of tracking error and control quantity between scheme 1 and scheme 2. It is obvious to see that the scheme 1 can obtain a higher accuracy tracking performance with consuming less energy than scheme the scheme 2.

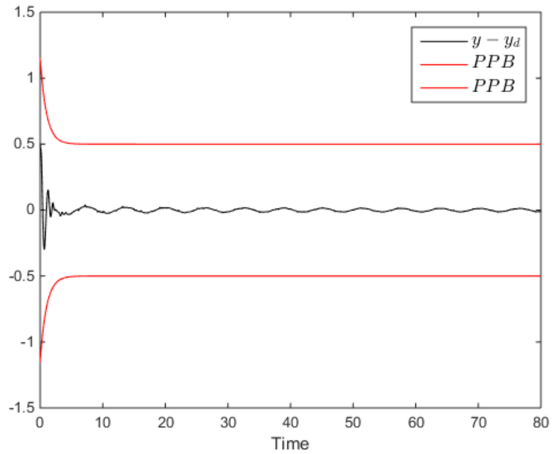


FIGURE 2. Curve of tracking error for scheme 1 in example 1.

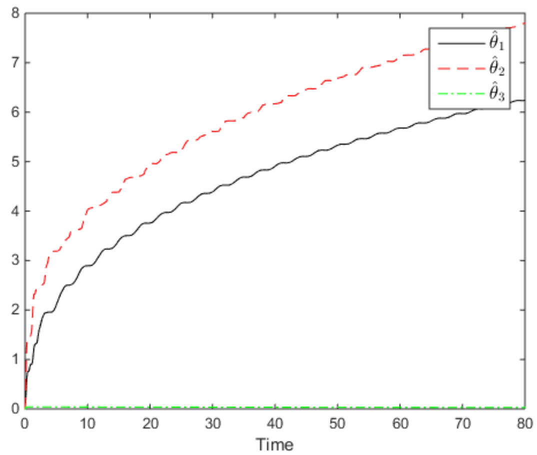


FIGURE 3. Curves of $\hat{\theta}_i$ for scheme 1 in example 1.

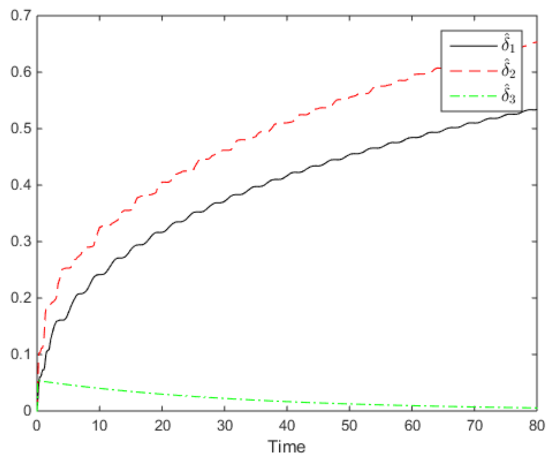


FIGURE 4. Curves of $\hat{\delta}_i$ for scheme 1 in example 1.

Example 2: Consider a one-link manipulator system

$$D\ddot{\varrho} + B\dot{\varrho} + N\sin(\varrho) = \emptyset + \emptyset_s, M\dot{\emptyset} + H\emptyset = u - K_m\dot{\varrho}, \tag{50}$$

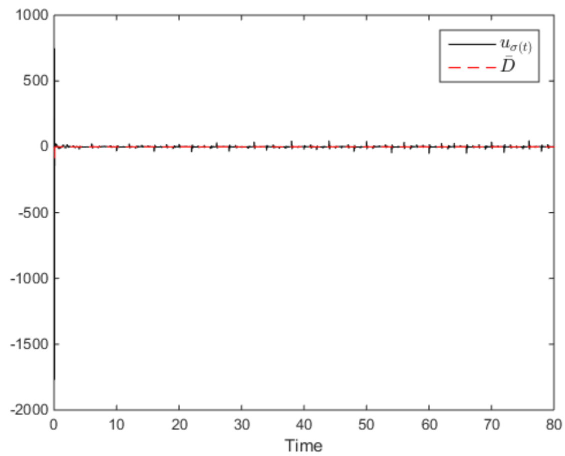


FIGURE 5. Curves of control inputs $u_{\sigma(t)}$ and $\bar{D}$ for scheme 1 in example 1.

