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A Genetic Algorithm (GA) Approach to the Portfolio Design Based on Market Movements and Asset Valuations

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ABSTRACT Vulnerable nature of price forecasts, such as an unpredictability of future and numbers of socio-economic factors that affect market stability, often makes investment risky. Earlier studies in Finance suggested that constructing a portfolio can promise risk-spread gains. While Fund Standardization improved the traditional theories by reducing the computational complexity and by associating every interaction in the portfolio, such a method still cannot become a winning strategy because it does not measure the current value or the relative price of each asset. Inspired by the works of finding returns per risk, we attempt to design an optimal portfolio by searching products that have potential to grow further. More specifically, we first analyze risk-adjusted returns in the previous periods and use their inertia as a momentum. However, because historic movements alone do not fully elucidate future changes nor guarantee positive returns, we scored the relative values of each stock to make more informed estimations. Using the Capital Asset Pricing Model, we measured the values of each stock and determined those undervalued. In this study, we applied a Genetic Algorithm to optimize portfolios while incorporating the momentum strategy and the asset valuations. The proposed GA model was tested in two separate markets, S&P500 and KOSPI200, and projected greater profits than that from both the previous method with momentum method and the market indexes. From the experimental results, the proposed CAPM+ method was found to be very effective in financial data analysis and to lay a groundwork for a sustainable investment execution.

INDEX TERMS Genetic algorithm, machine learning, portfolio optimization, modern portfolio theory, investment strategy, Sharpe ratio, capital asset pricing model, security market line.

I. INTRODUCTION

Guessing what will happen in the future, or making predictions always involve uncertainty. In the past, because we can never guarantee what the future will be like, betting on luck or simply praying has long been a way for predictions. Afterwards people noticed that patterns may exist when predicting the future, learned from previous mistakes and used those lessons to calculate odds for specific events. As a result, many started to make more informed estimations by measuring the likelihood, and many probability theories came into play. However, predictions with optimization processes or profit maximizations while calculating the chances make simple statistics no longer effective or cause the problem to be more complex.

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In finance, where many different factors affect the market, investment decision is even more intricate. For example, in equity markets, an annual net profit in cash flow statement alone does not fully represent this year's price movements. In addition to business performances, many externalities including market growths, geo-political issues, international trends and many more reasons affect stock prices.

Preferences for risk-margin ratio may differ with each individual investor, but as a rational person, the investor wants to win or maximize their risk-return tradeoffs. More extensively and thoroughly theorized, the ideas of maximizing the earnings in a given risk are compiled by researchers like Harry Markowitz. In Portfolio Theory [1], [2], he highlighted the importance of constructing portfolios to optimize expected return on an uncertainty.

Markowitz's Nobel Prize Winning theory is widely acclaimed, but at the same time some studies [28] refuted its practical usage. Researchers at National Chi Nan University of Taiwan, for example, supported legitimacy of the portfolio theory, but at the same time they highlighted its ineffective computational complexity as well as its unsatisfactory representation of correlation among financial products in the portfolio. Instead, they suggested Fund Standardization methodology to improve the complexity from $O(n^2)$ to $O(1)$ and to consider every relationship between equities in portfolio.

Studies of portfolio theory, risk analysis and many following investigations deserve acclaim for their contribution to analyze historic patterns and to measure volatility, but their applications as an investment strategy in the actual financial market is yet very limited. Technical analysis of historic prices within portfolio can be integrated with Momentum Strategy [10]–[14], where previous movement or positions are assumed to continue their trajectory. Being successful during the previous market period, however, does not always guarantee a portfolio's durability or continuity with respect to both its profit size and risk rate: prices may have already reached to its maximum point and be about to nosedive. Another possibility is that the repeated downtrend may finally hit the bottom and make all-time highs on the very next day. In that regard, technical analysis based on Momentum Strategy alone is often not satisfactory as an educated forecast or as an investment plan.

Therefore, we devised an investment strategy based on equity allocations and portfolio designs, which can sort out a list of sound assets in terms of rate of return on investment with respect to its risk while considering undervalued assets in the market that may retain growth potentials. More specifically, a gap between the actual rate of return of a given stock and the expected return from security market line in capital asset pricing model is expected to evaluate the accuracy of current market estimations through our research.

An ensemble of portfolio theory, risk-return analysis [3]–[5] and Capital Asset Pricing Model [6], [7] is effective only when they are actually calculated. Equity selection or portfolio optimization in the financial markets, where hundreds and thousands of different products exist, is often burdensome. Using a genetic algorithm [16], [17], our study attempts to examine the fitness of each individual in the market to design optimal portfolio as a promising investment initiative. Additionally, many previous studies [17]–[32] insisted on their computational excellence and use in finance, but not many of them have validated their applications in dynamic environments. Here, we tested our methodology in two different financial markets over a decade and intended to prove its merit for pragmatic usage.

II. BACKGROUND

As the size of the market expands over time, numbers of tradable products or indexes to be analyzed both in domestic and international scales also dramatically increase. Accordingly, investors, especially individual ones, and their capabilities have become more limited compared to the institutional investors. Many preceding attempts of trades and studies in the corresponding field, therefore, utilized computational power to acquire analytic competences. Automated trading methods often use machine learning techniques, such as Artificial Neural Network [18], [21], [25], [26], Support Vector Machine [19], [20], Reinforcement Learning [22], [23] or LSTM [24] and attention mechanism [17], based on technical analysis to forecast for specific quotes in the corresponding market.

Volatile and unpredictable characteristics of the financial market, however, often prevent such attempts to make accurate predictions. History may or may not repeat itself in the financial market. Moreover, unidentifiable noise in financial data also hinders the use of algorithmic trading. Unlike its prominent success in academic experiments, real-life investment and applications with many traditional machine learning in Finance [17]–[27], is known to be relatively less prevailing [33]–[36] compare to many successes in other fields of studies and works with traditional machine intelligence techniques. Even in the winning scenario of having 80% or 90% accuracy in the stock predictions, price pattern based forecasts may fail investors with large costs if they lose big in a single estimation with such approaches. As previously stated, noises, complex dimensionality in financial data, and various not-descriptive socio-economic factors often bring limitations in learning the patterns. For example, while the recent COVID-19 pandemic crashed the entire market, learning historic price data alone was not fully capable of projecting such sudden volatility into the forecasts.

However, unpredictability of future prices due to noise and dimensional complexity in traditional machine learning can be redeemed or hedged in fundamentals through diversifications and using asset valuation models. Building a portfolio or using a risk considered asset valuation models offer better returns per risk and to lay a groundwork for a sustainable investment strategy. Extensive but thorough studies [1]–[9] on how to measure risk balanced return on assets or how to evaluate the stock prices have been conducted by economists like H. Markowitz, W. F. Sharpe, J. Lintner and many more.

Although there is a distinction between the definitions of uncertainty and risk, with respect to their controllability, uncertainty in the financial market or any unknowns for future estimations all are considered as risk in this study. Likewise, volatility is also used as a risk. In short, risk in this research includes any unpredictable factor that may hinder price estimations in the market.

A. MODERN PORTFOLIO THEORY

Every rational investor is expected to pursue a higher return, but because 'there is no free lunch' in financial investment, obtaining a high return as a reward always incurs a larger cost, or a risk. In more simple terms, if a higher profit involves a bigger risk, the possibility of not making promising investment is more likely to happen. Then, not every investor will make the same decision of unconditionally following profits. Each may have different propensity for the level of risk taking: some may prefer high-risk high-return type of investment, and others may like a stable risk structure even if they cannot realize maximized return. It may be optimal to find a single product that offer a large profit with a low level of risk in the hypothetical situation. However, it is nearly impossible to find that single stock where market dynamically evolves while countless market factors are involved.

Modern portfolio theory [1], [2] assumes that risk-averse individuals in the market shall pursue a rational behavior of choosing the most profitable assets in a certain degree of risk. In other words, one prefers investment decisions on items that offer the most returns on the same level of risk or on less risky assets if expected returns are the same between two options.

Markowitz insists that maximized return for a given level of volatility can be achieved through diversification while constructing an efficient portfolio, a set of financial assets, instead of putting one's budget on a single commodity. In such circumstances, securities in each portfolio are considered to reduce uncertainty: compositions of stock offset or redeem individual return/risk and form a new synthetic overall return/risk. We can measure expected return on portfolio by summing up an individual equity's return based on its proportional weight and calculate risk through statistical measure and correlation via variance and covariance.

B. SHARPE RATIO

William F. Sharpe devised a new simplified model [3]–[5] which improves the practical aspect of Mean-Variance approach of Markowitz. Unlike the earlier model, which required costly and time-consuming computation of variance and covariance matrix for every individual stock in the portfolio, Sharpe's Diagonal Model assesses total risk of a portfolio in a simple regression analysis and eases the calculation workloads. Sharpe additionally developed a measure, the Sharpe ratio, to examine the return on investment per unit of risk. In equation (1), Sharpe defined the differential return while $\tilde{R}_A$ is the return on assets and $\tilde{R}_B$ is the return on the benchmark and the expected value of d and sigma d, the standard deviation of d, derive the expected differential return per unit of associated risk in equation (2).

$$\tilde{d} \equiv \tilde{R}_A - \tilde{R}_B \tag{1}$$

$$S \equiv \frac{\bar{d}}{\sigma_d} \tag{2}$$

In Sharpe ratio [5], which is also known as Reward-to-Variability Ratio, excess return of an asset over a benchmark or often riskless asset return delineates how much return is gained for the same level of risk. Therefore, larger Sharpe ratio indicates a portfolio with a better risk-adjusted performance, and negative values designate that a benchmark or risk-free asset offers a greater return than the selected equities.

C. FUND STANDARDIZATION

Modern portfolio theory [1], [2] and the Sharpe ratio [3]–[5] are criticized in some aspects despite their foundational contributions on finance and economics. Chou *et al.*, from National Chi Nan University of Taiwan, for example, highlighted that Mean-Variance model and the Sharpe ratio demand large amount of calculations to be carried out [28]. In the integrated field of computer science and finance, where performance with respect to computing resource and time spent to calculate is critical, having an inefficient complexity is a nuisance especially when complexity exponentially increases as the stocks in the portfolio accumulates.

