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Active Currents, Power Factor, and Apparent Power for Practical Power Delivery Systems

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ABSTRACT Concepts of apparent power and power factor as measures of a system's power delivery capability are over a century old but have not been defined in one general, rigorous and acceptable way. Instantaneous power is defined precisely, and average power measured over a selected period is widely accepted. The many ways of defining and measuring reactive and apparent power in single and three phase systems are based on different assumptions and give different results in real cases. Building on definitions in the IEEE Standard 1459-2010, this paper formulates in vector space linear algebra and the frequency domain, the active wire currents as those that cause the minimum losses in a network for the power delivered. Power factor measures the relative efficiency of power delivery as defined by the losses. Apparent power consistent with early terminology is the maximum power that can be sourced for the same original line losses and has the unit of power: Watt. It is identified without requiring the contentious concept of reactive and non-active power components. Measurements based on this approach are independent of assumptions about sinusoidal waveform, voltage and current balance, and frequency-dependent wire resistances, and apply to power delivery systems with any number of wires. The rigor of this novel general formulation is important for technical design of compensators and inverters; analyzing power system losses, delivery efficiency and voltage stability; and electricity cost allocation and pricing.

INDEX TERMS Active current, apparent power, harmonics, power theory, representational measurement, unbalance.

I. INTRODUCTION

Power theory has scientific, engineering and economic relevance. It identifies the relationships between parameters of power systems, such as voltages, currents, delivered power and the losses incurred, as explored in the literature review section. Despite the need for it, no general power theory adequately copes yet with the conditions of real power delivery systems. For example, many different definitions of reactive power, power factor (PF), and apparent power (AP) have been proposed in thousands of papers and engineering standards and implemented in meters and tariffs – but without general agreement and consistent technical rigor. The many different power theory formulations are based on different simplifications and assumptions that are violated in practical systems. Examples of violations include having a different number of wires from that used in formulating the definition of measurement, unbalanced voltages or currents instead of balanced, waveforms distorted by variable combinations of harmonics instead of only fundamental frequency sinusoids, and practical line impedances different from those assumed.

This paper presents a new, more general formulation of power theory for any poly-phase systems, including a neutral or without it, with any periodic non-sinusoidal waveforms and dc components, voltage and current unbalance, and unequal and/or frequency-dependent wire impedances.

The formulation is derived entirely in elementary vector space linear algebra [1]. It maintains rigorously the frame of reference of measurements and compliance with Kirchhoff's voltage and current laws and the law of Conservation of Energy. The calculations of a measurement are arithmetic and implemented easily in a simple spreadsheet, processor or program.

This power theory builds on two key interpretations of PF relative to power and losses in the IEEE Standard 1459-2010 [2]: that "unity power factor means minimum possible

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line losses for a given total active power transmitted'', and ''... the ratio (PF=) P/S is a utilization factor indicator''. These lead to a reformulation of the definition of AP, for which the units are shown to be Watts, as the maximum power that can be transmitted from the source to a specific point of a network for the same delivery loss. Further, it is shown that the widely used reactive power concept is incompatible with real systems with unbalanced and distorted waveforms.

Since the PF and AP can be identified with measurement made at a point of connection or common coupling (PCC) of a power supply to a load or other network, this power theory has many potential applications, such as in converters, loss reduction in delivery systems and loads, dispatching of embedded or distributed generation (DG) into systems, and in electricity trading and tariffs.

In the rest of the paper, Section II locates the challenge of a general definition in the context of power delivery, and Section III reviews some of the extensive literature describing various approaches to defining and measuring AP. Sections IV presents the theory development in two parts: as a transport problem of delivering power most efficiently to or from the source (identified in a Thévenin equivalent circuit) to the PCC. Section V extends the development with the concept of a controllable power processor such as a power electronic converter. Section VI demonstrates the arithmetic process of measurement in three examples with unbalanced voltages and currents and distortion. Section VII discusses the interpretation, implications and potential applications of the general power theory, and there is a brief conclusion in Section VIII.

II. PROBLEM STATEMENT

Electrical engineers recognized at an early stage that, compared with dc systems, loads supplied by ac were affected by an extra loss of power in the cables [3] and, later, that this depended on frequency, waveform distortion and unbalance. Bell [4, Discussion] identified the need to charge for capacity and not only energy when the PF is low, to which Steinmetz replied ''...it would be desirable to charge for a part of the wattless currents, a part sufficiently large to take care of the losses due to resistance of lines and transformers and generator capacity'' [4, Discussion]. Silsbee [5] wrote about the economic importance of capacity cost and the cost of losses for which it is important to define PF correctly in the presence of ''phase displacement, unbalance and wave form''. Much more recently, the IEEE Std 1459-2010 [2] identified the need to make accurate measurements so that the cost of maintaining the quality of electricity service could be distributed fairly.

The common needs expressed by all these writers can be represented by a source, a delivery system with any (real, >1) number M of wires (with or without a neutral wire) and a load, depicted in Fig. 1 by the Thévenin equivalent circuit. Further, consider that the currents drawn at the PCC of the delivery system and load are the result of unbalanced voltages and delivery system impedances and distorted by harmonic components, so that the waveforms of the currents and voltages at the PCC are such as depicted in Fig. 2. The following questions arise:

1) What re-distribution of the currents between wires and harmonics delivers the power most efficiently (with the lowest losses) to the PCC?
2) What AP or PF can be identified with this load as suggested by Bell, Steinmetz and Silsbee?
3) What if the load is replaced by a source feeding into the system instead of drawing currents from it?
4) What are the implications of the answers to these questions?

These questions have not yet been answered adequately in 100 years since the problems were first identified.

Note: For consistency, we refer to phases or conductors (including a ground neutral) as wires, and the elements of a Thévenin equivalent circuit as lines.

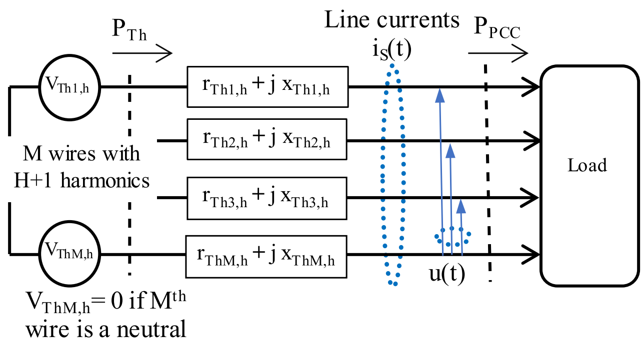

FIGURE 1. Dispatch of power P_{Th} from a Thévenin equivalent source through M x (H+1) lines representing a system of M wires with H+1 harmonics to deliver P_{PCC} at the PCC.

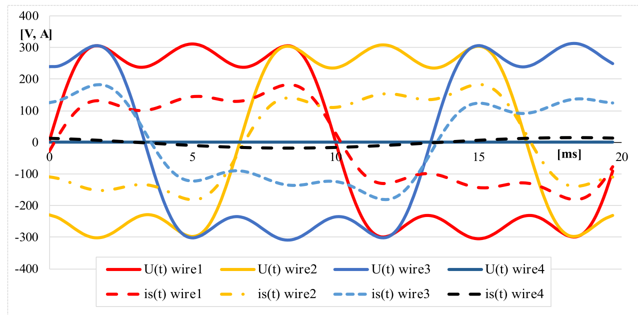

FIGURE 2. Wire current and voltage waveforms at the PCC, with dc and harmonic components, and v-i waveform phase-displacement of the fundamental frequency components.

III. LITERATURE REVIEW

In this section we review four aspects: recent papers in measurement theory; progress during the past decade towards determining active currents and AP; some of our previous research; and some approaches in the frequency domain.

A. RECENT MEASUREMENT THEORY

Representational measurements require close association between the concept model and the physical components being measured [6], so some power measurement problems can be attributed to an inadequate definition for reactive power measurement when the waveforms are no longer pure sinusoids [7]. Too many constraints or assumptions inherent in the definition of a parameter make it likely that measurements are operational and, in practice, violating the definition's conditions leads to invalid measurements.

