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Event-Triggered Dissipative Observer-Based Control for Delay Dependent T-S Fuzzy Singular Systems

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ABSTRACT This paper investigates the problem of event-triggered dissipative observer-based output feedback control for a class of Takagi-Sugeno (T-S) fuzzy singular systems with unavailable states, time-varying delays, and imprecise membership functions. The considered singular system is presented as a T-S fuzzy singular system which gives a more realistic presentation for a variety of non-linear dynamical systems than conventional state space representation. The general framework we designed is based on a fuzzy observer-based controller with time varying delay along with extended dissipativity, which provides the H_∞, L_2-L_∞ and dissipative performance indices and is robust against the disturbances and time-varying delay. Moreover, a novel Lyapunov-Krasovskii functional is adopted to analyze the closed loop stability of the fuzzy system. The solvability of Lyapunov-Krasovskii functional results in the formation of linear matrix inequalities. The controller and observer gains can be obtained by solving the linear matrix inequalities. Simulations are performed to validate the proposed scheme.

INDEX TERMS T-S fuzzy singular systems, fuzzy Lyapunov-Krasovskii functionals, time-varying delay, linear matrix inequalities (LMIs).

I. INTRODUCTION

To appropriately model the physical systems, it is appealing to describe both dynamic and static behaviors simultaneously. Singular systems have attained sufficient attention from the researchers in the past decades. In contrast to conventional state space representation, singular systems describe the practical demonstration of real-world applications which include large-scale systems, electric power systems, and economic systems. Singular systems are also known as descriptor systems whose dynamic part is represented by differential equations and relationships between different sections of the systems are described by algebraic equations [1]–[3].

In the last two decades, the model-based fuzzy control is widely used approach to handle the non-linear systems. The T-S fuzzy models transforms a nonlinear dynamical system into local linear models which dramatically attracts the attention of a huge number of researchers [4]. Usually, the ''event'' is described by an inequality, namely event-triggered condition, which consists of system states and a prescribed threshold parameter [5], [6]. It should be noted that the threshold parameter is closely related to data transmissions. However, the above-mentioned event triggered mechanism (ETMs), called as static ETMs, which are proposed with fixed constant threshold parameters, can not be adjusted along with the evolution of the systems and are difficult to be appropriately chosen in advance. In order to remove the limitations of static ETMs, a time-varying threshold parameter is considered in [7] for the event-triggered control problem of a singular system.

Significant research has been carried out for the design of closed-loop T-S fuzzy systems using parallel distribution compensation (PDC) approach for fuzzy state feedback controllers. Moreover, this type of controller based on fuzzy observers utilizes the same membership function [8]–[10]. Research related with the PDC approach can be found in [11], [12], leading to the complication of fuzzy rules and membership function. For design simplicity, imprecise

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premise design with PDC scheme is preferred. In this scheme, the fuzzy controller proposed in [13], [14] uses different membership functions and thus the robustness and design simplicity is increased. Moreover, imprecise premise design scheme results in less conservative results [15], [16].

The successful adaptation of T-S fuzzy systems compels the researchers to extend singular systems to T-S fuzzy singular systems [17], [18]. By using conventional T-S fuzzy models, the invertible solution of inertia matrix of Lagrangian systems is essentially required. The invertible solution to this matrix increases the number of fuzzy rules. On the contrary, the number of fuzzy rules will be reduced by using fuzzy singular systems. Moreover, the reduction of the number of fuzzy rules can decrease the risk of obtaining infeasible solution in LMI based control design [19].

Recently, network control systems (NCS) are getting too much attention in the modern research [20]–[22]. In NCS, the data from the sensor to the controller and from the controller to the actuator are transferred through a common network [23], [24]. Such type of mechanism is preferred in distributed control of modern industries including large power systems [25]. Comparing with traditional control, NCS is unpredictable due to time-varying delays in network and data dropouts. Therefore match premise requirement is not fulfilled in NCS [15]. As mentioned above in network controlled applications, time-varying delays occur due to limited capacities of data analysis and transmission between different sections of plants over a common network. So delay is considered as a key source of instability [26]–[29].

Currently, T-S fuzzy systems have got an extension to handle the time varying delays in singular systems [30], [31]. The stability of such type of systems is more challenging than normal regular systems, because the singular systems are required to be stable, regular and impulse free. Many researchers have considered the delay-dependent singular system, for details see [32], [33]. Handling of these concerns is one of the motivation. It is observed that most of the conventional control is time triggered and it wastes most of the communication resources and degrades the efficiency, so event-triggered communication over the network have got fruitful attention [34]–[40].

In most complex systems sometimes the internal states are unmeasurable. It mostly happens due to the cost of sensors or limited control techniques. In these situations, observer-based output feedback control is mostly adopted [41]. The observer design for singular systems is presented by [42]–[44]. The performance analysis of T-S fuzzy singular system is analysed by different techniques usually distinguished by input-output relationship, and plays a significant role in many practical applications, such as dissipative performance, L_2-L_∞ performance and H_∞ control Problem. In [45], the author discussed (Q-V-R)-α dissipative observer-based controller design. In [41], the similar problem is considered with uncertainties. Dissipative analysis for network-based singular system is considered in [46]. Delay- dependent stability and robust $L_2 - L_\infty$ control for fuzzy singular system is discussed in [33]. Dissipative control design for T-S fuzzy stochastic singular system is investigated in [47]. To present the performance indices in a unified manner the pioneer work was done by [48] which correlate all these performance indices. The concept of extended dissipativity was applied to neural networks in [49] and references therein but according to authors knowledge there is no similar work for event triggered singular systems for extended dissipative analysis in the literature. In spite of development reviewed above, to enhance the restricted network capacity, a trade-off is required between the computational complexities and the conservatism of NCS when time delay bounds are considered in NCS. To present these issues for fuzzy singular systems in an integrated model by maintaining a pre-specified control performance is the motivation of this work.

Based on the aforementioned discussions, this article is devoted to pursue event-triggered observer-based control design with state delays for T-S fuzzy singular systems, along with the derivation of the delay dependent conditions. One of the important derivations of this research article is the implementation of Lyapunov-Krasovskii fuzzy functionals (*LKF*) on fuzzy membership function also known as fuzzy *LKF* [50]. Such type of functions possess additional information about the behavior of the system and help in decreasing the conservatism of design and analysis for networked based fuzzy singular systems with event-triggered technique. This special type of fuzzy *LKF* addresses the limitations of common fuzzy *LKF* [51] by incorporating mode dependent integral terms which are coupled and its membership functions depend on non-integral terms in ordinary fuzzy *LKF*.

To the best of our knowledge, the considered event-triggered dissipative observer-based output feedback control for a class of T-S fuzzy singular systems with time-varying delay is not reported for T-S fuzzy singular systems. In the light of the above discussions, the authors are encouraged to pursue currently proposed research with the following listed novelties:

- A generalized non-linear system model for observer based T-S fuzzy singular system is derived for delay dependent NCS with event-triggered communication (ETC) and unavailable system states along with asynchronous premises due to the delay of the network.
- A uniform framework of dissipative control for fuzzy singular systems having capabilities to analyze performance indices for H_∞, $L_2 - L_\infty$ and dissipative is established.
- In contrast with [51], a novel LKF is considered to derive sufficient conditions to make system asymptotically stable in which fuzzy matrices in single and double integrals are employed which reduce conservatism under delay dependent conditions.

The remaining paper is organized as follows. In section II the system description and problem formulation is established along with ETC and T-S fuzzy observer design. In section III sufficient conditions for the stability of the closed loop system are proved. In section IV the effectiveness of the proposed design is presented through MATLAB/SIMULINK. The paper is concluded in section V.

A. ABBREVIATIONS AND ACRONYMS

The notation used in this paper is standard and can be found in the published literature. $\star$ is symmetric terms of matrices. $\mathfrak{L}_2[0,\infty]$ is space of integral vector over $[0,\infty]$. $\|\cdot\|$ represents the spectral norm for the matrices. N is the set of Positive integers.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider a continues-time T-S fuzzy singular model with time delayed network control system and event-triggered scheme described as:

P_i: IF $\varphi_1(x(t))$ is ν_1^i and if IF $\varphi_p(x(t))$ is ν_p^i THEN

$$\begin{cases} E\dot{x}(t) = \underline{A}_i x(t) + \underline{A}_{\hbar i} x(t-\hbar_1(t)) + \underline{B}_i u(t) + \underline{B}_{\omega i}\omega(t) \\ y(t) = \underline{C}_i x(t) + \underline{C}_{\hbar i} x(t-\hbar_1(t)) \\ z(t) = \underline{E}_i x(t) + \underline{E}_{\hbar i} x(t-\hbar_1(t)) \\ x(t) = \psi(t), t \in [-\bar{\hbar}_1, 0] \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, r$ denotes the number of fuzzy rules, $x(t) \in R^{n_x}$, $y(t) \in R^{n_y}$, $z(t) \in R^{n_z}$ and $u(t) \in R^{n_u}$ represent the state, the controlled output, the measured output and the control input, respectively, and $\omega(t) \in R^{n_\omega}$ represents the disturbance signal that belongs to $\mathfrak{L}_2[0,\infty]$. $E \in R^{n_x \times n_x}$ has rank$(E) = q \le n_x$. $\varphi_i(x(t))$ and ν_j^i ($i = 1, \cdots, r, j = 1, \cdots, p$) are premise variables and fuzzy sets. $\underline{A}_i$, $\underline{A}_{\hbar i}$, $\underline{B}_i$, $\underline{B}_{\omega i}$, $\underline{E}_i$, $\underline{E}_{\hbar i}$ $\underline{C}_i$ and $\underline{C}_{\hbar i}$ are the system matrices with proper dimensions. In system (1), $\psi(t)$ is the continuous vector-valued initial function defined on the interval $[-\bar{\hbar}_1, 0]$. $\hbar_1(t)$ is a time varying differentiable function that describes the delay of state. In order to avoid complication, $\varphi_j(x(t))$ can be written as $\varphi_j(x)$. By employing the fuzzy blending the whole fuzzy dynamical system in (1), we have

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r} \rho_i(\varphi(x))\{\underline{A}_i x(t) + \underline{A}_{\hbar i} x(t-\hbar_1(t)) \\ \qquad +\underline{B}_i u(t) + \underline{B}_{\omega i}\omega(t)\} \\ y(t) = \sum_{i=1}^{r} \rho_i(\varphi(x))\{\underline{C}_i x(t) + \underline{C}_{\hbar i} x(t-\hbar_1(t))\} \\ z(t) = \sum_{i=1}^{r} \rho_i(\varphi(x))\{\underline{E}_i x(t) + \underline{E}_{\hbar i} x(t-\hbar_1(t))\} \end{cases} \tag{2}$$

where $\varphi(x) = [\varphi_1(x), \varphi_2(x), \cdots, \varphi_p(x)]^T$ and $\rho_i(\varphi(x))$: $R^p \to [0\ 1]$, $i = 1, 2, \cdots, r$ represents the membership function of the fuzzy sets $\nu_1^i \times \cdots \times \nu_p^i$, defined as

$$\rho_i(\varphi(x)) = \prod_{j=1}^{p} \nu_j^i(\varphi(x)) \ge 0 \tag{3}$$

where $\nu_j^i(\varphi_j(x))$ designates as the grade membership function of $\varphi_j(x)$ in ν_j^i ($i = 1, 2, \cdots, r; j = 1, 2, \cdots, p$); the normalized weighted function can then be defined as:

$$\rho_i(\varphi_j(x)) = \frac{\mu_i(\varphi(x))}{\sum_{i=1}^{r}\mu_i(\varphi(x))} \quad \text{with} \ \sum_{i=1}^{r}\mu_i(\varphi(x)) = 1 \tag{4}$$

In this article, the measured output of the system is transmitted through common network channel.