The researchers at Chi Nan also stressed that the use of covariance in MPT to represent interactions among stocks does not denote every existing relationship in the portfolio. Markowitz's portfolio variance for the risk in equation (3), where w_i is the amount of assets allocated to stock i in the percentage, σ_{ij} is the covariance between two stocks i and j, accurately represents the correlation between any 2 stocks in the portfolio. However, if there exist more than 2 stocks in the portfolio, which is more likely to happen in real cases, Markowitz's approach is incapable of denoting interactions of multiple stocks more than 2. Here, as depicted in equation (4) and Fig.1, we provided the case of having 4 stocks and its risk where it does not consider the interactions of 3 and 4 stocks.

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij},\ where\ \sigma_{ij} = \sigma_i \sigma_j \rho_{ij} \tag{3}$$

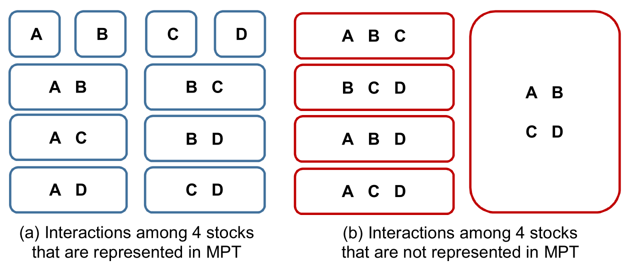

FIGURE 1. Correlation types among 4 stocks that are considered/ not considered due to covariance-approach in modern portfolio theory.

Chou, et. al proposed Fund Standardization, a measure for an individual return subtracted from transaction fee and tax for the allocated stocks, to assess portfolio risk more thoroughly, as precisely demonstrated in Table 4. Using simple additions and subtractions can help investors or portfolio managers to 1) take all interactions among stocks in portfolio set into account 2) simplify the calculations and improve the computation complexity from O(n^2) of modern portfolio theory to O(1).

$$\begin{aligned}\sigma_p^2 = {} & w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + w_D^2\sigma_D^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB} \\ & + 2w_A w_c \sigma_A \sigma_C \rho_{AC} + 2w_A w_D \sigma_A \sigma_D \rho_{AD} + 2w_B w_C \sigma_B \sigma_C \rho_{BC} \\ & + 2w_B w_D \sigma_B \sigma_D \rho_{BD} + 2w_C w_D \sigma_C \sigma_D \rho_{CD}\end{aligned} \tag{4}$$

D. CAPITAL ASSET PRICING MODEL AND SECURITY MARKET LINE

Return on investment is credited with a reward for taking risks. Based on many portfolio theories [1], [2], as earlier introduced, we can spread the risk by increasing the number of assets allocated in portfolio. Making investment on non-risky assets through portfolio design sounds promising but there still exist unidentified danger even after an investor hedge a risk through diversifications.

In finance and portfolio theory, risk, or total risk, is comprised of systematic risk and non-systematic risk. Markowitz and many researchers suggested the idea of eradicating the possibility of fluctuations through constructing a portfolio. The corresponding risk for assets volatility which can be mitigated through portfolio design is called a non-systematic risk, or an idiosyncratic risk. On the other hand, a systematic risk, also known as undiversifiable risk, cannot be spread because it is inherent to the market, not to the individual assets.

Expanding the assumptions on Markowitz's portfolio theory [8], [9], Capital Asset Pricing Model (CAPM), gives the expected return of an asset with respect to its systematic risk. In Fig.2, we made a graphical representation of CAPM, or a Security Market Line (SML) to depict and to evaluate what is the expected return of the market for different levels of market risk in Beta.

Beta or beta coefficient [6], [7] is a measure for the volatility of an individual stock based on its past performance relative to its market's movement as depicted in OLS regression of stock A in Fig.2 (a) or as in equation (5). In other words, beta value indicates how the stock moves compare to the market. A high-beta value larger than 1.0 means that the stock is more volatile, or riskier than the market, and low-beta stock of less than 1.0 is less likely to fluctuate. If a stock moves exactly same as the rest of market, it should have the beta value of 1.0.

$$\beta_i = \frac{Cov\,(R_i, R_m)}{Var(R_m)} \tag{5}$$

Having beta coefficient of the stock on the x-axis, expected return on the y-axis, we can graph the security market line to represent risk-return relationship of the capital asset pricing model. A reward for investors tolerating risks, risk return tradeoff or the risk-premium in Fig.2 (b) is the excess of the risk-free rate of return an investment and it is the slope value of the SML.

Adding risk-free rate of return to the risk premium multiplied by the beta-value, we can derive expected return at a

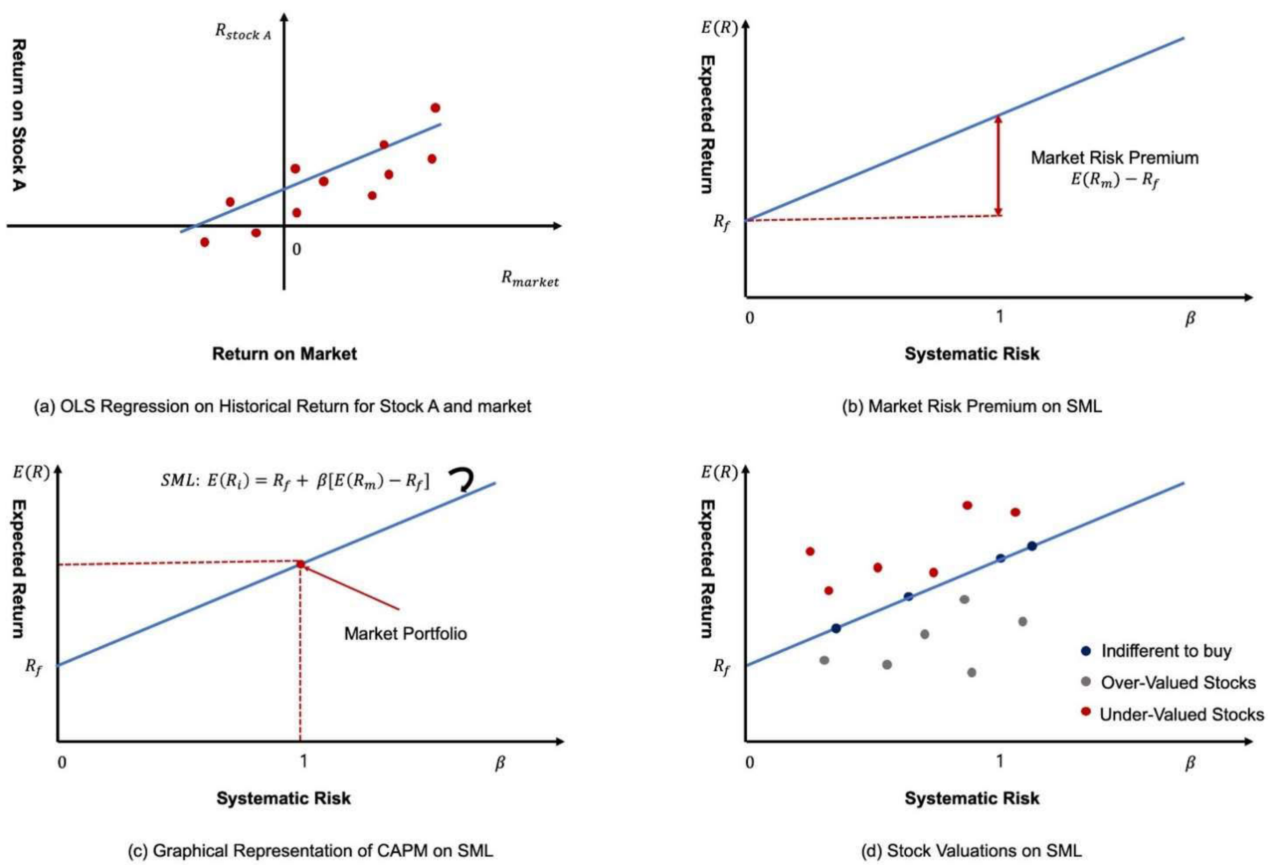

FIGURE 2. Representation of capital asset pricing model on security market line.

systematic risk. If a particular stock positions above the SML, it should provide a better return against its risk and assumed to be under-valued. On the other hand, stocks below the SML where the return is lower at a given risk, is over-valued, and investors should be reluctant to keep them in their buying list.

III. ENSEMBLE INVESTMENT STRATEGY USING GENETIC ALGORTIHM

To make investment profitable, we should buy stocks at low prices and in turn sell them at higher prices. To achieve such exchanges, we should first judge which stock can offer a more favorable return. However, it is nearly impossible for an investor to individually analyze every market factor, such as financial data, industry trends or economic outlook. Especially where hundreds of different but interrelated products fluctuate in real-time, individual investors, compare to institutional ones, are incapable of leading such a movement.

In the earlier sections, we have witnessed the following: 1) risk balanced returns are necessary to realize profits, and 2) both systematic and unsystematic risks can be reduced via diversifications and valuations. Although such approaches can be helpful when analyzing market data in a rapidly changing environment, analytics itself is not sufficient as an investment tool. In this section, we describe how to apply these analytical tools into an actual investment strategy.

A. PORTFOLIO DESIGN – MOMENTUM STRATEGY

Since Sir Isaac Newton introduced inertia in his first law of motion, we have observed the continuity of object movements in various fields. In finance, David Ricardo, a British economist and a successful trader, is known as one of the first scholars to develop a theory of continuous movements as an investment tool. Ricardo insisted that the prices of financial products tend to continue their previous actions and dropped a hint regarding future investment opportunities. A number of subsequent investors, like Jesse Livermore in *How to Trade in Stocks* [13] and Wyckoff in *The Richard D. Wyckoff Method of Trading in Stocks* [14] also supported this theory.

