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# Performance Evaluation of Relay-Aided CR-NOMA for Beyond 5G Communications

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**ABSTRACT** Non-orthogonal multiple access (NOMA) is considered as one of the most promising technologies to handle the issue of spectrum scarcity in beyond fifth-generation (5G) networks. Integrating NOMA in cognitive radio (CR) network is expected to usher a new era of reliable, seamless, and massive connectivity. In this regard, this work explores the relay selection problem in CR networks when operating in the spectrum sharing model. Specifically, the secondary users (SUs) in the CR-NOMA network opportunistically access the licensed spectrum resources to boost the number of accessible SUs sharing the limited and dynamic spectrum resources. Moreover, to improve the performance of far users, partial relay selection architecture is exploited at full-duplex (FD) and half duplex (HD) relays for both uplink and downlink communications. For in-depth performance evaluation, we provide closed-form expressions of the outage probabilities of the users in FD and HD relay-aided CR-NOMA networks. In addition to this, the analytical expressions of asymptotic outage probabilities and ergodic capacity are also provided which unveils the critical factors affecting the performance of CR-NOMA networks. To validate the derived expressions, extensive simulations are performed that demonstrate the accuracy of analytical expressions for FD and HD relay-aided CR-NOMA networks.

**INDEX TERMS** Cognitive radio network, full-duplex, relay selection, Non-orthogonal multiple access (NOMA).

## I. INTRODUCTION

Non-orthogonal multiple access (NOMA) has attracted increasing interest to make next-generation wireless communication systems come true. Two advantages of NOMA are the massive connectivity and capability of supporting higher spectral efficiency (SE) [1]. Many directions are introduced related to NOMA such as it has drawn significant attention in multiple-antenna systems due to the philosophy of diverse transceivers [2]–[4], in relaying networks [5], [6] in device-to-device (D2D) networks [7], and in downlink and uplink multi-cell networks as well [8]. In principle, during the same time and on the same frequency band simultaneous access is provided to multiple users in NOMA which is different from the traditional orthogonal multiple access (OMA).

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For example, by using power-domain multiplexing, NOMA exhibits an effective mechanism to serve massive connections. In the context of power-domain NOMA, the superimposed signal is processed at transmitter while successive interference cancellation (SIC) may be employed at the receivers to eliminate the mutual interference among different users [1]. A higher SE and energy efficiency (EE) are basic advantages of such a NOMA network and these characterizations are prominent relative to conventional OMA [8], [9].

## A. RELATED WORK

In recent years, a plethora of studies have emerged that show the importance of employing NOMA in wireless networks. In this regard, a NOMA network for cooperative communications is investigated in [10]. In such NOMA networks, amplify and forward (AF) and decode and forward (DF) modes are implemented to provide signal processing at

the relay. This helped in serving the far NOMA users with poor channel quality. In parallel, cognitive radio (CR) has emerged as a spectrum sharing architecture that concerns with enhancing the spectrum utilization by permitting a secondary user (SU) to occupy idle primary user (PU)'s spectrum resource. However, the normal communications of the PU must be guaranteed to isolate disturbance from the SU. The binary detection problem is employed in a situation that performs spectrum sensing from the SU who can sense idle spectrum resources [11], [12]. It is worth noting that the SU can only access the idle channel in the traditional schemes when the absence of the PU is awarded and has to deny signal from the SU if the presence of the PU is confirmed. In many scenarios of CR networks, not only the SU may access the spectrum bands of the PU, but also the interference caused by SUs is tolerable [13]. According to [14], they introduced three techniques including spectrum sharing, opportunistic spectrum access, and sensing-based enhanced spectrum sharing to employ CR in practice.

The combination of NOMA and CR can help in spectrum reuse and open new opportunities to cater to the growing demands of cellular users. The spectrum sharing method has been widely implemented and such technique benefits from its low implementation complexity. In [15]–[17], CR-NOMA is well analyzed to highlight system performance related to spectrum sharing. There is a significant increase in CR-NOMA being implemented in comparison to CR-OMA. Furthermore, a massive number of users can be served as integrating NOMA with CR network will raise its potential in further improving the spectrum efficiency [15], [18], [19]. The authors in [18] considered the impact of users pairing in the proposed CR-NOMA. They indicated that a SU is a user possessing strong channel condition and such SU squeezed into the spectrum owned by the PU user who met the situation of poor channel condition [18]. a special case of the CR system, in this case, is a model of NOMA. The stochastic geometry model is performed in underlay CR in which NOMA users are evaluated in terms of the outage probability as in [15]. The multiple-antenna CR-NOMA network is further examined together with a joint antenna selection problem as in [19].

Some relatively recent studies also investigate the inventive mechanism in cooperative NOMA and CR networks [20], [21]. Others use intermediate relays to improve the performance, e.g., the authors of [22] studied a full-duplex cooperative NOMA system. More specifically, they derived the closed-form expressions for the outage probabilities, ergodic rates, diversity orders, and system throughput in two transmission modes including delay-limited and delay-tolerant transmissions. The authors in [23] first introduced the advantages and disadvantages of uplink and downlink NOMA transmissions in a cellular system and they indicated key distinctions such as detection and decoding processing, implementation complexity, the intra-cell interference, and inter-cell interference.

In another trend of research, multi-relay NOMA networks have been studied to further improve the performance of cooperative NOMA networks. Relay selection (RS) is proposed and the full diversity gain for multi-relay networks remains intact as an effective method to reduce system complexity [24]. More system models regarding several RS schemes were considered in [25]–[28]. The two-stage max-min RS scheme is proposed in [26] and fixed power allocation (PA) is applied in the proposed relay-aided NOMA network. In such relay-aided NOMA network, a subset of relays is selected to satisfy the quality of service (QoS) at the low-rate user in the first stage, and then in the second stage max-min scheme is selected with a suitable relay from this subset to serve user possessing at the high data rate. Different from the traditional relay-aided OMA scheme, the two-stage max-min scheme was indicated with improved outage performance. The adaptive PA at the relays and QoS requirements at the users are studied in [27] and they showed that the proposed two-stage DF and AF schemes in such NOMA are better than that of the relay-aided NOMA scheme in [26]. Particularly, the price of higher overhead occurs if channel state varies in case of the PA coefficients known at each receiver. The optimal outage performance can not be achieved in the relay-aided NOMA schemes in [26], [27] as the users were sorted by their QoS requirements, regardless of their channel conditions. The partial RS and joint user and RS schemes are considered in [28] and [29].

## B. MOTIVATION AND CONTRIBUTION

The review of related studies highlights the issue that there are several benefits of employing CR-NOMA in the next-generation wireless networks. However, the performance of such networks is not well-understood, especially from the point of view of secondary source-destination pairing. To the best of the authors' knowledge, the existing literature lacks a relay aided CR-NOMA paradigm that provides the benefit of both spectrum efficiency and large-scale connectivity. Since both FD and half-duplex (HD) relays have their different applications, it is desirable to evaluate the performance of FD and HD relay-aided CR-NOMA networks. To further improve the performance, a partial relay selection (PRS) technique has been adopted to improve the diversity gains and to reduce the overhead of CR-NOMA networks. The major contribution of this work is summarized as follows:

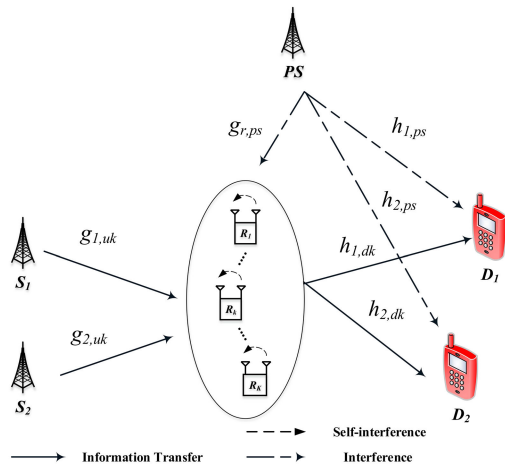
- 1) A novel uplink/downlink relay-aided communications architecture for CR-NOMA is considered. To make the model more practical, the limiting impact of interference from the primary source to the relays is considered during the relay selection process.
- 2) The PRS technique is employed to select the relay for forwarding the message to the destination. The PRS technique has shown to improve the performance of distant users. The degraded performance due to self-interference existing in FD relays as compared to HD

relays and the performance gains by employing PRS are also studied in this paper.

- 3) The closed-form expressions of outage probabilities for FD and HD relay-aided CR-NOMA networks are derived. In addition, the analytical expressions for asymptotic outage probabilities and ergodic capacity are presented. The derived expressions have been corroborated by performing Monte Carlo simulations.

**C. ORGANIZATION**

The rest of the paper is organized as follows. In Section II, we establish a system model and provide relevant details. The outage behavior of CR-NOMA is discussed in Section III. Section IV presents outage performance in CR-OMA. In Sections V and VI, the ergodic capacity analysis for CR-NOMA and CR-OMA is provided, respectively. Simulation results in Section VII are provided to assess the accuracy of derived expressions. Finally, Section VIII provides the conclusion and future research directions.



**FIGURE 1. Combining relay selection and uplink/downlink scheme in the secondary network of CR-NOMA.**

**II. SYSTEM MODEL**

Fig. 1 demonstrates a relay-aided CR-NOMA network having a primary source  $PS$  and several intermediate relays  $\{R_k | k = 1, 2, \dots, K\}$  employing relaying protocols [30], [31], [32], i.e. DF scheme. There are two source-destination pairs,  $G_1 = \{S_1, D_1\}$  and  $G_2 = \{S_2, D_2\}$ , operating in the same frequency band and using NOMA technique. We consider a worst-case communication scenario for the secondary sources  $S_1$  and  $S_2$ . Specifically, we assume there the direct link between secondary source-destination pairs undergo deep fading and the communication is only possible via intermediate relays. More specifically, there are two phases for uplink-downlink CR-NOMA transmission: (i) uplink phase indicated that  $S_1$  and  $S_2$  can form a NOMA pair [23] to transmit simultaneously to  $R_k$ , (ii) in downlink phase  $R_k$

can concurrently transmit a superimposed composite signal, consisting of the decoded symbols, transmitted by sources to  $D_1$  and  $D_2$ . In CR-NOMA, limiting interference is in noticeable terms since the  $PS$  make interference to multiple relays  $R_k$  and two destinations. In this case, both uplink and downlink require SIC performed similarly at both the source-destination pairs with respect to maximum achievable system capacity.

Wireless channels in such CR-NOMA are subjected to Rayleigh flat fading plus additive white Gaussian noise. The complex channel coefficients for the links  $S_1 \rightarrow R_k$ ,  $R_k \rightarrow D_1$ ,  $S_2 \rightarrow R_k$ ,  $R_k \rightarrow D_2$ ,  $PS \rightarrow R_k$ ,  $PS \rightarrow D_1$ ,  $PS \rightarrow D_2$  are represented by  $|g_{1,uk}|^2 \sim CN(0, \lambda_{1,uk})$ ,  $|h_{1,dk}|^2 \sim CN(0, \lambda_{1,dk})$ ,  $|g_{2,uk}|^2 \sim CN(0, \lambda_{2,uk})$ ,  $|h_{2,dk}|^2 \sim CN(0, \lambda_{2,dk})$ ,  $|g_{r,ps}|^2 \sim CN(0, \lambda_{r,ps})$ ,  $|h_{1,ps}|^2 \sim CN(0, \lambda_{1,ps})$  and  $|h_{2,ps}|^2 \sim CN(0, \lambda_{2,ps})$ , respectively.

Following the principle of uplink NOMA, both  $S_1$  and  $S_2$  transmit symbols  $x_1$  and  $x_2$  simultaneously during the considered time slots.<sup>1</sup> In NOMA, portion of allocated powers are  $a_1 P_S$  and  $a_2 P_S$  for two signals  $x_1$  and  $x_2$ , respectively. In this paper, the power allocation coefficients are  $a_1$  and  $a_2$  for the first hop transmission while  $a_3$  and  $a_4$  for the second one, and  $P_S$  is the total transmit power of sources. Concurrently, both destinations  $D_1$  and  $D_2$  are able to receive signal from relay  $R_k$  which transmits the superimposed composite signal  $x_S = \sqrt{a_3 P_R} x_1 + \sqrt{a_4 P_R} x_2$  with a processing delay  $t$  following the principle of downlink NOMA. It is noted that  $P_R$  is the total transmit power of  $R_k$ , and  $x_1, x_2$  are data symbols decoded at  $R_k$ . This constraint must be satisfied, i.e.  $a_1 + a_2 = 1$  and  $a_3 + a_4 = 1$  are the total transmission power requirement constraints that need to be satisfied for many practical scenarios. To implement advantages of FD mode,  $R_k$  can transmit/receive signals simultaneously from sources or destinations.