A. EVENT-TRIGGERED CONTROL SCHEME

As we have already discussed in the literature that we are considering a common communication channel for the measurement output. Due to the common communication channel, there is a limitation of bandwidth for NCS. In order to enhance the communication performance, an event-triggered communication-based transmitter is inserted. In addition, to decide the transmission of the current measured data $y(t)$ to the observer from the ETC transmitter is based on logic function [36]. As network-induced delays are considered between the sensor and the controller in our proposed scheme, so the premise variable for fuzzy rules are asynchronous for the system and the controller. The ETC is designed to handle the event generated by the instant $i_n h$, where h is the sampling period. The error between the current sampled data and the latest transmitted data can be expressed as:

$$e_n(t) = y(z_n h) - y(i_n h) \tag{5}$$

where $z_n h = i_n h + mh$, $m \in$ N and $z_n h$ denotes the sampling instant between two simultaneous instants. The future transmission instants are based on the ETC scheme which is expressed as follows:

$$\begin{aligned} i_{n+1}h = i_n h + \underbrace{\min}_{m\in \mathrm{N}}\{mh \mid e_n^T(t)\Psi e_n(t)\} \\ > \sqrt{\varrho}\, y^T(i_n h)\Psi y(i_n h) \end{aligned} \tag{6}$$

where $0 < \varrho < 1$ is a given scalar parameter and $\Psi > 0$ is a positive definite weighting matrix to be determined later. From the above mentioned condition (6), it is clear that the next transmitting instants are experienced by two important aspects, the trigger parameter and the system output. From [37], it is evident that the signals transmitted instant of $\{i_n h \mid i_n \in \mathrm{N}\}$, are the subset of sampled instants represented by $\{\jmath_n h \mid \jmath_n \in \mathrm{N}\}$ and here $i_0 h = 0$ represents the initial transmitting instant. In this paper we consider the network induced transmission delays, ℓ_{i_n} and $\ell_{i_{n+1}}$ at transmitting instants $i_n h$ and $i_{n+1}h$ respectively. Then $i_n h + \ell_{i_n}$ represents the instant when the transmitted signal arrives at zero-order-holder (ZOH).

It is examined that $\bar{y}(t)$ retains the value of $y(i_n h)$ with the interval $[i_n h + \ell_{i_n}, i_{n+1}h + \ell_{i_{n+1}})$ influenced by ZOH.

$$\bar{y}(t) = y(i_n h), \quad t \in [i_n h + \ell_{i_n}, i_{n+1}h + \ell_{i_{n+1}}) \tag{7}$$

The subset used to call the holding zone Ξ of ZOH [36]:

$$\Xi = [i_n h + \ell_{i_n}, i_{n+1}h + \ell_{i_{n+1}}) = \bigcup_{m=0}^{n_l} \Xi_m \tag{8}$$

where $\Xi_m = [i_n h + \ell_{i_n}, i_{n+1}h + h + \ell_{i_{n+1}})$, $m = 0, 1, 2, \cdots, n_l, n_l = i_{n+1} - i_n - 1$. The network delay could be defined as $\theta(t) = t - i_n h$, satisfying $0 \le \ell_{i_n} \le \theta(t) \le h + \bar{\tau} \equiv d_M$, where h and $\bar{\tau}$ represent, the sampling period and upper bound for the allowable network induced delay, respectively. Based on the above mentioned analysis, the original input of observer could be expressed as:

$$\bar{y}(t) = y(i_n h) = y(t - \theta(t)) - e_n(t) \tag{9}$$

B. FUZZY OBSERVER BASED CONTROLLER DESIGN THROUGH NON-PDC APPROACH

Unlike prevailing PDC approaches, the observer here is designed based on different premise membership function (MF). From the T-S fuzzy singular system (1), there exists a network control system. First, an observer with imprecise MF is considered to approximate the unavailable states. Then a fuzzy controller should be designed based on the approximated states from the observer to evaluate the control objective. By considering the aforementioned ETC scheme and time delay, the T-S fuzzy observer is given as below:

O_j: IF $g_1(\hat{x}(t))$ is F_1^j and $\cdots$ and $g_p(\hat{x}(t))$ is F_p^j THEN

$$\begin{cases} E\dot{\hat{x}}(t) = \underline{A}_j\hat{x}(t) + \underline{A}_{\hbar j}\hat{x}(t-\hbar_2(t)) + \underline{B}_j u(t) \\ +\underline{B}_{\omega j}\omega(t) + L_j(\bar{y}(t) - \hat{y}(t)) \\ \hat{y}(t) = \underline{C}_j\hat{x}(t) + \underline{C}_{\hbar j}\hat{x}(t-\hbar_2(t)) \\ \hat{z}(t) = \underline{E}_j\hat{x}(t) + \underline{E}_{\hbar j}\hat{x}(t-\hbar_2(t)) \\ \hat{x}(t) = \hat{\psi}(t), t \in [-\bar{\hbar}_2, 0] \end{cases} \quad (10)$$

where F_d^j $(j = 1, 2, \ldots, r, d = 1, 2, \ldots, p)$ denotes the fuzzy sets, $g_d(\hat{x}(t))$ denotes the premise variable. p and r denotes the number of IF-THEN fuzzy rules and premise variables respectively. $\hat{x}(t) \in R^n$ is the estimation of observer state and the measured signal via ETC is denoted by $\bar{y}(t) = y(i_n h) \in R^m$ for $t \in [i_n h + \ell_n, i_{n+1}h + \ell_{n+1})$ and $\hat{y}(t) \in R^m$ is the measured output. L_j is the observer gain needs to be determined.

The comprehensive design of singular observer based controller for the T-S fuzzy system could be expressed as follows:

$$\begin{cases} E\dot{\hat{x}}(t) = \sum_{j=1}^{r} \rho_j(g(\hat{x}(t))[\underline{A}_j\hat{x}(t) + \underline{A}_{\hbar j}\hat{x}(t-\hbar_2(t)) \\ +\underline{B}_j u(t) + \underline{B}_{\omega j}\omega(t) + L_j(\bar{y}(t) - \hat{y}(t))] \\ \hat{y}(t) = \sum_{j=1}^{r} \rho_j(g(\hat{x}(t))[\underline{C}_j\hat{x}(t) + \underline{C}_{\hbar j}\hat{x}(t-\hbar_2(t))] \\ \hat{z}(t) = \sum_{j=1}^{r} \rho_j(g(\hat{x}(t))[\underline{E}_j\hat{x}(t) + \underline{E}_{\hbar j}\hat{x}(t-\hbar_2(t))] \end{cases} \quad (11)$$

where $\rho_j(g(\hat{x}(t))) = \frac{\eta_j(g(\hat{x}))}{\sum_{j=1}^{r}\eta_j(g(\hat{x}))}$ and $\eta_j(g(\hat{x})) = \prod_{d=1}^{p} F_d^p(g_d(\hat{x}))$, $\sum_{j=1}^{r}\rho_j(g(\hat{x}(t))) = 1$, and $F_d^p(g_d(\hat{x}))$ corresponds to the membership function value of $g_d(\hat{x})$ in F_d^j. Due to the absence of network between the controller and observer, the assumption being made is to use the same premise variable for observer and controller.

The dynamics of the observer based fuzzy control law could be expressed as follows:

Controller s: IF $g_1(\hat{x}(t))$ is F_1^j and $\cdots$ and $g_p(\hat{x}(t))$ is F_p^j THEN

$$u(t) = K_s(\hat{x}(t)) \quad (12)$$

where the controller gain K_s will be determined later. The T-S fuzzy controller can be expressed as follows:

$$u(t) = \sum_{s=1}^{r} \rho_s(g(\hat{x}))K_s\hat{x}(t) \quad (13)$$

Defining the estimation error as $\tilde{E}\tilde{x}(t) \triangleq x(t) - \hat{x}(t)$, then observer error dynamics corresponds to

$$\tilde{E}\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{\hat{x}}(t) \quad (14)$$

By considering (11) and (13), the closed loop T-S fuzzy singular system could be expressed as

$$\begin{cases} \tilde{E}\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r} \rho_i(x)\rho_j(x)\rho_s(x)[\Omega_{ij}\hat{x}(t) \\ \quad +\underline{A}_{\hbar_i}\tilde{x}(t-\hbar_1(t) + \underline{A}_{\hbar_j}\hat{x}(t-\hbar_2(t)) \\ \quad +(\underline{B}_{\omega_i} - \underline{B}_{\omega_j})\omega(t) + \underline{A}_i\tilde{x}(t) - L_j(y(t - \hat{y}(t))] \end{cases} \quad (15)$$

where $\Omega_{ij} = \underline{A}_i - \underline{A}_j + (\underline{B}_i - \underline{B}_j)K_s$. Computing the error output $e(t) = z(t) - \hat{z}(t)$ and representing the state augmentation $\varsigma = \begin{bmatrix}\hat{x}(t) \\ \tilde{x}(t)\end{bmatrix}$, then the augmented system could be expressed as

$$\begin{cases} \tilde{E}\dot{\varsigma}(t) = \sum_{i,j,s=1}^{r} \rho_i(x)\rho_j(\hat{x})\rho_s(\hat{x})[\underline{A}_1\varsigma(t) \\ +\sum_{i=1}^{2} \underline{A}_{\hbar_i}\varsigma(t-\hbar_i(t)) \\ +\underline{A}_g X\varsigma(t-\vartheta(t)) \\ +\underline{B}_\omega\tilde{\omega}(t) + L_{ej}e_n(t)] \\ e(t) = \Sigma_{i,j=1}^{r}\rho_i(x)\rho_j[\underline{E}\varsigma(t) + \underline{E}_{\hbar_i}\varsigma(t-\hbar_i(t))] \end{cases} \quad (16)$$

where $\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & E\end{bmatrix}$, $\underline{A}_1 = \begin{bmatrix}\Delta_1 & 0 \\ \Delta_2 & \underline{A}_i\end{bmatrix}$, $\underline{A}_{\hbar 1} = \begin{bmatrix} 0 & 0 \\ \underline{A}_{\hbar i} & \underline{A}_{\hbar i}\end{bmatrix}$, $\underline{A}_{\hbar 2} = \begin{bmatrix}\underline{A}_{\hbar j} - L_j\underline{C}_{\hbar j} & 0 \\ -(\underline{A}_{\hbar j} - L_j\underline{C}_{\hbar j}) & 0\end{bmatrix}$ $\underline{A}_g = \begin{bmatrix} L_j\underline{C}_i \\ -L_j\underline{C}_i\end{bmatrix}$, $\underline{B}_\omega = \begin{bmatrix}\underline{B}_j \\ -\underline{B}_j\end{bmatrix}$, $L_{ej} = \begin{bmatrix}-L_j \\ L_j\end{bmatrix}$, $X = \begin{bmatrix} I & I\end{bmatrix}$, $\Delta_1 = \underline{A}_j + \underline{B}_j K_s - L_j\underline{C}_j$ $\Delta_2 = (\underline{A}_i - \underline{A}_j) + (\underline{B}_i - \underline{B}_j)K_s + L_j\underline{C}_j$, $\underline{E} = \begin{bmatrix}\underline{E}_i & \underline{E}_j\end{bmatrix}$, $\underline{E}_{\hbar_1} = \begin{bmatrix}\underline{E}_{\hbar i} & 0\end{bmatrix}$, $\underline{E}_{\hbar_2} = \begin{bmatrix} 0 & -\underline{E}_{\hbar_j}\end{bmatrix}$

It is observed that the augmented T-S fuzzy singular system (16) is experienced by delays with the following bounded conditions:

$$0 \le \hbar_g(t) \le \bar{\hbar}_g, \quad \dot{\hbar}_g \le \upsilon_g, g = 1, 2 \quad (17)$$

where υ_g and $\bar{\hbar}_g > 0$ are fixed scalar values.

