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Optimal Placement Algorithm of Multiple DGs Based on Model-Free Lyapunov Exponent Estimation

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ABSTRACT The implementation of distributed generation (DG) becomes more common while causing a challenge to maintain the voltage stability in a power system. The optimal placement of multiple DGs in a planning stage can effectively solve this problem. In many cases, the best placement has been determined by the small-signal study like the modal analysis. However, it is acceptable only if the linearization of large-scale nonlinear power system is accurate. To overcome this, this paper proposes the new optimal placement algorithm for multiple DGs based on the model-free Lyapunov exponent estimation to maintain the voltage of system stable. In other words, the individual Lyapunov exponents of all buses are firstly estimated to determine the candidates for optimal placement of DGs. Then, the maximum Lyapunov exponent for each candidate is calculated to decide which bus is the best place to improve the voltage stability of system. Several time-domain simulation studies are carried out to verify the effectiveness of proposed algorithm. In particular, its performance is compared with that of the conventional method. The results show that the optimal placement of multiple DGs determined by the proposed algorithm improves the voltage stability of system much more effectively.

INDEX TERMS Distributed generation, model-free Lyapunov exponent estimation, optimal placement, voltage stability.

I. INTRODUCTION

A. MOTIVATION AND INCITEMENT

The penetration of distributed generation (DG) based on renewable energies such as photovoltaic (PV) and wind to power grid continues to increase worldwide. Moreover, several developed countries plan to increase the generation dependency on renewable energies to even 50–70% in next two decades. In other words, the power systems will experience a great transition with the high penetration of renewable energies in the near future. Even though the high penetration of DG makes the operation and planning of power system more complex, its stability must be kept well while maximizing the effective use of DGs.

B. LITERATURE REVIEW

When a DG changes the power flow of power system, it is subject to exceed its operating limit. Several optimal placement algorithms [1]–[8] for multiple DGs in a planning stage were reported in order to minimize the power loss or improve the voltage stability. Optimal placement of DGs is one of the best ways to enhance the overall efficiency of power system. In particular, the single- or multi-objective optimization technique was recently applied [9]. For example, the multi-objective particle swarm optimization (MOPSO) method was used to minimize the power loss and reduce the voltage deviation. Also, the improved Harris hawks optimi-zation (IHHO) was suggested to find the optimal size and location of DG in distribution systems [10]. Moreover, the enhanced genetic algorithm (EGA), which is associated with the genetic algorithm and local search, was applied in [11].

One of the most representative optimal placement algorithms to improve the voltage stability is the modal analysis [12], by which the smallest positive eigenvalue corresponding to the voltage collapse can be obtained. Then, the associated participation factors at all buses can be calculated. Note that they indicate the remedial action

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on the buses in stabilizing an operating mode. Therefore, the bus with the highest participation factor can be selected as the best placement of DG. However, this method is based on the small-signal analysis, which is valid only when the linearization is accurate. To deal with this problem, the other methods [13] by the large-signal stability analysis were reported. In addition, the Lyapunov exponents [14]–[21] have been recently applied for both voltage and rotor angle stability studies. They are used to evaluate the relative stability contributions of all buses in various ways.

C. CONTRIBUTION AND PAPER ORGANIZATION

This paper proposes the new optimal placement algorithm based on the model-free Lyapunov exponent estimation. In particular, when a large fault occurs, the individual Lyapunov exponents for all buses determine the separate contribution of each bus to the voltage stability of entire system depending on the degree of improvement with the voltage measurements from all buses. Then, the maximum Lyapunov exponent of each candidate bus is evaluated to select the optimal placement of DG. In other words, if the maximum Lyapunov exponent is the smallest when a DG is placed at one of candidate buses, this corresponding bus becomes the best location for DG. The main advantage of the proposed algorithm is that it gives the better location for DG placement to improve voltage stability when compared to the conven-tional methods. The major contributions of this paper are summarized as follows:

- By applying the Lyapunov exponents to find the opti-mal location of DGs, the dynamic characteristic of system is considered.
- The Lyapunov exponent indicates the relative stability contribution to the voltage stability of system. Thus, it enables to give a standard to select the candidate buses for DG placement.
- The effectiveness of proposed algorithm is compared with that of conventional method (the modal analysis), on the IEEE 9-bus, 39-bus, and 118-bus test systems.

The paper is organized as follows. Section II describes the conventional modal analysis for selecting the optimal placement of DGs. Thereafter, the proposed algorithm based on the model-free Lyapunov exponent estimation is explained in detail. Then, Section III verifies the effectiveness of proposed algorithm with several case studies on the IEEE 9-bus, 39-bus, and 118-bus test systems. Also, the performances of two methods are compared. Finally, the conclusions are given in Section IV.

II. PROPOSED OPTIMAL DG PLACEMENT ALGORITHM

A. CONVENTIONAL MODAL ANALYSIS

As mentioned previously, several methods to select the optimal placement for DGs have been reported. They are implemented to achieve two different goals. One is to minimize the power loss, and the other is to improve the voltage stability of system. The modal analysis is mainly used to enhance the voltage stability by locating multiple DGs in a power system [22]. It can be firstly implemented with the linearized power flow equations by

$$\begin{bmatrix} \Delta \underline{P} \\ \Delta \underline{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J_{P\theta}} & \mathbf{J_{PV}} \\ \mathbf{J_{Q\theta}} & \mathbf{J_{QV}} \end{bmatrix} \begin{bmatrix} \Delta \underline{\theta} \\ \Delta \underline{V} \end{bmatrix} \tag{1}$$

where $\Delta \underline{P}$ and $\Delta \underline{Q}$ are the incremental changes of real and reactive powers of buses, receptively. $\Delta\theta$ and $\Delta \underline{V}$ are the incremental changes of voltage angle and magnitude of buses, respectively. Also, $\mathbf{J_{P\theta}}$, $\mathbf{J_{PV}}$, $\mathbf{J_{Q\theta}}$, and $\mathbf{J_{QV}}$ are the sub-matrices of Jacobian matrix, $\mathbf{J}$, which are the partial derivatives of real and reactive powers with respect to the voltage angle and magnitude of buses, respectively.

