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Finite-Time Non-Fragile Extended Dissipative Control of Periodic Piecewise Time-Varying Systems

YUN LIU, ZUOLIN DENG, PANSHUO LI, AND BIN ZHANG
School of Automation, Guangdong University of Technology, Guangzhou 510006, China
Guangdong Province Key Laboratory of Intelligent Decision and Cooperative Control, Guangdong University of Technology, Guangzhou 510006, China

Corresponding author: Panshuo Li (panshuoli812@gmail.com)

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ABSTRACT This paper studies the problem of finite-time non-fragile extended dissipative control for the periodic piecewise time-varying systems. First, based on the time-varying Lyapunov matrix and the matrix polynomial condition, a sufficient condition of finite-time boundedness for periodic piecewise time-varying subsystem is derived, and the finite-time extended dissipative performance is then analyzed. Furthermore, considering two types of time-varying controller perturbations, the non-fragile controllers are developed, which can guarantee the finite-time boundedness of periodic piecewise time-varying systems and satisfy the extended dissipative performance at the same time. Finally, numerical examples demonstrate the effectiveness of the proposed methods.

INDEX TERMS Periodic piecewise system, non-fragile control, extended dissipative control.

I. INTRODUCTION

Periodic characteristics extensively exist in various fields, such as vibration system [1], hypersonic cruise vehicle [2], spacecraft magnetic attitude control [3]. Among them, the stability analysis and synthesis of discrete-time periodic systems can be conducted based on the Floquet-Lyapunov theory and the Lifting techniques. However, for the continuous-time cases, the control problems are very difficult to solve due to lacking the closed-form solutions to Floquet factorisations [4]. Therefore, periodic piecewise system, as an effective approximating approach of continuous-time periodic systems, has attracted increasing interests [7]–[13], [32], [33].

Periodic piecewise system consists of several subsystems and has the fixed switching sequence and dwell time of each subsystems. Besides its valuable theoretical properties, periodic piecewise system also has many applications in practice, including but not limited to power converters [5], and vibration systems [6]. Many results have been reported in [7]–[9], where the time-invariant subsystems are considered. However, the time-invariant subsystem formulations may render loss of some dynamic properties of the original system [10]. Therefore, the time-varying subsystem description may be more promising in practice, as have been investigated in [10]–[13], [32], [33]. The stability, stabilisation and L_2-gain analysis of periodic piecewise time-varying systems were investigated in [10] based on the continuous time-varying Lyapunov function. The H_∞ tracking controller with time-varying periodic gains for periodic piecewise time-varying systems was developed in [11]. A less conservative result on reachable set estimation of periodic piecewise time-varying systems was derived in [12] based on the Lyapunov function with interval segmentation and a general reachable set estimation condition. Moreover, for periodic piecewise polynomial time-varying systems, the stability analysis and H_∞ control were studied in [13] and [14], respectively.

In practice, such as robot and missile control, the analysis and control of transient behaviour of a system over a finite time interval is of great significance. Many results have been reported like [15]–[19] since the concepts of finite-time stability and finite-time control were first proposed in [20].

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In [7], for periodic piecewise constant subsystem, the finite-time stability, L_2 performance, and H_∞ control strategy were investigated. On the other hand, the study of different system performances is also a focus in system analysis and control. In [21], the concept of extended dissipative performance was proposed, which can integrate several well-known performance indices such as H_∞, L_2-L_∞, passivity and (Q, S, R)-dissipativity into a unified framework. The desired performance can be transformed by adjusting the weighting matrix parameters. Many results have been reported for it [22]–[24]. The extended dissipative state estimation for Markov jump neural networks with unreliable links was studied in [23]. For a class of uncertain discrete time switched linear systems, the issues of finite-time extended dissipative performance condition was studied in [24]. However, the analysis of the finite-time extended dissipative performance for periodic piecewise time-varying system has not been reported so far.

In the controller development, controllers are assumed to accurately operate. However, in applications, the parameters of the controller are often prone to slight perturbation or deviation due to their own or external factors, thereby deteriorating the control performance, or even destroying the system [25], [26]. Therefore, non-fragile control has received increasing attention in engineering applications [26]–[31]. The data-driven non-fragile filtering problem of the cyber-physical system, and the non-fragile H_∞ multivariate *PID* controllers with derivative filters were reported in [27], [28]. The finite-time non-fragile control scheme of linear uncertain positive systems was presented in [29]. In [30], for linear time-invariant systems, the multiplicative non-fragile controller was designed, of which controller parameters could be transformed into solving the symmetric positive definite solutions to algebraic Riccati inequalities. The non-fragile synchronisation control scheme was provided in [31] for complex networks with time-varying coupling delay and missing data. Inspired by existing works, the non-fragile control strategies of periodic piecewise time-varying system were studied in [32], [33], but various performance indices are not taken into account.

Motivated by the above works, the problem of finite-time extended dissipative control for periodic piecewise time-varying systems is studied in this paper. The analysis of the finite-time extended dissipative performance combined with finite-time boundedness is first carried out. Then, two types of time-varying controller gain perturbations with periodic time-varying parameters are considered. Based on it, the non-fragile controllers to ensure the finite-time extended dissipative performance of the system are designed. The main contributions of this paper are twofold: (1) The condition of finite-time extended dissipativity for periodic piecewise time-varying systems is proposed for the first time, which provides a choice for analyzing multiple performance of the system simultaneously. (2) The non-fragile controllers which ensure the finite-time boundedness and the extended dissipative performance are designed. The controllers can be solved directly with the LMIs and are more desirable in engineering application. This paper is organized as follows. The problem descriptions of system are given in Section 2. The main results are given in Section 3. Numerical simulations are given in Section 4, and the conclusions in Section 5.

Notation: $\mathbb{R}^n$ represents the n-dimensional Euclidean space, $\|\cdot\|$ denotes the Euclidean vector norm of a matrix. $\lambda_{\max}(\cdot)$, $\lambda_{\min}(\cdot)$ refer to the maximum, minimum eigenvalues of a real symmetric matrix. I represents the identity matrix. Given a real symmetric matrix P, the notation $P \leq 0$ (respectively, $P < 0$) means that the matrix P is negative semi-definite (respectively, negative definite), and the superscripts P^T stands for the transpose of a matrix. $\mathrm{sym}(P) = P + P^T$, and $\mathrm{diag}\{\cdot\}$ stands for a diagonal matrix. In block symmetric matrices, we use $*$ as an ellipsis for the terms introduced, $\mathcal{D}^+(\cdot)$ denotes the upper right Dini derivative.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of continuous-time periodic piecewise time-varying systems:

$$\dot{x}(t) = A_i(t)x(t) + B_i(t)u(t) + E_i(t)w(t),$$
$$z(t) = C_i x(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^r$, $u(t) \in \mathbb{R}^m$ and $w(t) \in \mathbb{R}^d$ are the system state, control output, input and external disturbance, respectively. For $t > 0$, one has $A_i(t) = A_i(t + T_p)$, $B_i(t) = B_i(t + T_p)$, and $E_i(t) = E_i(t + T_p)$, where T_p is a fundamental period of the system and is composed of S subintervals $[t_{i-1}, t_i)$, $i \in \mathcal{S}$, $\mathcal{S} = \{1, 2, \ldots, S\}$. The dwell time of the ith subsystem is defined as $T_i = t_i - t_{i-1}$, where $\sum_{i=1}^{S} T_i = T_p$, $t_0 = 0$, and $t_S = T_p$. C_i is real constant matrices with appropriate dimensions. The linear time-varying matrices $A_i(t)$ is denoted by

$$A_i(t) = A_i + \frac{(t - \ell T_p - t_{i-1})}{T_i}(A_{i+1} - A_i), \tag{2}$$

and $B_i(t)$, $E_i(t)$ are given in the similar interpolation formulation, where $A_i, A_{i+1}, B_i, B_{i+1}, E_i, E_{i+1}$ are constant matrices.

Before stating the main contents in this section, an assumption is first introduced.

Assumption 1: Matrices $\psi_1, \psi_2, \psi_3, \psi_4$ satisfy the following conditions:

1) $\psi_1 = \psi_1^T \leq 0$, $\psi_3 = \psi_3^T > 0$, $\psi_4 = \psi_4^T \geq 0$;
2) $(\|\psi_1\| + \|\psi_2\|)\psi_4 = 0$.

To facilitate the conditions development, the following definitions and lemmas are then introduced.

Definition 1 (Finite-Time Boundedness [7]): Given three constants c_1, c_2, h with $0 < c_1 < c_2, 0 \leq h$, a time interval $0 < T_f$ and a positive definite matrix $0 < F$, the periodic piecewise time-varying system (1) with $u(t) = 0$ and the time-varying external disturbance $w(t)$ is said to be finite-time bounded with respect to (c_1, c_2, F, T_f, h), if

$$x^T(0)Fx(0) \leq c_1 \Rightarrow x^T(t)Fx(t) < c_2, \forall t \in [0, T_f],$$
$$\forall w(t) : \int_0^{T_f} w^T(t)w(t)dt \leq h.$$

Definition 2: *Given real matrices ψ_1, ψ_2, ψ_3 and ψ_4 satisfying Assumption 1, the periodic piecewise time-varying system (1) is said to be extended dissipative if the following inequality holds for any $0 \leq T_f$ and all $u(t) \in L_2[0,\infty]$:*

$$\int_0^{T_f} J(t)dt - \sup_{0\leq t\leq T_f} z^T(t)\psi_4 z(t) \geq 0, \forall t \in [0, T_f],$$
$$\forall x(t) : \int_0^{T_f} x^T(t)x(t)dt \leq d \tag{3}$$

where $J(t) = z^T(t)\psi_1 z(t) + z^T(t)\psi_2 w(t) + w^T(t)\psi_2^T z(t) + w^T(t)\psi_3 w(t)$, and d is a fixed sufficient large positive scalar.

