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PaDE-NPC: Parameter Adaptive Differential Evolution With Novel Parameter Control for Single-Objective Optimization

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ABSTRACT Single objective real-parameter optimization problems exist in many areas of the real world, and Differential Evolution (DE) is a powerful population based stochastic optimization approach for tackling such problems. There are many different mutation strategies mentioned in the literature, and each of them has its own advantage. In this paper, we propose combined mutation strategies which can make a full use of the advantage of each mutation strategy regarding a population diversity indicator during the evolution. Furthermore, Novel Parameter Control (NPC) for the three control parameters including the scale factor F, crossover rate CR and population size PS are also proposed in the paper. Different from employing the fitness value as a weight in recent proposed state-of-the-art DE variants, our PaDE-NPC algorithm can tackle a large optimization problems especially for those the fitness differences are unavailable; Moreover, a platform based population size reduction scheme is also involved in the NPC, which can get a better perception of the landscape at the early stage of the evolution while obtaining a balance between exploration and exploitation in the later part of the evolution. The novel PaDE-NPC algorithm is verified under 58 benchmark functions from CEC2013 and CEC2017 test suits for real-parameter optimization competitions and experiment results show that our proposed PaDE-NPC algorithm outperforms these recently proposed powerful DE variants.

INDEX TERMS Combined strategy, differential evolution, novel parameter control, single-objective optimization.

I. INTRODUCTION

Differential evolution (DE), as one of the most powerful and efficient evolutionary algorithms (EAs) [1]–[4] for global optimization, is first introduced by Price and Storn [5]–[10]. Because of its simple structure and feasibility for tough optimization problems [11]–[14], DE has gained a lot of focus and has been widely used in many real-world application, such as scientific and engineering applications [15]–[19]. In order to further improve the performance of DE, many researchers engaged in the study of DE and proposed lots of new DE variants in the past two decades [20]–[24]. Among these existing DE variants, most studies focus on developing new efficient mutation strategies and designing intelligent adaptation schemes for control parameters [25]–[30]. Brest *et al.* proposed a parameter control technique with self-adapting strategy [20], the values of both F and Cr were randomly altered within certain range when probability constraints were satisfied. Zhang and Sanderson proposed JADE algorithm [21] in which a new mutation strategy ‘‘DE/current-to-pbest’’ was invented. The mutation strategy can effectively maintain population diversity while keeping fast convergence property. Moreover, adaptation schemes of μ_F and μ_{Cr} are also applied to renew the parameters F and Cr of the individual. Tanabe and Fukunaga proposed a powerful DE variant named SHADE [31] which further extended the JADE algorithm. In SHADE, historical memory of successful control parameter settings are used to guide the selection of future control parameter values, and the fitness difference of the successful individuals are also incorporated in the adaptation schemes of control parameters. The authors further enhanced SHADE algorithm by introducing linear

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population size reduction [25], and this algorithm was named LSHADE which won the first place in the competitions. Brest *et al.* proposed iLSHADE [27] for the competition lunched in 2016, which is a further improvement of LSHADE algorithm. In iLSHADE, the generated control parameters F and Cr are readjusted according to different stages of the evolution, which is considered to be of too much human intervention and over-fitting problem. The jSO algorithm [29] proposed a new inertia weight based mutation strategy as well as incorporating stage-based control parameter readjustments in iLSHADE algorithm won the CEC2017 competition for single-objective real-parameter optimization among all the DE variants. Meng *et al.* proposed LPALMDE [28] which divided the control parameters into different groups, and all parameters were updated independently which tackled the misleading interaction among parameters. Moreover, the authors also proposed a HARD-DE [30] algorithm, an enhanced version of LPALMDE. A new hierarchical archive-based mutation strategy was introduced into it and better perception of landscapes of objective functions was obtained by the hierarchial mutation strategy.

In summary, the trial vector generation strategy dominates the overall optimization performance, and there are two components involved in the generation of trial vectors. One is the mutation strategy and the other is parameter control. As we know, all the mutation strategies mentioned in the literature have the advantages and disadvantages of their own and a single mutation strategy can not tackle all the problems. Here we propose combined mutation strategies which can make a full use of the advantage of each mutation strategy regarding a population diversity indicator during the evolution. For a given mutation strategy, the control parameters F, CR and PS are also very important for the optimization performance. According to these DE variants and some comparative studies of them [32]–[36], we can see that the fitness difference based parameter adaptation schemes are empirically very efficient in the enhancement of DE algorithm [37]–[39]. By introducing fitness difference in the adaptation of control parameters μ_F and μ_{Cr}, we can generate more accurate control parameters which lead to better optimization performance. That's also the reason why DE variants with fitness difference based adaptation schemes win recent competitions and show their superiority in improving the performance of DE algorithm [40]–[43]. Although fitness difference based adaptation schemes are very powerful, these kind of adaptation schemes are heavily dependent on the fitness values of the individuals, and there are the optimization cases that the exact fitness values are unavailable. To circumvent the above predicament, a new PaDE-NPC is proposed in this paper. The main innovation of the proposed algorithm is listed as follows:

1) The PaDE-NPC algorithm employs a combined mutation strategies which can make a full use of the advantage of each mutation strategy regarding a population diversity indicator, therefore, it is more likely to obtain better performance.
2) Novel Parameter Control (NPC) for the three control parameters including the scale factor F, crossover rate CR and population size PS are also proposed in the paper. Different from employing the fitness value as a weight in recent proposed state-of-the-art DE variants, our PaDE-NPC algorithm can tackle a large optimization problems especially for those the fitness differences are unavailable;
3) A platform based population size reduction scheme is also involved in the NPC, which can get a better perception of the landscape at the early stage of the evolution while obtaining a balance between exploration and exploitation in the later part of the evolution.
4) A test suite containing 58 benchmarks from CEC2013, and CEC2017 test suites on real-parameter single objective optimization is employed in the algorithm validation and the experiment results show the competitiveness of our PaDE-NPC algorithm.

The rest of this paper is organized as follows. In Section II, DE is briefly introduced. Section III presents the review of several powerful DE variants in the literature. Section IV presents the details of the proposed PaDE-NPC algorithm. Section V presents the experimental analysis under CEC2013 and CEC2017 test suits. Finally, conclusion is given in Section VI.

II. THE CLASSICAL DE ALGORITHM

DE is a stochastic population based trial-and-error method for the tackling of optimization problems by mimicking biological evolution. The whole optimization process can be divided into different stages of evolution: initialization and loop of mutation, crossover and selection.

A. INITIALIZATION

In general, the single objective optimization problem can be mathematically represented as searching for a global optimum point X^* in a D-dimensional space $\Re^D$:

$$\min f(X) \quad s.t. X \in S \tag{1}$$

where $f(X)$ is an objective function, S is the search region, and X is an D-dimensional vector restricted by lower bound $X_{min} = (x_{min,1}, x_{min,2}, \ldots, x_{min,D})$ and upper bound $X_{max} = (x_{max,1}, x_{max,2}, \ldots, x_{max,D})$. Typically, the initialization of the i^{th} individual can be generated as follows:

$$x_{i,j} = x_{min,j} + rand(0,1) \cdot (x_{max,j} - x_{min,j}) \qquad s.t.\ j \in \{1, 2, \ldots, D\} \tag{2}$$

where $rand(0, 1)$ denotes a uniformly distributed random number between 0 and 1, and the i^{th} individual of the population in the g^{th} generation is represented as $X_{i,g}$.

B. MUTATION

After initialization, the mutation operation is implemented to generate donor vector $V_{i,g} = (v_{i,1,g}, v_{i,2,g}, \ldots, v_{i,D,g})$, and four frequently used mutation strategies are presented below:

- DE/rand/1

$$V_{i,g} = X_{r1,g} + F \cdot (X_{r2,g} - X_{r3,g}) \quad (3)$$

- DE/best/1

$$V_{i,g} = X_{best,g} + F \cdot (X_{r1,g} - X_{r2,g}) \quad (4)$$

- DE/current-to-rand/1

$$V_{i,g}=X_{i,g}+F \cdot (X_{r1,g}-X_{i,g})+F \cdot (X_{r2,g} - X_{3,g}) \quad (5)$$

- DE/current-to-best/1

$$V_{i,g}=X_{i,g}+F \cdot (X_{best,g}-X_{i,g})+F \cdot (X_{r1,g} - X_{2,g}) \quad (6)$$

where $r_1, r_2, r_3 \in \{1, 2, \ldots, ps\}$ are the mutually exclusive indices randomly selected from the range $[1, ps]$, and all these indices are distinct from i. The scale factor $F \in \{0, 1\}$ is a real number which is used for the amplification of the difference vectors. $X_{best,g}$ denotes the individual with the best fitness value in the g^{th} generation.

C. CROSSOVER

After mutation operation, crossover operation is conducted to produce trial vector $U_{i,g}$ by selecting the components both from target vector $X_{i,g}$ and donor vector $V_{i,g}$ [44]–[46]. Commonly, the crossover operation in DE has two ways: exponential crossover and binomial crossover. According to some comparative studies, crossover in binomial way is generally more robust and effective than exponential way, and in most recently published papers, the binomial way is the commonly used crossover scheme in numerical optimization. For the crossover scheme, the operation is conducted on each dimension according to whether a randomly generated value $rand(0, 1)$ is no bigger than the crossover probability Cr or not, and it can be formulated as follows:

$$u_{i,j} = \begin{cases} v_{i,j}, & \text{if } \mathrm{rand}(0,1) \le Cr \ \text{ or } j = j_{\mathrm{rand}} \\ x_{i,j}, & \text{otherwise} \end{cases} \quad (7)$$

where j_{rand} is a random integer in the range $[1, D]$. Fig. 1 illustrated the relationship between target $X_{i,g}$, donor vector $V_{i,g}$ and trial vector candidates (i.e., $U_{i,g}$, $U'_{i,g}$ and $U''_{i,g}$) of the canonical DE in 2-D search domain.

D. SELECTION

As DE is based on the principle "Survival of the fittest", therefore the solution with better fitness value will be reserved while the one with relatively poor fitness value will be discarded in next generation. At this point, a comparison will be conducted between the trial vector $U_{i,g}$ and the target vector $X_{i,g}$ according to their function values, i.e., $f(X_{i,g})$ and $f(U_{i,g})$, and the one with better function value will survive into the next generation. For a minimization problem, the selection operator can be represented like this:

$$X_{i,g+1} = \begin{cases} U_{i,g}, & \text{if } f\left(U_{i,g}\right) \le f\left(X_{i,g}\right) \\ X_{i,g}, & \text{otherwise} \end{cases} \quad (8)$$

This evolution continues until the termination criterion is met.

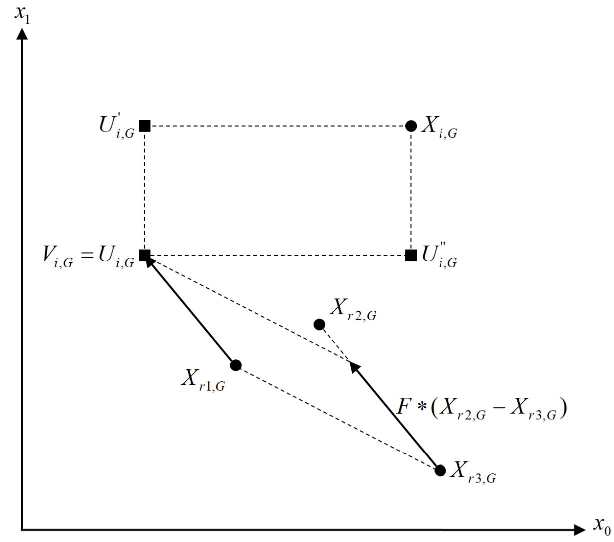


FIGURE 1. Relationship between target $X_{i,g}$, donor vector $V_{i,g}$ and trial vector candidates of the canonical DE in 2-D search domain.

III. RELATED WORK

The performance of the DE algorithm can be heavily influenced by the selection of different mutation strategies and control parameters. To further enhance the performance of classical DE, many approaches have been proposed in the literature, and the majority of these improvements includes developments in mutation strategies and developments in parameters control. In this section, literature reviews are presented from these two improvements of several state-of-the-art DE variants that closely related to our PaDE-NPC algorithm.

A. MUTATION STRATEGY

The four mutation strategies mentioned in Section II are canonical mutation strategies proposed in the early period of DE development. one branch of later studies mainly focused on developing new efficient mutation strategies to balance exploration and exploitation of them. Zhang and Sanderson proposed an effective mutation strategy in JADE, and the mutation strategy was denoted as "DE/current-to-$p$best/1/bin" in which an optional external archive was employed in diversity improvement of the individuals. In this new strategy, the top $100p\%$ solutions rather than the global best solution of the current generation were utilized to guide the evolution of the individuals. The "DE/current-to-pbest/1/bin" with external archive is presented below in Eq. 9:

$$V_{i,g} = X_{i,g} + F \cdot (X^{p}_{best,g} - X_{i,g}) + F \cdot (X_{r_1,g} - \widetilde{X}_{r_2,g}) \quad (9)$$

where $X_{i,g}$ is the target vector, $V_{i,g}$ is the donor vector, and $X^{p}_{best,g}$ is a vector randomly selected form the top $100p\%$ solutions for the current population. A denotes an optional external archive that records the inferior solutions of the evolution, and $\widetilde{X}_{r_2,g}$ is a vector randomly selected from the union of the current population P and external archive A.

Brest *et al.* further improved the mutation strategy of JADE by incorporating an inertia weight in a new DE variant, the jSO algorithm [29], meanwhile, the parameter control of jSO is similar as its former algorithm, the iLSHADE algorithm [27]. The mutation strategy in jSO is also presented below in Eq. 10:

$$V_{i,g}=X_{i,g}+F_w\cdot F\cdot(X^p_{best,g}-X_{i,g})+F\cdot(X_{r_1,g}-\widetilde{X}_{r_2,g}) \quad (10)$$

However, a single powerful mutation strategy can not tackle all the problems, and combined mutation strategies may show the superiority because they can make full use of the advantages of different mutation strategies. Meng *et al.* proposed a DE variant with a new hierarchical archive based mutation strategies in HARD-DE algorithm [30]. The novel hierarchical archive based mutation strategies were verified to be able to obtain good perception of objectives at different stages of the evolution. The details of the mutation strategy is presented as follows:

$$\begin{aligned} V_{i,g} &= X_{i,g}+F\cdot(X^p_{best,g}-X_{i,g}) \\ &\quad +F_1\cdot(X_{r_1,g}-\widetilde{X}_{r_2,g})+F_2\cdot(X_{r_1,g}-\widetilde{X}_{r_3,g}) \end{aligned} \quad (11)$$

where same symbols have already defined in the earlier part, and $\widetilde{X}_{r_3,G}$ denotes a randomly chosen individual from the union $P\bigcup B$, where P is the current population and B denotes the set of individuals of former population, moreover, B is restored in the second part of the hierarchical archive of HARD-DE algorithm. The mutation strategies of the novel PaDE-NPC algorithm in this paper is a further development of the ones in HARD-DE algorithm.

B. PARAMETER CONTROL

Considerable research has also been carried out to develop effective parameter control. Tanabe and Fukunaga in LSHADE proposed a novel parameter control mechanism in which historical superior control parameters were used to guide the distribution of these control parameters [25], moreover, the fitness differences of successful individuals are also incorporated into the update of the μ_F and μ_{Cr}. There are H entries in the memory pool, and each entry was assigned a μ_F and μ_{Cr} pair within it. This setting can enhance the robustness of the control parameters and only an entry recording the μ_F–μ_{Cr} pair is renewed during each generation. Eq. 12 and Eq. 13 present the details of the parameter control in the LSHADE algorithm:

$$\begin{cases} w_k = \dfrac{\Delta f_k}{\sum_{k=1}^{|S_F|}\Delta f_k} \\ \Delta f_j = f(X_{j,G})-f(U_{j,G}) \\ mean_{WL}(S_F)=\dfrac{\sum_{k=1}^{|S_F|} w_k\cdot S^2_{F,k}}{\sum_{k=1}^{|S_F|} w_k\cdot S_{F,k}} \\ \mu_{F,G+1}=\begin{cases} mean_{WL}(S_F), & if\ S_F\neq\emptyset \\ \mu_{F,G}, & otherwise \end{cases} \end{cases} \quad (12)$$

$$\begin{cases} w_k = \dfrac{\Delta f_k}{\sum_{k=1}^{|S_{Cr}|}\Delta f_k} \\ \Delta f_j = f(X_{j,G})-f(U_{j,G}) \\ mean_{WA}(S_{Cr})=\sum_{k=1}^{|S_{Cr}|} w_k\cdot S_{Cr,k} \\ \mu_{Cr,k,G+1}=\begin{cases} mean_{WA}(S_{Cr}), & if\ S_{Cr}\neq\emptyset \\ \mu_{Cr,k,G}, & otherwise \end{cases} \end{cases} \quad (13)$$

Brest et al proposed stage-based adaptation of control parameters in iL-SHADE [27], and these readjustments of control parameters were also inherited into a novel jSO algorithm that secured the first place among DE competitors in CEC2017 competition. Meng et al proposed a new DE variant which introduced a Parameters with Adaptive Learning Mechanism (LPALMDE) [28], in which control parameters were separated into different groups to tackle the misleading interaction among control parameters, and the adaptation scheme for the scale factor F is based on the success values of the corresponding individuals while the adaptation scheme for crossover rate Cr is based on its success probability. Although the parameter control in this algorithm tried to employ fitness-independent ways in the adaptation of μ_{Cr}, the adaptation of μ_F still employed the fitness-difference. All the DE variants with fitness-difference dependent parameter control can not tackle the optimization applications that the fitness values were unavailable. That's also the contribution of our novel PaDE-pet algorithm in this paper.

IV. THE NOVEL PaDE-NPC ALGORITHM

In this section, we provide an overall description of the novel PaDE-NPC algorithm, and the PaDE-NPC algorithm can be separated into the following four parts: mutation strategy, the grouping strategy, the novel parameter control, and the population size reduction.

A. MUTATION STRATEGY

As mentioned in Section III, mutation strategy can significantly influence the overall optimization of DE variants. It is known to all that the "DE/target-to-pbest/1/bin" mutation strategy, firstly introduced in JADE [21], is a very effective mutation strategy. By incorporating an optional external archive with inferior solutions, a good population diversity can be achieved but the fast convergence speed can also be maintained during evolution. Recently, many state-of-the-art DE variants, including LSHADE [25], iLSHADE [27], QUATRE-EAR [35], [46], jSO [29] and LPALMDE [28] etc., employed the same or similar strategy as the "DE/target-to-pbest/1/bin" mutation strategy. As is mentioned above, a single powerful mutation strategy can not tackle all the optimization problems and a combined mutation strategy can solve a much wilder optimization applications. Therefore, in our novel PaDE-NPC algorithm combined mutation strategies with population diversity indicator are proposed and the details of the mutation strategies are given in Eq.14 and Eq.15 respectively:

$$V_{i,g}=X_{i,g}+F\cdot(X^p_{best,g}-X_{i,g})+F\cdot(X_{r_1,g}-\widetilde{X}_{r_2,g}) \quad (14)$$

$$V_{i,g} = X_{i,g} + F \cdot (X^{p}_{best,g} - X_{i,g}) + F_1 \cdot (X_{r_1,g} - \widetilde{X}_{r_2,g}) + F_2 \cdot (X_{r_1,g} - \widetilde{X}_{r_3,g}) \tag{15}$$

where same symbols have already defined in the former parts. For the relationship among the three scale factors in Eq. 15, $F_1 = 0.9 \cdot F$ and $F_2 = 0.7 \cdot F$ are employed in the proposed PaDE-NPC algorithm, which are the same as the former HARD-DE algorithm. Moreover, two parameters r^{arc} and r^{har} are used for delimiting the size of archive A and H, and the relationship between the population size of g^{th} generation and the size of archive always satisfy the following equation:

$$\begin{cases} |A| = r^{arc} \cdot ps_g \\ |H| = r^{hrc} \cdot ps_g \end{cases} \tag{16}$$

where $|A|$ and $|H|$ denote the size of these two archives, and the default values of r^{arc} and r^{hrc} are set to $r^{arc} = 1.0$ and $r^{hrc} = 3.0$. Especially, when the number of solutions exceeds the fixed maximum of the corresponding archive, randomly selected inferior individuals will be discarded from the archive to keep the number of solutions is no bigger than the fixed maximum size. For the population diversity indictor, DM is proposed for the choice of a certain mutation strategy at current population diversity. Then the evolution can be divided into two parts according to the following equation:

$$\begin{cases} LD_g/LD_1 > DM & \rightarrow \text{ Earlier evolution stage} \\ LD_g/LD_1 \leq DM & \rightarrow \text{ Later evolution stage} \end{cases} \tag{17}$$

where LD_g can be calculated via:

$$LD_g = \frac{1}{ps} \cdot \sqrt{\sum_{i=1}^{ps} \left\| X_{i,G} - \frac{1}{ps} \cdot \sum_{j=1}^{ps} X_{j,g} \right\|^2} \tag{18}$$

LD_1 denotes the density of the population at the initialization stage of the evolution while LD_g denotes the population diversity in the g^{th} generation. In our PaDE-NPC algorithm, the hierarchical archive based mutation strategy in Eq. 15 is employed at the earlier stage and the mutation strategy in Eq. 14 is employed at the later stage of the evolution.