According to uplink NOMA,  $R_k$  first decodes  $x_1$  with better channel conditions by treating  $x_2$ , having worse channel conditions, as noise. SIC is then carried out at  $R_k$  to obtain symbol  $x_2$ . Therefore, the signal to interference plus noise ratio (SINR) received at  $R_k$ , associated with  $x_1$  and  $x_2$  under the impact of interference from the  $PS$ , are respectively given by

$$\gamma_{x_1}^{S_1-R} = \frac{a_1 \rho_S |g_{1,uk}|^2}{a_2 \rho_S |g_{2,uk}|^2 + \rho_P |g_{r,ps}|^2 + I_R + 1}, \quad (1)$$

$$\gamma_{x_2}^{S_1-R} = \frac{a_2 \rho_S |g_{2,uk}|^2}{a_1 \rho_S |\tilde{g}_{1,uk}|^2 + \rho_P |g_{r,ps}|^2 + I_R + 1}, \quad (2)$$

where  $\rho_S = \frac{P_S}{\sigma^2}$ ,  $\rho_P = \frac{P_P}{\sigma^2}$  with  $P_P$  is transmit power at the primary source  $PS$ , interference channel related to SIC

<sup>1</sup>The system model does not consider any PU as there are already many studies focusing on the outage performance of PUs. However, the performance of relay aided secondary source-destination pairs is not well explored in the literature. Therefore, this work mainly focuses on the performance of NOAM-enabled secondary user/destination. The joint performance of primary and secondary user/destination can be evaluated in future studies.

imperfection characterized by  $\tilde{g}_{1,uk} \sim CN(0, \tau_1 \lambda_{1im})$  and  $\sigma^2$  is denoted as noise variance. Here, residual self-interference is  $I_R$ , which can be considered a constant as in [17] for simplicity of analysis. Moreover, SIC imperfection level at the relay is related to level of residual interference, and such term is denoted by  $\tau_1$  ( $0 \leq \tau_1 \leq 1$ ). Two main cases are  $\tau_1 = 0$  and  $\tau_1 = 1$  corresponding to perfect SIC and imperfect SIC, respectively.

On the contrary, according to the principle of downlink NOMA,  $D_1$  decodes  $x_1$  by treating  $x_2$  as noise. Therefore, the SINR received at  $D_1$  under the impact of interference from the  $PS$  is computed by

$$\gamma_{x_1}^{R-D1} = \frac{a_3 \rho_R |h_{1,dk}|^2}{a_4 \rho_R |h_{1,dk}|^2 + \rho_P |h_{1,ps}|^2 + 1}. \quad (3)$$

Conversely,  $D_2$  considers its own low powered symbol  $x_2$  as noise to decode high powered symbol  $x_1$ , and then, SIC is enabled at  $D_2$  to achieve  $x_2$ . Therefore, the SINRs received at  $D_2$ , associated with  $x_1$  and  $x_2$  under interference impact of the  $PS$ , are given by

$$\gamma_{x_1 \rightarrow x_2}^{R-D2} = \frac{a_3 \rho_R |h_{2,dk}|^2}{a_4 \rho_R |h_{2,dk}|^2 + \rho_P |h_{2,ps}|^2 + 1}, \quad (4)$$

$$\gamma_{x_2}^{R-D2} = \frac{a_4 \rho_R |h_{2,dk}|^2}{a_3 \rho_R |\tilde{h}_{2,dk}|^2 + \rho_P |h_{2,ps}|^2 + 1}, \quad (5)$$

where  $\rho_R = \frac{P_R}{\sigma^2}$ , corresponding interference channel of imperfect SIC  $\tilde{h}_{2,dk} \sim CN(0, \tau_2 \lambda_{2im})$ , and  $\gamma_{x_1 \rightarrow x_2}^{R-D2}$  represents the SINR required at  $D_2$  to decode symbol  $x_1$ . The level of residual interference at  $D_2$  because of SIC imperfection is denoted by  $\tau_2$ , which indicates similar behavior to  $\tau_1$ .

Regarding PRS, the helping relay is able to select based on only the CSI of the  $S_1 \rightarrow R_k$  links and  $S_2 \rightarrow R_k$  links. In particular, the index of the selected relay  $R_{k^*}$  using PRS scheme are given by

$$k_1^* = \arg \max_{k_1=1, \dots, K} (|g_{1,uk}|^2), \quad (6)$$

and

$$k_2^* = \arg \max_{k_2=1, \dots, K} (|g_{2,uk}|^2). \quad (7)$$

### III. OUTAGE PROBABILITY ANALYSIS FOR RELAY-AIDED CR-NOMA

This section provides details of the derivation of outage probability for both FD and HD relay-aided CR-NOMA.

#### A. THE FD RELAY-AIDED CR-NOMA

According to the required quality of service,  $R_1$  and  $R_2$  are assumed to be the predefined target rate thresholds of  $G_1$  and  $G_2$ , respectively. Then,  $\gamma_{th1}^{FD}$ ,  $\gamma_{th2}^{FD}$  are signal to noise ratio (SNR) thresholds. The exact outage probability of  $G_1$  and  $G_2$  are computed in the following subsections.

#### 1) OUTAGE PROBABILITY OF $G_1$

The exact outage probability of  $G_1$  can be written as

$$\begin{aligned} OP_1^{FD} &= 1 - \Pr \left( \gamma_{x_1}^{S1-Rk^*} \geq \gamma_{th1}^{FD} \cap \gamma_{x_1}^{Rk^*-D1} \geq \gamma_{th1}^{FD} \right) \\ &= 1 - \underbrace{\Pr \left( \gamma_{x_1}^{S1-Rk^*} \geq \gamma_{th1}^{FD} \right)}_{A_1} \times \underbrace{\Pr \left( \gamma_{x_1}^{Rk^*-D1} \geq \gamma_{th1}^{FD} \right)}_{A_2}. \end{aligned} \quad (8)$$

*Proposition 1:* The closed-form expression for the outage probability of the  $G_1$  is derived as

$$\begin{aligned} OP_1^{FD} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\ &\quad \times \frac{j \rho_R \lambda_{1,uk} \lambda_{1,uk} \lambda_{1,dk}}{(i \alpha_2^{FD} \lambda_{2,uk} + j \lambda_{1,uk}) (i \alpha_3^{FD} \lambda_{r,ps} + \lambda_{1,uk})} \\ &\quad \times \frac{(a_3 - \gamma_{th1}^{FD} a_4)}{(\gamma_{th1}^{FD} \rho_P \lambda_{1,ps} + (a_3 - \gamma_{th1}^{FD} a_4) \rho_R \lambda_{1,dk})} \\ &\quad \times \exp \left( -\frac{i \alpha_4^{FD}}{\lambda_{1,uk}} - \frac{\gamma_{th1}^{FD}}{(a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{1,dk}} \right), \end{aligned} \quad (9)$$

where  $\gamma_{th1}^{FD} = 2^{R_1} - 1$ ,  $\alpha_1^{FD} = \gamma_{th1}^{FD} (I_R + 1)$ ,  $\alpha_2^{FD} = \frac{\gamma_{th1} a_2}{a_1}$ ,  $\alpha_3^{FD} = \frac{\gamma_{th1} \rho_P}{a_1 \rho_S}$  and  $\alpha_4^{FD} = \frac{\alpha_1}{a_1 \rho_S}$ .

*Proof:* See Appendix A.

#### 2) OUTAGE PROBABILITY OF $G_2$

The exact outage probability of  $G_2$  with imperfect SIC can be written as

$$OP_{2-isc}^{FD} = 1 - \Pr \left( \gamma_{x_2}^{S1-Rk^*} \geq \gamma_{th2}^{FD} \cap \gamma_{x_2}^{Rk^*-D2} \geq \gamma_{th2}^{FD} \right. \\ \left. \cap \gamma_{x_1 \rightarrow x_2}^{Rk^*-D2} \geq \gamma_{th1}^{FD} \right). \quad (10)$$

*Proposition 2:* The closed-form expression for the outage probability of the  $G_2$  is derived as

$$\begin{aligned} OP_{2-isc}^{FD} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\ &\quad \times \frac{j \lambda_{2,uk} \lambda_{2,uk}}{(i \beta_2^{FD} \tau_1 \lambda_{1im} + j \lambda_{2,uk}) (i \beta_3^{FD} \lambda_{r,ps} + \lambda_{2,uk})} \\ &\quad \times \frac{\lambda_{2,dk} \lambda_{2,dk}}{(m_1^{FD} \tau_2 \lambda_{2im} + \lambda_{2,dk}) (m_2^{FD} \lambda_{2,ps} + \lambda_{2,dk})} \\ &\quad \times \frac{(a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{2,dk}}{\gamma_{th1}^{FD} \rho_P \lambda_{2,ps} + (a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{2,dk}} \\ &\quad \times \exp \left( -\frac{i \beta_4^{FD}}{\lambda_{2,uk}} - \frac{m_3^{FD}}{\lambda_{2,dk}} - \frac{\gamma_{th1}^{FD}}{(a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{2,dk}} \right), \end{aligned} \quad (11)$$

where  $\gamma_{th2}^{FD} = 2^{R_2} - 1$ ,  $\beta_1^{FD} = \gamma_{th2}^{FD} (I_R + 1)$ ,  $\beta_2^{FD} = \frac{\gamma_{th2}^{FD} a_1}{\lambda_{2,dk}}$ ,  $\beta_3^{FD} = \frac{\gamma_{th2}^{FD} \rho_P}{a_2 \rho_S}$ ,  $\beta_4^{FD} = \frac{\beta_1^{FD}}{a_2 \rho_S}$ ,  $m_1^{FD} = \frac{\gamma_{th2}^{FD} a_3}{a_4}$ ,  $m_2^{FD} = \frac{\gamma_{th2}^{FD} \rho_P}{a_4 \rho_R}$  and  $m_3^{FD} = \frac{\gamma_{th2}^{FD}}{a_4 \rho_R}$ .

*Proof:* See Appendix B.

Then, by substituting  $\tau_1 = \tau_2 = 0$  in (10) and (11), the exact outage probability of  $G_2$  under perfect SIC (psic) can be obtained as

$$\begin{aligned}
 OP_{2-psic}^{FD} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\lambda_{2,uk}\lambda_{2,uk}}{(i\beta_2^{FD}\lambda_2 + j\lambda_{2,uk})(i\beta_3^{FD}\lambda_{r,ps} + \lambda_{2,uk})} \\
 &\times \frac{\lambda_{2,dk}\lambda_{2,dk}}{\lambda_{2,dk}(m_2^{FD}\lambda_{2,ps} + \lambda_{2,dk})} \\
 &\times \frac{(a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{2,dk}}{\gamma_{th1}^{FD}\rho_P\lambda_{2,ps} + (a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{2,dk}} \\
 &\times \exp\left(-\frac{i\beta_4^{FD}}{\lambda_{2,uk}} - \frac{m_3^{FD}}{\lambda_{2,dk}} - \frac{\gamma_{th1}^{FD}}{(a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{2,dk}}\right). \tag{12}
 \end{aligned}$$

**B. THE HD RELAY-AIDED CR-NOMA**

1) OUTAGE PROBABILITY OF  $G_1$

Similar to (8) and (9), and we replace  $I_R = 0$ , the outage probability of  $G_1$  in HD-NOMA is given by

$$\begin{aligned}
 OP_1^{HD} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\rho_R\lambda_{1,uk}\lambda_{1,uk}\lambda_{1,dk}}{(i\alpha_2^{HD}\lambda_{2,uk} + j\lambda_{1,uk})(i\alpha_3^{HD}\lambda_{r,ps} + \lambda_{1,uk})} \\
 &\times \frac{(a_3 - \gamma_{th1}^{HD}a_4)}{(\gamma_{th1}^{HD}\rho_P\lambda_{1,ps} + (a_3 - \gamma_{th1}^{HD}a_4)\rho_R\lambda_{1,dk})} \\
 &\times \exp\left(-\frac{i\alpha_4^{HD}}{\lambda_{1,uk}} - \frac{\gamma_{th1}^{HD}}{(a_3\rho_R - \gamma_{th1}^{HD}a_4\rho_R)\lambda_{1,dk}}\right), \tag{13}
 \end{aligned}$$

where  $\gamma_{th1}^{HD} = 2^{2R_1} - 1$ ,  $\alpha_2^{HD} = \frac{\gamma_{th1}^{HD}a_2}{a_1}$ ,  $\alpha_3^{HD} = \frac{\gamma_{th1}^{HD}\rho_P}{a_1\rho_S}$ ,  $\alpha_4^{HD} = \frac{\gamma_{th1}^{HD}}{a_1\rho_S}$ .

2) OUTAGE PROBABILITY OF  $G_2$

The outage probability of  $G_2$  in HD-NOMA with isic is expressed by

$$\begin{aligned}
 OP_{2-isic}^{HD} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\lambda_{2,uk}\lambda_{2,uk}}{(i\beta_2^{HD}\tau_1\lambda_{1im} + j\lambda_{2,uk})(i\beta_3^{HD}\lambda_{r,ps} + \lambda_{2,uk})} \\
 &\times \frac{\lambda_{2,dk}\lambda_{2,dk}}{(m_1^{HD}\tau_2\lambda_{2im} + \lambda_{2,dk})(m_2^{HD}\lambda_{2,ps} + \lambda_{2,dk})}
 \end{aligned}$$

$$\begin{aligned}
 &\times \frac{(a_3\rho_R - \gamma_{th1}^{HD}a_4\rho_R)\lambda_{2,dk}}{\gamma_{th1}^{HD}\rho_P\lambda_{2,ps} + (a_3\rho_R - \gamma_{th1}^{HD}a_4\rho_R)\lambda_{2,dk}} \\
 &\times \exp\left(-\frac{i\beta_4^{HD}}{\lambda_{2,uk}} - \frac{m_3^{HD}}{\lambda_{2,dk}} - \frac{\gamma_{th1}^{HD}}{(a_3\rho_R - \gamma_{th1}^{HD}a_4\rho_R)\lambda_{2,dk}}\right), \tag{14}
 \end{aligned}$$

where  $\gamma_{th2}^{HD} = 2^{2R_2} - 1$ ,  $\beta_2^{HD} = \frac{\gamma_{th2}^{HD}a_1}{a_2}$ ,  $\beta_3^{HD} = \frac{\gamma_{th2}^{HD}\rho_P}{a_2\rho_S}$ ,  $\beta_4^{HD} = \frac{\gamma_{th2}^{HD}}{a_2\rho_S}$ ,  $m_1^{HD} = \frac{\gamma_{th2}^{HD}a_3}{a_4}$ ,  $m_2^{HD} = \frac{\gamma_{th2}^{HD}\rho_P}{a_4\rho_R}$ ,  $m_3^{HD} = \frac{\gamma_{th2}^{HD}}{a_4\rho_R}$ .