Even though the voltage stability in a power system is affected by changes in both the real and reactive powers, it is reasonable to assume that the real power can be kept constant while focusing on the relationship between the reactive power and voltage magnitude. Therefore, by setting $\Delta \underline{P}$ to zero, the equation (1) can be reduced as

$$\Delta \underline{Q} = (\mathbf{J}_{\mathrm{QV}} - \mathbf{J}_{\mathrm{Q}\theta} \cdot \mathbf{J}_{\mathrm{P}\theta}) \cdot \Delta \underline{V} \tag{2}$$

From (2), the reduced Jacobian matrix, $\mathbf{J_{reduced}}$ is defined as

$$\mathbf{J_{reduced}} = \mathbf{J_{QV}} - \mathbf{J_{Q\theta}} \cdot \mathbf{J_{P\theta}^{-1}} \cdot \mathbf{J_{PV}} \tag{3}$$

By using $\mathbf{J_{reduced}}$ in (2), the following linearized power flow equation can be derived.

$$\Delta \underline{Q} = \mathbf{J_{reduced}} \cdot \Delta \underline{V} \tag{4}$$

$$\Delta \underline{V} = \mathbf{J_{reduced}^{-1}} \cdot \Delta \underline{Q} \tag{5}$$

In particular, $\mathbf{J_{reduced}}$ can be formulated by the eigenvalue analysis as

$$\mathbf{J_{reduced}} = \mathbf{Z} \cdot \Lambda \cdot \mathbf{H} \tag{6}$$

where $\mathbf{Z}$ is the right eigenvector matrix of $\mathbf{J_{reduced}}$, $\mathbf{H}$ is the left eigenvector matrix of $\mathbf{J_{reduced}}$, and Λ is its diagonalized matrix. By taking the inverse of $\mathbf{J_{reduced}}$ as (7) and inserting it to (5), the equation (8) is obtained.

$$\mathbf{J_{reduced}^{-1}} = \mathbf{Z} \cdot \Lambda^{-1} \cdot \mathbf{H} \tag{7}$$

$$\Delta \underline{V} = \mathbf{Z} \cdot \Lambda^{-1} \cdot \mathbf{H} \cdot \Delta \underline{Q} \tag{8}$$

Then, it is expressed as

$$\Delta V = \sum_i \frac{z_i \cdot h_i}{\lambda_i} \cdot \Delta \underline{Q} \tag{9}$$

where $\underline{z_i}$ is the i-th column vector of $\mathbf{Z}$, $\underline{h_i}$ is the i-th row vector of $\mathbf{H}$, and λ_i is the i-th eigenvalue of $\mathbf{J_{reduced}}$. Thereafter, k_{im}, which is the participation factor of bus m in mode i, is calculated as

$$k_{im} = z_{im} \cdot h_{im} \tag{10}$$

where z_{im} and h_{im} are the m-th elements of $\underline{z_i}$ and $\underline{h_i}$, respectively. Participation factor determines the most crit-ical area which leads the power system to instability.

Therefore, this participation factor is useful to evaluate the relative contribution of each bus to the voltage stability of system. A large value of participation factor means that the corresponding bus has better restorative action in securing the mode.

Also, among several indices to evaluate the performance of optimal placement algorithm of a DG, the voltage stability margin (VSM) [23] has been mostly used. It is defined with the difference between the amount of load consumption, P_0 and its loadability limit, $P_{\max}$ in MW as

$$M = P_{\max} - P_0 \tag{11}$$

where M is the VSM. The higher VSM means the more stable system. Then, the P–V curve according to the VSM is shown in Fig. 1. Note that the continuous power flow calculation is required to obtain the VSM while finding successive solutions from an initial load condition to the point where the voltage collapses.

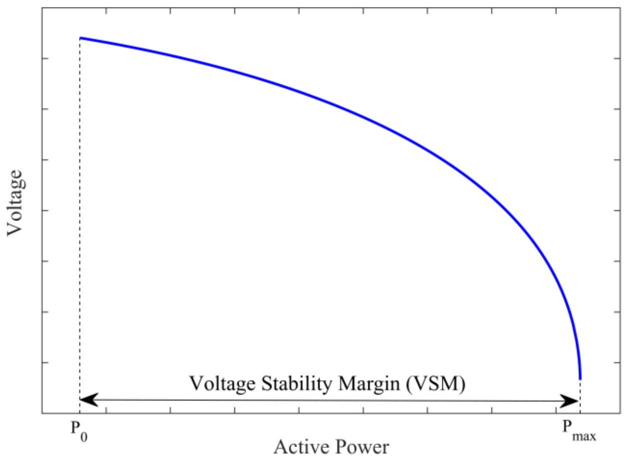

FIGURE 1. ***P*– *V* curve according to the VSM.**

In summary, the continuous power flow is firstly carried out. Then, the bus, which is the most sensitive to the voltage collapse, is selected. This means that the voltage drops most seriously. Therefore, this bus becomes one of the candidates for the optimal placement of DG. Next, the modal analysis is made to find the weak modes, which have small positive eigenvalues. In other words, small positive eigenvalues are close to the unstable modes because the small changes of reactive power cause the large variations in the voltage magnitude by (9). In particular, the participation factors of all buses in the weakest mode are calculated to select the optimal placement of DGs. Finally, the bus with the highest participation factor becomes one of the candidates for DG placement. During this process, the VSM can be computed to evaluate the degree of improvement. If its value is increased, this bus is selected as the optimal placement of DG. Then, the modal analysis is newly made again from the beginning whenever the new DGs are connected to the grid.