B. GROUPING STRATEGY OF CONTROL PARAMETERS

For the control parameters in our PaDE-NPC algorithm, the whole population are divided into k groups in the evolution, and each group in PaDE-NPC records two parameters, the selection probability $P(\cdot)$ and the center of the distribution of crossover rate μ_{Cr}. At the beginning of the evolution, the initial value of the selection probability for each group is assigned to $1/k$. Then, all individuals of the population in each generation can be divided into the different groups by implementing stochastic universal selection [28]. Moreover, the selection probability of each group will be renewed at the end of each generation and μ_{Cr} of the group with smallest selection probability will replaced. The update scheme of selection probability is shown in Eq 19:

$$\begin{cases} r_j = \begin{cases} \dfrac{ns_j^2}{ns \cdot (ns_j + nf_j)}, & \text{if } ns_j > 0 \\ \epsilon, & \text{otherwise} \end{cases} \\ ns = \sum_{j=1}^{k} ns_j \\ P(j) = \dfrac{r_j}{\sum_{j=1}^{k} (r_j)} \end{cases} \tag{19}$$

where ns_j denotes the number of individuals in the jth group that do found better solutions, while ns_j denotes the number of individuals in the jth group that do not found better solutions. Moreover, a small value $\epsilon = 0.01$ is implemented here to avoid possible null values of probability. After the update of selection probabilities, we can find the group with lowest selection probability, and then, we will update the μ_{Cr} value of this group according the corresponding parameter adaptation scheme. Especially, the update of μ_{Cr} is only carried out in one group at each generation, i.e. when there are more than one group are associated with the same smallest probability, we only select one group to renew its corresponding μ_{Cr} value at random, and the index of the selected group is indicated as idx. The adaptation scheme for the control parameter $\mu_{Cr_{idx}}$ will be illustrated in the next part. The grouping strategy of PaDE-NPC is illustrated in Fig. 2.

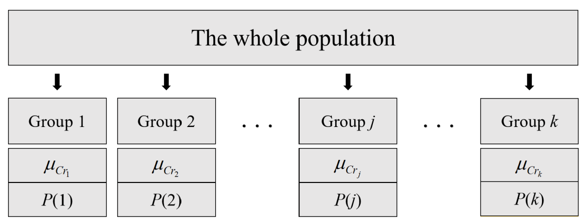


FIGURE 2. Different groups of individuals in the PaDE-NPC algorithm.

C. THE NOVEL PARAMETER CONTROL

As is mentioned above, the winner DE variants in the recent competitions all employed the fitness difference based adaptation scheme for control parameters, however, fitness differences were unavailable in some real-world optimization problems. Fig. 3 illustrated a black-box model in real world application which the fitness differences of the objective function are unavailable, and in order to tackle this problem, a novel location information based parameter adaptation scheme is proposed in this paper.

The main idea of the proposed adaptation scheme is to adapt the scale factor F and the crossover rate Cr by using the location information of the population. In our PaDE-NPC algorithm, a novel adaption scheme for the parameter μ_F and

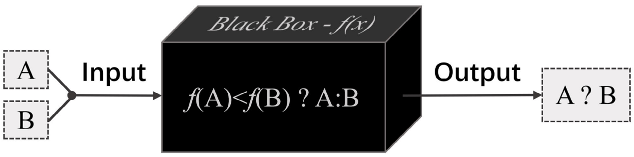


FIGURE 3. A black box model in real-world application: when input *A* is better, the black-box function outputs A; otherwise outputs B. We can see that there is no fitness value available in the black box objectives and the fitness value based DE variants can not tackle this problem.

μ_{Cr} are presented in Eq. 20 and Eq. 21:

$$\begin{cases} w_k = \dfrac{\text{std}(\Delta Loc_k)}{\sum_{l=1}^{|S_F|} \text{std}(\Delta Loc_l)} \\ \Delta Loc_k = Loc(X_{k,g}) - Loc(U_{k,g}) \\ \text{mean}_{WL}\ (S_F) = \dfrac{\sum_{k=1}^{|S_F|} w_k \cdot S_{F,k}^2}{\sum_{k=1}^{|S_F|} w_k \cdot S_{F,k}} \\ \mu_{F,g+1} = \begin{cases} 0.5 \cdot (mean_{WL}\ (S_F) + \mu_{F,g}), & \text{if } S_F \neq \varnothing \\ \mu_{F,g}, & \text{otherwise} \end{cases} \end{cases} \tag{20}$$

$$\begin{cases} w_k = \dfrac{\text{std}(\Delta Loc_k)}{\sum_{l=1}^{|S_{Cr}|} \text{std}(\Delta Loc_l)} \\ \Delta Loc_k = Loc(X_{k,g}) - Loc(U_{k,g}) \\ \text{mean}_{WL}\ (S_{Cr}) = \dfrac{\sum_{k=1}^{|S_{Cr}|} w_k \cdot S_{Cr,k}^2}{\sum_{k=1}^{|S_{Cr}|} w_k \cdot S_{Cr,k}} \\ \mu_{Cr,idx,g+1} = \begin{cases} 0.5 \cdot (mean_{WL}\ (S_{Cr}) + \mu_{Cr,g}), \\ \text{if } S_{Cr} \neq \varnothing \& \max\{S_{Cr}\} > 0 \\ 0, \\ \text{if } S_{Cr} \neq \varnothing \& \mu_{Cr_{idx},g} = 0 \\ \mu_{Cr,g+1}, \\ \text{otherwise} \end{cases} \end{cases} \tag{21}$$

where S_F denotes the set of the corresponding scale factor F of success individuals, S_{Cr} denotes the set of the corresponding crossover rate of Cr of success individuals. Both $|S_F|$ and $|S_{Cr}|$ denotes the size of the set S_F and set S_{Cr}. ΔLoc_k denotes the location differences, i.e. the coordinates differences, of the kth individual in the success individual set. std$(\cdot)$ denotes the operation that calculate the standard deviation of all different variables of the location differences. mean$_{WL}(S)$ is the weighted Lehmer mean of the success set S. And the parameter μ_F is the mean of the Cauchy distribution obeyed by the scale factor F, $F \sim C(\mu_F, 0.1)$, while the parameter μ_{Cr} is the mean of the Normal distribution obeyed by the crossover rate Cr, $Cr \sim N(\mu_{Cr}, 0.1)$. Furthermore, if the generated F_i and Cr_i values is out the range (0,1], it should be adjust according to Eq. 22 and Eq. 23.

$$F_i = \begin{cases} C_i\,(\mu_F, \sigma_F)\,, & \text{while } F_i \leq 0 \\ 1, & \text{if } F_i > 1 \\ F_i, & \text{otherwise} \end{cases} \tag{22}$$

$$Cr_i = \begin{cases} 0, & \text{while } \mu_{Cr} \leq 0\tilde{o}r\ \ Cr_i < 0 \\ 1, & \text{if } Cr_i > 1 \\ Cr_i, & \text{otherwise} \end{cases} \tag{23}$$

D. POPULATION SIZE REDUCTION STRATEGY

A novel platform based linear population size reduction strategy is also proposed in the PaDE-NPC algorithm. Population size reduction strategy [47] has been shown to be powerful in improving DE performance. However, the quick reduction of population size at the beginning of the evolution usually lead to bad perception of the landscape of most objective functions. Therefore, in our PaDE-NPC algorithm, the population at the early stage of the evolution is kept fixed and the linear reduction begins when the current number of function evaluation is bigger than $plat \cdot 100\%$ maximum number of function evaluation, i.e. $nfe > plat \cdot nfe_{max}$. The detailed equation of the novel population size reduction scheme is presented in Eq. 24:

$$ps_{g+1} = \lfloor \frac{nfe - plat \cdot nfe_{max}}{(1 - plat) \cdot nfe_{max}} \cdot (ps_{min} - ps_{max}) + ps_{ini} \rfloor \tag{24}$$

where ps_{ini} and ps_{min} denote the initial and minimum value of population size, nfe_{max} and nfe denote the maximum number of function evaluation and current number of function evaluation respectively, $floor[\cdot]$ denotes the operation that rounding down to the nearest value. The pseudo code of the PaDE-NPC is presented in Algorithm 1.

V. EXPERIMENT ANALYSIS

Generally, it is difficult to determine how "good" an optimization algorithm is because of the lack of related theoretical knowledge, and benchmark functions play a very important role in performance evaluations of different evolutionary algorithms. Thus, here in this paper, 58 benchmark functions in CEC2013 [48] and CEC2017 [49] test suit are employed in verifying the proposed PaDE-NPC algorithm. Functions in CEC2013 test suit fall into three major types: there are five uni-modal functions (f_1–f_5), fifteen basic multi-modal functions ($f_6 - f_{20}$) and eight composition functions ($f_{21} - f_{28}$); Functions in CEC2017 test suit can be categorized into four major types: there are three uni-modal functions ($f'_1 - f'_3$), thirteen basic multi-modal functions ($f'_4 - f'_{16}$), five hybrid functions ($f'_{17} - f'_{22}$) and eight composition functions ($f'_{23} - f'_{30}$). In our conducted experiments, the maximum number of function evaluations (nfe_{max}) for these compared algorithm are all set to $10000 * D$, where D is the dimensions of the benchmark function. Moreover, the mean and standard deviation of fitness errors $\Delta f = f_i - f_i^*$ of these benchmark functions are collected, compared, and analyzed in the conducted experiment. The overall performance of a certain algorithm under 51-run is evaluated under Wilcoxon Signed Rank test with the significant level $\alpha = 0.05$ in comparison with our PaDE-NPC algorithm on a PC with Intel(R) Core(TM) i7-9700k CPU @ 3.6GHz on Windows 10 Professional Edition Operating System with 16GB of RAM, and all these algorithms are implemented in Matlab

Algorithm 1 Pseudo Code of PaDE-NPC

Input: Bound constraints $[R^D_{min}, R^D_{max}]$, the fixed maximum number of function evaluations nfe_{max};
Output: Best individual X_{gbest}, Best fitness value $f_{X_{gbest}}$, number of function evaluations $nfes$;
Initialize the population size $ps = ps_{ini}$, all individuals $X = X_1, X_2, \ldots, X_{ps}$, $k = 4$, $A = \varnothing$, $\mu_F = 0.3$, $\mu_{Cr_1} = \mu_{Cr_2} = \cdots = \mu_{Cr_k} = 0.8$, $p = 0.11$, $r^{arc} = 1$, $r^{hrc} = 3$, $P(1) = P(2) = \cdots = P(k) = \frac{1}{k}$, $G = 1$;
while $nfe \leq nfe_{max}$ **do**
  for $i = 1; j \leq ps; i++$ **do**
    Generate $X^p_{best,g}$, $X_{r_1,g}$, $\widetilde{X}_{r_2,g}$ and $\widetilde{X}_{r_3,g}$;
  end for
  if G > 2 **then**
    Adjust the individuals of the population;
    Adjust storage A and B according to Eq. 16 ;
  end if
  Categorize ps individuals into k groups by stochastic universal selection in ALgorithm 1;
  for $j = 1; j \leq k; j++$ **do**
    Generate F and Cr of individuals in the jth group: $F \sim C(\mu_F, 0.1)$, $Cr \sim N(\mu_{Cr_j}, 0.1)$;
    Readjust F and Cr into the bound constrains if necessary;
  end for
  for $I = 1; i \leq ps; i++$ **do**
    Generate $X_{r_1,g}$, $X_{r_2,g}$ and $X_{r_3,g}$;
    Generate donor vector $V_{i,g}$ and trial vector $U_{i,g}$;
    Calculate fitness value $f(U_{i,g})$;
  end for
  $nfe = nfe + ps$;
  for $i = 1; i \leq ps; i++$ **do**
    if $f(U_{i,g}) \leq f(X_{i,g})$ **then**
      $X_{i,g+1} = U_{i,g}$;
    else
      $X_{i,g+1} = X_{i,g}$;
    end if
  end for
  if $S_F \neq \varnothing$ **then**
    Update μ_F according to Eq. 20;
    Update $P(\cdot)$ according to Eq. 19 ;
    Update $\mu_{Cr_{idx}}$ according to Eq. 21 ;
  end if
  $G++$;
  Update archive A and B;
  Label $X_{gbest,g}$ and the corresponding $f(X_{gbest,g})$;
  Adjust population size according to Eq. 24 if necessary;
end while
$f(X_{gbest}) = f(X_{gbest,g})$, $X_{gbest} = X_{gbest,g}$;
return $f(X_{gbest})$, (X_{gbest}) and nfe;

2019a Unix version. The fitness errors that smaller than $eps = 2.2204e - 16$ are regarded as 0 here.

A. PARAMETER SETTINGS

Five recently proposed powerful DE variants including LSHADE, iLSHADE, jSO, LPALMDE and HARD-DE are examined in comparison with the novel PaDE-NPC algorithm. These DE variants all have close relationship with our algorithm not only from the mutation strategy perspective but also from the parameter control. The settings of these algorithms in the paper are the default ones recommended by their authors, and they are summarized in Table 1.

In SHADE algorithm, scale factor F and crossover rate obey Cauchy distribution $F \sim C(\mu_F, 0.1)$ and Normal distribution $Cr \sim N(\mu_{Cr}, 0.1)$ respectively. $\mu_F = 0.5$ and $\mu_{Cr} = 0.5$ are employed as the initial values at the beginning of evolution, and they can be dynamically updated during the evolution. A linear population size reduction scheme is incorporated into LSHADE with the initial size $ps_{ini} = 15 \cdot D$ and terminal size $ps_{min} = 4$. Moreover, a H-entry memory pool is employed to store the historical value of μ_F and μ_{Cr}, and $H = 6$ is the default value in LSHADE. The ratio of top superior solutions $p = 0.11$ and the factor of external archive $r^{arc} = 2.6$ is employed in the mutation strategy. In iLSHADE algorithm, the same distribution of control parameter F and Cr are utilized, but the initial values of μ_F and μ_{Cr} are different to and LSHADE, $\mu_F = 0.8$ and $\mu_{Cr} = 0.5$. The population size also decreased from $ps_{init} = 12 \cdot D$ to $ps_{min} = 4$ in a linear way during the evolution. Unlike LSHADE, the parameter p in iLSHADE dynamically decreased from 0.2 to 0.1. For jSO, the distribution of control parameter μ_F, μ_{Cr} and the archive factor r^{arc} are also the same as LSHADE while the values of μ_F and μ_{Cr} are initialized to 0.3 and 0.8. Moreover, a different decreasing interval of p is employed, $p \in [0.25, 0.125]$. The historical pool size setting is $H = 5$ and population size decreases from $ps_{init} = 25 \cdot \ln D \cdot \sqrt{D}$ to 4 in jSO. In LPALMDE algorithm, control parameter F and Cr obey the same distribution with the same initial μ_f and μ_{Cr} values as LSHADE. Group number K equals to 8, and population size is dynamically decreased from $ps_{init} = 23 \cdot D$ to $ps_{min} = K$. The time stamp $T_0 = 70$ and the ratio of top superior individuals $p = 0.11$ are the default values in LPALMDE algorithm. In HARD-DE algorithm, scale factor F and crossover rate Cr obey the same distribution as LSHADE while the initial value of μ_F and μ_{Cr} are set to $\mu_F = 0.3$ and $\mu_{Cr} = 0.8$. Moreover, the parameter p employs the same value, $p = 0.11$, as LSHADE. The setting of initial population size ps_{init} is the same as jSO, $ps_{init} = 25 \ln(D) \cdot \sqrt{D}$, and a smaller number of groups is employed in HARD-DE algorithm, $K = 4$. For the factor of external archives, $r^{arc} = 1$ and $r^{hrc} = 3$ is employed. In our PaDE-NPC algorithm, control parameter F and Cr also obey semi-fixed distributions, $F \sim randc(\mu_F, 0.1)$, $Cr \sim randc(\mu_{Cr}, 0.1)$ and the initial values of μ_F and μ_{Cr} are the same as HARD-DE algorithm. A fixed $p = 0.11$ defining the ratio of top superior individuals is employed in PaDE-NPC, and the factor $r^{arc} = 1.0$ and $r^{hrc} = 3.0$ are the default values here. The number of groups

TABLE 1. Recommended parameter settings of all these contrasted algorithms.

Algorithms.	Parameter settings of the algorithms
LSHADE	$\mu_F = 0.5$, $F \sim C(\mu_F, 0.1)$, $\mu_{CR} = 0.5$, $CR \sim N(\mu_{CR}, 0.1)$, $PS = 18 \cdot D \sim 4$, $p = 0.11$, $r^{arc} = 2.6$, $H = 6$
iLSHADE	H, F, CR & r^{arc} same as LSHADE, $\mu_F = 0.5$, $\mu_{CR} = 0.8$, $\mu_{F_H} = \mu_{F_{CR}} = 0.9$, $PS = 12 \cdot D \sim 4$, $p = 0.2 \sim 0.1$
jSO	F, CR & r^{arc} same as iLSHADE, $\mu_F = 0.3$, $\mu_{CR} = 0.8$, $H = 5$, $PS = 25ln(D)\sqrt{D} \sim 4$, $p = 0.25 \sim 0.125$
LPalmDE	$F_j = 0.8$, $F_{j,i} \sim C(F_j, 0.1)$, $\mu_{CR} = 0.6$, $CR \sim N(\mu_{CR}, 0.1)$, $K = 20$, $PS = 23 \cdot D \sim 4$, $p = 0.11$, $r^{rac} = 1.6$, $T_0 = 70$
HARD-DE	$\mu_F = 0.3$, $F \sim C(\mu_F, 0.1)$, $\mu_{CR} = 0.8$, $CR \sim N(\mu_{CR}, 0.1)$, $K = 4$, $p = 0.11$, $PS = 25 \cdot ln(D)\sqrt{D} \sim 4$, $r^{arc} = 3.0$
PaDE-NPC	F & Cr & p same as LSHADE, $\mu_F = 0.3$, $\mu_{Cr} = 0.8$, $r^{arc} = 1.0$, $r^{hrc} = 3.0$, $ps = 25ln(D)\sqrt{D} \sim 4$, $plat = 0.15$, $DM = 2/3$

TABLE 2. Mean and standard deviation (Mean/Std) of fitness errors on 10D optimization over 51 runs are presented here, and the symbols behind Mean/Std denotes the results in comparison with the proposed PaDE-NPC algorithm under Wilcoxon signed rank test on CEC2013 test bed.

10D	LSHADE	iLSHADE	jSO	LPALMDE	HARD-DE	EBLSHADE	PaDE-NPC
f_1	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_2	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_3	1.9091E-001/9.7404E-001(<)	1.1193E-002/2.6209E-002(<)	*1.3992E-003/9.9919E-003*(>)	*1.3992E-003/9.9919E-003*(>)	5.5967E-003/1.9375E-002(<)	4.1975E-003/1.6957E-002(≈)	4.1975E-003/1.6957E-002
f_4	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_5	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_6	5.1948E+000/4.9465E+000(<)	6.1568E+000/4.7914E+000(<)	1.3468E+000/3.4102E+000(<)	2.1164E+000/4.0760E+000(<)	**0/0**(>)	3.4632E+000/4.7359E+000(<)	5.7720E-001/2.3318E+000
f_7	1.4148E-005/2.3149E-005(<)	1.4712E-005/4.1995E-005(<)	3.4153E-005/1.0479E-004(<)	1.1676E-005/3.8326E-005(<)	3.3804E-003/6.5293E-003(<)	1.4373E-005/4.6400E-005(<)	**8.6353E-006/1.5061E-005**
f_8	2.0230E+001/1.5088E-001(<)	2.0338E+001/9.3426E-002(<)	2.0358E+001/8.3752E-002(<)	**2.0186E+001/1.037E-001**(>)	2.0208E+001/1.2112E-001(<)	2.0215E+001/1.6319E-001(<)	2.0199E+001/1.4439E-001
f_9	2.3294E+000/1.6767E+000(<)	5.8850E-001/8.7988E-001(>)	7.0117E-001/8.6243E-001(>)	**5.5580E-001/6.7395E-001**(>)	2.5237E+000/1.1282E+000(<)	2.3807E+000/1.4357E+000(<)	9.3667E-001/1.2262E+000
f_{10}	1.0334E-002/1.2181E-002(>)	6.0844E-003/8.2879E-003(>)	1.7401E-003/3.8597E-003(>)	1.1630E-002/1.6007E-002(<)	**5.4299E-006/3.8271E-005**(>)	5.8582E-003/8.6119E-003(>)	1.1102E-002/1.2286E-002
f_{11}	*0/0*(≈)	1.9509E-002/1.3932E-001(<)	*0/0*(≈)	*0/0*(≈)	7.8020E-015/1.9755E-014(<)	*0/0*(≈)	*0/0*
f_{12}	2.1393E+000/7.9528E-001(>)	2.0906E+000/7.7796E-001(>)	2.3801E+000/8.2236E-001(>)	2.3411E+000/1.0867E+000(>)	2.1331E+000/8.6644E-001(>)	**1.8411E+000/8.0162E-001**(>)	2.6590E+000/1.2955E+000
f_{13}	2.0873E+000/1.1803E+000(<)	1.7814E+000/8.5561E-001(<)	2.1201E+000/1.0770E+000(<)	2.1391E+000/1.3736E+000(<)	2.5298E+000/9.7643E-001(<)	1.8253E+000/9.3845E-001(<)	**1.5803E+000/9.7655E-001**
f_{14}	2.8166E-002/5.0451E-002(<)	2.8030E-001/6.4621E-001(<)	3.6075E-002/4.3136E-002(<)	1.2246E-003/8.7454E-003(>)	**6.5983E-013/4.0992E-013**(>)	2.8166E-002/4.0114E-002(<)	1.4695E-002/2.6756E-002
f_{15}	3.0419E+002/1.1924E+002(>)	**2.5327E+002/1.1714E+002**(>)	2.8348E+002/1.1125E+002(>)	3.6513E+002/1.4990E+002(<)	3.6552E+002/1.5347E+002(<)	3.1869E+002/1.0631E+002(>)	3.2822E+002/1.4604E+002
f_{16}	2.9314E-001/1.6262E-001(>)	8.3311E-001/3.0579E-001(<)	1.0942E+000/2.0347E-001(<)	**2.1236E-001/1.8834E-001**(>)	2.9639E-001/1.6617E-001(>)	2.8683E-001/1.6458E-001(>)	3.8283E-001/3.6107E-001
f_{17}	*1.0122E+001/7.9877E-015*(≈)	1.0126E+001/6.6650E-003(<)	1.0123E+001/8.5962E-004(<)	*1.0122E+001/1.1223E-014*(≈)	*1.0122E+001/1.7940E-015*(≈)	*1.0122E+001/2.1894E-014*(≈)	*1.0122E+001/1.8501E-014*
f_{18}	1.3860E+001/1.2415E+000(>)	**1.3379E+001/1.2681E+000**(>)	1.6434E+001/1.9716E+000(<)	1.5154E+001/2.2040E+000(<)	1.5253E+001/1.7140E+000(<)	1.3707E+001/1.3475E+000(>)	1.5065E+001/2.4200E+000
f_{19}	**2.2556E-001/3.2446E-002**(>)	3.1047E-001/5.9523E-002(<)	2.7388E-001/4.9680E-002(<)	2.7388E-001/9.7294E-002(<)	2.4165E-001/5.5838E-002(<)	2.3504E-001/3.6690E-002(<)	2.3115E-001/3.8759E-002
f_{20}	1.9943E+000/3.8533E-001(<)	1.8281E+000/5.2566E-001(<)	1.7248E+000/3.1470E-001(<)	1.6703E+000/3.2317E-001(<)	1.8424E+000/3.7433E-001(<)	1.9209E+000/3.5648E-001(<)	**1.5004E+000/2.3152E-001**
f_{21}	4.0019E+002/0(≈)	4.0019E+002/0(≈)	3.9627E+002/2.8033E+001(>)	4.0019E+002/0(≈)	**3.8842E+002/4.7573E+001**(>)	3.9627E+002/2.8033E+001(>)	4.0019E+002/0
f_{22}	1.1607E+001/2.3918E+001(<)	2.2267E+001/3.2619E+001(<)	6.5426E+000/4.8560E+000(<)	3.8005E+000/3.8893E+000(>)	**3.4106E+000/3.8802E+000**(>)	1.2081E+001/2.3752E+001(<)	5.0878E+000/5.3489E+000
f_{23}	2.8705E+002/1.5536E+002(<)	2.2308E+002/1.1810E+002(<)	2.2153E+002/1.1252E+002(<)	2.9356E+002/1.6600E+002(<)	3.3960E+002/1.7191E+002(<)	2.5224E+002/1.2496E+002(<)	**2.1968E+002/1.4159E+002**
f_{24}	2.0074E+002/1.4231E+001(<)	2.0109E+002/1.1946E+001(<)	1.9992E+002/1.1073E+001(<)	2.0000E+002/0(<)	2.0000E+002/9.4284E-004(<)	2.0221E+002/3.1558E+000(<)	**1.9618E+002/1.9111E+001**
f_{25}	1.9963E+002/1.3963E+001(<)	2.0054E+002/1.7517E+000(<)	2.0009E+002/6.3574E-001(<)	2.0000E+002/3.2155E-014(<)	**1.9813E+002/1.3267E+001**(>)	1.9931E+002/7.5837E+000(>)	1.9818E+002/1.2970E+001
f_{26}	1.6054E+002/4.7715E+001(<)	1.2384E+002/4.0550E+001(<)	**1.0258E+002/1.8040E+000**(>)	1.1060E+002/2.6445E+001(<)	1.0278E+002/1.1268E+000(>)	1.3769E+002/4.6833E+001(<)	1.0678E+002/1.9091E+001
f_{27}	*3.0000E+002/0*(≈)	3.1415E+002/4.9002E+001(<)	*3.0000E+002/0*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/0*
f_{28}	2.9216E+002/3.9208E+001(≈)	3.0000E+002/0(<)	3.0000E+002/0(<)	3.0000E+002/0(<)	3.0000E+002/0(<)	**2.8824E+002/4.7527E+001**(>)	2.9216E+002/3.9208E+001
w/d/l	6/9/13	5/5/18	7/6/15	7/8/13	9/6/13	9/7/12	-/-/-