Although not expressed in the context of representational measurement, Ghassemi [8] identified the requirements for a rigorous definition of electrical power parameters as having to include applicability to systems of all topologies and conditions, with any number of phases, balanced or unbalanced, sinusoidal or distorted waveforms, and without violating any principle of electrical engineering.

B. POWER PARAMETERS RESEARCH DURING THE PAST 10 YEARS

For many years, a rigorous, comprehensive power theory of reactive power, PF and AP has been a controversial topic, even in single phase systems [9]. The IEEE Standard 1459-2010 [2] might have resolved the problems, but it too left several issues unresolved [10], [11].

In 2011, a review of definitions of various power parameters used in meters [12] identified eleven formulae for the calculation and measurement of reactive power (termed VAR) in single-phase elements. It was shown by testing with 14 different waveforms that the measured VAR varied widely, differing even in sign between some approaches. The AP in single phase elements was calculated by three methods. In three-phase supplies, the AP and reactive power are the sum of the single elements, or a vector sum of the power, AP and VAR as calculated by a method specified by the manufacturer. To summarize, the survey found many different methods being used to measure reactive power and AP and they gave different measurement results. None of the measurements included the resistance of the wires of the delivery system.

We have scanned many (>200) of the apparently most relevant papers published since 2014 and most appear to follow three well-established approaches: that of Current Physical Components (CPC) proposed by Czarnecki, or Conservative Power Theory (CPT) following Tenti, or variations of the p-q instantaneous power theory developed by Akagi.

The CPC power theory identifies active, reactive, scattered, load generated, and three unbalanced sequence current components [13]. Various applications are constrained by different assumptions, such as loads being linear and time-invariant, or by decomposition into symmetrical components of a three-phase system [14]–[16]. Some characteristics of the CPT approaches, with five or six component currents [17], [18], are like the CPC theory and suffer from similar limitations. Despite the limitations requiring a modified formulation according to each system topology, CPT and CPC approaches have been applied recently to filter design [19] and to reactive compensation [16]. The IEEE Std 1459-2010 [2] adopted a similar approach in identifying several reactive and distortion power components.

A recent formulation in geometric algebra [20] proposed a different set of apparent power components.

The p-q theory was developed to define instantaneous power. None of the more recent p-q approaches appears to have addressed fully the limitations of its assumptions. An instantaneous approach to mitigate harmonic distortion [21] is interesting for its depiction of the distorted grid but a solution that does not depend on the grid parameters. To overcome the limitations of the instantaneous power theory, the Lagrange multiplier technique has been used to identify the active current using digital signal processors [22]. This appears to be one of relatively few power theory approaches tested in hardware and not only simulation, and although the control algorithms were promising they could not "keep the grid currents balanced". The work illustrates the difference between grid current balancing and the compensation of non-active power, which is discussed in Section V.

Some of the papers reviewed propose alternative methods for specific studies, including applications of optimization algorithms, fuzzy controllers or neural networks (comprehensively reviewed in [23]), but do not appear to have been followed up with general approaches. Such approaches cannot reflect the structure and physics of the system and cannot qualify as measurements or definitions of any parameters other than as defined in the artificial intelligence approach used.

In all the papers reviewed, the constraints on the various definitions, by the number and resistances of phase and neutral wires or by specified non-physical current components or by any other assumptions, limit all the approaches to being operational measurements of PF and AP.

C. GENERAL POWER THEORY IN TIME DOMAIN

We proposed a general power theory developed in linear algebra for multiple wire systems with consistency between the instantaneous and average time domain [24]–[26] and complying with all the requirements identified by Ghassemi [8]. We reviewed in the paper on instantaneous power [24] the various proposals of Akagi, Buchholtz, Dai, Depenbrock, Ferrero, Fryze, Nabae, Peng, Rossetto, Salmerón, and Willems and their co-authors, and in the paper on average power [25] the concepts of AP, including papers by Czarnecki, Depenbrock, Emanuel, Ferrero, Filipski, Jeon, Mayordona, Morsi, and Willems, and their co-authors. Although some of those papers introduced unequal resistances [27] or distortion, unbalance and multi-wire systems [28], none of the papers proposed approaches meeting all the requirements. We concluded that agreement on reactive or non-active components required a unique definition of AP for systems for all the variables of any number of wires of any resistance, and any conditions of unbalance, harmonic distortion, and direct current components.

We explained the requirement for a resistance-weighted null point as the correct reference for the voltage measurements, else the minimum theoretical line losses cannot be identified [24], [25]. The derivations were consistent with the link between power theory and delivery losses that requires the wire resistances to be included in the analysis [29], [30], and the definition of unity PF as the minimum loss condition [31].

The derivation showed compensating currents have only two components and that they are physically realizable, being a portion instantaneously transferred between wires and a portion that needs energy storage during the period of a wavelength [25]. Later [32], we extended the compensation reference to the Thévenin point, maintaining a consistent frame of reference for the null point of voltage measurement, and reported the successful testing of compensator hardware.

However, until recently, our approach did not provide for frequency dependent Thévenin equivalent impedances consistent with the harmonic distortion of waveforms. This led us to consider a frequency domain approach.

D. ACTIVE CURRENTS IN THE FREQUENCY DOMAIN

According to Czarnecki [33] and Depenbrock [34], Fryze and Buchholz defined the active current in a sinusoidal system in the time domain as:

$$\mathrm{i_a}(t) = \mathrm{P}\big/\|u(t)\|^2 u(t)$$

where P is the power delivered, u is the load (and source) voltage and ll$u(t)$ll is its rms value.

Expressed in the frequency domain it is equivalent to:

$$I_{A,m} = \mathrm{P}\big/\|U\|^2 U_{\mathrm{m}} = \mathrm{Ge}U_{\mathrm{m}} \tag{1}$$

where Ge is a common, constant of proportionality, and a real number termed 'equivalent admittance' of the load and its components, U_{m} and $I_{\mathrm{A,m}}$ are M complex root mean square (CRMS) values representing each of the m$^{\mathrm{th}}$ wire voltages and current components, and

$$\|U\|^2 = \sum\nolimits_1^M \|U_{\mathrm{m}}\|^2 \tag{2}$$

In the case of bandwidth limited non-sinusoids the active CRMS components values for each m-wire includes also H harmonics components [10], [33]:

$$I_{A,m,h} = \mathrm{P}\big/\|U\|^2 U_{\mathrm{m,h}} = \mathrm{Ge}U_{\mathrm{m,h}}$$

and

$$\|U\|^2 = \sum\nolimits_1^M \sum\nolimits_0^H \left|U_{\mathrm{m,h}}\right|^2 \tag{3}$$

where m = 1 to M wires and h = 0 to H.

At the PCC, all $U_{\mathrm{m,h}}$ for every m and h are measured from a common reference that is generally not defined, or is 'arbitrary' [2], and therefore may result in incorrect and inconsistent calculation of active currents. A 'fictitious null-point' from which all the wire voltages add to zero, which effectively eliminates the zero sequence has been proposed [35]. In the case where not all the wires have equal resistances, the 'null reference' must be calculated in such a way the line voltages are weighted in proportion to the inverse of their respective line resistances [24], [25], [30], [31].

Jeon [36] proposed a generalized theory in the frequency domain, based on Buchholz and a 'fictitious' neutral. The approach allows transmission lines to have different and frequency-dependent resistances by determining a reference resistance and an effective current and voltage. In what is effectively a superposition approach, compensation currents are rearranged among the subsystems of the whole power system to minimize the total transmission loss. All components are defined at the PCC.

An approach combining frequency components with the CPC theory [37] and referencing the supply voltage to an artificial zero, still does not consider the impedances of the delivery system.