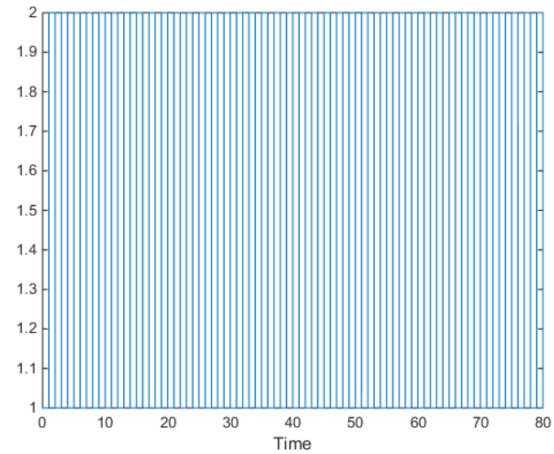


FIGURE 6. Curve of switching signal for scheme 1 in example 1 and example 2.

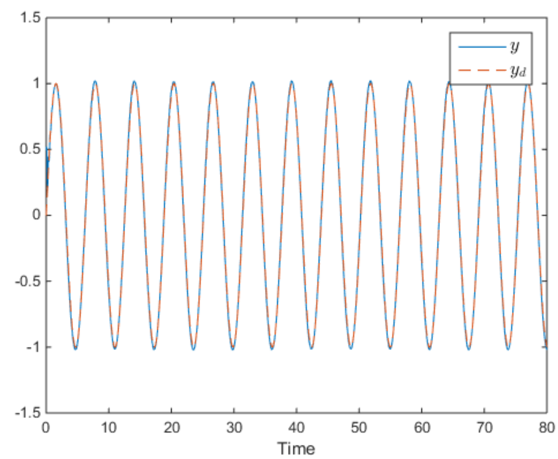


FIGURE 7. Curves of y and y_d for scheme 2 in example 1.

where $D = 1kg/m^2$ represents mechanical inertia, $M = 0.2H$ is inductance, $B = 0.1Nms/rad$ denotes the coefficient of viscous friction, $H = 1\Omega$ represents resistance, $K_m = 0.2Nm/A$ is back-emf coefficient, and $N = 10$ represents positive

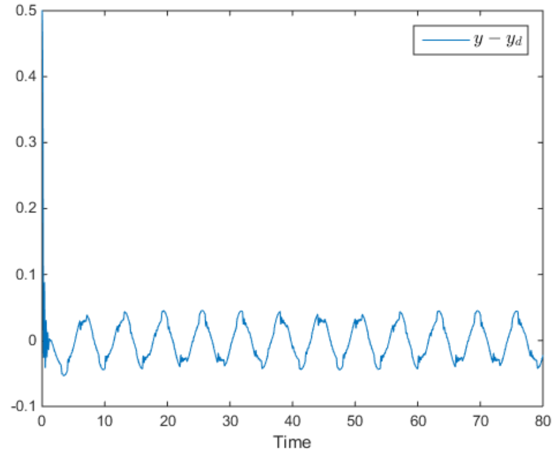


FIGURE 8. Curve of tracking error for scheme 2 in example 1.

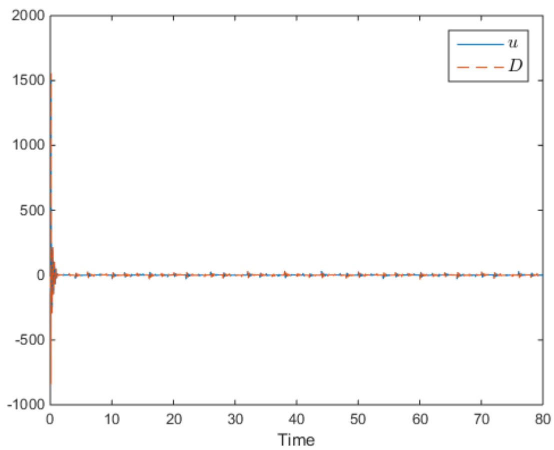


FIGURE 9. Curves of u and D for scheme 2 in example 1.