TABLE 3. Mean and standard deviation (Mean/Std) of fitness errors on 30D optimization over 51 runs are presented here, and the symbols behind Mean/Std denotes the results in comparison with the proposed PaDE-NPC algorithm under Wilcoxon signed rank test on CEC2013 test bed.

30D	LSHADE	iLSHADE	jSO	LPALMDE	HARD-DE	EBLSHADE	PaDE-NPC
f_1	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_2	2.8399E-012/1.2034E-011(<)	2.0823E-009/1.0525E-008(<)	3.7115E-010/7.7174E-010(<)	5.0379E-013/5.3093E-013(<)	2.3362E-012/2.3555E-012(<)	8.3816E-013/1.0900E-012(<)	**3.7896E-013/2.5991E-13**
f_3	2.4652E-001/1.4341E+000(<)	5.3028E-001/2.7556E-002(<)	2.3793E-009/1.5886E-008(<)	1.4576E-002/9.5931E-002(<)	2.2960E-012/3.8844E-012(<)	4.5933E-005/2.1043E-004(<)	**6.0633E-013/2.8405E-012**
f_4	**8.4708E-014/1.1103E-013**(>)	3.9679E-013/2.5255E-013(<)	2.5457E-012/1.7191E-012(<)	1.1592E-013/1.1480E-013(>)	2.9425E-013/1.5283E-013(<)	1.7833E-013/1.3112E-013(<)	2.1846E-013/1.0129E-013
f_5	*1.1369E-013/0*(≈)	*1.1369E-013/0*(≈)	*1.1369E-013/0*(≈)	*1.1369E-013/0*(≈)	*1.1369E-013/0*(≈)	*1.1369E-013/0*(≈)	*1.1369E-013/0*
f_6	5.4436E-012/3.6491E-011(<)	1.0356E+000/5.1769E+000(<)	1.0356E+000/5.1769E+000(<)	4.4895E-012/2.8476E-011(<)	1.1049E-009/4.2715E-009(<)	4.3605E-011/1.7827E-010(<)	**1.6942E-013/6.9610E-014**
f_7	1.7157E-001/3.1288E-001(<)	2.8039E-001/2.2748E-001(<)	4.9068E-002/1.5007E-001(<)	1.4780E-001/2.3292E-001(<)	4.3935E-002/1.0781E-001(<)	4.4311E-001/2.7015E-001(<)	**9.7875E-003/2.5433E-002**
f_8	2.0852E+001/1.2136E-001(<)	2.0833E+001/1.0683E-001(<)	2.0957E+001/4.8471E-002(<)	2.0816E+001/1.2389E-001(>)	2.0833E+001/1.2659E-001(<)	**2.0804E+001/1.5493E-001**(>)	2.0825E+002/1.2327E-001
f_9	2.6226E+001/1.4189E+000(<)	**2.0158E+001/4.9504E+000**(>)	2.3704E+001/2.6676E+000(>)	2.0447E+001/3.4790E+000(>)	2.5028E+001/2.5150E+000(>)	2.6560E+001/1.6467E+000(<)	2.5688E+001/1.6852E+000
f_{10}	1.0635E-003/2.7123E-003(<)	*0/0*(≈)	*0/0*(≈)	9.6659E-004/2.7479E-003(<)	*0/0*(≈)	1.7395E-003/3.8897E-003(<)	*0/0*
f_{11}	6.4645E-014/2.5471E-014(<)	6.5760E-014/2.0878E-014(<)	1.6496E-013/7.3021E-014(<)	**3.3437E-014/3.0455E-014**(>)	1.6161E-013/3.9989E-014(<)	6.9104E-014/2.8566E-014(<)	5.2385E-014/1.5434E-014
f_{12}	6.1472E+000/1.4810E+000(>)	7.0237E+000/2.0623E+000(<)	9.0538E+000/2.4992E+000(<)	9.9324E+000/2.6199E+000(<)	1.1127E+001/1.6733E+000(<)	**5.4673E+000/1.5879E+000**(>)	6.5062E+000/1.8251E+000
f_{13}	6.9643E+000/2.9226E+000(>)	1.0136E+001/5.7137E+000(<)	1.0715E+001/5.2616E+000(<)	1.2793E+001/5.6339E+000(<)	1.9049E+001/5.9231E+000(<)	**5.4401E+000/2.6080E+000**(>)	7.8386E+000/2.9310E+000
f_{14}	3.2727E-002/2.2168E-002(>)	4.6537E-002/2.9907E-002(>)	8.4381E+000/4.4500E+000(>)	9.3891E-003/1.3372E-002(>)	**8.5726E-003/1.3283E-002**(>)	3.1841E-002/2.7098E-002(<)	1.9941E+001/7.8542E+001
f_{15}	2.6208E+003/2.9263E+002(>)	**2.5137E+003/2.7530E+002**(>)	2.6816E+003/3.3684E+002(>)	3.0182E+003/4.5995E+002(<)	2.8906E+003/2.8315E+002(>)	2.6655E+003/2.6516E+002(>)	2.9075E+003/3.3899E+002
f_{16}	7.0875E-001/2.0045E-001(>)	9.2240E-001/4.3183E-001(>)	2.3393E+000/3.0407E-001(<)	5.7656E-001/2.7061E-001(>)	7.3832E-001/4.4400E-001(>)	7.6585E-001/1.6331E-001(<)	1.0946E+000/7.1704E-001
f_{17}	*3.0434E+001/9.4299E-007*(≈)	*3.0434E+001/2.1913E-006*(≈)	3.0669E+001/1.0979E-001(<)	*3.0434E+001/1.8285E-006*(≈)	*3.0434E+001/4.4578E-006*(≈)	*3.0434E+001/9.4299E-007*(≈)	*3.0434E+001/2.1913E-006*
f_{18}	5.2234E+001/2.8773E+000(>)	**4.4237E+001/4.0750E+000**(>)	5.6911E+001/6.2991E+000(>)	4.4979E+001/5.3281E+000(>)	6.1596E+001/4.8775E+000(<)	5.1855E+001/2.1847E+000(>)	5.7775E+001/5.5980E+000
f_{19}	1.1755E+000/8.9966E-002(<)	**1.0496E+000/1.3757E-001**(>)	1.2898E+000/1.0311E-001(<)	1.1806E+000/1.4382E-001(<)	1.1775E+000/9.3495E-002(<)	1.1620E+000/9.3623E-002(<)	1.1159E+000/9.6361E-002
f_{20}	1.1820E+001/2.3304E+000(<)	1.0761E+001/1.5341E+000(<)	9.7472E+000/3.8117E-001(<)	9.2179E+000/4.3779E+000(<)	9.7054E+000/4.4241E-001(<)	1.0825E+001/2.0697E+000(<)	**9.1158E+000/3.6179E-001**
f_{21}	3.0171E+002/3.4946E+001(<)	3.0316E+002/6.2148E+001(<)	3.0708E+002/5.8493E+001(<)	2.9412E+002/2.3764E+001(>)	2.9949E+002/5.3368E+001(>)	**2.9020E+002/3.0033E+001**(>)	2.9975E+002/3.7738E+001
f_{22}	1.0688E+002/9.8060E-001(<)	1.0676E+002/1.1981E+000(<)	1.1976E+002/3.4337E+000(<)	1.0696E+002/2.5209E-001(<)	**1.0601E+002/6.2829E-001**(>)	1.0835E+002/2.3572E+000(<)	1.0619E+002/8.3259E-001
f_{23}	2.6094E+003/2.9320E+002(>)	**2.1826E+003/4.0411E+002**(>)	2.3659E+003/3.3190E+002(>)	2.6700E+003/4.4407E+002(>)	2.9878E+003/2.9060E+002(<)	2.4831E+003/2.8163E+002(>)	2.7828E+003/3.6969E+002
f_{24}	2.0002E+002/3.6200E-002(<)	2.0069E+002/4.1575E+000(<)	2.0003E+002/3.2709E-002(<)	2.0003E+002/6.5237E-002(<)	2.0001E+002/7.2523E-003(<)	2.0010E+002/1.4004E+000(<)	**2.0000E+002/4.3059E-003**
f_{25}	2.2410E+002/2.2390E+001(<)	2.4000E+002/4.6140E+000(<)	2.3924E+002/6.8915E+000(<)	2.3029E+002/2.1074E+001(<)	**2.0677E+002/1.5845E+001**(>)	2.4024E+002/7.4889E+000(<)	2.0698E+002/1.5266E+001
f_{26}	*2.0000E+002/1.4352E-013*(≈)	2.0784E+002/2.7153E+001(<)	*2.0000E+002/0*(≈)	*2.0000E+002/1.5055E-013*(≈)	*2.0000E+002/0*(≈)	*2.0000E+002/1.3422E-013*(≈)	*2.0000E+002/1.4262E-013*
f_{27}	3.0069E+002/1.1959E+000(<)	3.0104E+002/2.6188E+000(<)	3.0083E+002/9.8159E-001(<)	3.0034E+002/7.7094E-001(<)	**3.0015E+002/1.4219E-001**(>)	3.0210E+002/3.3980E+000(<)	3.0019E+002/2.9047E-001
f_{28}	*3.0000E+002/2.1254E-013*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/6.4311E-013*(≈)	*3.0000E+002/2.7047E-013*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/1.8801E-013*
w/d/l	8/5/15	7/5/16	5/5/18	9/5/14	8/6/14	9/4/15	-/-/-

and the initial population size in PaDE-NPC are same as HARD-DE.

B. OPTIMIZATION ACCURACY AND CONVERGENCE SPEED

We first examine the optimization performance under CEC2013 test suite on 10D, 30D, 50D, 100D optimization, and the results are presented in Table 2–Table 4 respectively. Symbols ''>'', ''≈'' and ''<'' in the parentheses behind the mean/std mean ''Better Performance'', ''Similar Performance'' and ''Worse Performance'' of the total 51-run under Wilcoxon signed rank test with the significant level $\alpha = 0.05$.

For Table 2 on 10D optimization, we can find that all these DE variants perform equally well on f_1, f_2, f_4 and f_5 which are all unimodal functions, and the global optima of these functions can also be found by each independent run. LSHADE, jSO, LPALMDE and PaDE-NPC can always find the global optima on function f_{11} each run. The proposed PaDE-NPC algorithm outperforms these DE variants on function f_7, f_{13}, f_{20}, f_{23} and f_{24}. Moreover, it obtains 22 better

TABLE 4. Mean and standard deviation (Mean/Std) of fitness errors on 50D optimization over 51 runs are presented here, and the symbols behind Mean/Std denotes the results in comparison with the proposed PaDE-NPC algorithm under Wilcoxon signed rank test on CEC2013 test bed.

50D	LSHADE	iLSHADE	jSO	LPALMDE	HARD-DE	EBLSHADE	PaDE-NPC
f_1	2.2292E-014/4.8286E-014(<)	4.4583E-014/9.1172E-014(<)	2.6750E-014/7.3986E-014(<)	6.2416E-014/1.0248E-013(<)	1.1146E-013/1.1480E-013(<)	8.9166E-014/1.1212E-013(<)	**4.4583E-015/3.1839E-014**
f_2	1.4036E+003/1.6680E+003(<)	3.4366E+003/3.9672E+003(<)	2.2197E+002/1.6572E+002(<)	3.5139E+003/6.7612E+003(<)	1.9177E+002/6.0687E+002(<)	1.1567E+003/1.1866E+003(<)	**7.0408E+001/1.9402E+002**
f_3	6.6993E+004/1.7655E+005(<)	4.9184E+003/1.7221E+004(<)	1.3171E+002/8.0003E+002(<)	4.2645E+003/1.1343E+004(<)	1.8254E+003/1.0164E+004(<)	1.3048E+003/3.2175E+003(<)	**1.2424E+001/4.6480E+001**
f_4	**3.8029E-011/8.0764E-011**(>)	1.6106E-009/3.6941E-009(>)	4.7411E-009/7.7628E-009(<)	2.7489E-010/7.2919E-010(>)	1.3238E-009/3.1395E-009(>)	3.3826E-009/5.8063-009(>)	3.5394E-009/7.1460E-009
f_5	1.8279E-013/5.6058E-014(<)	1.7164E-013/6.1737E-014(<)	2.0954E-013/4.7545E-014(<)	1.5827E-013/536058E-014(<)	1.3821E-013/4.7224E-014(<)	1.2929E-013/3.9511E-014(<)	**1.2260E-013/3.0869E-014**
f_6	*4.3447E+001/0*(≈)	*4.3447E+001/0*(≈)	*4.3447E+001/0*(≈)	*4.3447E+001/0*(≈)	*4.3447E+001/0*(≈)	*4.3447E+001/1.6078E-014*(≈)	*4.3447E+001/0*
f_7	1.6569E+000/1.2973E+000(<)	4.5763E-001/4.7477E-001(<)	1.4960E-001/1.2100E-001(<)	1.5297E+000/1.3176E+000(<)	2.7876E-001/3.3636E-001(<)	2.3761E+000/1.2198E+000(<)	**1.2917E-001/1.5098E-001**
f_8	2.1091E+001/8.5035E-002(<)	2.1062E+001/7.5472E-002(<)	2.1123E+001/4.4397E-002(<)	2.1068E+001/9.7926E-002(<)	**2.1030E+001/1.1946E-001**(>)	2.1041E+001/1.3079E-001(>)	2.1060E+001/9.654E-002
f_9	5.3319E+001/1.9029E+000(<)	**4.0393E+001/8.6307E+000**(>)	4.7942E+001/5.2101E+000(>)	4.0614E+001/6.8815E+000(>)	4.6825E+001/1.8323E+000(>)	5.3198E+001/2.0848E+000(<)	5.0399E+001/4.2998E+000
f_{10}	6.0887E-003/7.2157E-003(<)	**4.1078E-003/4.8959E-003**(>)	4.5698E-014/2.2793E-014(>)	1.4635E-002/1.0288E-002(<)	4.4938E-003/5.8245E-003(>)	1.6992E-002/1.3927E-002(<)	4.7838E-003/5.0832E-003
f_{11}	8.3785E-011/9.8714E-011(<)	8.4373E-013/6.2676E-013(>)	5.9902E-009/1.9493E-008(<)	**1.0254E-013/5.9095E-014**(>)	3.5221E-013/4.8260E-014(>)	1.3035E-010/1.9697E-010(<)	1.5036E-012/1.3949E-012
f_{12}	**1.2194E+001/2.4613E+000**(>)	1.2747E+001/2.9177E+000(>)	1.5065E+001/3.4463E+000(>)	2.2041E+001/5.3198E+000(<)	2.5027E+001/3.4523E+000(<)	1.4521E+001/2.2107E+000(>)	1.6883E+001/2.5767E+000
f_{13}	**1.6538E+001/6.5736E+000**(>)	1.8351E+001/8.3932E+000(>)	1.8860E+001/1.0259E+001(>)	4.5578E+001/1.5756E+001(<)	5.9910E+001/1.0352E+001(<)	2.0477E+001/8.4196E+000(>)	2.9300E+001/1.0218E+001
f_{14}	2.2459E-001/4.8029E-002(<)	2.5666E-001/6.8396E-002(<)	5.9348E+001/1.4388E+001(<)	1.2339E-001/4.6738E-002(<)	**4.0487E-002/1.8009E-002**(>)	2.5178E-001/5.9578E-002(<)	9.6221E-002/2.8945E-002
f_{15}	6.2702E+003/3.6560E+002(>)	**5.3857E+003/5.6631E+002**(>)	5.9500E+003/5.6668E+002(>)	6.0556E+003/5.9295E+002(>)	6.5896E+003/3.8730E+002(>)	6.3873E+003/3.8202E+002(>)	6.7313E+003/4.0129E+002
f_{16}	1.2660E+000/1.5780E-001(>)	1.3932E+000/6.1530E-001(>)	3.1480E+000/3.3212E-001(<)	**1.0047E+000/3.9696E-001**(>)	1.1268E+000/5.1483E-001(>)	1.2154E+000/1.8229E-001(>)	1.5891E+000/8.5024E-001
f_{17}	**5.0708E+001/2.0207E-003**(>)	5.0787E+001/2.1396E-003(<)	5.2434E+001/4.6550E-001(<)	5.0786E+001/1.1487E-010(≈)	5.0786E+001/1.1487E-010(≈)	5.0788E+001/5.6820E-003(<)	5.0786E+001/6.7595E-005
f_{18}	1.0623E+002/6.4447E+000(>)	8.4367E+001/8.8521E+000(>)	1.1175E+002/8.8481E+000(>)	**7.6622E+001/8.9696E+000**(>)	1.1937E+002/9.2856E+000(<)	1.1304E+002/5.8414E+000(<)	1.1233E+002/7.6272E+000
f_{19}	2.5057E+000/1.7443E-001(<)	**2.3122E+000/2.0796E-001**(>)	2.6998E+000/1.4630E-001(<)	2.3411E+000/2.6241E-001(>)	2.4163E+000/1.5435E-001(>)	2.5088E+000/1.4256E-001(<)	2.5029E+000/1.6997E-001
f_{20}	1.8545E+001/5.3900E-001(<)	1.8579E+001/4.4491E-001(<)	1.9189E+001/4.2809E-001(<)	1.7857E+001/5.6649E-001(<)	1.8629E+001/3.2887E-001(<)	1.8195E+001/6.1586E-001(<)	**1.7770E+001/4.8209E-001**
f_{21}	8.5095E+002/4.2437E+002(<)	7.9671E+002/4.4508E+002(<)	6.8389E+002/4.4920E+002(<)	9.7753E+002/3.3871E+002(<)	8.4663E+002/4.0890E+002(<)	6.8822E+002/4.6488E+002(<)	**5.6292E+002/4.4285E+002**
f_{22}	1.3566E+001/1.4444E+000(<)	1.3775E+001/1.4722E+000(<)	5.9697E+001/1.2125E+001(<)	**1.1458E+001/8.5004E-001**(>)	1.1583E+001/6.0933E-001(>)	1.3975E+001/1.5177E+000(<)	1.2083E+001/1.1023E+000
f_{23}	5.3110E+003/4.4341E+002(>)	**4.7427E+003/5.5371E+002**(>)	5.0592E+003/6.1531E+002(>)	5.8268E+003/5.5792E+002(>)	6.6357E+003/5.1804E+002(<)	5.6396E+003/4.2677E+002(>)	6.4798E+003/5.0351E+002
f_{24}	2.1139E+002/7.3310E+000(<)	2.0943E+002/1.3038E+001(<)	2.0453E+002/1.0014E+001(<)	2.0576E+002/3.9215E+000(<)	2.0039E+002/7.6989E-001(<)	2.0977E+002/4.5837E+000(<)	2.0017E+002/2.3045E-001
f_{25}	2.7692E+002/5.5531E+000(>)	2.7591E+002/6.8752E+000(>)	**2.7417E+002/7.7462E+000**(>)	2.8806E+002/8.1324E+000(<)	2.8358E+002/7.3162E+000(<)	2.7881E+002/7.0066E+000(>)	2.8208E+002/6.3855E+000
f_{26}	2.4876E+002/5.2279E+001(<)	2.9033E+002/3.3377E+001(<)	2.3174E+002/4.7404E+001(<)	2.7407E+002/5.0668E+001(<)	2.3860E+002/5.0620E+001(<)	2.5663E+002/5.3964E+001(<)	**2.1616E+002/3.7850E+001**
f_{27}	4.5605E+002/9.9901E+001(<)	4.9832E+002/1.4892E+002(<)	4.4705E+002/1.4816E+002(<)	3.6689E+002/3.7037E+001(<)	3.1654E+002/1.5274E+001(<)	4.0696E+002/4.5157E+001(<)	**3.1133E+002/1.1240E+001**
f_{28}	*4.0000E+002/2.8433E-013*(≈)	*4.0000E+002/2.8705E-013*(≈)	*4.0000E+002/2.8705E-013*(≈)	*4.0000E+002/2.8159E-013*(≈)	*4.0000E+002/2.8159E-013*(≈)	*4.0000E+002/2.5862E-013*(≈)	*4.0000E+002/2.8433E-013*
w/d/l	9/2/17	12/2/14	8/2/18	9/3/16	10/3/15	8/2/18	-/-/-

TABLE 5. Mean and standard deviation (Mean/Std) of fitness errors on 100D optimization over 51 runs are presented here, and the symbols behind Mean/Std denotes the results in comparison with the proposed PaDE-NPC algorithm under Wilcoxon signed rank test on CEC2013 test bed.