In a series of seven papers, Lev-Ari, Stanković and co-authors calculate the near-optimum shunt compensator compensation for an induction machine load using the Hilbert transform and frequency domain. In early papers they refer to weighting the Thévenin side wire voltages with the inverse wire resistances, respectively, [38], and to voltage measurements from a common point or the neutral wire [39]; though the concepts of a weighted null point are not explicitly included, even later [40]. These papers are not the only ones using analysis in the Hilbert space. Other authors have applied geometric algebra, but then adopt power components that make the definition of AP "impossible" [20] or other simplifications that limit their approach.

E. SUMMARY OF DEFINITIONS OF PF AND AP

Although different forms of PF have been identified, it is generally agreed that PF is an index of relative delivery efficiency and a measure of utilization [2], [30] consistent with the meaning implied by Bell [4]. It is also generally agreed that PF = P/AP and PF = 1 when only active currents are delivered, incurring the minimum loss [2], [31] [34].

However, the values of PF < 1 are not identified uniquely and vary with the definition of AP.

Formulations of AP have followed different approaches [41]. They are based on various models constrained by assumptions, such as voltages being invariant after compensation [10], or that AP is a geometric sum of components of power and non-active power, such as in the IEEE Std 1459-2010 [2] and the CPC and CPT approaches.

Therefore, better, more general, unique definitions are needed of PF and AP. Any new definition should apply in the context of measurement theory, meet the needs of various practical applications, and conform to all physical electrical principles. The review suggests a novel solution might be found in the frequency domain with a Thévenin equivalent circuit decomposed into lines of wire-harmonic components, and with careful attention to a line-resistance-dependent reference for all voltage components at all frequencies, to comply with Kirchhoff's current and voltage laws. The approach must allow unequal frequency-dependent line resistances and inductances in the physical model of the power system and always lead to optimal (minimum loss for the delivered power) dispatch by redistribution of the active currents. We have not identified an approach incorporating all these fundamental concepts.

IV. GENERAL DERIVATION OF ACTIVE CURRENTS IN THE FREQUENCY DOMAIN AS TRANSPORTATION PROBLEM

The problem can be considered as a transportation problem where the same total energy as before is delivered to a PCC over a specific time interval. This is achieved through the re-assignment of the current components' CRMS values between all wires. Re-assignment allows the same energy as before to be transported with minimal power losses. This leads to the following relationships:

- the power consumed (or generated) at the PCC remains constant,
- the objective of identifying minimal losses requires a specified reference for resistance-weighting the voltage measurements [24], [25], which includes maintaining a consistent frame of reference over the extent of the system;
- there is a physical model of currents that can be redistributed between wires and/or temporarily extracted, stored and returned to a wire, which allows that a wire can be represented by Thévenin equivalent lines for each frequency component, such that the dimensions of vectors $\boldsymbol{U}_{\mathrm{m}}$ and $\boldsymbol{I}_{\mathrm{A}}$ in (1) and (2) are increased; and
- Kirchhoff's current law applies to all wire currents and also to their respective line current components.

The approach transfers the point of reference for all voltages to the Thévenin equivalent point. The formulation of (1) is generalized as:

$$\boldsymbol{I}_{\mathrm{A}} = \mathrm{K}_{\mathrm{A}} \boldsymbol{V}_{\mathrm{Th(null)}} \mathbf{R}^{-1} \tag{4}$$

where $\boldsymbol{V}_{\mathrm{Th(null)}}$ is a voltage vector representing all the equivalent Thévenin CRMS voltages components of the equivalent circuit of the network (source). Each voltage component, h, differs from the others in that each has its own reference, not necessarily the same for all frequency components. This "multiple" reference for all h satisfies Kirchhoff's current law that the sum of all wires' weighted harmonic current components respectively add to zero.

Accordingly, this approach differs in several aspects from previous formulations of active currents, namely:

1) The key voltages are the Thévenin point voltages measured from h different references.
2) The optimal total power is at the Thévenin point after re-assigning current components, and not the power at the load/PCC.
3) Weighted voltage references are specific to each frequency voltage component, so that

$$\sum_{1}^{M} \frac{V_{Thm,h}}{Rm,h} = 0 \tag{5}$$

is always true for all h.

4)

$$\mathrm{K}_{\mathrm{A}} = \frac{\mathrm{P}_{\mathrm{Th(OPT)}}}{||\boldsymbol{V}'_{\mathrm{Th(null)}}||^2} \text{ (and not } \frac{\mathrm{P}_{\mathrm{PCC}}}{||\boldsymbol{U}||^2} \text{ as in (1))} \tag{6}$$

where $\mathrm{P}_{\mathrm{Th(OPT)}} = \mathrm{P}_{\mathrm{PCC}} +/- \left|\left|\boldsymbol{I}'_{\mathrm{A}}\right|\right|^2$ where $\left|\left|\boldsymbol{I}'_{\mathrm{A}}\right|\right|^2$ is the minimal line loss for the same power delivery, and

$$||\boldsymbol{I}'_{\mathrm{A}}|| = 1/2(-||\boldsymbol{V}'_{\mathrm{Th(null)}}|| - / + \sqrt{(||\boldsymbol{V}'_{\mathrm{Th(null)}}||^2 + 4\,\mathrm{P}_{\mathrm{PCC}})})$$

and K_{A} replaces Ge of (1) to allow for additional variables; where K_{A} and Ge will only be the same when three limiting conditions apply: the line voltages do not change at the PCC after the current components are re-assigned, the line resistances are equal and negligible, and there are no weighted zero sequence voltage components.

The main stages of the theoretical development are:

- setting up the input data structures in the form of CRMS vectors and a resistance square matrix;
- identifying the Thévenin side CRMS voltage vector $\boldsymbol{V}_{\mathrm{Th}}$;
- calculating the voltage components' reference CRMS offsets from the PCC-side reference;
- finding $\boldsymbol{V}_{\mathrm{Th\,(null)}}$ by subtracting the offset voltages from $\boldsymbol{V}_{\mathrm{Th}}$;
- identifying the weighted Thévenin voltage vector and finding the minimum possible $\mathrm{P}_{\mathrm{(OPT)}}$ at the Thévenin point by adding the minimum line loss to $\mathrm{P}_{\mathrm{PCC}}$;
- identifying K_{A} and the individual CRMS solution current components $\boldsymbol{I}_{\mathrm{A\,m,h}}$ from (4).

These stages are developed in the following sub-sections.

A. SETTING UP THE INPUT MATRICES

Consider the network shown in Fig. 1 with M wires described in terms of the equivalent Thévenin circuit in which the voltages vector $\boldsymbol{U}(\mathrm{t})$ and uncompensated currents vector $\boldsymbol{I}_{\mathrm{S}}(\mathrm{t})$ at the PCC point are known.

Similar to (1), each M wire's time-variable bandwidth-limited voltage $\boldsymbol{U}_{\mathrm{m}}$ and current $\boldsymbol{I}_{\mathrm{m}}$ during a chosen time interval T can be expressed in a condensed form, by Fourier, with a finite set of h CRMS values, namely:

$$(U_{\mathrm{m},0}, U_{\mathrm{m},1}, \ldots U_{\mathrm{m},,\mathrm{H}}) \text{ and } (I_{\mathrm{m},0}, I_{\mathrm{m},1}, \ldots I_{\mathrm{m},,\mathrm{H}})$$

where m = 1 to M and h = 0 to H (includes dc h = 0).