Before proceeding to the main results of our proposed methodology, we are going to present the important definition and lemma which is helpful in the derivation of our main results.

Let us consider a delayed T-S fuzzy singular system with $u(t) = 0$

$$\begin{cases} E\dot{x}(t) = \underline{A}_i x(t) + \underline{A}_{\hbar i}x(t-\hbar(t)) \\ x(t) = \psi(t), t \in [-\bar{\hbar}_i, 0] \end{cases} \quad (18)$$

Definition 1 [41]: System (18) is known as impulse free and regular, if the pair $(E, \underline{A}_i)$ *is impulse free and regular.*

Lemma 1 [52]: Let us consider $\hbar(t) \in [0, \hbar_M]$ and $\mathbf{g}_1$, $\mathbf{g}_2$ and $\mho$ with appropriate dimension matrices, then:

$$(\hbar_M - \hbar(t))\mathbf{g}_1 + \hbar(t)\mathbf{g}_2 + \mho < 0$$

if and only if

$$\hbar_M \mathbf{g}_1 + \mho < 0$$
$$\hbar_M \mathbf{g}_2 + \mho < 0$$

Lemma 2 [15]: Rank of Matrix $(\underline{C}_i) = m$, $\underline{C}_i \in R^{m\times n}$, the single value decomposition (SVD) for C can be expressed as $C = O\begin{bmatrix} S & 0\end{bmatrix}\mathbb{V}^t$, where $O.O^T = I$ and $\mathbb{V}.\mathbb{V}^T = I$, Assume $\mathbb{X}_i > 0$, $\mathbb{M} \in \mathcal{R}^{m\times m}$. Then, there exist $\tilde{\mathbb{X}}_i$ such that $\underline{C}_i\mathbb{X}_i = \tilde{\mathbb{X}}_i\underline{C}_i$ if and only if

$$\mathbb{X}_i = \mathbb{V} * diag\{\mathrm{M}, \mathrm{N}\} * \mathbb{V}^T$$

Definition 2 [48]: Suppose $\Lambda_0 \geq 0$, $\Lambda_1 \leq 0$, Λ_2 and $\Lambda_3 > 0$ are matrices, that assure $(\| \Lambda_1 \| + \| \Lambda_2 \|) \| \Lambda_0 \| = 0$. Then, the T-S fuzzy singular system (16) is called to be extended dissipative if there exists a scalar ϑ such that the following inequality holds for any $t_f \geq 0$ and all $\omega(t) \in \mathfrak{L}_2[0, \infty]$:

$$\int_0^{t_f} \mathbb{J}(t)dt \geq \vartheta + \sup_{0\leq t\leq t_f} e(t)^T \Lambda_0 e(t) \tag{19}$$

where

$$\mathbb{J}(t) = e(t)^T \Lambda_1 e(t) + 2e(t)^T \Lambda_2 \omega(t) + \omega(t)^T \Lambda_3 \omega(t)$$

Remark 1: In Definition 2, Λ_0, Λ_1, Λ_2 and Λ_3 are the weighting matrices. It is clear from [50] that the performance notation includes the H_∞ performance, L_2-L_∞ performance, dissipativity and passivity performance as different cases when the weighting matrices Λ_0, Λ_1, Λ_2 and Λ_3 are selected as special forms. It is observed that $\Lambda_0 \geq 0$ and $\Lambda_1 \leq 0$ can guarantee that there always exists matrices $\tilde{\Lambda}_0$ and $\tilde{\Lambda}_1$ such that

$$\Lambda_0 = \tilde{\Lambda}_0^T \tilde{\Lambda}_0, \quad \Lambda_1 = -\tilde{\Lambda}_1^T \tilde{\Lambda}_1 \tag{20}$$

This decomposition is basis for the establishment of the main results of this work.

III. MAIN RESULTS

In this section, we are going to present that the closed loop system (16) is asymptotically stable, regular, impluse free and dissipative.

Theorem 1: For the given $0 < \xi_g < 1$, and $\sum_{g=1}^{3} \xi_g = 1$, $\bar{\hbar}_g$, d_M, and γ, the closed-loop system (16) under the event-triggered scheme given by (6) is regular, impulse free and dissipative for the time-varying delay $\hbar_g(t)$ governed by the condition (17), if there exist symmetric positive-definite matrices $\mathcal{G}_{gi} > 0$, $\mathcal{G} > 0$, $R_{gi} > 0$, $U > 0$, $R > 0$, $\mathbb{S}_g > 0$, $\mathbb{Z}_{gi} > 0$, $\mathbb{P}_i > 0$, $\Psi > 0$, $\mathbb{W}_g > 0$, $\mathbb{M}_{gi}$ such that $\mathbf{G}_{gi} := (\dot{\mathcal{G}}_{gi} + \dot{R}_{gi} - \mathbb{S}_g) < 0$, $\mathbf{R}_{gi} := (\dot{R}_{gi} - \mathbb{S}_g) < 0$, $\mathbf{Z}_{gi} := (\dot{\mathbb{Z}}_{gi} - \bar{\hbar}_g^{-1}\mathbb{W}_g) < 0$, $(U - \mathbb{P}_i) < 0$, $\begin{bmatrix} \mathbb{Z}_{gi} & -\mathbb{M}_{gi} \\ -\mathbb{M}_{gi} & \mathbb{Z}_{gi}\end{bmatrix} > 0$, $g = 1, 2$, $i = 1, 2, \cdots, r$ and the following inequalities hold:

$$\tilde{E}^T\mathbb{P}_i = \mathbb{P}_i^T\tilde{E} \geq 0 \tag{21}$$

$$\Phi_{ij} = \begin{bmatrix} \Phi_{ij}^{11} & \Phi_{ij}^{12} \\ \star & -I \end{bmatrix} < 0, \tag{22}$$

$$\begin{aligned}
\Phi_{ij}^{11} &= \mathrm{diag}\{-\xi_1 U, -\xi_2 U, -\xi_3 U\} \\
\Phi_{ij}^{12} &= \mathrm{col}\{\underline{E}^T \tilde{\Lambda}_0, \underline{E}_{\hbar_1}^T \tilde{\Lambda}_0, \underline{E}_{\hbar_2}^T \tilde{\Lambda}_0\} \\
\Upsilon_{ij}^{11} &= \dot{\mathbb{P}}_i + \mathbb{P}_i\underline{A}_1 + \underline{A}_1^T\mathbb{P}_i \\
&\quad + \sum_{g=1}^{2}\left(\mathcal{G}_{gi} + R_{gi} - \mathbb{Z}_{gi} + a_g\mathbb{S}_g\right) \\
&\quad + \mathcal{G} + \mathcal{S}_{ij}^1 + \mathcal{S}_{ij}^{1^T} \\
\wp_{\ell i} &= -(1-\upsilon_\ell)\mathcal{G}_{\ell i} \quad \ell = 1, 2 \\
\varphi_{\ell i} &= -\tilde{E}^T R_{\ell i}\tilde{E} - \tilde{E}^T\mathbb{Z}_{\ell i}\tilde{E}^T, \quad \ell = 1, 2 \\
\Upsilon_{ij}^{14} &= \mathbb{P}_i\underline{A}_g - \mathcal{S}_{ij}^1 + \mathcal{S}_{ij}^{6^T} \\
\Upsilon_{ij}^{66} &= \sqrt{\varrho}\underline{C}_i^T\Psi\underline{C}_i - \mathcal{S}_{ij}^6 - \mathcal{S}_{ij}^{6^T} + \mathcal{T}_{ij}^6 + \mathcal{T}_{ij}^{6^T} \\
\Upsilon_{ij}^{77} &= -\mathcal{G} - \mathcal{T}_{ij}^7 - \mathcal{T}_{ij}^{7^T} \\
\Theta_{ij}^{12} &= \begin{bmatrix} \mathbb{P}_i L_{ej} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\Theta_{ij}^{13} &= \begin{bmatrix} \digamma_i & -\Lambda_2^T\underline{E}_{\hbar_1} & 0 & -\Lambda_2^T\underline{E}_{\hbar_2} & 0 & 0 & 0 \end{bmatrix} \\
\Theta_{ij}^{14} &= \begin{bmatrix} \mathbb{P}_i\underline{A}_1 & \mathbb{P}_i\underline{A}_{\hbar_1} & 0 & \mathbb{P}_i\underline{A}_{\hbar_2} & 0 & \mathbb{P}_i\underline{A}_g & 0 \end{bmatrix} \\
\Theta_{ij}^{15} &= \begin{bmatrix} \tilde{\Lambda}_1\underline{E} & \tilde{\Lambda}_1\underline{E}_{\hbar_1} & 0 & \tilde{\Lambda}_1\underline{E}_{\hbar_2} & 0 & 0 & 0 \end{bmatrix} \\
\Theta_{ij}^{16}(1) &= \tilde{E}^T\sqrt{d_M}\mathcal{S}_{ij}\tilde{E}, \quad \Theta_{ij}^{16}(2) = \tilde{E}^T\sqrt{d_M}\mathcal{T}_{ij}\tilde{E} \\
\mathcal{S}_{ij} &= [\mathcal{S}_{ij}^1, 0, 0, 0, 0, \mathcal{S}_{ij}^6, 0] \\
\mathcal{T}_{ij} &= [0, 0, 0, 0, 0, \mathcal{T}_{ij}^6, \mathcal{T}_{ij}^7] \\
\mathbb{Z}_i &= \sum_{g=1}^{2}\left(\mathbb{Z}_{gi} + 1/2\bar{\hbar}_g^{2}\mathbb{W}_g\right) + d_M R \\
\digamma_i &= -\Lambda_2^T\underline{E} + \underline{B}_w^T\mathbb{P}_i
\end{aligned} \tag{24}$$

Proof: This proof is comprised on two main parts. First we will show that the system (16) is impulse free and regular. While in the second part we are concerned about the stability of the T-S fuzzy singular system.

Since $\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & E\end{bmatrix}$ and the rank $E = r \leq n$, then the rank $\tilde{E} = 2r$ so there must exist two nonsingular matrices $\hat{G}$ and $\hat{H} \in R^{n_x \times n_x}$ such that

$$\hat{G}\tilde{E}\hat{H} = \begin{bmatrix} I_{2r} & 0 \\ 0 & 0\end{bmatrix}, \quad \hat{G}\underline{A}_i\hat{H} = \begin{bmatrix} \underline{A}_{i1} & \underline{A}_{i2} \\ \underline{A}_{i3} & \underline{A}_{i4}\end{bmatrix} \tag{25}$$

Corresponding to the method applied in [53], which yields that $\underline{A}_{i4}$ is a nonsingular matrix, which follows from Definition 1 that the T-S fuzzy singular system in (16) is regular and impulse free.