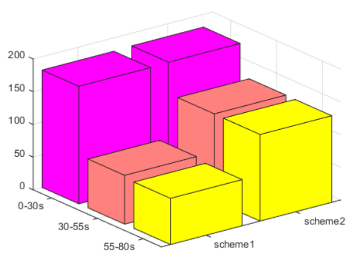


FIGURE 10. The performance comparison of tracking error for example 1.

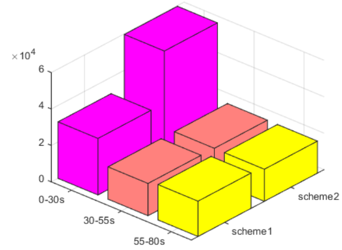


FIGURE 11. The comparison of control energy for example 1.

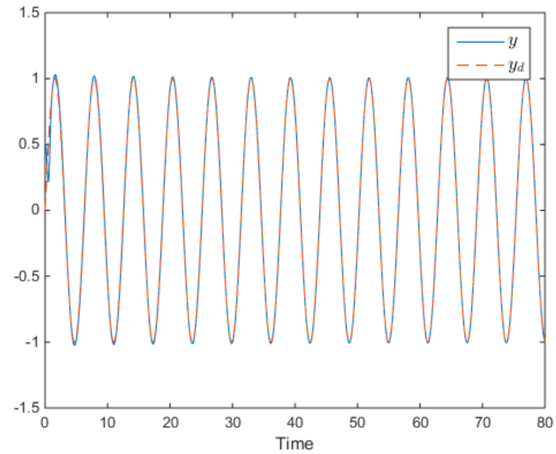


FIGURE 12. Curves of y and y_d for scheme 1 in example 2.

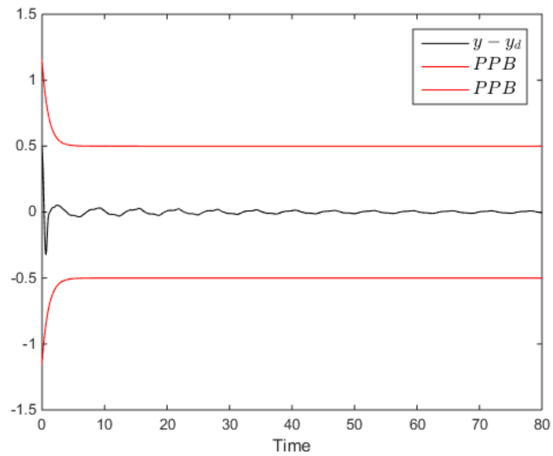


FIGURE 13. Curve of tracking error for scheme 1 in example 2.

constant. ϱ represents the position of link, $\dot{\varrho}$ is velocity, and $\ddot{\varrho}$ is acceleration. u represents the control input, $\emptyset$ and $\emptyset_s$ represent torque and torque perturbations, respectively.

Let $x_1=\varrho$, $x_2=\dot{\varrho}$, $x_3=\emptyset$. As is shown in [2], the system (50) can be rewritten as switched systems with dead zone input :

$$\begin{aligned}\dot{x}_1 &= g_{1,\sigma(t)}x_2+f_{1,\sigma(t)}(x_1)+d_1(t),\\ \dot{x}_2 &= g_{2,\sigma(t)}x_3+f_{2,\sigma(t)}(x_1,x_2)+d_2(t),\\ \dot{x}_3 &= g_{3,\sigma(t)}u(D_{\sigma(t)}(t))+f_{3,\sigma(t)}(x_1,x_2,x_3)+d_3(t),\end{aligned}$$

