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Improved Study on the Fluctuation Velocity of High-Speed Railway Catenary Considering the Influence of Accessory Parts

XIAOQIANG CHEN[1,3], XI ZHANG[1], YING WANG[1], AND XINYI WANG[2]

[1]School of Automation and Electrical Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China
[2]School of Electrical Engineering, Southwest Jiaotong University, Chengdu 610031, China
[3]Key Laboratory of Opto-Technology and Intelligent Control Ministry of Education, Lanzhou 730070, China

Corresponding author: Xi Zhang (18845095018@163.com)

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ABSTRACT To improve the prediction accuracy of the wave propagation velocity of high-speed railway catenary, a method for identifying the fluctuation velocity of catenary (FVC) considering the influence of the accessory parts of catenary (APC) is proposed. Based on the classical Bernoulli-Euler beam vibration model, the single Euler beam model under ideal conditions is improved considering the influence of the inertial forces exerted by accessory parts on the catenary. The calculation expression of FVC considering the influence of APC is deduced according to the theory of elastomer vibration mechanics. The key parameters α and β that affect the FVC are obtained, and the relation curves between the fluctuation velocity and the key parameters α and β are analyzed respectively. Through the fluctuation velocity test for the actual catenary, the theoretical fluctuation velocity of contact wire considering the influence of APC are compared and verified with the measured fluctuation velocity. Research results show that the relative error between the theoretical fluctuation velocity obtained from the modified fluctuation velocity expression of contact wire considering the influence of APC and the measured fluctuation velocity is less than 10%. Compared with the fluctuation velocity theoretical formula based on the string/cable and Euler beam theory, the modified expression of the fluctuation velocity of contact wire (FVCW) is more accurate in identifying the FVCW, and the recognition accuracy is increased by 3.75%, which verifies the accuracy of the recognition method and provides a more accurate theoretical basis of fluctuation velocity for designing the structural parameters of catenary.

INDEX TERMS High-speed railway, catenary, accessory part, fluctuation velocity, test verification.

I. INTRODUCTION

With the rapid development of electrified railway, the requirement of dynamic contact performance between pantograph and catenary is increasing day by day. Good dynamic performance of pantograph-catenary is the key factor to ensure safe, stable and efficient operation of electric multiple unit (EMU) [1]. When an EMU runs at high speed, the fluctuation performance of catenary becomes the dominant factor affecting the dynamic contact performance of pantograph-catenary system [2], [3]. As the main measurement factor to measure the fluctuation performance of catenary, the accurate identification of wave propagation velocity is of great significance for the reasonable design of the catenary structural parameters.

As the only power source of electric locomotive, the interaction performance of pantograph-catenary system has always been of great interest to many scholars. Normally, the pantograph-catenary interaction is affected by the external environment and disturbance of the system itself. Some researchers investigate the influence of external factors, such as the wind load [4]–[6], the iced load [7], the locomotive excitation [8], and the electromagnetic interference [9] on the pantograph-catenary interaction. The internal disturbance

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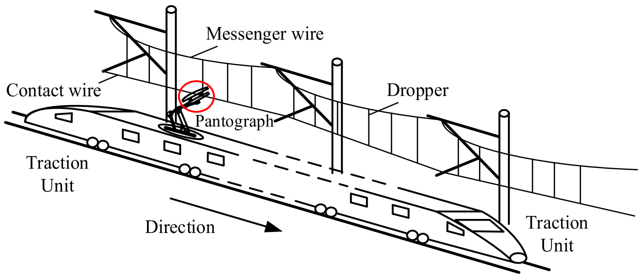


FIGURE 1. Schematic of pantograph-catenary sliding electric contact.

normally includes the propagation of elastic wave in the catenary wires [10], wear and irregularity of contact wire [11], and the role of accessory parts [12].

With the continuous improvement of the operating speed of high-speed trains, the fluctuating phenomenon of catenary caused by the sliding contact of high-speed pantograph, as shown in Fig. 1, has attracted wide attention from researchers at home and abroad [13], [14]. On the one hand, the fluctuation velocity of catenary (FVC) limits the maximum operating speed of EMU and affects the amplitude of contact force [15] and the structure stress [16]. For this reason, theoretical studies on the FVC are carried out in [17] and [18]. However, due to the great uncertainty in the selection of the wavelet mother function and its transformation coefficient and the limited influence of air damping on the fluctuation of contact wire, the theoretical FVC results they obtained need to be further verified. On the other hand, the propagation process of the elastic wave itself has a significant influence on the dynamic performance of pantograph-catenary system. The wave characteristics of the vibration wave of catenary based on the beam model was studied for the first time in [19], and the wave propagation law of vibration wave on the Euler beam was explained in detail. Hayasaka [20] analyzed the incident and reflection processes of vibration wave of contact wire at the joint of the catenary anchor segment based on the principle of row wave mechanics, and proposed that a damper should be installed at the joint of the anchor segment to reduce the wave amplitude of the fluctuation of contact wire, so as to improve the current collection quality between pantograph and catenary. Cho [21] adopted a finite element method to verify that the fluctuation velocity of high-frequency vibration was larger than that of low-frequency vibration in catenary, and proposed a nonlinear suspender model. Zhou and Zhang [22] analyzed the main factors affecting the dynamic performance of pantograph-catenary under the action of double pantographs based on the theory of pantograph-catenary dynamics, and made a reasonable explanation of the research results through the theoretical analysis of vibration wave propagation process of contact wire. Lv [23] proposed a method of fluctuation velocity recognition based on displacement contour map, and carried out a fluctuation velocity test on the actual 3-span catenary through camera measuring equipment, and compared the method with the actual measured fluctuation velocity to verify the accuracy of this identification method.