Now we construct the following Lyapunov-Krasovskii function candidate for system (16) with the form of

$$V(x_t,t)=\varsigma(t)^T\tilde{E}^T\mathbb{P}_i\tilde{E}\varsigma(t)+\sum_{g=1}^{3}V_g(t) \tag{26}$$

where

$$\begin{aligned}V_1(t)&=\sum_{g=1}^{2}\left(\int_{t-\hbar_g(t)}^{t}\varsigma(s)^T\mathcal{G}_{gi}\varsigma(s)\right)ds\\&+\int_{t-d_M}^{t}\varsigma(s)^T\mathcal{G}\varsigma(s)ds\\&+\sum_{g=1}^{2}\left(\int_{t-\bar{\hbar}_g}^{t}\varsigma(s)^TR_{gi}\varsigma(s)\right)ds\\V_2(t)&=\sum_{g=1}^{2}\bar{\hbar}_g\int_{-\bar{\hbar}_g}^{0}\int_{t+\alpha}^{t}\dot{\varsigma}(s)^T\tilde{E}^T\mathbb{Z}_{gi}\tilde{E}\dot{\varsigma}(s)dsd\gamma\\&+\sum_{g=1}^{2}\left(\int_{-\bar{\hbar}_g}^{0}\int_{t+\alpha}^{t}\varsigma(s)^T\mathbb{S}_g\varsigma(s)\right)dsd\gamma\\&+\int_{-d_M}^{0}\int_{t+\alpha}^{t}\dot{\varsigma}(s)^T\tilde{E}^TR\tilde{E}\dot{\varsigma}(s)dsd\gamma\\V_3(t)&=\sum_{g=1}^{2}\left(\int_{-\bar{\hbar}_g}^{0}\int_{\theta}^{0}\int_{t+\alpha}^{t}\dot{\varsigma}(v)^T\mathbb{W}_g\dot{\varsigma}(s)\right)dsd\gamma d\theta\end{aligned}$$

Taking the time derivative of $V(x_t,t)$ along the trajectories of the system given in (16) is computed as:

$$\begin{aligned}&V(x_t,t)\\&=\varsigma(t)^T(\tilde{E}^T\dot{\mathbb{P}}_i\tilde{E}+\sum_{g=1}^{2}(\mathcal{G}_{gi}+\tilde{E}^TR_{gi}\tilde{E}+\bar{\hbar}_g\mathbb{S}_g)+\mathcal{G})\varsigma(t)\\&\quad+2\varsigma(t)^T\tilde{E}^T\mathbb{P}_i\tilde{E}\dot{\varsigma}(t)\\&\quad-\sum_{g=1}^{2}(1-\dot{\hbar}_g(t))\varsigma(t-\hbar_g(t))^T\mathcal{G}_{gi}\varsigma(t-\hbar_g(t))\\&\quad-\sum_{g=1}^{2}\varsigma(t-\bar{\hbar}_g)^T\tilde{E}^TR_{gi}\tilde{E}\varsigma(t-\bar{\hbar}_g)\\&\quad-\varsigma(t-d_M)^T\mathcal{G}\varsigma(t-d_M)\\&\quad-\dot{\varsigma}(t)^T\left(\sum_{g=1}^{2}\left(\bar{\hbar}_g^2\tilde{E}^T\mathbb{Z}_{gi}\tilde{E}+0.5\bar{\hbar}_g^2\mathbb{W}_g\right)+d_MR\right)\dot{\varsigma}(t)\\&\quad+\int_{t-d_M}^{t}\dot{\varsigma}^T(s)\mathrm{R}\dot{\varsigma}(s)ds\\&\quad-\sum_{g=1}^{2}\bar{\hbar}_g\int_{t-\bar{\hbar}_g}^{t}\dot{\varsigma}(s)^T\tilde{E}^T\mathbb{Z}_{gi}\tilde{E}\dot{\varsigma}(s)ds\\&\quad+\sum_{g=1}^{2}\int_{t-\hbar_g(t)}^{t}\varsigma(s)^T\mathbf{G}_{gi}\varsigma(s)ds\\&\quad+\sum_{g=1}^{2}\int_{t-\bar{\hbar}_g}^{t-\hbar_g(t)}\varsigma(s)^T\mathbf{R}_{gi}\varsigma(s)ds\\&\quad+\sum_{g=1}^{2}\bar{\hbar}_g\int_{-\bar{\hbar}_g}^{0}\int_{t+\alpha}^{t}\dot{\varsigma}(s)^T\mathbf{Z}_{gi}\dot{\varsigma}(s)dsd\gamma\end{aligned}$$

Notice that:

$$\sum_{g=1}^{2}\bar{\hbar}_g\int_{t-\bar{\hbar}_g}^{t}\dot{\varsigma}(s)^T\tilde{E}^T\mathbb{Z}_{gi}\tilde{E}\dot{\varsigma}(s)ds\geq\zeta^T\tilde{E}^T\begin{bmatrix}\mathbb{Z}_{gi}&-\mathbb{M}_{gi}\\-\mathbb{M}_{gi}&\mathbb{Z}_{gi}\end{bmatrix}\tilde{E}\zeta \tag{27}$$

where $\zeta=\begin{bmatrix}\varsigma(t)\\\varsigma(t-\bar{\hbar}_g)\end{bmatrix}$. To deal with the above mentioned inequalities, we are employing free weighting matrices with proper dimension [23]:

$$\begin{aligned}&2\zeta^T(t)\mathcal{S}_{ij}[\tilde{E}\varsigma(t)-\tilde{E}\varsigma(t-\theta(t))-\int_{t-\theta(t)}^{t}\tilde{E}\dot{\varsigma}(s)ds]\\&=0\\&2\zeta^T(t)\mathcal{T}_{ij}[\tilde{E}\varsigma(t-\tilde{E}\theta(t))-\tilde{E}\varsigma(t-d_M)-\int_{t-d_M}^{t-\theta(t)}\tilde{E}\dot{\varsigma}(s)ds]\\&=0\end{aligned} \tag{28}$$

$$\Theta_{ij}=\begin{bmatrix}\Theta_{ij}^{11}&\Theta_{ij}^{12^T}&\Theta_{ij}^{13^T}&\Theta_{ij}^{14^T}&\Theta_{ij}^{15^T}&\Theta_{ij}^{16^T}(s)\\\star&-\Psi&0&0&0&0\\\star&\star&-\Lambda_3&\underline{B}_{\omega}^T\mathbb{P}_i&0&0\\\star&\star&\star&\mathbb{Z}_i-2\mathbb{P}_i&0&0\\\star&\star&\star&\star&-I&0\\\star&\star&\star&\star&\star&-R\end{bmatrix}<0,\ s=1,2 \tag{23}$$

$$\Theta_{ij}^{11}=\begin{bmatrix}\Upsilon_{ij}^{11}&\mathbb{P}_i\underline{A}_{\hbar_1}&\tilde{E}^T\mathbb{M}_{1i}\tilde{E}&\mathbb{P}_i\underline{A}_{\hbar_2}&\tilde{E}^T\mathbb{M}_{2i}\tilde{E}&\Upsilon_{ij}^{14}&0\\\star&\wp_{1i}&0&0&0&0&0\\\star&\star&\varphi_{1i}&0&0&0&0\\\star&\star&\star&\wp_{2i}&0&0&0\\\star&\star&\star&\star&\varphi_{2i}&0&0\\\star&\star&\star&\star&\star&\Upsilon_{ij}^{66}&-\mathcal{T}_{ij}^6+\mathcal{T}_{ij}^{7^T}\\\star&\star&\star&\star&\star&\star&\Upsilon_{ij}^{77}\end{bmatrix}$$

where $\mathcal{S}_{ij}$ and $\mathcal{T}_{ij}$ are the matrices with suitable dimensions. We acquire, utilizing the Lemma 1:

$$-2\zeta^T(t)\mathcal{S}_{ij}\int_{t-\theta(t)}^{t}\tilde{E}\dot{\varsigma}(s)ds \leq \int_{t-\theta(t)}^{t}\dot{\varsigma}^T(s)\tilde{E}^T R\tilde{E}\dot{\varsigma}(s)ds + \theta(t)\zeta^T(t)\mathcal{S}_{ij}R^{-1}\mathcal{S}_{ij}^T\zeta(t)$$
$$-2\zeta^T(t)\mathcal{T}_{ij}\int_{t-d_M}^{t-\theta(t)}\tilde{E}\dot{\varsigma}(s)ds \leq \int_{t-d_M}^{t-\theta(t)}\dot{\varsigma}^T(s)\tilde{E}^T R\tilde{E}\dot{\varsigma}(s)ds + (d_M-\theta(t))\zeta^T(t)\mathcal{T}_{ij}R^{-1}\mathcal{T}_{ij}^T\zeta(t) \quad (29)$$

From the triggering condition (6), $t \in \Xi_m$, we have:

$$e_n^T(i_nh)\Psi e_n(i_nh) \leq \sqrt{\varrho}y(t-\theta(t))^T\Psi y(t-\theta(t))$$

similarly can be expressed as

$$y(t-\theta(t))^T\Psi y(t-\theta(t)) = \varsigma(t-\theta(t))^T H^T \underline{C}(\rho_\hbar)^T \Psi\underline{C}(\rho_\hbar)H\varsigma(t-\theta(t))$$

Now, characterize the augmented matrix

$$\zeta(t) = \text{col}[\varsigma(t), \varsigma(t-\hbar_1(t), \varsigma(t-\bar{\hbar}_1), \varsigma(t-\hbar_2(t), \varsigma(t-\bar{\hbar}_2), \zeta(t-\theta(t), \zeta(t-d_M), e_n(t), \omega(t)]$$