100D	LSHADE	iLSHADE	jSO	LPALMDE	HARD-DE	EBLSHADE	PaDE-NPC
f_1	*2.2737E-013/0*(≈)	*2.2737E-013/0*(≈)	*2.2737E-013/0*(≈)	*2.2737E-013/0*(≈)	*2.2737E-013/0*(≈)	*2.2737E-013/0*(≈)	*2.2737E-013/0*
f_2	1.5915E+005/3.5492E+004(<)	1.6230E+005/5.5430E+004(<)	**1.4092E+005/2.6555E+004**(>)	1.4809E+005/3.5693E+004(>)	1.4412E+005/4.3301E+004(>)	1.7021E+005/4.9454E+004(<)	1.5899E+005/5.8480E+004
f_3	2.4740E+006/2.7368E+006(<)	8.4378E+005/1.1534E+006(<)	6.5655E+005/7.6617E+005(<)	2.9123E+006/2.9632E+006(<)	8.1556E+005/1.0231E+006(<)	1.7159E+006/2.9453E+006(<)	**6.3525E+005/9.7446E+005**
f_4	1.3144E-004/1.2223E-004(>)	1.1497E-004/1.3134E-004(>)	1.2555E-004/1.3208E-004(>)	**1.0949E-004/8.2789E-005**(>)	5.8949E-004/4.7110E-004(>)	4.3609E-004/3.5078E-004(>)	2.8327E-003/2.7561E-003
f_5	4.2800E-013/6.6633E-014(<)	3.4552E-013/4.5251E-014(<)	3.6335E-013/6.4390E-014(<)	4.0125E-013/7.3160E-014(<)	4.2577E-013/7.1336E-014(<)	3.4552E-013/7.1761E-014(<)	**3.1654E-013/5.2413E-014**
f_6	1.5230E+002/4.4164E+001(<)	1.7877E+002/5.7628E+001(<)	1.9312E+002/5.3028E+001(<)	1.5321E+002/4.6052E+001(<)	1.5368E+002/5.4437E+001(<)	1.5906E+002/3.7840E+001(<)	**1.1201E+002/6.2226E+001**
f_7	5.3555E+000/1.2973E+000(<)	2.8821E+000/1.2602E+000(>)	**1.7559E+000/6.7735E-001**(>)	8.3934E+000/2.4188E+000(<)	3.6956E+000/1.3676E+000(<)	6.9493E+000/1.4882E+000(<)	3.2591E+000/1.3463E+000
f_8	2.1276E+001/5.3753E-002(<)	2.1297E+001/3.9187E-002(<)	2.1307E+001/2.5600E-002(<)	2.1274E+001/4.4710E-002(<)	**2.1222E+001/8.1758E-002**(>)	2.1275E+001/3.8110E-002(<)	2.1263E+001/7.1563E-002
f_9	1.3213E+002/2.7386E+000(<)	1.3368E+002/4.1812E+000(<)	1.3235E+002/6.1672E+000(<)	1.3290E+002/8.1515E+000(<)	1.3205E+002/3.0456E+000(<)	1.3268E+002/2.6578E+000(<)	**1.3148E+002/2.9714E+000**
f_{10}	1.3135E-002/1.1909E-002(>)	1.1154E-002/1.2709E-002(>)	**1.1719E-003/7.7418E-003**(>)	1.5213E-002/1.2926E-002(<)	1.2700E-002/1.1462E-002(>)	1.5789E-002/1.4844E-002(<)	1.4148E-002/1.5119E-002
f_{11}	1.3988E-003/1.3822E-003(<)	1.6738E+001/5.2606E+000(<)	2.3879E+001/4.4362E+000(<)	**4.4774E-011/1.8709E-010**(>)	1.3121E-008/1.5065E-008(>)	1.9134E-003/9.5017E-004(<)	1.5365E-006/1.4960E-006
f_{12}	6.7399E+001/9.1807E+000(<)	**3.1818E+001/7.4559E+000**(>)	3.2230E+001/5.4639E+000(>)	6.7760E+001/9.2569E+000(<)	7.7765E+001/8.1774E+000(<)	6.3986E+001/1.1522E+001(<)	5.8402E+001/7.1947E+000
f_{13}	1.6813E+002/2.5723E+001(<)	1.1040E+002/3.3915E+001(>)	**9.6341E+001/3.2549E+001**(>)	1.8403E+002/3.6376E+001(<)	2.0421E+002/2.4844E+001(<)	1.6834E+002/2.3516E+001(<)	1.5516E+002/2.3595E+001
f_{14}	6.6834E+001/1.4451E+001(>)	7.9223E+002/2.1458E+002(<)	1.7967E+003/3.0239E+002(<)	1.9721E+001/7.8511E+000(>)	**4.7542E+000/1.5592E+000**(>)	9.3465E+001/1.7525E+001(<)	5.2203E+002/1.2861E+003
f_{15}	1.5563E+004/5.2107E+002(>)	1.5815E+004/7.8933E+002(<)	1.6534E+004/1.3502E+003(<)	**1.3359E+004/1.0501E+003**(>)	1.5176E+004/7.0949E+002(>)	1.5700E+004/7.5855E+002(<)	1.5624E+004/6.9641E+002
f_{16}	1.8687E+000/1.4262E-001(>)	2.7980E+000/5.4015E-001(<)	3.2698E+000/2.9087E-001(<)	**1.5660E+000/5.3405E-001**(>)	1.6884E+000/4.6056E-001(>)	1.9014E+000/1.8553E-001(>)	2.1506E+000/9.7929E-001
f_{17}	1.0308E+002/3.2867E-001(<)	1.1422E+002/2.6401E+000(<)	1.2547E+002/3.4342E+000(<)	**1.0164E+002/4.9669E-002**(>)	1.0183E+002/5.2957E-002(<)	1.0335E+002/3.7163E-001(<)	1.0174E+002/7.0045E-002
f_{18}	2.8483E+002/1.2600E+001(<)	3.0870E+002/1.6835E+001(<)	3.1694E+002/2.0990E+001(<)	**1.8225E+002/3.4030E+001**(>)	2.9667E+002/1.5671E+001(<)	2.8169E+002/1.3524E+001(>)	2.8301E+002/1.3711E+001
f_{19}	7.3629E+000/3.2179E-001(<)	9.3938E+000/6.3598E-001(<)	9.8075E+000/6.8103E-001(<)	**6.3794E+000/6.3782E-001**(>)	6.4936E+000/2.9162E-001(>)	7.3577E+000/3.3361E-001(<)	6.8235E+000/3.3436E-001
f_{20}	5.0000E+001/2.6888E-004(<)	**4.9981E+001/9.5521E-002**(>)	4.9998E+001/1.1750E-002(<)	4.9996E+001/1.3567E-001(<)	4.9990E+001/6.8653E-002(≈)	4.9998E+001/1.3372E-002(<)	4.9990E+001/6.8641E-002
f_{21}	4.0000E+002/4.6236E-013(<)	3.9216E+002/2.7152E+001(<)	3.9608E+002/1.9604E+001(<)	3.8824E+002/3.2540E+001(<)	3.9412E+002/2.3764E+001(<)	**3.7255E+002/4.5071E+001**(>)	3.8235E+002/3.8501E+001
f_{22}	8.9697E+001/1.2589E+001(<)	4.9526E+002/1.5372E+002(<)	9.1456E+002/2.3059E+002(<)	3.5359E+001/7.0663E+000(>)	**2.3477E+001/1.7610E+000**(>)	1.1814E+002/2.3505E+001(<)	4.5629E+001/3.4214E+001
f_{23}	1.4826E+004/8.0158E+002(>)	1.4199E+004/6.8510E+002(>)	1.4276E+004/1.1573E+003(>)	**1.3465E+004/1.2285E+003**(>)	1.6132E+004/8.9004E+002(<)	1.5194E+004/6.9824E+002(>)	1.6105E+004/7.7255E+002
f_{24}	2.2697E+002/5.0431E+000(<)	2.1573E+002/5.8576E+000(<)	2.1527E+002/4.6198E+000(<)	2.4234E+002/7.9357E+000(<)	2.1677E+002/5.2718E+000(<)	2.3822E+002/7.0418E+000(<)	2.1326E+002/4.4954E+000
f_{25}	4.0960E+002/1.2457E+001(<)	3.8880E+002/1.0774E+001(>)	**3.8709E+002/1.1132E+001**(>)	4.1440E+002/1.5370E+001(<)	4.0289E+002/1.3065E+001(<)	3.9650E+002/1.0551E+001(>)	3.9931E+002/2.6917E+001
f_{26}	3.4605E+002/5.8923E+000(>)	3.1646E+002/4.3866E+000(>)	**3.1012E+002/4.0047E+000**(>)	3.4840E+002/6.9427E+000(>)	3.5059E+002/8.0966E+000(<)	3.4942E+002/4.7967E+000(<)	3.4908E+002/7.3333E+000
f_{27}	6.0614E+002/5.0009E+001(<)	4.5439E+002/6.4277E+001(>)	**3.8412E+002/3.5153E+001**(>)	7.7349E+002/8.7988E+001(<)	5.2240E+002/6.2301E+001(<)	7.4238E+002/9.3334E+001(<)	5.2133E+002/6.7756E+001
f_{28}	**2.5165E+004/1.9653E+001**(>)	2.7994E+003/7.5734E+002(>)	2.7007E+003/6.7077E+002(>)	2.8055E+003/7.2045E+002(>)	2.7803E+003/7.2319E+002(>)	3.1086E+003/8.5749E+002(<)	2.8538E+004/8.0170E+002
w/d/l	8/1/19	11/1/16	11/1/16	13/1/14	11/2/15	6/1/21	-/-/-

or similar performance improvements out of 28 benchmark functions in comparison with LSHADE algorithm; it obtains 23 better or similar performance improvements out of 28 benchmarks in contrast with iLSHADE algorithm; obtains 21 better or similar performance improvements in contrast with jSO algorithm; obtains 21 better or similar performance improvements in comparison with LPALMDE; obtains 19 better or similar performance improvements in comparison with HARD-DE algorithm. To sum up, the novel PaDE-NPC algorithm secures overall better performance on 10D optimization under the tested benchmark functions.

For Table 3 on 30D optimization, we can find that all these DE variants perform equally well on f_1, f_5 and f_{28}, and the global optima of f_1 can be found during each run. Furthermore, the proposed PaDE-NPC algorithm achieves 20 better or similar performance improvements out of 28 benchmark function in contrast with LSHADE algorithm; it also achieves 21 better or similar performance improvements in contrast with iLSHADE algorithm; achieves 23 better or similar performance improvements in contrast with jSO algorithm, 19 better or similar performance improvements in contrast with LPALMDE algorithm, 20 better or similar performance improvements in contrasted with HARD-DE algorithm. In general, the proposed PaDE-NPC algorithm reveals overall much better performance on 30D optimization under the tested benchmark functions.

For Table 4 on 50D optimization, we can find that the proposed PaDE-NPC algorithm outperforms the other DE variants on $f_1 - f_3$, f_5, f_7, f_{20}, f_{21}, f_{26} and f_{27}, and the novel PaDE-NPC reveals 19 better or similar performance improvements in contrast with LSHADE algorithm, it also reveals 16 better or similar performance improvements in contrast with iLSHADE algorithm; reveals 20 better or similar performance improvements in contrast with jSO algorithm, 19 better or similar performance improvements in contrast with LPALMDE algorithm, 18 better or similar

TABLE 6. Mean and standard deviation (Mean/Std) of fitness errors on 10D optimization over 51 runs are presented here, and the symbols behind Mean/Std denotes the results in comparison with the proposed PaDE-NPC algorithm under Wilcoxon signed rank test on CEC2017 test bed.

10D	LSHADE	iLSHADE	jSO	LPALMDE	HARD-DE	EBLSHADE	PaDE-NPC
f_1'	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*	*0/0*
f_2'	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*	*0/0*
f_3'	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*	*0/0*
f_4'	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*	*0/0*
f_5'	2.4605E+000/8.2820E-001(>)	**1.9711E+000/8.3282E-001**(>)	2.1660E+000/9.8948E-001(>)	2.3216E+000/1.0654E+000(>)	2.3061E+000/7.0440E-001(>)	2.5966E+000/8.9162E-001(<)	2.5437E+000/1.6553E+000
f_6'	4.4583E-015/2.2287E-014(>)	*0/0*(>)	1.3375E-014/3.993E-014(>)	1.7833E-014/4.1756E-014(>)	2.0062E-014/4.3771E-014(≈)	*0/0*(>)	2.0062E-014/4.3771E-014
f_7'	1.2034E+001/7.0465E-001(>)	**1.1997E+001/5.8191E-001**(>)	1.2589E+001/7.7781E-001(<)	1.2792E+001/1.3159E+000(<)	1.2573E+001/7.8338E-001(<)	1.2230E+001/8.3266E-001	1.2496E+001/1.3130E+000
f_8'	2.3231E+000/7.3572E-001(<)	**1.9320E+000/7.2914E-001**(>)	2.5167E+000/7.5387E-001(<)	2.5752E+000/1.0749E+000(<)	2.8925E+000/1.1128E+000(<)	2.5574E+000/1.0195E+000(<)	2.2309E+000/1.2155E+000
f_9'	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_{10}'	**3.9726E+001/5.2782E+001**(>)	4.4008E+001/6.4195E+001(>)	8.5905E+001/1.0381E+002(<)	6.0081E+001/6.7817E+001(<)	4.2145E+001/5.2571E+001(>)	1.9951E+001/3.2810E+001(>)	5.6587E+001/8.2649E+001
f_{11}'	3.3296E-001/6.3656E-001(<)	7.2577E-002/5.1830E-001(<)	4.1908E-013/1.6549E-012(>)	**0/0**(>)	1.6514E+000/5.0293E-001(<)	2.6125E-001/5.2322E-001(<)	1.2325E-002/8.8019E-002
f_{12}'	3.0628E+001/5.2461E+001(<)	5.7164E+001/5.9695E+001(<)	7.6218E+000/2.8438E+001(<)	3.2991E+001/5.3743E+001(<)	2.1349E+001/4.5929E+001(<)	2.2932E+001/4.7810E+001(<)	**2.8569E-001/1.7642E-001**
f_{13}'	1.7109E+000/2.2716E+000(>)	3.4599E+000/2.7185E+000(<)	1.3989E+000/2.0892E+000(>)	1.8653E+000/2.4115E+000(>)	**1.1766E+000/1.7414E+000**(>)	3.9645E+000/2.1062E+000(<)	1.9870E+000/2.4366E+000
f_{14}'	2.5361E-001/4.5440E-001(>)	7.8444E-001/3.9211E+000(<)	**8.1603E-002/2.7030E-001**(>)	1.3656E-001/3.4579E-001(>)	2.7741E-001/3.7433E-001(>)	4.9894E-001/5.2103E-001(<)	4.6154E-001/7.5000E-001
f_{15}'	1.7760E-001/2.0050E-001(<)	2.7469E-001/2.0760E-001(<)	3.1955E-001/2.0114E-001(<)	1.3893E-001/1.9930E-001(<)	2.1824E-001/1.3237E-001(<)	**1.1409E-001/1.7746E-001**(>)	1.2481E-001/1.8502E-001
f_{16}'	38554E-001/1.8179E-001(>)	3.4985E-001/2.4186E-001(>)	6.4697E-001/3.3068E-001(<)	**2.3555E-001/1.8324E-001**(>)	4.5039E-001/2.0038E-001(<)	3.5177E-001/2.0576E-001(>)	4.3193E-001/1.6510E-001
f_{17}'	**1.3290E-001/1.5179E-001**(>)	1.4125E+000/4.7569E+000(<)	4.2828E+000/7.6945E+000(<)	1.9363E-001/3.0306E-001(<)	3.1078E-001/2.2623E-001(<)	1.2449E-001/1.6038E-001(>)	1.5259E-001/1.8801E-001
f_{18}'	2.2008E-001/2.0193E-001(<)	2.5016E-001/1.9343E-001(<)	3.6028E-001/1.9595E-001(<)	2.6642E-001/2.1470E-001(<)	2.9795E-001/1.9912E-001(<)	1.6741E-001/1.8873E-001(<)	**1.4460E-001/1.8587E-001**
f_{19}'	8.3787E-003/1.0307E-002(>)	1.3548E-002/2.6891E-002(<)	1.2917E-002/1.7122E-002(<)	9.9239E-003/1.1942E-002(>)	2.8761E-002/9.7825E-003(<)	**8.1990E-003/1.0022E-002**(>)	1.0954E-002/1.0960E-002
f_{20}'	*0/0*(≈)	3.8973E-001/2.7833E+000(<)	1.3892E+000/4.7377E+000(<)	*0/0*(≈)	2.7507E-008/1.9644E-007(<)	6.1210E-003/4.3713E-002(<)	*0/0*
f_{21}'	1.4529E+002/4.8777E+001(<)	1.8053E+002/4.2654E+001(<)	1.5072E+002/5.2239E+001(<)	1.4281E+002/5.1680E+001(<)	**1.2379E+002/4.2859E+001**(>)	1.6643E+002/2.0671E+002(<)	1.3653E+002/4.9958E+001
f_{22}'	1.0000E+002/8.9223E-014(>)	1.0005E+002/1.2759E-001(≈)	1.0001E+002/4.0101E-002(>)	1.0000E+002/1.4838E-013(>)	**9.8039E+001/1.4003E+001**(>)	1.0005E+002/1.3112E-001(≈)	1.0006E+002/1.4011E-001
f_{23}'	3.0171E+002/1.6178E+000(<)	3.0051E+002/1.1209E+000(>)	**2.9479E+002/4.2126E+001**(>)	3.0135E+002/1.7951E+000(<)	3.0268E+002/1.4888E+000(<)	3.0274E+002/1.6706E+000(<)	3.0075E+002/1.2969E+000
f_{24}'	2.7794E+002/9.2961E+001(<)	2.8207E+002/9.0922E+001(<)	2.8393E+002/9.1751E+001(<)	2.8646E+002/8.1744E+001(<)	2.6641E+002/9.8827E+001(<)	2.9422E+002/8.4614E+001(<)	**2.5979E+002/1.0426E+002**
f_{25}'	4.1305E+002/2.1634E+001(<)	4.2036E+002/2.3061E+001(<)	4.0417E+002/1.5787E+001(>)	4.1395E+002/2.1926E+001(<)	**4.0414E+002/1.5787E+001**(>)	4.1410E+002/2.2029E+001(<)	4.0772E+002/1.8867E+001
f_{26}'	*3.0000E+002/0*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/0*(≈)	*3.0000E+002/0*
f_{27}'	3.9288E+002/1.8896E+000(<)	3.9266E+002/2.0809E+000(<)	3.9093E+002/2.0505E+000(>)	3.9313E+002/1.6469E+000(<)	3.9200E+002/2.1395E+000(<)	**3.8938E+002/2.2504E-001**(>)	3.9146E+002/2.3941E+00
f_{28}'	3.0556E+002/3.9732E+001(<)	3.7398E+002/1.2788E+002(<)	3.4793E+002/1.0718E+002(<)	3.0556E+002/3.9732E+001(<)	*3.0000E+002/0*(≈)	3.5534E+002/1.1659E+002(<)	*3.0000E+002/0*
f_{29}'	2.3478E+002/3.7851E+000(<)	2.3462E+002/3.9792E+000(<)	2.3803E+002/4.0478E+000(<)	**2.3200E+002/2.7401E+000**(>)	2.3943E+002/3.9195E+000(<)	2.3329E+002/2.2644E+000(<)	2.3314E+002/4.2694E+000
f_{30}'	3.9759E+002/1.1523E+001(<)	3.9687E+002/2.1541E+001(<)	*3.9451E+002/2.7928E-002*(>)	3.9946E+002/1.4869E+001(<)	*3.9451E+002/6.4970E-003*(>)	3.9642E+004/1.1443E+001(<)	3.9545E+002/6.7435E+000
w/d/l	10/7/13	7/7/16	10/6/14	9/7/15	8/8/14	8/7/15	-/-/-

TABLE 7. Mean and standard deviation (Mean/Std) of fitness errors on 30D optimization over 51 runs are presented here, and the symbols behind Mean/Std denotes the results in comparison with the proposed PaDE-NPC algorithm under Wilcoxon signed rank test on CEC2017 test bed.