From the literature mentioned above, it can be noted that the current researches on the fluctuation characteristics of catenary mainly focus on the analysis of the propagation process of elastic wave, while the researches on the FVC are relatively few, and most of them are based on the simple beam model and ignore the influence of the accessory parts of catenary (APC). For this reason, a recognition method to identify the FVC considering the influence of APC is proposed in this paper. Its recognition accuracy is verified by the fluctuation velocity test.

FVC is an important index to control the maximum operating speed of high-speed trains from the perspective of engineering design, and the accurate identification of its magnitude is a necessary condition for the reasonable design of structural parameters of catenary. For this reason, this paper attempts to make a contribution to this topic. The rest of this paper is organized as follows. Section II describes the fluctuation velocity formula of the beam model based on Euler beam vibration theory. In Section III, considering the influence of APC, the single Euler beam model is improved. Based on the theory of elastic vibration mechanics, the calculation expression of FVC considering the influence of APC is derived, and the relation curves between the fluctuation velocity and the key parameters α and β are analyzed. In Section IV, the theoretical fluctuation velocity obtained by the modified expression is compared with the measured fluctuation velocity to verify the accuracy of the recognition method. Finally, Section V reaches conclusions.

II. FVC UNDER THE EULER BEAM MODEL

Due to its own flexible performance and the rigid performance of catenary under the action of pantograph, the contact wire or the messenger wire is generally equivalent to an Euler beam with flexural stiffness, linear density and constant tension at both ends when establishing the mathematical model of catenary [1], [24], [25]. In order to study the vibration characteristics of catenary, a differential segment dx is taken from the contact wire or the messenger wire for force analysis, as shown in Fig. 2.

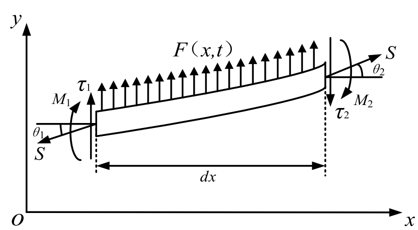


FIGURE 2. Force analysis diagram of a differential segment *dx* of the contact wire or the messenger wire.

In Fig. 2, S represents the tension at both ends of the differential segment. θ is the angle between the tension and the horizontal plane. M is the bending moment of the section of the differential segment. τ is the shear stress of this section.

$F(x,t)$ is the external force on the unit length of the differential segment, which can be expressed by the Dirac delta function:

$$F\ (x,t) = P\delta\ (x - v_0 t) \tag{1}$$

where P is the equivalent amplitude of lifting force exerted on the contact wire by the pantograph (which is 0N on the messenger wire). x represents the transverse position, t is for time. v_0 is the operating speed of pantograph. δ is the Dirac delta function.

According to the Newton's second law and the bending moment balance theorem, the vibration differential equation of the contact wire can be deduced as:

$$\rho\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} - S\frac{\partial^2 y}{\partial x^2} + C\frac{\partial y}{\partial t} = P\delta(x - v_0 t) \tag{2}$$

where ρ is the linear density of the differential segment. EI is the bending stiffness. C is the self-damping coefficient. y is the vertical displacement generated by the contact wire or the messenger wire with time and transverse position, $y = y(x,t)$. Thus, the homogeneous equation corresponding to equation (2) can be obtained as:

$$\rho\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} - S\frac{\partial^2 y}{\partial x^2} + C\frac{\partial y}{\partial t} = 0 \tag{3}$$

This equation is a linear homogeneous partial differential equation, which can be solved by the method of separating variables. Let its general solution be:

$$y = q(t)\varphi(x) \tag{4}$$

By substituting equation (4) into equation (3), the following two equations can be obtained:

$$\frac{d^2q(t)}{dt^2} + 2\xi\omega\frac{dq(t)}{dt} + \omega^2 q(t) = 0 \tag{5}$$

$$EI\frac{d^4\varphi(x)}{dx^4} - S\frac{d^2\varphi(x)}{dx^2} - \rho\omega^2\varphi(x) = 0 \tag{6}$$

where ω is the intrinsic circular frequency of free vibration of catenary ($\omega = 2\pi f$, f is the vibration frequency), and $\zeta = C/2\rho\omega$. It can be obtained by solving equation (5):

$$q(t) = e^{-\xi\omega t}(A_1\cos\omega_r t + A_2\sin\omega_r t) = Ae^{-\xi\omega t}\cos(\omega_r t + \theta') \tag{7}$$

where $\omega_r = \omega\sqrt{1-\xi^2}$, θ' is the phase angle. By solving equation (6), we can obtain:

$$\varphi(x) = K_1 ch\alpha x + K_2 sh\alpha x + K_3\cos\beta x + K_4\sin\beta x \tag{8}$$

By using the boundary conditions of zero displacement and zero moment of fixed hinge support, we can obtain: $K_1 = K_2 = K_3 = 0$, $K_4 \neq 0$. Thus:

$$\beta = \frac{m\pi}{L} = \sqrt{\frac{\sqrt{S^2 + 4EI\rho\omega^2} - S}{2EI}} \quad (m = 1, 2, \cdots, \infty) \tag{9}$$

where L represents the length of the anchor segment of catenary.