B. PROPOSED OPTIMAL PLACEMENT ALGORITHM BASED ON MODEL-FREE LYAPUNOV EXPONENT ESTIMATION

In general, the Lyapunov exponent provides the quantifying information if a dynamic system is in a chaotic condition. In particular, the given system is considered chaotic if it is difficult to predict the future behavior of system even when there is a small perturbation in an initial condition. In quantifying this dynamic behavior, the Lyapunov exponent represents the exponential sensitivity to the initial condition. By its definition, the Lyapunov exponent can be calculated by

$$\|\Delta v(t)\| = e^{xt} \cdot \|\Delta v_0\| \tag{12}$$

where Δv_0 is the voltage deviation at an initial condition, $\Delta v(t)$ is the voltage deviation at time t, and x is the Lyapunov exponent. By taking the limit when t goes to infinity, the equation (12) is expressed as

$$x = \lim_{t\to\infty} \frac{1}{t} \cdot \log \frac{\|\Delta v(t)\|}{\|\Delta v_0\|} \tag{13}$$

Note that the n-dimensional dynamic system has n number of Lyapunov exponents, and the largest Lyapunov exponent gives the important information about system stability. In (13), the sign of Lyapunov exponent is positive when $\|\Delta v(t)\|$ is larger than $\|\Delta v_0\|$. This means that the voltage deviation diverges. In contrast, the sign of Lyapunov exponent is negative when $\|\Delta v(t)\|$ is smaller than $\|\Delta v_0\|$ such that the voltage deviation converges. In other words, the positive maximum Lyapunov exponent implies the system is unstable. On the contrary, the negative maximum Lyapunov exponent indicates that the system is stable because the signs of all Lyapunov exponents are negative. In this respect, the Lyapunov exponent in nonlinear system can be considered as the eigenvalue in linear system.

The Lyapunov exponent can be estimated by using the Jacobian approach [24] or model-free method [16] in a power system. In this study, the model-free direct method is used to estimate the Lyapunov exponent with the time-series data measurements when a fault occurs. Then, the maximum Lyapunov exponent, X can be estimated as

$$X(k \cdot \Delta t) = \frac{1}{N.k.\Delta t} \sum_{m=1}^{N} \log \frac{\|\underline{V}_{k+m\cdot\Delta t} - \underline{V}_{(k+m-1)\cdot\Delta t}\|}{\|\underline{V}_{m\cdot\Delta t} - \underline{V}_{(m-1)\cdot\Delta t}\|}, \; k > N \tag{14}$$

where $\underline{V}$ is the voltage magnitudes of all buses, Δt is the sampling period of time-series data, and N is the number of initial conditions. Similarly to (13), the $\|V_{m\Delta t}\|$ $\|V_{(m-1)\Delta t}\|$ and $\|V_{(k+m)\Delta t}\|$ $\|V_{(k+m-1)\Delta t}\|$ in (14) correspond to the voltage deviations at an initial condition and at time t, respectively. Therefore, the voltage stability of power system can be determined by observing the sign of X based on the time-series data measurements.

The value of Lyapunov exponent depends on N. For example, if the small N is chosen, the fast estimation of stability is possible. In contrast, if the large N is used, it requires the long estimation time even though the accuracy of prediction increases significantly. In summary, the proper N must be carefully selected to obtain the desired result by considering the trade-off between the accuracy and speed of estimation. The individual Lyapunov exponent, x_i of i-th bus in a power system can also be estimated as

$$x_i(k \cdot \Delta t) = \frac{1}{N.k.\Delta t} \sum_{m=1}^{N} \log \frac{|V^i_{k+m\cdot\Delta t} - V^i_{(k+m-1)\cdot\Delta t}|}{|V^i_{m\cdot\Delta t} - V^i_{(m-1)\cdot\Delta t}|}, \ k > N \tag{15}$$

where x_i is the individual Lyapunov exponent of i-th bus, and v_i is the voltage measurement for i-th bus. For the arbitrary small numbers, ε_1 and ε_2 $(0 < \varepsilon_1 < \varepsilon_2)$, the integer N can be chosen such that $\varepsilon_1 < |v_{m\Delta t} - v_{(m-1)\Delta t}| < \varepsilon_2$ for $m = 1, 2, \ldots, N$. Then, x in (15) estimates the stability contribution of each bus to the entire system. It gives the chance to determine the rank with the degree of stability contribution. Then, the coherent buses, which have the similar Lyapunov exponents can also be identified. It is important to note that the fault locations must be far from the candidate buses for DG placement for the proper Lyapunov exponent estimation. Consequently, the buses having the small exponents contribute to the voltage stability of system more than those with the large exponents. Therefore, it can be maximized when the DG is connected to one of these candidate buses, which can be determined by

$$S_i = x_{i+1} - x_i, \quad 1 \le i \le \left[\frac{n}{2}\right] + 1 \tag{16}$$

where n is the number of buses in a system, and $[\cdot]$ denotes the greatest integer function. The number of candidate buses for DG placement must be small enough to reduce the computation efforts. Thus, it is divided in half. Also, x_i satisfies the following condition for all i.

$$x_{i+1} \ge x_i \tag{17}$$

From (17), it is obvious that S_i in (16) is always positive. Then, the bus, m, which is the smallest i satisfying (18) can be found to reduce the number of candidate buses. Finally, the buses from $i = 1$ to $i = m+1$ become the candidate buses for DG placement.