30D	LSHADE	iLSHADE	jSO	LPALMDE	HARD-DE	EBLSHADE	PaDE-NPC
f_1'	*0/0*(>)	*0/0*(>)	1.9505E-015/4.9939E-015(<)	1.1146E-015/3.8586E-015(<)	6.6875E-015/7.1637E-015(<)	5.5729E-016/2.7859E-015(<)	2.7864E-016/1.9899E-015
f_2'	1.6719E-015/6.7540E-015(<)	9.4739E-015/1.5739E-014(<)	1.8948E-014/2.6459E-014(<)	2.7864E-015/8.5358E-015(<)	5.0156E-015/3.5819E-014(<)	3.9010E-015/9.8777E-015(<)	**5.5729E-016/3.9798E-015**
f_3'	**3.3437E-015/1.3508E-014**(>)	5.5729E-015/1.7072E-014(>)	3.3437E-014/2.8254E-014(<)	1.1146E-014/2.2793E-014(>)	1.5604E-014/2.5620E-014(<)	1.0031E-014/2.1885E-014(>)	1.2260E-014/2.3612E-014
f_4'	5.8351E+001/1.0183E+000(<)	5.7681E+001/8.3031E+000(<)	5.6265E+001/1.1480E+001(>)	**5.0138E+001/2.1924E+001**(>)	5.9215E+001/1.8078E+000(<)	5.8779E+001/1.0892E+000(<)	5.6810E+001/1.1711E+001
f_5'	**7.2931E+000/1.4709E+000**(>)	7.7579E+000/1.6820E+000(>)	9.5923E+000/2.5129E+000(<)	1.1492E+001/3.5611E+000(<)	1.2847E+001/1.8534E+000(<)	6.2333E+000/1.3830E+000(>)	8.1236E+000/1.3420E+000
f_6'	5.3677E-009/2.6833E-008(<)	1.2077E-008/5.0208E-008(<)	1.1247E-007/6.5329E-007(<)	7.8083E-008/5.5751E-007(<)	4.0946E-008/1.7106E-007(<)	*1.1369E-013/0*(≈)	*1.1369E-013/0*
f_7'	**3.7073E+001/1.3195E+000**(>)	3.7892E+001/1.4661E+000(>)	3.9084E+001/2.3758E+000(<)	4.1199E+001/3.1368E+000(<)	4.2453E+001/2.2784E+000(<)	3.7169E+001/1.1761E+000(>)	3.8637E+001/1.3210E+000
f_8'	8.0941E+000/1.4854E+000(>)	7.4480E+000/1.7729E+000(>)	9.4778E+000/2.8537E+000(<)	1.2700E+001/3.1374E+000(<)	1.3448E+001/2.3598E+000(<)	**7.0513E+000/1.4264E+000**(>)	9.1736E+000/1.5568E+000
f_9'	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_{10}'	**1.4373E+003/1.9107E+002**(>)	1.7368E+003/3.1329E+002(<)	1.6220E+003/4.0675E+002(>)	1.6101E+003/2.9307E+002(>)	1.4927E+003/2.2856E+002(>)	1.4807E+003/1.7409E+002(>)	1.6810E+003/2.2005E+002
f_{11}'	1.0251E+001/1.6235E+001(<)	1.4194E+001/2.2473E+001(<)	5.9899E+000/1.4077E+001(<)	9.5296E+000/1.6492E+001(<)	7.6263E+000/1.1703E+001(<)	2.3849E+001/2.7322E+001(<)	**3.4355E+000/1.8043E+000**
f_{12}'	1.0056E+003/4.1683E+002(<)	8.8011E+002/3.9093E+002(<)	**3.0842E+002/1.5982E+002**(>)	1.0802E+003/3.4218E+002(<)	5.2112E+002/2.7212E+002(<)	9.1251E+002/3.2942E+002(<)	3.3866E+002/2.0256E+002
f_{13}'	**1.3787E+001/5.8058E+000**(>)	1.8498E+001/8.4564E+000(<)	1.5875E+001/7.0381E+000(<)	1.5447E+001/6.7461E+000(<)	1.4985E+001/6.5261E+000(<)	1.5231E+001/6.5127E+000(<)	1.4374E+001/7.8372E+000
f_{14}'	2.1943E+001/2.8573E+000(>)	2.1786E+001/1.0796E+000(>)	2.1724E+001/1.0776E+000(>)	2.0486E+001/6.0206E+000(>)	2.2465E+001/4.3292E+000(>)	**2.0138E+001/5.9250E+000**(>)	2.3348E+001/1.2219E+000
f_{15}'	2.8371E+000/1.3450E+000(<)	3.6957E+000/1.9741E+000(<)	**1.3478E+000/9.9484E-001**(>)	3.0487E+000/1.8599E+000(<)	2.3090E+000/1.1690E+000(<)	2.6465E+000/1.4872E+000(<)	1.8498E+000/8.9666E-001
f_{16}'	1.0827E+002/9.5197E+001(>)	4.8754E+001/6.9117E+001(>)	7.0278E+001/8.3611E+001(>)	1.3114E+002/1.2189E+002(>)	1.8818E+002/8.5053E+001(<)	**3.3312E+001/1.9211E+001**(>)	1.4877E+002/1.0156E+002
f_{17}'	**3.1251E+001/5.5560E+000**(>)	3.8068E+001/5.2253E+000(<)	3.3132E+001/7.1891E+000(>)	3.3916E+001/1.2487E+001(>)	3.5672E+001/8.2123E+000(>)	3.3165E+001/4.4298E+000(>)	3.7166E+001/7.1560E+000
f_{18}'	2.2055E+001/1.0562E+000(<)	2.1435E+001/8.0543E-001(<)	2.0739E+001/2.3340E-001(<)	2.2496E+001/1.4841E+000(<)	2.0673E+001/4.6518E-001(<)	2.1013E+001/2.9269E+000(<)	**2.0482E+001/2.9100E+000**
f_{19}'	5.2897E+000/1.5221E+000(<)	8.5276E+000/1.9607E+000(<)	6.8840E+000/2.1626E+000(<)	5.3748E+000/1.9060E+000(<)	7.0429E+000/1.5852E+000(<)	5.2604E+000/1.6686E+000(<)	**4.2680E+000/1.2765E+000**
f_{20}'	3.9241E+001/7.7126E+000(<)	4.7966E+001/1.8459E+001(<)	4.7584E+001/1.0330E+001(<)	3.5851E+001/2.8843E+001(>)	4.5613E+001/1.7334E+001(<)	**3.2691E+001/5.9544E+000**(>)	3.9078E+001/8.2006E+000
f_{21}'	2.0752E+002/1.4667E+000(>)	2.0824E+001/1.6415E+000(>)	2.0977E+002/2.9848E+000(<)	2.1119E+002/2.9636E+000(<)	2.1276E+002/1.6281E+000(<)	**2.0680E+002/1.1470E+000**(>)	2.0843E+002/1.8333E+000
f_{22}'	*1.0000E+002/1.0047E-013*(≈)	*1.0000E+002/1.0047E-013*(≈)	*1.0000E+002/1.0047E-013*(≈)	*1.0000E+002/1.0047E-013*(≈)	*1.0000E+002/1.3713E-013*(≈)	*1.0000E+002/1.0047E-013*(≈)	*1.0000E+002/1.0047E-013*
f_{23}'	3.4489E+002/3.4048E+000(<)	3.5058E+002/4.4819E+000(<)	3.5299E+002/5.2180E+000(<)	3.4979E+002/4.7944E+000(<)	3.4844E+002/4.0376E+000(<)	3.4931E+002/2.8677E+000(<)	**3.4270E+002/3.7215E+000**
f_{24}'	4.2119E+002/2.1633E+000(<)	4.2531E+002/2.6884E+000(<)	4.2604E+002/3.6101E+000(<)	4.2279E+002/2.9262E+000(<)	4.2162E+002/3.3418E+000(<)	4.2539E+002/1.6970E+000(<)	**4.2066E+002/3.1876E+000**
f_{25}'	3.8675E+002/2.6782E-002(<)	3.8673E+002/2.3683E-002(≈)	**3.8670E+002/9.6970E-003**(>)	3.8677E+002/2.4526E-002(<)	3.8673E+002/2.1175E-002(≈)	3.8675E+002/2.6875E-002(<)	3.8673E+002/1.7944E-002
f_{26}'	8.8031E+002/3.0735E+001(<)	9.0822E+002/4.6641E+001(<)	9.0537E+002/4.4540E+001(<)	9.2519E+002/6.5520E+001(<)	9.1477E+002/4.4632E+001(<)	9.0213E+002/4.9548E+001(<)	**8.6834E+002/3.7640E+001**
f_{27}'	5.0545E+002/6.5924E+000(<)	5.0416E+002/6.8481E+000(<)	5.0242E+002/6.5970E+000(>)	5.0842E+002/6.0369E+000(<)	**4.9844E+002/7.2804E+000**(>)	5.0115E+002/6.1407E+000(>)	5.0309E+002/7.6204E+000
f_{28}'	3.1641E+002/3.8453E+001(<)	3.1886E+002/4.1194E+001(<)	3.1257E+002/3.4812E+001(>)	3.3970E+002/5.5071E+001(<)	**3.1096E+002/3.3615E+001**(>)	3.2219E+002/4.7158E+001(<)	3.1299E+002/3.5970E+001
f_{29}'	4.3257E+002/7.3701E+000(>)	4.4648E+002/1.0443E+001(<)	4.4357E+002/1.4053E+001(<)	4.4932E+002/9.6137E+000(<)	4.4093E+002/1.0124E+001(<)	**4.3230E+002/1.0218E+001**(>)	4.3912E+002/1.1401E+001
f_{30}'	2.0041E+003/5.2556E+001(<)	2.0296E+003/5.8781E+001(<)	**1.9871E+003/2.7501E+001**(>)	2.0723E+003/7.4082E+001(<)	1.9971E+003/3.3366E+001(>)	2.0008E+003/4.0182E+001(<)	1.9996E+003/3.7968E+001
w/d/l	12/2/16	8/3/20	11/2/17	7/2/21	6/3/21	12/3/15	-/-/-

performance improvements in contrasted with HARD-DE algorithm. In summary, the proposed PaDE-NPC algorithm obtains overall better performance on 50D optimization under the tested benchmark functions.

For Table 5 on 100D optimization, we can find that the proposed PaDE-NPC algorithm outperforms the other DE variants on f_1, f_3, f_5, f_6 and f_9, and the novel PaDE-NPC reveals 20 better or similar performance improvements in contrast with LSHADE algorithm, it also reveals 17 better or similar performance improvements in contrast with iLSHADE algorithm and jSO; reveals 15 better or similar performance improvements in contrast with LPALMDE algorithm, 17 better or similar performance improvements in contrast with HARD-DE algorithm. In summary, the proposed PaDE-NPC algorithm obtains overall better performance on 100D optimization under the tested benchmark functions.

We also examine our PaDE-NPC algorithm under the CEC2017 test suite in comparison with these state-of-the-art DE algorithms on 10D, 30D, 50D and 100D optimization. The comparison results are presented in Table 6, Table 7, Table 8 and Table 9 under the 51 independent runs with the maximum number of function evaluation equalling to

TABLE 8. Mean and standard deviation (Mean/Std) of fitness errors on 50D optimization over 51 runs are presented here, and the symbols behind Mean/Std denotes the results in comparison with the proposed PaDE-NPC algorithm under Wilcoxon signed rank test on CEC2017 test bed.

50D	LSHADE	iLSHADE	jSO	LPALMDE	HARD-DE	EBLSHADE	PaDE-NPC
f_1'	2.0898E-014/7.1637E-015(>)	**1.9226E-014/6.8587E-015**(>)	2.9815E-014/9.1016E-015(<)	2.2013E-014/7.1416E-015(<)	3.3159E-014/8.8367E-015(<)	2.0619E-014/7.6863E-015(>)	2.1734E-014/7.1637E-015
f_2'	1.3375E-013/2.0818E-013(<)	**4.4583E-014/3.1779E-014**(>)	2.7642E-012/1.8686E-011(<)	7.8020E-014/2.2864E-013(>)	6.4171E-001/4.5827E+000(<)	2.8422E-014/1.7053E-014(>)	9.6968E-014/5.1449E-013
f_3'	1.7722E-013/5.6441E-014(>)	1.6273E-013/5.2122E-014(>)	2.7307E-013/7.5428E-014(<)	**1.5270E-013/4.7545E-014**(>)	2.8310E-013/6.7727E-014(<)	1.8168E-013/5.9095E-014(>)	2.6750E-013/7.3056E-014
f_4'	8.2638E+001/4.3569E+001(>)	6.7712E+001/5.1010E+001(>)	7.7991E+001/5.1819E+001(>)	7.2917E+001/4.8941E+001(>)	7.0579E+001/4.5615E+001(>)	**6.7105E+001/4.6270E+001**(>)	8.3236E+001/4.3358E+001
f_5'	1.3305E+001/2.1177E+000(>)	1.2228E+001/3.0667E+000(>)	1.5600E+001/3.7475E+000(>)	2.2752E+001/4.2620E+000(<)	2.7215E+001/2.3598E+000(<)	**1.2189E+001/1.7523E+000**(>)	1.6774E+001/2.3404E+000
f_6'	9.9025E-008/2.8913E-007(<)	3.6035E-008/6.3691E-008(<)	3.7956E-007/6.4809E-007(<)	3.9656E-004/2.0272E-003(<)	1.7497E-007/2.8302E-007(<)	3.4305E-005/2.4056E-004(<)	**3.3926E-008/7.6633E-008**
f_7'	6.3941E+001/1.7283E+000(<)	6.6161E+001/2.0233E+000(<)	6.5593E+001/4.0246E+000(<)	7.1086E+001/4.2874E+000(<)	7.3628E+001/3.4112E+000(<)	6.5286E+001/1.6128E+000(<)	**6.3902E+001/1.7246E+000**
f_8'	1.3419E+001/2.0742E+000(>)	**1.2078E+001/2.4784E+000**(>)	1.4978E+001/3.5213E+000(>)	2.3705E+001/4.9813E+000(<)	2.5878E+001/3.3379E+000(<)	1.2535E+001/1.9141E+000(>)	1.7396E+001/2.6225E+000
f_9'	**2.8979E-014/5.0038E-014**(>)	1.7833E-014/4.1756E-014(>)	6.0187E-014/5.7310E-014(<)	1.7555E-003/1.2536E-002(<)	9.5854E-014/4.1756E-014(<)	1.7555E-003/1.2536E-002(<)	4.4583E-014/5.6058E-014
f_{10}'	**3.1112E+003/3.2073E+002**(>)	3.6085E+003/3.4604E+002(<)	4.0027E+003/7.3271E+002(<)	3.3905E+003/3.8563E+002(<)	3.1984E+003/2.8815E+002(>)	3.1913E+003/2.9140E+002(>)	3.3513E+003/2.6400E+002
f_{11}'	5.4847E+001/8.5566E+000(<)	3.6370E+001/6.5888E+000(<)	**2.5668E+001/3.4890E+000**(>)	7.0368E+001/1.2183E+001(<)	4.0976E+001/6.7729E+000(<)	4.5864E+001/8.5705E+000(<)	3.0351E+001/4.0150E+000
f_{12}'	2.2602E+003/5.5373E+002(<)	2.0529E+003/5.2684E+002(<)	2.0017E+003/4.9303E+002(<)	2.2057E+003/4.2758E+002(<)	1.9738E+003/4.8212E+002(<)	2.1269E+003/4.7645E+002(<)	**1.8680E+003/6.2302E+002**
f_{13}'	6.9853E+001/3.3457E+001(<)	5.2260E+001/2.6905E+001(<)	**3.2973E+001/1.7082E+001**(>)	7.2190E+001/3.2991E+001(<)	4.3437E+001/1.9865E+001(>)	5.1591E+001/2.7311E+001(<)	4.7394E+001/2.9452E+001
f_{14}'	3.1217E+001/3.8058E+000(<)	2.5625E+001/1.6382E+000(>)	**2.3545E+001/1.4663E+000**(>)	3.1219E+001/3.9469E+000(<)	3.0648E+001/3.0766E+000(<)	2.7794E+001/2.1940E+000(<)	2.6258E+001/2.1847E+000
f_{15}'	4.6395E+001/1.5035E+001(<)	2.9510E+001/6.2238E+000(<)	**2.3336E+001/3.2839E+000**(>)	4.9879E+001/1.6159E+001(<)	3.1060E+001/6.7132E+000(<)	3.3175E+001/6.7454E+000(<)	2.6399E+001/4.0029E+000
f_{16}'	3.8406E+002/1.1932E+002(>)	**2.9663E+002/1.1955E+002**(>)	4.3460E+002/1.5915E+002(>)	4.3267E+002/1.1646E+002(>)	4.2674E+002/1.1130E+002(>)	3.8033E+002/1.2941E+002(>)	4.4730E+002/1.2416E+002
f_{17}'	2.5879E+002/6.2217E+001(>)	2.4654E+002/8.7589E+001(>)	2.8770E+002/8.8839E+001(>)	3.2102E+002/1.1479E+002(>)	3.6945E+002/8.9655E+001(<)	**2.2391E+002/7.6225E+001**(>)	3.4495E+002/7.8644E+001
f_{18}'	5.0283E+001/1.7870E+001(<)	3.1908E+001/5.9069E+000(<)	**2.4434E+001/2.0326E+000**(>)	5.8167E+001/3.8742+001(<)	2.8642E+001/3.3892E+000(<)	3.0979E+001/7.0121E+000(<)	2.5788E+001/2.5734E+000
f_{19}'	3.4233E+001/1.1328E+001(<)	1.9326E+001/3.1872E+000(>)	**1.4134E+001/2.4317E+000**(>)	3.9049E+001/1.3702E+001(<)	2.0662E+001/3.7582E+000(<)	1.8481E+001/3.7625E+000(<)	1.4378E+001/2.5034E+000
f_{20}'	1.5728E+002/5.4246E+001(>)	**1.3378E+002/3.5771E+001**(>)	1.4051E+002/6.1084E+001(>)	1.5129E+002/1.0633E+002(>)	2.3284E+002/8.6946E+001(<)	1.6455E+002/6.1894E+001(>)	2.0697E+002/7.6256E+001
f_{21}'	2.1548E+002/2.4791E+000(>)	2.1431E+002/2.8921E+000(>)	2.1836E+002/4.6772E+000(<)	2.2283E+002/4.8226E+000(<)	2.2799E+002/3.2311E+000(<)	**2.1279E+002/2.6985E+000**(>)	2.1773E+002/2.6541E+000
f_{22}'	6.9168E+002/1.3627E+003(<)	1.1178E+003/1.7767E+003(<)	4.5121E+002/1.2167E+003(<)	6.9399E+002/1.3919E+003(<)	1.0317E+002/1.0685E+001(<)	2.3266E+003/1.7213E+003(<)	**1.0288E+002/1.7165E+001**
f_{23}'	4.2842E+002/5.5681E+000(<)	4.3981E+002/8.1671E+000(<)	4.3986E+002/9.5792E+000(<)	4.3432E+002/8.7671E+000(<)	4.2921E+002/5.9307E+000(<)	4.2839E+002/4.7177E+000(<)	**4.1914E+002/7.4600E+000**
f_{24}'	5.0496E+002/4.9801E+000(<)	5.5528E+002/5.6708E+000(<)	5.0673E+002/5.8449E+000(<)	5.0500E+002/5.1297E+000(<)	4.9861E+002/4.4250E+000(<)	5.0510E+002/2.9934E+000(<)	**4.9780E+002/5.2351E+000**
f_{25}'	4.8173E+002/5.7965E+000(>)	4.8358E+002/1.3066E+001(>)	**4.8072E+002/2.2707E+000**(>)	4.9948E+002/2.8711E+001(<)	4.9061E+002/1.8825E+001(>)	4.8359E+002/8.3221E+000(>)	4.9592E+002/2.4701E+001
f_{26}'	1.0664E+003/5.5955E+001(<)	1.0657E+003/7.6915E+001(<)	1.0793E+003/8.5906E+001(<)	1.1480E+003/6.7297E+001(<)	1.1420E+003/6.8646E+001(<)	1.1197E+003/4.9405E+001(<)	1.0613E+003/7.3774E+001
f_{27}'	5.4051E+002/1.0026E+001(<)	5.2718E+002/6.1334E+000(<)	**5.1806E+002/6.2981E+000**(>)	5.4281E+002/1.2234E+001(<)	5.2262E+002/1.0358E+001(>)	5.2493E+002/1.3924E+001(<)	5.2309E+002/8.0877E+000
f_{28}'	4.9524E+002/2.1499E+001(<)	4.7896E+002/2.4279E+001(<)	**4.6076E+002/9.5757E+000**(>)	4.9524E+002/2.1499E+001(<)	4.7226E+002/2.2015E+001(<)	4.7705E+002/2.3851E+001(<)	4.6651E+002/1.7941E+001
f_{29}'	**3.4710E+002/1.1296E+001**(>)	3.8113E+002/1.4711E+001(<)	3.9201E+002/1.9161E+001(<)	3.5293E+002/1.7288E+001(>)	3.7144E+002/1.2998E+001(<)	3.5069E+002/9.5484E+000(>)	3.6674E+002/1.3474E+001
f_{30}'	6.1686E+005/3.2219E+004(<)	6.1786E+005/3.5173E+004(<)	6.0574E+005/3.0529E+004(<)	6.2874E+005/3.2086E+004(<)	6.1055E+005/3.1183E+004(<)	6.4487E+005/5.6879E+004(<)	**6.0001E+005/2.8680E+004**
w/d/l	13/0/17	14/0/16	15/0/15	7/0/23	6/0/24	13/0/17	-/-/-

TABLE 9. Mean and standard deviation (Mean/Std) of fitness errors on 100D optimization over 51 runs are presented here, and the symbols behind Mean/Std denotes the results in comparison with the proposed PaDE-NPC algorithm under Wilcoxon signed rank test on CEC2017 test bed.