To sum up, we substitute (7)-(9) into (4) to obtain:

$$y(x,t) = Ae^{-\xi\omega t}\cos(\omega_r t + \theta')K_4\sin\frac{m\pi}{L}x \tag{10}$$

By converting (10) with the relational formula of trigonometric function (11), we can have:

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] \tag{11}$$

$$\begin{aligned} y(x,t) &= AK_4 e^{-\xi\omega t}\cdot\frac{1}{2}[\sin(\frac{m\pi}{L}x + \omega_r t + \theta') \\ &\quad + \sin(\frac{m\pi}{L}x - \omega_r t - \theta')] \\ &= \frac{1}{2}AK_4 e^{-\xi\omega t}\cdot[\sin\frac{m\pi}{L}(x + \frac{L}{m\pi}\omega_r t + \frac{L}{m\pi}\theta') \\ &\quad + \sin\frac{m\pi}{L}(x - \frac{L}{m\pi}\omega_r t - \frac{L}{m\pi}\theta')] \end{aligned} \tag{12}$$

According to the mechanical theory of elastomer vibration and the general expression of plane wave: $y(x,t) = F(x - vt) + \varphi(x + vt)$ [26], the FVC under the Euler beam model can be obtained as:

$$v = \frac{\omega_r}{\frac{m\pi}{L}} = \frac{\omega_r}{\sqrt{\frac{\sqrt{S^2+4EI\rho\omega^2}-S}{2EI}}} \tag{13}$$

By substituting $\omega_r = \omega\sqrt{1-\xi^2}$ and $\omega = 2\pi f$ into (13), we can obtain:

$$v = \frac{2\pi f\sqrt{1-\xi^2}}{\sqrt{\sqrt{\left(\frac{S}{2EI}\right)^2 + \frac{\rho(2\pi f)^2}{EI}} - \frac{S}{2EI}}} \tag{14}$$

In general, the damping coefficient C of contact wire or messenger wire is very small, which can be ignored [10], [20]. In other words, when $\zeta = 0$, the theoretical formula of FVC is

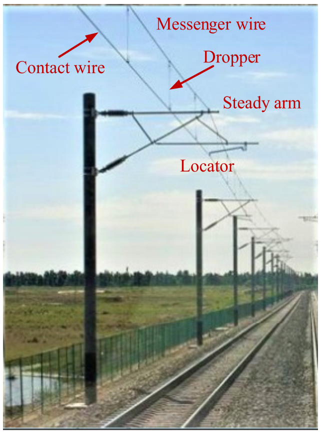
$$v = \frac{2\pi f}{\sqrt{\sqrt{\left(\frac{S}{2EI}\right)^2 + \frac{\rho(2\pi f)^2}{EI}} - \frac{S}{2EI}}} \tag{15}$$

It can be seen from (15) that the fluctuation velocity v is related not only to the tension S at both ends, but the bending stiffness EI, line density ρ, and the vibration frequency f are also the main factors affecting the fluctuation velocity.

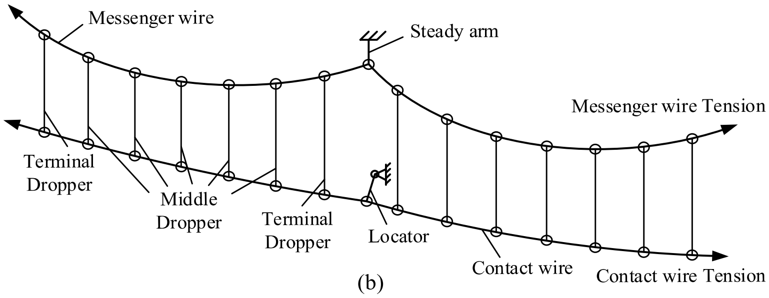
III. FVC CONSIDERING THE INFLUENCE OF APC

A. FVCW CONSIDERING THE INFLUENCE OF APC

Catenary can be regarded as a complex structure with the messenger/contact wire as the main body attached to droppers, locators and steady arm [1], [27], [28], as shown in Fig. 3(a). When the EMU is operating at high speed, the influence of the inertia force exerted by APC on the fluctuation performance of catenary cannot be ignored. Therefore, considering the role of APC, the single Euler beam model under ideal conditions is improved, as shown in Fig. 3(b). The actual 3-span catenary parameters of the State-Key Laboratory of Traction Power in China, as shown in Fig. 3(c),



(a)



(b)

- Stagger: (±0.2m); Structure height: (1.6m);
- Messenger wire: Line density(1.07kg/m), Tension(20580N), Flexural rigidity(114.5N/m^2)
- Contact wire: Line density(1.07kg/m), Tension(26460N), Flexural rigidity(148.9N/m^2);
- Dropper: Line density(0.2kg/m); Locator quality(2.0kg);
- Intervals between droppers: 4.3m/8.33m/7.04m/7.83m/7.58m/7.42/7.18/4.32m

(c)

FIGURE 3. Nonlinear catenary model. (a) real catenary line; (b) its geometry structure; (c) real parameters.

are adopted to verify subsequently the correctness of the proposed fluctuation velocity identification method. In this paper, the subscript a is used to represent the physical quantity related to the contact wire, and the subscript b is used to represent the physical quantity related to the messenger wire.