Remark 1 [21]: *By designing appropriate parameters weighted matrices, the extended dissipativity can evolve into multiple performance indices. To be specific, (3) stands for the H_∞ performance when $\psi_1 = -I, \psi_2 = 0, \psi_3 = \gamma^2 I$ and $\psi_4 = 0$; (3) becomes the $L_2 - L_\infty$ performance when $\psi_1 = \psi_2 = 0, \psi_3 = \gamma^2 I$ and $\psi_4 = I$; (3) turns into the passivity performance when $\psi_1 = 0, \psi_2 = I, \psi_3 = \gamma I$ and $\psi_4 = 0$; (3) transforms into the (Q, S, R)-dissipativity performance when $\psi_1 = Q, \psi_2 = S, \psi_3 = R - \beta I$ and $\psi_4 = 0$.*

Definition 3 [33]: *Consider that there are two types of gain perturbations in the controller, which are given as follows. (1) Norm-bounded additive perturbations:*

$$\Delta K_i(t) = H_i G_i(t) X_i(t), \quad i \in \mathcal{S}, \tag{4}$$

(2) Norm-bounded multiplicative perturbations:

$$\Delta K_i(t) = H_i G_i(t)\widetilde{X}_i(t) K_i(t), \quad i \in \mathcal{S}, \tag{5}$$

and consider the time-varying matrices therein:

$$X_i(t) = X_i + \frac{(t - \ell T_p - t_{i-1})}{T_i}(X_{i+1} - X_i),$$
$$\widetilde{X}_i(t) = \widetilde{X}_i + \frac{(t - \ell T_p - t_{i-1})}{T_i}(\widetilde{X}_{i+1} - \widetilde{X}_i) \tag{6}$$

where matrices H_i, X_i, X_{i+1}, $\widetilde{X}_i$ and $\widetilde{X}_{i+1}$ are known matrices with appropriate dimensions. $G_i(t)$ for $i \in \mathcal{S}$, are unknown time-varying matrices and satisfying

$$G_i(t) \in \Phi \triangleq \{G_i(t) | G_i(t)G_i^T(t) \leq I\}. \tag{7}$$

Remark 2 [21]: *Note that the $X_i(t)$ and $\widetilde{X}_i(t)$ are time-varying which introduces more free variables, and giving rise to more plentiful perturbation characteristic of the controller.*

Lemma 1 [10]: *Consider the matrix polynomial $f(\delta_1, \delta_2, \ldots, \delta_m)$ given as,*

$$f(\delta_1, \delta_2, \ldots, \delta_m) = \Sigma_0 + \delta_1\Sigma_1 + \delta_1\delta_2\Sigma_2 + \cdots + \Big(\prod_{q=1}^{m}\delta_q\Big)\Sigma_m \tag{8}$$

where $m \in \mathbb{N}^+$ and $m \geq 2$, $\Sigma_k \in \mathbb{R}^{r\times r}$, $k = 0, 1, \ldots, m$ are real symmetric matrices, $\delta_q, q = 1, 2, \cdots, m$ are variables and $\delta_q \in [0, 1]$. If

$$\sum_{q=0}^{d}\Sigma_q < 0, \quad d = 0, 1, \ldots, m,$$

then the matrix polynomial $f(\delta_1, \delta_2, \ldots, \delta_m) < 0$.

Lemma 2 [28]: *Given a real symmetric matrix $\boldsymbol{\mu}$, let $\boldsymbol{h}$ and $\boldsymbol{n}$ be real matrices with appropriate dimensions, then*

$$\boldsymbol{\mu} + \boldsymbol{h}\xi\boldsymbol{n} + \boldsymbol{n}^T\xi^T\boldsymbol{h}^T < 0,$$

for all ξ satisfying $\xi\xi^T \leq I$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$\boldsymbol{\mu} + \varepsilon\boldsymbol{h}\boldsymbol{h}^T + \varepsilon^{-1}\boldsymbol{n}^T\boldsymbol{n} < 0 \tag{9}$$

Lemma 3 [34]: *For any real matrices α, β of appropriate dimensions, we have $\beta^T\alpha + \alpha^T\beta \leq \beta^T\beta + \alpha^T\alpha$.*

III. MAIN RESULTS

Constructing a Lyapunov function with periodic time-varying matrices:

$$V(t) = V_i(t) = x^T(t)P_i(t)x(t) \tag{10}$$

where

$$P_i(t) = P_i + \frac{t - \ell T_p - t_{i-1}}{T_i}(P_{i+1} - P_i),$$
$$t \in [\ell T_p + t_{i-1},\ \ell T_p + t_i) \tag{11}$$

with $P_i(t) = P_i(t + T_p)$, $i \in \mathcal{S}$, and $P_i > 0, P_{i+1} > 0$ are constant matrices, $P_{S+1} = P_1$, and

$$\sup_{i\in\mathcal{S}} \lambda_{\max}(P_i)I \geq P_i(t) \geq \inf_{i\in\mathcal{S}} \lambda_{\min}(P_i)I > 0. \tag{12}$$

A. FINITE-TIME BOUNDEDNESS ANALYSIS

In this subsection, the finite-time boundedness of periodic piecewise time-varying system is studied.

Theorem 1: *Consider periodic piecewise time-varying system (1) with $u(t) = 0$, given scalars $\alpha_i > 0, \alpha_{\max} = \max_{i\in\mathcal{S}}\{\alpha_i\}, i \in \mathcal{S}$, $0 < c_1 < c_2, h > 0$, a time interval $T_f > 0$ and a matrix $F > 0$, the system (1) is finite-time bounded with respect to (c_1, c_2, F, T_f, h), if there exist scalars $\lambda_1, \lambda_2, \lambda_3$ and matrices P_i, $U_i > 0$, $i \in \mathcal{S}$, with $P_{S+1} = P_1$, such that*

$$\begin{bmatrix} \Sigma_i^0 & \Omega_i^0 \\ * & \Lambda_i^0 \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \Sigma_i^0 & \Omega_i^0 \\ * & \Lambda_i^0 \end{bmatrix} + \begin{bmatrix} \Sigma_i^1 & \Omega_i^1 \\ * & 0 \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} \Sigma_i^0 & \Omega_i^0 \\ * & \Lambda_i^0 \end{bmatrix} + \begin{bmatrix} \Sigma_i^1 & \Omega_i^1 \\ * & 0 \end{bmatrix} + \begin{bmatrix} \Sigma_i^2 & \Omega_i^2 \\ * & 0 \end{bmatrix} < 0, \tag{15}$$

$$\lambda_1 F \leq P_i \leq \lambda_2 F, \tag{16}$$

$$U_i \leq \lambda_3 I, \tag{17}$$

$$(\lambda_2 c_1 + \lambda_3 h)e^{\alpha_{\max}T_f} < \lambda_1 c_2 \tag{18}$$

where

$$\Sigma_i^0 = A_i^T P_i + P_i A_i + \frac{1}{T_i}(P_{i+1} - P_i) - \alpha_i P_i,$$

$$\begin{aligned}
\Sigma_i^1 &= \mathrm{sym}(A_i^T P_{i+1} + A_{i+1}^T P_i - 2A_i^T P_i) \\
&\quad -\alpha_i(P_{i+1} - P_i), \\
\Sigma_i^2 &= \mathrm{sym}(A_{i+1}^T P_{i+1} + A_i^T P_i - A_i^T P_{i+1} - A_{i+1}^T P_i), \\
\Omega_i^0 &= P_i E_i, \\
\Omega_i^1 &= P_i E_{i+1} + P_{i+1} E_i - 2P_i E_i, \\
\Omega_i^2 &= P_{i+1} E_{i+1} - P_{i+1} E_i - P_i E_{i+1} + P_i E_i, \\
\Lambda_i^0 &= -U_i, \\
\lambda_1 &= \inf_{i\in\mathcal{S}} \lambda_{\min}(\widehat{P}_i),\ \lambda_2 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\widehat{P}_i), \\
\lambda_3 &= \sup_{i\in\mathcal{S}} \lambda_{\max}(U_i),\ \widehat{P}_i = F^{-\frac{1}{2}} P_i F^{-\frac{1}{2}}.
\end{aligned}$$