The exact outage probability of  $G_2$  under perfect SIC can be obtained as

$$\begin{aligned}
 OP_{2-psic}^{HD} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\lambda_{2,uk}\lambda_{2,uk}}{j\lambda_{2,uk}(i\beta_3^{HD}\lambda_{r,ps} + \lambda_{2,uk})} \\
 &\times \frac{\lambda_{2,dk}\lambda_{2,dk}}{\lambda_{2,dk}(m_2^{HD}\lambda_{2,ps} + \lambda_{2,dk})} \\
 &\times \frac{(a_3\rho_R - \gamma_{th1}^{HD}a_4\rho_R)\lambda_{2,dk}}{\gamma_{th1}^{HD}\rho_P\lambda_{2,ps} + (a_3\rho_R - \gamma_{th1}^{HD}a_4\rho_R)\lambda_{2,dk}} \\
 &\times \exp\left(-\frac{i\beta_4^{HD}}{\lambda_{2,uk}} - \frac{m_3^{HD}}{\lambda_{2,dk}} - \frac{\gamma_{th1}^{HD}}{(a_3\rho_R - \gamma_{th1}^{HD}a_4\rho_R)\lambda_{2,dk}}\right). \tag{15}
 \end{aligned}$$

**C. ANALYSIS ON ASYMPTOTIC OUTAGE PROBABILITY**

Based on the previous results, an asymptotic analysis for both  $D_1$  and  $D_2$  will be carried out to evaluate the outage behavior, i.e.,  $OP_1^{FD}$  and  $OP_2^{FD}$ , respectively. Particularly, the following expressions provide insightful details for the proposed system in the high SNR regime.

1) ASYMPTOTIC OUTAGE PROBABILITY FD IN THE FD RELAY-AIDED CR-NOMA

a) Asymptotic Outage Probability FD at  $G_1$

Based on the above analytical results in (9), by using  $\exp(-x) = 1 - x$  the asymptotic outage probability of  $D_1$  with is given by

$$\begin{aligned}
 OP_1^{FD-asym} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\rho_R\lambda_{1,uk}\lambda_{1,uk}\lambda_{1,dk}}{(i\alpha_2^{FD}\lambda_{2,uk} + j\lambda_{1,uk})(i\alpha_3^{FD}\lambda_{r,ps} + \lambda_{1,uk})} \\
 &\times \frac{(a_3 - \gamma_{th1}^{FD}a_4)}{(\gamma_{th1}^{FD}\rho_P\lambda_{1,ps} + (a_3 - \gamma_{th1}^{FD}a_4)\rho_R\lambda_{1,dk})} \\
 &\times \left(1 - \frac{i\alpha_4^{FD}}{\lambda_{1,uk}} - \frac{\gamma_{th1}^{FD}}{(a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{1,dk}}\right). \tag{16}
 \end{aligned}$$



To look at the lower bound, when  $\rho_S, \rho_R, \rho_P \rightarrow \infty$ , the asymptotic outage probability of  $G_1$  is obtained as

$$OP_1^{FD-floor} = 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \times \frac{j\lambda_{1,uk}\lambda_{1,uk}}{(i\alpha_2^{FD}\lambda_{2,uk} + j\lambda_{1,uk}) \left( \frac{\gamma_{th1}^{FD}}{a_1} \lambda_{r,ps} + \lambda_{1,uk} \right)} \times \frac{(a_3 - \gamma_{th1}^{FD} a_4) \lambda_{1,dk}}{\gamma_{th1}^{FD} \lambda_{1,ps} + (a_3 - \gamma_{th1}^{FD} a_4) \lambda_{1,dk}}. \quad (17)$$

### b) Asymptotic Outage Probability FD at $G_2$

The asymptotic outage probability of  $G_2$  under imperfect SIC is given by

$$OP_{2-istic}^{FD-asym} = 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \times \frac{j\lambda_{2,uk}\lambda_{2,uk}}{(i\beta_2^{FD} \tau_1 \lambda_{1im} + j\lambda_{2,uk}) (i\beta_3^{FD} \lambda_{r,ps} + \lambda_{2,uk})} \times \frac{\lambda_{2,dk}\lambda_{2,dk}}{(m_1^{FD} \tau_2 \lambda_{2im} + \lambda_{2,dk}) (m_2^{FD} \lambda_{2,ps} + \lambda_{2,dk})} \times \frac{(a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{2,dk}}{\gamma_{th1}^{FD} \rho_P \lambda_{2,ps} + (a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{2,dk}} \times \left( 1 - \frac{i\beta_4^{FD}}{\lambda_{2,uk}} - \frac{m_3^{FD}}{\lambda_{2,dk}} - \frac{\gamma_{th1}^{FD}}{(a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{2,dk}} \right). \quad (18)$$

Similarly, the asymptotic outage probability of  $G_2$  under perfect SIC is computed by

$$OP_{2-psic}^{FD-asym} = 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \times \frac{j\lambda_{2,uk}\lambda_{2,uk}}{j\lambda_{2,uk} (i\beta_3^{FD} \lambda_{r,ps} + \lambda_{2,uk})} \times \frac{\lambda_{2,dk}\lambda_{2,dk}}{\lambda_{2,dk} (m_2^{FD} \lambda_{2,ps} + \lambda_{2,dk})} \times \frac{(a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{2,dk}}{\gamma_{th1}^{FD} \rho_P \lambda_{2,ps} + (a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{2,dk}} \times \left( 1 - \frac{i\beta_4^{FD}}{\lambda_{2,uk}} - \frac{m_3^{FD}}{\lambda_{2,dk}} - \frac{\gamma_{th1}^{FD}}{(a_3 \rho_R - \gamma_{th1}^{FD} a_4 \rho_R) \lambda_{2,dk}} \right). \quad (19)$$

When  $\rho_S, \rho_R, \rho_P \rightarrow \infty$ , the asymptotic outage probability of  $G_2$  under imperfect SIC with is determined by

$$OP_{2-istic}^{FD-floor} = 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \times \frac{j\lambda_{2,uk}\lambda_{2,uk}}{(i\beta_2^{FD} \tau_1 \lambda_{1im} + j\lambda_{2,uk}) \left( \frac{i\gamma_{th2}^{FD} \lambda_{r,ps}}{a_2} + \lambda_{2,uk} \right)} \times \frac{\lambda_{2,dk}\lambda_{2,dk}}{(m_1^{FD} \tau_2 \lambda_{2im} + \lambda_{2,dk}) \left( \frac{\gamma_{th2}^{FD} \lambda_{2,ps}}{a_4} + \lambda_{2,dk} \right)} \times \frac{(a_3 - \gamma_{th1}^{FD} a_4) \lambda_{2,dk}}{\gamma_{th1}^{FD} \lambda_{2,ps} + (a_3 - \gamma_{th1}^{FD} a_4) \lambda_{2,dk}}, \quad (20)$$

and  $G_2$  under ipsic case is determined by

$$OP_{2-istic}^{FD-floor} = 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \times \frac{j\lambda_{2,uk}\lambda_{2,uk}}{j\lambda_{2,uk} \left( \frac{i\gamma_{th2}^{FD} \lambda_{r,ps}}{a_2} + \lambda_{2,uk} \right)} \times \frac{\lambda_{2,dk}\lambda_{2,dk} (a_3 - \gamma_{th1}^{FD} a_4) \lambda_{2,dk}}{\lambda_{2,dk} \left( \frac{\gamma_{th2}^{FD} \lambda_{2,ps}}{a_4} + \lambda_{2,dk} \right) (\gamma_{th1}^{FD} \lambda_{2,ps} + (a_3 - \gamma_{th1}^{FD} a_4) \lambda_{2,dk})}. \quad (21)$$

## 2) ASYMPTOTIC OUTAGE PROBABILITY HD IN THE HD RELAY-AIDED CR-NOMA

### a) Asymptotic Outage Probability HD at $G_1$

Based on the above analytical results in (13), by using  $\exp(-x) = 1 - x$ , the asymptotic outage probability of  $G_1$  is obtained as

$$OP_1^{HD-asym} = 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \times \frac{j\rho_R \lambda_{1,uk} \lambda_{1,uk} \lambda_{1,dk}}{(i\alpha_2^{HD} \lambda_{2,uk} + j\lambda_{1,uk}) (i\alpha_3^{HD} \lambda_{r,ps} + \lambda_{1,uk})} \times \frac{(a_3 - \gamma_{th1}^{HD} a_4)}{(\gamma_{th1}^{HD} \rho_P \lambda_{1,ps} + (a_3 - \gamma_{th1}^{HD} a_4) \rho_R \lambda_{1,dk})} \times \left( 1 - \frac{i\alpha_4^{HD}}{\lambda_{1,uk}} - \frac{\gamma_{th1}^{HD}}{(a_3 \rho_R - \gamma_{th1}^{HD} a_4 \rho_R) \lambda_{1,dk}} \right). \quad (22)$$

To look the lower bound, when  $\rho_S, \rho_R, \rho_P \rightarrow \infty$ , the asymptotic outage probability of  $G_1$  is

obtained as

$$\begin{aligned}
 OP_1^{HD-floor} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\rho_R \lambda_{1,uk} \lambda_{1,dk}}{(i\alpha_2^{HD} \lambda_{2,uk} + j\lambda_{1,uk}) (i\alpha_3^{HD} \lambda_{r,ps} + \lambda_{1,uk})} \\
 &\times \frac{(a_3 - \gamma_{th1}^{HD} a_4)}{(\gamma_{th1}^{HD} \rho_P \lambda_{1,ps} + (a_3 - \gamma_{th1}^{HD} a_4) \rho_R \lambda_{1,dk})}.
 \end{aligned} \tag{23}$$

b) Asymptotic Outage Probability HD at  $G_2$

Similar to FD, the asymptotic outage probability of HD scenario at  $G_2$  under isic case is determined by

$$\begin{aligned}
 OP_{2-isic}^{HD-asym} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\lambda_{2,uk} \lambda_{2,dk}}{(i\beta_2^{HD} \tau_1 \lambda_{1im} + j\lambda_{2,uk}) (i\beta_3^{HD} \lambda_{r,ps} + \lambda_{2,uk})} \\
 &\times \frac{\lambda_{2,dk} \lambda_{2,dk}}{(m_1^{HD} \tau_2 \lambda_{2im} + \lambda_{2,dk}) (m_2^{HD} \lambda_{2,ps} + \lambda_{2,dk})} \\
 &\times \frac{(a_3 \rho_R - \gamma_{th1} a_4 \rho_R) \lambda_{2,dk}}{\gamma_{th1}^{HD} \rho_P \lambda_{2,ps} + (a_3 \rho_R - \gamma_{th1}^{HD} a_4 \rho_R) \lambda_{2,dk}} \\
 &\times \left( 1 - \frac{i\beta_4^{HD}}{\lambda_{2,uk}} - \frac{m_3^{HD}}{\lambda_{2,dk}} - \frac{\gamma_{th1}^{HD}}{(a_3 \rho_R - \gamma_{th1}^{HD} a_4 \rho_R) \lambda_{2,dk}} \right),
 \end{aligned} \tag{24}$$

and asymptotic outage probability in HD mode at  $G_2$  with psic is obtained as

$$\begin{aligned}
 OP_{2-psic}^{HD-asym} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\lambda_{2,uk} \lambda_{2,dk}}{j\lambda_{2,uk} (i\beta_3^{HD} \lambda_{r,ps} + \lambda_{2,uk})} \times \frac{\lambda_{2,dk} \lambda_{2,dk}}{\lambda_{2,dk} (m_2^{HD} \lambda_{2,ps} + \lambda_{2,dk})} \\
 &\times \frac{(a_3 \rho_R - \gamma_{th1} a_4 \rho_R) \lambda_{2,dk}}{\gamma_{th1}^{HD} \rho_P \lambda_{2,ps} + (a_3 \rho_R - \gamma_{th1}^{HD} a_4 \rho_R) \lambda_{2,dk}} \\
 &\times \left( 1 - \frac{i\beta_4^{HD}}{\lambda_{2,uk}} - \frac{m_3^{HD}}{\lambda_{2,dk}} - \frac{\gamma_{th1}}{(a_3 \rho_R - \gamma_{th1}^{HD} a_4 \rho_R) \lambda_{2,dk}} \right).
 \end{aligned} \tag{25}$$