Dropper is the key component connecting the messenger wire with the contact wire, and its effect on the contact wire can be regarded as a concentrated mass block with a mass of m_{ap} (p is the serial number of droppers, $p = 1, 2, 3, \ldots, N$) applied to the contact wire. When the contact wire vibrates under the external force, the force exerted by droppers on the contact wire can be decomposed into an inertial force $m_{ap}\ddot{y}_a$ and a tensile force F_p. y_a and $\ddot{y}_a$ respectively represents the vertical displacement and the vertical acceleration of contact wire. Similarly, the force exerted by droppers on the messenger wire is equivalent to an inertial force $m_{bp}\ddot{y}_b$ and a tensile force F_{p} applied to the messenger wire. y_b and $\ddot{y}_b$ respectively represents the vertical displacement and the vertical acceleration of messenger wire. It is generally believed that the vertical force exerted by a locator on the contact wire is small and negligible [3], [27]. Therefore, the effect of a locator on the contact wire is only equivalent to attaching a concentrated mass block M_{aq} (q is the serial number of locators, $q = 1, 2, 3, \ldots, R$) to the contact wire. The locator exerts an inertial force $M_{aq}\ddot{y}_a$ to the contact wire when the contact wire vibrates. Thus, considering the influence of the load force of droppers and locators, equation (2) can be modified as:

$$\begin{aligned}\rho_a\frac{\partial^2 y_a}{\partial t^2} + EI_a\frac{\partial^4 y_a}{\partial x^4} - S_a\frac{\partial^2 y_a}{\partial x^2} + C_a\frac{\partial y_a}{\partial t} &= -\sum_{p=1}^{N} m_{ap}\frac{\partial^2 y_a}{\partial t^2}\delta\left(x - x_p\right) - \sum_{q=1}^{R} M_{aq}\frac{\partial^2 y_a}{\partial t^2}\delta\left(x - x_q\right) \\ &\quad + \sum_{p=1}^{N} F_p\delta(x - x_p) + P\delta\left(x - vt\right)\end{aligned} \tag{16}$$

where N and R are the total number of droppers and locators in the anchor segment, respectively.

The corresponding homogeneous equation of (16) is:

$$\begin{aligned}\rho_a\frac{\partial^2 y_a}{\partial t^2} + EI_a\frac{\partial^4 y_a}{\partial x^4} - S_a\frac{\partial^2 y_a}{\partial x^2} + C_a\frac{\partial y_a}{\partial t} &= -\sum_{p=1}^{N} m_{ap}\frac{\partial^2 y_a}{\partial t^2}\delta\left(x - x_p\right) - \sum_{q=1}^{R} M_{aq}\frac{\partial^2 y_a}{\partial t^2}\delta\left(x - x_q\right)\end{aligned} \tag{17}$$

According to the theory of partial differential equations, the solution of (17) can be set as:

$$y_a\left(x, t\right) = \sum_{j=1}^{\infty} w(t)\sin\frac{j\pi x}{L} \tag{18}$$

By substituting (18) into (17), we can find:

$$\begin{aligned}&\sum_{j=1}^{\infty}\{\rho_a\ddot{w}(t) + c_a\dot{w}(t) + [EI_a(\frac{j\pi}{L})^4 + S_a(\frac{j\pi}{L})^2]w(t)\}\cdot \\ &\sin\frac{j\pi x}{L} = -\sum_{p=1}^{N} m_{ap}\frac{\partial^2 y_a}{\partial t^2}\delta\left(x - x_p\right) \\ &\qquad - \sum_{q=1}^{R} M_{aq}\frac{\partial^2 y_a}{\partial t^2}\delta\left(x - x_q\right)\end{aligned} \tag{19}$$

Multiply both ends of (19) by $\sin\frac{k\pi x}{L}$, and integrate the equation on $x \in [0, L]$ to have:

$$\ddot{w}(t) + \frac{c_a}{\rho_a + \frac{2}{L}(Q_1 + Q_2)}\dot{w}(t) + \frac{EI_a(\frac{k\pi}{L})^4 + S_a(\frac{k\pi}{L})^2}{\rho_a + \frac{2}{L}(Q_1 + Q_2)}w(t) = 0 \tag{20}$$

where

$$Q_1 = \sum_{p=1}^{N} m_{ap}\sin^2\frac{k\pi x_p}{L} \quad k = 1, 2, \cdots, \infty \tag{21}$$

$$Q_2 = \sum_{q=1}^{R} M_{aq}\sin^2\frac{k\pi x_q}{L} \quad k = 1, 2, \cdots, \infty \tag{22}$$

In (21) and (22), x_p is the coordinate of the p-th dropper, and x_q is the coordinate of the q-th locator.

$$\omega^2 = \frac{EI_a(\frac{k\pi}{L})^4 + S_a(\frac{k\pi}{L})^2}{\rho} \tag{23}$$

$$\frac{C_a}{\rho} = 2\xi\omega \tag{24}$$

where $\rho = \rho_a + \frac{2}{L}(Q_1 + Q_2)$, $\xi = \frac{C_a}{2\rho\omega}$.