Proof: To conduct the finite-time boundedness analysis over $[0, T_f]$, we first take the derivative for the function (10), it will evolve into the following form:

$$\begin{aligned}
&\dot{V}(t) - \alpha_i V(t) - w^T(t) U_i w(t) \\
&= \dot{x}^T(t) P_i(t) x(t) + x^T(t) \mathcal{D}^+ P_i(t) x(t) \\
&\quad + x^T(t) P_i(t) x(t) - \alpha_i V(t) - w^T(t) U_i w(t), \\
&= x^T(t)\Big(A_i^T(t) P_i(t) + P_i(t) A_i(t) - \alpha_i P_i(t) \\
&\quad + \mathcal{D}^+ P_i(t)\Big) x(t) + w^T(t) E_i^T(t) P_i(t) x(t) \\
&\quad + x^T(t) P_i(t) E_i(t) w(t) - w^T(t) U_i w(t), \qquad (19)
\end{aligned}$$

which can be rewritten as

$$\dot{V}(t) - \alpha_i V(t) - w^T U_i w(t) = \chi^T(t) \Phi(t) \chi(t) \qquad (20)$$

where

$$\chi(t) = [x^T(t) \ \ w^T(t)]^T, \quad \Phi(t) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ * & -U_i \end{bmatrix}$$

with

$$\begin{aligned}
\Phi_{11} &= A_i(t)^T P_i(t) + P_i(t) A_i(t) - \alpha_i P_i(t) + \mathcal{D}^+ P_i(t), \\
\Phi_{12} &= P_i(t) E_i(t).
\end{aligned}$$

For $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i)$, $\ell = 0, 1, 2 \ldots$, $i \in \mathcal{S}$, since $0 \leq \frac{t - \ell T_p - t_{i-1}}{T_i} \leq 1$, with (13)-(15) and Lemma 1, one has

$$\begin{bmatrix} \rho(t) & P_i(t) E_i(t) \\ * & -U_i \end{bmatrix} < 0 \qquad (21)$$

where

$$\rho(t) = A_i^T(t) P_i(t) + P_i(t) A_i(t) + \mathcal{D}^+ P_i(t) - \alpha_i P_i(t).$$

Then, one has

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ * & -U_i \end{bmatrix} < 0. \qquad (22)$$

Since condition (22) holds, it is obvious that

$$\dot{V}(t) < \alpha_i V(t) + w^T(t) U_i w(t). \qquad (23)$$

By integrating (23) from $\ell T_p + t_{i-1}$ to $\ell T_p + t_i$, one obtains

$$\begin{aligned}
V(t) &< e^{\alpha_i(t - \ell T_p - t_{i-1})} V(\ell T_p + t_{i-1}) \\
&\quad + \lambda_{\max}(U_i) \int_{\ell T_p + t_{i-1}}^{t} e^{\alpha_i(t-\tau)} w^T(\tau) w(\tau) d\tau.
\end{aligned}$$

Note that the above $\alpha_i > 0$, it thus has

$$\begin{aligned}
&\int_{\ell T_p + t_{i-1}}^{t} e^{\alpha_i(t-\tau)} w^T(\tau) w(\tau) d\tau \\
&\qquad \leq e^{\alpha_i(t - \ell T_p - t_{i-1})} \int_{\ell T_p + t_{i-1}}^{t} w^T(\tau) w(\tau) d\tau.
\end{aligned}$$

Combining with the similar arguments in [7], one has

$$\begin{aligned}
V(t) &< e^{\alpha_i(t - \ell T_p - t_{i-1})} \Big\{ e^{\alpha_i(t_{i-1} - t_{i-2})} V(\ell T_p \\
&\quad + t_{i-2}) + \lambda_{\max}(U_i) \int_{\ell T_p + t_{i-2}}^{\ell T_p + t_{i-1}} e^{\alpha_i(\ell T_p + t_{i-1} - \tau)} \\
&\quad \times w^T(\tau) w(\tau) d\tau \Big\} + \lambda_{\max}(U_i) e^{\alpha_i(t - \ell T_p - t_{i-1})} \\
&\quad \times \int_{\ell T_p + t_{i-1}}^{t} w^T(\tau) w(\tau) d\tau \\
&< e^{\alpha_i(t - \ell T_p - t_{i-2})} \Big\{ e^{\alpha_i(t_{i-2} - t_{i-3})} V(\ell T_p + t_{i-3}) \\
&\quad + \lambda_{\max}(U_i) \int_{\ell T_p + t_{i-3}}^{\ell T_p + t_{i-2}} e^{\alpha_i(\ell T_p + t_{i-2} - \tau)} \\
&\quad \times \int_{\ell T_p + t_{i-1}}^{t} w^T(\tau) w(\tau) d\tau + \lambda_{\max}(U_i) \\
&\quad \times \int_{\ell T_p + t_{i-2}}^{\ell T_p + t_{i-1}} e^{\alpha_i(\ell T_p + t_{i-1} - \tau)} w^T(\tau) w(\tau) d\tau \Big\} \\
&\quad + \lambda_{\max}(U_i) e^{\alpha_i(t - \ell T_p - t_{i-1})} \int_{\ell T_p + t_{i-1}}^{t} w^T(\tau) w(\tau) d\tau \\
&\quad \vdots \\
&< e^{\alpha_i t} V(0) + \lambda_{\max}(U_i) \\
&\quad \times \Big\{ e^{\alpha_i(t_1 - 0)} \int_{\ell T_p + 0}^{\ell T_p + t_1} w^T(\tau) w(\tau) d\tau \\
&\quad + e^{\alpha_i(t_2 - t_1)} \int_{\ell T_p + t_1}^{\ell T_p + t_2} w^T(\tau) w(\tau) d\tau \\
&\quad + \ldots + e^{\alpha_i(t - t_{i-1})} \int_{\ell T_p + t_{i-1}}^{t} w^T(\tau) w(\tau) d\tau \Big\} \\
&< e^{\alpha_{\max} t} \{ V(0) + \sup_{i\in\mathcal{S}} \lambda_{\max}(U_i) h \}. \qquad (24)
\end{aligned}$$

Let $\widehat{P}_i = F^{-\frac{1}{2}} P_i F^{-\frac{1}{2}}$, and scalars $c_1 = x(0)^T x(0)$, $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ satisfying

$$\lambda_1 = \inf_{i\in\mathcal{S}} \lambda_{\min}(\widehat{P}_i),\ \ \lambda_2 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\widehat{P}_i),\ \ \lambda_3 = \sup_{i\in\mathcal{S}} \lambda_{\max}(U_i).$$

With conditions (16)-(18) and (12), given a finite $t = T_f > 0$, one has

$$x^T(T_f) F x(T_f) < \frac{\lambda_2 c_1 + \lambda_3 h}{\lambda_1} e^{\alpha_{\max} T_f} < c_2.$$

Therefore, the system (1) is finite-time bounded with repect to (c_1, c_2, F, T_f, h). The proof is completed.

Remark 3: It is worth mentioning that the finite-time boundedness of periodic piecewise time-invariant system can *be found in [7]. However, that is only valid in the time-invariant subsystem case and cannot be applied for the time-varying subsystem case as in this work.*

B. FINITE-TIME EXTENDED DISSIPATIVE ANALYSIS

In this subsection, the condition of finite-time extended dissipative performance of periodic piecewise time-varying system is derived.

Theorem 2: Consider periodic piecewise time-varying system (1) with $u(t) = 0$, *given scalars* $\alpha_i > 0, \alpha_{\max} = \max_{i\in\mathcal{S}}\{\alpha_i\}, i \in \mathcal{S}, 0 < c_1 < c_2, h > 0, d > 0$, *a time interval* $T_f > 0$ *and matrices* $\psi_1, \psi_2, \psi_3, \psi_4, F > 0$, *the system (1) is finite-time bounded and satisfies the extended dissipative performance, if there exist scalars* $\lambda_1, \lambda_2, \lambda_4, \lambda_5, \lambda_6$ *and matrices* $P_i,\ i \in \mathcal{S}$, *with* $P_{S+1} = P_1$, *such that*

$$\begin{bmatrix} \Sigma_{i,0} & \Omega_{i,0} \\ * & \Lambda_{i,0} \end{bmatrix} < 0, \tag{25}$$

$$\begin{bmatrix} \Sigma_{i,0} & \Omega_{i,0} \\ * & \Lambda_{i,0} \end{bmatrix} + \begin{bmatrix} \Sigma_{i,1} & \Omega_{i,1} \\ * & 0 \end{bmatrix} < 0, \tag{26}$$

$$\begin{bmatrix} \Sigma_{i,0} & \Omega_{i,0} \\ * & \Lambda_{i,0} \end{bmatrix} + \begin{bmatrix} \Sigma_{i,1} & \Omega_{i,1} \\ * & 0 \end{bmatrix} + \begin{bmatrix} \Sigma_{i,2} & \Omega_{i,2} \\ * & 0 \end{bmatrix} < 0, \tag{27}$$

$$C_i^T \psi_4 C_i - \frac{1}{e^{\alpha_{\max} T_f}} P_i < 0, \tag{28}$$

$$C_i^T \psi_4 C_i - \frac{1}{e^{\alpha_{\max} T_f}} P_{i+1} < 0, \tag{29}$$

$$\lambda_1 F \le P_i \le \lambda_2 F, \tag{30}$$

$$e^{\alpha_{\max} T_f}[\lambda_4 d + (\lambda_5 + \lambda_6)h] < \lambda_1 c_2 \tag{31}$$

where

$$\begin{aligned}
\Sigma_{i,0} &= A_i^T P_i + P_i A_i + \frac{1}{T_i}(P_{i+1} - P_i) \\
&\quad -\alpha_i P_i - C_i^T \psi_1 C_i, \\
\Sigma_{i,1} &= \mathrm{sym}(A_i^T P_{i+1} + A_{i+1}^T P_i - 2A_i^T P_i) \\
&\quad -\alpha_i (P_{i+1} - P_i), \\
\Sigma_{i,2} &= \mathrm{sym}(A_{i+1}^T P_{i+1} + A_i^T P_i - A_i^T P_{i+1} - A_{i+1}^T P_i), \\
\Omega_{i,0} &= P_i E_i - C_i^T \psi_2, \\
\Omega_{i,1} &= P_i E_{i+1} + P_{i+1} E_i - 2P_i E_i, \\
\Omega_{i,2} &= P_{i+1} E_{i+1} - P_{i+1} E_i - P_i E_{i+1} + P_i E_i, \\
\Lambda_{i,0} &= -\psi_3, \\
\lambda_1 &= \inf_{i\in\mathcal{S}} \lambda_{\min}(\widehat{P}_i),\ \lambda_2 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\widehat{P}_i), \\
\widehat{P}_i &= F^{-\frac{1}{2}} P_i F^{-\frac{1}{2}}, \lambda_4 = \sup_{i\in\mathcal{S}} \lambda_{\max}(C_i^T C_i), \\
\lambda_5 &= \sup_{i\in\mathcal{S}} \lambda_{\max}(\psi_2^T \psi_2), \lambda_6 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\psi_3).
\end{aligned}$$