$$S_{i+1} > S_i \tag{18}$$

For example, in a power system with four buses, how to determine the candidate buses by the proposed algorithm is illustrated in Fig. 2. Firstly, when a fault occurs, the dynamic voltage responses at all buses are measured, as shown in Fig. 2(a). Then, the individual Lyapunov exponents of four buses are estimated by (15), and all buses are ranked by (17) from the smallest to the largest exponent, as shown in Fig. 2(b). From the result, bus 2 has the smallest exponent of 0.8324 and bus 3 has the largest exponent of 1.0871. Finally, the candidate buses for DG placement are determined by S_i in (16), as shown in Fig. 2(c). As S_2 is bigger than S_1,

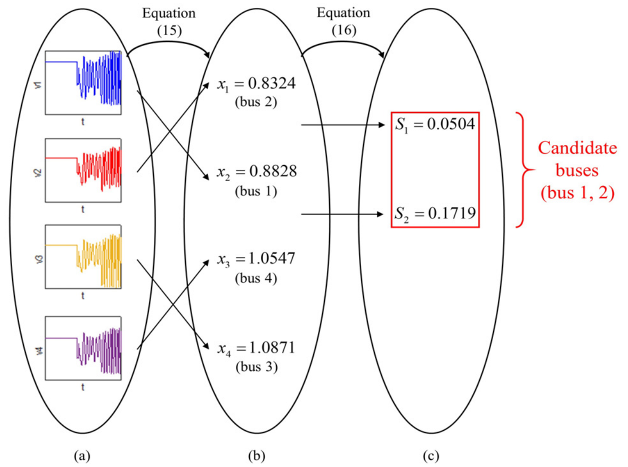

FIGURE 2. **An example to determine the candidate buses. (a) Dynamic voltage measurements of buses after a fault occurs. (b) Individual Lyapunov exponent of buses. (c) Determination of the candidate buses by calculating S_i.**

it satisfies (18). Thus, the buses, 1 and 2 corresponding to $i = 2$ and $i = 1$, respectively, become the candidate buses.

The next step is to find the most optimal placement for DG among the candidate buses. To do so, the maximum Lyapunov exponent is calculated with the DG located at each candidate bus. Then, the bus with the minimum Lyapunov exponent becomes the best location for DG to improve the voltage stability. This is illustrated in Fig. 3. With the results from Fig. 2, the candidate buses, 1 and 2 are shown in Fig. 3(a). Firstly, a DG is connected to bus 1. Then, the maximum Lyapunov exponent of system is calculated by (14). Its value is −0.3122, as shown in Fig. 3(b).

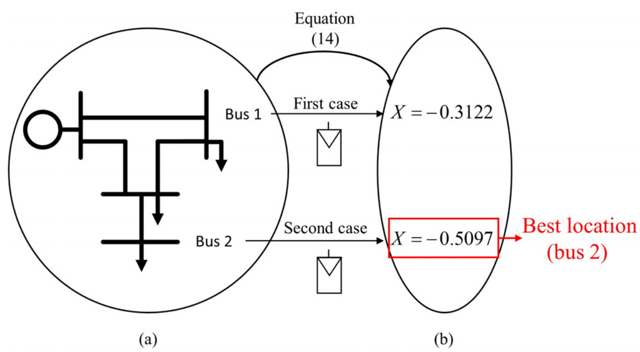

FIGURE 3. **An example to find the most optimal placement for DGs. (a) The candidate buses for DG placement. (b) The maximum Lyapunov exponent in each case.**

Next, when the same DG is now placed at bus 2, the value of maximum Lyapunov exponent by (14) is −0.5097, which is lower than −0.3122. Therefore, bus 2 is finally selected as the most optimal placement for DG.

For the multiple DGs, the proposed algorithm can be repeatedly applied whenever the additional DG is newly connected to the system. The overall procedure to implement

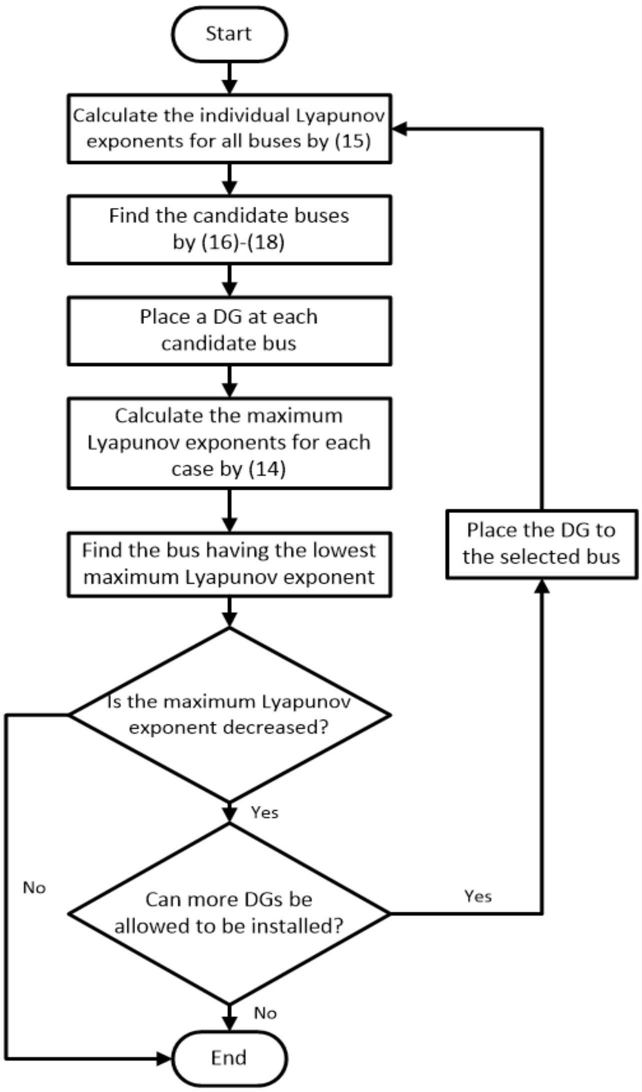

FIGURE 4. The overall procedure to implement the proposed optimal placement algorithm for multiple DGs.

the proposed optimal placement algorithm for multiple DGs is summarized in Fig. 4.