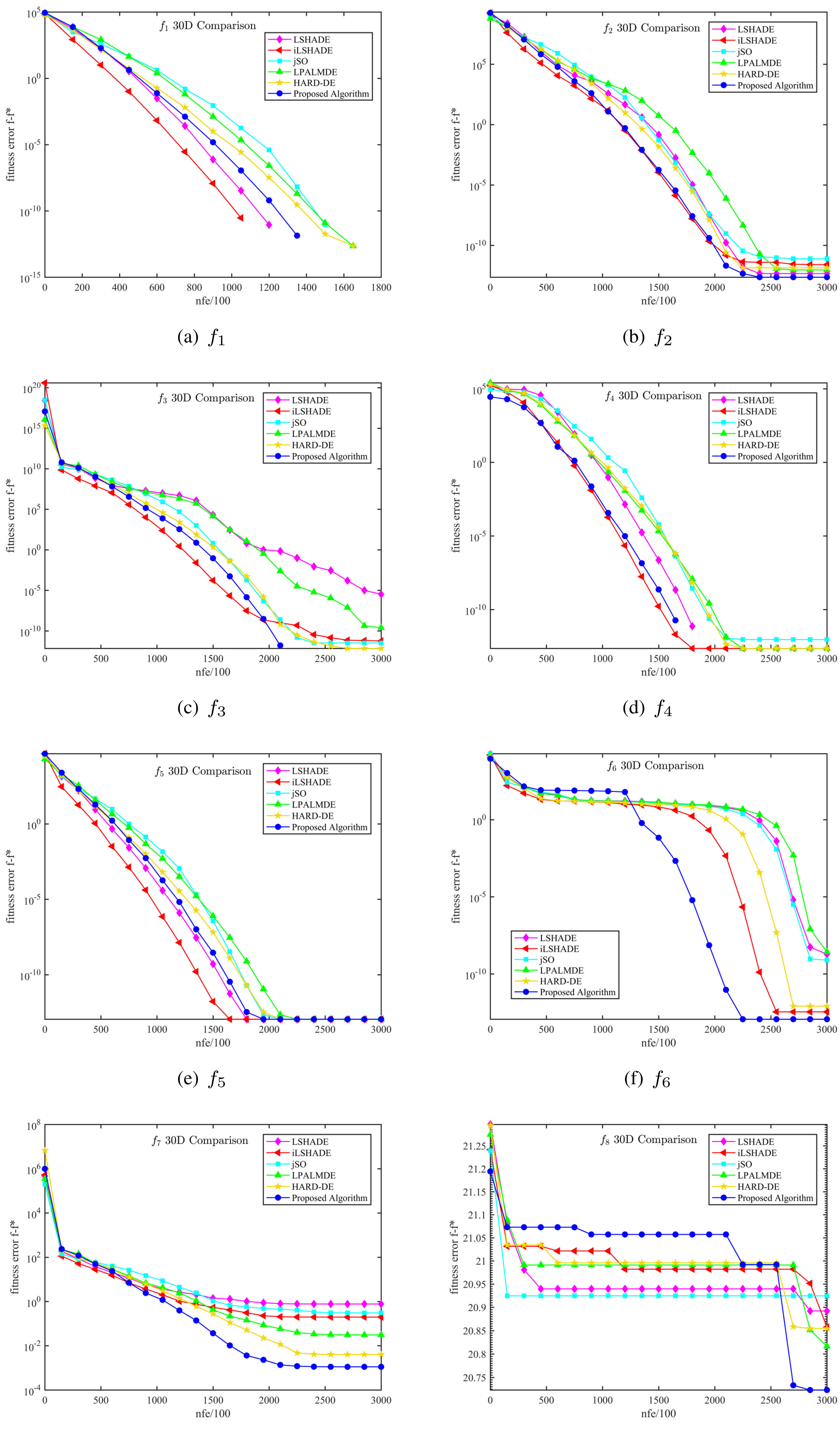
100D	LSHADE	iLSHADE	jSO	LPALMDE	HARD-DE	EBLSHADE	PaDE-NPC
f_1'	1.4211E-012/2.7082E-012(<)	1.5186E-013/835761E-014(>)	5.4809E-013/5.9679E-013(<)	**1.3542E-013/8.8032E-014**(>)	6.3856E-011/1.3280E-010(<)	4.2190E-011/1.4544E-010(<)	2.6443E-013/3.3932E-013
f_2'	1.1594E+004/8.2767E+004(<)	2.3715E-004/1.4302E-003(>)	2.2386E+002/1.5032E+003(<)	**7.0068E-008/2.9160E-007**(>)	1.3257E+004/9.1457E+004(<)	1.9506E+002/1.3825E+003(<)	3.7892E-001/2.7061E+000
f_3'	5.4005E-007/5.9670E-007(>)	**3.3446E-007/3.8462E-007**(>)	4.7528E-007/9.1321E-007(>)	1.4509E-007/2.4168E-007(>)	2.9761E-006/4.6367E-006(>)	2.0563E-006/2.5437E-006(>)	8.6898E-005/9.9677E-005
f_4'	1.2345E+002/7.0464E+001(<)	1.9279E+002/1.7334E+001(<)	1.9194E+002/1.4574E+001(<)	1.0820E+002/6.5443E+001(<)	1.5090E+002/5.4893E+001(<)	1.7141E+002/4.5627E+001(<)	**8.9445E+001/6.9318E+001**
f_5'	4.7457E+001/5.3652E+000(<)	**2.3040E+001/3.4025E+000**(>)	3.8948E+001/5.7517E+000(>)	5.4815E+001/7.8280E+000(<)	7.1422E+001/6.3527E+000(<)	4.0710E+001/6.8255E+000(>)	4.6537E+001/5.6288E+000
f_6'	6.9127E-003/5.6732E-003(<)	8.9942E-004/9.5757E-004(<)	**1.4874E-004/3.3555E-004**(>)	1.6898E-002/1.1637E-002(<)	1.5572E-003/1.7937E-003(<)	1.1532E-002/8.5668E-003(<)	8.1653E-004/1.1631E-003
f_7'	1.4247E+002/4.7725E+000(<)	**1.3393E+002/6.0520E+000**(>)	1.3831E+002/8.7469E+000(<)	1.5072E+002/8.0181E+000(<)	1.5675E+002/5.0006E+000(<)	1.4014E+002/4.1727E+000(<)	1.3691E+002/4.6284E+000
f_8'	4.7379E+001/4.5236E+000(>)	**2.5960E+001/4.0702E+000**(>)	3.6805E+001/5.5087E+000(>)	5.7337E+001/7.9098E+000(<)	7.1303E+001/5.9827E+000(<)	4.1783E+001/5.1211E+000(>)	4.7544E+001/5.0748E+000
f_9'	3.8662E-001/3.2520E-001(<)	1.2629E-001/2.3589E-001(<)	**3.5109E-003/1.7551E-002**(>)	9.8827E-001/7.0390E-001(<)	9.9092E-002/1.5838E-001(<)	7.1419E-001/5.7678E-001(<)	9.5842E-002/2.6803E-001
f_{10}'	1.0364E+004/5.3037E+002(<)	1.0845E+004/7.6545E+002(<)	1.1409E+004/8.9231E+002(<)	**9.3784E+003/7.8105E+002**(>)	9.8770E+003/4.9999E+002(>)	1.0299E+004/5.9054E+002(<)	1.0201E+004/4.2849E+002
f_{11}'	5.2336E+002/9.0464E+001(<)	2.5398E+002/6.2286E+001(<)	**1.1945E+002/2.6619E+001**(>)	6.2588E+002/9.0721E+001(<)	2.9301E+002/8.9063E+001(<)	4.3196E+002/1.1348E+002(<)	1.7902E+002/4.9694E+001
f_{12}'	2.0960E+004/7.1116E+003(<)	2.0551E+004/9.9623E+003(<)	1.8940E+004/7.7258E+003(<)	2.3015E+004/1.0628E+004(<)	1.8031E+004/6.9024E+003(<)	2.2231E+004/8.1301E+003(<)	**1.6251E+004/7.2153E+003**
f_{13}'	6.6225E+002/4.5017E+002(<)	3.0907E+002/8.3148E+001(<)	2.3685E+002/7.4725E+001(<)	1.2931E+003/7.8023E+002(<)	2.8155E+002/7.4996E+001(<)	2.3396E+002/6.0571E+001(<)	**1.6614E+002/5.4161E+001**
f_{14}'	2.4919E+002/3.2456E+001(<)	2.0179E+002/2.9469E+001(<)	7.8312E+001/1.6924E+001(<)	2.7512E+002/3.6641E+001(<)	1.7522E+002/3.0242E+001(<)	2.3193E+002/2.9265E+001(<)	**7.4801E+001/1.5087E+001**
f_{15}'	2.5424E+002/4.5844E+001(<)	2.6857E+002/5.3568E+001(<)	**2.1195E+002/4.0595E+001**(>)	2.5293E+002/4.9710E+001(<)	2.4181E+002/4.4167E+001(<)	2.5420E+002/4.8601E+001(<)	2.3279E+002/5.3515E+001
f_{16}'	1.6946E+003/2.5993E+002(>)	**1.2473E+003/3.4444E+002**(>)	1.8008E+003/2.8950E+002(>)	1.7089E+003/3.0760E+002(>)	1.9296E+003/2.3942E+002(<)	1.5273E+003/2.3680E+002(>)	1.8507E+003/2.2794E+002
f_{17}'	1.1472E+003/1.9937E+002(>)	**9.3359E+002/2.3571E+002**(>)	1.1367E+003/2.3649E+002(>)	1.2282E+003/2.224E+002(>)	1.3585E+003/2.0425E+002(<)	1.1090E+003/1.7398E+002(>)	1.3324E+003/2.2249E+002
f_{18}'	2.1488E+002/5.2954E+001(<)	2.2610E+002/4.3816E+001(<)	2.0912E+002/4.0385E+001(<)	2.2779E+002/4.9561E+001(<)	2.1383E+002/4.9754E+001(<)	2.3299E+002/5.2045E+001(<)	**2.0240E+002/4.1373E+001**
f_{19}'	1.7418E+002/2.3287E+001(<)	1.8544E+002/3.0728E+001(<)	**1.4185E+002/1.8501E+001**(>)	1.8325E+002/3.214E+001(<)	1.7983E+002/2.4530E+001(<)	1.7157E+002/2.3049E+001(<)	1.6283E+002/2.4871E+001
f_{20}'	1.6428E+003/1.9887E+002(>)	**1.2757E+003/1.9859E+002**(>)	1.4299E+003/2.1101E+002(>)	1.3284E+003/3.0248E+002(>)	1.5803E+003/2.0432E+002(>)	1.5102E+003/2.0511E+002(>)	1.7259E+003/2.3684E+002
f_{21}'	2.6913E+002/5.6031E+000(<)	**2.5367E+002/4.7692E+000**(>)	2.6776E+002/6.5972E+000(<)	2.8267E+002/9.4343E+000(<)	2.8992E+002/6.8868E+000(<)	2.5709E+002/6.7581E+000(>)	2.6442E+002/5.6623E+000
f_{22}'	1.1459E+004/6.5298E+002(>)	1.1042E+004/8.3198E+002(>)	1.1704E+004/8.7544E+002(<)	1.0689E+004/8.0972E+002(>)	**1.0546E+004/2.6336E+003**(>)	1.1353E+004/5.1092E+002(>)	1.1613E+004/5.0858E+002
f_{23}'	5.8704E+002/1.2330E+001(<)	5.8989E+002/1.0420E+001(<)	5.9166E+002/1.1416E+001(<)	5.8644E+002/1.2927E+001(<)	5.7024E+002/8.4550E+000(<)	5.7387E+002/8.2666E+000(<)	**5.6737E+002/1.1316E+001**
f_{24}'	8.9687E+002/1.0027E+001(<)	9.0911E+002/1.0206E+001(<)	9.0277E+002/9.4153E+000(<)	9.3688E+002/1.4942E+001(<)	9.0437E+002/1.6965E+001(<)	9.0582E+002/8.8593E+000(<)	**8.7082E+002/1.2822E+001**
f_{25}'	7.4622E+002/2.7438E+001(<)	7.3375E+002/3.7469E+001(<)	7.2358E+002/4.2583E+001(<)	73884E+002/3.5890E+001(<)	7.3165E+002/3.5400E+001(<)	7.4233E+002/3.0068E+001(<)	**7.1419E+002/4.1045E+001**
f_{26}'	3.2058E+003/9.5845E+001(<)	3.1420E+003/8.3181E+001(<)	3.1950E+003/8.9570E+001(<)	3.5372E+003/1.5782E+002(<)	3.3233E+003/1.2648E+002(<)	3.2464E+003/8.7494E+001(<)	**3.0167E+003/1.1712E+002**
f_{27}'	6.3870E+002/1.6299E+001(<)	6.3239E+002/1.7838E+001(<)	6.0995E+002/1.5812E+001(<)	6.4624E+002/1.6248E+001(<)	6.1058E+002/1.8779E+001(<)	6.1621E+002/1.9986E+001(<)	**6.0957E+002/1.9927E+001**
f_{28}'	5.3427E+002/3.5232E+001(<)	**5.2405E+002/2.9386E+001**(>)	5.2490E+002/2.9441E+001(>)	5.3321E+002/3.0215E+001(<)	5.2674E+002/3.1478E+001(>)	5.3364E+002/3.4093E+001(<)	5.3204E+002/3.4522E+001
f_{29}'	1.1573E+003/1.7586E+002(<)	1.3735E+003/1.8172E+002(<)	1.2090E+003/2.1291E+002(<)	1.4256E+003/2.0165E+002(<)	1.37702E+003/1.9965E+002(<)	1.1139E+003/1.6241E+002(<)	**1.1072E+003/1.3907E+002**
f_{30}'	2.4912E+003/1.5320E+002(<)	2.4749E+003/1.4882E+002(<)	2.4288E+003/1.3568E+002(>)	2.5012E+003/1.5214E+002(<)	2.4586E+003/1.4048E+002(<)	**2.4119E+003/1.6847E+002**(>)	2.4341E+003/1.3134E+002
w/d/l	6/0/14	12/0/18	13/0/17	8/0/22	5/0/25	9/0/21	-/-/-

$nfe_{max} = 10000 \cdot D$, and it shows the competitiveness of our PaDE-NPC algorithm in comparison with these state-of-the-art DE variants.

Prof. Ali W. Mohamed reviewed our paper in the second round of view, and he recommended us to take the EBLSHADE algorithm [50] into comparison, therefore, we add this algorithm in these tables and we can see that our PaDE-NPC algorithm obtains 12 performance improvements, 15 performance improvements, 18 performance improvements and 21 performance improvements on 10D, 30D, 50D and 100D optimization under CEC2013 test suite; it also obtains 15 performance improvements, 15 performance improvements, 17 performance improvements and 21 performance improvements on 10D, 30D, 50D and 100D optimization under CEC2017 test suite in comparison with EBLSHADE. Again, our PaDE-NPC algorithm shows its competitiveness.

Furthermore, we also examine the convergence speed of our PaDE-NPC algorithm in comparison with these five famous DE variants on 30D optimization under the CEC2013 test suit, and these figures are plotted in Figs. 4- 7.

We can see from the graphs that our proposed PaDE-NPC algorithm outperforms other advanced algorithms on f_2-f_3, f_6-f_7, f_{12}-f_{13}, f_{20}, and it still competitive and no worse



(a) f_1

(b) f_2

(c) f_3

(d) f_4

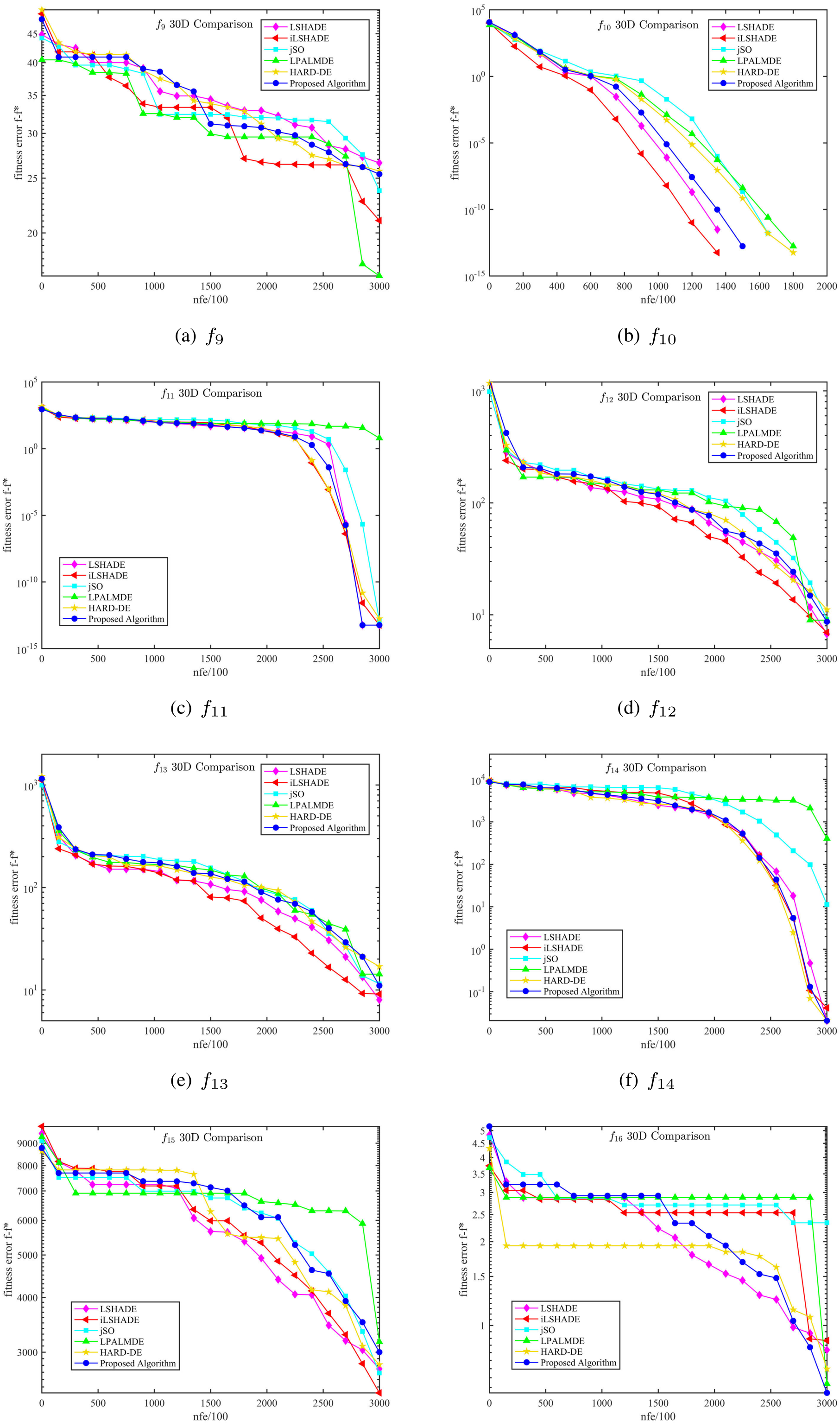
(e) f_5

(f) f_6

(g) f_7

(h) f_8

FIGURE 4. Here presents the convergence speed comparison by employing the median value of 51 runs obtained by each algorithm on 30*D* optimization. There are total 28 comparison figures and the first 8 figures are presented here.



(a) f_9 (b) f_{10}

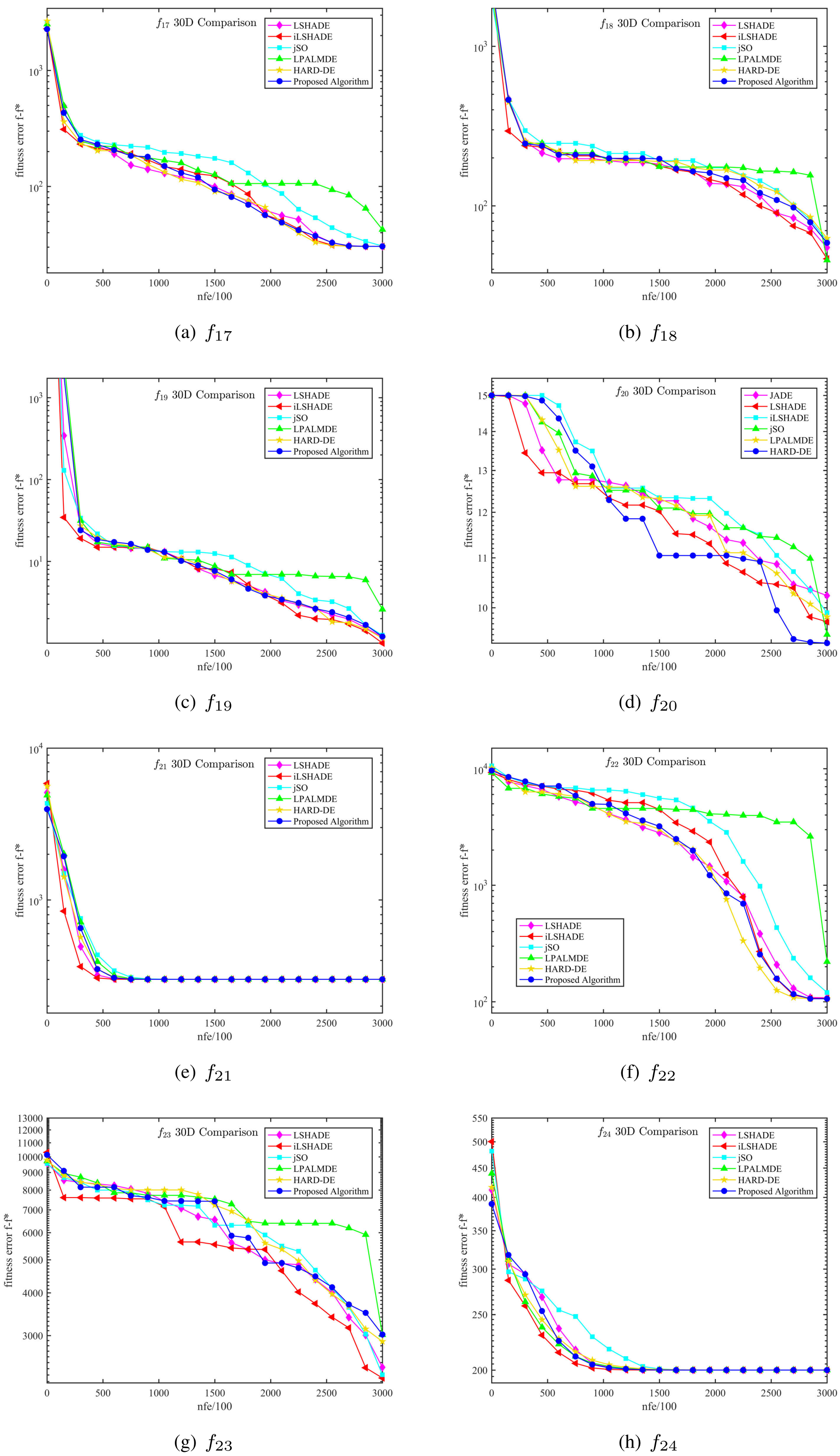
(c) f_{11} (d) f_{12}

(e) f_{13} (f) f_{14}

(g) f_{15} (h) f_{16}

FIGURE 5. As a continued part from Fig. 4, comparisons on benchmarks f_9-f_{16}.

than other advanced algorithms on f_1, f_4-f_5, f_8, f_{10}-f_{11}, f_{14}, f_{17}, f_{21}-f_{22}, f_{24}-f_{28}. We also summarize the best performance and the tier best performance obtained by a certain algorithm in Table 10, and from this perspective of view, our PaDE-NPC algorithm is also competitive with the contrasted algorithms.