Proof: To analyze the finite-time boundedness and the extended dissipativity over $t \in [0, T_f]$, let us first consider $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i), \ell = 0, 1, 2 \ldots, i \in \mathcal{S}$. Constructing the Lyapunov function as (10) with continuous time-varying Lyapunov matrix (11). Since $0 \le \frac{t-\ell T_p - t_{i-1}}{T_i} \le 1$, with (25)-(27) and Lemma 1, one has

$$\begin{bmatrix} \pi(t) & P_i(t)E_i(t) - C_i^T \psi_2 \\ * & -\psi_3 \end{bmatrix} < 0 \tag{32}$$

where

$$\begin{aligned}
\pi(t) = A_i^T(t)P_i(t) + P_i(t)A_i(t) - \alpha_i P_i(t) \\
+\mathcal{D}^+ P_i(t) - C_i^T \psi_1 C_i
\end{aligned}$$

The inequality (32) can be rewritten as

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ * & -\psi_3 \end{bmatrix} < 0 \tag{33}$$

where

$$\begin{aligned}
\Psi_{11} &= A_i(t)^T P_i(t) + P_i(t)A_i(t) - \alpha_i P_i(t) \\
&\quad +\mathcal{D}^+ P_i(t) - C_i^T \psi_1 C_i, \\
\Psi_{12} &= P_i(t)E_i(t) - C_i^T \psi_2,
\end{aligned}$$

According to the similar derivation of (19), one obtains

$$\dot{V} - \alpha_i V(t) - J(t) = \chi^T(t)\Psi(t)\chi(t)$$

where

$$\chi(t) = [x^T(t)\ w^T(t)]^T, \quad \Psi(t) = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ * & -\psi_3 \end{bmatrix}.$$

It is manifest that $\dot{V} - \alpha_i V(t) - J(t) < 0$ holds or not, which is determined by inequality (33). Hence, following the proof line of (24), it is easy to obtain the following form

$$V(t) < e^{\alpha_{\max} t}\{V(0) + \int_0^t J(s)ds\},$$

under zero initial condition $V(0) = 0$, one has

$$V(t) < e^{\alpha_{\max} t} \int_0^t J(s)ds, \tag{34}$$

In what follows, let us now consider the left-hand side of inequality (3) in Definition 2, that is, $\int_0^{T_f} J(t)dt - \sup_{0\le t\le T_f} z^T(t)\psi_4 z(t)$, which is denoted by $\boldsymbol{\phi}(t)$. Since $\int_0^{T_f} J(t)dt \ge 0$, one can observe that $\boldsymbol{\phi}(t) \ge 0$ when $\psi_4 = 0$. Note, moreover, when $\psi_4 > 0$, set the parameters in Assumption 1 to $\psi_1 = 0, \psi_2 = 0, \psi_3 > 0$, it follows that

$$\int_0^t J(s)ds = \int_0^t w^T(s)\psi_3 w(s)ds.$$

Combining with (34), it is thus that for $\forall t \in [0, T_f]$, one has

$$\begin{aligned}
\int_0^{T_f} J(s)ds > \int_0^t J(s)ds > \frac{V(t)}{e^{\alpha_{\max} T_f}} \\
\ge \frac{1}{e^{\alpha_{\max} t}} x^T(t)P_i(t)x(t) > 0,
\end{aligned}$$

with (28)-(29), then one obtains

$$\begin{aligned}
\int_0^{T_f} J(s)ds \ge \frac{1}{e^{\alpha_{\max} t}} x^T(t)P_i(t)x(t) \\
\ge x^T(t)C_i^T \psi_4 C_i x(t) = z^T(t)\psi_4 z(t).
\end{aligned}$$

Hence $\boldsymbol{\phi}(t) \geq 0$, that is,

$$\int_0^{T_f} J(s)ds - \sup_{0 \geq t \geq T_f} z^T(t)\psi_4 z(t) \geq 0.$$

The proof of extended dissipativity is completed. Next, what has to be proven is the finite-time boundedness. In the light of inequality (34), combining with the the similar reasoning of [22], it holds that

$$V(t) \leq e^{\alpha_{\max} T_f} \int_0^{T_f} J(s)ds,$$

when $\psi_1 \leq 0$, one obtains

$$\int_0^{T_f} J(s)ds \leq \int_0^{T_f} [z^T(s)\psi_2 w(s) + w^T(s)\psi_2^T z(s) + w^T(s)\psi_3 w(s)]ds,$$

which implies

$$V(t) \leq e^{\alpha_{\max} T_f} [\int_0^{T_f} [z^T(s)\psi_2 w(s) + w^T(s)\psi_2^T z(s) + w^T(s)\psi_3 w(s)]ds].$$

Hence, given $\widehat{P}_i = F^{-\frac{1}{2}} P_i F^{-\frac{1}{2}}$, one has

$$\begin{aligned} x^T(t)Fx(t) &\leq \frac{V(t)}{\lambda_1} \\ &\leq \frac{e^{\alpha_{\max} T_f}}{\lambda_1} [\int_0^{T_f} [x^T(s)C_i^T \psi_2 w(s) \\ &\quad + w^T(s)\psi_2^T C_i x(s) \\ &\quad + w^T(s)\psi_3 w(s)]ds] \end{aligned} \tag{35}$$

where $\lambda_1 = \inf_{i \in \mathcal{S}} \lambda_{\min}(\widehat{P}_i) > 0$. Utilizing the Lemma 3, one has

$$x^T(s)C_i^T \psi_2 w(s) + w^T(s)\psi_2^T C_i x(s) \leq x^T(s)C_i^T C_i x(s) + w^T(s)\psi_2^T \psi_2 w(s).$$

Combining with (35), one obtains

$$\begin{aligned} x^T(t)Fx(t) &< \frac{V(t)}{\lambda_1} < \frac{e^{\alpha_{\max} T_f}}{\lambda_1} [\int_0^{T_f} [x^T(s)C_i^T C_i x(s) \\ &\quad + w^T(s)\psi_2^T \psi_2 w(s) + w^T(s)\psi_3 w(s)]ds \\ &< \frac{e^{\alpha_{\max} T_f}}{\lambda_1} [\lambda_4 d + (\lambda_5 + \lambda_6)h] \end{aligned}$$

where

$$\lambda_4 = \sup_{i \in \mathcal{S}} \lambda_{\max}(C_i^T C_i), \ \lambda_5 = \sup_{i \in \mathcal{S}} \lambda_{\max}(\psi_2^T \psi_2),$$
$$\lambda_6 = \sup_{i \in \mathcal{S}} \lambda_{\max}(\psi_3).$$

Then, one has

$$x^T(t)Fx(t) < \frac{e^{\alpha_{\max} T_f}}{\lambda_1} [\lambda_4 d + (\lambda_5 + \lambda_6)h] < c_2.$$

Relying on Definition 1, it is apparent that the system (1) satisfies both finite-time boundedness and extended dissipative performance. The proof is completed.

C. FINITE-TIME NON-FRAGILE EXTENDED DISSIPATIVE CONTROL

Based on the results of previous subsections, the non-fragile controllers of the periodic piecewise time-varying system (1) are further studied in this subsection. Consider a non-fragile periodic time-varying control law $u(t) = \big(K_i(t) + \Delta K_i(t)\big)x(t)$, $t \in [t_{i-1}, t_i)$, $i \in \mathcal{S}$, the corresponding closed-loop system is described as

$$\begin{aligned} \dot{x}(t) &= A_{ci}(t)x(t) + E_i(t)w(t), \\ z(t) &= C_i x(t) \end{aligned} \tag{36}$$

where $A_{ci}(t) = A_i(t) + B_i(t)\big(K_i(t) + \Delta K_i(t)\big)$, $i \in \mathcal{S}$. $K_i(t) = K_i(t + \ell T_p)$, and $\Delta K_i(t)$ is the uncertain perturbation of controller gain for the ith subsystem therein.

Based on Lemmas 1-3 and the above Theorems, for the additive gain perturbations, that is, $\Delta K_i(t) = H_i G_i(t) X_i(t)$, $i \in \mathcal{S}$, a non-fragile controller is developed below.