III. CASE STUDIES

A. TEST ON IEEE 9-BUS SYSTEM

The simulation test is firstly carried out on the IEEE 9-bus system by using the DIgSILENT Power Factory® software. The single line diagram of system is shown in Fig. 5. It consists of 3 generators and 9 buses. The total load is 315 $+j115$ MVA. The DG connected to the system is assumed to be 30 MW.

To find the candidate buses to improve the voltage stability of system, the individual Lyapunov exponents for all buses are estimated. To do so, a general fault case must be selected. Even though the IEEE 9-bus system has a symmetric structure with 3 generators, they generate different output powers. In other words, the output powers from G2 and G1 are maximum and minimum, respectively. Therefore, a three-phase short circuit of 250 ms is applied to the center of line between buses, 7 and 8. Note that this fault location is close to G2. Then, the average values of individual Lyapunov exponents for 5 s at load buses are shown in Fig. 6. Also, they are listed in Table 1 by the ascending order of their individual Lyapunov exponents.

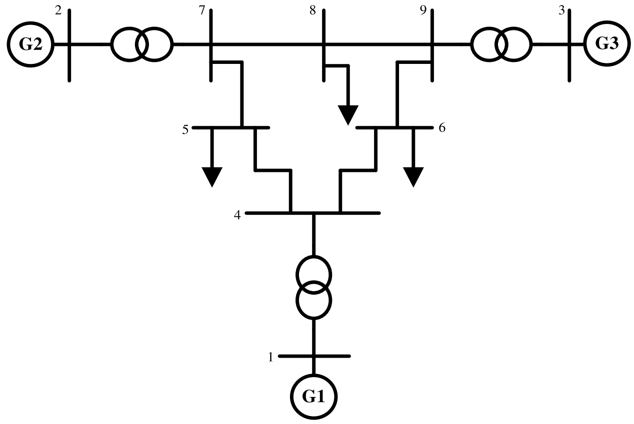

FIGURE 5. IEEE 9-bus test system.

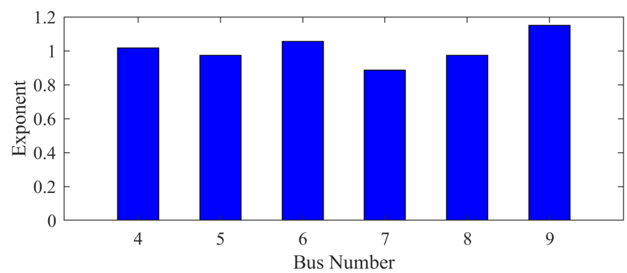

FIGURE 6. Average values of individual Lyapunov exponents at load buses for 5 s after a fault occurs in the IEEE 9-bus test system.

TABLE 1. Lyapunov exponents of candidate buses.

Location	i	x_i	S_i	X
Bus 7	1	0.8848	**0.0876**	**0.9677**
Bus 5	2	0.9724	**0.0006**	1.0587
Bus 8	3	0.9730	**0.0434**	1.0114
Bus 4	4	1.0164	-	-

As shown in Table 1, S_3 is larger than S_2. Therefore, the buses, 5, 7, and 8 (for $i = 3$) become the candidate buses with the lower individual Lyapunov exponents than the other buses. Again, this means that they contribute most significantly to the voltage stability of system. Among three buses, 5, 7, and 8, one bus must be selected for the best location of DG. Then, the maximum Lyapunov exponent is estimated for each candidate. The results are also shown in Table 1. The same fault is applied, and the average values of maximum Lyapunov exponents for 5 s is calculated.

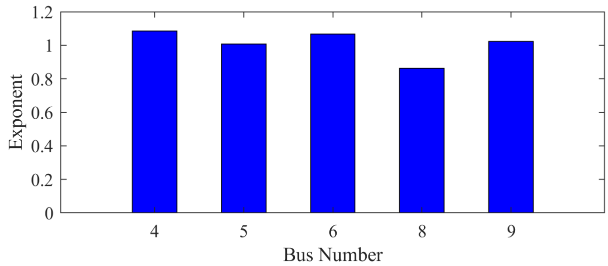

FIGURE 7. Average values of individual Lyapunov exponents at load buses for 5 s to determine the next optimal location of second DG.

Finally, bus 7 with the lowest maximum Lyapunov exponent is selected for DG placement.

After selecting bus 7 as the best location of DG, the same optimization process is repeated for the next optimal placement of the second DG. As the results in Fig. 6, the average values of individual Lyapunov exponents for 5 s at load buses except for bus 7 are shown in Fig. 7. Likewise, they are listed in Table 2 by the ascending order of their individual Lyapunov exponents. As shown in Table 2, S_3 is still larger than S_2. Therefore, the buses, 5, 8, and 9 (for $i = 3$) become the candidate buses with the lower individual Lyapunov exponents than the other buses.

TABLE 2. Lyapunov exponents of candidate buses for additional DG.