By substituting (23) and (24) into (20), we can obtain:

$$\ddot{w}(t) + 2\xi\omega\dot{w}(t) + \omega^2 w(t) = 0 \tag{25}$$

By solving (25), we have:

$$w(t) = Ae^{-\xi\omega t}\cos(\omega_r t + \theta'') \tag{26}$$

where $\omega_r = \omega\sqrt{1-\xi^2}$, θ'' is the phase angle. According to the fluctuation velocity formula of (13), the expression of the FVCW affected by the APC can be obtained as:

$$v_a = \frac{\omega_r}{\frac{j\pi}{L}} = \frac{\omega\sqrt{1-\xi^2}}{\sqrt{\frac{\sqrt{S_a^2+4EI_a\rho\omega^2}-S_a}{2EI_a}}} \tag{27}$$

where $\omega = 2\pi f$, $\rho = \rho_a + \frac{2}{L}(Q_1 + Q_2)$, $\xi = \frac{C_a}{2\rho\omega}$. Then

$$v_a = \frac{2\pi f\sqrt{1-\xi^2}}{\sqrt{\sqrt{(\frac{S_a}{2EI_a})^2 + \frac{(2\pi f)^2[\rho_a+\frac{2}{L}(Q_1+Q_2)]}{EI_a}} - \frac{S_a}{2EI_a}}} \tag{28}$$

It is generally believed that the damping coefficient C_a of contact wire itself is very small. Therefore, the FVCW can be expressed as:

$$v_a = \frac{2\pi f}{\sqrt{\sqrt{(\frac{S_a}{2EI_a})^2 + \frac{(2\pi f)^2[\rho_a+\frac{2}{L}(Q_1+Q_2)]}{EI_a}} - \frac{S_a}{2EI_a}}} \tag{29}$$

It can be seen from (29) that the FVCW is not only related to its own tension, bending stiffness, linear density and vibration frequency, but also closely related to the quality, quantity and position of droppers and locators as well as the length of the anchor segment. When the length of the anchor segment is certain, compared with the formula of FVCW under the Euler beam model, the influence of APC on it is mainly reflected in the part of linear density.

Given the tension at both ends, line density, bending stiffness and the length of the anchor segment of the actual contact wire of a high-speed railway in China: $S = 27$kN, $\rho_a = 1.083$kg/m, $EI = 150$N$\cdot$ m^2, $L = 480$m, and considering that the contact wire vibrates mainly at low frequency [9], the vibration frequency range of contact wire is set to be 0~20Hz, and the relation curve between the FVCW, the frequency f and the key parameter $\alpha(\alpha = Q_1 + Q_2)$ can be drawn, as shown in Fig. 4.

As can be seen from Fig. 4, when the key parameter α is constant, the FVCW is less affected by the frequency, and the variation amplitude of its corresponding value tends to

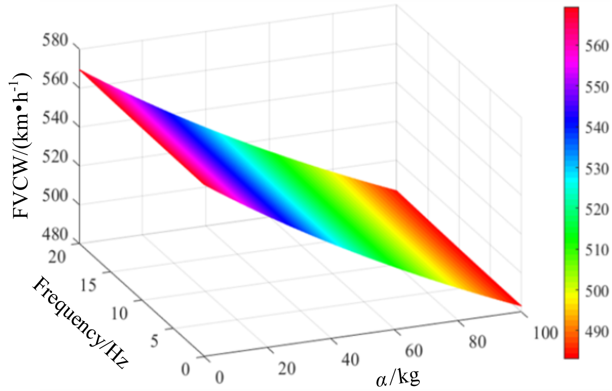


FIGURE 4. The relation curve between the FVCW and the frequency f and the key parameter α.

be stable. Under the certain frequency conditions, the FVCW decreases gradually with the increase of α, and the corresponding change rate of it shows a linear trend. This is because the main mode $\sum \sin^2 \frac{j\pi x}{L}$ of Q_1 and Q_2 is always greater than zero, thus the FVCW will gradually decrease with the increase of the quality or quantity of droppers and locators.

To sum up, the inertia force exerted by the APC on the contact wire has a relatively obvious inhibitory effect on the FVCW, and this inhibitory effect becomes more obvious with the increase of the quality or quantity of APC.

B. FVMW CONSIDERING THE INFLUENCE OF APC

Messenger wire is an important conductor bearing the dual function of power transmission and suspending the contact wire, and its fluctuation performance also affects the effect of catenary matching design and the quality of dynamic current collection of pantograph-catenary system [29]. In the process of high-speed operation of EMU, the influence of APC connected to messenger wire on the fluctuation performance of messenger wire is also not negligible.

In addition to the role of droppers, the elastic force of steady arm will also affect the fluctuation velocity of messenger wire (FVMW). Steady arm mainly plays the role of fixed support to the messenger wire, and the main form of the force exerted on the messenger wire is the elastic supporting force, which is expressed as $K_{bq}y_b$. K_b is the elastic coefficient of the steady arm at the supporting point. Therefore, the differential equation of vibration of messenger wire under the Euler beam model can be modified as:

$$\begin{aligned}\rho_b\frac{\partial^2 y_b}{\partial t^2} + EI_b\frac{\partial^4 y_b}{\partial x^4} - S_b\frac{\partial^2 y_b}{\partial x^2} + C_b\frac{\partial y_b}{\partial t} \\ = -\sum_{p=1}^{N} m_{bp}\frac{\partial^2 y_b}{\partial t^2}\delta(x-x_p) + \sum_{q=1}^{R} K_{bq}y_b\delta(x-x_q) \\ - \sum_{p=1}^{N} F_p\delta(x-x_p)\end{aligned} \tag{30}$$

where N and R are the total number of droppers and steady arms in the anchor segment, respectively.