When  $\rho_S, \rho_R, \rho_P \rightarrow \infty$ , the asymptotic outage probability of  $G_2$  under isic is obtained as

$$\begin{aligned}
 OP_{2-isic}^{HD-floor} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\lambda_{2,uk} \lambda_{2,dk}}{(i\beta_2^{HD} \tau_1 \lambda_{1im} + j\lambda_{2,uk}) (i\beta_3^{HD} \lambda_{r,ps} + \lambda_{2,uk})}
 \end{aligned}$$

$$\begin{aligned}
 &\times \frac{\lambda_{2,dk} \lambda_{2,dk}}{(m_1^{HD} \tau_2 \lambda_{2im} + \lambda_{2,dk}) (m_2^{HD} \lambda_{2,ps} + \lambda_{2,dk})} \\
 &\times \frac{(a_3 \rho_R - \gamma_{th1} a_4 \rho_R) \lambda_{2,dk}}{\gamma_{th1}^{HD} \rho_P \lambda_{2,ps} + (a_3 \rho_R - \gamma_{th1}^{HD} a_4 \rho_R) \lambda_{2,dk}},
 \end{aligned} \tag{26}$$

and asymptotic outage probability of  $G_2$  under psic is obtained as

$$\begin{aligned}
 OP_{2-psic}^{HD-floor} &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\times \frac{j\lambda_{2,uk} \lambda_{2,dk}}{j\lambda_{2,uk} (i\beta_3^{HD} \lambda_{r,ps} + \lambda_{2,uk})} \times \frac{\lambda_{2,dk} \lambda_{2,dk}}{\lambda_{2,dk} (m_2^{HD} \lambda_{2,ps} + \lambda_{2,dk})} \\
 &\times \frac{(a_3 \rho_R - \gamma_{th1} a_4 \rho_R) \lambda_{2,dk}}{\gamma_{th1}^{HD} \rho_P \lambda_{2,ps} + (a_3 \rho_R - \gamma_{th1}^{HD} a_4 \rho_R) \lambda_{2,dk}}.
 \end{aligned} \tag{27}$$

#### IV. OUTAGE PROBABILITY FOR RELAY-AIDED CR-OMA

This phase resembles uplink CR-OMA,  $R_k$  first decodes  $x_1$  with better channel conditions by treating  $x_2$  having worse channel conditions, as noise. SIC is then carried out at  $R_k$  to obtain symbol  $x_2$ . Therefore, the SINRs received at  $R_k$ , associated with  $x_1$  and  $x_2$  under the impact of interference from the  $PS$ , are respectively given by

$$\gamma_{x1-OMA}^{S1-R} = \frac{\rho_S |g_{1,uk}|^2}{\rho_P |g_{r,ps}|^2 + I_R + 1}, \tag{28}$$

$$\gamma_{x2-OMA}^{S1-R} = \frac{\rho_S |g_{2,uk}|^2}{\rho_P |g_{r,ps}|^2 + I_R + 1}. \tag{29}$$

By contrast, according to the principle of downlink OMA,  $D_1$  decodes  $x_1$ . Therefore, the received SINR at  $D_1$  is given by

$$\gamma_{x1-OMA}^{R-D1} = \frac{\rho_R |h_{1,dk}|^2}{\rho_P |h_{1,ps}|^2 + 1}. \tag{30}$$

Therefore, the received SINR at  $D_2$  is given by

$$\gamma_{x2-OMA}^{R-D2} = \frac{\rho_R |h_{2,dk}|^2}{\rho_P |h_{2,ps}|^2 + 1}. \tag{31}$$

#### A. THE FD RELAY-AIDED CR-OMA

##### 1) OUTAGE PROBABILITY OF $G_1$

The exact outage probability of  $G_1$  can be written as

$$OP_{1-OMA}^{FD} = 1 - \Pr \left( \begin{aligned} &\gamma_{x1-OMA}^{S1-Rk*} \geq \gamma_{th1}^{FD-O} \\ &\cap \gamma_{x1-OMA}^{Rk*-D1} \geq \gamma_{th1}^{FD-O} \end{aligned} \right), \tag{32}$$

where  $\gamma_{th1}^{FD-O} = 2^{2R_1} - 1$ .

**Proposition 3:** The closed-form expression for the outage probability of the  $G_1$  is derived as

$$\begin{aligned}
 OP_{1-OMA}^{FD} &= 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \\
 &\times \frac{\rho_S \rho_R \lambda_{1,uk} \lambda_{1,dk}}{\left( i \gamma_{th1}^{FD-O} \rho_P \lambda_{r,ps} + \rho_S \lambda_{1,uk} \right) \left( \gamma_{th1}^{FD-O} \rho_P \lambda_{1,ps} + \rho_R \lambda_{1,dk} \right)} \\
 &\times \exp \left( -\frac{i}{\lambda_{1,uk}} \left( \frac{\gamma_{th1}^{FD-O} I_R}{\rho_S} + \frac{\gamma_{th1}^{FD-O}}{\rho_S} \right) - \frac{\gamma_{th1}^{FD-O}}{\rho_R \lambda_{1,dk}} \right). \tag{33}
 \end{aligned}$$

*Proof:* See Appendix C.

2) OUTAGE PROBABILITY OF  $G_2$

The exact outage probability of  $G_2$  can be written as

$$\begin{aligned}
 OP_{2-OMA}^{FD} &= 1 - \Pr \left( \begin{matrix} \gamma_{x2-OMA}^{S1-Rk*} \geq \gamma_{th2}^{FD-O} \\ \cap \gamma_{x2-OMA}^{Rk*-D2} \geq \gamma_{th2}^{FD-O} \end{matrix} \right) \\
 &= 1 - \Pr \left( \underbrace{\gamma_{x2-OMA}^{S1-Rk*} \geq \gamma_{th2}^{FD-O}}_{D_1} \right) \\
 &\times \Pr \left( \underbrace{\gamma_{x2-OMA}^{Rk*-D2} \geq \gamma_{th2}^{FD-O}}_{D_2} \right), \tag{34}
 \end{aligned}$$

where  $\gamma_{th2}^{FD-O} = 2^{2R_2} - 1$ .

Similar the computations of  $C_1$  and  $C_2$  from appendix applicable to  $D_1$  and  $D_2$ , the closed-form expression for the outage probability of  $G_2$  is derived as

$$\begin{aligned}
 OP_{2-OMA}^{FD} &= 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \\
 &\times \frac{\rho_S \rho_R \lambda_{2,dk} \lambda_{2,uk}}{\left( i \gamma_{th2}^{FD-O} \rho_P \lambda_{r,ps} + \rho_S \lambda_{2,uk} \right) \left( \gamma_{th2}^{FD-O} \rho_P \lambda_{2,ps} + \rho_R \lambda_{2,dk} \right)} \\
 &\times \exp \left( -\frac{i}{\lambda_{2,uk}} \left( \frac{\gamma_{th2}^{FD-O} I_R}{\rho_S} + \frac{\gamma_{th2}^{FD-O}}{\rho_S} \right) - \frac{\gamma_{th2}^{FD-O}}{\rho_R \lambda_{2,dk}} \right). \tag{35}
 \end{aligned}$$

**B. THE HD RELAY-AIDED CR-OMA**

Similar to (32) and (33) and by substituting  $I_R = 0$ , the outage probability of  $G_1$  in HD-NOMA network is expressed as

$$\begin{aligned}
 OP_{1-OMA}^{HD} &= 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \\
 &\times \frac{\rho_S \rho_R \lambda_{1,uk} \lambda_{1,dk}}{\left( i \gamma_{th1}^{HD-O} \rho_P \lambda_{r,ps} + \rho_S \lambda_{1,uk} \right) \left( \gamma_{th1}^{HD-O} \rho_P \lambda_{1,ps} + \rho_R \lambda_{1,dk} \right)} \\
 &\times \exp \left( -\frac{i \gamma_{th1}^{HD-O}}{\rho_S \lambda_{1,uk}} - \frac{\gamma_{th1}^{HD-O}}{\rho_R \lambda_{1,dk}} \right), \tag{36}
 \end{aligned}$$

and outage probability of  $G_2$  in HD-NOMA network is obtained as

$$\begin{aligned}
 OP_{2-OMA}^{HD} &= 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \\
 &\times \frac{\rho_S \rho_R \lambda_{2,dk} \lambda_{2,uk}}{\left( i \gamma_{th2}^{HD-O} \rho_P \lambda_{r,ps} + \rho_S \lambda_{2,uk} \right) \left( \gamma_{th2}^{HD-O} \rho_P \lambda_{2,ps} + \rho_R \lambda_{2,dk} \right)} \\
 &\times \exp \left( -\frac{i \gamma_{th2}^{HD-O}}{\rho_S \lambda_{2,uk}} - \frac{\gamma_{th2}^{HD-O}}{\rho_R \lambda_{2,dk}} \right), \tag{37}
 \end{aligned}$$

where  $\gamma_{th1}^{HD-O} = 2^{4R_1} - 1$  and  $\gamma_{th2}^{HD-O} = 2^{4R_2} - 1$ .

**C. ANALYSIS ON ASYMPTOTIC OUTAGE PROBABILITY**

1) CASE OF FD IN RELAY-AIDED CR-OMA

Based on the above analytical results in (33) and with the help of  $\exp(-x) = 1 - x$ , the asymptotic outage probability of  $G_1$  with is obtained as

$$\begin{aligned}
 OP_{1-OMA}^{FD-asym} &= 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \\
 &\times \frac{\rho_S \rho_R \lambda_{1,uk} \lambda_{1,dk}}{\left( i \gamma_{th1}^{FD-O} \rho_P \lambda_{r,ps} + \rho_S \lambda_{1,uk} \right) \left( \gamma_{th1}^{FD-O} \rho_P \lambda_{1,ps} + \rho_R \lambda_{1,dk} \right)} \\
 &\times \left( 1 - \frac{i}{\lambda_{1,uk}} \left( \frac{\gamma_{th1}^{FD-O} I_R}{\rho_S} + \frac{\gamma_{th1}^{FD-O}}{\rho_S} \right) - \frac{\gamma_{th1}^{FD-O}}{\rho_R \lambda_{1,dk}} \right). \tag{38}
 \end{aligned}$$

The lower bound can be determined, when  $\rho_S, \rho_R, \rho_P \rightarrow \infty$ , the asymptotic outage probability of  $G_1$  is obtained as

$$\begin{aligned}
 OP_{1-OMA}^{FD-floor} &= 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \\
 &\times \frac{\lambda_{1,uk} \lambda_{1,dk}}{\left( i \gamma_{th1}^{FD-O} \lambda_{r,ps} + \lambda_{1,uk} \right) \left( \gamma_{th1}^{FD-O} \lambda_{1,ps} + \lambda_{1,dk} \right)}. \tag{39}
 \end{aligned}$$

Similarly, we have  $G_2$

$$\begin{aligned}
 OP_{2-OMA}^{FD-asym} &= 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \\
 &\times \frac{\rho_S \rho_R \lambda_{2,dk} \lambda_{2,uk}}{\left( i \gamma_{th2}^{FD-O} \rho_P \lambda_{r,ps} + \rho_S \lambda_{2,uk} \right) \left( \gamma_{th2}^{FD-O} \rho_P \lambda_{2,ps} + \rho_R \lambda_{2,dk} \right)} \\
 &\times \left( 1 - \frac{i}{\lambda_{2,uk}} \left( \frac{\gamma_{th2}^{FD-O} I_R}{\rho_S} + \frac{\gamma_{th2}^{FD-O}}{\rho_S} \right) - \frac{\gamma_{th2}^{FD-O}}{\rho_R \lambda_{2,dk}} \right), \tag{40}
 \end{aligned}$$



and

$$OP_{2-OMA}^{FD-floor} = 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \times \frac{\lambda_{2,dk} \lambda_{2,uk}}{\left(i\gamma_{th2}^{FD-O} \lambda_{r,ps} + \lambda_{2,uk}\right) \left(\gamma_{th2}^{FD-O} \lambda_{2ps} + \lambda_{2,dk}\right)} \quad (41)$$

2) CASE OF HD IN RELAY-AIDED CR-OMA

Based on the analytical results in (36) and by using  $\exp(-x) = 1 - x$ , the asymptotic outage probability of  $G_1$  is obtained as

$$OP_{1-OMA}^{HD-asym} = 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \times \frac{\rho_S \rho_R \lambda_{1,uk} \lambda_{1,dk}}{\left(i\gamma_{th1}^{HD-O} \rho_P \lambda_{r,ps} + \rho_S \lambda_{1,uk}\right) \left(\gamma_{th1}^{HD-O} \rho_P \lambda_{1,ps} + \rho_R \lambda_{1,dk}\right)} \times \left(1 - \frac{i\gamma_{th1}^{HD-O}}{\rho_S \lambda_{1,uk}} - \frac{\gamma_{th1}^{HD-O}}{\rho_R \lambda_{1,dk}}\right) \quad (42)$$

With regard to the lower bound, when  $\rho_S, \rho_R, \rho_P \rightarrow \infty$ , the asymptotic outage probability of  $G_1$  with is obtained as

$$OP_{1-OMA}^{HD-floor} = 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \times \frac{\lambda_{1,uk} \lambda_{1,dk}}{\left(i\gamma_{th1}^{HD-O} \lambda_{r,ps} + \lambda_{1,uk}\right) \left(\gamma_{th1}^{HD-O} \lambda_{1,ps} + \lambda_{1,dk}\right)} \quad (43)$$

In a similar manner, we can obtain

$$OP_{2-OMA}^{HD-asym} = 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \times \frac{\rho_S \rho_R \lambda_{2,dk} \lambda_{2,uk}}{\left(i\gamma_{th2}^{HD-O} \rho_P \lambda_{r,ps} + \rho_S \lambda_{2,uk}\right) \left(\gamma_{th2}^{HD-O} \rho_P \lambda_{2ps} + \rho_R \lambda_{2,dk}\right)} \times \left(1 - \frac{i\gamma_{th2}^{HD-O}}{\rho_S \lambda_{2,uk}} - \frac{\gamma_{th2}^{HD-O}}{\rho_R \lambda_{2,dk}}\right) \quad (44)$$

and

$$OP_{2-OMA}^{HD-floor} = 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \times \frac{\lambda_{2,dk} \lambda_{2,uk}}{\left(i\gamma_{th2}^{HD-O} \lambda_{r,ps} + \lambda_{2,uk}\right) \left(\gamma_{th2}^{HD-O} \lambda_{2ps} + \lambda_{2,dk}\right)} \quad (45)$$