Theorem 3: Consider periodic piecewise time-varying system (36), given scalars $\alpha_i > 0, \alpha_{\max} = \max_{i \in \mathcal{S}}\{\alpha_i\}$, $i \in \mathcal{S}$, $0 < c_1 < c_2, h > 0, d > 0$, a time interval $T_f > 0$ and matrices $F > 0, \psi_1, \psi_2, \psi_3, \psi_4, H_i, X_i, X_{i+1}$, $i \in \mathcal{S}$, the system (36) is finite-time bounded and satisfies the extended dissipative performance, if there exist scalars $\varepsilon, \lambda_1, \lambda_2, \lambda_4, \lambda_5, \lambda_6$ and matrices $W_i > 0, Q_i$, $i \in \mathcal{S}$, with $W_{S+1} = W_1$, such that

$$\boldsymbol{H}_{ac,i,0} < 0, \tag{37}$$
$$\boldsymbol{H}_{ac,i,0} + \boldsymbol{H}_{ac,i,1} < 0, \tag{38}$$
$$\boldsymbol{H}_{ac,i,0} + \boldsymbol{H}_{ac,i,1} + \boldsymbol{H}_{ac,i,2} < 0, \tag{39}$$
$$\begin{bmatrix} -\frac{1}{e^{\alpha_{\max} T_f}} W_i & W_i C_i^T \psi_4 \\ * & -\psi_4 \end{bmatrix} < 0, \tag{40}$$
$$\begin{bmatrix} -\frac{1}{e^{\alpha_{\max} T_f}} W_{i+1} & W_{i+1} C_i^T \psi_4 \\ * & -\psi_4 \end{bmatrix} < 0, \tag{41}$$
$$\frac{1}{\lambda_2} F^{-1} \leq W_i \leq \frac{1}{\lambda_1} F^{-1}, \tag{42}$$
$$e^{\alpha_{\max} T_f} [\lambda_4 d + (\lambda_5 + \lambda_6)h] < \lambda_1 c_2 \tag{43}$$

where

$$\boldsymbol{H}_{ac,i,0} = \begin{bmatrix} \Sigma_{i0} & \Omega_{i0} & \Xi_{i0} & \Pi_{i0} \\ * & -\psi_3 & 0 & 0 \\ * & * & -\varepsilon I & 0 \\ * & * & * & \psi_1 \end{bmatrix},$$

$$\boldsymbol{H}_{ac,i,1} = \begin{bmatrix} \Sigma_{i1} & \Omega_{i1} & \Xi_{i1} & \Pi_{i1} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix},$$

$$\boldsymbol{H}_{ac,i,2} = \begin{bmatrix} \Sigma_{i2} & 0 & 0 & \Pi_{i2} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}$$

with

$$\begin{aligned}
\Sigma_{i0} &= \mathrm{sym}(A_iW_i + B_iQ_i) - \frac{1}{T_i}(W_{i+1} - W_i) \\
&\quad -\alpha_i W_i + \varepsilon B_i H_i H_i^T B_i^T, \\
\Sigma_{i1} &= \mathrm{sym}(A_iW_{i+1} + A_{i+1}W_i - 2A_iW_i + B_iQ_{i+1} \\
&\quad +B_{i+1}Q_i - 2B_iQ_i) - \alpha_i(W_{i+1} - W_i) \\
&\quad +\varepsilon B_iH_iH_i^TB_{i+1}^T + \varepsilon B_{i+1}H_iH_i^TB_i^T \\
&\quad -2\varepsilon B_iH_iH_i^TB_i^T, \\
\Sigma_{i2} &= \mathrm{sym}(A_{i+1}W_{i+1} - A_{i+1}W_i - A_iW_{i+1} + A_iW_i \\
&\quad +B_{i+1}Q_{i+1} - B_{i+1}Q_i - B_iQ_{i+1} + B_iQ_i) \\
&\quad +\varepsilon B_{i+1}H_iH_i^TB_{i+1}^T - \varepsilon B_{i+1}H_iH_i^TB_i^T \\
&\quad -\varepsilon B_iH_iH_i^TB_{i+1}^T + \varepsilon B_iH_iH_i^TB_i^T, \\
\Omega_{i0} &= E_i - W_iC_i^T\psi_2, \\
\Omega_{i1} &= E_{i+1} - E_i - W_{i+1}C_i^T\psi_2 + W_iC_i^T\psi_2, \\
\Xi_{i0} &= W_iX_i^T, \\
\Xi_{i1} &= W_iX_{i+1}^T + W_{i+1}X_i^T - 2W_iX_i^T, \\
\Xi_{i2} &= W_{i+1}X_{i+1}^T - W_{i+1}X_i^T - W_iX_{i+1}^T + W_iX_i^T, \\
\Pi_{i0} &= W_iC_i^T\psi_1, \\
\Pi_{i1} &= W_{i+1}C_i^T\psi_1 - W_iC_i^T\psi_1, \\
\lambda_1 &= \inf_{i\in\mathcal{S}} \lambda_{\min}(\widehat{W}_i),\ \lambda_2 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\widehat{W}_i), \\
\widehat{W}_i &= F^{-\frac{1}{2}}W_i^{-1}F^{-\frac{1}{2}}, \lambda_4 = \sup_{i\in\mathcal{S}} \lambda_{\max}(C_i^TC_i), \\
\lambda_5 &= \sup_{i\in\mathcal{S}} \lambda_{\max}(\psi_2^T\psi_2), \lambda_6 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\psi_3).
\end{aligned}$$

Then the periodic controller gain is given as

$$K_i(t) = Q_i(t)W_i^{-1}(t), t \in [\ell T_p + t_{i-1}, \ell T_p + t_i), \quad \ell = 0, 1, 2, \ldots, \ i \in \mathcal{S} \tag{44}$$

where $Q_i(t)$ and $W_i(t)$ are continuous periodic time-varying matrix functions and given as

$$Q_i(t) = Q_i + \frac{t - \ell T_p - t_{i-1}}{T_i}(Q_{i+1} - Q_i), \tag{45}$$

$$W_i(t) = W_i + \frac{t - \ell T_p - t_{i-1}}{T_i}(W_{i+1} - W_i). \tag{46}$$

Proof: Constructing a Lyapunov function with periodic time-varying matrices:

$$V(t) = V_i(t) = x^T(t)Y_i(t)x(t), t \in [t_{i-1}, t_i) \tag{47}$$

where $Y_i(t) = W_i^{-1}(t)$. For $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i)$, $\ell = 0, 1, 2, \ldots,\ i \in \mathcal{S}$, since $0 \leq \frac{t-\ell T_p - t_{i-1}}{T_i} \leq 1$, with (37)-(41) and Lemma 1, one has

$$\begin{bmatrix} \mathcal{E}(t) & \mathcal{T}(t) & W_i(t)X_i^T(t) & W_i(t)C_i^T\psi_1 \\ * & -\psi_3 & 0 & 0 \\ * & * & -\varepsilon I & 0 \\ * & * & * & \psi_1 \end{bmatrix} < 0 \tag{48}$$

where

$$\begin{aligned}
\mathcal{E}(t) &= \mathrm{sym}\big(A_i(t)W_i(t) + B_i(t)Q_i(t)\big) - \mathcal{D}^+W_i(t) \\
&\quad -\alpha_iW_i(t) + \varepsilon B_i(t)H_iH_i^TB_i^T(t), \\
\mathcal{T}(t) &= E_i(t) - W_i(t)C_i^T\psi_2.
\end{aligned}$$

Note that, for $W_i(t)$, $i \in \mathcal{S}$, W_i and W_{i+1} are constant matrices, and $W_{S+1} = W_1$. One can observe that $W_i(t)$ is continuous, and since $W_i(t) > 0$, one has $W_i^{-1}(t) > 0$ and continuous. Moreover, since $\psi_1 = \psi_1^T \leq 0$, utilizing Schur complement, inequality (48) is equivalent to

$$\begin{bmatrix} \boldsymbol{\alpha}(t) & \mathcal{T}(t) & W_i(t)X_i^T(t) \\ * & -\psi_3 & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \tag{49}$$

where

$$\begin{aligned}
\boldsymbol{\alpha}(t) &= \mathrm{sym}\big(A_i(t)W_i(t) + B_i(t)Q_i(t)\big) - \mathcal{D}^+W_i(t) \\
&\quad -\alpha_iW_i(t) + \varepsilon B_i(t)H_iH_i^TB_i^T(t) \\
&\quad -W_iC_i^T\psi_1C_iW_i(t).
\end{aligned}$$

Using the Schur complement again, then one obtains

$$\begin{bmatrix} \boldsymbol{\delta}(t) & \mathcal{T}(t) \\ * & -\psi_3 \end{bmatrix} < 0$$

where

$$\begin{aligned}
\boldsymbol{\delta}(t) &= \mathrm{sym}\big(A_i(t)W_i(t) + B_i(t)Q_i(t)\big) - \mathcal{D}^+W_i(t) \\
&\quad -\alpha_iW_i(t) + \varepsilon B_i(t)H_iH_i^TB_i^T(t) \\
&\quad -W_iC_i^T\psi_1C_iW_i(t) \\
&\quad +\varepsilon^{-1}W_i(t)X_i^T(t)X_i(t)W_i(t).
\end{aligned}$$