From these, let row vectors $\boldsymbol{I}_{\mathrm{S}}$ and $\boldsymbol{U}$ be constructed so that each consists of two M x (H+1) CRMS values:

$$\boldsymbol{U} = \{(U_{1,0}, U_{1,1}, \ldots U_{1,\mathrm{H}}), (U_{2,0}, U_{2,1}, \ldots U_{2,\mathrm{H}}), \ldots, (U_{\mathrm{M},0}, U_{\mathrm{M},1}, \ldots U_{\mathrm{M},,\mathrm{H}})\} \tag{7a}$$

$$\boldsymbol{I}_{\mathrm{S}} = \{(I_{1,0}, I_{1,1}, \ldots I_{1,\mathrm{H}}), (I_{2,0}, I_{2,1}, \ldots I_{2,\mathrm{H}}), \ldots (I_{\mathrm{M},0}, I_{\mathrm{M},1}, \ldots I_{\mathrm{M},,\mathrm{H}})\} \tag{7b}$$

Further, construct a square matrix $\mathbf{R}$ with M x (H+1) rows and M x (H+1) columns, where the diagonal vector is

$$\mathrm{R(diag)} = \{(r_{1,0}, r_{1,1}, \ldots r_{1,\mathrm{H}}), (r_{2,0}, r_{2,1}, \ldots r_{2,\mathrm{H}}), \ldots, (r_{\mathrm{M},0}, r_{\mathrm{M},1}, \ldots r_{\mathrm{M},\mathrm{H}})\} \tag{8}$$

and where all other elements of the **R** square matrix are zero.

Vectors $\boldsymbol{U}$ and $\boldsymbol{I}_{\mathrm{S}}$ and matrix **R** represent all the parameters in convenient data structures, to which can be applied the properties of linear algebra to find a solution current vector called $\boldsymbol{I}_{\mathrm{A}}$.

B. RELATIONSHIPS BETWEEN PCC AND THÉVENIN POINT

We propose a Theorem: A unique vector $\boldsymbol{I}_{\mathrm{A}}$ consisting of M x H+1 optimal line CRMS current components $I_{\mathrm{Am,h}}$ values exists, which delivers the same total power to the $\mathrm{P_{PCC}}$ with minimal losses, can be identified and computed with (4):

$$\boldsymbol{I}_{A} = \mathrm{K_A}\, \boldsymbol{V}_{\mathrm{Th(null)}}\mathbf{R}^{-1} \qquad (9)\text{..same as (4)}$$

where $\mathrm{K_A}$ is a single real value constant throughout the interval chosen and applicable to all M wires and H+1 components:

$$\mathrm{K_A} = \frac{\mathrm{P_{Th(OPT)}}}{||\boldsymbol{V}'_{\mathrm{Th(null)}}||^2} \qquad (10)$$

in which $\mathrm{P_{Th(OPT)}}$ and $\boldsymbol{V}'_{\mathrm{Th(null)}}$ are calculated with (24) and (31) below.

The proof of the theorem is developed in the rest of this section and up to (36) at the end of III.H. It is necessary both for rigor and to identify the equations needed for calculating a measurement quantity. A reader more interested in the application of the theory may pick up the general story in Section III.I.

Given the CRMS vectors $\boldsymbol{U}$, $\boldsymbol{I}_{\mathrm{S}}$ representing the original time domain line voltages $\mathrm{u_m(t)}$ and currents $\mathrm{i_{s\,m}(t)}$, and the physical constraints of the system namely Kirchhoff's current law (the sum of current components in all lines respectively must sum to zero) and that the power delivered to the PCC remains the same after re-assigning the current components, then $\mathrm{P_{PCC}}$, the power flow at the PCC point, can be calculated from classical power theory as:

$$\mathrm{P_{PCC}} = \langle \boldsymbol{U}, \boldsymbol{I}_{\mathrm{S}} \rangle \qquad (11)$$

where $\langle \boldsymbol{U}, \boldsymbol{I}_{\mathrm{S}} \rangle$ is the real part of the inner product of the two vectors defined as the dot product of the complex vectors $\boldsymbol{U}$ and $\boldsymbol{I}^*_{\mathrm{S}}$ where $\boldsymbol{I}^*_{\mathrm{S}}$ is the conjugate of $\boldsymbol{I}_{\mathrm{S}}$.

It is important to note that the power $\mathrm{P_{PCC}}$ is independent of the reference chosen for measuring the voltages at the PCC, shown as $\mathrm{Ref_{PCC}}$ in (a) of Fig. 3. Different voltage references may even be used for each voltage component, such as measuring the voltages between phases of a three phase, three wire system. In practice, it is common to use one of the wires of the system or a ground point.

Also, given (measured) the sets of all M wires equivalent Thévenin resistances $r_{\mathrm{m,h}}$ and equivalent inductances: $l_{\mathrm{m,h}}$ for all m = 1 to M and h = 0 to H, then…

…using classical complex power theory, one can calculate the Thévenin side voltages with respect to the PCC common arbitrarily chosen voltage reference $\mathrm{Ref_{PCC}}$ (see (b) in Fig. 3).

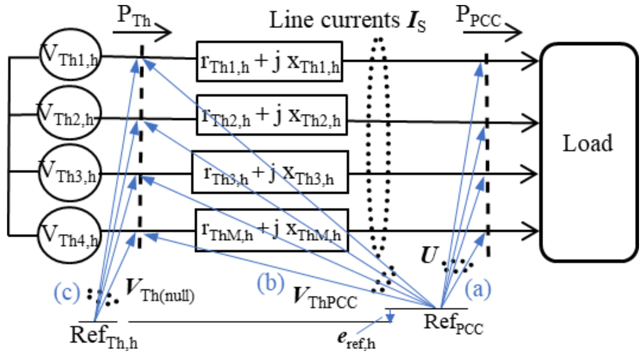

FIGURE 3. Mx(H+1) line Thévenin equivalent circuit as seen from PCC, with (a) the PCC side voltages measured from a common arbitrarily chosen reference, possibly the potential of one of the wires; (b) the voltages at the Thévenin point referenced to $\mathrm{Ref_{PCC}}$; and (c) the null reference at the Thévenin point, offset from $\mathrm{Ref_{PCC}}$ by $\mathrm{e_{ref}}$.

C. THÉVENIN SIDE VOLTAGE VECTOR

Each of the Thévenin side CRMS voltage components $V_{\mathrm{Th\,m,h}}$ for all m and h with respect to the original single voltage PCC-side reference $\mathrm{Ref_{PCC}}$ are calculated using standard power theory as follows:

$$V_{\mathrm{Th\,m,h}} = U_{\mathrm{m,h}} + z_{\mathrm{m,h}} I_{\mathrm{m,h}}$$

where $z_{\mathrm{m,h}} = r_{\mathrm{m,h}} + \mathrm{j}2\pi f \mathrm{h}\, l_{\mathrm{m,h}}$ for m = 1 to M, and h = 0 to H.

Construct the M x H+1 Thévenin complex voltage vectors $\boldsymbol{V}_{\mathrm{Th}}$:

$$\begin{aligned}\boldsymbol{V}_{\mathrm{Th}} = \{&(V_{\mathrm{Th1,0}}, V_{\mathrm{Th1,1}}, \ldots V_{\mathrm{Th1,H}}), (V_{\mathrm{Th2,0}}, V_{\mathrm{Th2,1}}, \ldots \\ &V_{\mathrm{Th2,H}}), \ldots, (V_{\mathrm{ThM,0}}, V_{\mathrm{ThM,1}}, \ldots V_{\mathrm{ThM,H}})\} \qquad (12)\end{aligned}$$

The total power $\mathrm{P_{Th}}$ drawn at the equivalent Thévenin voltage side (point) can then be calculated as:

$$\mathrm{P_{Th}} = \langle \boldsymbol{V}_{\mathrm{Th}}, \boldsymbol{I}_{\mathrm{S}} \rangle \qquad (13)$$

This total power can be positive or negative depending of the direction of the total power flow. The difference of power between $\mathrm{P_{PCC}}$ and $\mathrm{P_{Th}}$ is the transmission losses consisting of the sum of the losses due to all current components. Further, the component powers may be individually positive or negative and may not be flowing in the same direction.