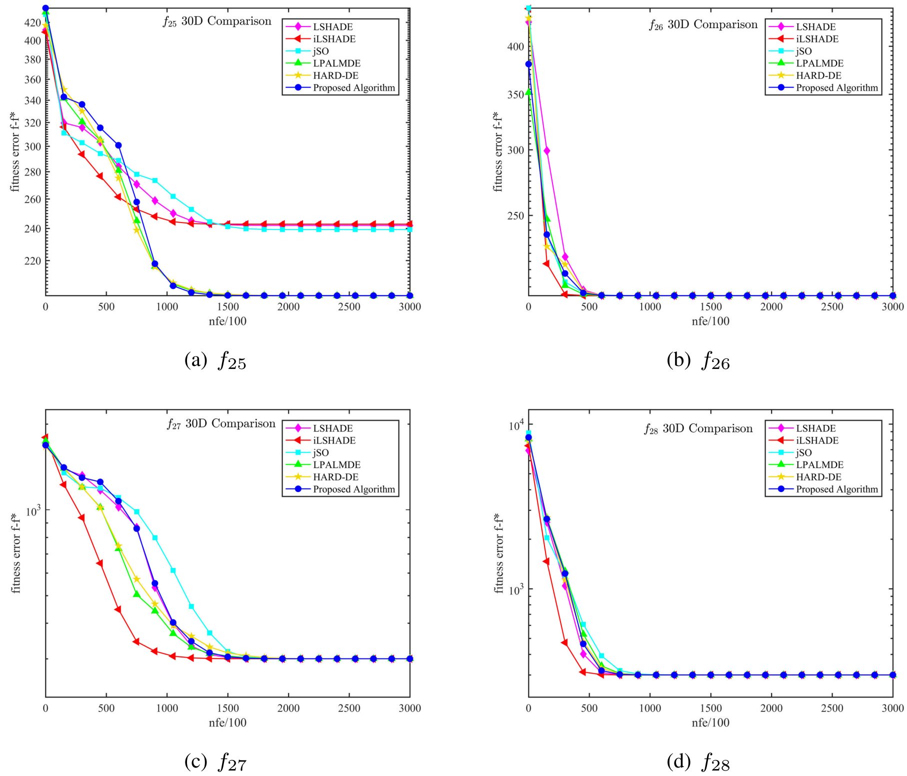


(a) f_{17} (b) f_{18} (c) f_{19} (d) f_{20} (e) f_{21} (f) f_{22} (g) f_{23} (h) f_{24}

FIGURE 6. As a continued part from Fig. 5, comparisons on benchmarks f_{17}-f_{24}.

C. NEW PARAMETERS IN OUR PaDE-NPC

There are three additional parameters, p, $plat$ and DM. In the proposed PaDE-NPC. $p\%$ is the percentage of top superior individuals which is utilized to improve the exploration capacity of mutation strategy in comparison with the global best. According to the former study in JADE



(a) f_{25}

(b) f_{26}

(c) f_{27}

(d) f_{28}

FIGURE 7. As a continued part from Fig. 6, the last comparisons are figured out here for analysis.

TABLE 10. Convergence speed comparison by employing the median value of 51 runs obtained by each algorithm on 30*D* optimization.

Algorithms.	Best(Tier Best) Performance
LSHADE	f_1, f_4, f_{12}, f_{17}, f_{21}, f_{22}, f_{24}, f_{26}-f_{28}
iLSHADE	f_5, f_{10}, f_{15}, f_{17}, f_{19}, f_{21}-f_{24}, f_{26}-f_{28}
jSO	f_{17}, f_{21}, f_{24}, f_{26}-f_{28}
LPALMDE	f_8, f_9, f_{11}, f_{16}, f_{18}, f_{21}, f_{24}-f_{28}
HARD-DE	f_{14}, f_{17}, f_{21}, f_{22}, f_{24}-f_{28}
PaDE-NPC	f_2, f_3, f_5-f_7, f_{10}, f_{12}-f_{14}, f_{17}, f_{20}-f_{22}, f_{24}-f_{28}

and LSHADE algorithm, the value of p is generally close to 0.10. In this part, several potential values of p, $p \in \{0.08, 0.09, 0.10, 0.11, 0.12, 0.13\}$, are selected to test which one is suitable for the proposed PaDE-NPC algorithm. The "Mean" and "Standard Deviation" of fitness errors over 51 independent runs on each benchmark of the CEC2013 test suit under different p settings are compared in Table 11 with the maximum number of function evaluation equaling to $nfe_{max} = 10000 \cdot D$. From the comparison results, we can find that $p = 0.11$ is a good choice for our PaDE-NPC algorithm, and $p = 0.11$ is taken as the default setting of parameter p.

The parameter *plat* determines the percentage of the whole evolution that employs fixed population size in our PaDE-NPC algorithm, and we examine several different settings of the parameter $plat \in \{0.00, 0.05, 0.10, 0.15, 0.20, 0.25\}$. As we know, fixed population size usually performs poor in optimization tasks especially for those multi-modal objectives and complex systems. Meanwhile, a relative large population at the earlier stage of the evolution helps to get a full perception of the landscapes of the objective or system. Therefore the percentage *plat* should be a large value. The first test case $plat = 0$ means that the platform based reduction scheme of population size is actually degraded into linear population size reduction in this case, moreover, the left cases have different number of generations employing fixed population size. Experiments results are presented in Table 12 with 51 independent runs under CEC2013 test suit. From the table, we can find that $plat = 0.15$ is a good choice which is taken as the default setting of parameter *plat* in our PaDE-NPC algorithm.

The new parameter *DM* is employed as a threshold in the choice of a certain mutation strategy among the combined mutation strategies. In this part, we present a simple experiment analysis of the value of *DM*. $DM = 0$ and $DM = 1$ are the boundary setting of the choice of mutation strategies: mutation strategy in Eq. 15 is employed during the whole evolution when $DM = 0$, and mutation strategy in Eq. 14 is employed during the whole evolution when $DM = 1$. We also examine other different cases, i.e. $DM \in \{\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}\}$ in our experiment. and the experiment results of "Mean" and "Standard Deviation" of fitness errors over 51 independent runs on 30D optimization under CEC2013 test suit are presented in Table 13.

TABLE 11. Optimization results comparison of different *p* values under CEC2013 test suite.

30D	$p = 0.08$	$p = 0.09$	$p = 0.10$	$p = 0.12$	$p = 0.13$	$p = 0.11$
NO.	mean/std	mean/std	mean/std	mean/std	mean/std	mean/std
f_1	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_2	4.5475E-013/5.6249E-013(<)	3.8556E-013/2.5756E-013(<)	4.0125E-013/2.8612E-013(<)	3.9921E-013/1.5928E-013(<)	4.1908E-013/3.7876E-013(<)	**3.7896E-013/2.5991E-013**
f_3	5.6434E-009/3.8682E-008(<)	**2.6304E-013/2.8170E-013**(>)	1.8208E-011/1.2834E-010(<)	6.1079E-013/1.9509E-012(<)	2.3553E-010/1.6478E-009(<)	6.0633E-013/2.8405E-012
f_4	2.2737E-013/9.0949E-014(<)	2.1400E-013/8.3996E-014(>)	2.1400E-013/1.1515E-013(>)	2.1400E-013/1.0579E-013(>)	**2.0954E-013/1.0992E-013**(>)	2.1846E-013/1.0129E-013
f_5	**1.0700E-013/2.7016E-014**(>)	1.1369E-013/0(≈)	1.1146E-013/1.5919E-014(>)	1.1146E-013/1.5919E-014(>)	1.1369E-013/0(≈)	1.1369E-013/0
f_6	2.3852E-013/3.2891E-013(<)	2.0731E-013/1.3200E-013(<)	2.3406E-013/1.9482E-013(<)	4.9933E-013/1.4009E-012(<)	2.8087E-013/3.7661E-013(<)	**1.6942E-013/6.9610E-014**
f_7	1.8860E-002/3.3869E-002(<)	1.5408E-002/4.7878E-002(<)	2.3690E-002/5.7420E-002(<)	1.6422E-002/3.9331E-002(<)	**6.3544E-003/1.4895E-002**(>)	9.7875E-003/2.5433E-002
f_8	**2.0809E+001/1.2393E-001**(>)	2.0834E+001/1.1437E-001(<)	2.0827E+002/1.3472E-001(<)	2.0816E+001/1.4163E-001(>)	2.0842E+001/1.4609E-001(<)	2.0825E+002/1.2327E-001
f_9	2.5845E+001/1.9688E+000(<)	2.6000E+001/1.5210E+00(<)	2.6322E+001/1.4990E+000(<)	**2.5622E+001/1.9997E+000**(>)	2.5719E+001/2.3891E+000(<)	2.5688E+001/1.6852E+000
f_{10}	1.4502E-004/1.0357E-003(>)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	0/0
f_{11}	7.1333E-014/3.5668E-014(<)	6.2416E-014/2.8433E-014(<)	7.8020E-014/4.8181E-014(<)	7.0218E-014/4.0336E-014(<)	6.4645E-014/4.4059E-014(<)	**5.2385E-014/1.5434E-014**
f_{12}	6.9501E+000/1.5678E+000(<)	6.5151E+000/1.7071E+000(<)	6.6058E+000/1.9464E+000(<)	6.7925E+000/1.7500E+000(<)	6.8804E+000/1.4771E+000(<)	**6.5062E+000/1.8251E+000**
f_{13}	8.2643E+000/3.4760E+000(<)	7.5472E+000/2.7414E+000(>)	7.6594E+000/3.9106E+000(>)	**7.4852E+000/2.7748E+000**(>)	8.0722E+000/3.2980E+000(<)	7.8386E+000/2.9310E+000
f_{14}	8.4238E+000/2.8025E+001(>)	**5.2621E+000/1.9205E+001**(>)	1.2961E+001/3.6381E+001(>)	1.4927E+001/5.3563E+001(>)	3.6174E+001/1.0233E+002(<)	1.9941E+001/7.8542E+001
f_{15}	2.9159E+003/2.6329E+002(<)	**2.8725E+003/3.1814E+002**(>)	2.8749E+003/30672E+002(>)	2.9183E+003/4.0566E+002(<)	2.9118E+003/3.1609E+002(<)	2.9075E+003/3.3899E+002
f_{16}	**1.0527E+000/7.1056E-001**(>)	1.3761E+000/7.4873E-001(<)	1.2289E+000/7.6794E-001(<)	1.1675E+000/7.5428E-001(<)	1.1008E+000/6.9919E-001(<)	1.0946E+000/7.1704E-001
f_{17}	*3.0434E+001/1.6003E-006*(≈)	*3.0434E+001/1.3202E-006*(≈)	*3.0434E+001/1.3202E-006*(≈)	*3.0434E+001/9.4299E-007*(≈)	*3.0434E+001/9.4299E-007*(≈)	*3.0434E+001/2.1913E-006*
f_{18}	5.7498E+001/5.8761E+000(>)	**5.6729E-001/6.1667E+000**(>)	5.7023E+001/6.4648E+000(>)	5.7459E+001/4.9647E+000(>)	5.7779E+001/8.1403E+000(<)	5.7775E+001/5.5980E+000
f_{19}	1.2118E+000/9.9825E-002(<)	1.2136E+000/1.0552E-001(<)	1.2161E+000/1.0647E-001(<)	1.1878E+000/1.0634E-001(<)	1.2005E+000/1.1248E-001(<)	**1.1159E+000/9.6361E-002**
f_{20}	9.1249E+000/3.5861E-001(<)	9.1048E+000/3.7166E-001(>)	**9.0907E+000/3.8255E-001**(>)	9.1247E+000/3.8073E-001(<)	9.1211E+000/3.5531E-001(<)	9.1158E+000/3.6179E-001
f_{21}	3.0367E+002/3.1788E+001(<)	3.0256E+002/4.2774E+001(<)	3.0427E+002/5.5153E+001(<)	3.0819E+002/5.0912E+001(<)	3.0194E+002/4.8177E+001(<)	**2.9975E+002/3.7738E+001**
f_{22}	1.0639E+002/1.1077E+000(<)	1.0634E+002/1.3227E+000(<)	1.0630E+002/1.4227E+000(<)	1.0639E+002/1.6441E+000(<)	1.0982E+002/1.6708E+001(<)	**1.0619E+002/8.3259E-001**
f_{23}	2.8413E+003/4.0100E+002(<)	2.8002E+003/3.5655E+002(<)	2.9415E+003/3.2895E+002(<)	2.8577E+003/3.6939E+002(<)	2.8903E+003/3.1722E+002(<)	**2.7828E+003/3.6969E+002**
f_{24}	*2.0000E+002/6.4620E-003*(≈)	*2.0000E+002/7.2678E-003*(≈)	*2.0000E+002/4.6197E-003*(≈)	*2.0000E+002/3.3411E-003*(≈)	*2.0000E+002/5.9239E-003*(≈)	*2.0000E+002/4.3059E-003*
f_{25}	2.1024E+002/1.8744E+001(<)	2.0966E+002/1.7687E+001(<)	2.0786E+002/1.6129E+001(<)	2.0660E+002/1.5513E+001(>)	**2.0585E+002/1.4906E+001**(>)	2.0698E+002/1.5266E+001
f_{26}	*2.0000E+002/1.4352E-013*(≈)	*2.0000E+002/1.4352E-013*(≈)	*2.0000E+002/1.4262E-013*(≈)	*2.0000E+002/1.4262E-013*(≈)	*2.0000E+002/1.4171E-013*(≈)	*2.0000E+002/1.4262E-013*
f_{27}	3.0019E+002/2.9163E-001(≈)	**3.0015E+002/2.6941E-001**(>)	3.0017E+002/4.3248E-001(>)	3.0022E+002/5.5428E-001(<)	3.0021E+002/2.9775E-001(<)	3.0019E+002/2.9047E-001
f_{28}	*3.0000E+002/2.5050E-013*(≈)	*3.0000E+002/1.9871E-013*(≈)	*3.0000E+002/2.3996E-013*(≈)	*3.0000E+002/1.9209E-013*(≈)	*3.0000E+002/2.3229E-013*(≈)	*3.0000E+002/1.8801E-013*
w/d/l	6/6/16	8/7/13	8/6/14	8/6/14	3/7/18	-/-/-

TABLE 12. Optimization results comparison of different *plat* values under CEC2013 test suite.

30D	$plat = 0.00$	$plat = 0.05$	$plat = 0.10$	$plat = 0.20$	$plat = 0.25$	$plat = 0.15$
NO.	mean/std	mean/std	mean/std	mean/std	mean/std	mean/std
f_1	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_2	5.3054E-013/4.3776E-013(<)	734454E-013/1.3891E-012(<)	4.4137E-013/3.8697E-013(<)	2.9425E-013/1.8911E-013(>)	**2.8533E-013/1.4267E-013**(>)	3.7896E-013/2.5991E-013
f_3	1.4684E-005/8.9493E-005(<)	2.5858E-013/4.0167E-013(>)	4.3147E-011/3.3915E-010(<)	2.0062E-013/1.4124E-013(>)	*1.9617E-013/3.5766E-013*(>)	6.0633E-013/2.8405E-012
f_4	2.4967E-013/9.3803E-014(<)	2.3400E-013/8.3996E-014(<)	2.3183E-013/1.4729E-013(<)	*1.8279E-013/1.1157E-013*(>)	2.0062E-013/1.0806E-013(>)	2.1846E-013/1.0129E-013
f_5	1.1369E-013/0(≈)	*1.1146E-013/1.5919E-014*(>)	*1.1146E-013/1.5919E-014*(>)	1.1369E-013/0(≈)	*1.1146E-013/1.5919E-014*(>)	1.1369E-013/0
f_6	**1.3821E-013/4.7224E-014**(>)	1.7610E-013/1.3889E-013(<)	2.1177E-013/2.0466E-013(<)	1.0432E-012/5.9734E-012(<)	7.0218E-013/2.3766E-012(<)	1.6942E-013/6.9610E-014
f_7	2.3186E-002/4.3969E-002(<)	1.2064E-002/2.2882E-002(<)	2.1766E-002/4.1462E-002(<)	1.2064E-002/2.2882E-002(<)	2.1060E-002/5.1000E-002(<)	**9.7875E-003/2.5433E-002**
f_8	2.0821E+001/1.1311E-001(>)	**2.0800E+001/1.4491E-001**(>)	2.0829E+001/1.4331E-001(<)	2.0828E+001/1.4127E-001(<)	2.0825E+001/1.2043E-001(≈)	2.0825E+002/1.2327E-001
f_9	2.5522E+001/1.7898E+000(>)	2.5958E+001/1.4048E+000(>)	2.5622E+001/2.8046E+000(>)	2.5532E+001/1.6202E+000(>)	**2.4328E+001/4.0249E+000**(>)	2.5688E+001/1.6852E+000
f_{10}	2.9004E-004/1.4499E-003(<)	1.4502E-004/1.0357E-003(<)	*0/0*(≈)	*0/0*(≈)	1.4502E-004/1.0357E-003(<)	*0/0*
f_{11}	5.3500E-014/4.7439E-014(<)	6.9104E-014/3.4695E-014(<)	6.1302E-014/3.3808E-014(<)	6.9104E-014/3.4695E-014(<)	8.4708E-014/4.4574E-014(<)	**5.2385E-014/1.5434E-014**
f_{12}	6.5562E+000/1.7223E+000(<)	6.8764E+000/1.5876E+000(<)	6.5924E+000/1.5447E+000(<)	6.7402E+000/1.7333E+000(<)	7.7382E+000/1.4449E+000(<)	**6.5062E+000/1.8251E+000**
f_{13}	9.0415E+000/3.7988E+000(<)	7.9944E+000/3.3913E+000(<)	7.8507E+000/3.8413E+000(<)	8.0352E+000/2.9181E+000(<)	8.1373E+000/3.8564E+000(<)	**7.8386E+000/2.9310E+000**
f_{14}	5.2801E+000/2.5092E+001(<)	4.1475E+000/2.7771E+001(<)	**6.1900E-001/2.3535E+000**(>)	2.7931E+001/1.0509E+002(<)	2.7268E+001/8.1637E+001(<)	1.9941E+001/7.8542E+001
f_{15}	**2.7992E+003/2.9516E+002**(>)	2.8697E+003/2.9281E+002(>)	2.8837E+003/5.2895E+002(>)	2.9880E+003/3.6252E+002(<)	2.9580E+003/3.4378E+002(<)	2.9075E+003/3.3899E+002
f_{16}	1.05360E+000/7.4864E-001(>)	9.9260E-001/7.3834E-001(>)	1.0209E+000/6.7931E-001(>)	1.0994E+000/6.9880E-001(<)	**9.4091E-001/6.3171E-001**(>)	1.0946E+000/7.1704E-001
f_{17}	*3.0434E+001/4.6005E-014*(≈)	*3.0434E+001/9.4299E-007*(≈)	*3.0434E+001/5.3952E-014*(≈)	*3.0434E+001/1.8285E-006*(≈)	*3.0434E+001/1.8284E-006*(≈)	*3.0434E+001/2.1913E-006*
f_{18}	**5.4488E+001/3.3457E+000**(>)	5.5390E+001/4.8652E+000(>)	5.7634E+001/5.1191E+000(>)	5.9328E+001/6.3139E+000(<)	6.0114E+001/5.3090E+000(<)	5.7775E+001/5.5980E+000
f_{19}	1.2148E+000/1.0203E-001(<)	1.1566E+000/7.8187E-002(<)	1.1834E+000/1.0832E-001(<)	1.2395E+000/1.0526E-001(<)	1.2501E+000/1.2657E-001(<)	**1.1159E+000/9.6361E-002**
f_{20}	9.1918E+000/3.9560E-001(<)	**8.9624E+000/3.7929E-001**(>)	9.1119E+000/3.1644E-001(>)	9.1869E+000/3.9556E-001(<)	9.2285E+000/3.2588E-001(<)	9.1158E+000/3.6179E-001
f_{21}	3.0563E+002/2.8140E+001(<)	3.0538E+002/4.7105E+001(<)	3.0452E+002/4.01925E+001(<)	2.9693E+002/3.1662E+001(>)	**2.9105E+002/3.9120E+001**(>)	2.9975E+002/3.7738E+001
f_{22}	1.0609E+002/6.0905E-001(>)	**1.0599E+002/3.8268E-001**(>)	1.0672E+002/1.9690E+000(<)	1.1335E+002/2.9398E+001(<)	1.1526E+002/6.0218E+001(<)	1.0619E+002/8.3259E-001
f_{23}	2.8162E+003/3.9742E+002(<)	2.8540E+003/3.9166E+002(<)	**2.7656E+003/3.5496E+002**(>)	2.9047E+003/3.0345E+002(<)	2.9016E+003/2.8227E+002(<)	2.7828E+003/3.6969E+002
f_{24}	*2.0000E+002/5.9282E-003*(≈)	*2.0000E+002/8.8933E-003*(≈)	*2.0000E+002/7.4953E-003*(≈)	*2.0000E+002/2.0377E-003*(≈)	*2.0000E+002/1.5390E-003*(≈)	*2.0000E+002/4.3059E-003*
f_{25}	2.0582E+002/1.4813E+001(>)	2.0750E+002/1.6494E+001(<)	2.0939E+002/1.8193E+001(<)	2.0541E+002/1.3738E+001(>)	**2.0485E+002/1.3461E+001**(>)	2.0698E+002/1.5266E+001
f_{26}	*2.0000E+002/1.4262E-013*(≈)	*2.0000E+002/1.4352E-013*(≈)	*2.0000E+002/1.4352E-013*(≈)	*2.0000E+002/1.4262E-013*(≈)	*2.0000E+002/1.4352E-013*(≈)	*2.0000E+002/1.4262E-013*
f_{27}	3.0030E+002/4.9641E-001(<)	3.0028E+002/4.8714E-001(<)	*3.0010E+002/1.6724E-001*(>)	3.0019E+002/5.7484E-001(≈)	*3.0010E+002/2.3901E-001*(>)	3.0019E+002/2.9047E-001
f_{28}	*3.0000E+002/1.7072E-013*(≈)	*3.0000E+002/1.6297E-013*(≈)	*3.0000E+002/1.7015E-013*(≈)	*3.0000E+002/2.3006E-013*(≈)	*3.0000E+002/2.1086E-013*(≈)	*3.0000E+002/1.8801E-013*
w/d/l	8/6/14	9/5/14	9/6/13	6/8/14	9/6/13	-/-/-

TABLE 13. Optimization results comparison of different *DM* values under CEC2013 test suite.