Integrating (27-29) with event triggering mechanism (6), we get

$$\begin{aligned}
&V(x_t,t)-\mathbb{J}(t)\\
&\leq \varsigma(t)^T(\tilde{E}^T\dot{\mathbb{P}}_i\tilde{E}+\sum_{g=1}^{2}(\mathcal{G}_{gi}+\tilde{E}^TR_{gi}\tilde{E}+\hbar_g\mathbb{S}_g)\\
&\quad +\mathcal{G})\varsigma(t)+2\varsigma(t)^T\tilde{E}^T\mathbb{P}_i\tilde{E}\dot{\varsigma}(t)\\
&\quad -\sum_{g=1}^{2}(1-\dot{\hbar}_g(t))\varsigma(t-\hbar_g(t))^T\mathcal{G}_{gi}\varsigma(t-\hbar_g(t))\\
&\quad -\sum_{g=1}^{2}\varsigma(t-\bar{\hbar}_g)^T\tilde{E}^TR_{gi}\tilde{E}\varsigma(t-\bar{\hbar}_g)\\
&\quad -\varsigma(t-d_M)^T\mathcal{G}\varsigma(t-d_M)\\
&\quad -\dot{\varsigma}(t)^T[\sum_{g=1}^{2}(\bar{\hbar}_g^2\mathbb{Z}_{gi}+0.5\bar{\hbar}_g^2\mathbb{W}_g)+d_MR]\dot{\varsigma}(t)\\
&\quad +\int_{t-d_M}^{t}\dot{\varsigma}^T(s)R\dot{\varsigma}(s)ds\\
&\quad +\sum_{g=1}^{2}\int_{t-\hbar_g(t)}^{t}\varsigma(s)^T\mathbf{G}_{gi}\varsigma(s)ds\\
&\quad +\sum_{g=1}^{2}\int_{t-\bar{\hbar}_g}^{t-\hbar_g(t)}\varsigma(s)^T\mathbf{R}_{gi}\zeta(s)ds\\
&\quad +\sum_{g=1}^{2}\bar{\hbar}_g\int_{-\bar{\hbar}_g}^{0}\int_{t+\alpha}^{t}\dot{\varsigma}(s)^T\tilde{E}^T\mathbf{Z}_{gi}\tilde{E}\dot{\varsigma}(s)dsd\gamma\\
&\quad +\sum_{g=1}^{2}\begin{bmatrix}\varsigma(t)\\ \varsigma(t-\bar{\hbar}_g)\end{bmatrix}^T\tilde{E}^T
\begin{bmatrix}\mathbb{Z}_{gi} & -\mathbb{M}_{gi}\\ -\mathbb{M}_{gi} & \mathbb{Z}_{gi}\end{bmatrix}\tilde{E}\begin{bmatrix}\varsigma(t)\\ \varsigma(t-\bar{\hbar}_g)\end{bmatrix}\\
&\quad +\underline{C}_i^Tx^T(t-\theta(t))\Psi\underline{C}_ix(t-\theta(t))\\
&\quad -\sqrt{\varrho}e_n^T(t)\Psi e_n(t)\\
&\quad +\theta(t)\zeta^T(t)\mathcal{S}_{ij}R^{-1}\mathcal{S}_{ij}^T\zeta(t)\\
&\quad +(d_M-\theta(t))\zeta^T(t)\mathcal{T}_{ij}R^{-1}\mathcal{T}_{ij}^T\zeta(t)
\end{aligned}$$

At this point, we conclude that

$$V(x_t,t)-\mathbb{J}(t) \leq \zeta^T(t)\Theta_{ij}\zeta(t)$$

Note that

$$\mathbb{Z}_i = \mathbb{P}_i[\mathbb{P}_i\mathbb{Z}_i^{-1}\mathbb{P}_i]\mathbb{P}_i \leq \mathbb{P}_i[2\mathbb{P}_i-\mathbb{Z}_i]^{-1}\mathbb{P}_i$$

This implies that

$$\begin{aligned}
\Theta_{ij} \leq &\begin{bmatrix}\Theta_{ij}^{11} & \Theta_{ij}^{12^T} & \Theta_{ij}^{13^T}\\ \star & -\Psi & 0\\ \star & \star & -\Lambda_3\end{bmatrix}\\
&+\begin{bmatrix}\Theta_{ij}^{14^T}\\ 0\\ B_w^T\end{bmatrix}\mathbb{P}_i[2\mathbb{P}_i-\mathbb{Z}_i]^{-1}\mathbb{P}_i\begin{bmatrix}\Theta_{ij}^{14^T}\\ 0\\ B_w^T\end{bmatrix}^T\\
&+\begin{bmatrix}\Theta_{ij}^{15^T}\\ 0\\ 0\end{bmatrix}\tilde{\Lambda}_1^T\tilde{\Lambda}_1\begin{bmatrix}\Theta_{ij}^{15^T}\\ 0\\ 0\end{bmatrix}^T\\
&+\begin{bmatrix}\Theta_{ij}^{16^T}\\ 0\\ 0\end{bmatrix}\begin{bmatrix}\Theta_{ij}^{16^T}\\ 0\\ 0\end{bmatrix}^T
\end{aligned} \quad (30)$$

By applying schur complement results in (22). The matrix on right hand side of (30) is negative definite which confirms that $\Theta_{ij} \leq 0$. This along with (27-29) concludes that

$$V(x_t,t)-\mathbb{J}(t) \leq 0 \quad (31)$$

Therefore further calculations are similar to [50], Theorem 1) so it is not difficult to formulate that the system error dynamics (16) is extended dissipative from definition 1. Thus the proof is completed. □

Assumption 1: Real constant scalars λ_i exist, such that $\dot{\rho}_i \leq \lambda_i, i = 1, 2, 3, \cdots, r$.

Remark 2: Although, the LMI conditions in Theorem 1 are based on the membership functions, which are usually complicated to be solved. Thus, it is compulsory to convert into the conditions in Theorem 1 to strict LMIs. To this end, we have to make an assumption on the rate of function λ.

Based on Theorem 1, the existence of controller and observer design is described in Theorem 2 as a sufficient condition as follows.

Theorem 2: For the given $0 < \xi_g < 1$, and $\sum_{g=1}^{3}\xi_g = 1$, $\bar{\hbar}_g$, d_M, and γ, the closed-loop system (16) under the event-triggered scheme given by (6) is regular, impulse free and dissipative for the time-varying delay $\hbar_g(t)$ governed by the condition (17), if there exist symmetric positive-definite matrices $\tilde{\mathcal{G}}_{gi} > 0$, $\tilde{\mathcal{G}} > 0$, $\tilde{R}_{gi} > 0$, $\tilde{U} > 0$ $\tilde{R} > 0$, $\tilde{\mathbb{S}}_g > 0$, $\tilde{\mathbb{Z}}_{gi} > 0$, $\tilde{\mathbb{P}}_i > 0$, $\tilde{\Psi} > 0$, $\tilde{\mathbb{W}}_g > 0$, $\mathbf{X}_0$, $\tilde{\mathbb{M}}_{gi}$, L_g, K_g, N_g, $\mathcal{Y}_j$, *and $\mathcal{F}_j$ with appropriate dimensions such that the following LMIs (32)–(36,38) hold for $g = 1, 2$ and $i, j, k = 1, 2, \cdots, r$:*

$$\tilde{U} - \tilde{\mathbb{P}}_i < 0, \qquad (\mathbb{X}_i + \mathbf{X}_0) > 0 \tag{32}$$
$$\begin{bmatrix} \tilde{\mathbb{Z}}_{gi} & -\tilde{\mathbb{M}}_{gi} \\ \star & \tilde{\mathbb{Z}}_{gi} \end{bmatrix} > 0, \tag{33}$$
$$\mathcal{A}_n < 0, \mathcal{B}_n > 0, \qquad n = 1, 2, 3 \tag{34}$$
$$\hat{\Phi}_{ijg} + \hat{\Phi}_{jig} < 0, \quad i < j \tag{35}$$
$$\tilde{\Theta}_{ijg} + \tilde{\Theta}_{jig} < 0, \quad i < j \tag{36}$$

where

$$\begin{aligned}
\hat{\Phi}_{ijg} &= \begin{bmatrix} \tilde{\Phi}_{ij}^{11} & \tilde{\Phi}_{ij}^{12} \\ \star & -I \end{bmatrix} < 0, \\
\tilde{\Phi}_{ij}^{11} &= \mathrm{diag}\{-\xi_1 \tilde{U}, -\xi_2 \tilde{U}, -\xi_3 \tilde{U}\} \\
\tilde{\Phi}_{ij}^{12} &= \mathrm{col}\{\sigma_{ij}^{(a)} \tilde{\Lambda}_0, \sigma_{ij}^{(b)} \tilde{\Lambda}_0, \sigma_{ij}^{(c)} \tilde{\Lambda}_0\} \\
\tilde{\Upsilon}_{ij}^{11} &= \sum_{i=1}^{r} \left(\begin{bmatrix} I \\ 0 \end{bmatrix} (\sum_{i=1}^{r} \lambda_i (\tilde{\mathbb{X}}_i + \mathbf{X}_0) \begin{bmatrix} I & 0 \end{bmatrix} \right) \\
&\quad + \sigma_{ij}^{(I)} + \sigma_{ij}^{(I)^T} + \tilde{\mathcal{G}} + \tilde{\mathcal{S}}_{ij}^1 \tilde{\mathcal{S}}_{ij}^{1^T} \\
&\quad + \sum_{g=1}^{2} \left(\tilde{\mathcal{G}}_{gi} + \tilde{E}^T \tilde{R}_{gi} \tilde{E} - \tilde{E}^T \tilde{\mathbb{Z}}_{gi} \tilde{E} + \hbar_g \tilde{\mathbb{S}}_g \right) \\
\tilde{\Upsilon}_{ij}^{14} &= \sigma_{ij}^{(IV)} - \tilde{\mathcal{S}}_{ij}^1 + \tilde{\mathcal{S}}_{ij}^{6^T} \\
\tilde{\Upsilon}_{ij}^{66} &= \sqrt{\varrho} \underline{C}_i^T \tilde{\Psi} \underline{C}_i - \tilde{\mathcal{S}}_{ij}^6 - \tilde{\mathcal{S}}_{ij}^{6^T} + \tilde{\mathcal{T}}_{ij}^6 + \tilde{\mathcal{T}}_{ij}^{6^T} \\
\tilde{\Upsilon}_{ij}^{77} &= -\tilde{\mathcal{G}} - \tilde{\mathcal{T}}_{ij}^7 - \tilde{\mathcal{T}}_{ij}^{7^T} \\
\tilde{\Theta}_{ij}^{12} &= \mathrm{col}\{\mathbb{L}_{ej}, 0, 0, 0, 0, 0, 0\} \\
\tilde{\Theta}_{ij}^{13} &= \mathrm{col}\{-\sigma_{ij}^{(a)} \Lambda_2^T + \underline{B}_w, -\sigma_{ij}^{(b)} \Lambda_2^T, 0, -\sigma_{ij}^{(c)} \Lambda_2^T, 0, 0, 0\} \\
\tilde{\Theta}_{ij}^{14} &= \mathrm{col}\{\sigma_{ij}^{(I)^T}, \sigma_{ij}^{(II)^T}, 0, \sigma_{ij}^{(III)^T}, 0, \sigma_{ij}^{(IV)^T}, 0\} \\
\tilde{\Theta}_{ij}^{15} &= \mathrm{col}\{\sigma_{ij}^{(a)} \Lambda_1^T, \sigma_{ij}^{(b)} \Lambda_1^T, 0, \sigma_{ij}^{(c)} \Lambda_1^T, 0, 0, 0\} \\
\tilde{\wp}_{\ell i} &= -(1 - \upsilon_\ell) \tilde{\mathcal{G}}_{\ell i} \\
\tilde{\varphi}_{\ell i} &= -\tilde{E}^T \tilde{R}_{\ell i} \tilde{E} - \tilde{E}^T \tilde{\mathbb{Z}}_{\ell i} \tilde{E}, \ \ell = 1, 2 \\
\mathcal{A}_1 &= \sum_{i=1}^{r} \lambda_i [\tilde{\mathcal{G}}_{gi} + R_{gi} + \mathrm{L}_g] - \tilde{\mathbb{S}}_g \\
\mathcal{B}_1 &= \tilde{\mathcal{G}}_{gi} + R_{gi} + \mathrm{L}_g \\
(\mathcal{A}_2, \mathcal{B}_2) &= \left(\sum_{i=1}^{r} \lambda_i [\tilde{R}_{gi} + \mathrm{K}_g] - \tilde{\mathbb{S}}_g, \quad \tilde{R}_{gi} + \mathrm{K}_g \right) \\
(\mathcal{A}_3, \mathcal{B}_3) &= \left(\sum_{i=1}^{r} \lambda_i [\tilde{\mathbb{Z}}_{gi} + \mathrm{N}_g] - \frac{1}{\hbar_g} \tilde{\mathbb{W}}_g, \quad \tilde{\mathbb{Z}}_{gi} + \mathrm{N}_g \right) \\
\tilde{\Theta}_{ij}^{16}(1) &= \tilde{E}^T \sqrt{d_M} \tilde{\mathcal{S}}_{ij} \tilde{E}, \quad \tilde{\Theta}_{ij}^{16}(2) = \tilde{E}^T \sqrt{d_M} \tilde{\mathcal{T}}_{ij} \tilde{E}, \\
\sigma_{ij}^{(I)} &= \begin{bmatrix} \underline{A}_j \mathbb{X}_j + \underline{B}_j \mathcal{Y}_j - \mathcal{F}_j C_j & 0 \\ (\underline{A}_i - \underline{A}_j) \mathbb{X}_j + (\underline{B}_i - \underline{B}_j) \mathcal{Y}_j + \mathcal{F}_j \underline{C}_j & \underline{A}_i \mathbb{X}_j \end{bmatrix} \\
\sigma_{ij}^{(II)} &= \begin{bmatrix} 0 & 0 \\ \underline{A}_{\hbar_i} \mathbb{X}_j & \underline{A}_{\hbar_i} \mathbb{X}_j \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\sigma_{ij}^{(III)} &= \begin{bmatrix} \underline{A}_{\hbar_j} \mathbb{X}_j - \mathcal{F}_j \underline{C}_{\hbar_j} & 0 \\ -(\underline{A}_{\hbar_j} \mathbb{X}_j - \mathcal{F}_j \underline{C}_{\hbar_j}) & 0 \end{bmatrix} \\
\sigma_{ij}^{(IV)} &= \begin{bmatrix} \mathcal{F}_j \underline{C}_{\hbar_j} \\ -\mathcal{F}_j \underline{C}_{\hbar_j} \end{bmatrix} H, \quad \sigma_{ij}^{(V)} = \begin{bmatrix} -\mathcal{F}_j \\ \mathcal{F}_j \end{bmatrix} \\
\sigma_{ij}^{(a)} &= \begin{bmatrix} \mathbb{X}_i \underline{E}_i^T \\ -\mathbb{X}_i \underline{E}_j^T \end{bmatrix}, \quad \sigma_{ij}^{(b)} = \begin{bmatrix} \mathbb{X}_i \underline{E}_{\hbar_i}^T \\ 0 \end{bmatrix} \\
\sigma_{ij}^{(c)} &= \begin{bmatrix} 0 \\ -\mathbb{X}_i \underline{E}_{\hbar_j}^T \end{bmatrix}, \quad \mathbb{L}_{ej} = \begin{bmatrix} -\mathcal{F}_j \\ \mathcal{F}_j \end{bmatrix} \\
\tilde{\mathcal{S}}_{ij} &= [\tilde{\mathcal{S}}_{ij}^1, 0, 0, 0, 0, \tilde{\mathcal{S}}_{ij}^6, 0] \\
\tilde{\mathcal{T}}_{ij} &= [0, 0, 0, 0, 0, \tilde{\mathcal{T}}_{ij}^6, \tilde{\mathcal{T}}_{ij}^7] \\
\tilde{\mathbb{Z}}_i &= \sum_{g=1}^{2} \left(\tilde{\mathbb{Z}}_{gi} + 0.5 \bar{\hbar}_g^2 \tilde{\mathbb{W}}_g \right) + d_M \tilde{R}
\end{aligned}$$