$$y = x_1, \tag{51}$$

where $g_{1,1}=g_{1,2}=1$, $g_{2,1}=g_{2,2}=1/D$, $g_{3,1}=g_{3,2}=1/M$, $f_{1,1}=f_{1,2}=0$, $f_{2,1}=-\frac{N}{D}\sin x_1-\frac{B}{D}x_2+\sin x_1\cos x_2$, $f_{2,2}=-\frac{N}{D}\sin x_1-\frac{B}{D}x_2+\sin x_1 x_2$, $f_{3,1}=-\frac{K_m}{M}x_2-\frac{H}{M}x_3$, $f_{3,2}=-\frac{K_m}{M}x_2-\frac{H}{M}x_3+\frac{x_1}{20}+x_3$, $d_1(t)=0$, $d_2(t)=\frac{1}{D}\cos t$, $d_3(t)=\frac{1}{D}\sin t\cos t$.

Choose the tracking signal as $y_d=\sin t$. Dead-zone parameters are selected as $\iota=1$, $D_1=0.1$, $D_2=0.2$. Define $\zeta(t)=0.65e^{-t}+0.5$ with $\gamma_1=\gamma_2=1$. The following

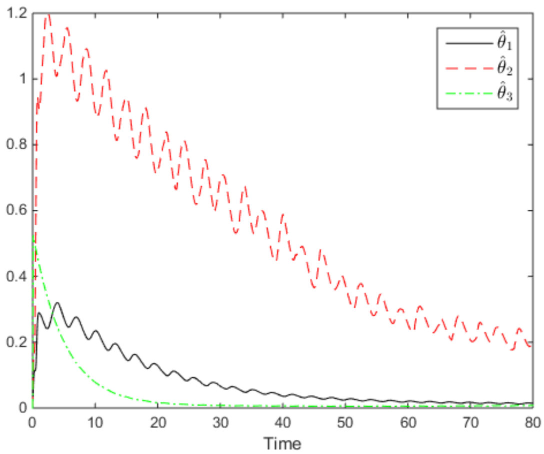


FIGURE 14. Curves of $\hat{\theta}_i$ for scheme 1 in example 2.

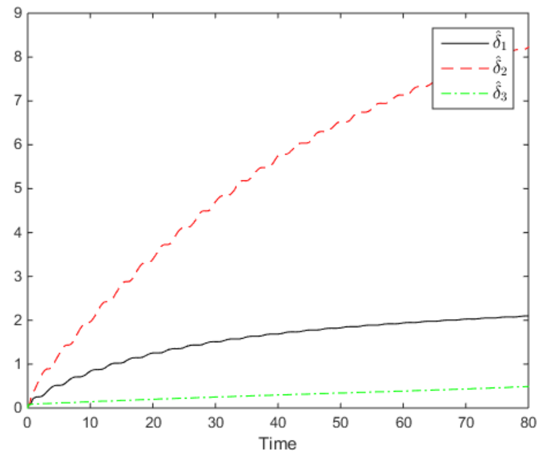


FIGURE 15. Curves of $\hat{\delta}_i$ for scheme 1 in example 2.

parameters are selected as :$\hbar_1 = \hbar_2 = \hbar_3 = 0.2$, $\varpi_1 = \varpi_2 = \varpi_3 = 0.05$, $\mu_1 = \mu_2 = \mu_3 = 1$, $a_1 = a_2 = a_3 = 0.2$, $\beta_1 = \beta_2 = \beta_3 = 0.2$, $\rho_1 = \rho_2 = \rho_3 = 0.001$, $p_1 = p_2 = p_3 = 0.8$, $c_1 = 1$, $c_2 = 2$, $c_3 = 5$, initial conditions are set as $x_1(0) = 0.5$, $x_2(0) = 0.2$, $x_3(0) = -0.5$, $\hat{\theta}_1(0) = \hat{\theta}_2(0) = \hat{\theta}_3(0) = 0$, $\hat{\delta}_1(0) = \hat{\delta}_2(0) = \hat{\delta}_3(0) = 0$. In Figures 12-16, the simulation results are shown. In Figure 12, curves of the output y and track signal y_d are given. Figure 13 shows the tracking errors. From these two figures, it is seen that the proposed control scheme can make the output tracks the tracking signal and the tracking error satisfies the prescribed performance. In Figures 14-15, curves of the $\hat{\theta}_i$ and $\hat{\delta}_i$ are shown. Figure 16 depicts curves of control input $u_{\sigma(t)}$ and $\bar{D}$. In this example, we also make a comparison with [22]. Under scheme 2, Figures 17-18 plot curves of the output y, tracking signal y_d, and tracking error, respectively. Curves of control input is presented in Figure 19. The performance comparison of tracking error and control energy between scheme 1 and scheme 2 are shown in Figures 20-21, respectively. It is easy to see from these Figures that the scheme 1 consumes