Later, A. Cowles and H. E. Jones drafted the first academic work on momentum [10]. They articulated a number of favorable observations related to continuity in the movement of prices, quoting that "the tendency is very pronounced for stocks which have exceeded the median in one year to exceed it also in the year following." Additionally, N. Jegadeesh and S. Titman examined price inertia and realized excess average return obtained from purchasing past-outperforming stocks and selling past-underperforming stocks [12].

Positive experiment results from investors following the market and taking advantage of the existing trends are not only found in the equity market but also in different financial markets, like bonds, commodities, or foreign exchange markets. Moreover, compared to value investing, investors may also obtain profits with a momentum strategy in a relatively short term. This strategy is also known to be very accessible without a deliberate financial analysis.

B. PORTFOLIO DESIGN – CAPM STRATEGY

Despite of its strong theoretical reasoning and evidences, the use of momentum investing is sometimes criticized for its incompetency in the flat market. Additionally, in line with the fact that Newton's first law of motion applies under some constraint of not being "compelled to change its state by the action of an external force," momentum strategy in the financial market also operates under certain conditions: stock prices can be reversed due to, for example, domestic and international economic matters, or a corporate's own financial issues. In such cases, investors with a momentum-only strategy may suffer and lose big, as it were. Because a trend-following method lacks deliberate analysis, an investor can never completely forecast market whims.

Therefore, instead of simply wishing the same return-and-risk trends to repeat in the coming periods, it is more reasonable to score the relative values of stocks. One of the most prescriptive ways of pricing a stock or portfolio takes the help of the capital asset pricing model [8], [9]. According to Sharpe and Lintner, we can price the market values of shares by measuring their risks and their relations to the market. In CAPM, return on an individual stock i, or $E(R_i)$, is a value of risk-free-rate and a premium for accrued risks.

Although the risk-free-rate of return is a return with zero-risk in theory, because no such investments exist, the Treasury Bill rate of 2% often replaces it in practice. The premium here consists of a stock's relative volatility and market premium. Market premium is the expected return from the market minus the risk-free-rate and involves how an individual stock or a portfolio reacts with respect to the market, denoted as Beta. Sharpe's model can be found in equation (6) and can be converted to equation (7):

$$\frac{E(R_i) - R_f}{\beta_i} = E(R_m) - R_f \tag{6}$$

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \tag{7}$$

With the adjusted close prices of individual stocks, KOSPI200, and the S&P500 index, we initially calculated their monthly percent changes. We can easily find a single Beta value for each company. However, CAPM often encounters criticism for using a single fixed beta value for different time periods. Therefore, we dynamically computed the 3-year-monthly beta of each stock, as shown in Table 1, that changes over time throughout the testing periods of 10 years in our experiments. After gathering the Beta values of every stock, we can complete equation (7) and measure the price based on its behavior in an efficient market.

C. PORTFOLIO OPTIMIZATION – GENETIC ALGORITHM

Granting funds to the optimal portfolio, which returns risk-adjusted profits the most while remaining underestimated till date, is going to be a goal of our investment strategy. However, it is nearly impossible for a human to manually analyze all the existing items in a market and decide whether or not to include a specific stock in a portfolio.

TABLE 1. Pseudo-code for 3-year-monthly beta calculation.

Beta calculation (3-Year-Monthly)

Input: *Stock and Index Prices for each period*
Output: *Beta Values of every individual stock for each period*

For index = 1 to m // for each month m in test periods:
Get monthly prices of Stock A for 3-year periods;
Get percent change as a monthly return for Stock A;
Get monthly prices of Market Index for 3-year periods;
Get percent change as a monthly return for index;
Drop missing row; //first row doesn't have monthly return;
Make an OLS regression model with Stock A and Index;
Save the coefficient as Beta Value

Instead, we can more easily find the best combination by using a genetic algorithm (GA).

Inspired by Darwinian ideas, this heuristic search algorithm [15], [16] follows natural selections to discover an optimal solution among viable candidates. By passing the best existing ones down to the succeeding generations or a group of populations, we can gradually find better fitness-scored solutions. A detailed process of this stochastic optimization methodology is provided below in Fig. 3 and Table 2. Using such optimization procedures, our goal is to design a portfolio with optimal stocks for the investment periods to follow.

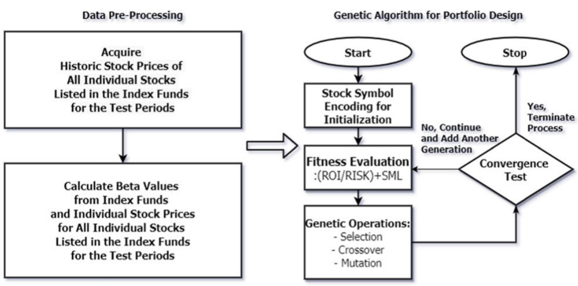

FIGURE 3. Flow chart of overall process and a genetic algorithm.

TABLE 2. Pseudo-code for a genetic algorithm.

Genetic Algorithm

Input: $Population_n$, mutation rate
Output: *Best_Population*

Population ← initialization($Population_n$);
Fitness calculation(*Population*); // Table IV
While *generations* < *max_generations*:
$Child_{pre}$ ← Tournament Selection(*Population, fitness*); // Table VI
$Child_{cross}$ ← 1-point Crossover($Child_{pre}$); // Table VI
$Child_{mutation}$ ← Mutation($Child_{cross}$, *mutation rate*); // Table VI
Fitness calculation($Child_{mutation}$);
Population ← initialization($Child_{mutation}$);
end
Best_Population ← GetBestPopulation(*Population, fitness*);
end
Return *Best_Population*;

1) ENCODING AND INITIALIZATION

As a preliminary step of a genetic algorithm, we should define a problem and an objective function: build a portfolio with maximum risk-balanced returns that are composed of undervalued stocks. Then, a defined problem should be transformed into its genotypic form. By assigning each stock from the index fund to a chromosome in a string or list, phenotypic variables are encoded as genotypic representations. For the simplification and fast-computing purposes of this study, this is done via binary representations.

Table 3 presents a reduced example of six companies from S&P500 companies under chromosomal representation. Similar to numerous earlier studies on portfolio management with a GA [28]–[32], having 1 for Chromosome indicates that the given company is included in a portfolio. On the contrary, 0 denotes that a corresponding company does not contribute to the plan for the coming investing period. For example, in Table 3, only two companies, American Airline Group and Amerisource Bergen Corporation, are included in the portfolio for the next term.

TABLE 3. An example of stocks in genotypic representations.

Company Name	American Airline Group	Advance Auto Parts, Inc.	AbbVie, Inc.
Symbol	AAL	AAP	ABBV
Chromosome	1	0	0
Company Name	Agilent Technologies, Inc.	Amerisource Bergen Corporation	Apple, Inc.
Symbol	A	ABC	AAP:
Chromosome	0	1	0

The experiment conducted in our study deals with data from two separate markets, KOSPI200 and S&P500, which include approximately 200 and 500 stocks, respectively. Therefore, with genetic encoding, each stock is represented in 200 and 500 binary-bits and each gene group is identified as a chromosome. During the evolutionary process, a number of different chromosomes form a population. While later generations' populations will acquire their chromosomes and binary values through genetic operations, the binary genotypic value in the initial stage is given arbitrarily.

2) FITNESS CALCULATION

Using a computationally effective portfolio design provided by a fund standardization approach [28], we optimized stock allocations. However, the key differences between our methodology and the traditional GA approach with portfolio are in the objective functions and the notion of survival of the fittest. While previous methods, such as Chou's, focuses on optimizing a portfolio with high returns and low- risk stocks, our understanding of portfolio as an investment strategy also highlights the aspects of stock valuations.

First, we raise either capital or set the amount of initial budget before the trade executions. Then, we equally distribute the budget over the stocks that are randomly selected as a part of a portfolio. For instance, if the total budget is \$100 and two random stocks are selected, we put \$50 on each stock. After finding the price of those allocated stocks and

TABLE 4. Pseudo-code for fitness calculation.

```
Fitness Calculation(Population)
Get number of stocks selected in the chromosome;
If number of stocks != 0:
    Allocate funds = Total fund amount/ Number of stocks;          // Allocate funds to stocks
Else:
    Allocated funds = 0
Remainder of funds = Initial allocated Fund – Amount spent on stock purchase;

For index = 1 to m: // for all stocks in the fund
    Share amount = Allocated funds / Price per share;       // Num of shares to be bought with a given budget for each stock
    Handling fee = Share × Stock price × Rates;             // Handling fees include transaction costs for each stock
    Remaining budget = Allocated funds – (Stock price × Share) – Handling fee;   // Deduct assets and handling fees from the budgets
    Return = (Stock Price at termination date × Share);
    Tax = Stock price at termination date × Share × Tax rate;
    Fund standardization = Return – Handling fee – Tax + Remaining budget;
    Fund CAPM = Get CAPM values for all selected stocks;

For index = 1 to m: // for all stocks in the fund
    Portfolio fund standardization = Sum of all elements in fund standardization;
    Portfolio CAPM = Sum all elements from fund CAPM;

ROI = Portfolio fund standardization – Initial funds/ Initial funds × 100;
Risk = Standard deviation of portfolio fund standardization / Average of portfolio fund standardization;
Fitness = (ROI – Risk-free-rate) / Risk + Portfolio CAPM;
Population.fitness = Fitness
```

computing how many stocks can be bought within a budget, we deduct the incurred transaction fees and taxes to acquire the fund standardization of each stock. Similarly, we should also find CAPM values for every selected stock. Adding fund standardization and CAPM values of all selected stocks and remaining budgets, we can get Portfolio Fund Standardization and Portfolio CAPM. Finally, using the Sharpe ratio and Portfolio CAPM, we can measure the fitness of a given chromosome. More detailed procedures of the fitness calculations can be found in Table 4.