Substituting $\mathcal{D}^+Y_i^{-1}(t)$ by $-Y_i^{-1}(t)\mathcal{D}^+Y_i(t)Y_i^{-1}(t)$, and $Q_i(t)$ by $K_i(t)W_i(t)$, respectively. Then, multiply both sides of it by $\mathrm{diag}\{Y_i(t), I\}$. One has

$$\begin{bmatrix} \boldsymbol{m}(t) & \boldsymbol{y}(t) \\ * & -\psi_3 \end{bmatrix} + \varepsilon \begin{bmatrix} \boldsymbol{k}(t) & 0 \\ 0 & 0 \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} X_i^T(t)X_i(t) & 0 \\ * & 0 \end{bmatrix} < 0 \tag{50}$$

where

$$\begin{aligned}
\boldsymbol{m}(t) &= A_{ni}^T(t)Y_i(t) + Y_i(t)A_{ni}(t) - \alpha_iY_i(t) + \mathcal{D}^+Y_i(t) \\
&\quad -C_i^T\psi_1C_i, \\
\boldsymbol{y}(t) &= Y_i(t)E_i(t) - C_i^T\psi_2, \\
\boldsymbol{k}(t) &= Y_i(t)B_i(t)H_iH_i^TB_i(t)^TY_i(t)
\end{aligned}$$

with

$$A_{ni}(t) = A_i(t) + B_i(t)K_i(t).$$

It is noted that $G_i(t)$ for $i \in \mathcal{S}$ satisfying $G_i(t)G_i^T(t) \leq I$ in (7), combining with Lemma 2, (50) is equivalent to

$$\begin{bmatrix} \boldsymbol{m}(t) & \boldsymbol{y}(t) \\ * & -\psi_3 \end{bmatrix} + \begin{bmatrix} Y_i(t)B_i(t)H_i \\ 0 \end{bmatrix} G_i(t) \begin{bmatrix} X_i^T(t) \\ 0 \end{bmatrix}^T + \begin{bmatrix} X_i^T(t) \\ 0 \end{bmatrix} G_i(t)^T \begin{bmatrix} Y_i(t)B_i(t)H_i \\ 0 \end{bmatrix}^T < 0. \tag{51}$$

Then, one has

$$\begin{bmatrix} \boldsymbol{\varpi}_{11} & \boldsymbol{\varpi}_{12} \\ * & -\psi_3 \end{bmatrix} < 0 \tag{52}$$

where

$$\varpi_{11} = A_{ci}^T(t)Y_i(t) + Y_i(t)A_{ci}(t) - \alpha_i Y_i(t) + \mathcal{D}^+ Y_i(t) - C_i^T \psi_1 C_i,$$
$$\varpi_{12} = Y_i(t)E_i(t) - C_i^T \psi_2$$

with

$$A_{ci}(t) = A_i(t) + B_i(t)\big(K_i(t) + H_i G_i(t) X_i\big).$$

Combining with inequality (33), upon the proof lines of Theorem 2, one has

$$\dot{V} - \alpha_i V(t) - J(t) = \chi^T(t)\daleth(t)\chi(t) < 0$$

where

$$\chi(t) = [x^T(t)\ w^T(t)]^T,$$
$$\daleth(t) = \begin{bmatrix} \varpi_{11} & \varpi_{12} \\ * & -\psi_3 \end{bmatrix}.$$

Following the same arguments in the proof of Theorem 2, it can be easily obtained that the LMI conditions (37)-(43) can ensure the closed-loop system (36) satisfying both the finite-time boundedness and extended dissipative performance under the non-fragile controller, which completes the proof.

Consider the multiplicative gain perturbations $\Delta K_i(t) = H_i G_i(t)\widetilde{X}_i(t)K_i(t)$, $i \in \mathcal{S}$, we can obtain the following theorem.

Theorem 4: Consider periodic piecewise time-varying system (36), given scalars $\alpha_i > 0, \alpha_{\max} = \max_{i\in\mathcal{S}}\{\alpha_i\}, i \in \mathcal{S}$, $0 < c_1 < c_2, h > 0, d > 0$, a time interval $T_f > 0$ and matrices $F > 0, \psi_1, \psi_2, \psi_3, \psi_4, H_i, \widetilde{X}_i, \widetilde{X}_{i+1}, i \in \mathcal{S}$, the system (36) is finite-time bounded and satisfies the extended dissipative performance, if there exist scalars $\varepsilon, \lambda_1, \lambda_2, \lambda_4, \lambda_5, \lambda_6$ and matrices $W_i > 0, Q_i$, $i \in \mathcal{S}$, with $W_{S+1} = W_1$, such that (40)-(43) hold and

$$\boldsymbol{M}_{mc,i,0} < 0, \tag{53}$$
$$\boldsymbol{M}_{mc,i,1} + \boldsymbol{M}_{mc,i,1} < 0, \tag{54}$$
$$\boldsymbol{M}_{mc,i,0} + \boldsymbol{M}_{mc,i,1} + \boldsymbol{M}_{mc,i,2} < 0 \tag{55}$$

where

$$\boldsymbol{M}_{mc,i,0} = \begin{bmatrix} \Sigma_{0i} & \Omega_{0i} & \Xi_{0i} & \Pi_{0i} \\ * & -\psi_3 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 \\ * & * & * & \psi_1 \end{bmatrix},$$
$$\boldsymbol{M}_{mc,i,1} = \begin{bmatrix} \Sigma_{1i} & \Omega_{1i} & \Xi_{1i} & \Pi_{1i} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix},$$
$$\boldsymbol{M}_{mc,i,2} = \begin{bmatrix} \Sigma_{2i} & 0 & 0 & \Pi_{2i} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}$$

with

$$\Sigma_{0i} = \mathrm{sym}(A_i W_i + B_i Q_i) - \frac{1}{T_i}(W_{i+1} - W_i) - \alpha_i W_i + \varepsilon B_i H_i H_i^T B_i^T,$$
$$\Sigma_{1i} = \mathrm{sym}(A_i W_{i+1} + A_{i+1} W_i - 2A_i W_i + B_i Q_{i+1} + B_{i+1} Q_i - 2B_i Q_i) - \alpha_i (W_{i+1} - W_i) + \varepsilon B_i H_i H_i^T B_{i+1}^T + \varepsilon B_{i+1} H_i H_i^T B_i^T - 2\varepsilon B_i H_i H_i^T B_i^T,$$
$$\Sigma_{2i} = \mathrm{sym}(A_{i+1} W_{i+1} - A_{i+1} W_i - A_i W_{i+1} + A_i W_i + B_{i+1} Q_{i+1} - B_{i+1} Q_i - B_i Q_{i+1} + B_i Q_i) + \varepsilon B_{i+1} H_i H_i^T B_{i+1}^T - \varepsilon B_{i+1} H_i H_i^T B_i^T - \varepsilon B_i H_i H_i^T B_{i+1}^T + \varepsilon B_i H_i H_i^T B_i^T,$$
$$\Omega_{0i} = E_i - W_i C_i^T \psi_2,$$
$$\Omega_{1i} = E_{i+1} - E_i - W_{i+1} C_i^T \psi_2 + W_i C_i^T \psi_2,$$
$$\Xi_{0i} = Q_i^T \widetilde{X}_i^T,$$
$$\Xi_{1i} = Q_i^T \widetilde{X}_{i+1}^T + Q_{i+1}^T \widetilde{X}_i^T - 2Q_i^T \widetilde{X}_i^T,$$
$$\Xi_{2i} = Q_{i+1}^T \widetilde{X}_{i+1}^T - Q_{i+1}^T \widetilde{X}_i^T - Q_i^T \widetilde{X}_{i+1}^T + Q_i^T \widetilde{X}_i^T,$$
$$\Pi_{0i} = W_i C_i^T \psi_1,$$
$$\Pi_{1i} = W_{i+1} C_i^T \psi_1 - W_i C_i^T \psi_1,$$
$$\lambda_1 = \inf_{i\in\mathcal{S}} \lambda_{\min}(\widehat{W}_i),\ \lambda_2 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\widehat{W}_i)$$
$$\widehat{W}_i = F^{-\frac{1}{2}} W_i^{-1} F^{-\frac{1}{2}}, \lambda_4 = \sup_{i\in\mathcal{S}} \lambda_{\max}(C_i^T C_i),$$
$$\lambda_5 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\psi_2^T \psi_2), \lambda_6 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\psi_3)$$

where the controller gain is given in (44). The proof is omitted here for brevity.

Remark 4: It should be noted that compared with the norm-bounded additive perturbation, the multiplicative perturbation is related to the change of the controller gain, it thus has a more general and realistic and may be more desirable in applications.

The nominal controller without gain perturbation is synthesized in the following corollary and is used in the comparison of the aftermentioned simulation subsection to illustrate the robustness of the non-fragile control. With the similar arguments in the proof of Theorem 3, the condition is obtained as follow.

Corollary 1: Consider periodic piecewise time-varying system (36), given scalars $\alpha_i > 0, \alpha_{\max} = \max_{i\in\mathcal{S}}\{\alpha_i\}, i \in \mathcal{S}$, $0 < c_1 < c_2, h > 0, d > 0$, a time interval $T_f > 0$ and matrices $\psi_1, \psi_2, \psi_3, \psi_4, F > 0$, the system (36) is finite-time bounded and satisfies the extended dissipative performance, if there exist scalars $\lambda_1, \lambda_2, \lambda_4, \lambda_5, \lambda_6$ and matrices $W_i > 0, Q_i$, $i \in \mathcal{S}$, with $W_{S+1} = W_1$, such that (40)-(43) hold and

$$\boldsymbol{E}_{c,i,0} < 0, \tag{56}$$
$$\boldsymbol{E}_{c,i,0} + \boldsymbol{E}_{c,i,1} < 0, \tag{57}$$
$$\boldsymbol{E}_{c,i,0} + \boldsymbol{E}_{c,i,1} + \boldsymbol{E}_{c,i,2} < 0 \tag{58}$$

TABLE 1. Optimized variables of four performances.