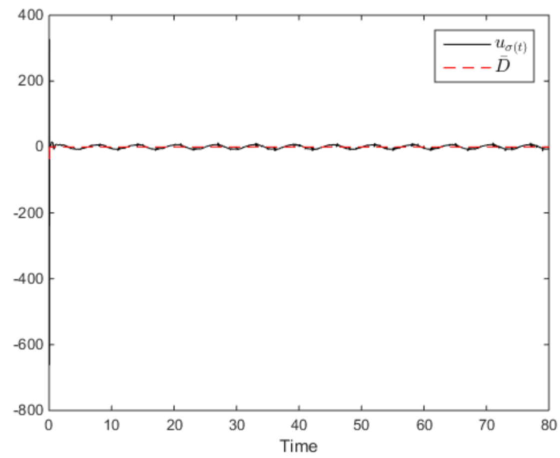


FIGURE 16. Curves of control inputs $u_{\sigma(t)}$ and $\bar{D}$ for scheme 1 in example 2.

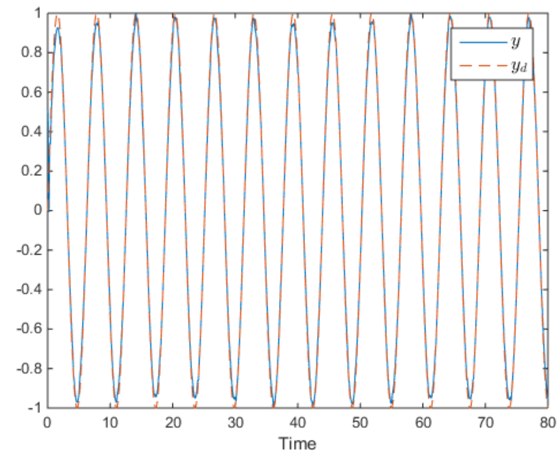


FIGURE 17. Curves of y and y_d for scheme 2 in example 2.

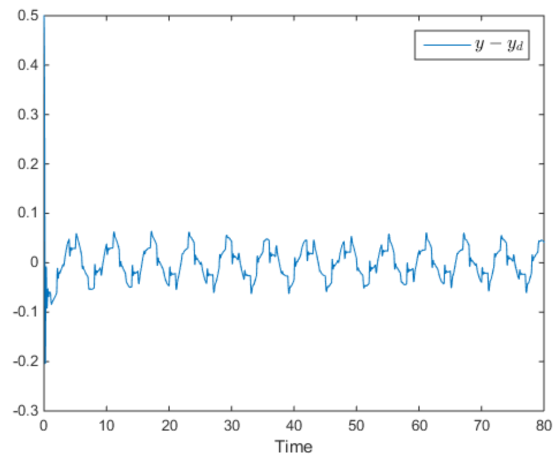


FIGURE 18. Curve of tracking error for scheme 2 in example 2.

less energy to obtain a better tracking performance than the scheme 2.