Then let the following weighted Thévenin voltage vectors be defined as: $\boldsymbol{V}'_{\mathrm{Th}} = \boldsymbol{V}_{\mathrm{Th}}\mathrm{R}^{-1/2}$; namely:

$$\begin{aligned}V'_{\mathrm{Th}} = \{&(V_{\mathrm{Th1,0}}r_{1,0}^{-1/2}, V_{\mathrm{Th1,1}}r_{1,1}^{-1/2}, \ldots V_{\mathrm{Th1,H}}r_{1,\mathrm{H}}^{-1/2}), \\ &(V_{\mathrm{Th2,0}}r_{2,1}^{-1/2}, V_{\mathrm{Th2,1}}r_{2,1}^{-1/2}, \ldots V_{\mathrm{Th2,H}}\, r_{2,\mathrm{H}}^{-1/2}), (\ldots), \\ &(V_{\mathrm{ThM,0}}r_{\mathrm{M},0}^{-1/2}, V_{\mathrm{ThM,1}}r_{\mathrm{M},1}^{-1/2}, \ldots V_{\mathrm{ThM,H}}r_{\mathrm{M,H}}^{-1/2})\} \quad (14)\end{aligned}$$

and the weighted current vector be defined as the product of a vector and a matrix:

$$\boldsymbol{I}'_{\mathrm{S}} = \boldsymbol{I}_{\mathrm{S}}\mathbf{R}^{1/2};$$

namely:

$$\begin{aligned}\boldsymbol{I}'_{\mathrm{S}} = \{&(I_{1,0}r_{1,0}^{1/2}, I_{1,1}r_{1,1}^{1/2}, \ldots I_{1,\mathrm{H}}r_{1,\mathrm{H}}^{1/2}), \\ &(I_{2,0}r_{2,0}^{1/2}, I_{2,1}r_{2,1}^{1/2}, \ldots I_{2,\mathrm{H}}r_{2,\mathrm{H}}^{1/2}), (\ldots), (\ldots), \\ &(I_{\mathrm{M},0}r_{\mathrm{M},0}^{1/2}, I_{\mathrm{M},1}\, r_{\mathrm{M},1}^{1/2}, \ldots I_{\mathrm{M,H}}r_{\mathrm{M,H}}^{1/2})\} \qquad (15)\end{aligned}$$

D. REFERENCE OFFSETS FROM THE PCC SIDE

It can be seen from (15) that the norm square $<\boldsymbol{I}'_{\mathrm{S}}, \boldsymbol{I}'_{\mathrm{S}}>$, where the inner product is defined as the real part of the product of $\boldsymbol{I}'_{\mathrm{S}}$ and its conjugate I'^{*}_{S}, is equal to the wire losses $\mathrm{P}_{\mathrm{S(loss)}}$ since it is the sum of all I^2r of all wires for all current components:

$$\mathrm{P}_{\mathrm{S(loss)}} = <\boldsymbol{I}'_{\mathrm{S}}, \boldsymbol{I}'_{\mathrm{S}}> = ||\boldsymbol{I}'_{\mathrm{S}}||^2 \tag{16}$$

The solution approach consists of finding first the weighted vector $\boldsymbol{I}'_{\mathrm{A}}$ and then $\boldsymbol{I}_{\mathrm{A}}$, the current vector representing the physical currents required to flow through the wires. This current vector with the minimum norm in the solution subspace will deliver the same power $\mathrm{P}_{\mathrm{PCC}}$ as delivered with the set of non-optimal currents.

The mathematical technique to find such minimum norm vector is to project the original weighted current I'_S onto a solution subspace representing all the physical constrains of the M x (H+1) sub-vector weighted solution space, in two steps.

First, Kirchhoff's current law at PCC, is expressed mathematically for each h from 0 to H as:

$$\sum\nolimits_{1}^{M} I_{\mathrm{Th,m,h}} = 0 \tag{17}$$

This can be expressed also as the equivalent inner products for each and every h being equal to zero:

$$<\boldsymbol{I}_{\mathrm{A}}, \mathbf{1}_{\mathrm{h}}> = 0$$

where the vectors $\mathbf{1}_{\mathrm{h}}$ vector structures also have M x (H+1) elements.

$$\begin{aligned}
\mathbf{1}_1 &= \{(1, 1, \ldots 1), (0, 0, \ldots, 0), \ldots, (0, 0, \ldots, 0)\}\\
\mathbf{1}_2 &= \{(0, 0, \ldots 0), (1, 1, \ldots, 1), \ldots, (0, 0, \ldots, 0)\}\\
&\ldots\\
\mathbf{1}_3 &= \{(0, 0, \ldots 0), (0, 0, \ldots, 0), \ldots, (1, 1, \ldots, 1)\}
\end{aligned} \tag{18}$$

It can now be seen that the following inner product of two weighted vectors is also equivalent for any h = 0 to H:

$$<\boldsymbol{I}'_{\mathrm{A}}, \mathbf{1}'_{\mathrm{h}}> = 0 \tag{19}$$

where $\mathbf{1}'_{\mathrm{h}} = \mathbf{1}_{\mathrm{h}}\mathbf{R}^{-1/2}$.

The vectors are $\mathbf{1}'_0, \mathbf{1}'_1, \ldots, \mathbf{1}'_{\mathrm{H}}$ where:

$$\begin{aligned}
\mathbf{1}'_0 &= \{(\mathrm{r}_{1,0}^{-1/2}, \mathrm{r}_{1,1}^{-1/2}, \ldots \mathrm{r}_{1,\mathrm{H}}^{-1/2}), (0, 0, \ldots, 0), \ldots,\\
&\qquad (0, 0, \ldots, 0)\}\\
\mathbf{1}'_2 &= \{(0, 0, \ldots, 0), (\mathrm{r}_{2,0}^{-1/2}, \mathrm{r}_{2,1}^{-1/2}, \ldots, \mathrm{r}_{2,\mathrm{M}}^{-1/2}), \ldots,\\
&\qquad (0, 0, \ldots, 0)\}\\
&\ldots\\
\mathbf{1}'_{\mathrm{H}} &= \{(0, 0, \ldots, 0), \ldots, (\mathrm{r}_{\mathrm{M},0}^{-1/2}, \mathrm{r}_{\mathrm{M},1}^{-1/2}, \ldots, \mathrm{r}_{\mathrm{M,H}}^{-1/2})\}.
\end{aligned} \tag{20}$$

Therefore, $\mathbf{1}'_0$, $\mathbf{1}'_1$, …, $\mathbf{1}'_{\mathrm{H}}$ are some of the necessary weighted vectors defining the weighted solution subspace where the optimal weighted current vector $\boldsymbol{I}'_{\mathrm{A}}$ must reside.

Furthermore $\mathbf{1}'_0, \mathbf{1}'_1, \ldots, \mathbf{1}'_{\mathrm{H}}$ can be seen to be orthogonal to each other, such that H+1 coordinates must be part of the definition of the weighted solution subspace.

Secondly, the following equation must also remain true after current component re-assignment where the line current has changed to I_A, which still needs to be found.

$$\mathrm{P}_{\mathrm{PTh(OPT)}} = <\boldsymbol{V}_{\mathrm{Th}}, \boldsymbol{I}_{\mathrm{A}}> \tag{21}$$

Clearly, this can also be calculated as the inner product of the weighted vectors $\boldsymbol{V}'_{\mathrm{Th}}, \boldsymbol{I}'_{\mathrm{A}}$ as defined above:

$$\mathrm{P}_{\mathrm{PTh(OPT)}} = <\boldsymbol{V}'_{\mathrm{Th}}, \boldsymbol{I}'_{\mathrm{A}}>$$

Therefore, $\boldsymbol{V}'_{\mathrm{Th}}$ must also reside in the solution weighted vector subspace needed to find the solution current vector. However, $\boldsymbol{V}'_{\mathrm{Th}}$ as measured from the PCC ref, is not necessarily orthogonal to the other solution vector space's coordinates vector $\mathbf{1}'_0, \mathbf{1}'_1, \ldots, \mathbf{1}'_{\mathrm{H}}$: and is not a coordinate vector.