The corresponding homogeneous equation of (30) is:

$$\rho_b \frac{\partial^2 y_b}{\partial t^2} + EI_b \frac{\partial^4 y_b}{\partial x^4} - S_b \frac{\partial^2 y_b}{\partial x^2} + C_b \frac{\partial y_b}{\partial t} = -\sum_{p=1}^{N} m_{bp} \frac{\partial^2 y_b}{\partial t^2} \delta\left(x - x_p\right) + \sum_{q=1}^{R} K_{bq} y_b \delta\left(x - x_q\right) \tag{31}$$

According to the theory of partial differential equations, the solution of (31) can be set as:

$$y_b\left(x, t\right) = \sum_{m=1}^{\infty} q(t) \sin \frac{m\pi x}{L} \tag{32}$$

In the same way as the derivation process in section 2.1, it can be obtained as:

$$\ddot{q}(t) + \frac{C_b}{\rho_b + \frac{2}{L} Q_3} \dot{q}(t) + \frac{EI_b(\frac{k\pi}{L})^4 + S_b(\frac{k\pi}{L})^2 - \frac{2}{L} Q_4}{\rho_b + \frac{2}{L} Q_3} q(t) = 0 \tag{33}$$

where

$$Q_3 = \sum_{p=1}^{N} m_{bp} \sin^2 \frac{k\pi x_p}{L} \quad k = 1, 2, \cdots, \infty \tag{34}$$

$$Q_4 = \sum_{q=1}^{R} K_{bq} \sin^2 \frac{k\pi x_q}{L} \quad k = 1, 2, \cdots, \infty \tag{35}$$

In (34) and (35), x_p is the coordinate of the p-th dropper, and x_q is the coordinate of the q-th steady arm.

$$\omega^2 = \frac{EI_b(\frac{k\pi}{L})^4 + S_b(\frac{k\pi}{L})^2}{\rho} - \frac{2Q_4}{L\rho} \tag{36}$$

$$\frac{C_b}{\rho} = 2\xi\omega \tag{37}$$

$$\omega_m^2 = \omega^2 + \frac{2Q_4}{L\rho} \tag{38}$$

where $\rho = \rho_b + \frac{2}{L} Q_3$, $\xi = \frac{C_b}{2\rho\omega}$.

Thus, the expression of FVMW considering the influence of APC can be obtained as:

$$v_b = \frac{\omega_r}{\frac{m\pi}{L}} = \frac{\omega\sqrt{1-\xi^2}}{\sqrt{\frac{\sqrt{S_b^2 + 4EI_b\rho\omega_m^2} - S_b}{2EI_b}}} \tag{39}$$

where $\omega = 2\pi f$. Then

$$v_b = \frac{2\pi f\sqrt{1-\xi^2}}{\sqrt{\sqrt{(\frac{S_b}{2EI_b})^2 + \frac{(2\pi f)^2}{EI_b}\{\rho_b + \frac{2}{L}[Q_3 + \frac{Q_4}{(2\pi f)^2}]\}} - \frac{S_b}{2EI_b}}} \tag{40}$$

Since the damping coefficient C_b of messenger wire itself is also small, the FVMW can be expressed as:

$$v_b = \frac{2\pi f}{\sqrt{\sqrt{(\frac{S_b}{2EI_b})^2 + \frac{(2\pi f)^2}{EI_b}\{\rho_b + \frac{2}{L}[Q_3 + \frac{Q_4}{(2\pi f)^2}]\}} - \frac{S_b}{2EI_b}}} \tag{41}$$

It can be seen from (41) that tension, flexural stiffness, linear density and vibration frequency are the main factors affecting the FVMW. In addition, the quantity and position of droppers and steady arm, the quality of droppers, the elastic coefficient at the supporting point and the length of anchor segment also have an impact on the FVMW. When the length of the anchor segment is constant, the comparison with the formula of fluctuation velocity under the Euler beam model shows that the influence of APC on the FVMW is mainly reflected in the part of linear density, but different from the modified expression of FVCW, the variable part of linear density is affected not only by APC but also by the vibration frequency.

In order to study the specific influence of APC on the FVMW, considering that the main frequency band of the vibration of messenger wire is mainly low-frequency, and its range is generally between 1 and 2Hz. Thus, given: $S = 21$kN, $\rho_b = 1.068$kg/m, $EI = 120$N·m^2, $f = 1.5$Hz, $L = 480$m, the relation curve between the FVMW and the key parameter $\beta(\beta = Q_3 + Q_4/4\pi^2 f^2)$ can be obtained, as shown in Fig. 5.