V. CONCLUSION

In this paper, the tracking control problem for a class of switched nonlinear systems has been studied, and a group

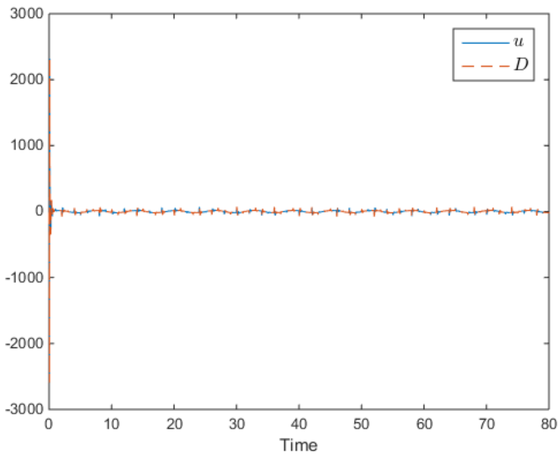


FIGURE 19. Curves of *u* and *D* for scheme 2 in example 2.

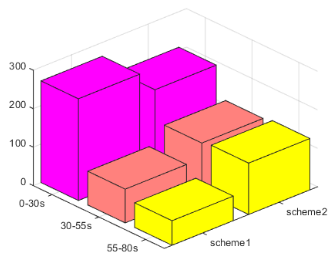


FIGURE 20. The performance comparison of tracking error for example 2.

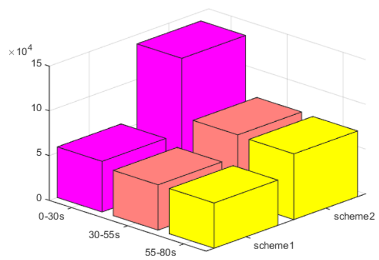


FIGURE 21. The comparison of control energy for example 2.

of PI tracking controllers have been designed, which have a simple structure and a clear physical meaning. The tracking performance has greatly improved by using prescribed performance technique. By employing common Lyapunov function theory, it has been demonstrated that all the signals of closed-loop system were bounded. Simulation results have been employed to verify the feasibility of the solution. In this paper, the backstepping method is used to construct the adaptive PI controller. In order to enhance the disturbance rejection performance of the closed-loop systems. The sliding mode control (SMC) technique provides a feasible way [34], [35]. Inspired by the above references, we will employ SMC scheme to design controllers for switched/Marokovian Jump nonlinear systems with strict feedback form or non-strict feedback form.

REFERENCES

[1] R.-H. Wang, B.-X. Xue, and J.-B. Zhao, "Time-varying H_∞ control for discrete-time switched systems with admissible edge-dependent average dwell time," *Int. J. Control, Autom. Syst.*, vol. 17, no. 8, pp. 1921–1934, Aug. 2019.

[2] Y. Li and S. Tong, "Adaptive fuzzy output-feedback stabilization control for a class of switched nonstrict-feedback nonlinear systems," *IEEE Trans. Cybern.*, vol. 47, no. 4, pp. 1007–1016, Apr. 2017.

[3] J. Fu, R. Ma, and T. Chai, "Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers," *Automatica*, vol. 54, pp. 360–373, Apr. 2015.

[4] Y. Li, S. Tong, L. Liu, and G. Feng, "Adaptive output-feedback control design with prescribed performance for switched nonlinear systems," *Automatica*, vol. 80, pp. 225–231, Jun. 2017.

[5] B. Niu and J. Zhao, "Barrier Lyapunov functions for the output tracking control of constrained nonlinear switched systems," *Syst. Control Lett.*, vol. 62, no. 10, pp. 963–971, Oct. 2013.

[6] X. Zhao, P. Shi, X. Zheng, and L. Zhang, "Adaptive tracking control for switched stochastic nonlinear systems with unknown actuator dead-zone," *Automatica*, vol. 60, pp. 193–200, Oct. 2015.

[7] T.-T. Han, S. Sam Ge, and T. Heng Lee, "Adaptive neural control for a class of switched nonlinear systems," *Syst. Control Lett.*, vol. 58, no. 2, pp. 109–118, Feb. 2009.

[8] T. C. Lee and Z. P. Jiang, "Uniform asymptotic stability of nonlinear switched systems with an application to mobile robots," *IEEE Trans. Autom. Control*, vol. 53, no. 5, pp. 1235–1252, Jun. 2008.