Another orthogonal coordinate vector $\boldsymbol{V}'_{\mathrm{Th(null)}}$ (with a different reference) that is orthogonal to $\mathbf{1}'_0, \mathbf{1}'_1, \ldots, \mathbf{1}'_{\mathrm{H}}$ and also in the solution subspace can be obtained using the Gram-Schmidt method [1]. This linear algebra method consists of subtracting from $\boldsymbol{V}_{\mathrm{Th}}$ all the components $\mathbf{e}'_{\mathrm{ref,h}}$ for every h in the direction of $\mathbf{1}'_{\mathrm{h}}$, where $\mathbf{e}'_{\mathrm{ref,h}}$ the projection of $\boldsymbol{V}'_{\mathrm{Th}}$ is calculated as:

$$\mathbf{e}'_{\mathrm{refh}} = <\boldsymbol{V}'_{\mathrm{Th}}, \mathbf{1}'_{\mathrm{h}}) > \mathbf{1}'_{\mathrm{h}}/||\mathbf{1}'_{\mathrm{h}}||^2 \tag{22}$$

for h = 1 to H, and

$$\mathbf{e}'_{\mathrm{refh}} = (\sum\nolimits_{m=1}^{m} \boldsymbol{V}_{\mathrm{Th,m,h}}\mathrm{R}_{\mathrm{m,h}}{}^{-1} / \sum\nolimits_{m=1}^{m} \mathrm{R}_{\mathrm{m,h}}^{-1})1'_{\mathrm{h}} \tag{23}$$

E. $V_{Th(null)}$ BY SUBTRACTING THE OFFSETS

It can then be seen (shown as (c) in Fig. 3) that $\boldsymbol{V}'_{\mathrm{Th(null)}}$ can be obtained directly by changing the voltage component reference from the original common $\mathrm{Ref}_{\mathrm{PCC}}$ to a new reference displaced by a constant value $\mathbf{e}'_{\mathrm{ref,h}}$ for all the wires and their respective frequency components h.

Since the calculation of power is independent of the voltage reference chosen, it applies to any of the power components. An offset can be subtracted from each component's reference without changing the power calculated at the Thévenin point:

$$\mathrm{P}_{\mathrm{Th(OPT)}} = <\boldsymbol{V}'_{\mathrm{Th(null)}}, \boldsymbol{I}'_{\mathrm{A}}> = <\boldsymbol{V}_{\mathrm{Th}}, \mathbf{I}_{\mathrm{A}}> \tag{24}$$

F. WEIGHTED THÉVENIN VECTOR V'_{Th}, I'_A

Now

$$\begin{aligned}
\boldsymbol{V}'_{\mathrm{Th(null)}} &= \boldsymbol{V}'_{\mathrm{Th}} - \mathbf{e}'_{\mathrm{ref}}\\
&= (\mathbf{V}_{\mathrm{Th}} - \mathbf{e}_{\mathrm{ref}})\mathbf{R}^{-1/2} = \boldsymbol{V}_{\mathrm{Th(null)}}\mathbf{R}^{-1/2}
\end{aligned} \tag{25}$$

Namely:

$$\begin{aligned}
V'_{Th(null)} = \{&((V_{1,0} - \mathrm{e}_{\mathrm{ref}\,0})\mathrm{r}_{1,0}^{-1/2}, (V_{1,1} - \mathrm{e}_{\mathrm{ref}\,1})\mathrm{r}_{1,1}^{-1/2}, \ldots,\\
&(V_{1,\mathrm{H}} - \mathrm{e}_{\mathrm{ref\,H}})\mathrm{r}_{1,\mathrm{H}}^{-1/2}),\\
&((V_{2,0} - \mathrm{e}_{\mathrm{ref}\,0})\mathrm{r}_{2,0}^{-1/2}, (V_{2,1} - \mathrm{e}_{\mathrm{ref}\,1})\mathrm{r}_{2,1}^{1/2}, \ldots,\\
&(V_{2,\mathrm{H}} - \mathrm{e}_{\mathrm{ref\,H}})\mathrm{r}_{2,\mathrm{H}}^{-1/2}),\\
&(\ldots\ldots\ldots\ldots\ldots\ldots\ldots,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \ldots,\\
&(\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots),\\
&((V_{\mathrm{M},0} - \mathrm{e}_{\mathrm{ref}\,0})\mathrm{r}_{\mathrm{M},0}^{-1/2}, (V_{\mathrm{M},1} - \mathrm{e}_{\mathrm{ref}\,1})\mathrm{r}_{\mathrm{M},1}^{-1/2}, \ldots,\\
&(V_{\mathrm{M,H}} - \mathrm{e}_{\mathrm{ref\,H}})\mathrm{r}_{\mathrm{M,H}}^{-1/2})\}
\end{aligned} \tag{26}$$

where for $h = 0$ to H and $m = 1$ to M:

$$e_{\mathrm{ref\,h}} = (\sum_{m=1}^{M} V_{\mathrm{Th,m,h}} r_{\mathrm{m,h}}{}^{-1} / \sum_{m=1}^{M} r_{\mathrm{m,h}}^{-1})$$

From which

$$||\boldsymbol{V}'_{\mathrm{Th(null)}}||^2 = <\boldsymbol{V}'_{\mathrm{Th(null)}}, \boldsymbol{V}'_{\mathrm{Th(null)}}> \tag{27}$$

This is the sum of all $V_{\mathrm{Th(null)m,h}}$ (weighted) magnitude squared divided by respective resistance.

G. THÉVENIN POWER AND MULTIPLIER K_A

The weighted solution vector space is now completely determined by the set of H+1 orthogonal coordinates $\mathbf{1}'_0, \mathbf{1}'_1, \ldots, \mathbf{1}'_H$ representing Kirchhoff's law applied to the current components and the weighted Thévenin voltage $\boldsymbol{V}'_{\mathrm{Th(null)}}$ which is also mutually orthogonal to $\mathbf{1}'_0, \mathbf{1}'_1, \ldots, \mathbf{1}'_H$.

The solution optimal weighted current vector defined $\boldsymbol{I}'_A$ which has minimum length can now be found by projecting the non-optimal weighted original line current vector $\boldsymbol{I}'_S$ onto the weighted solution vector space. This projection vector (by the property of linear algebra), which has minimum length (i.e. $||\boldsymbol{I}'_A||^2$ is the minimum possible sum of squares, representing line losses) and conforms to the solution space constraints, namely Kirchhoff's current law (18..16), will deliver the same power P_{PCC} as the original line current vector $\boldsymbol{I}_S$ (13).

The coordinates of $\boldsymbol{I}'_A$ in $\mathbf{1}'_0, \mathbf{1}'_1, \ldots, \mathbf{1}'_H$ are all zero since:

$$<\boldsymbol{I}'_A, \mathbf{1}'_h> = 0 \text{ for all h } = 0 \text{ to H} \tag{28}$$

And, since all current components add to zero for each respective harmonic, this implies that the solution vector $\boldsymbol{I}'_A$ is in the direction $\boldsymbol{V}'_{\mathrm{Th(null)}}$ only, as it is the only other coordinate left which is not zero. Therefore:

$$\boldsymbol{I}'_A = K_A \boldsymbol{V}'_{\mathrm{Th(null)}} \tag{29}$$

where K_A is a real value and constant throughout the duration of T chosen and common for all components.

From (21) and (29):

$$P_{\mathrm{Th(OPT)}} = <K_A \boldsymbol{V}'_{\mathrm{Th(null)}}, \boldsymbol{V}'_{\mathrm{Th(null)}}>$$

Hence:

$$K_A = P_{\mathrm{Th(OPT)}} / ||\boldsymbol{V}'_{\mathrm{Th(null)}}||^2 \tag{30}$$

And since $= <\boldsymbol{I}'_A, \boldsymbol{I}'_A>$ represents the sum of all line components losses, then:

$$P_{\mathrm{Th(OPT)}} = P_{PCC} +/- ||\boldsymbol{I}'_A||^2 \tag{31}$$

where +/− depends on the direction of total power flow.

$$||\boldsymbol{I}'_A||^2 - ||\boldsymbol{V}'_{\mathrm{Th(null)}}|| \, ||\boldsymbol{I}'_A|| + P_{PCC} = 0 \tag{32}$$

Expanding, rearranging and solving the quadratic equation:

$$||\boldsymbol{I}'_A|| = 1/2(-||\boldsymbol{V}'_{\mathrm{Th(null)}}|| -/+ (||\boldsymbol{V}'_{\mathrm{Th(null)}}||^2 + 4P_{PCC})^{1/2} \tag{33}$$

Now knowing $||\boldsymbol{I}'_A||$, $P_{\mathrm{Th(OPT)}}$ can be calculated from (31) and K_A can be calculated too.