Controller	Nominal ($\Delta K_i(t) \equiv 0$)	Additive $\Delta K_i(t)$	Multiplicative $\Delta K_i(t)$
H_∞ Performance	$\gamma_{\min} = 0.3446$	$\gamma_{\min} = 7.7129$	$\gamma_{\min} = 7.6142$
$L_2 - L_\infty$ Performance	$\gamma_{\min} = 8.4662$	$\gamma_{\min} = 9.2332$	$\gamma_{\min} = 8.6079$
Passivity	$\gamma_{\min} = 11.856$	$\gamma_{\min} = 12.351$	$\gamma_{\min} = 11.943$
Dissipativity	$\beta_{\min} = 6.3323 * 10^{-12}$	$\beta_{\min} = 1.8451 * 10^{-12}$	$\beta_{\min} = 1.3555 * 10^{-12}$

where

$$\boldsymbol{E}_{c,i,0} = \begin{bmatrix} \Sigma_{0,i} & \Omega_{0,i} & \Pi_{0,i} \\ * & \Lambda_{0,i} & 0 \\ * & * & \psi_1 \end{bmatrix},$$

$$\boldsymbol{E}_{c,i,1} = \begin{bmatrix} \Sigma_{1,i} & \Omega_{1,i} & \Pi_{0,i} \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix}, \boldsymbol{E}_{c,i,2} = \begin{bmatrix} \Sigma_{2,i} & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix}$$

with

$$\begin{aligned}
\Sigma_{0,i} &= \text{sym}(A_iW_i + B_iQ_i) - \frac{1}{T_i}(W_{i+1} - W_i) \\
&\quad -\alpha_i W_i, \\
\Sigma_{1,i} &= \text{sym}(A_iW_{i+1} + A_{i+1}W_i - 2A_iW_i + B_iQ_{i+1} \\
&\quad +B_{i+1}Q_i - 2B_iQ_i) - \alpha_i(W_{i+1} - W_i), \\
\Sigma_{2,i} &= \text{sym}(A_{i+1}W_{i+1} - A_{i+1}W_i - A_iW_{i+1} + A_iW_i \\
&\quad +B_{i+1}Q_{i+1} - B_{i+1}Q_i - B_iQ_{i+1} + B_iQ_i), \\
\Omega_{0,i} &= E_i - W_iC_i^T\psi_2, \\
\Omega_{1,i} &= E_{i+1} - E_i - W_{i+1}C_i^T\psi_2 + W_iC_i^T\psi_2, \\
\Pi_{0,i} &= W_iC_i^T\psi_1, \\
\Pi_{1,i} &= W_{i+1}C_i^T\psi_1 - W_iC_i^T\psi_1, \\
\Lambda_{0,i} &= -\psi_3, \\
\lambda_1 &= \inf_{i\in\mathcal{S}} \lambda_{\min}(\widehat{W_i}), \ \lambda_2 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\widehat{W_i}) \\
\widehat{W_i} &= F^{-\frac{1}{2}}W_i^{-1}F^{-\frac{1}{2}}, \lambda_4 = \sup_{i\in\mathcal{S}} \lambda_{\max}(C_i^TC_i), \\
\lambda_5 &= \sup_{i\in\mathcal{S}} \lambda_{\max}(\psi_2^T\psi_2), \lambda_6 = \sup_{i\in\mathcal{S}} \lambda_{\max}(\psi_3)
\end{aligned}$$

where the controller gain is given in (44). The proof is omitted here for brevity.

IV. NUMERICAL EXAMPLE

In this section, simulation results are given to verify the effectiveness of our proposed criteria.

Firstly, four cases of the extended dissipativity are given as follows:

- Case 1: $\psi_1 = -I, \psi_2 = 0, \psi_3 = \gamma^2 I, \psi_4 = 0$; ($H_\infty$)
- Case 2: $\psi_1 = \psi_2 = 0, \psi_3 = \gamma^2 I, \psi_4 = I$; ($L_2$-$L_\infty$)
- Case 3: $\psi_1 = 0, \psi_2 = I, \psi_3 = \gamma I, \psi_4 = 0$; (Passivity)
- Case 4: $\psi_1 = -I, \psi_2 = I, \psi_3 = 100 - \beta I, \psi_4 = 0$. (Dissipativity)

Consider a periodic piecewise time-varying system consisting of three subsystems, of which the period of the system is $T_p = 2$, and the dwell time for each subsystem is set to $T_1 = 0.4, T_2 = 0.6, T_3 = 1$. To the light of the linear interpolation formula, the related parameters of the system can be described as

$$\begin{aligned}
A_1(t) &= \begin{bmatrix} -1.8 & 0.12 \\ 5.02 & -0.17 \end{bmatrix} + 2.5(t - \ell T_p) \\
&\quad \times \begin{bmatrix} 1.96 & 1.32 \\ -2.79 & -0.08 \end{bmatrix}, \\
A_2(t) &= \begin{bmatrix} 0.16 & 1.44 \\ 2.23 & -0.25 \end{bmatrix} + \frac{5(t - \ell T_p - 0.4)}{3} \\
&\quad \times \begin{bmatrix} -9.02 & -0.31 \\ -2.82 & -2.83 \end{bmatrix}, \\
A_3(t) &= \begin{bmatrix} -8.86 & 1.13 \\ -0.59 & -3.08 \end{bmatrix} + (t - \ell T_p - 1) \\
&\quad \times \begin{bmatrix} 7.06 & -1.01 \\ 5.61 & 2.91 \end{bmatrix}, \\
B_1(t) &= \begin{bmatrix} -5.1 \\ 3.01 \end{bmatrix} + 2.5(t - \ell T_p) \begin{bmatrix} 1.1 \\ -2.51 \end{bmatrix}, \\
B_2(t) &= \begin{bmatrix} -4 \\ 0.5 \end{bmatrix} + \frac{5(t - \ell T_p - 0.4)}{3} \begin{bmatrix} 2 \\ 2.52 \end{bmatrix}, \\
B_3(t) &= \begin{bmatrix} -2 \\ 3.02 \end{bmatrix} + (t - \ell T_p - 1) \begin{bmatrix} -3.1 \\ -0.1 \end{bmatrix}, \\
E_1(t) &= \begin{bmatrix} -1 \\ 0.5 \end{bmatrix} + 2.5(t - \ell T_p) \begin{bmatrix} 4 \\ -2.5 \end{bmatrix}. \\
E_2(t) &= \begin{bmatrix} 3 \\ -2 \end{bmatrix} + \frac{5(t - \ell T_p - 0.4)}{3} \begin{bmatrix} -4.5 \\ 3 \end{bmatrix}. \\
E_3(t) &= \begin{bmatrix} -1.5 \\ 1 \end{bmatrix} + (t - \ell T_p - 1) \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0.9 \\ 0.7 \end{bmatrix}^T, C_2 = \begin{bmatrix} 0.8 \\ 1 \end{bmatrix}^T, C_3 = \begin{bmatrix} 1.01 \\ -0.69 \end{bmatrix}^T,
\end{aligned}$$

and choose the disturbance $w(t) = e^{-0.5t}$, let the initial condition $x_0 = [0, 0]^T$. Given $T_f = 5, F = I, \alpha_1 = 0.5, \alpha_2 = 0.4, \alpha_3 = 0.8$. It is easily seen that $\alpha_{\max} = 0.8$. Based on the designed auxiliary matrix values, as shown in cases 1-4, and according to Corollary 1, four optimized performance indices under the nominal controller are listed in the second column of Table 1. The corresponding variation of controller gains $K_i(t)$ is shown in Figure 1. One can see that system the nominal controller has a good control effect for guaranteeing the finite-time extended dissipative performance of the periodic piecewise time-varying system.

Then, consider that there are two types of gain perturbations in the controller of the investigated system, that is, the norm-bounded additive form (4) and the norm-bounded multiplicative form (5).

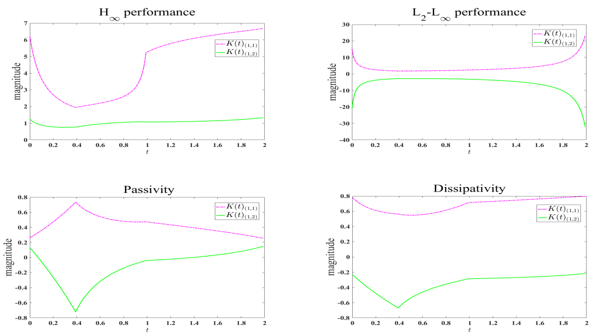

FIGURE 1. Variation of $K_i(t)$ under nominal controller over a period.