[9] J. M. Toibero, F. Roberti, R. Carelli, and P. Fiorini, "Switching control approach for stable navigation of mobile robots in unknown environments," *Robot. Comput.-Integr. Manuf.*, vol. 27, no. 3, pp. 558–568, Jun. 2011.

[10] E. R. Westervelt, J. W. Grizzle, and C. C. de Wit, "Switching and pi control of walking motions of planar biped walkers," *IEEE Trans. Autom. Control*, vol. 48, no. 2, pp. 308–312, Feb. 2003.

[11] A. Balluchi, L. Benvenuti, M. D. di Benedetto, C. Pinello, and A. L. Sangiovanni-Vincentelli, "Automotive engine control and hybrid systems: Challenges and opportunities," *Proc. IEEE*, vol. 88, no. 7, pp. 888–912, Jul. 2000.

[12] J.-L. Wu, "Stabilizing controllers design for switched nonlinear systems in strict-feedback form," *Automatica*, vol. 45, no. 4, pp. 1092–1096, Apr. 2009.

[13] R. Ma and J. Zhao, "Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings," *Automatica*, vol. 46, no. 11, pp. 1819–1823, Nov. 2010.

[14] X. Zhao, X. Zheng, B. Niu, and L. Liu, "Adaptive tracking control for a class of uncertain switched nonlinear systems," *Automatica*, vol. 52, pp. 185–191, Feb. 2015.

[15] G. Lai, Z. Liu, Y. Zhang, C. L. Philip Chen, and S. Xie, "Adaptive backstepping-based tracking control of a class of uncertain switched nonlinear systems," *Automatica*, vol. 91, pp. 301–310, May 2018.

[16] D. Zhai, L. An, J. Dong, and Q. Zhang, "Switched adaptive fuzzy tracking control for a class of switched nonlinear systems under arbitrary switching," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 585–597, Apr. 2018.

[17] Y. Li and S. Tong, "Command-Filtered-Based fuzzy adaptive control design for MIMO-switched nonstrict-feedback nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 3, pp. 668–681, Jun. 2017.

[18] Y. Li, S. Sui, and S. Tong, "Adaptive fuzzy control design for stochastic nonlinear switched systems with arbitrary switchings and unmodeled dynamics," *IEEE Trans. Cybern.*, vol. 47, no. 2, pp. 403–414, Feb. 2017.

[19] Y. Song, Z. Shen, L. He, and X. Huang, "Neuroadaptive control of strict feedback systems with full-state constraints and unknown actuation characteristics: An inexpensive solution," *IEEE Trans. Cybern.*, vol. 48, no. 11, pp. 3126–3134, Nov. 2018.

[20] Y. Song, J. Guo, and X. Huang, "Smooth neuroadaptive PI tracking control of nonlinear systems with unknown and nonsmooth actuation characteristics," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 9, pp. 2183–2195, Sep. 2017.

[21] S. Tong, L. Zhang, and Y. Li, "Observed-based adaptive fuzzy decentralized tracking control for switched uncertain nonlinear large-scale systems with dead zones," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 1, pp. 37–47, Jan. 2016.

[22] R.-B. Li, X.-M. Wang, X.-M. Liu, B. Niu, and D. Yang, "Adaptive neural tracking control for uncertain switched nonlinear non-lower triangular system with disturbances and dead-zone input," *Int. J. Control, Autom. Syst.*, vol. 18, no. 6, pp. 1445–1452, Jun. 2020.

[23] L. Ma, X. Huo, X. Zhao, and G. Zong, "Adaptive fuzzy tracking control for a class of uncertain switched nonlinear systems with multiple constraints: A small-gain approach," *Int. J. Fuzzy Syst.*, vol. 21, no. 8, pp. 2609–2624, Nov. 2019.

[24] X. Liu, D. Zhai, J. Dong, and Q. Zhang, "Adaptive fault-tolerant control with prescribed performance for switched nonlinear pure-feedback systems," *J. Franklin Inst.*, vol. 355, no. 1, pp. 273–290, Jan. 2018.