The controller and observer gains are given as below:

$$K_s = \mathcal{Y}_j \mathbb{X}_i^{-1}, \quad L_j = \mathcal{F}_j O S \mathbb{X}_i^{-1} S^{-1} O^{-1}. \tag{37}$$

Proof: Define $\mathbb{X}_i = \tilde{\mathbb{P}}_i^{-1}$, $\mathbb{X}_i \mathcal{G}_{gi} \mathbb{X}_i = \tilde{\mathcal{G}}_{gi}$, $\mathbb{X}_i \mathcal{G} \mathbb{X}_i = \tilde{\mathcal{G}}$, $\mathbb{X}_i R_{gi} \mathbb{X}_i = \tilde{R}_{gi}$, $\mathbb{X}_i R \mathbb{X}_i = \tilde{R}$, $\mathbb{X}_i \mathbb{Z}_{gi} \mathbb{X}_i = \tilde{\mathbb{Z}}_{gi}$, $\mathbb{X}_i \mathbb{M}_{gi} \mathbb{X}_i = \tilde{\mathbb{M}}_{gi}$, $\mathbb{X}_i \Psi \mathbb{X}_i = \tilde{\Psi}$, $\mathbb{X}_i \mathcal{S}_{ij} \mathbb{X}_i = \tilde{\mathcal{S}}_{ij}$, $\mathbb{X}_i \mathcal{T}_{ij} \mathbb{X}_i = \tilde{\mathcal{T}}_{ij}$. For $\mathbb{X}_i = \mathcal{V} \begin{bmatrix} \mathbb{X}_{1i} & \star \\ \star & \mathbb{X}_{2i} \end{bmatrix} \mathcal{V}^T$, according to Lemma 2 in [15], there exists $\tilde{\mathbb{X}}_i = O S \mathbb{X}_{1i} S^{-1} O^{-1}$. Let $\underline{C}_i \mathbb{X}_i = \tilde{\mathbb{X}}_i \underline{C}_i$ where $\tilde{\mathbb{X}}_i^{-1} = O S \mathbb{X}_{1i}^{-1} S^{-1} O^{-1}$. Pre- and post-multiplying (22) by $\{\mathbb{X}_i, \mathbb{X}_i, \mathbb{X}_i, I\}$ and its transpose, (35) can be obtained. Likewise, pre- and post multiplying (23) by $\{\underbrace{\mathbb{X}_i, \cdots, \mathbb{X}_i}_{6}, I, \mathbb{X}_i, I, \mathbb{X}_i\}$ and its transpose, (36) can be obtained.

Now, defining $\mathbb{X}(\rho) = \sum_{i=1}^{r} \rho_i \mathbb{X}_i$, $\tilde{\mathcal{G}}_g(\rho) = \sum_{i=1}^{r} \rho_i \tilde{\mathcal{G}}_{gi}$, $\tilde{R}_g(\rho) = \sum_{i=1}^{r} \rho_i \tilde{R}_{gi}$. Then it is confirmed that $\mathbb{X}_i > 0$, $\tilde{\mathcal{G}}_{gi} > 0$, $\tilde{R}_{gi} > 0$. It is noticed that $\sum_{i=1}^{r} \rho_i = 1$ results in $\sum_{i=1}^{r} \dot{\rho}_i = 0$. Based on Assumption 1 and the inequality condition $(\mathbb{X}_i + \mathbf{X}_0) > 0$, mentioned in (32), we obtain

$$\dot{\mathbb{X}}(\rho) = \sum_{i=1}^{r} \dot{\rho}_i \mathbb{X}_i = \sum_{i=1}^{r} \dot{\rho}_i (\mathbb{X}_i + \mathbf{X}_0) \le \sum_{i=1}^{r} \lambda_i (\mathbb{X}_i + \mathbf{X}_0)$$

Since $\dot{\rho}_i \le \lambda_i$, $\rho_i > 0$, and $\sum_{i=1}^{r} \dot{\rho}_i = 0$, one can obtain

$$\begin{aligned}
\dot{\mathbb{X}}(\rho) &= \sum_{i=1}^{r} \dot{\rho}_i \mathbb{X}_i = \sum_{i=1}^{r} \dot{\rho}_i (\mathbb{X}_i + \mathbf{X}_0), \\
&\le \sum_{i=1}^{r} \lambda_i \underbrace{(\mathbb{X}_i + \mathbf{X}_0)}_{>0},
\end{aligned}$$

Computing further

$$\Theta^{11}(\rho) \le \sum_{i=1}^{r} \sum_{j=1}^{r} \rho_i \rho_j \Theta_{ij}^{11} \tag{39}$$

Assume $\tilde{\Theta}(\rho)$ corresponds to matrix on the left hand side of (36). So concerned to (39), we have

$$\tilde{\Theta}(\rho) \leq \sum_{fij=1}^{r} h_f \rho_i \rho_j \Theta_{ijf} = \sum_{f=1}^{r} \rho_f [\sum_{i=1}^{r} \rho_i^2 \Theta_{iif} + \sum_{i=1}^{r-1} \sum_{j=i+1}^{r} \rho_i \rho_j (\Theta_{ijf} + \Theta_{jif})]$$

Combining this with (36), results in

$$\tilde{\Theta}(\rho) < 0$$

The inequality condition in (36) is fulfilled. Similarly, it is not difficult to prove that the condition (22) is fulfilled when the condition (35) holds. On the other side with Assumption 1 and the condition (34), we have that

$$\begin{aligned}\dot{\tilde{\mathcal{G}}}_g(h) + \dot{\tilde{R}}_g(h) - \tilde{\mathbb{S}}_g &= \sum_{i=1}^{r} \dot{\rho}_i(\tilde{\mathcal{G}}_{gi} + \tilde{R}_{gi}) - \tilde{\mathbb{S}}_g \\ &= \sum_{i=1}^{r} \dot{\rho}_i(\tilde{\mathcal{G}}_{gi} + \tilde{R}_{gi} + \mathrm{L}_g) - \tilde{\mathbb{S}}_g \\ &\leq \sum_{i=1}^{r} \lambda_i \underbrace{(\tilde{\mathcal{G}}_{gi} + \tilde{R}_{gi} + \mathrm{L}_g)}_{>0} - \tilde{\mathbb{S}}_g \\ &< 0\end{aligned}$$

Similar procedure can be adopted for the inequalities in (34) to show

$$\begin{aligned}\dot{\tilde{R}}_{gi} - \tilde{\mathbb{S}}_g &= \sum_{i=1}^{r} \dot{\rho}_i \tilde{R}_{gi} - \tilde{\mathbb{S}}_g \\ &= \sum_{i=1}^{r} \dot{\rho}_i(\tilde{R}_{gi} + \mathrm{K}_g) - \tilde{\mathbb{S}}_g \\ &\leq \sum_{i=1}^{r} \lambda_i \underbrace{(\tilde{R}_{gi} + \mathrm{K}_g)}_{>0} - \tilde{\mathbb{S}}_g \\ &< 0\end{aligned}$$

$$\begin{aligned}\dot{\tilde{\mathbb{Z}}}_{gi} - \bar{\hbar}_2^{-1} \tilde{\mathbb{W}}_g &= \sum_{i=1}^{r} \dot{\rho}_i \tilde{\mathbb{Z}}_{gi} - \bar{\hbar}_2^{-1} \tilde{\mathbb{W}}_g \\ &= \sum_{i=1}^{r} \dot{\rho}_i(\tilde{\mathbb{Z}}_{gi} + \mathrm{N}_g) - \bar{\hbar}_2^{-1} \tilde{\mathbb{W}}_g \\ &\leq \sum_{i=1}^{r} \lambda_i \underbrace{(\tilde{\mathbb{Z}}_{gi} + \mathrm{N}_g)}_{>0} - \bar{\hbar}_2^{-1} \tilde{\mathbb{W}}_g \\ &< 0\end{aligned}$$

When the LMIs in (33-36) are feasible, all the mentioned conditions in Theorem 2 are fulfilled. The proof is then completed. □

Remark 3: As we are using fuzzy Lyapunov-Krasovskii functional method, which can grant us more information on membership function in the designing process. So the conditions of Theorem 2 are relying on Assumption 1, where it is assumed that the membership functions are differentiable and bounded. But it is important to note that when in the situations where the membership functions are not differentiable, it is difficult to use matrices dependent on the membership function in the Lyapunov-Krasovskii functional. In such cases the quadratic Lyapunov-Krasovskii functional with matrices independent of the membership functions can be used.