H. CRMS SOLUTION CURRENT COMPONENTS

Finally, the active current components are given by multiplying (29) by $\mathbf{R}^{-1/2}$:

$$\boldsymbol{I}'_A \mathbf{R}^{-1/2} = K_A \boldsymbol{V}'_{\mathrm{Th(null)}} \mathbf{R}^{-1/2} \tag{34}$$

The optimal CRMS solution (active) current vector is:

$$\boldsymbol{I}_A = K_A \boldsymbol{V}_{\mathrm{Th(null)}} \mathbf{R}^{-1} \tag{35}$$

And the individual unweighted CRMS solution current components are:

$$I_{A(m,h)} = K_A \frac{V_{\mathrm{Th(null)(m,h)}}}{r(m,h)} \text{ for all m and h.} \tag{36}$$

I. IDENTIFYING PF AND AP

By definition:

$$PF = ||\boldsymbol{I}'_A|| / ||\boldsymbol{I}'_S|| \text{or} P_{\mathrm{Th(OPT)}} / P_{\mathrm{Th(bef)}} \tag{37}$$

where $P_{\mathrm{Th(OPT)}} = ||\boldsymbol{V}'_{\mathrm{Th(null)}}|| \, ||\boldsymbol{I}'_A||$ is the minimum power needed to be transmitted from the Thévenin source to deliver P_{PCC} with optimal delivery loss; and

$$AP = P_{\mathrm{Th(bef)}} = ||\boldsymbol{V}'_{\mathrm{Th(null)}}|| \, ||\boldsymbol{I}'_S|| \tag{38}$$

is the maximum optimally distributed power that could be transmitted from the source with the original delivery loss.

J. PRACTICAL APPLICATION

The order of the steps followed to calculate the minimal loss current CRMS vector, representing the optimal current components that will deliver P_{PCC} power to a PCC point with minimal losses, is not the same as the order of the theory development and are illustrated by examples in Section VI.

V. POWER PROCESSOR (COMPENSATOR) CONCEPT

The applications of power theory include the calculation of active currents and the control with compensators and inverters of the currents in power systems to reduce the delivery losses.

With the advent of modern, flexible, and low-cost power processing devices (PPD), we use this term to include all forms of compensators, converters and inverters, irrespective of their topology.

PPDs that process the electrical energy can be represented simply as a combination of bi-directional current sources, voltage sources and temporary energy storage (capacitors, inductances and batteries) connected at the PCC, from which power can be received from the Thévenin side during an interval T (typically a cycle), illustrated in Fig. 4. For example the converter could be a double conversion converter drawing the specific currents $\boldsymbol{I}_A$ and reissuing the original voltages $\boldsymbol{U}$ on the output. This converter would have a zero average power requirement apart from the internal losses of the power PPD. The power capacity of this converter could be minimal depending on the topology used.

The input currents of the PPD are processed as input current sinks, and the compensation purpose of the PPD is

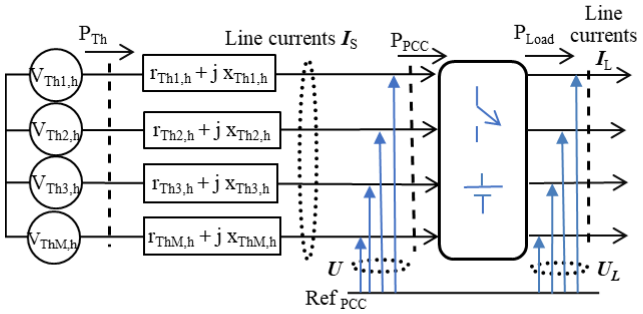

FIGURE 4. **PPD connected at the PCC changes the currents from the Thévenin point to minimize delivery losses, and control the output voltages as needed.**

to optimize the power delivery to the PCC by reducing the delivery losses. However, the result of the re-distribution of current components will change voltages at the PCC from the original voltage $\boldsymbol{U}$ to $\boldsymbol{U}_{(OPT)}$. It can be shown that:

$$<U_{(OPT)}, I_C> + <U - U_{(OPT)}, I_S> = 0$$

so that no power apart from compensator loss is needed if the output voltage is restored to the original voltage. The output voltages can be processed independently to deliver the power P_{Load} according to the application. They may be restored to the original values or balanced voltages as needed.

The flow of power out of the PPD could deliver the same power P_{Load} as received, less the PPD losses or plus power from any local source such as solar. The output voltages and currents depend on the nature of the load circuit, which may be another network, and can be controlled independently according to the type of PPD, such as balancing the output voltages and/or restoring an acceptable voltage range.

Thus, the role of a PPD at the PCC is to modify an original set of line currents ($\boldsymbol{I}_S$) transporting power by injecting and extracting compensation currents ($\boldsymbol{I}_C$) in such a way that the same quantity of power still reaches the PCC, with lower losses or, preferably, minimal possible losses, as identified by the transportation solution identifying the active currents ($\boldsymbol{I}_A$) described in Section IV.

In essence, the PPD approach is that the losses will be reduced to a minimum by the compensation current. For the system, this can be written as loss before rearrangement:

$$||\boldsymbol{I}'_S||^2 = ||\boldsymbol{I}'_A||^2 + ||\boldsymbol{I}'_C||^2 + 2<\boldsymbol{I}'_A, \boldsymbol{I}'_C> \tag{39}$$

And the loss after rearrangement is $||\boldsymbol{I}'_A||^2$ giving a loss reduction of $||\boldsymbol{I}'_C||^2 + 2<\boldsymbol{I}'_A, \boldsymbol{I}'_C>$ in which $2<\boldsymbol{I}'_A, \boldsymbol{I}'_C>$ can be a negative or positive value. This real value of power is attributable to the voltage change at the PCC point and can be positive or negative depending on the circumstances. The value will be zero only if $\boldsymbol{I}'_A$ and $\boldsymbol{I}'_C$ are orthogonal. Note that $||\boldsymbol{I}'_C||^2$ is the loss attributed to the "non-necessary" currents conventionally associated with the concept of reactive or non-active power.

These steps lead to reconciliation of the losses without and with (or before and after) compensation and, therefore, identify the PF before compensation. After compensation, only active current is delivered and the loss is minimized for the power sent out, so PF = 1.

VI. MEASUREMENT EXAMPLES

Electrical quantities defined by mathematical models can be measured by modern instruments incorporating processors and programs [2].

Although the algebraic notation and the length of some equations of Section III might appear complex, the measurement breaks down into simple arithmetic steps, for which the operations are written easily in a spreadsheet or a program. The steps are shown in Table 1, referenced to the cells of the full spreadsheet (shown in Example 2).