Location	i	x_i	S_i	X
Bus 8	1	0.8633	**0.1447**	1.0010
Bus 5	2	1.0080	**0.0159**	**0.9967**
Bus 9	3	1.0239	**0.0418**	1.0012
Bus 6	4	1.0657	-	-

Then, after calculating the maximum Lyapunov exponents for the candidate buses, 5, 8, and 9, bus 5 is finally selected as the optimal location of second DG. Note that the value of maximum Lyapunov exponent for two DGs of 30 MW at buses, 5 and 7 is larger than that for one DG of 30 MW at bus 7. Therefore, it can stop applying the proposed algorithm at this point.

B. TEST ON IEEE 39-BUS TEST SYSTEM

The performance of proposed algorithm is evaluated by the simulation test on the IEEE 39-bus system with 10 generators and 39 buses in Fig. 8, and it is compared with that of conventional modal analysis. The total load is 6097 $+j1409$ MVA. The DG connected to the system is assumed to be 80 MW.

As mentioned before, the continuous power flow is firstly carried out. Bus 7, which is the most sensitive to the voltage collapse, is selected, and it becomes one of the candidate buses. Then, the conventional modal analysis is made to find the weakest mode of 0.0010887 pu/MVAr.

In this weakest mode, the participation factors for all load buses are calculated, and the result is shown in Fig. 9.

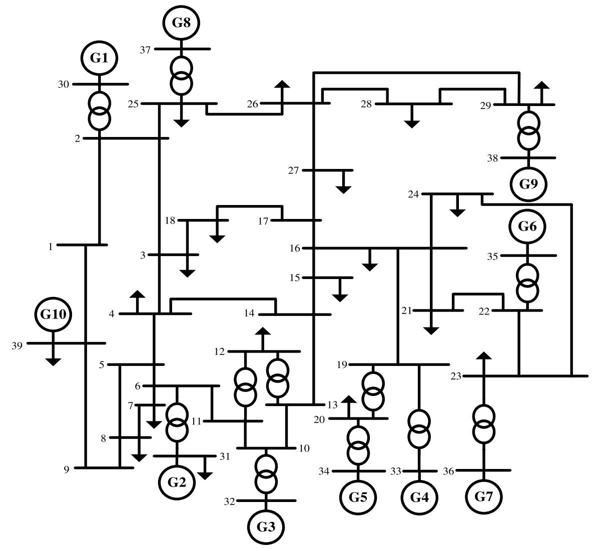

FIGURE 8. IEEE 39-bus test system.

Bus 12 has the highest participation factor, and it becomes the candidate bus for DG placement. Finally, for these candidate buses, 7 and 12, the values of VSMs in (11) are calculated. When the DG of 80 MW is placed at bus 7, the value of VSM is higher than that at bus 12. Therefore, it is concluded that the best location for DG placement by the conventional modal analysis is bus 7. On the other hand, when the proposed optimal placement algorithm is used, the individual Lyapunov exponent is firstly estimated. Their average values are shown in Fig. 10. After a 230 ms three-phase short circuit is applied to the middle of line between buses, 16 and 24, they are ranked in the ascending order, and S_i is calculated. After that, two buses, 19 and 21 become the candidate buses. In other words, the values of S_1 and S_2 are 0.0101 and 0.0408 for buses, 19 and 21, respectively. For these candidate buses, when the DG of 80 MW is connected to bus 21, its maximum Lyapunov exponent is lower than that at bus 19. As the result, the best location for DG placement by the proposed optimal placement algorithm becomes bus 21.

For two buses, 7 and 21, which are selected as the best locations by the conventional modal analysis and proposed algorithm, respectively, the voltage stability depending on different locations is evaluated when a three-phase short circuit of 300 ms is applied to the middle of line between buses, 4 and 14 at 0.5 s. The dynamic responses of voltage at bus 6 are shown in Fig. 11. It is clearly observed that the system becomes unstable after the fault occurs if the DG of 80 MW is connected to bus 7 selected by the conventional modal analysis. In contrast, when the same DG is installed at bus 21 determined by the proposed algorithm, the operation of system is still kept to be stable.

Also, when the DG is connected to two candidate buses (buses, 7 and 12 by the conventional modal analysis and buses, 19 and 21 by the proposed optimal placement algorithm) obtained by two methods, the values of VSMs are

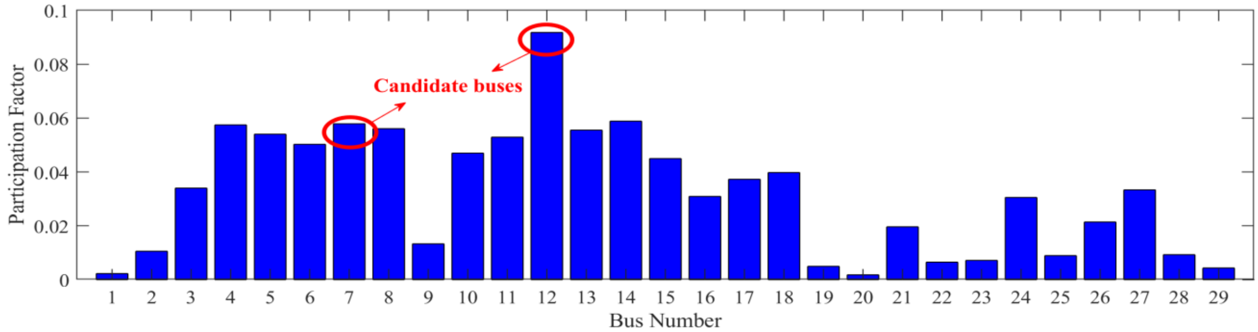

FIGURE 9. Participation factors of load buses obtained by the conventional modal analysis in the IEEE 39-bus test system.