[25] B.-W. Chen and L.-G. Tan, "Adaptive anti-saturation tracking control with prescribed performance for hypersonic vehicle," *Int. J. Control, Autom. Syst.*, vol. 18, no. 2, pp. 394–404, Feb. 2020.

[26] L. Tang and J. Zhao, "Neural network based adaptive prescribed performance control for a class of switched nonlinear systems," *Neurocomputing*, vol. 230, pp. 316–321, Mar. 2017.

[27] S. Qi, D. Zhang, L. Guo, and L. Wu, "Adaptive dynamic surface control of nonlinear switched systems with prescribed performance," *J. Dyn. Control Syst.*, vol. 24, no. 2, pp. 269–286, Apr. 2018.

[28] P. Gawthrop, "Self-tuning PID controllers: Algorithms and implementation," *IEEE Trans. Autom. Control*, vol. 31, no. 3, pp. 201–209, Mar. 1986.

[29] H. K. Khalil, "Universal integral controllers for minimum-phase nonlinear systems," *IEEE Trans. Autom. Control*, vol. 45, no. 3, pp. 490–494, Mar. 2000.

[30] R. D. Nussbaum, "Some remarks on a conjecture in parameter adaptive control," *Syst. Control Lett.*, vol. 3, no. 5, pp. 243–246, Nov. 1983.

[31] Z. Liu, F. Wang, Y. Zhang, and C. L. P. Chen, "Fuzzy adaptive quantized control for a class of stochastic nonlinear uncertain systems," *IEEE Trans. Cybern.*, vol. 46, no. 2, pp. 524–534, Feb. 2016.

[32] S. S. Ge, F. Hong, and T. H. Lee, "Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients," *IEEE Trans. Syst., Man Cybern. B, Cybern.*, vol. 34, no. 1, pp. 499–516, Feb. 2004.

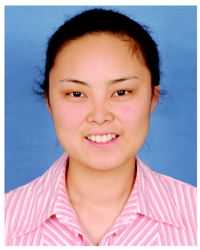
[33] L.-X. Wang and J. M. Mendel, "Fuzzy basis functions, universal approximation, and orthogonal least-squares learning," *IEEE Trans. Neural Netw.*, vol. 3, no. 5, pp. 807–814, Sep. 1992.

[34] C. Du, F. Li, and C. Yang, "An improved homogeneous polynomial approach for adaptive sliding-mode control of Markov jump systems with actuator faults," *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 955–969, Mar. 2020.

[35] F. Li, C. Du, C. Yang, L. Wu, and W. Gui, "Finite-time asynchronous sliding mode control for Markovian jump systems," *Automatica*, vol. 109, Nov. 2019, Art. no. 108503.

HONGYAO LI was born in Linyi, China, in 1996. He received the bachelor's degree from the School of Mathematical Science, Liaocheng University, in 2018. He is currently pursuing the master's degree with the School of Engineering, Qufu Normal University. His current research interest includes switched nonlinear systems.

HAIBIN SUN (Member, IEEE) was born in 1982. He received the Ph.D. degree in control theory and application from Southeast University, in 2013. He held a postdoctoral position with the School of Automation, Beihang University, Beijing, from 2013 to 2014. In 2017, he was a Visiting Scholar with the School of Engineering, RMIT University, Melbourne, Australia. In 2019, he was a Visiting Scholar with the School of EEE, Nanyang Technological University, Singapore. He is currently an Associate Professor at the School of Engineering, Qufu Normal University. His research interests include switched control, anti-disturbance control, and its application.



LINLIN HOU received the master's and Ph.D. degrees from the Institute of Automation, Qufu Normal University, Qufu, China, in 2009 and 2012, respectively. She was a Visiting Scholar with the School of Engineering, RMIT University, Melbourne, Australia. She is currently an Associate Professor at the School of Information Science and Engineering, Qufu Normal University, China. Her research interests include analysis and control of switched systems, time-delay systems, and networked control systems.