V. ERGODIC CAPACITY ANALYSIS FOR RELAY-AIDED CR-NOMA

A. THE FD RELAY-AIDED CR-NOMA

1) ERGODIC CAPACITY OF  $S_1$  TO  $D_1$

By using (1), (3), and (4), the achievable capacity associated with symbol  $x_1$  is given as

$$C_{G_1}^{FD} = \log_2 \left(1 + \min \left(\gamma_{x_1}^{S1-Rk*}, \gamma_{x_1}^{Rk*-D1}, \gamma_{x_1 \rightarrow x_2}^{Rk*-D2}\right)\right) \quad (46)$$

$$\bar{C}_{G_1}^{FD} = E \left(\log_2 (1 + H_1)\right) = \frac{1}{\ln 2} \int_0^\infty \frac{1 - F_{H_1}(x)}{1 + x} dx \quad (47)$$

where  $H_1 = \min \left(\gamma_{x_1}^{S1-Rk*}, \gamma_{x_1}^{Rk*-D1}, \gamma_{x_1 \rightarrow x_2}^{Rk*-D2}\right)$ . The CDF of  $H_1$  can be written as

$$F_{H_1}(x) = \Pr \left(\min \left(\gamma_{x_1}^{S1-Rk*}, \gamma_{x_1}^{Rk*-D1}, \gamma_{x_1 \rightarrow x_2}^{Rk*-D2}\right) < x\right) = 1 - \left(1 - F_{\gamma_{x_1}^{S1-Rk*}}(x)\right) \left(1 - F_{\gamma_{x_1}^{Rk*-D1}}(x)\right) \times \left(1 - F_{\gamma_{x_1 \rightarrow x_2}^{Rk*-D2}}(x)\right) \quad (48)$$

Base on (76),  $F_{\gamma_{x_1}^{S1-Rk*}}(x)$  can be written as

$$F_{\gamma_{x_1}^{S1-Rk*}}(x) = 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \times \frac{j\lambda_{1,uk} \lambda_{1,dk}}{\left(i\varepsilon_1 \lambda_{2,uk} x + j\lambda_{1,uk}\right) \left(i\varepsilon_2 \lambda_{r,ps} x + \lambda_{1,uk}\right)} \times \exp\left(-\frac{i\varepsilon_3^{FD} x}{\lambda_{1,uk}}\right) \quad (49)$$

where  $\varepsilon_1 = \frac{a_2}{a_1}, \varepsilon_2 = \frac{\rho_P}{a_1 \rho_S}, \varepsilon_3^{FD} = \frac{I_R + 1}{a_1 \rho_S}$ .

Based on (77),  $F_{\gamma_{x_1}^{Rk*-D1}}(x)$  can be written as

$$F_{\gamma_{x_1}^{Rk*-D1}}(x) = 1 - \frac{(a_3 \rho_R - a_4 \rho_R x) \lambda_{1,dk}}{\rho_P \lambda_{1,ps} x + (a_3 \rho_R - a_4 \rho_R x) \lambda_{1,dk}} \times \exp\left(-\frac{x}{(a_3 \rho_R - a_4 \rho_R x) \lambda_{1,dk}}\right) \quad (50)$$

From (82),  $F_{\gamma_{x_1 \rightarrow x_2}^{Rk*-D2}}(x)$  can be written as

$$F_{\gamma_{x_1 \rightarrow x_2}^{Rk*-D2}}(x) = 1 - \frac{(a_3 \rho_R - a_4 \rho_R x) \lambda_{2,dk}}{\rho_P \lambda_{2,ps} x + (a_3 \rho_R - a_4 \rho_R x) \lambda_{2,dk}} \times \exp\left(-\frac{x}{(a_3 \rho_R - a_4 \rho_R x) \lambda_{2,dk}}\right) \quad (51)$$

From (49)-(51),  $F_{H_1}(x)$  can be written as

$$F_{H_1}(x) = 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} q_1 q_2 q_3 \times \exp\left(-\frac{i\varepsilon_3^{FD} x}{\lambda_{1,uk}} - \frac{x}{(a_3 \rho_R - a_4 \rho_R x) \lambda_{1,dk}}\right) \quad (52)$$

where  $q_1 = \frac{j\lambda_{1,uk} \lambda_{1,dk}}{\left(i\varepsilon_1 \lambda_{2,uk} x + j\lambda_{1,uk}\right) \left(i\varepsilon_2 \lambda_{r,ps} x + \lambda_{1,uk}\right)}, q_2 = \frac{(a_3 \rho_R - a_4 \rho_R x) \lambda_{1,dk}}{\rho_P \lambda_{1,ps} x + (a_3 \rho_R - a_4 \rho_R x) \lambda_{1,dk}}, q_3 = \frac{(a_3 \rho_R - a_4 \rho_R x) \lambda_{2,dk}}{\rho_P \lambda_{2,ps} x + (a_3 \rho_R - a_4 \rho_R x) \lambda_{2,dk}}$ .

Then, the expression for the ergodic capacity of the  $G_1$  can be derived as

$$\begin{aligned} \bar{C}_{G_1}^{FD} &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \frac{1}{\ln 2} \\ &\times \int_0^{\frac{a_3}{a_4}} \frac{q_1 q_2 q_3}{1+x} \exp\left(-\frac{i\xi_3^{FD} x}{\lambda_{1,uk}} - \frac{x}{(a_3 \rho_R - a_4 \rho_R x) \lambda_{1,dk}}\right) dx. \end{aligned} \quad (53)$$

## 2) ERGODIC CAPACITY OF $S_2$ TO $D_2$

By using (2) and (5), the achievable capacity associated with symbol  $x_2$  is given as

$$C_{G_2-isc}^{FD} = \log_2 \left( 1 + \min \left( \gamma_{x_2}^{S1-Rk*}, \gamma_{x_2}^{Rk*-D2} \right) \right), \quad (54)$$

$$\begin{aligned} \bar{C}_{G_2-isc}^{FD} &= E \left( \log_2 (1 + H_2) \right) \\ &= \frac{1}{\ln 2} \int_0^\infty \frac{1 - F_{H_2}(x)}{1+x} dx, \end{aligned} \quad (55)$$

where  $H_2 = \min \left( \gamma_{x_2}^{S1-Rk*}, \gamma_{x_2}^{Rk*-D2} \right)$ . The cumulative distribution function (CDF) of  $H_2$  can be written as

$$\begin{aligned} F_{H_2}(x) &= \Pr \left( \min \left( \gamma_{x_2}^{S1-Rk*}, \gamma_{x_2}^{Rk*-D2} \right) < x \right) \\ &= 1 - \left( 1 - F_{\gamma_{x_2}^{S1-Rk*}}(x) \right) \left( 1 - F_{\gamma_{x_2}^{Rk*-D2}}(x) \right). \end{aligned} \quad (56)$$

Based on (79),  $F_{\gamma_{x_2}^{S1-Rk*}}(x)$  is obtained as

$$\begin{aligned} F_{\gamma_{x_2}^{S1-Rk*}}(x) &= 1 - \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\ &\times \frac{j \lambda_{2,uk} \lambda_{2,dk}}{(i \xi_1 \lambda_{1im} x + j \lambda_{2,uk}) (i \xi_2 \lambda_{r,ps} x + \lambda_{2,uk})} \\ &\times \exp\left(-\frac{i \xi_3^{FD} x}{\lambda_{2,uk}}\right), \end{aligned} \quad (57)$$

where  $\xi_1 = \frac{a_1}{a_2}$ ,  $\xi_2 = \frac{\rho p}{a_2 \rho_S}$  and  $\xi_3^{FD} = \frac{I_R + 1}{a_2 \rho_S}$ .

Similarly, from result of (80),  $F_{\gamma_{x_2}^{Rk*-D2}}(x)$  can be written as

$$\begin{aligned} F_{\gamma_{x_2}^{Rk*-D2}}(x) &= 1 - \frac{\lambda_{2,dk} \lambda_{2,dk}}{(v_1 \lambda_{2im} x + \lambda_{2,dk}) (v_2 \lambda_{2,ps} x + \lambda_{2,dk})} \\ &\times \exp\left(-\frac{x}{a_4 \rho_R \lambda_{2,dk}}\right). \end{aligned} \quad (58)$$

Finally, replacing (57) and (58) into (55), the expression for the ergodic capacity of the  $G_2$  can be derived as

$$\begin{aligned} \bar{C}_{G_2-isc}^{FD} &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \frac{1}{\ln 2} \\ &\times \int_0^\infty \frac{s_1 s_2}{1+x} \exp\left(-\frac{i \xi_3^{FD} x}{\lambda_{2,uk}} - \frac{x}{a_4 \rho_R \lambda_{2,dk}}\right) dx, \end{aligned} \quad (59)$$

where  $s_1 = \frac{j \lambda_{2,uk} \lambda_{2,dk}}{(i \xi_1 \lambda_{1im} x + j \lambda_{2,uk}) (i \xi_2 \lambda_{r,ps} x + \lambda_{2,uk})}$ ,  $s_2 = \frac{\lambda_{2,dk} \lambda_{2,dk}}{(v_1 \lambda_{2im} x + \lambda_{2,dk}) (v_2 \lambda_{2,ps} x + \lambda_{2,dk})}$ .

Now, by substituting  $\tau_1 = \tau_2 = 0$  in (59), the exact ergodic capacity of  $G_2$  under psic can be obtained as

$$\begin{aligned} \bar{C}_{G_2-psic}^{FD} &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \frac{1}{\ln 2} \\ &\times \int_0^\infty \frac{s_3 s_4}{1+x} \exp\left(-\frac{i \xi_3^{FD} x}{\lambda_{2,uk}} - \frac{x}{a_4 \rho_R \lambda_{2,dk}}\right) dx, \end{aligned} \quad (60)$$

where  $s_3 = \frac{\lambda_{2,uk}}{(i \xi_2 \lambda_{r,ps} x + \lambda_{2,uk})}$  and  $s_4 = \frac{\lambda_{2,dk}}{(v_2 \lambda_{2,ps} x + \lambda_{2,dk})}$ .

## B. THE HD RELAY-AIDED CR-NOMA

### 1) ERGODIC CAPACITY OF $S_1$ TO $D_1$

Similar to (47) and (53), when we have  $I_R = 0$ , the ergodic capacity of  $G_1$  in HD-NOMA is formulated by

$$\begin{aligned} \bar{C}_{G_1}^{HD} &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \frac{1}{2 \ln 2} \\ &\times \int_0^{\frac{a_3}{a_4}} \frac{q_1 q_2 q_3}{1+x} \exp\left(-\frac{i \xi_3^{HD} x}{\lambda_{1,uk}} - \frac{x}{(a_3 \rho_R - a_4 \rho_R x) \lambda_{1,dk}}\right) dx, \end{aligned} \quad (61)$$

where  $\xi_3^{HD} = \frac{1}{a_1 \rho_S}$ .

### 2) ERGODIC CAPACITY OF $S_2$ TO $D_2$

Similar to (55) and (59), by substituting  $I_R = 0$ , ergodic capacity of  $G_2$  under isic in HD-NOMA is given by

$$\begin{aligned} \bar{C}_{G_2-isc}^{HD} &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \frac{1}{2 \ln 2} \\ &\times \int_0^\infty \frac{s_1 s_2}{1+x} \exp\left(-\frac{i \xi_3^{HD} x}{\lambda_{2,uk}} - \frac{x}{a_4 \rho_R \lambda_{2,dk}}\right) dx, \end{aligned} \quad (62)$$

where  $\xi_3^{HD} = \frac{1}{a_2 \rho_S}$ .