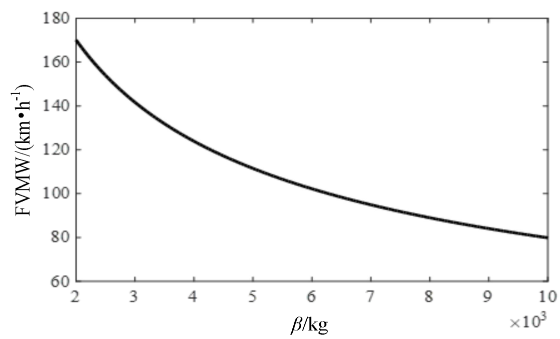


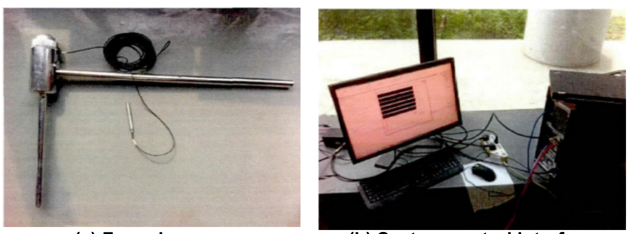
FIGURE 5. The relation curve between the FVMW and the key parameter β.

It can be seen from Fig. 5 that the FVMW decreases gradually with the increase of β, and the corresponding change rate of it has a downward trend, which indicates that the APC have a relatively obvious inhibitory effect on the FVMW, and with the increase of the quantity and quality of droppers and the increase of the elastic coefficient at the supporting point, the FVMW will further decrease and tend to be stable. In addition, the FVMW is significantly reduced compared with that of contact wire, which is caused by the fact that the elastic force of a steady arm at the supporting point of messenger wire is much larger than the inertial force at the connection point of a locator, and it is more capable of maintaining the original vibration state of messenger wire.

IV. TEST VERIFICATION

In order to further analyze the correctness of the recognition method above, it is necessary to carry out the field test on the wave propagation velocity of actual catenary. Based on the principle of photogrammetry, three sets of industrial cameras and related test equipment were used to test the actual fluctuation velocity of the 3-span contact wire on the test site through the experimental platform of catenary in the State-Key Laboratory of Traction Power [13], [23]. This test is mainly used to verify the accuracy of a fluctuation velocity recognition method based on displacement contour map proposed in the above paper. Based on the test data, by taking into account the influence of APC, this paper also verifies the correctness of the proposed recognition method of fluctuation velocity. The experimental platform of catenary is shown in Fig. 6, and the test equipment is shown in Fig. 7.

FIGURE 6. The experimental platform of catenary.



(a) Force hammer (b) System control interface

FIGURE 7. The experimental equipment.

According to the different positions of camera testing points, the fluctuation velocity test of contact wire is divided into three groups of operating conditions, and three different excitations are set for each group, which are applied at the first locator of each span. These three excitations include the initial uplift displacement excitation with fixed value and the fixed-point force hammer excitation exerted by two kinds of force hammers with different stiffness. The positions of the measuring points are the first dropper, midpoint and the second locator at each span separately, and the measured fluctuation velocity of the elastic wave of contact wire under all operating conditions is calculated according to the distance between the measuring points and the time difference between the starting vibration of the measuring points. The mean value of 138.87 m$\cdot$ s^{-1} is taken as the final result of fluctuation velocity c_{test} in contact wire test to reduce the influence of measurement error. The parameters of the test catenary are shown in Fig. 3(c), and the test results are shown in Table 1.

TABLE 1. Test results of the FVCW.

OCN	FVCW	OCN	FVCW
1-1	139.24	2-3	137.00
1-2	137.90	3-1	138.00
1-3	139.13	3-2	139.78
2-1	137.85	3-3	140.00
2-2	140.96		

Note: the front label of the operating condition sequence number (OCN) represents the catenary span number, and the back label represents the location sequence number of the test point.

Take the shape of the first order vibration wave to study, that is, take k = 1. By substituting the quality and position parameters of droppers into (21), the main vibration function value corresponding to each dropper in Q_1 can be obtained. Summing the values we can find:

$$Q_1 = \sum_{p=1}^{21} m_{ap} \sin^2 \frac{\pi}{3 \cdot 54} x_p = 3.05 \tag{42}$$

Similarly, substituting the quality and position parameters of locators into (22) leads to:

$$Q_2 = \sum_{q=1}^{4} M_{aq} \sin^2 \frac{\pi(q-1)}{3} = 3 \tag{43}$$

By substituting (42) and (43) into (29), the FVCW affected by the APC can be calculated as 152.03m$\cdot$ s^{-1}. Meanwhile, according to the parameters of contact wire in Fig. 3(c) and the main frequency in the test signal, the FVCW under the single string/cable model and Euler beam model is calculated, and the results obtained from the fluctuation velocity expression of contact wire after modification above are compared with the measured fluctuation velocity, as shown in Table 2.

As can be seen from Table 2, in the case of low-frequency vibration of catenary, the FVCW calculated based on string/cable vibration theory and Euler beam vibration theory is larger than the actual test result, and the relative error is 13.23%. As for the reasons, on the one hand, compared with the contact wire regarded as a single string/cable or Euler beam, the accessory parts connected to the actual contact wire, such as droppers and locators, improve the ability of contact wire to maintain its original vibration state, that is, to increase its inertial force, but do not change the pre-added tension in contact wire, that is, do not affect its elastic force. In general, the fluctuation velocity of elastic wave is proportional to the elastic force of elastic medium and inversely proportional to the inertial force of medium. Therefore, the fluctuation velocity calculated based on these two

TABLE 2. The comparison results of FVCW under different identification methods.