For the additive $\Delta K_i(t)$, the parameters that satisfy the perturbation definition are selected as follows:

$$H_1 = 0.01, H_2 = 0.08, H_3 = -0.03, G_i(t) = \begin{bmatrix} G_{i1}(t) \\ G_{i2}(t) \end{bmatrix}^T,$$
$$X_1(t) = \begin{bmatrix} 0.09 & 0 \\ 0.15 & 0.2 \end{bmatrix} + 2.5(t - \ell T_p)\begin{bmatrix} -1.29 & 0 \\ -0.15 & 0 \end{bmatrix},$$
$$X_2(t) = \begin{bmatrix} -1.2 & 0 \\ 0 & 0.2 \end{bmatrix} + \frac{5(t - \ell T_p - 0.4)}{3}\begin{bmatrix} 1.7 & 0 \\ 0 & -0.4 \end{bmatrix},$$
$$X_3(t) = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.2 \end{bmatrix} + (t - \ell T_p - 1)\begin{bmatrix} -0.41 & 0 \\ 0.15 & 0.4 \end{bmatrix}.$$

For the multiplicative $\Delta K_i(t)$, the parameters that satisfy the perturbation definition are selected as follows:

$$H_1 = 0.13, H_2 = 0.15, H_3 = -0.06, G_i(t) = \begin{bmatrix} G_{i1}(t) \\ G_{i2}(t) \end{bmatrix}^T,$$
$$\widetilde{X}_1(t) = \begin{bmatrix} 1.05 \\ 0.12 \end{bmatrix} + 2.5(t - \ell T_p)\begin{bmatrix} -2.65 \\ -0.38 \end{bmatrix},$$
$$\widetilde{X}_2(t) = \begin{bmatrix} -1.6 \\ 0.5 \end{bmatrix} + \frac{5(t - \ell T_p - 0.4)}{3}\begin{bmatrix} 1.9 \\ 0 \end{bmatrix},$$
$$\widetilde{X}_3(t) = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix} + (t - \ell T_p - 1)\begin{bmatrix} 0.75 \\ -0.38 \end{bmatrix}.$$

The above values of $G_i(t)$ are assumed to be within the interval $(-1, 1)$, which can be obtained by setting $G_{i1}(t) = 2*rand(1,1)-1$, $G_{i2}(t) = -\sqrt{1 - G_{i1}^2(t)} + 2*\sqrt{1 - G_{i1}^2(t)} * rand(1,1)$.

TABLE 2. The values $G_i(t)$ of additive perturbations.

Controller	Additive $\Delta K_i(t)$		
H_∞	$G_1 = \begin{bmatrix} -0.2522 \\ -0.4033 \end{bmatrix}^T$	$G_2 = \begin{bmatrix} 0.9433 \\ -0.1564 \end{bmatrix}^T$	$G_3 = \begin{bmatrix} 0.3222 \\ -0.7858 \end{bmatrix}^T$
$L_2 - L_\infty$	$G_1 = \begin{bmatrix} 0.0627 \\ 0.9080 \end{bmatrix}^T$	$G_2 = \begin{bmatrix} -0.1247 \\ 0.3405 \end{bmatrix}^T$	$G_3 = \begin{bmatrix} 0.9434 \\ -0.0845 \end{bmatrix}^T$
Passivity	$G_1 = \begin{bmatrix} 0.9939 \\ 0.0823 \end{bmatrix}^T$	$G_2 = \begin{bmatrix} -0.3429 \\ -0.2799 \end{bmatrix}^T$	$G_3 = \begin{bmatrix} -0.1067 \\ -0.6196 \end{bmatrix}^T$
Dissipativity	$G_1 = \begin{bmatrix} 0.1756 \\ -0.7034 \end{bmatrix}^T$	$G_2 = \begin{bmatrix} -0.2506 \\ -0.1221 \end{bmatrix}^T$	$G_3 = \begin{bmatrix} 0.8037 \\ -0.3581 \end{bmatrix}^T$

Under the considered $G_i(t)$ in Table 2 and Table 3 listed, according to Theorem 3 and Theorem 4, the obtained $K_i(t)$ is

TABLE 3. The values $G_i(t)$ of multiplicative perturbations.

Controller	Multiplicative $\Delta K_i(t)$		
H_∞	$G_1 = \begin{bmatrix} 0.9247 \\ -0.0746 \end{bmatrix}^T$	$G_2 = \begin{bmatrix} -0.8714 \\ 0.1641 \end{bmatrix}^T$	$G_3 = \begin{bmatrix} -0.3787 \\ -0.3253 \end{bmatrix}^T$
$L_2 - L_\infty$	$G_1 = \begin{bmatrix} 0.1035 \\ -0.6407 \end{bmatrix}^T$	$G_2 = \begin{bmatrix} -0.4156 \\ 0.7585 \end{bmatrix}^T$	$G_3 = \begin{bmatrix} 0.9029 \\ -0.1777 \end{bmatrix}^T$
Passivity	$G_1 = \begin{bmatrix} -0.7744 \\ -0.1921 \end{bmatrix}^T$	$G_2 = \begin{bmatrix} 0.3519 \\ -0.3973 \end{bmatrix}^T$	$G_3 = \begin{bmatrix} -0.3916 \\ 0.5240 \end{bmatrix}^T$
Dissipativity	$G_1 = \begin{bmatrix} 0.0316 \\ -0.8181 \end{bmatrix}^T$	$G_2 = \begin{bmatrix} -0.9780 \\ 0.1289 \end{bmatrix}^T$	$G_3 = \begin{bmatrix} -0.5553 \\ 0.0592 \end{bmatrix}^T$

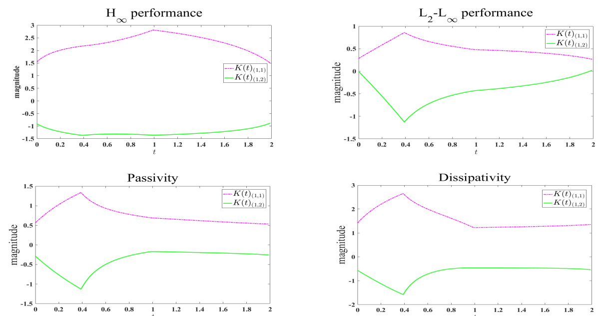

FIGURE 2. Variation of $K_i(t)$ under non-fragile controller (additive) over a period.

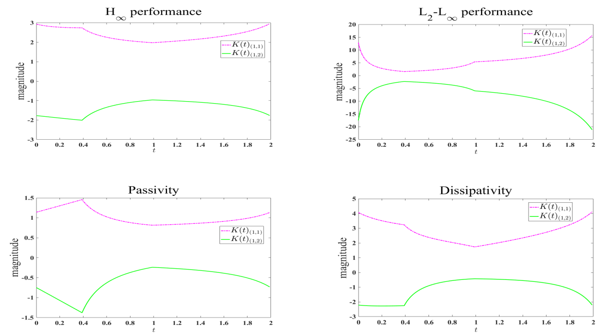

FIGURE 3. Variation of $K_i(t)$ under non-fragile controller (multiplicative) over a period.

shown in Figures 2-3 for additive perturbation and multiplicative perturbation, respectively. With the obtained controller, four corresponding optimized performance indices under two types of non-fragile controllers can be obtained, as shown in the third and fourth column of Table 1, respectively. Compared with the performance with nominal controller, one can observe that both the proposed additive and multiplicative non-fragile controllers are reliable and robust under the gain perturbations.

V. CONCLUSION

In this paper, the finite-time non-fragile extended dissipative control of periodic piecewise time-varying system is investigated. Based on a continuous periodic time-varying Lyapunov function, and a condition of matrix polynomial, sufficient conditions of finite-time boundedness and extended dissipativity for periodic piecewise time-varying system are presented. Then, considering two types of norm-bounded controller gain perturbations with time-varying periodic parameters, the non-fragile controllers that allows one to optimize multiple performances conveniently and to satisfy finite-time boundedness are developed for periodic piecewise time-varying system. The gain of the proposed controller could be obtained by solving LMIs. Finally, numerical examples are given to verify the effectiveness of the proposed methods. The results may be extended to the time-delay system [35]–[38] and nonlinear jumping system [39], [40] in the future.

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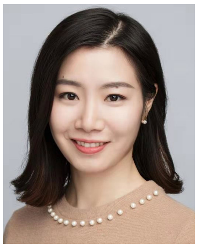
YUN LIU was born in Jiangxi, China. He is currently pursuing the M.S. degree in control science and engineering with the Guangdong University of Technology, Guangzhou, China. His current research interests include periodic systems, switched systems, and networked control systems.

ZUOLIN DENG was born in Guangdong, China. He is currently pursuing the M.S. degree in control science and engineering with the Guangdong University of Technology, Guangzhou, China. His current research interests include switched systems, periodic systems, and networked control systems.



PANSHUO LI received the B.S. degree in mechanical engineering from Donghua University, Shanghai, China, in 2009, the M.S. degree from Shanghai Jiao Tong University, Shanghai, in 2012, and the Ph.D. degree in mechanical engineering from The University of Hong Kong, Hong Kong, in 2016. Since 2016, she has been an Associate Professor with the School of Automation, Guangdong University of Technology, Guangzhou, China. Her current research interests include switched systems, periodic systems, intelligent vehicle control, and vibration control.

BIN ZHANG received the Ph.D. degree in control science and engineering from Central South University, Changsha, China, in 2016. Since 2017, she has been with the Guangdong University of Technology, Guangzhou, China, where she is currently a Lecturer with the School of Automation. Her current research interests include complex process modeling and optimization and intelligent algorithms and their applications.