Table 5 provides an exemplary demonstration of Table 4, which shows how fitness calculation was carried out with six stocks, AAL, AAP, ABBV, A, ABC, and AAPL, in S&P500 in January 2008. At first, we encoded them into their genotypic forms and chose the stocks to trade. Once the encoding was done, we split the total fund of \$10,000 into two and allocated \$5,000 to selected stocks (AAL and ABC) in this particular scenario. Thereafter, we acquired the necessary information for the transactions, such as stock prices, handling fees, transaction taxes on particular dates. Then, we calculated how much the portfolio is worth after the investment, precisely after 21 days (the available trading dates in January 2008).

As a result, investing on AAL and ABC as an optimal chromosome collected from genetic operations with Jan 2008 financial data, returned \$10,389.89 from the initial capital of \$10,000.00, giving approximately 3.90% profits with a risk rate of 3.02. The CAPM value, obtained from security market line differences and beta calculation, is 0.13, and the final Fitness Scores for this portfolio are 1.42 and 1.29 with and without CAPM information, respectively.

TABLE 5. An example of fitness calculation of trading 6 stocks from S&P500 in January 2008.

AAL	AAL	AAP	ABBV	A	ABC	AAPL
Selected	1	0	0	0	1	0
Allocated Fund	5000	0	0	0	5000	0
Dates	21	21	0	21	21	21
Stock Prices (Day 0)	12.68	36.24	0	24.34	18.96	24.38
Stock Prices (Day 21)	13.18	34.50	0	22.71	19.82	16.94
Shares	394	0	0	0	263	0
Handling Fees (Day 0)	0.07	0	0	0	0.07	0
Handling Fees (Day 21)	0.08	0	0	0	0.08	0
Remainder	2	0	0	0	14	0
Return (Day 0)	4997	0	0	0	4985	0
Return (Day 21)	5192	0	0	0	5212	0
Security Transaction Tax (Day 0)	14.99	0	0	0	14.96	0
Security Transaction Tax (Day 21)	15.58	0	0	0	15.64	0
Fund Standardization (Day 0)	4999	0	0	0	4999	0
Fund Standardization (Day 21)	5179	0	0	0	5210	0
Portfolio Fund Standardization (Day 0)					9999.85	
Portfolio Fund Standardization (Day 21)					10389.89	
ROI					3.90	
RISK					3.02	
Portfolio CAPM					0.13	
Fitness Value (With CAPM)					1.42	
Fitness Value (Without CAPM)					1.29	

3) GENETIC OPERATIONS

Darwinian natural selections that iteratively refine species and realize the "survival of the fittest" start with random guesses through a process known as initialization or encoding. Then, a group of chromosomes composed of stocks or a population in each generation either evolves or makes natural variations via genetic operations. The evolutionary stages of a genetic algorithm that helps to move chromosomes toward the ultimate solution of finding the best portfolio in a given period include three steps, which are also witnessed in the field of biology: selection, crossover and mutation. The repeated steps of genetic operations produce offspring with better fitness score values and ultimately lead to global optimal solutions. In our case, each genetic operation gradually searches an optimal portfolio that is under-valued and has a strong positive momentum. This will be done by pursuing better fitness scores in the following genetic operations. A detailed description of the algorithm used in this research is stated in Table 6.

TABLE 6. Pseudo-code for 3 genetic operations.

```
Tournament Selection (Population)
-----------------------------------------------
For index = 0 to population size:
  Generate a random number; // for position
  if Pop[index]. fitness > Pop[random number].fitness:
    add Pop[index] to new offspring population;
  else:
    add Pop[random number] to new offspring population;

1-point Crossover (Population)
-----------------------------------------------
For index = 0 to population size/2:
  Offspring1 = Chromosome1; //copy Chromosome1
  Offspring2 = Chromosome2; //copy Chromosome2
  Generate a random number between 0 and population size/2;
  Offspring1[random:] = Chromosome2[random:];
  Offspring2[random:] = Chromosome1[random:];
  Add Offspring1, Offspring2 to new offspring population;

Mutation (Population, mutation rate)
-----------------------------------------------
For index = 0 to m: // for every individual stock
  Generate a random number; //
  if random number < Mutation rate:
    if Chromosome[index] == 0:
      Chromosome[index] = 1;
    else:
      Chromosome[index] = 0;
```

- Selection: After the first populations are created, their genetic information is passed down to the second generation. As in genetics and biology, the dominant species tend to survive longer. By following such an evolutionary theory and preservation for favorable variations, species with better fitness are devised to have a better chance of speciating in the next generations. In other words, a genetic algorithm performs the operation of searching chromosomes with better fitness scores and choosing them as more likable candidates for the eventual breeding. This process is called selection and it increases desirable results via repetition in the succeeding generations. Among many, we used tournament selection to reduce early convergence without re-scaling. Moreover, tournament selection is expected to have a better takeover time, compared to the proportional selection methods.
- Crossover: Another type of genetic operation executed is crossover. This process, also known as recombination, imitates the genetic inheritance and the selected parents' chromosomes by recombining their segments for the next generations and exploring additional mixtures of stocks. While selection may largely contribute to the preservation and breeding of dominant genes, crossover is expected to ensure the exploration of search space in a stock list and induce variations in our portfolio combinations. However, without the information on the building blocks at present, we decided to emphasize more on exploitations and on sustaining parental genetic information while reproducing variations with a two-point crossover, rather than the larger-size crossovers. In our case, each crossover got various arbitrary crossover positions.
- Mutation: Another method of reproduction is to modify the alleles of individuals. Similar to biological mutation, which alters the nucleotide sequences, a portion of the genes in some offspring is subject to be flipped. The realization of genetic mutation fosters diversity, or at least prevents early convergence by introducing changes and generating novel offspring.
- Overlap: We also implemented the idea of overlap to avoid eradicating the best individuals acquired from genetic operations. For instance, we forced certain portions of the best chromosomes in the previous generation to the next one, in order to guarantee that the best individuals in the coming generation would always be better than or at least equal to the ones before in terms of their fitness scores.

We attached examples of genetic operations performed with six stocks of AAL, AAP, ABBV, A, ABC, and AAPL from S&P500 in Fig. 4. Because the figure only demonstrates the general procedures of the genetic operations, chromosome size, population size, selection, evolution rates were all modified in the later experiments. We will state more of the actual rates and the parameters used in the following section.

IV. EXPERIMENTAL RESULTS

There are many previous studies on the financial applications of Machine Intelligence with equity investments using neural networks [17], [18], [21], [24]–[26] support vector machines [19], [20], or a genetic algorithm [27]–[32], [37]–[39]. Most of those price prediction problems, however, were not very successful in the real world, unlike the expectations [33]–[36]. For instance, even when the designed model recognized the earlier patterns well enough, we could not precisely estimate the actual prices in a different timeframe. Furthermore, many regularization techniques seem ineffective in this particular field of studies. Such errors in equity price forecast are even compared to "A Monkey Throwing Darts."

BEFORE SELECTION

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
0	0	0	0	0	1	0	1.88
1	0	0	0	0	1	0	1.88
2	0	1	0	0	0	0	-0.86
3	0	1	0	0	1	1	-2.35
4	1	0	1	0	0	0	-8.45
5	0	0	0	0	0	0	-999999

PROCESS OF TOURNAMENT SELECTION

INDEX 0	1.88	SURVIVALS	
RANDOM INDEX 3	-2.35	INDEX 0	1.88
INDEX 1	1.88	SURVIVALS	
RANDOM INDEX 4	-8.45	INDEX 1	1.88
INDEX 2	-0.86	SURVIVALS	
RANDOM INDEX 0	1.88	RANDOM INDEX 0	1.88
INDEX 3	-2.35	SURVIVALS	
RANDOM INDEX 2	-0.86	RANDOM INDEX 2	-0.86
INDEX 4	-8.45	SURVIVALS	
RANDOM INDEX 3	-2.35	RANDOM INDEX 3	-2.35
INDEX 5	-9999999	SURVIVALS	
RANDOM INDEX 4	-8.45	RANDOM INDEX 4	-8.45

AFTER SELECTION

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
0	0	0	0	0	1	0	1.88
1	0	0	0	0	1	0	1.88
2	0	0	0	0	1	0	1.88
3	0	1	0	0	0	0	-0.86
4	0	1	0	0	1	1	-2.35
5	1	0	1	0	0	0	-8.45

BEFORE CROSSOVER

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
0	1	0	0	0	0	0	0.62
1	0	0	1	0	0	0	1
2	1	1	0	0	0	1	-2.23
3	0	1	1	1	0	1	-6.07
4	0	1	1	0	0	0	-8.88
5	1	1	1	1	1	0	-10.02

PROCESS OF CROSSOVER #1

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
0	1	0	0	0	0	0	0.62
1	0	0	1	0	0	0	1
AFTER CROSSOVER at POSITION INDEX 3 (RANDOMLY GENERATED POSITON)							
0	1	0	0	0	0	0	0.62
1	0	0	1	0	0	0	1

PROCESS OF CROSSOVER #2

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
2	1	1	0	0	0	1	-2.23
3	0	1	1	1	0	1	-6.07
AFTER CROSSOVER at POSITION INDEX 4 (RANDOMLY GENERATED POSITON)							
2	1	1	0	0	0	1	-2.23
3	0	1	1	1	0	1	-6.07

PROCESS OF CROSSOVER #3

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
4	0	1	1	0	0	0	-8.88
5	1	1	1	1	1	0	-10.02
AFTER CROSSOVER at POSITION INDEX 3 (RANDOMLY GENERATED POSITON)							
4	0	1	1	1	1	0	-13.38
5	1	1	1	0	0	0	-7.62

AFTER CROSSOVER

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
0	1	0	0	0	0	0	0.62
1	0	0	1	0	0	0	1
2	1	1	0	0	0	1	-2.23
3	0	1	1	1	0	1	-6.07
4	0	1	1	1	1	0	-13.38
5	1	1	1	0	0	0	-7.62