Now, by substituting  $\tau_1 = \tau_2 = 0$  in (62), the exact ergodic capacity of  $G_2$  under isic can be obtained as

$$\begin{aligned} \bar{C}_{G_2-psic}^{HD} &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \frac{1}{2 \ln 2} \\ &\times \int_0^\infty \frac{s_3 s_4}{1+x} \exp\left(-\frac{i \xi_3^{HD} x}{\lambda_{2,uk}} - \frac{x}{a_4 \rho_R \lambda_{2,dk}}\right) dx. \end{aligned} \quad (63)$$

## VI. ERGODIC CAPACITY ANALYSIS FOR RELAY-AIDED CR-OMA

### A. THE FD RELAY-AIDED CR-OMA

#### 1) ERGODIC CAPACITY OF $S_1$ TO $D_1$

The exact ergodic capacity of  $G_1$  can be written as

$$C_{G_1-OMA}^{FD} = \frac{1}{2} \log_2 \left( 1 + \min \left( \gamma_{x1-OMA}^{S1-Rk*}, \gamma_{x1-OMA}^{Rk*-D1} \right) \right), \quad (64)$$

$$\begin{aligned} \bar{C}_{G_1-OMA}^{FD} &= E\left(\frac{1}{2}\log_2\left(1 + H_1^{OMA}\right)\right) \\ &= \frac{1}{2\ln 2} \int_0^\infty \frac{1 - F_{H_1^{OMA}}(x)}{1+x} dx, \end{aligned} \quad (65)$$

where  $H_1^{OMA} = \min(\gamma_{x1-OMA}^{S1-Rk*}, \gamma_{x1-OMA}^{Rk*-D1})$ . The CDF of  $H_1$  can be written as

$$\begin{aligned} F_{H_1^{OMA}}(x) &= \Pr\left(\min\left(\gamma_{x1-OMA}^{S1-Rk*}, \gamma_{x1-OMA}^{Rk*-D1}\right) < x\right) \\ &= 1 - \left(1 - F_{\gamma_{x1-OMA}^{S1-Rk*}}(x)\right)\left(1 - F_{\gamma_{x1-OMA}^{Rk*-D1}}(x)\right). \end{aligned} \quad (66)$$

Based on (85),  $F_{\gamma_{x1-OMA}^{S1-Rk*}}(x)$  is obtained as

$$\begin{aligned} F_{\gamma_{x1-OMA}^{S1-Rk*}}(x) &= 1 - \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \\ &\quad \times \frac{\rho_S \lambda_{1,uk}}{i\rho_P \lambda_r \rho_S x + \rho_S \lambda_{1,uk}} \exp\left(-\frac{ix}{\lambda_{1,uk}} \left(\frac{I_R}{\rho_S} + \frac{1}{\rho_S}\right)\right). \end{aligned} \quad (67)$$

Based on (86),  $F_{\gamma_{x1-OMA}^{Rk*-D1}}(x)$  is obtained as

$$\begin{aligned} F_{\gamma_{x1-OMA}^{Rk*-D1}}(x) &= 1 - \frac{\rho_R \lambda_{1,dk}}{\rho_P \lambda_{1,ps} x + \rho_R \lambda_{1,dk}} \\ &\quad \times \exp\left(-\frac{x}{\rho_R \lambda_{1,dk}}\right). \end{aligned} \quad (68)$$

The the ergodic capacity of the  $G_1$  is formulated as

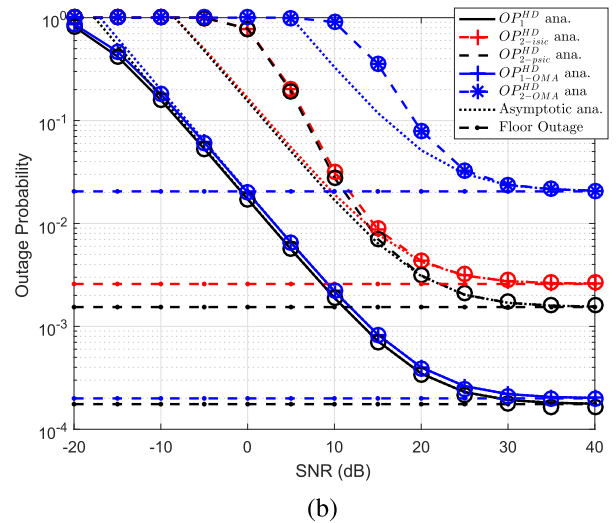
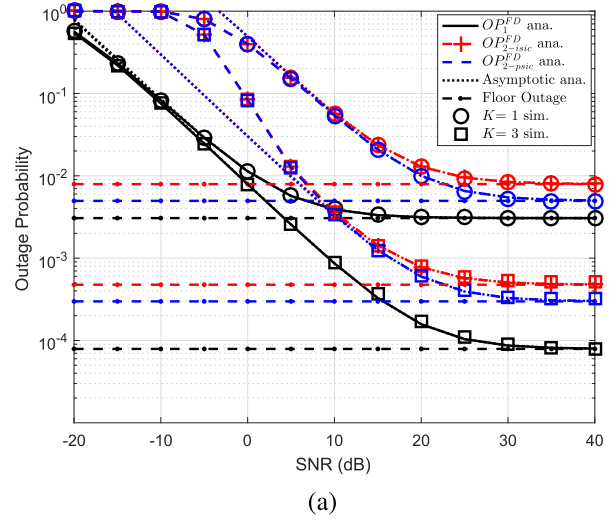
$$\begin{aligned} \bar{C}_{G_1-OMA}^{FD} &= \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \frac{1}{2\ln 2} \\ &\quad \times \int_0^\infty \frac{q_1^{OMA} q_2^{OMA}}{1+x} \exp\left(-\left(\frac{I_R}{\rho_S} + \frac{1}{\rho_S}\right) \frac{ix}{\lambda_{1,uk}} - \frac{x}{\rho_R \lambda_{1,dk}}\right) dx, \end{aligned} \quad (69)$$

where  $q_1^{OMA} = \frac{\rho_S \lambda_{1,uk}}{i\rho_P \lambda_r \rho_S x + \rho_S \lambda_{1,uk}}$  and  $q_2^{OMA} = \frac{\rho_R \lambda_{1,dk}}{\rho_P \lambda_{1,ps} x + \rho_R \lambda_{1,dk}}$ .

## 2) ERGODIC CAPACITY OF $S_2$ TO $D_2$

Similarly with computations of  $G_1$ , the closed-form expression for the ergodic capacity of the  $G_2$  can be derived as

$$\begin{aligned} \bar{C}_{G_2-OMA}^{FD} &= \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \frac{1}{2\ln 2} \\ &\quad \times \int_0^\infty \frac{q_3^{OMA} q_4^{OMA}}{1+x} \\ &\quad \times \exp\left(-\left(\frac{I_R}{\rho_S} + \frac{1}{\rho_S}\right) \frac{ix}{\lambda_{2,uk}} - \frac{x}{\rho_R \lambda_{2,dk}}\right) dx, \end{aligned} \quad (70)$$



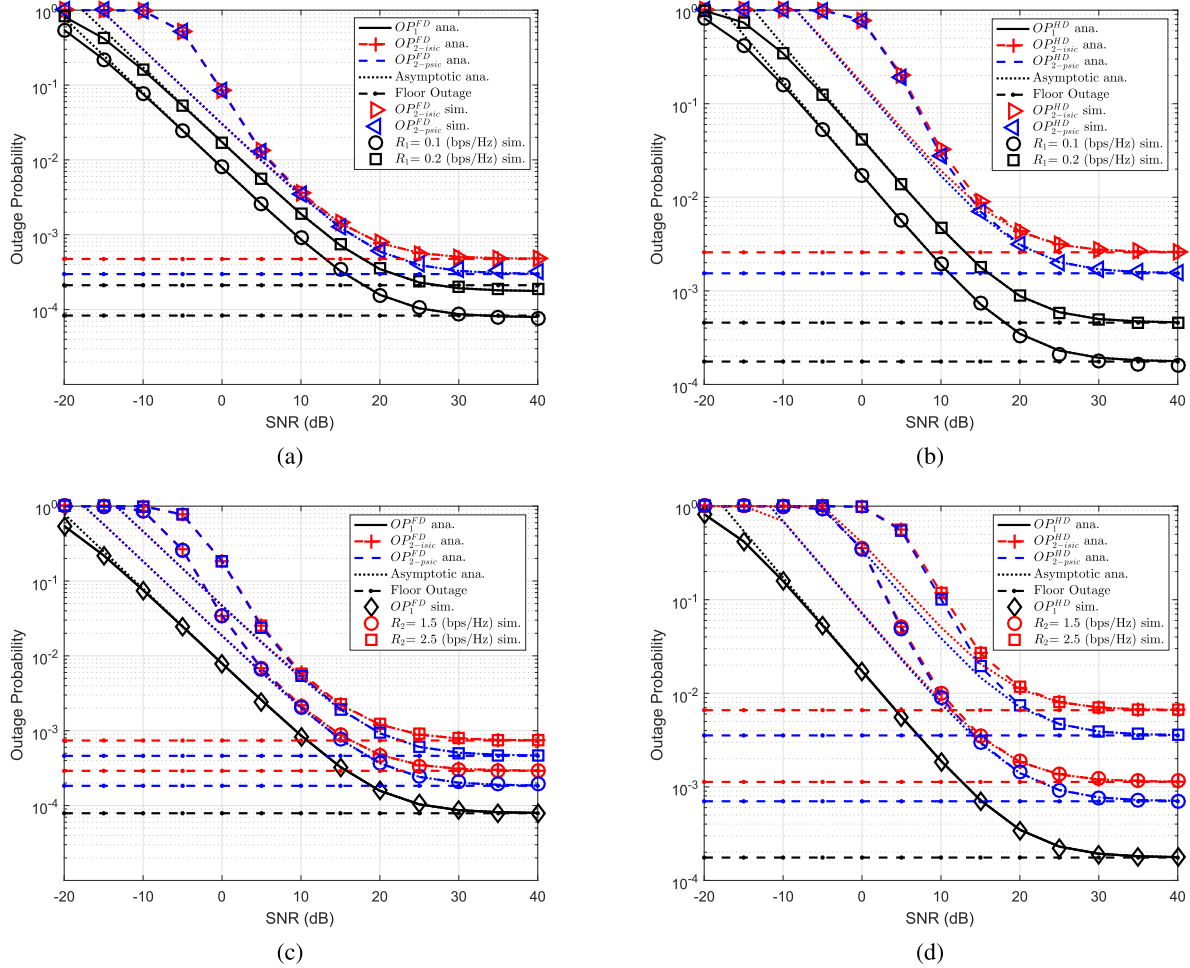
**FIGURE 2.** Comparison study on outage performance of  $D_1$  and  $D_2$  (imperfect SIC and perfect SIC) for varying  $K$ , where  $\alpha_1 = \alpha_3 = 0.6$ ,  $R_1 = 0.1$  (bps/Hz),  $R_2 = 2$  (bps/Hz). (a) FD relay-aided NOMA and relay-aided OMA, where  $K = 3$ .

where  $q_3^{OMA} = \frac{\rho_S \lambda_{2,uk}}{i\rho_P \lambda_r \rho_S x + \rho_S \lambda_{2,uk}}$  and  $q_4^{OMA} = \frac{\rho_R \lambda_{2,dk}}{\rho_P \lambda_{2,ps} x + \rho_R \lambda_{2,dk}}$ .

## B. THE HD RELAY-AIDED CR-OMA

Similar to (69) and (70), the condition  $I_R = 0$  results in ergodic capacity of  $G_1$  and  $G_2$  in HD-OMA respectively as

$$\begin{aligned} \bar{C}_{G_1-OMA}^{HD} &= \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \frac{1}{4\ln 2} \\ &\quad \times \int_0^\infty \frac{q_1^{OMA} q_2^{OMA}}{1+x} \exp\left(-\frac{ix}{\rho_S \lambda_{1,uk}} - \frac{x}{\rho_R \lambda_{1,dk}}\right) dx, \end{aligned} \quad (71)$$



**FIGURE 3.** Comparison study on outage performance of  $D_1$  and  $D_2$  (imperfect SIC and perfect SIC) for (a) FD relay-aided CR-NOMA for varying  $R_1$  ( $\sigma_1 = \sigma_3 = 0.6$ ,  $R_2 = 2$  (bps/Hz),  $I_R = -20$  (dB),  $K = 3$ ). (b) HD relay-aided CR-NOMA for varying  $R_1$  ( $\sigma_1 = \sigma_3 = 0.6$ ,  $R_2 = 2$  (bps/Hz),  $I_R = -20$  (dB),  $K = 3$ ). (c) FD relay-aided CR-NOMA for varying  $R_2$  ( $\sigma_1 = \sigma_3 = 0.6$ ,  $R_1 = 0.1$  (bps/Hz),  $I_R = -20$  (dB),  $K = 3$ ). (d) HD relay-aided CR-NOMA for varying  $R_2$  ( $\sigma_1 = \sigma_3 = 0.6$ ,  $R_1 = 0.1$  (bps/Hz),  $I_R = -20$  (dB),  $K = 3$ ).

and

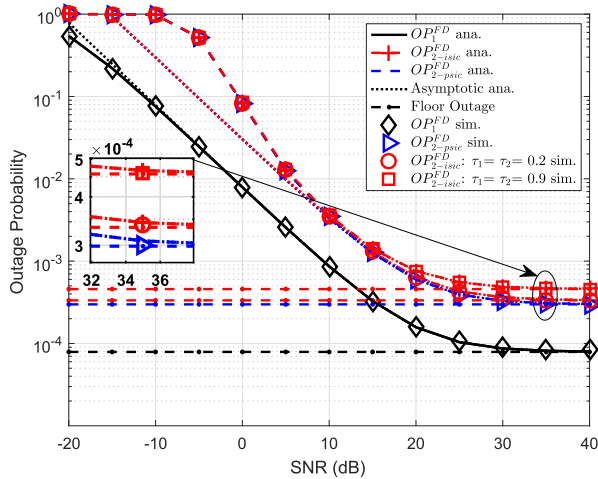
$$\begin{aligned} & \bar{C}_{G_2-OMA}^{HD} \\ &= \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \frac{1}{4 \ln 2} \\ & \times \int_0^\infty \frac{q_3^{OMA} q_4^{OMA}}{1+x} \exp\left(-\frac{ix}{\rho_S \lambda_{2,uk}} - \frac{x}{\rho_R \lambda_{2,dk}}\right) dx. \end{aligned} \quad (72)$$

**VII. NUMERICAL RESULTS**

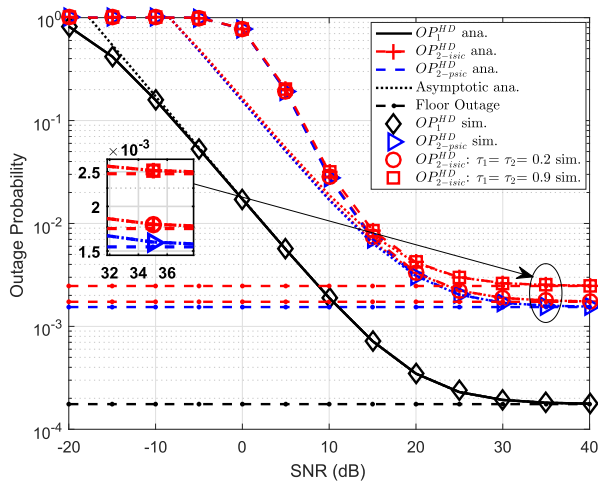
This section provides numerical results and relevant discussion. Unless stated otherwise, the simulation parameters and their values are given as follows  $\lambda_{1,uk} = d_{1,uk}^{-\nu}$ ,  $\lambda_{1,dk} = d_{1,dk}^{-\nu}$ ,  $\lambda_{2,uk} = d_{2,uk}^{-\nu}$ ,  $\lambda_{2,dk} = d_{2,dk}^{-\nu}$ . The path loss exponent is expressed by  $\nu = 4$  and  $d_{1,uk} = 0.25$ ,  $d_{2,uk} = 0.5$  and  $d_{2,dk} = 0.25$  denote the distance in meters from  $S_1$  to Relay, Relay to  $D_1$ ,  $S_2$  to Relay and Relay to  $D_2$ , respectively.  $\lambda_{r,ps} = \lambda_{1,ps} = \lambda_{2,ps} = 0.01$ .