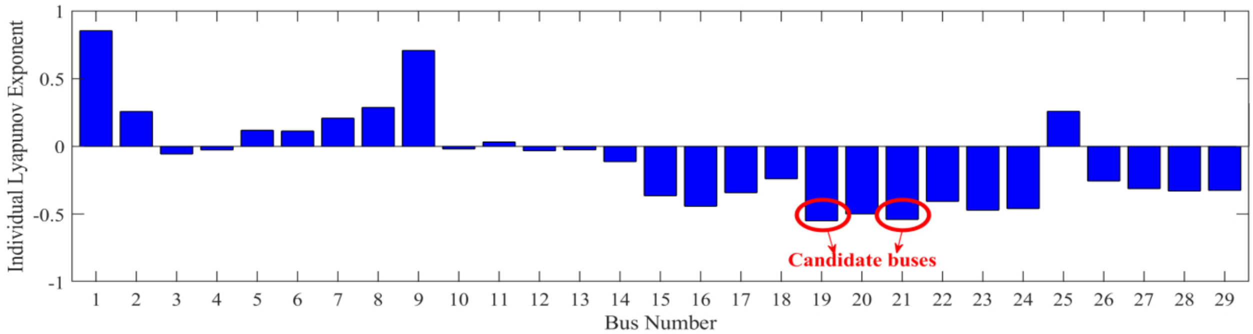

FIGURE 10. Individual Lyapunov exponents of load buses obtained by the proposed optimal placement algorithm in the IEEE 39-bus test system.

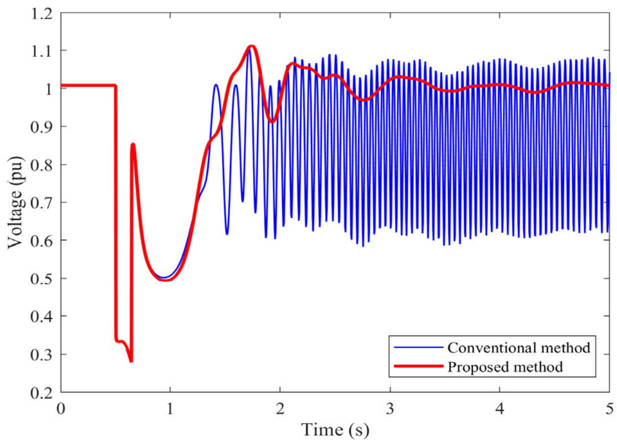

FIGURE 11. The dynamic responses of voltage at bus 6 when a three-phase short circuit of 300 ms is applied to the middle of line between buses, 4 and 14 in the IEEE 39-bus test system.

calculated. Then, their corresponding *P–V* curves at bus 6 are shown in Fig. 12. When no DG is connected to the system (this is the base case), the corresponding value of VSM is 1772 MW. When the DG of 80 MW is connected to buses, 7 and 12, they are 1840 MW and 1832 MW, respectively. When the DG of 80 MW is connected to buses, 19 and 21, they are 1843 MW and 1850 MW, respectively. This clearly verifies that the voltage stability of system is significantly improved when the DG is connected to the location selected optimally by the proposed method.

Finally, when the conventional modal analysis is applied, the relationship between the participation factor and VSMs in the IEEE 39-bus system is shown in Fig. 13. It is observed that they have the linear relationship when the value of VSM is smaller than 1835 MW. However, the participation factor is inversely proportional to the VSM after its value becomes bigger than 1835 MW. This means that the conventional modal analysis is not effective any more. Generally speaking, it is suggested that a DG must be connected to the bus having relatively higher participation factor by the conventional modal analysis.

However, the analysis shown in Fig. 13 indicates that the bus with higher participation factor does not always have higher value of VSM. On the contrary, the relationship between the individual Lyapunov exponent and the VSMs is shown in Fig. 14. It is observed that they have the clear tendency. That is, the individual Lyapunov exponents obtained by the proposed algorithm are inversely proportional to the VSMs in overall ranges.

In other words, it is expected that the value of VSM is increased if a DG is connected to the bus having relatively lower value of individual Lyapunov exponent, and therefore the voltage stability of system can be effectively improved. In summary, the proposed optimal placement algorithm for DGs can be more preferably used for improving the voltage stability when compared to the conventional modal analysis.

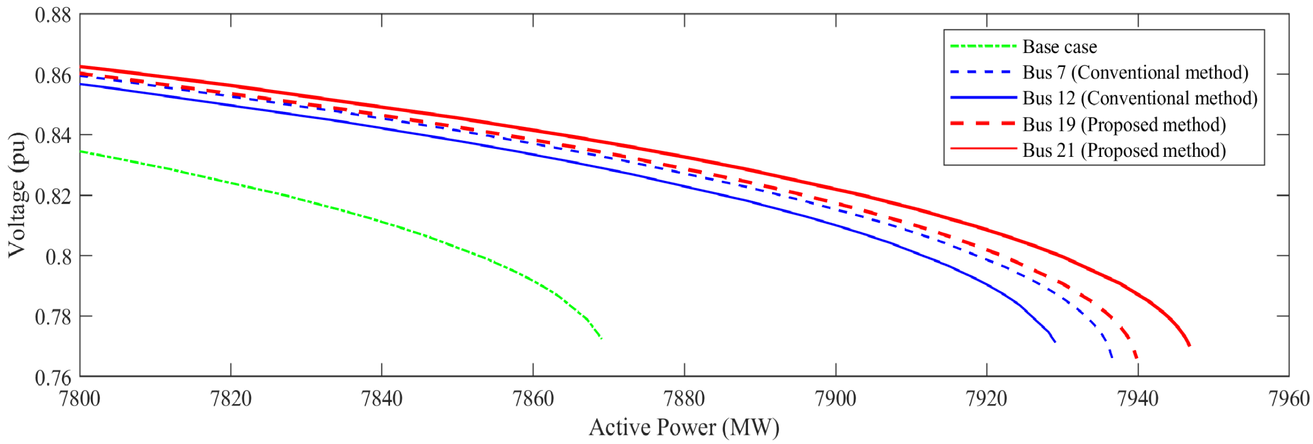

FIGURE 12. ***P– V*** **curves at bus 6 when the DG is connected to the candidate buses obtained by two methods in the IEEE 39-bus test system.**