BEFORE MUTATION

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
0	0	0	0	1	0	0	-1.96
1	0	1	0	0	0	0	-0.86
2	0	1	0	0	0	1	-2.32
3	0	0	0	1	0	0	-1.96
4	0	0	1	0	0	0	1
5	0	1	0	1	0	1	-2.42

PROCESS OF MUTATION #2

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
2	0	1	0	0	0	1	-2.32
index: 1 -random_num: 0.08 MUTATION IS EXECUTED.							
index: 3 -random_num: 0.03 MUTATION IS EXECUTED.							
2	0	0	0	1	0	1	-2.42

PROCESS OF MUTATION #3

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
3	0	0	0	1	0	0	-1.96
index: 0 -random_num: 0.17 MUTATION IS EXECUTED.							
index: 2 -random_num: 0.17 MUTATION IS EXECUTED.							
index: 5 -random_num: 0.04 MUTATION IS EXECUTED.							
3	1	0	1	0	0	1	-7.57

PROCESS OF MUTATION #4

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
4	0	0	1	0	0	0	1
index: 1 -random_num: 0.09 MUTATION IS EXECUTED.							
index: 3 -random_num: 0.15 MUTATION IS EXECUTED.							
index: 4 -random_num: 0.05 MUTATION IS EXECUTED.							
4	0	1	1	1	1	0	-13.38

PROCESS OF MUTATION #5

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
5	0	1	0	1	0	1	-2.42
index: 0 -random_num: 0.12 MUTATION IS EXECUTED.							
5	1	1	0	1	0	1	-2.35

AFTER MUTATION

POPULATION#	AAL	AAP	ABBV	A	ABC	AAPL	FITNESS
0	0	0	0	1	0	0	-1.96
1	0	1	0	0	0	0	-0.86
2	0	0	0	1	0	1	-2.42
3	1	0	1	0	0	1	-7.57
4	0	1	1	1	1	0	-13.38
5	1	1	0	1	0	1	-2.35

FIGURE 4. An example of genetic operation with selection, crossover, and mutation processes of six stocks from S&P 500.

Noise in the data is believed to be one of many reasons behind this unpredictability. This is because not just past price patterns, but various socio-economic factors also affect the prices, provided that the data available may not be enough to make good investments. Moreover, because a designed model has to deal with the events that did not occur yet, it is surely overwhelming to forecasts solely from historical data.

Therefore, we designed a genuine investment strategy that identifies and reduces both systematic and non-systematic risks to succeed in the financial market via machine intelligence, particularly using a genetic algorithm. In the following sub-sections, we validate portfolios with CAPM and momentum ensembled strategy, with three different analyses: CAPM Effects Validity Test, Dynamic Market Test, and Overall Application Analysis. In general, we show the gross and annual profits as well as the overall analysis.

A. DYNAMIC MARKET ENVIRONMENTS

1) MARKET ENVIRONMENTS

Before digging the details out, we would like to discuss the experimental universe in which our ensembled portfolio strategy unfolds. Indeed, we conducted experiments on two equity lists in different financial markets: the constituent stocks of KOSPI200 from Korea and S&P500 from the United States. Many applications which expect their models to identify the market-oriented patterns in a single market, already exist. However, multi-market application is too challenging to be completed while using such approaches of detecting patterns. Our ensembled strategy, on the other hand, does not concern a single model but a machine intelligence that captures and analyzes the fluctuating mechanisms of the market using a theory-based valuation technique. Therefore, we expect that if our methodology works in one market, it should be applicable in others as well. The operations of our methodology in two different equity markets are investigated further in the ''Dynamic Market Validity Test'' section.

First, we performed the ensembled investment strategy test on the Korean market. According to Korea Exchange (KRX), the domestic equity market in Korea comprised 1,789 companies as of 2012 and 2,111 companies as of 2018. Despite its repeated bull-and-bear markets, the Korean stock market continued its growth and was worth \$2.456 trillion as of 2018. The Korea Composite Stock Price Index 200 (KOSPI 200) is the representative index in Korea, which indicates 200 large capitalization companies and their equities.

According to the World Bank and World Federation of Exchanges database, the world's total traded stocks is worth approximately \$68.212 trillion. Our next target market is the largest stock market, the U.S. stock market, which takes up a 48.41% share of the world market and is worth about \$33.027 trillion. Since the operation of its first official stock market, the Board of Brokers of Philadelphia in 1790, and the Buttonwood Agreement in 1792, which turned into today's New York Stock Exchange, the U.S. Stock Market has also witnessed repeated expansions and recessions. The Standard & Poor's 500 Index (S&P500 Index or S&P500) is a representative indication of the U.S. stock market: its index, comprising 500 large-cap companies that trade on either the NYSE or NASDAQ, takes up 80% of the total market value.

We picked these two markets due to a number of reasons. First, both markets are large enough, well-institutionalized, and not severely manipulated. Extreme market movements from political corruptions, or market manipulations that may arise in many smaller markets, could hinder legitimate analysis from the informed estimations and discourage reasonable investment opportunities. Additionally, arbitrarily repeated uptrends, downtrends, or flats in both markets made the test universe complicated in relation to analysis and provided real-world like environments for future analyses.

Last, the independence of the markets with respect to their price movements can be also helpful when verifying the applications of our strategy in multiple markets. Although the Korean stock market is often influenced by the U.S. economy and its financial market, its own geo-political and industrial backgrounds, with exclusive strengths and weaknesses, make it a unique and independent market in itself.

We conducted experiments with stock price data listed on both KOSPI 200 and S&P500, particularly for 2008 and 2018. This specific timeframe includes both the global 2008 financial crisis, the worst economic disaster since the Great Depression of 1929, and the stock markets' record rally of both markets repeatedly reaching all-time highs. Under these two extreme investing environments, we investigated how our methodology survives and operates.

2) PARAMETERS AND SETTINGS

Throughout the tests, we set out to incur trading fees and transaction taxes of 0.015% and 0.3%, respectively. Although there is a popular trend where many stock brokerage firms or trading platforms that offer 0% commissions and different countries may have different standard for taxes on trading, we settled those values as a conventionally incurring fare in both the Korean and U.S. markets. Additionally, all tests were conducted with stock price data listed in KOSPI200 or S&P500 for 2008 and 2018. Some stocks, however, did not go public before 2008 and may have missed several prices for some periods in our dataset. Selecting those missing value stocks in a given period is probably futile, and in most cases the heuristic optimization will automatically not select those stocks with no prices in certain periods.

For operations in a genetic algorithm, we also set a mutation rate of 1% and a crossover rate of 100% [40], [41], except for two overlapping individuals who are forced into the next generation to maintain the previous generation's best values. Because we use only the best individuals in the next trading period after the GA analysis, not many identical individuals, even when they have good fitness values, are required to pass down to the coming generations.

In the following experiments, we used financial data in 1- 3- 6- month time frame for a GA analysis. For example, as depicted in Fig. 6, we first perform a genetic algorithm in timestamp t_0. After the proposed genetic operation is completed with financial data in time t_0, we use a set of stocks that are returned from the analysis to make investments in the next period of t_1. Such process is repeated until t_{n+1} when last investment is made based on a GA analysis from time t_n.

B. ENSEMBLED EFFECTS VALIDITY TEST

In this section, we compare the results following the methodologies of CAPM and Momentum Ensembled Strategy (CAPM+) and Momentum Only Strategy (NO-CAPM) to the movements of stock index prices during 2008 and 2018, using KOSPI 200 data. Although this data often comprised 200 large-cap stocks, for data continuity, we acquired the stock price data of 190 companies during a given time

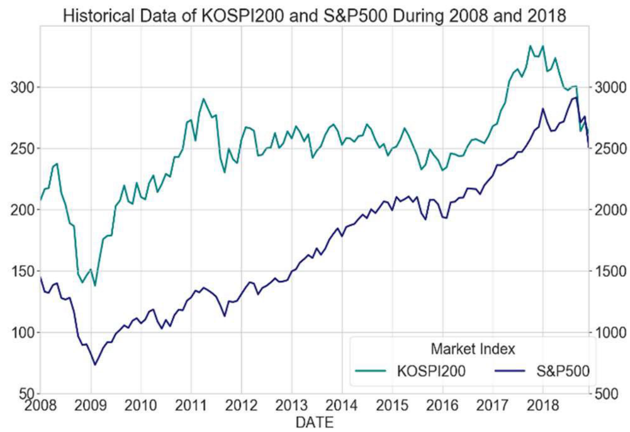

FIGURE 5. Historical price movements data of KOSPI200 and S&P500.

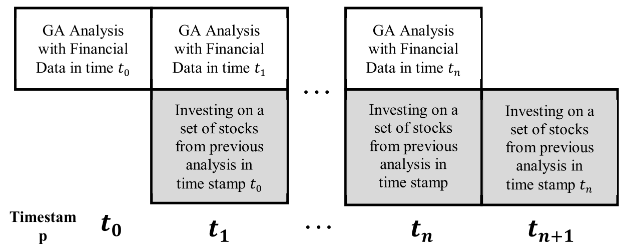

FIGURE 6. An example of time windows for CAPM+ and momentum analysis and investments in each timestamp.

period (tickers of constituent companies are presented in the Appendix). Using this comparison, we examined the validation of our approach and expected to see CAPM+'s superiority, in terms of generating profits, over a momentum-only strategy, while such NO-CAPM strategy still outperformed the movements of the index fund.