Fig. 2 demonstrates outage probability as a function of increasing values of SNR of the system. It can be noticed that an increase in SNR reduces the probability of occurrence of an outage event. It can be observed in Fig. 2 (a) that the FD relay-aided CR-NOMA outperforms the conventional FD relay-aided CR-OMA. Moreover, as the values of  $K$  increase from 1 to 3, the overall outage probability drops significantly. Similarly, for HD relay-aided CR-NOMA in Fig. 2 (b), the outage probability also reduces. However, the gap between the curves of HD relays is high as compared to the gap between the curves for FD.

Fig. 3 illustrates the impact of different values of  $R_1$  and  $R_3$  for both FD and HD relay-aided CR-NOMA/ CR-OMA transmissions. Here, we can note that the simulation curves closely follow the analytical curves. This validates the accuracy of our derived closed-form expressions and asymptotic analysis. From Figs. 3 (a) & (b), we can observe that an increase in FD relay-aided NOMA network considerably outperforms the HD relay-aided CR-NOMA network. This is



(a)

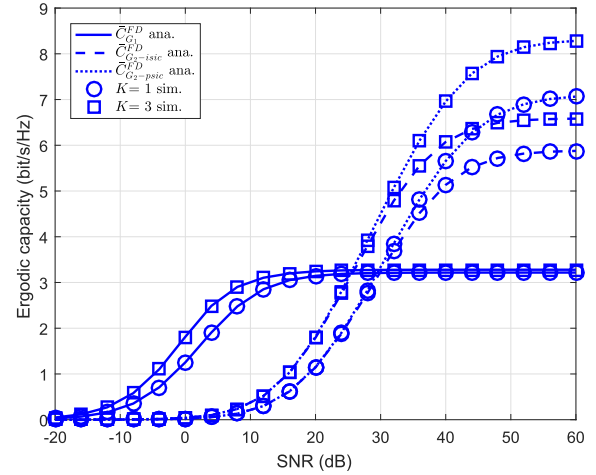


(b)

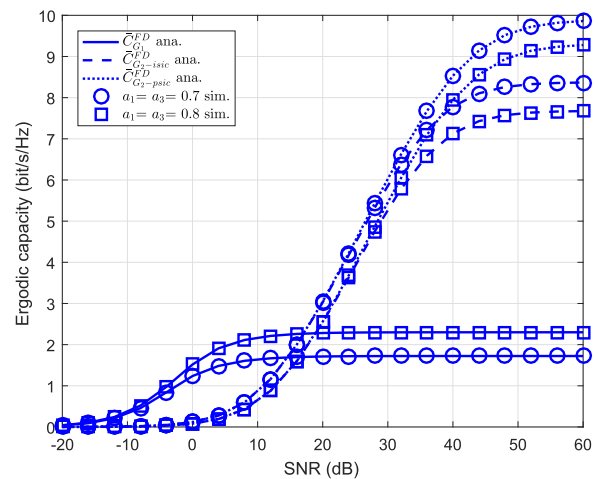
**FIGURE 4.** Comparison study on outage performance of  $D_1$  and  $D_2$  (imperfect SIC and perfect SIC) for varying  $\tau_1 = \tau_2$ , where  $(\alpha_1 = \alpha_3 = 0.6, R_1 = 0.1$  (bps/Hz),  $R_2 = 2$  (bps/Hz),  $I_R = -20$  (dB),  $K = 3$ ). (a) FD relay-aided CR-NOMA (b) HD relay-aided CR-NOMA.

especially true for smaller values of SNR, thus, indicating the utility of FD relay-aided CR-NOMA. Similar trends can be observed in Figs. 3 (c) & (d) where  $R_2$  is varied and  $R_1$  is kept constant. It can be noted that when  $R_2$  increases from 1.5 to 2.5, the overall outage performance increases significantly for the HD relay-aided CR-NOMA network. By contrast, the increase for FD relay-aided CR-NOMA is negligible, especially at higher values of SNR.

Fig. 4 demonstrates the outage probability for varying values of  $\tau_1 = \tau_2$  to clarify the impact of  $\tau_1$  and  $\tau_2$  on the outage performance. It can be observed that the FD relay-aided CR-NOMA network again outperforms the conventional FD relay-aided CR-NOMA. However, when comparing Figs. 4 (a) & (b), it becomes clear that a change in  $\tau_1 = \tau_2$  has more impact on the HD relay-aided CR-NOMA as compared to FD relay-aided CR-NOMA. Moreover, at higher values of SNR, the asymptotic results closely follow the analytical and simulation results that verify their accuracy.



**FIGURE 5.** Comparison study on ergodic capacity performance of  $D_1$  and  $D_2$  (imperfect SIC and perfect SIC) for FD relay-aided CR-NOMA, with varying  $K$ , where  $\alpha_1 = \alpha_3 = 0.9, I_R = 20$  (dB).

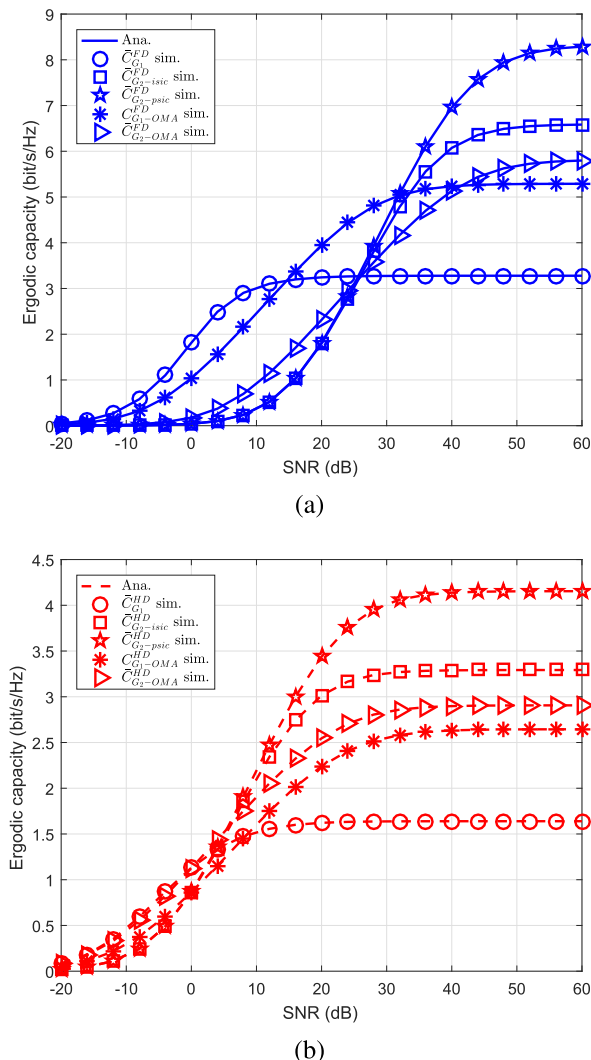


**FIGURE 6.** Comparison study on ergodic capacity performance of  $D_1$  and  $D_2$  (imperfect SIC and perfect SIC) for FD relay-aided CR-NOMA with varying  $\alpha_1 = \alpha_3$ , where  $K = 3, I_R = 20$  (dB).

Fig. 5 shows the ergodic capacity against the increasing values of SNR. It can be noted that an increase in the value of  $K$  generally results in increasing the ergodic capacity of the system. However, this impact is far more significant for  $G_2$  as compared to  $G_1$ . More specifically, at high SNR, the distance between the curves of  $G_1$  becomes negligible for different values of  $K$ . Whereas, for  $G_2$ , the distance between different curves of  $K$  grows indicating the higher impact of change in  $K$ .

To illustrate the effects of  $\alpha_1$  and  $\alpha_3$  on the ergodic capacity of FD relay-aided CR-NOMA networks, Fig. 6 shows the ergodic capacity for different values of  $\alpha_1 = \alpha_3$ . It can be seen that the performance for imperfect SIC scenario is significantly lower than that for the perfect SIC scenario. Moreover, an increase in the values of  $\alpha_1$  and  $\alpha_3$  results in decreasing the performance of the network. The analytical results closely follow the simulations that validates the analytical expressions.





**FIGURE 7.** Comparison study on ergodic capacity performance of  $D_1$  and  $D_2$  (imperfect SIC and perfect SIC) for HD/FD relay-aided CR-NOMA and relay-aided CR-OMA, where  $\alpha_1 = \alpha_3 = 0.9$ ,  $K = 3$ ,  $I_R = 20$  (dB). (a) FD relay-aided NOMA and relay-aided OMA. (b) HD relay-aided NOMA and relay-aided OMA.

Fig. 7 presents a comparative analysis of FD and HD relay-aided CR-NOMA network against FD and HD relay-aided CR-OMA network. It can be noted that the ergodic capacity of  $G_1$  is better than  $G_2$  at higher values of SNR. Moreover, it is also worth highlighting that the FD relay aided the CR-NOMA network outperforms the HD relay aided CR-OMA network. The difference between FD and HD increases more at higher values of SNR. The ceiling is reached at very high SNR values that are because of the limiting effect of interference.

**VIII. CONCLUSION**

Relay-aided CR-NOMA networks are going to play a critical role in the beyond 5G networks. In this regard, this paper has provided a performance analysis for FD and HD relay-aided CR-NOMA networks. Specifically, the closed-form expressions for FD and HD relay-aided CR-NOMA networks have

been derived by considering the PRS framework. Besides, the analytical expressions for ergodic capacity and asymptotic outage probabilities are also derived to provide more insights on such networks. The results demonstrate that the changes in the outage probability of the FD relay-aided CR-NOMA network are negligible, especially at higher values of SNR. Moreover, the difference between FD and HD ergodic capacity increases more at higher values of SNR and a ceiling is reached at very high SNR values due to the effect of interference. We anticipate that the analytical expressions provided in this work will be helpful in the practical realization of relay-aided CR-NOMA networks.

**APPENDIX A  
PROOF OF THE PROPOSITION 1**

By implementing order statistics of separated components,  $OP_{1-isc}^{FD}$  can be obtained as

$$OP_{1-isc}^{FD} = 1 - \underbrace{\Pr\left(\gamma_{x1}^{S1-Rk*} \geq \gamma_{th1}^{FD}\right)}_{A_1} \times \underbrace{\Pr\left(\gamma_{x1}^{Rk*-D1} \geq \gamma_{th1}^{FD}\right)}_{A_2}. \quad (73)$$

To compute such outage,  $A_1$  can be first calculated as

$$\begin{aligned} A_1 &= \Pr\left(\frac{\alpha_1 \rho_S |g_{1,uk*}|^2}{\alpha_2 \rho_S |g_{2,uk*}|^2 + \rho_P |g_{r,ps}|^2 + I_R + 1} \geq \gamma_{th1}^{FD}\right) \\ &= \Pr\left(|g_{1,uk*}|^2 \geq \alpha_2^{FD} |g_{2,uk*}|^2 + \alpha_3^{FD} |g_{r,ps}|^2 + \alpha_4^{FD}\right) \\ &= \int_0^\infty \int_0^\infty \left(1 - F_{|g_{1,uk*}|^2}(\alpha_2^{FD} x + \alpha_3^{FD} y + \alpha_4^{FD})\right) \\ &\quad \times f_{|g_{2,uk*}|^2}(x) f_{|g_{r,ps}|^2}(y) dy. \end{aligned} \quad (74)$$

By using partial integration and the CDF and probability density function (PDF),  $A_1$  can be first calculated as

$$\begin{aligned} A_1 &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \frac{j}{\lambda_{2,uk}} \frac{1}{\lambda_{r,ps}} \\ &\quad \times \exp\left(-\frac{i\alpha_4^{FD}}{\lambda_{1,uk}}\right) \int_0^\infty \int_0^\infty \exp\left(-\left(\frac{i\alpha_2^{FD}}{\lambda_{1,uk}} + \frac{j}{\lambda_{2,uk}}\right)x\right) dx \\ &\quad \times \exp\left(-\left(\frac{i\alpha_3^{FD}}{\lambda_{1,uk}} + \frac{1}{\lambda_{r,ps}}\right)y\right) dy \\ &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \frac{j}{\lambda_{2,uk}} \frac{1}{\lambda_{r,ps}} \\ &\quad \times \exp\left(-\frac{i\alpha_4^{FD}}{\lambda_{1,uk}}\right) \int_0^\infty \exp\left(-\left(\frac{i\alpha_2^{FD}}{\lambda_{1,uk}} + \frac{j}{\lambda_{2,uk}}\right)x\right) dx \\ &\quad \times \int_0^\infty \exp\left(-\left(\frac{i\alpha_3^{FD}}{\lambda_{1,uk}} + \frac{1}{\lambda_{r,ps}}\right)y\right) dy \end{aligned}$$

$$\begin{aligned}
 &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\quad \times \frac{j\lambda_{1,uk}\lambda_{1,uk}}{(i\alpha_2^{FD}\lambda_{2,uk} + j\lambda_{1,uk})(i\alpha_3^{FD}\lambda_{r,ps} + \lambda_{1,uk})} \\
 &\quad \times \exp\left(-\frac{i\alpha_4^{FD}}{\lambda_{1,uk}}\right). \tag{75}
 \end{aligned}$$