30D	$DM = 0$	$DM = 1/6$	$DM = 2/6$	$DM = 3/6$	$DM = 5/6$	$DM = 1$	$DM = 4/6$
NO.	mean/std	mean/std	mean/std	mean/std	mean/std	mean/std	mean/std
f_1	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_2	9.9866E-013/1.1595E-012(>)	6.6875E-013/4.5341E-013	7.6683E-013/4.7468E-013	4.4583E-013/3.1163E-013	2.5412E-013/1.2575E-013	**2.4075E-013/1.1515E-013**	3.7896E-013/2.5991E-013
f_3	6.8658E-013/2.0732E-012	4.8239E-012/3.0580E-011	5.3054E-013/1.0256E-012	**2.0062E-013/1.8554E-013**	7.1645E-012/4.9606E-011	4.3961E-005/3.1394E-004	6.0633E-013/2.8405E-012
f_4	2.4521E-013/1.0992E-013	2.7642E-013/1.5296E-013	3.2546E-013/19429E-013	2.2737E-013/1.2031E-013	1.3821E-013/1.1212E-013	**1.2929E-013/1.2249E-013**	2.1846E-013/1.0129E-013
f_5	**1.0700E-013/2.7016E-014**	1.1146E-013/1.5919E-014	1.1369E-013/0	1.1369E-013/0	1.1369E-013/0	1.1369E-013/0	1.1369E-013/0
f_6	6.9666E-010/4.8218E-009	2.7196E-013/2.2287E-013	1.3420E-012/3.1079E-012	4.2577E-013/1.5125E-012	1.9839E-013/1.8025E-013	2.4298E-013/5.7297E-013	**1.6942E-013/6.9610E-014**
f_7	**4.3054E-003/1.9380E-002**	1.2363E-002/2.8059E-002	1.6023E-002/3.2206E-002	1.3562E-002/2.9257E-002	8.2681E-003/1.8515E-002	1.3678E-002/3.1831E-002	9.7875E-003/2.5433E-002
f_8	2.0953E+001/4.6246E-002	2.0921E+001/6.3004E-002	2.0933E+001/6.6782E-002	2.0889E+001/7.1702E-002	2.0858E+001/1.4488E-001	2.0836E+001/1.1986E-001	**2.0825E+002/1.2327E-001**
f_9	2.5610E+001/1.6563E+000	2.5634E+001/1.9414E+000	2.5284E+001/2.5439E+000	2.4777E+001/3.2020E+000	2.4450E+001/4.3731E+000	**2.4335E+001/2.4856E+000**	2.5688E+001/1.6852E+000
f_{10}	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*(≈)	*0/0*
f_{11}	2.0174E-013/4.1482E-014	1.8168E-013/3.2149E-014	7.8036E-002/3.9010E-001	7.2447E-014/4.4146E-014	6.7989E-014/3.9414E-014	6.1302E-014/2.7481E-014	**5.2385E-014/1.5434E-014**
f_{12}	1.1370E+001/2.0755E+000	1.0656E+001/2.1674E+000	1.0095E+001/2.0365E+000	6.7967E+000/1.5958E+000	6.6450E+000/1.9174E+000	6.5814E+000/1.5972E+000	**6.5062E+000/1.8251E+000**
f_{13}	1.9023E+001/5.0852E+000	1.8631E+001/7.1664E+000	1.6953E+001/5.4259E+000	7.5694E+000/3.3634E+000	**7.3140E+000/2.7427E+000**	7.9191E+000/2.7661E+000	7.8386E+000/2.9310E+000
f_{14}	133333E+001/6.1862E+000	7.8885E-002/4.2779E-002	**2.4085E-002/2.4412E-002**	1.8778E-002/1.8271E-002	2.4493E-002/1.8947E-002	3.5107E-002/2.5495E-002	1.9941E+001/7.8542E+001
f_{15}	3.0121E+003/3.9900E+002	3.0606E+003/4.0505E+002	3.0563E+003/4.9933E+002	2.9877E+003/2.6116E+002	**2.7476E+003/3.2553E+002**	2.8104E+003/3.0440E+002	2.9075E+003/3.3899E+002
f_{16}	1.9478E+000/4.7612E-001	1.8424E+000/5.1323E-001	1.6209E+000/6.0985E-001	1.3262E+000/7.1027E-001	9.7221E-001/8.6132E-001	**9.1432E-001/7.9131E-001**	1.0946E+000/7.1704E-001
f_{17}	3.0963E+001/1.6399E-001	*3.0434E+001/9.4297E-007*(≈)	*3.0434E+001/6.4898E-005*(≈)	*3.0434E+001/1.7977E-006*(≈)	*3.0434E+001/5.5052E-004*(≈)	*3.0434E+001/1.3204E-006*(≈)	3.0434E+001/2.1913E-006
f_{18}	6.4449E+001/5.8579E+000	6.5981E+001/1.0615E+001	5.7700E+001/5.8990E+000	5.7540E+001/7.6221E+000	5.6447E+001/5.1233E+000	**5.6526E+001/5.1414E+000**	5.7775E+001/5.5980E+000
f_{19}	1.2772E+000/1.0572E-001	1.2609E+000/1.0242E-001	1.2025E+000/1.0817E-001	1.2004E+000/1.0430E-001	1.2101E+000/1.0885E-001	1.2492E+000/8.9426E-002	**1.1159E+000/9.6361E-002**
f_{20}	9.5781E+000/4.3237E-001	9.5966E+000/4.9302E-001	9.4461E+000/3.0045E-001	9.3145E+000/4.6320E-001	9.1474E+000/3.4970E-001	9.1546E+000/3.8996E-001	**9.1158E+000/3.6179E-001**
f_{21}	3.0648E+002/3.7329E+001	**2.9693E+002/3.1662E+001**	2.9975E+002/3.7738E+001	3.0452E+002/4.0192E+001	3.0930E+002/4.1955E+001	3.0256E+002/4.2774E+001	2.9975E+002/3.7738E+001
f_{22}	1.2088E+002/4.4303E+000	1.0643E+002/1.2802E+000	**1.0605E+002/9.6280E-001**	1.1825E+002/2.2639E+001	1.0645E+002/1.6828E+000	1.0647E+002/1.6377E+000	1.0619E+002/8.3259E-001
f_{23}	2.9985E+003/4.0565E+002	3.0494E+003/4.1127E+002	3.0118E+003/5.3933E+002	2.9274E+003/3.5611E+002	2.7985E+003/3.4243E+002	**2.7117E+003/2.7974E+002**	2.7828E+003/3.6969E+002
f_{24}	*2.0000E+002/2.2105E-003*(≈)	*2.0000E+002/17.2859E-003*(≈)	*2.0000E+002/4.6098E-003*(≈)	*2.0000E+002/2.6003E-003*(≈)	*2.0000E+002/2.0870E-003*(≈)	*2.0000E+002/5.2453E-003*(≈)	*2.0000E+002/4.3059E-003*
f_{25}	2.0476E+002/1.3175E+001	2.0713E+002/1.7.90E+001	2.0461E+002/1.2762E+001	**2.0416E+002/1.2750E+001**	2.0869E+002/1.6832E+001	2.1184E+002/1.8609E+001	2.0698E+002/1.5266E+001
f_{26}	*2.0000E+002/1.4352E-013*(≈)	*2.0000E+002/1.4352E-013*(≈)	*2.0000E+002/1.4352E-013*(≈)	*2.0000E+002/1.4353E-013*(≈)	*2.0000E+002/1.4262E-013*(≈)	*2.0000E+002/1.4171E-013*(≈)	*2.0000E+002/1.4262E-013*
f_{27}	**3.0014E+002/2.7943E-001**	3.0048E+002/7.8815E-001	3.0036E+002/6.7610E-001	3.0038E+002/7.8995E-001	3.0026E+002/4.9419E-001	3.0020E+002/1.7263E-001	3.0019E+002/2.9047E-001
f_{28}	*3.0000E+002/1.7813E-013*(≈)	*3.0000E+002/2.6357E-013*(≈)	*3.0000E+002/2.1254E-013*(≈)	*3.0000E+002/2.1375E-013*(≈)	*3.0000E+002/2.1615E-013*(≈)	*3.0000E+002/2.5114E-013*(≈)	*3.0000E+002/1.8801E-013*
w/d/l	6/5/17	4/6/18	7/7/14	7/6/15	9/7/12	8/7/13	-/-/-

TABLE 14. Optimization comparison between fitness based adaptation scheme and the adaptations in the NPC under CEC2013 test suite on 30D and 50D optimization.

30D	PaDE-Fit		PaDE-NPC		50D	PaDE-Fit		PaDE-NPC	
NO.	Best	Mean/Std	Best	Mean/Std	NO.	Best	Mean/Std	Best	Mean/Std
f_1	*0*(≈)	*0/0*(≈)	*0*	*0/0*	f_1	*0*(≈)	4.9041E-014/9.4449E-014(<)	*0*	**4.4583E-015/3.1839E-014**
f_2	*0*(≈)	**3.3437E-013/2.2897E-013**(>)	*0*	3.7896E-013/2.5991E-013	f_2	1.0362E+001(<)	1.4947E+003/1.9925E+003(<)	**1.5260E-001**	**7.0408E+001/1.9402E+002**
f_3	*0*(≈)	7.1654E-003/3.0792E-002(<)	*0*	**6.0633E-013/2.8405E-012**	f_3	3.3659E-006(<)	1.6918E+003/5.4115E+003(<)	**1.1933E-009**	**1.2424E+001/4.6480E+001**
f_4	*0*(≈)	**2.0062E-013/7.3986E-014**(>)	*0*	2.1846E-013/1.0129E-013	f_4	3.8199E-011(<)	7.5313E-009/1.4354E-008(<)	**3.7517E-011**	**3.5394E-009/7.1460E-009**
f_5	**0**(>)	**1.0923E-013/2.2287E-014**(>)	1.1369E-013	1.1369E-013/0	f_5	*1.1369E-013*(≈)	1.3820E-013/4.7224E-014(<)	*1.1369E-013*	**1.2260E-013/3.0869E-014**
f_6	**0**(>)	4.6366E-013/1.1231E-012(<)	1.1369E-013	**1.6942E-013/6.9610E-014**	f_6	*4.3447E+001*(≈)	*4.3447E+001/0*(≈)	*4.3447E+001*	*4.3447E+001/0*
f_7	6.5640E-005(<)	9.7253E-002/1.1139E-001(<)	**1.1270E-005**	**9.7875E-003/2.5433E-002**	f_7	1.1096E-001(<)	1.3520E+000/1.0599E+000(<)	**1.5548E-002**	**1.2917E-001/1.5098E-001**
f_8	2.0433E+001(<)	2.0836E+001/1.2095E-001(<)	**2.0392E+001**	**2.0825E+002/1.2327E-001**	f_8	**2.0728E+001**(>)	**2.1041E+001/1.1371E-001**(>)	2.0802E+001	2.1060E+001/9.654E-002
f_9	**1.5447E+001**(>)	2.5773E+001/2.1841E+000(<)	2.0864E+001	**2.5688E+001/1.6852E+000**	f_9	4.5522E+001(<)	5.1580E+001/2.2159E+000(<)	**2.9794E+001**	**5.0399E+001/4.2998E+000**
f_{10}	*0*(≈)	2.1751E-003/4.0549E-003(<)	*0*	**0/0**	f_{10}	5.6843E-014(<)	1.7100E-002/1.1384E-002(<)	**0**	**4.7838E-003/5.0832E-003**
f_{11}	*0*(≈)	**4.5698E-014/2.2793E-014**(>)	*0*	5.2385E-014/1.5434E-014	f_{11}	**1.1369E-013**(>)	**6.5760E-013/5.7211E-013**(>)	3.4106E-013	1.5036E-012/1.3949E-012
f_{12}	3.6993E+000(<)	6.7749E+000/1.2756E+000(<)	**2.4172E+000**	**6.5062E+000/1.8251E+000**	f_{12}	1.3539E+001(<)	1.8030E+001/2.3988E+000(<)	**1.1481E+001**	**1.6883E+001/2.5767E+000**
f_{13}	**1.6623E+000**(>)	8.5379E+000/4.7907E+000(<)	3.0022E+000	**7.8386E+000/2.9310E+000**	f_{13}	1.6489E+001(<)	3.4615E+001/9.8639E+000(<)	**1.0743E+001**	**2.9300E+001/1.0218E+001**
f_{14}	2.1828E-011(<)	2.1175E+001/6.0463E+001(<)	**3.6380E-012**	**1.9941E+001/7.8542E+001**	f_{14}	9.4396E-002(<)	8.9307E+001/2.1793E+002(<)	**2.7593E-002**	**9.6221E-002/2.8945E-002**
f_{15}	2.1665E+003(<)	2.9200E+003/2.9229E+002(<)	**2.1380E+003**	**2.9075E+003/3.3899E+002**	f_{15}	**5.3788E+003**(>)	**6.6992E+003/4.1831E+002**(>)	5.5110E+003	6.7313E+003/4.0129E+002
f_{16}	**6.0501E-002**(>)	1.2692E+000/7.0534E-001(<)	1.1276E-001	**1.0946E+000/7.1704E-001**	f_{16}	2.8747E-001(<)	1.8243E+000/8.8902E-001(<)	**2.2609E-001**	**1.5891E+000/8.5024E-001**
f_{17}	*3.0434E+001*(≈)	*3.0434E+001/1.3202E-006*(≈)	*3.0434E+001*	*3.0434E+001/2.1913E-006*	f_{17}	*5.0786E+001*(≈)	*5.0786E+001/4.8166E-004*(≈)	*5.0786E+001*	*5.0786E+001/6.7595E-005*
f_{18}	4.7263E+001(<)	**5.5337E+001/4.3834E+000**(>)	**3.7921E+001**	5.7775E+001/5.5980E+000	f_{18}	**8.5376E+001**(>)	**1.0712E+002/6.4832E+000**(>)	8.9115E+001	1.1233E+002/7.6272E+000
f_{19}	9.5500E-001(<)	1.1772E+000/9.3676E-002(<)	**9.2477E-001**	**1.1159E+000/9.6361E-002**	f_{19}	2.1691E+000(<)	**2.4945E+000/1.3096E-001**(>)	**2.0569E+000**	2.5029E+000/1.6997E-001
f_{20}	8.5422E+000(<)	9.3000E+000/3.4073E-001(<)	**8.3148E+000**	**9.1158E+000/3.6179E-001**	f_{20}	1.6416E+001(<)	1.7975E+001/5.2839E-001(<)	**1.6379E+001**	**1.7770E+001/4.8209E-001**
f_{21}	*2.0000E+002*(≈)	**2.9949E+002/5.3368E+001**(>)	*2.0000E+002*	2.9975E+002/3.7738E+001	f_{21}	*2.0000E+002*(≈)	8.4790E+002/3.9472E+002(<)	*2.0000E+002*	**5.6292E+002/4.4285E+002**
f_{22}	**1.0517E+002**(>)	1.0714E+002/2.4765E+000(<)	1.0520E+002	**1.0619E+002/8.3259E-001**	f_{22}	9.6516E+000(<)	2.1901E+001/4.9455E+001(<)	**8.9687E+000**	**1.2083E+001/1.1023E+000**
f_{23}	2.1089E+003(<)	2.8283E+003/3.0758E+002(<)	**1.9368E+003**	**2.7828E+003/3.6969E+002**	f_{23}	**5.1234E+003**(>)	**6.3341E+003/5.4386E+002**(>)	5.2848E+003	6.4798E+003/5.0351E+002
f_{24}	*2.0000E+002*(≈)	2.0004E+002/6.5210E-002(<)	*2.0000E+002*	**2.0000E+002/4.3059E-003**	f_{24}	2.0032E+002(<)	2.0365E+002/2.4766E+000(<)	**2.0000E+002**	**2.0017E+002/2.3045E-001**
f_{25}	*2.0000E+002*(≈)	2.1191E+002/1.9657E+001(<)	*2.0000E+002*	**2.0698E+002/1.5266E+001**	f_{25}	2.7537E+002(<)	2.8884E+002/7.5967E+000(<)	**2.6826E+002**	**2.8208E+002/6.3855E+000**
f_{26}	*2.0000E+002*(≈)	*2.0000E+002/1.4262E-013*(≈)	*2.0000E+002*	*2.0000E+002/1.4262E-013*	f_{26}	*2.0000E+002*(≈)	2.3620E+002/5.1750E+001(<)	*2.0000E+002*	**2.1616E+002/3.7850E+001**
f_{27}	*3.0000E+002*(≈)	3.0137E+002/1.8534E+000(<)	*3.0000E+002*	**3.0019E+002/2.9047E-001**	f_{27}	3.1200E+002(<)	3.5205E+002/2.6861E+001(<)	**3.0063E+002**	**3.1133E+002/1.1240E+001**
f_{28}	*3.0000E+002*(≈)	*3.0000E+002/1.4975E-013*(≈)	*3.0000E+002*	*3.0000E+002/1.8801E-013*	f_{28}	*4.0000E+002*(≈)	*4.0000E+002/2.8705E-013*(≈)	*4.0000E+002*	*4.0000E+002/2.8433E-013*
w/d/l	6/13/9	6/4/18	-/-/-	-/-/-		5/7/16	6/3/19	-/-/-	-/-/-

From the table we can see that the PaDE-NPC with $DM = 2/3$ obtains best performance in compared with other DM settings, and we take this value as the the default setting in our PaDE-NPC algorithm.

D. THE NOVEL NPC VERSUS FITNESS BASED ADAPTATION

One of the main contributions in our Novel Parameter Control (NPC) is that the adaptation schemes of control parameters F and Cr are fitness independent, which is distinct from the adaptation schemes in the recent proposed state-of-the-art DE variants for single-objective real-parameter optimization. In this part we present a simple experiment analysis between these two adaptation schemes, and these two schemes in our algorithm are denoted as PaDE-Fit and PaDE-NPC respectively, and the experiment results are arranged in Table 14.

From the table we can see that the proposed PaDE-NPC obtains superior performance in comparison with the algorithm with fitness based adaptation schemes not only from the ''Best'' perspective of view but also from the ''Mean/Std'' perspective. The proposed PaDE-NPC achieves 9 better and 13 similar performance improvements in comparison with the PaDE-Fit on 30D optimization and achieves 16 better and 7 similar performance improvements on 50D optimization from ''Best'' perspective of view. The proposed PaDE-NPC obtains 18 better and 4 similar performance improvements in comparison with the PaDE-Fit on 30D optimization and achieves 19 better and 3 similar performance improvements on 50D optimization from ''Mean/Std'' view. As a result, our PaDE-NPC shows excellent performance and can tackle a relative large optimization applications especially for those that the fitness values are unavailable.

E. TIME COMPLEXITY ANALYSIS

Time consumption of basic arithmetic expressions in CEC2013 competition recommendation is recorded as T_0, the time consumption of 200000 function evaluations for 30D optimization on benchmark function f_{14} from CEC2013 test suit is recorded as T_1, and the overall cost of a certain algorithm optimizing f_{14} is recorded as T_2. 51 independent runs are conducted to get the average T_0, T_1 and T_2, and then, $\frac{\widehat{T_2 - T_1}}{T_0}$ is collected for complexity evaluation. The time complexity comparisons among these DE variants are presented in Table 15, and we can see that the proposed PaDE-NPC consumes more time in comparison with other advanced DE variants because the calculation of location is more time consuming in comparison with the fitness-error based DE variants, but it is tolerable for the optimization applications especially the exact fitness value is unavailable e.g. the example shown in Fig. 3.

TABLE 15. Algorithm time complexity comparison on 30D optimization under CEC2013 benchmark f_{14}.

Algorithms.	T_0	T_1	$\widehat{T_2}$	$\frac{\widehat{T_2 - T_1}}{T_0}$
LSHADE	0.0968	0.7182	1.5865	8.97
iLSHADE			1.6765	9.90
jSO			1.6439	9.56
LPALM-DE			2.9156	22.70
HARD-DE			2.0438	13.69
PaDE-NPC			3.7774	31.61

VI. CONCLUSION

In this paper, we proposed a novel PaDE-NPC algorithm for real-parameter single objective optimization. In the PaDE-NPC algorithm, combined mutation strategies were employed in the generation of trial vectors, and this can make better use of the advantages of each mutation strategy regarding a novel proposed population diversity indictor. Furthermore, Novel Parameter Control (NPC) schemes were also proposed: 1) different from the recently proposed state-of-the-art DE variants, fitness independent parameter control schemes for crossover rate CR and scale factor F were proposed in the PaDE-NPC algorithm. These techniques can broaden the domain and improve the optimization performance of DE, because the PaDE-NPC algorithm was verified to be able to obtain better performance under the test suite and it also can tackle optimization applications with unknown fitness value. 2) A platform based population size reduction was also proposed in the NPC, which can make a balance between the exploration and exploitation capacity while getting a better perception of the landscape at the early stage of the evolution. Moreover, the calculation of positions of the individuals in the adaptations of control parameters usually cost more time in comparison with the fitness difference based adaptations for control parameters in the recently proposed state-of-the-art DE variants, and this can be tolerable because it broadens the range of optimization applications that the DE algorithm can tackle and improves the performance of DE as well.

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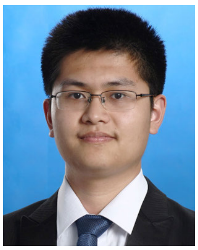

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