Before analyzing the results of each investment plan, we confirmed whether the GA can achieve optimal portfolio allocations, that includes the devised fitness evaluation methodologies on CAPM and fund standardization. Each sub-plot in Fig. 7 shows the changes in average, maximum, or minimum fitness values of the designed portfolio over generations. In this example of genetic operations, different color lines represent distinct months of the year 2013. Although the values were all different, throughout the 11 months of

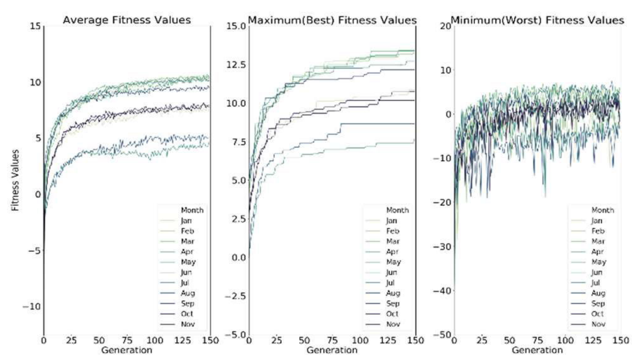

FIGURE 7. An example of 1-month analysis in Jan, 2013.

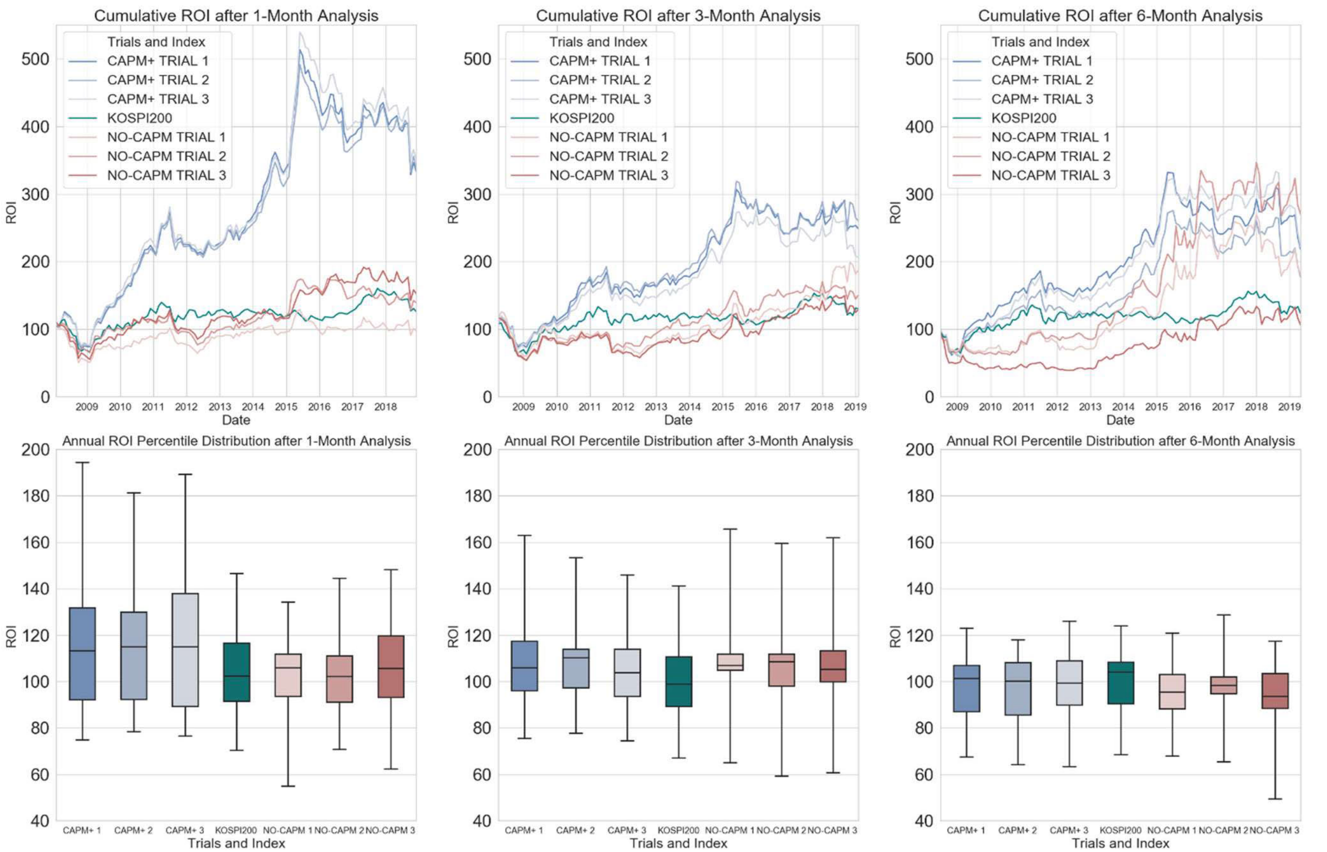

FIGURE 8. **Cumulative ROI movements and annual ROI percentile Distributions between 2008 and 2018 in KOSPI market.**

analysis, their scores improved with successive generations and finally converged before the 150th generation. As mentioned earlier, overlapped individuals guaranteed the best fitness scores and the maximum fitness value never worsened across generations. Additionally, because the intended variations induced by the genetic operations, minimum fitness values are not as stable as average and maximum fitness values, showing that the implemented algorithm preserves variations but prevents premature convergences. We can conclude that our GA can complete optimal stock allocations.

Fig. 8 shows how the proposed trading technique performed throughout 2008 and 2018, compared to how the index fund changed. Among six subplots, each column refers to an analysis period of one, three, or six months. While the three graphs on the top row demonstrate the cumulative Returns on Investment (ROI), the three boxplots on the bottom row show the annual ROI percentile distributions. Within each analysis, we conducted three identical trials for CAPM+ and applied the NO-CAPM strategy on the same data. This was done to settle concerns about a GA having non-deterministic features. Although the results did not perfectly match, regardless of the analysis periods, each trial provided very similar results.

Except for one case, all our investments based on both CAPM+ and NO-CAPM strategies made positive returns. However, returns from the index funds for 11 consecutive years outperformed some of our investment trials. For example, when using the NO-CAPM momentum strategy, although we can find some positive results and obtain positive profits, it cannot evidently outperform index funds nor be treated as a good investment plan. Especially, the ROIs on the NO-CAPM strategy was found to drop severely when the market was either on a downturn or did not offer strong upward trends during the flat market.

On the contrary, CAPM+ initiatives provided distinctive ROIs. In all three different trials of the one-month CAPM analysis, we obtained over 400% returns over 10 years. While a recent market instability caused a significant drop in 2018 even with our methodology, trading a portfolio of less-risky and undervalued assets was still twofold better compared to the returns from index fund investments.

Although it is less profitable than a 1-month model, a 3-month CAPM+ analysis still offered more profits compared to returns from both the NO-CAPM strategy and the index trading. Here, we observed that it is less likely to make better margins if the periods of analysis become larger with CAPM+. We believe that this is due to the characteristics of valuation in capital asset pricing model. The stock price converges to a point where an efficient market witnesses decisions made according to theories on CAPM and beta. Therefore, selecting undervalued stocks no longer remains effective as we consider a longer period of time.

The fact that a 6-month NO-CAPM analysis outperforms CAPM+ also supports the notion that a CAPM-based approach is more prominent only if it is analyzed for a shorter term. Because the momentum-only approach of NO-CAPM does not concern market- and beta-based valuations as much as CAPM analysis, NO-CAPM works better for longer periods. At the same time, it is also notable that a NO-CAPM method based on a portfolio composed of outstanding stocks of the last six months can provide good investment outcomes. We believe that momentum-only trading in a longer term made the portfolio more stable, while avoiding the selection of stocks that have good scores only temporarily from a sharp increase during short-term fluctuations.

Investors in most real-world cases, however, would not put their funds for 10+ consecutive years and wait for the cumulative returns to become 500%. They are more likely to expect successful results from their investments on a monthly or at least on a yearly basis. Therefore, we assessed how our method works in relation to annual returns.

The boxplots above draw the percentile distribution of annual returns and compares three individual trials of each technique with index fund investment. Similar to the cumulative return results, the three boxplots with whiskers, from the minimum to the maximum of annual returns as per our tactics, display a preferable investment opportunity. Here, we realized that not only the average values but the overall quartiles from the boxplot of the 1-month CAPM analysis also provides better annual returns than the market index, on average. Moreover, both the maximum and minimum annual returns of our devised method are always better than those of the NO-CAPM strategy or KOSPI200. We found a similar result for the CAPM+ of the 3-month analysis, but it was not as effective as the 1-month version. Likewise, as the period in study gets longer, momentum strategy becomes more desirable. However, it seems that we need more cumulative years, instead of executing investment every other year, to make a suitable 6-month momentum analysis of the cumulative ROI case.

In order to acquire such boxplots, we first obtained the annual return during 2008 and 2018 using different models. Then, we measured how much we had gained in percentage throughout each year; every year's investment started in January and its trade was terminated by the end of December.

We recorded how much profit we generated after one month of CAPM+ and NO-CAPM GA analyses, compare to the annual returns from the index funds presented in Table 7. Although there were a few exceptions, especially during the bear market years, where we did not win it, CAPM+ mostly beat the market and gave good annual returns. Similar to the cumulative returns, we observed that CAPM+ outperforms NO-CAPM. However, unlike the earlier tests of cumulative returns, analyzing six months of inertia effect seemed less promising with this annual measure and did not lead to a win.

In short, the CAPM+ of finding undervalued and historically outstanding stocks prevailed with respect to the market index fund and the momentum-only strategy of using risk and returns only. It worked better if the stock price data was long enough to find a valuation of the portfolio, and the outcome became worse if analysis became too long. On the contrary, the momentum-only strategies worked better when analyzing longer periods of data. With efficiently designed portfolios (as a result of employing a genetic algorithm) via historic price data analysis, we were able to discover viable options to outperform the index.