Besides,  $A_2$  can be expressed by

$$\begin{aligned}
 A_2 &= \Pr\left(\frac{a_3\rho_R|h_{1,dk}|^2}{a_4\rho_R|h_{1,dk}|^2 + \rho_P|h_{1,ps}|^2 + 1} \geq \gamma_{th1}^{FD}\right) \\
 &= \Pr\left(|h_{1,dk}|^2 \geq \frac{\gamma_{th1}^{FD}\rho_P|h_{1,ps}|^2 + \gamma_{th1}^{FD}}{a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R}\right). \tag{76}
 \end{aligned}$$

It is worth noting that  $A_2$  satisfies  $a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R > 0 \rightarrow a_3 > \gamma_{th1}^{FD}a_4$ , then it can be further computed as

$$\begin{aligned}
 A_2 &= \int_0^\infty \left(1 - F_{|h_{1,dk}|^2}\left(\frac{\gamma_{th1}^{FD}\rho_P x + \gamma_{th1}^{FD}}{a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R}\right)\right) \\
 &\quad \times f_{|h_{1,ps}|^2}(x) dx \\
 &= \int_0^\infty \exp\left(-\frac{\gamma_{th1}^{FD}\rho_P x + \gamma_{th1}^{FD}}{(a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{1,dk}}\right) \frac{1}{\lambda_{1,ps}} \\
 &\quad \times \exp\left(-\frac{x}{\lambda_{1,ps}}\right) dx \\
 &= \frac{(a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{1,dk}}{\gamma_{th1}^{FD}\rho_P\lambda_{1,ps} + (a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{1,dk}} \\
 &\quad \times \exp\left(-\frac{\gamma_{th1}^{FD}}{(a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{1,dk}}\right). \tag{77}
 \end{aligned}$$

Finally, replacing (74) and (77) into (73), the outage probability for  $D_2$  in this mode can be computed.

It completes the proof.

### APPENDIX B

#### PROOF OF THE PROPOSITION 2

Similarly,  $OP_{2-isic}^{FD}$  will be processed as

$$\begin{aligned}
 OP_{2-isic}^{FD} &= 1 - \underbrace{\Pr\left(\gamma_{x2}^{S1-Rk*} \geq \gamma_{th2}^{FD}\right)}_{B_1} \\
 &\quad \times \underbrace{\Pr\left(\gamma_{x2}^{Rk*-D2} \geq \gamma_{th2}^{FD}\right)}_{B_2} \times \underbrace{\Pr\left(\gamma_{x1 \rightarrow x2}^{Rk*-D2} \geq \gamma_{th1}^{FD}\right)}_{B_3}. \tag{78}
 \end{aligned}$$

To compute such outage,  $B_1$  can be first calculated as

$$\begin{aligned}
 B_1 &= \Pr\left(\gamma_{x2}^{S1-Rk*} \geq \gamma_{th2}^{FD}\right) \\
 &= \Pr\left(\frac{a_2\rho_S|g_{2,uk*}|^2}{a_1\rho_S|g_{1,uk*}|^2 + \rho_P|g_{r,ps}|^2 + I_R + 1} \geq \gamma_{th2}^{FD}\right) \\
 &= \Pr\left(|g_{2,uk*}|^2 \geq \frac{\gamma_{th2}^{FD}a_1\rho_S}{a_2\rho_S}|g_{1,uk*}|^2 + \frac{\gamma_{th2}^{FD}\rho_P}{a_2\rho_S}|g_{r,ps}|^2\right) \\
 &\quad + \frac{\gamma_{th2}^{FD}(I_R+1)}{a_2\rho_S}. \tag{79}
 \end{aligned}$$

Similar to  $A_1$ ,  $B_1$  can be simplified as

$$\begin{aligned}
 B_1 &= \sum_{i=1}^K \sum_{j=1}^K \binom{K}{i} \binom{K}{j} (-1)^{i+j-2} \\
 &\quad \times \frac{j\lambda_{2,uk}\lambda_{2,uk}}{(i\beta_2^{FD}\lambda_{1im} + j\lambda_{2,uk})(i\beta_3^{FD}\lambda_{r,ps} + \lambda_{2,uk})} \\
 &\quad \times \exp\left(-\frac{i\beta_4^{FD}}{\lambda_{2,uk}}\right). \tag{80}
 \end{aligned}$$

Then,  $B_2$  is formulated as

$$\begin{aligned}
 B_2 &= \Pr\left(\frac{a_4\rho_R|h_{2,dk}|^2}{a_3\rho_R|h_{2,dk}|^2 + \rho_P|h_{2,ps}|^2 + 1} \geq \gamma_{th2}^{FD}\right) \\
 &= \Pr\left(|h_{2,dk}|^2 \geq \frac{m_1^{FD}|h_{2,dk}|^2 + m_2^{FD}|h_{2,ps}|^2 + m_3^{FD}}{a_3\rho_R - \gamma_{th2}^{FD}a_4\rho_R}\right) \\
 &= \int_0^\infty \int_0^\infty \left(1 - F_{|h_{2,dk}|^2}\left(m_1^{FD}x + m_2^{FD}y + m_3^{FD}\right)\right) \\
 &\quad \times f_{|h_{2,dk}|^2}(x) dx f_{|h_{2,ps}|^2}(y) dy \\
 &= \frac{1}{\lambda_{2im}} \frac{1}{\lambda_{2,ps}} \exp\left(-\frac{m_3^{FD}}{\lambda_{2,dk}}\right) \\
 &\quad \times \int_0^\infty \exp\left(-\left(\frac{m_1^{FD}}{\lambda_{2,dk}} + \frac{1}{\lambda_{2im}}\right)x\right) dx \\
 &\quad \times \int_0^\infty \exp\left(-\left(\frac{m_2^{FD}}{\lambda_{2,dk}} + \frac{1}{\lambda_{2,ps}}\right)y\right) dy \\
 &= \frac{\lambda_{2,dk}\lambda_{2,dk}}{(m_1^{FD}\lambda_{2im} + \lambda_{2,dk})(m_2^{FD}\lambda_{2,ps} + \lambda_{2,dk})} \exp\left(-\frac{m_3^{FD}}{\lambda_{2,dk}}\right). \tag{81}
 \end{aligned}$$

Next,  $B_3$  can be computed as

$$\begin{aligned}
 B_3 &= \Pr\left(\frac{a_3\rho_R|h_{2,dk}|^2}{a_4\rho_R|h_{2,dk}|^2 + \rho_P|h_{2,ps}|^2 + 1} \geq \gamma_{th1}^{FD}\right) \\
 &= \Pr\left(|h_{2,dk}|^2 \geq \frac{\gamma_{th1}^{FD}\rho_P|h_{2,ps}|^2 + \gamma_{th1}^{FD}}{a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R}\right). \tag{82}
 \end{aligned}$$

Noticing that  $B_3$  satisfies  $a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R > 0 \rightarrow a_3 > \gamma_{th1}^{FD}a_4$ , then  $B_3$  is given as

$$\begin{aligned}
 B_3 &= \int_0^\infty \left(1 - F_{|h_{2,dk}|^2}\left(\frac{\gamma_{th1}^{FD}\rho_P x + \gamma_{th1}^{FD}}{a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R}\right)\right) f_{|h_{2,ps}|^2}(x) dx \\
 &= \int_0^\infty \exp\left(-\frac{\gamma_{th1}^{FD}\rho_P x + \gamma_{th1}^{FD}}{(a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{2,dk}}\right) \frac{1}{\lambda_{2,ps}} \\
 &\quad \times \exp\left(-\frac{x}{\lambda_{2,ps}}\right) dx \\
 &= \frac{(a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{2,dk}}{\gamma_{th1}^{FD}\rho_P\lambda_{2,ps} + (a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{2,dk}} \\
 &\quad \times \exp\left(-\frac{\gamma_{th1}^{FD}}{(a_3\rho_R - \gamma_{th1}^{FD}a_4\rho_R)\lambda_{2,dk}}\right). \tag{83}
 \end{aligned}$$

Finally, replacing (80), (81) and (83) into (78), the outage probability for  $D_2$  in this mode can be computed.

It completes the proof.

## APPENDIX C

### PROOF OF THE PROPOSITION 3

$OP_{1-OMA}^{FD}$  is processed as

$$OP_{1-OMA}^{FD} = 1 - \underbrace{\Pr\left(\gamma_{x1-OMA}^{S1-Rk*} \geq \gamma_{th1}^{FD-O}\right)}_{C_1} \times \underbrace{\Pr\left(\gamma_{x1-OMA}^{Rk*-D1} \geq \gamma_{th1}^{FD-O}\right)}_{C_2}. \quad (84)$$

To compute such outage probability,  $C_1$  is calculated as

$$\begin{aligned} C_1 &= \Pr\left(\frac{\rho_S |g_{1,uk*}|^2}{\rho_P |g_{r,ps}|^2 + I_R + 1} \geq \gamma_{th1}^{FD-O}\right) \\ &= \Pr\left(|g_{1,uk*}|^2 \geq \frac{\gamma_{th1}^{FD-O} \rho_P}{\rho_S} |g_{r,ps}|^2 + \frac{\gamma_{th1}^{FD-O} I_R}{\rho_S} + \frac{\gamma_{th1}^{FD-O}}{\rho_S}\right) \\ &= \int_0^\infty \left(1 - F_{|g_{1,uk*}|^2}\left(\frac{\gamma_{th1}^{FD-O} \rho_P}{\rho_S} x + \frac{\gamma_{th1}^{FD-O} I_R}{\rho_S} + \frac{\gamma_{th1}^{FD-O}}{\rho_S}\right)\right) \\ &\quad \times f_{|g_{r,ps}|^2}(x) dx \\ &= \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \exp\left(-\frac{i}{\lambda_{1,uk}} \left(\frac{\gamma_{th1}^{FD-O} I_R}{\rho_S} + \frac{\gamma_{th1}^{FD-O}}{\rho_S}\right)\right) \\ &\quad \times \frac{1}{\lambda_{r,ps}} \int_0^\infty \exp\left(-\left(\frac{i \gamma_{th1}^{FD-O} \rho_P}{\rho_S \lambda_{1,uk}} + \frac{1}{\lambda_{r,ps}}\right) x\right) dx \\ &= \sum_{i=1}^K \binom{K}{i} (-1)^{i-1} \frac{\rho_S \lambda_{1,uk}}{i \gamma_{th1}^{FD-O} \rho_P \lambda_{r,ps} + \rho_S \lambda_{1,uk}} \\ &\quad \times \exp\left(-\frac{i}{\lambda_{1,uk}} \left(\frac{\gamma_{th1}^{FD-O} I_R}{\rho_S} + \frac{\gamma_{th1}^{FD-O}}{\rho_S}\right)\right). \quad (85) \end{aligned}$$

Then,  $C_2$  is expressed by

$$\begin{aligned} C_2 &= \Pr\left(\frac{\rho_R |h_{1,dk}|^2}{\rho_P |h_{1,ps}|^2 + 1} \geq \gamma_{th1}^{FD-O}\right) \\ &= \Pr\left(|h_{1,dk}|^2 \geq \frac{\gamma_{th1}^{FD-O} \rho_P}{\rho_R} |h_{1,ps}|^2 + \frac{\gamma_{th1}^{FD-O}}{\rho_R}\right) \\ &= \int_0^\infty \left(1 - F_{|h_{1,dk}|^2}\left(\frac{\gamma_{th1}^{FD-O} \rho_P}{\rho_R} x + \frac{\gamma_{th1}^{FD-O}}{\rho_R}\right)\right) \\ &\quad \times f_{|h_{1,ps}|^2}(x) dx \end{aligned}$$

$$\begin{aligned} &= \int_0^\infty \exp\left(-\frac{1}{\lambda_{1,dk}} \left(\frac{\gamma_{th1}^{FD-O} \rho_P}{\rho_R} x + \frac{\gamma_{th1}^{FD-O}}{\rho_R}\right)\right) \\ &\quad \times \frac{1}{\lambda_{1,ps}} \exp\left(-\frac{x}{\lambda_{1,ps}}\right) dx \\ &= \frac{\rho_R \lambda_{1,dk}}{\gamma_{th1}^{FD-O} \rho_P \lambda_{1,ps} + \rho_R \lambda_{1,dk}} \exp\left(-\frac{\gamma_{th1}^{FD-O}}{\rho_R \lambda_{1,dk}}\right). \quad (86) \end{aligned}$$

Finally, substituting (85) and (86) into (84), the outage behavior for  $D_2$  can be computed.

It completes the proof.

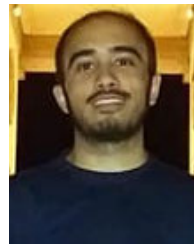
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