Identification method	Formula mode	FVCW (m/s)	Relative error(%)
String/Cable theory [10]	$\sqrt{S/\rho}$	157.25	13.23
Euler beam theory [18]	$\frac{2\pi f}{\sqrt{\sqrt{\left(\frac{S}{2EI}\right)^2+\frac{\rho(2\pi f)^2}{EI}}-\frac{S}{2EI}}}$	157.25	13.23
Equation (29)	$\frac{2\pi f}{\sqrt{\sqrt{(\frac{S}{2EI})^2+\frac{(2\pi f)^2[\rho+\frac{2}{L}(Q_1+Q_2)]}{EI}}-\frac{S}{2EI}}}$	152.03	9.48
Experimental test	$\Delta L/\Delta t$	138.87	

Note: the test is only conducted on the contact wire, and the main frequency of the signal is 1.189Hz.

vibration theories will overestimate the actual FVCW. On the other hand, compared with a single contact line, the elastic wave in the actual contact wire will reflect at the droppers and the locators during the propagation process. Hence, it takes longer for the same amount of elastic wave energy to travel the same distance on the actual contact wire. Therefore, the test result of fluctuation velocity is lower than the calculated value of fluctuation velocity theoretically. By comparing the theoretical FVCW considering the influence of APC with the measured fluctuation velocity, the relative error Δ between the theoretical fluctuation velocity and the measured fluctuation velocity can be obtained as:

$$\Delta = \frac{(152.03 - 138.87)}{138.87} \times 100\% = 9.48\%$$

Thus, it can be seen that by considering the inertial force exerted by droppers and locators on the contact wire, the relative error of the result calculated by the proposed modified expression considering the influence of APC is reduced by 3.75% compared with that of the result calculated based on the string/cable and Euler beam theory, which shows that the recognition method can effectively improve the prediction accuracy of FVCW.

V. CONCLUSION

In this paper, a recognition method is proposed to improve the recognition accuracy of FVC, which can provide a more reliable verification means for the matching design of the catenary structure parameters, so as to improve the current collection quality of pantograph-catenary and realize the speed upgrade. The single ideal Euler beam structure is improved by considering the effects of the inertial force exerted by the accessory parts on the catenary, and the FVC is modified through theoretical derivation. Moreover, the influence of the key parameters α and β on the FVC are explored. Through the fluctuation velocity test, the theoretical fluctuation velocity is compared and verified with the measured result. The conclusions are as follows:

(1) When the key parameter α is constant, the FVCW is less affected by the frequency, and the corresponding value changes in amplitude tend to be stable. Under certain frequency conditions, the inertial force exerted by the APC on the contact wire has a relatively obvious inhibition effect on the wave propagation velocity of contact wire, and the inhibition effect becomes more obvious with the increase of the quality and quantity of droppers and locators.

(2) Compared with the contact wire, the FVMW decreases significantly. The inertia force applied on the messenger wire by the APC also has a relatively obvious inhibitory effect on the FVMW. With the increase of the quantity and quality of droppers and the increase of the elastic coefficient at the supporting point, the FVMW will further decrease and become stable.

(3) It is found from the fluctuation velocity test of actual catenary that the relative error between the theoretical fluctuation velocity obtained by the fluctuation velocity modified expression of contact wire considering the influence of APC and the measured fluctuation velocity is less than 10%. Compared with the theoretical fluctuation velocity formula based on the string/cable and Euler beam theory, the recognition method proposed in this paper is more accurate in identifying the FVCW, and the recognition accuracy is improved by 3.75%.

In fact, in addition to the inertial force exerted on the catenary wires by the APC, the elastic wave of catenary will reflect at their points in the propagation process, which leads to a certain deviation between the theoretical and actual FVC. In the future, we will focus on exploring the propagation process of the elastic wave in the catenary, analyze the reflection mechanism of elastic wave at the points of APC from the theoretical perspective, and further modify the theoretical expression of FVC, so as to provide a more accurate verification means for the matching design of the catenary structure parameters.

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XIAOQIANG CHEN received the Ph.D. degree from the School of Electrical Engineering and Automation, Lanzhou Jiaotong University, Lanzhou, in 2010. He is currently a Professor with the School of Electrical Engineering and Automation, Lanzhou Jiaotong University. His research interests include electrical relationship of EMUs and traction and detection and assessment of pantograph-catenary in high-speed railway.

XI ZHANG received the B.S. degree in electrical engineering and its automation from Heilongjiang University, Harbin, China, in 2018. He is currently pursuing the M.S. degree in electric engineering with Lanzhou Jiaotong University, Lanzhou, China. His research interests include electrical contact and dynamic performance of pantograph-catenary.

YING WANG received the B.S. degree from the School of Electrical Engineering and Automation, Chongqing University, Chongqing, China, and the Ph.D. degree from the School of Electrical Engineering, Southwest Jiaotong University, Chengdu, China. His main research interests include reliable power supply control and prediction for high-speed railway EMUs.

XINYI WANG received the B.S. degree from the School of Electrical Engineering and Automation, Hebei University of Engineering, Handan, China, in 2019. She is currently pursuing the M.S. degree in electric engineering with Southwest Jiaotong University, Chengdu, China. Her research interests include structural dynamics and dynamic relationship of pantograph-catenary.