TABLE 7. **Annual ROI in percentage after 1 month analysis.**

	CAPM +1	CAPM +2	CAPM +3	KOSPI 200	NO-CAPM1	NO-CAPM2	NO-CAPM3
2008	75.0	78.5	80.1	70.4	55.0	70.7	62.3
2009	194.5	181.3	189.3	146.6	134.2	144.5	148.3
2010	153.5	154.0	154.5	128.9	121.6	111.9	123.9
2011	100.5	101.3	97.1	87.2	87.2	84.5	87.9
2012	99.8	98.7	97.5	102.7	114.6	101.7	115.5
2013	116.2	114.9	116.1	102.4	106.0	110.1	98.6
2014	126.7	123.4	127.4	96.5	99.8	107.6	100.8
2015	137.0	136.6	148.6	96.2	107.4	140.1	138.8
2016	84.56	86.2	81.5	112.0	98.3	89.6	105.7
2017	113.3	117.6	115.0	121.1	109.2	102.4	110.2
2018	76.5	78.4	76.5	78.6	88.9	92.8	81.6

C. DYNAMIC MARKET VALIDITY TEST

In order to verify our CAPM+ portfolio works in different markets, as briefly introduced earlier, we conducted the same tests in the U.S. stock market. At first, we did not change any condition except the stock price data from KOSPI200 to the S&P500 index. However, we obtained some results that were not as successful as the KOSPI cases after completing the identical genetic operations. After reviewing the acquired numerical data, we realized that the chromosome of the second test is longer, because they have about 300 more listed stocks in S&P500 compared to those in KOSPI200. Thus, the best solutions we found at the 150th generation may not have yet reached the global optimum point within the search space.

Thus, we performed additional tests with expanded iterative periods of generations, specifically 300th and 450th. Because it is widely known that the generation size is proportional to the problem size, or the number stocks in the index for our study, linearly increasing or tripling the size of generations seems reasonable. However, because the computation time also accordingly increases as the iterating procedure becomes more complicated, we tested the case of convergences at the 300th generation, to find a difference in value between the initial attempt and the improved one.

While the basic structure of graphs, three line-plots of cumulative returns on the top and the annual ROI box plots on the bottom remain the same, while each line and box in the subplot represents a different value. Unlike the previous plot in Fig. 8, where each line and box indicate the different trials

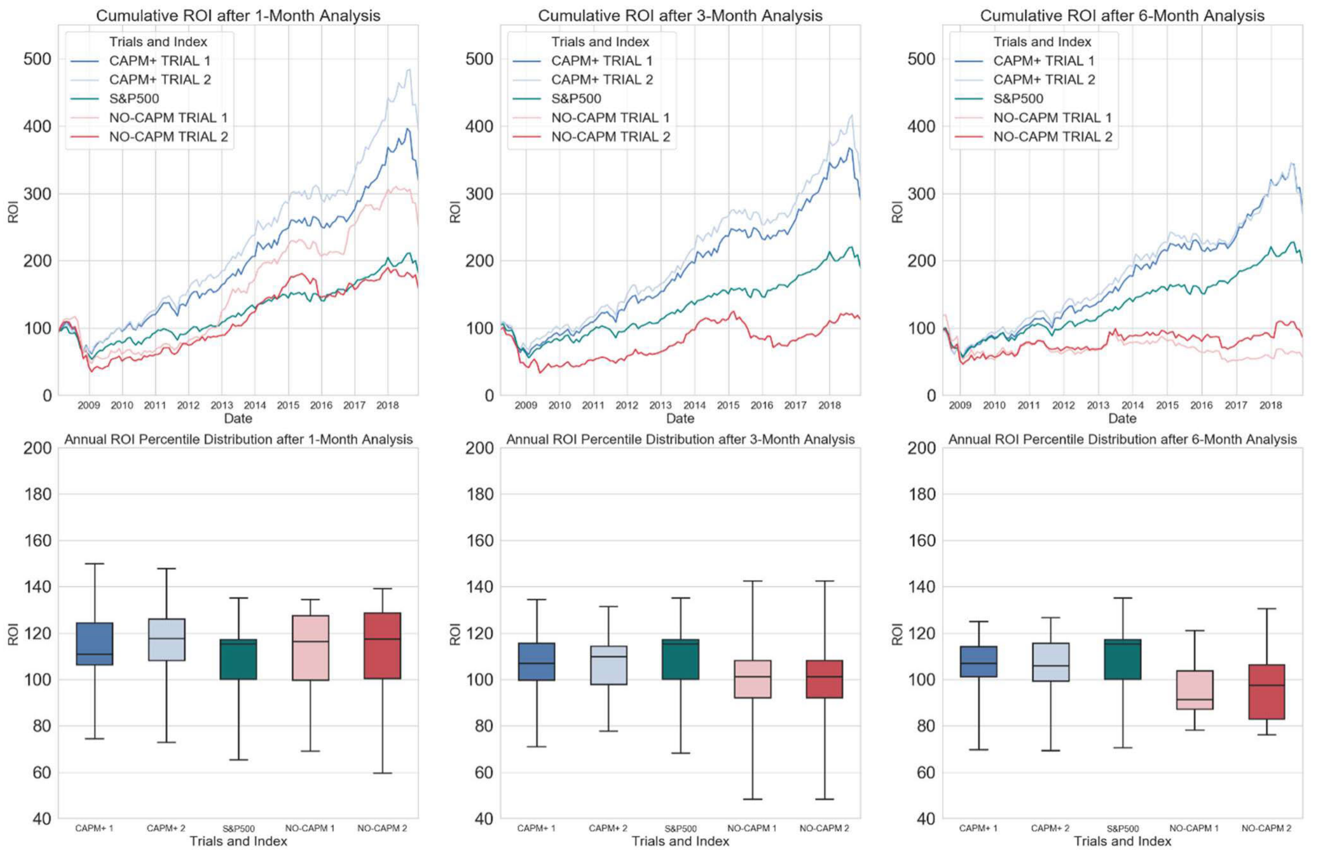

FIGURE 9. Cumulative ROI movements and annual ROI percentile distributions between 2008 and 2018 in S&P500 market.

of each strategy, the lines and boxes in Fig. 9 denote different trails based on the number of GA generations iterated.

Similar to the KOSPI200 case, although S&P500 usually includes 500 large-cap stocks, here we used only 464 actively traded stocks during a given test period (tickers of the constituent companies are presented in the Appendix).

In general, the CAPM+ experiments on S&P500 listed stocks, similar to those on KOSPI 200, offered positive ROIs. Due to the years witnessing the bull market rallies, especially after the 2008 recession, we had a very strong and winning momentum in the market. In the winning market, as long as we could identify which stocks had the potential of continued growth in the next trading period, we were able to profit from our predictions. For example, the 1-month analysis of CAPM+ methodology provided more than 400% profits. Although it was less profitable, investors still could have earned about 300% of cumulative returns from 11 years of investments with either the 3- or 6-month analyses of CAPM+. However, having more stocks in the list or having greater market growth do not necessarily entail or guarantee greater returns than those from obtained stock portfolios in the KOSPI market.

The NO-CAPM methodology of using the momentum-only strategy did not do any better even under several years of such upward trends. First, the line plots seem to have been able to recover from the initial drop during 2008 and 2009. Similar to the KOSPI 200 case, ROIs from the 1-month NO-CAPM merely converged to that generated by the index fund trading after long periods, with S&P500 data. Additionally, unlike the KOSPI case, longer periods of analysis did not aid the momentum-only approach work better in these particular market circumstances. Although they were able to capture the inertia in the market and followed the herds, the NO-CAPM strategy returns stayed behind the movements and remained more vulnerable to any downturn signals.

Although the recession years led to negative returns annually, the Annual ROI Percentile Distribution with month-long use of both CAPM+ and NO-CAPM strategies showed positive returns in most of the years. It seems that having longer periods of analysis is less effective in relation to the Annual ROI box, similar to the KOSPI 200 scenario.

Having a larger generation size definitely proved to be helpful in the search for optimal portfolios with S&P data in this research. For instance, the case of 450^{th} or 300^{th} generation size returned better profits as per both Cumulative ROI and Annual ROI. However, first two lines and boxes in each graph, which respectively represent the genetic operations with 450^{th}- or 300^{th}- generation sizes, did not show a big difference between two. We believe that this indicates, after a certain level and around the 300^{th} generation, that the fitness values of the portfolio allocation have reached their convergence point and that additional generation is not required. Without further optimization, extra generations might just consume the computing resources unless an additional convergence test helps them exit the process. Therefore, we observed that the generation size of 300 for 464 stock optimizations worked more efficiently while providing almost the same results.

D. SELF-ANALYSIS

To sum up, we were able to successfully utilize the CAPM+ strategy while outperformed both KOSPI200 and S&P500, the representative index funds. Moreover, it is noteworthy that we succeeded in both bull (the U.S.) and flat (the Korean) markets and the both produced over 300% gains over 11 years. Considering the fact that we were able to grasp promising investment opportunities even without any positive impacts or market trends, we believe that our strategy is very strong. Although we were able to earn some gains from investments with the NO-CAPM methodology as well, it was neither as profitable as the CAPM+ nor significantly better than the market index trading in either the Korean or the U.S. markets.

We also tested our works in light of the more realistic scenarios of annually evaluating the investments and gained impressive profits (>15%) in most of the test years. One impressive result from the experiments on annual ROI is that our trading strategy worked better than the market index in both markets for the year 2008, when the returns on index trading dropped by more than 20% due to the global economic recessions. Although our strategy could not turn the negative movements into positives, it could have reduced the degrees or the impacts of negative market shifts by searching those combinations of stocks that have more potential in the future.

In short, our method of designing a portfolio, constituted by risk-return balanced stocks with high potential, is capable of generating profits in both bull and flat markets. Suffering along with the market is inevitable when recessions or crises come, regardless of our informed estimations, because our tasks and assumptions depend on market movements. Nevertheless, we can provide a combination of stocks with a future that can at least reduce the amount of losses incurred.

In course of these experiments, we also observed that having a greater number of stocks in the list or larger market growths did not meaningfully make our approach become more successful. Because we simply identify stocks that are low-risk with high return and have high potentials in relation to the CAPM+ strategy, market growth or the number of stocks is not necessarily related to the profits generated by our method.

TABLE 8. List of stock codes(tickers) used in the experiments.