Remark 4: In the structure of Lyapunov-Krasovskii functional, mostly authors implement the common quadratic LKF. In our work, our main part is implemented the weight-dependent LKF. As presented in [51], we believe that in such type of fuzzy LKF have more grip on the information about the nonlinear dynamic plant due to the dependence on the membership function. Which results in less conservative results because the conservatism depends on the LKF along with the inequality bounding techniques for the establishment of results. As per authors knowledge such type of LKF with free weighting matrices has not reported previously for T-S fuzzy singular systems.

Remark 5: As the proposed method has some limitations as mentioned in Remark 3, for that reason, we can only

$$\tilde{\Theta}_{ijg} = \begin{bmatrix} \tilde{\Theta}_{ij}^{11} & \tilde{\Theta}_{ij}^{12} & \tilde{\Theta}_{ij}^{13} & \tilde{\Theta}_{ij}^{14} & \tilde{\Theta}_{ij}^{15} & \tilde{\Theta}_{ij}^{16}(s) \\ \star & -\tilde{\Psi} & 0 & 0 & 0 & 0 \\ \star & \star & -\Lambda_3 & B_w^T & 0 & 0 \\ \star & \star & \star & \tilde{\mathbb{Z}}_i - 2\tilde{\mathbb{P}}_i & 0 & 0 \\ \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & -\tilde{R} \end{bmatrix} < 0, \qquad s = 1, 2 \tag{38}$$

$$\Theta_{ij}^{11} = \begin{bmatrix} \tilde{\Upsilon}_{ij}^{11} & \sigma_{ij}^{(II)} & \tilde{E}^T \tilde{\mathbb{M}}_{1i} \tilde{E} & \sigma_{ij}^{(III)} & \tilde{E}^T \tilde{\mathbb{M}}_{2i} \tilde{E} & \tilde{\Upsilon}_{ij}^{14} & 0 \\ \star & \tilde{\wp}_{1i} & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \tilde{\varphi}_{1i} & 0 & 0 & 0 & 0 \\ \star & \star & \star & \tilde{\wp}_{2i} & 0 & 0 & 0 \\ \star & \star & \star & \star & \tilde{\varphi}_{2i} & 0 & 0 \\ \star & \star & \star & \star & \star & \tilde{\Upsilon}_{ij}^{66} & -\tilde{\mathcal{T}}_{ij}^{6} + \tilde{\mathcal{T}}_{ij}^{7^T} \\ \star & \star & \star & \star & \star & \star & \tilde{\Upsilon}_{ij}^{77} \end{bmatrix}$$

exploit the quadratic LKF (i.e., Lyapunov matrices are not dependent upon the membership functions). In this scenario, the obtained conditions in Theorem 2 are valid by setting ρ_i to be appropriately small and by limiting matrices variables. $\tilde{\mathbb{P}}_i > 0$, $\tilde{\mathbb{S}}_g > 0$, $\tilde{\mathcal{G}}_{gi} > 0$, $\tilde{R}_{gi} > 0$, $\tilde{\mathbf{Z}}_{gi} > 0$, $\tilde{\mathbb{W}}_g > 0$, $\tilde{\mathbb{M}}_{gi}$, L_g, K_g, N_g, *to* $\tilde{\mathbb{P}} > 0$, $\tilde{\mathbb{S}} > 0$, $\tilde{\mathcal{G}} > 0$, $\tilde{R} > 0$, $\tilde{\mathbf{Z}} > 0$, $\tilde{\mathbb{W}} > 0$, $\tilde{\mathbb{M}}$, L, K, N,. *However, this may lead to some limitation, when these variables are delimited to special cases.*

IV. SIMULATION EXAMPLES

Now in this section, we are going to present the numerical examples to illustrate the advantage and effectiveness of our proposed method. The first two examples are used to demonstrate the enhancement of our results regarding the existing schemes. The third example is presented to show the applicability of the proposed controller design approach.

A. EXAMPLE 1

Let us consider the T-S fuzzy system borrowed from [54] under input $u(t) = 0$ and $\omega(t) = 0$ with two fuzzy rules. The system parameters are listed as below:

$$\underline{A}_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix} \underline{A}_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix},$$
$$\underline{A}_{\hbar_1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} \underline{A}_{\hbar_2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}$$

For the considered example the system is non-singular and the time-derivative of the delay $\hbar(t)$ is unknown with lower bound is considered as zero, and we set $E = I_{2\times 2}$ similarly presented in [54]. This fuzzy system is studied most frequently in the published literature, and the objective is to compute the maximum upper bound delay $\bar{\hbar}$ for a given γ at which the system remains in the stable region. It is also evident from the comparison of Table 1, that the maximum allowable delay ensures the asymptotic stability of the system and is also compared with some recent researches. It is also evident from the compared results that the proposed results obtained from Theorem 2 with time varying delay are less conservative than [54]–[58]. So the results obtained under Theorem 2 provides large upper bound delay with improvement of 69% from [58].

TABLE 1. Maximum upper bounds of time-delay $\bar{\hbar}$.

Method	[57]	[56]	[58]	[55]	[59]	Proposed
$\bar{\hbar}$	1.597	2.2943	2.5932	3.2712	3.5260	5.9858

B. EXAMPLE 2

To demonstrate the quality and potency of our results, we are considering the T-S fuzzy singular system presented in [41]. The system parameters are listed as below:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\underline{A}_1 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad \underline{A}_2 = \begin{bmatrix} 0 & a \\ -2 & -2 \end{bmatrix},$$

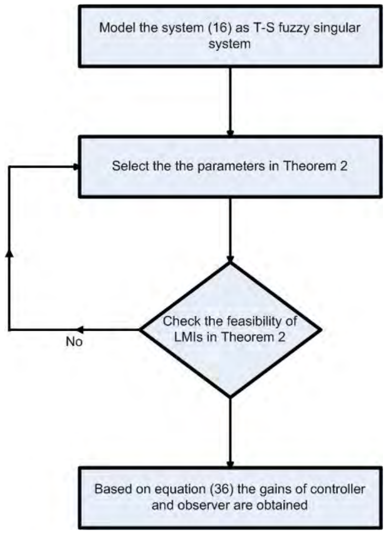

FIGURE 1. Algorithm to calculate the controller and observer gains.

TABLE 2. Minimum allowable γ for different values of a and b.

a	2			5	
b	0.3	0.7	0.95	0.8	0.83
Proposed	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
Theorem 2 [41]	0.1908	0.4763	12.5166	2.8888	18.5937
Theorem 2 [60]	0.2913	0.6714	–	10.8072	–

$$\underline{A}_{\hbar_1} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \quad \underline{A}_{\hbar_2} = \begin{bmatrix} 0.1 & 0 \\ b & -0.5 \end{bmatrix},$$
$$\underline{B}_{\omega 1} = \underline{B}_{\omega 2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \underline{E}_1 = \underline{E}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

To illustrate the proposed scheme is less conservative than the schemes proposed in [59] and [41], we have adopted the H_∞ performance index to measure the system performance. The minimum attenuation level is computed for different values of a and b and the corresponding comparison is tabulated in Table. 2. It is evident from the comparison that our proposed scheme is better than [59] and [41].

C. EXAMPLE 3

Consider a mass-spring-damper system [60], [61], the differential equations of the system are given as below:

$$\mathbb{M}\ddot{s} + \mathfrak{g}(s, \dot{s}) + \mathfrak{f}(s) + q_1(s)\omega = q_2(s)u \tag{40}$$

where $\mathbb{M}$ is the mass; ω is the disturbance applied externally; s is the displacement; u is the applied force; $\mathfrak{g}(s, \dot{s})$, $\mathfrak{f}(s)$, $q_1(s)$ and $q_2(s)$ represents the nonlinearities with respect to the damper, the spring, ω and u, respectively.

Assume that $\mathbb{M} = 1$, $\mathfrak{g}(s, \dot{s}) = -0.75\dot{s}$, $f(s) = 0.67s^3 - 0.05s$, $q_1(s) = -0.5 - 0.1\, s^2$, and $q_2(s) = 1 - 0.1\, s^2$. Let

$x_1 = s$ and $x_2 = \dot{s}$. Then from (40), we have

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.67x_1^3 + 0.5x_1 + 0.75x_2 + (0.5 + 0.1x_1^2)\omega \\ +(1 - 0.1x_2^2)u \\ z = x_2(t) + 0.5u \end{cases} \quad (41)$$

For the system (41), consider the operational range, $\Im = x :| x_1 |\leq 1.5, | x_2 |\leq 2.5$. By choosing the premise variable as $\chi = x_1$ and the universe of discourse as $\Im_\chi = \chi :| \chi |\leq 1.5$. Likewise [60], [61], the dynamics system (41) can be represented by the following T-S fuzzy model:

Plant Rule 1: *IF* χ is about 0, *THEN*

$$\begin{aligned} E\dot{x}(t) &= \underline{A}_1 x(t) + \underline{B}_1 u(t) + \underline{B}_{\omega 1}\omega(t) \\ z(t) &= \underline{E}_1 x(t) + \underline{D}_1 u(t) \end{aligned} \quad (42)$$

Plant Rule 2: *IF* χ is about ± 1.5, *THEN*

$$\begin{aligned} E\dot{x}(t) &= \underline{A}_2 x(t) + \underline{B}_2 u(t) + \underline{B}_{\omega 2}\omega(t) \\ z(t) &= \underline{E}_2 x(t) + \underline{D}_2 u(t) \end{aligned} \quad (43)$$

Membership functions are $\rho_1(\chi) = \cos(\chi_1(t))^2$ and $\rho_2(\chi) = 1 - \rho_1(\chi)$. The system matrices (42) are given as

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \underline{A}_1 & \underline{B}_1 & \underline{B}_{w_1} \\ \underline{A}_2 & \underline{B}_2 & \underline{B}_{w_2} \end{bmatrix} = \left[\begin{array}{cc|c|c} 0 & 1 & 0 & 0 \\ 0.05 & 0.75 & 1 & 0.5 \\ \hline 0 & 1 & 0 & 0 \\ 0.05 & 0.75 & 0.775 & 0.725 \end{array}\right]$$

$$\underline{E}_1 = \underline{E}_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \underline{D}_1 = \underline{D}_2 = 0$$

Remaining parameters of system (1) are considered as follows:

$$\begin{bmatrix} \underline{A}_{\hbar 1} \\ \underline{A}_{\hbar 2} \end{bmatrix} = \left[\begin{array}{cc} 0 & 1 \\ 0.05 & 0.75 \\ \hline 0 & 1 \\ 0.05 & 0.75 \end{array}\right]$$

$$\begin{bmatrix} \underline{C}_1 & \underline{C}_{\hbar 1} & \underline{E}_{\hbar 1} \\ \underline{C}_2 & \underline{C}_{\hbar 2} & \underline{E}_{\hbar 2} \end{bmatrix} = \left[\begin{array}{cc|cc|cc} 0 & 1 & -3 & 0.5 & 0 & 0 \\ \hline 0 & 1 & -2.5 & 0.6 & 0 & 0 \end{array}\right]$$

The T-S fuzzy observer-based controller can be designed as below.