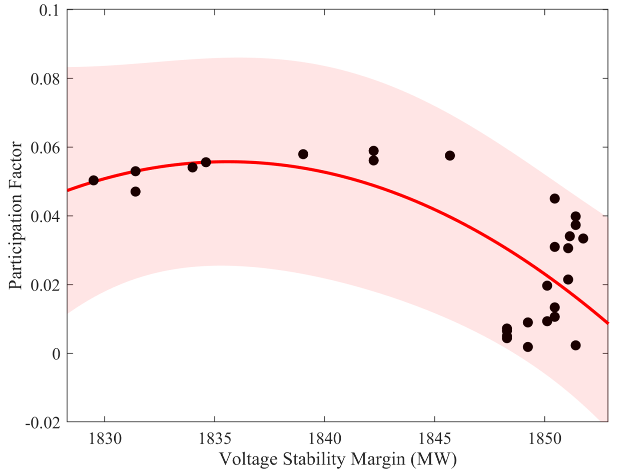

FIGURE 13. The relationship between the participation factors and VSMs in the IEEE 39-bus system.

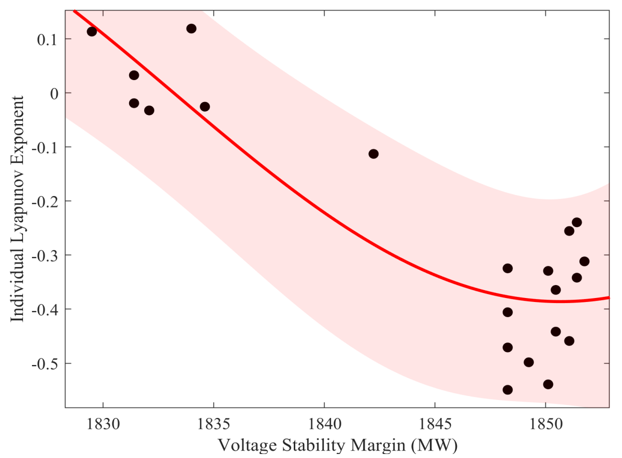

FIGURE 14. The relationship between individual Lyapunov exponents and the VSMs in the IEEE 39-bus system.

C. TEST ON IEEE 118-BUS TEST SYSTEM

The performance of proposed algorithm is evaluated by the case study on the IEEE 118-bus system [25] with 54 generators and 118 buses, and it is compared with that by the conventional modal analysis. The total load is 3668 $+j1438$ MVA. The power capacity of DG connected to this system is assumed to 50 MW.

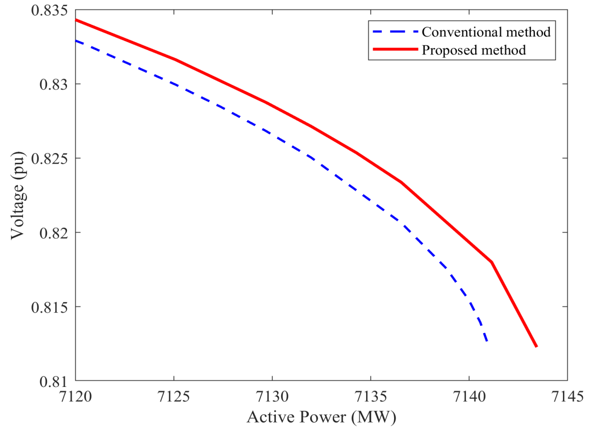

FIGURE 15. ***P – V*** **curves at bus 47 when the DG is connected to the candidate buses selected by two methods in the IEEE 118-bus test system.**

When the conventional method is applied, bus 21 is chosen as the best location of DG placement with the largest participation factor of 0.2165. In contrast, bus 17 is selected as the best candidate bus for DG placement by the proposed method. Note that a three-phase short circuit is applied to the middle of line between buses 26 and 30 for calculating the Lyapunov exponents. Thereafter, when the DG of 50 MW is placed at each candidate bus, the VSM is calculated to compare the performances of two methods. Their corres-ponding P–V curves at bus 47, which is the critical bus, are shown in Fig. 15. It is clearly observed that the VSM becomes larger when the best location is selected by the proposed method. This verifies that the voltage stability of system can be still improved effectively even in large scale power system when the proposed method is used.

IV. CONCLUSION

This study proposed the new optimal placement algorithm for distributed generations (DGs) based on the model-free Lyapunov exponent estimation. After a severe fault was applied, the individual Lyapunov exponents of all buses were firstly estimated with the voltage measurements to evaluate the separate contribution of each bus to the voltage stability of entire system depending on the degree of improvement. Then, the effective utilization of maximum Lyapunov exponent index was developed to select the optimal placement of DG. That is, if the maximum Lyapunov exponent is the smallest when a DG is connected to one of the candidate buses, then the corresponding bus becomes the best location for DG placement.

To verify the effectiveness of proposed algorithm, the several case studies were carried out on the IEEE 9-bus, 39-bus, and 118-bus test systems. The results showed that voltage stability of system is more significantly improved when a DG is connected to the candidate buses selected by the proposed algorithm than those determined by the conventional modal analysis. In particular, the proposed algorithm has the clear tendency for the relationship between the individual Lyapunov exponent and the voltage stability margin (VSM) without regard to the amount of DG, which is connected to the power system.

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