KOSPI200 Constituent Stocks
000070.KS,000080.KS,000100.KS,000120.KS,000150.KS,000210.KS,000240.KS,000270.KS,0006
40.KS,000660.KS,000670.KS,000720.KS,000810.KS,000880.KS,000990.KS,001040.KS,001060.KS
,001120.KS,001230.KS,001430.KS,001450.KS,001520.KS,001680.KS,001740.KS,001800.KS,0022
40.KS,002270.KS,002350.KS,002380.KS,002790.KS,002960.KS,003000.KS,003240.KS,003410.KS
,003490.KS,003520.KS,003550.KS,003620.KS,003850.KS,003920.KS,004000.KS,004020.KS,0041
70.KS,004370.KS,004490.KS,004800.KS,004990.KS,005180.KS,005250.KS,005300.KS,005380.KS
,005440.KS,005490.KS,005610.KS,005830.KS,005850.KS,005930.KS,005940.KS,006120.KS,0062
60.KS,006280.KS,006360.KS,006390.KS,006400.KS,006650.KS,006800.KS,006840.KS,007070.KS
,007310.KS,007570.KS,008060.KS,008560.KS,008770.KS,008930.KS,009150.KS,009240.KS,0094
20.KS,009540.KS,009830.KS,010060.KS,010120.KS,010130.KS,010140.KS,010620.KS,010780.KS
,010950.KS,011070.KS,011170.KS,011210.KS,011780.KS,011790.KS,012330.KS,012450.KS,0126
30.KS,012750.KS,014680.KS,014820.KS,014830.KS,015760.KS,016360.KS,017800.KS,018250.KS
,018260.KS,018880.KS,019680.KS,020000.KS,020150.KS,021240.KS,023530.KS,024110.KS,0255
40.KS,025860.KS,026960.KS,027410.KS,028050.KS,028260.KS,028670.KS,029780.KS,031430.KS
,032830.KS,033780.KS,034020.KS,034220.KS,034730.KS,035250.KS,036460.KS,042660.KS,0426
70.KS,047040.KS,047050.KS,047810.KS,049770.KS,051600.KS,051900.KS,051910.KS,052690.KS
,055550.KS,057050.KS,060980.KS,064350.KS,064960.KS,066570.KS,068270.KS,069260.KS,0696
20.KS,069960.KS,071050.KS,071840.KS,073240.KS,078930.KS,079430.KS,079550.KS,081660.KS
,086280.KS,086790.KS,088350.KS,090430.KS,093050.KS,093370.KS,096760.KS,096770.KS,0979
50.KS,103140.KS,105560.KS,105630.KS,108670.KS,111770.KS,114090.KS,115390.KS,120110.KS
,128940.KS,138930.KS,139480.KS,145990.KS,161390.KS,161890.KS,170900.KS,185750.KS,1924
00.KS,192820.KS,204320.KS,207940.KS,241560.KS,267250.KS,271560.KS,282330.KS,285130.KS
,294870.KS,298040.KS,316140.KS
S&P500 Constituent Stocks
DOV,HP,AAPL,SYMC,IDXX,MCK,MAR,COG,LIN,CF,MMM,ADSK,CMI,BR,BBT,ACN,ETFC,M
OS,MAS,CNC,TWTR,IQV,GPC,INTU,ATVI,GWW,APH,GE,NFLX,AEE,CPB,LYB,CCL,STT,CBS,
DISH,KO,AVGO,CHD,FLT,DLR,PEP,CINF,LNT,FTNT,FANG,BAC,AEP,MMC,ATO,LKQ,XOM,
AAP,NEM,IFF,ADS,BHGE,SBAC,LLY,AMCR,BAX,ABBV,FDX,NOV,PSA,NDAQ,FOXA,PKI,CI,P
PL,ROL,ADI,PCAR,SNA,KIM,BIIB,ETR,PXD,ORLY,AMP,PPG,NWSA,T,ROK,CNP,APA,SYF,AM
G,AZO,PNR,HAS,BBY,CPRI,DVA,DAL,APD,PRU,SYY,SCHW,PNC,INCY,FISV,PNW,EIX,DRI,PV
H,LH,CSX,MCHP,NKTR,TFX,AVY,BLK,SRE,ES,TRIP,JNPR,SBUX,RMD,HCA,ALB,GPS,CMS,M
AC,FLIR,UDR,ADP,NSC,GM,COP,CVS,SWKS,LW,MAA,QCOM,NTAP,MLM,DHR,UA,TDG,UA
A,CTL,TMUS,DOW,BWA,BA,O,ICE,DHI,MXIM,OMC,DGX,PFE,SEE,HSY,AMT,KLAC,FBHS,LH
X,BEN,PH,ILMN,JWN,CMCSA,HSIC,CFG,JNJ,HD,DISCK,HAL,PKG,HOG,AIG,CTSH,EXPD,D
XC,MGM,MDLZ,MSCI,FIS,AKAM,NCLH,EQR,HON,JCI,MET,IR,TPR,EVRG,TSS,ANSS,SWK,BK
,RHI,TRV,GLW,HES,ED,PEG,IT,SPG,MS,CMG,STI,CE,MRO,TROW,L,DTE,EXR,ANET,KEY,CM
A,SNPS,AFL,JBHT,CERN,FLS,PM,EOG,CAG,C,LMT,MPC,DISCA,USB,NWS,TMO,FRC,STX,CA
H,JPM,NWL,AAL,MTB,CELG,GPN,CXO,ALK,ADBE,IVZ,BSX,TIF,DD,GD,UAL,PFG,JEC,COTY
,KMI,GIS,F,DUK,LEG,MSI,ABMD,MSFT,NI,REGN,HOLX,NRG,ISRG,AON,PHM,PYPL,NVDA,A
LGN,CTVA,UPS,OXY,GILD,NTRS,TXT,QRVO,GOOG,RTN,RE,MDT,ESS,SJM,PG,NOC,AIZ,ROS
T,STZ,FE,INTC,LEN,BF_B,D,DVN,FTV,TTWO,COF,JEF,CRM,ARNC,NKE,UNP,NBL,HFC,ARE,
BXP,CLX,EBAY,MHK,EXC,COO,AWK,CHRW,COST,AVB,CPRT,HST,KMX,TSN,GRMN,HBI,SL
B,IRM,SYK,IP,HPE,RCL,AGN,HUM,AMAT,XEC,BDX,PAYX,ITW,GOOGL,AMZN,FFIV,FRT,AL
XN,URI,KHC,ORCL,DE,M,AIV,IPG,ABT,APTV,KSU,EMR,AJG,BMY,HBAN,SHW,TGT,TAP,CB,
ECL,CL,ANTM,FL,ULTA,OKE,AMGN,CVX,NUE,MU,CME,HIG,EW,DFS,CAT,REG,LNC,CTAS,
TXN,DRE,PRGO,JKHY,PWR,EXPE,NEE,FMC,ABC,DLTR,KEYS,RL,CCI,EA,LRCX,NLSN,AOS,R
OP,SO,CSCO,ALL,SLG,FOX,PBCT,RSG,RJF,AMD,MCO,MA,MO,UNH,EFX,MKTX,LOW,IBM,F
AST,HRB,PSX,MYL,MTD,CTXS,ADM,KR,HLT,KSS,TJX,SPGI,ETN,SIVB,PLD,KMB,PGR,AME,A
LLE,BKNG,AES,AXP,EMN,TEL,BLL,RF,K,FB,FCX,MNST,MRK,HRL,LUV,INFO,MCD,DG,TSC
O,A,LB,CDNS,FTI,GS,EL,HII,BRK_B,EQIX,FITB,XRAY,CBRE,HPQ,MKC,CHTR,CBOE,IPGP,H
CP, APC, GL

V. CONCLUSION

The recent years of bull market rallies provided winning opportunities to many investors in both the U.S. and Korean stock markets. For example, investing on widely-renowned companies like Amazon (AMZN) or Netflix (NFLX) could have brought investors abundant profits in the last 10 years, while their stock price went up more than 2500% and 3500%, respectively. Throughout those years, many public media continuously announced the public how good the market was. Additionally, companies like Regeneron Pharmaceuticals, Inc. (REGN) or Alexion Pharmaceutical, Inc. (ALXN) could have brought investors profits worth more than their initial funds multiplied by 24, during a recent bull rally in the S&P market.

At the same time, the values of companies like Apache (APA), CenturyLink (CTL), Devon Energy (DVN), and Exelon (EXC) all significantly dropped despite the bull rallies. There are many more companies which lost their market capitals and their statuses as 500-large cap companies. Many companies in bio/pharmaceuticals or technology industry rallied, while companies in the energy sector started to lose their market caps. Such analysis of who did well or bad, unfortunately, can be completed only after the results came out. Investors, especially individual ones, cannot easily figure out which stocks will be the future winners of the market, so to speak. Without insider information or industry expertise, investments are often compared to no better than "A Monkey Throwing Darts."

Through the experiments, we first acquired historic stock price data from the Korean and U.S. stock markets. With the obtained data, we calculated beta values for each stock and measured its valuations. After, we applied them in a genetic algorithm to search which stocks have potential to grow further and to form an optimal portfolio in a next period. Thereafter, we observed how the combinations of such stocks performed in the succeeding trading periods. Although the proposed method did not offer 3000% profits, it hedged the risk by building a portfolio and by selecting undervalued assets. As a result, our ensembled investment strategy of asset valuations and momentum strategy was considered as capable of offering a better opportunity of searching out an optimal portfolio that entails high returns, low risks and has the potential to grow without deliberate analysis of experts or domain knowledge.

In the next step, we plan to test our investment strategy with real-time data, in order to verify its feasibility and market usability. We believe that our attempts to find an optimal portfolio can be further extended to Fama-French's Factor Model, which improves Capital Asset Pricing Model and more empirically used in practice. In the future works, market volatility indexes can also be used to analyze the overall market behavior and act as a part of a multi-objective genetic algorithm.

APPENDIX

See Table 8.

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