$\mathfrak{Rule}_1$ *IF* $\psi(t)$ is about 0, *THEN*

$$u(t) = K_1\hat{x}(t)$$

$\mathfrak{Rule}_2$ *IF* $\psi(t)$ is about ± 1.5, *THEN*

$$u(t) = K_2\hat{x}(t)$$

where $\psi(t) = \hat{\theta}(t)$. The membership functions are given as belows:

$$\rho_1(\psi(t)) = \frac{\psi(t)^2}{22.25} \quad \text{and} \quad \rho_2(\psi(t)) = 1 - \rho_1(\psi(t)).$$

The time-varying delay is given as $\hbar_\kappa(t) = 2\sin(0.25\bar{\hbar}_\kappa) + 2, \quad \kappa = 1, 2$. The delay is bounded by condition (17).

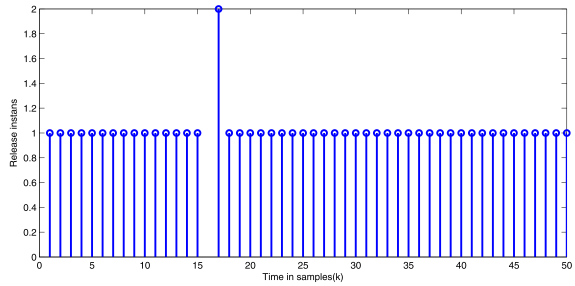

FIGURE 2. Release instants and release intervals by ETC for Example-2 in Case-I.

In order to verify the robustness of the system, the disturbance induced in the system is given as belows:

$$\omega(t) = \begin{cases} 1, & 3 \leq t \leq 7 \\ -1, & 10 \leq t \leq 15 \\ 0, & \text{elsewhere} \end{cases} \quad (44)$$

In the following we present three cases: the H_∞ control, dissipative control and $L_2 - L_\infty$ control.

Case-I: H_∞ Controller:

Let $\Lambda_\circ = 0$, $\Lambda_1 = -1$, $\Lambda_2 = 0$ and $\Lambda_3 = \gamma^2$. The value of γ is also chosen to be 2.5. The results obtained from the feasible LMI (32)–(36,38) for controller and observer gain with trigger matrix $\tilde{\Psi} = 24.9941$ are as follows:

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -19.6548 & -12.2567 \\ -6.2257 & -4.1476 \end{bmatrix}$$

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}^T = \begin{bmatrix} -48.2989 & -3.3210 \\ -44.8941 & -11.7604 \end{bmatrix}^T$$

The control constraints of H_∞ can be acquired in the form (37). By considering the initial conditions $\psi = [0.5\pi, 0.75\pi]$, the state estimation of mass-spring system are presented in Figure 3. (a-b). As it is clear from the figures that the controller designed based on T-S fuzzy dynamic output feedback is successful for the system. In order to verify proposed control scheme, our applied event triggered scheme with the given parameters $\varrho = 0.5$, induced communication delay $d_M = 0.6$ ms and by choosing the sampling period $h = 0.2$ ms, the system states and transmission instants and intervals are shown in the Figure 2. So the system is stable with desired control performance. Based on system non-linearities disturbance and delay, the system under consideration well behaves with managed communication resources. By using H_∞ controller, the error is improved and decline in error estimate results in better performance.

Case-II: Dissipative Controller:

Let $\Lambda_\circ = 0$, $\Lambda_1 = -1$, $\Lambda_2 = 1$ and $\Lambda_3 = \gamma$. The value of γ is also chosen to be 2.5. It is observed that the feasible solution of LMI (32)–(36,38) with trigger matrix $\tilde{\Psi} = 24.6521$ results in the controller and observer gain given as below:

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -19.5098 & -11.8366 \\ -6.2263 & -4.0709 \end{bmatrix}$$

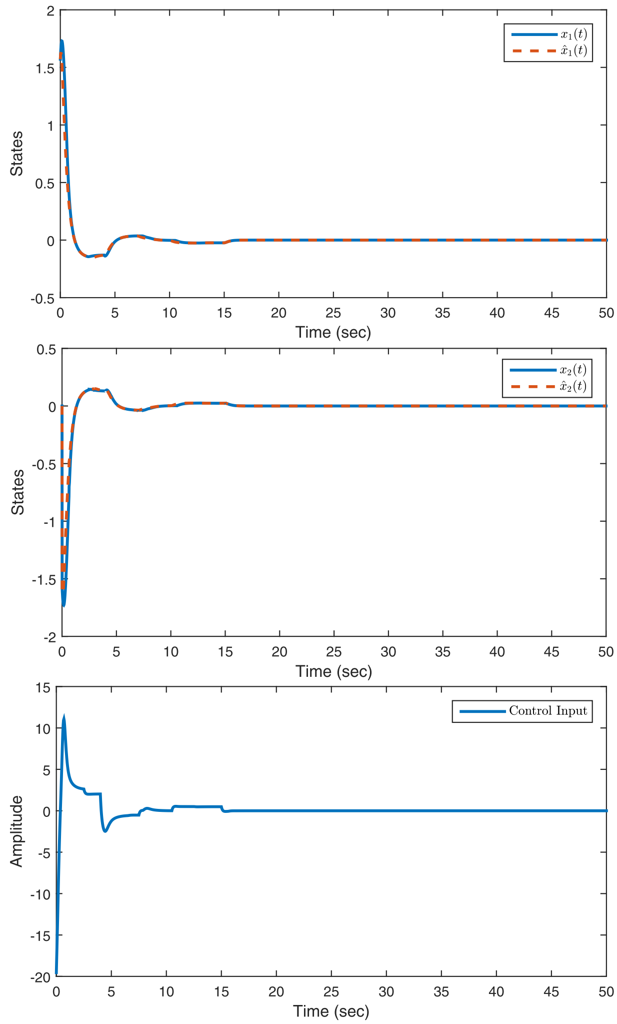

FIGURE 3. **(a) Responses of $x_1(t)$ and its estimation, (b) responses of $x_2(t)$ and its estimation, and (c) control input, for Example 2 in Case-I.**

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}^T = \begin{bmatrix} 0.0507 & 0.4267 \\ 1.0686 & 1.0652 \end{bmatrix}^T$$

The parameters of the controller by considering dissipative analysis are calculated in (37) with the initial condition $\psi = [0.5\pi, 0.75\pi]$. The estimation error and measured output with its estimation is presented in Fig.4 (a-b) respectively and it is clearly observed that as the time tends to infinity the error corresponds to zero. Basically, the dissipative analysis is the relation of applied energy to the system with energy stored in the system, that is why we analyze this issue in our paper.

Case-III: $L_2 - L_\infty$ Controller:

Let $\Lambda_\circ = 1$, $\Lambda_1 = 0$, $\Lambda_2 = 0$ and $\Lambda_3 = \gamma^2$. The value of γ is also chosen to be 2.5. It is observed that the feasible solution of LMI (32)–(36,38) with trigger matrix $\tilde{\Psi} = 24.9764$ results in the controller and observer gain given as below:

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -19.6548 & -12.2567 \\ -6.2257 & -4.1476 \end{bmatrix}$$

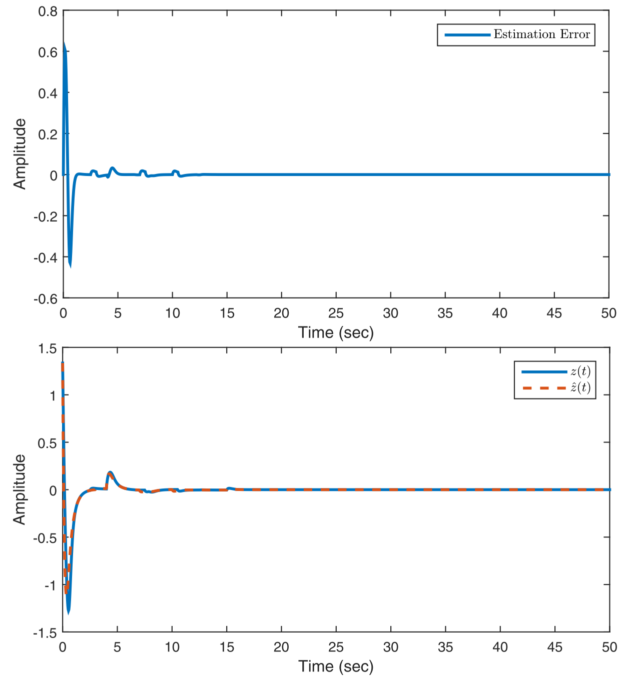

FIGURE 4. **(a) Response of estimation error (b) responses of $z(t)$ and $\hat{z}(t)$ for Example 2 in Case-II.**

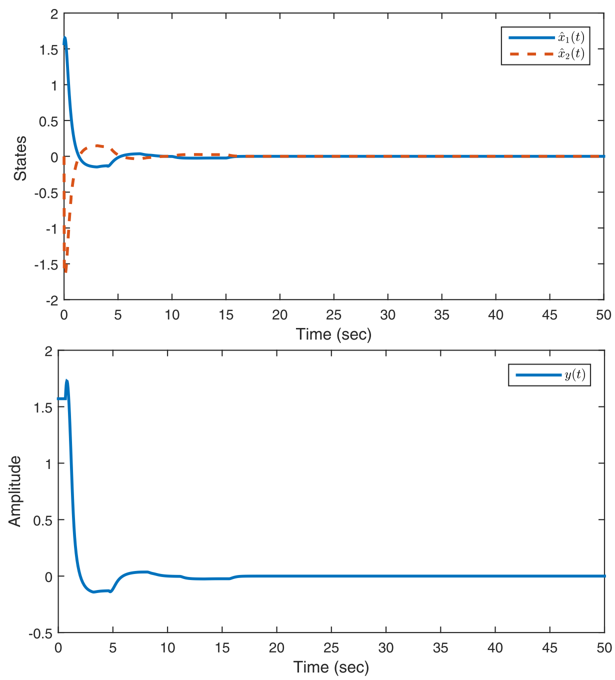

FIGURE 5. **(a) Estimation of system states (b) response of system output $y(t)$ for Example 2 in Case-III.**

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}^T = \begin{bmatrix} 0.0574 & 0.5848 \\ 1.4599 & 1.4767 \end{bmatrix}^T$$

The parameters of the controller by considering $L_2 - L_\infty$ analysis are calculated in (37) with the initial condition $\psi = [0.5\pi, 0.75\pi]$, the estimation of states are presented in

Figure 5. It can be observed from Fig. 5 that the same control objective is also fulfilled.

V. CONCLUSIONS

In this paper, a dissipative observer-based output feedback controller with imprecise premise match is designed between the controlled plant and event-based networked observer for T-S fuzzy singular systems with delay. The event triggered scheme is used for the effective utilization of bandwidth. The unavailable states are estimated through delayed fuzzy observer with imprecise matching. The control objective is achieved by the successful design of T-S fuzzy observer with delay. The criteria for stability and stabilization is based on non-PDC approach for the system under consideration which results in the formation of LMI. The proposed method is validated through a practical simulation example: the mass-spring-damper system. One of the important aspects of network control is packet loss which effects the control performance of the systems. Combining the effect of packet loss and delay is an interesting challenging problem to extend the proposed method to T-S fuzzy multiagent systems.

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