TABLE 1. **Calculation process.**

Rows A3..A18: Inputs at PCC: $U_{m,h}$, $I_{Sm\,h}$, Thév $r_{m,h}$, $x_{m,h}$: Convert values of Table I to CRMS values, constructing two M x (H+1) dimensional vectors $\boldsymbol{U}$ and $\boldsymbol{I}_S$ according to (4), (7). Insert Thévenin equivalent impedances $r_{m,h}$ and $x_{m,h}$.
A19..29: inputs give P_{PCC} (11), loss, $\|\|\boldsymbol{I}_S'\|\|$ and P_{Th} without compensation, and conventional S and PF.
A30..34: CRMS $V_{Th\,m,h}$: Using (11) and (12), calculate $V_{Th\,m,h}$ line voltages with respect to original PCC side common reference.
A35..44: Calc offsets e_{ref_h} and $V_{Th(null)(m,h)}$: From (23) calculate $\boldsymbol{e}_{ref\,h}$ reference offsets from original PCC side, and $\boldsymbol{V}_{Th(null)}$ by subtracting the respective $e_{ref,h}$ offsets from $V_{Th\,m,h}$. (26), then $\|\|\boldsymbol{V}'_{Th\,null}\|\|^2$ (27).
A45..50: Calc loss, PF and AP: Use (33) to calculate $\|\|\boldsymbol{I}'_A\|\|^2$; calculate PF (37), AP (38) and loss reduction.
A51..A60: Calculate K_A and optimal $\boldsymbol{I}_A$: Use (31) to calc. $P_{PTh(OPT)} = P_{PCC} + \|\|\boldsymbol{I}'_A\|\|^2$ and $K_A = \frac{P_{Th(OPT)}}{\|\|\boldsymbol{V}'_{Th\,(null)}\|\|2}$ (30) Using (35) and (36) calculate $\boldsymbol{I}_A = K_A \boldsymbol{V}_{Th(null)} \mathbf{R}^{-1}$, checking min. loss by $\boldsymbol{I}_A$.
A61..A69: New PCC-side line voltages: Calculate new PCC line voltages from $\boldsymbol{V}_{Th}$, wire impedances and now optimal line currents. Check.
A70..A75: Compensation currents $I_{Cm,h}$: Calculate from $I_{Sm,h}$ and $I_{Am,h}$.
A76..A84: Loss reduction and PF: Reconcile loss reduction (39); calculate PF by two alternative ratios.

The spreadsheets not only illustrate the process of the calculation, but act as a benchmark for others using the approach. The contents of the key cells with the more complex formulae of each table are expressed in their arithmetic form in the Appendix, from which is should be easy to identify the calculation steps, adapt the equations for the similar cells and translate them into other software, as we have done already into JavaScript, Matlab measurement blocks, and a processor to control a compensator.

Example 1 shows the solution of a measurement with only fundamental frequency waveforms and unbalanced wire impedances. In Example 2 some harmonic content is added, including a dc component, and the full spreadsheet is presented. Example 3 compares our approach with another approach that is similar in some respects.

A. EXAMPLE 1: FUNDAMENTAL FREQUENCY ONLY

The first example considers three phases with a neutral wire and with only fundamental frequency waveforms (no harmonics). The example is compiled using arbitrary sinusoidal waveforms and phase shifts to give unbalanced currents, illustrated in Fig. 5, and unequal line resistances and inductances.

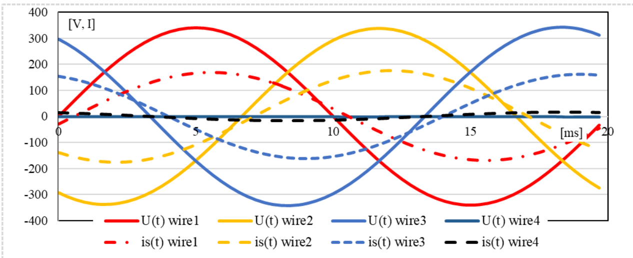

FIGURE 5. Example 1 line voltages $u_m(t)$ and currents $i_{Sm}(t)$ at PCC before component current re-assignment, with sinusoidal waveforms and unbalance.

Since this example is constrained to the fundamental waveforms, frequency dependent Thévenin impedances are not relevant. A time domain solution is possible following our past work [25], [32] and using the 16 samples of the six waveforms over period T in Table 2.

TABLE 2. Measurements (from Fig. 5).

t ms	U1(t)	U2(t)	U3(t)	U4(t)	is1(t)	is2(t)	is3(t)	is4(t)
0.00	0.00	-291.49	296.39	0	-29.47	-139.30	154.67	14.10
1.25	129.89	-333.70	208.34	0	36.73	-170.35	123.67	9.95
2.50	240.00	-325.11	88.58	0	97.34	-175.46	73.83	4.29
3.75	313.58	-267.03	-44.67	0	143.13	-153.86	12.76	-2.03
5.00	339.41	-168.29	-171.12	0	167.13	-108.83	-50.26	-8.04
6.25	313.58	-43.93	-271.52	0	165.68	-47.24	-105.62	-12.82
7.50	240.00	87.11	-330.58	0	139.01	21.54	-144.91	-15.65
8.75	129.89	204.90	-339.31	0	91.18	87.05	-162.13	-16.10
10.00	0.00	291.49	-296.39	0	29.47	139.30	-154.67	-14.10
11.25	-129.89	333.70	-208.34	0	-36.73	170.35	-123.67	-9.95
12.50	-240.00	325.11	-88.58	0	-97.34	175.46	-73.83	-4.29
13.75	-313.58	267.03	44.67	0	-143.13	153.86	-12.76	2.03
15.00	-339.41	168.29	171.12	0	-167.13	108.83	50.26	8.04
16.25	-313.58	43.93	271.52	0	-165.68	47.24	105.62	12.82
17.50	-240.00	-87.11	330.58	0	-139.01	-21.54	144.91	15.65
18.75	-129.89	-204.90	339.31	0	-91.18	-87.05	162.13	16.10

Alternatively, using the frequency domain approach, the first step is to take the sampled line currents and voltages measured from an arbitrary reference and derive the frequency components, in this case the fundamental frequency waveforms.

Thereafter, the process of the calculation follows the sequence detailed in Table 1, using the input parameters of Table 3. The results are reviewed in Section VII.

Were re-assignment or compensation implemented, the avoidable losses would be reduced to zero, making the relative efficiency PF = 1. Although the power P_{PCC} delivered to the PCC without and with compensation does not change, compensation would change the voltages, according to the reallocated wire currents and loss reduction. The voltages and currents at the PCC after compensation, if implemented, are illustrated in Fig. 6, showing that the currents are not necessarily in phase with the voltages at the PCC when the delivery loss is minimized, even though the PF = 1.

TABLE 3. Input parameters of Example 1.

Inputs at PCC	Wire m	Fund. freq h1	
CRMS voltages		Um,h [Vrms]	α [deg]
	1	240.00	0.00
	2	238.00	-120.00
	3	242.00	-240.00
	4	0.00	0.00
CRMS currents		Is m,h [A]	α [deg]
	1	120.00	-10.00
	2	125.00	-128.00
	3	115.00	-252.00
	4	-11.47	-60.31
R, X		r(m,1)	x(m,1)
	1	0.020	0.040
	2	0.030	0.060
	3	0.010	0.020
	4	0.030	0.010

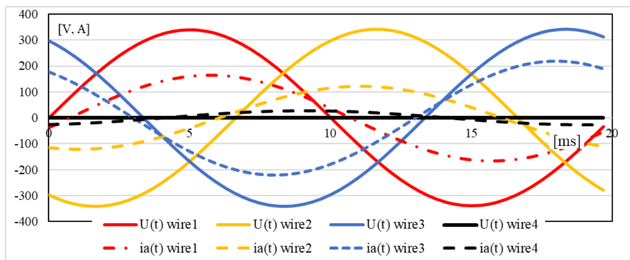

FIGURE 6. Example 1 voltage and current waveforms at the PCC after compensation.

B. EXAMPLE 2: SEVERAL FREQUENCY COMPONENTS AND FREQUENCY-DEPENDENT IMPEDANCES

The second example comprises a three-wire four-wire system with fundamental, dc and 3rd and 5th harmonic components, and with frequency-dependent impedances. The input values are the CRMS voltages measured from PCC side and line currents at the PCC, and line resistances $R_{m,h}$ and $L_{m.h}$ for all lines from m = 1 to M and harmonics 0 to H.

To demonstrate the relative effects of unbalance and the harmonics, all the values of the fundamental frequency components are the same as those in Example 1, with the dc and two harmonic components added to shape the voltages towards a square wave (to illustrate the power delivery capability of the harmonic components), with arbitrary added current components.

The waveforms before compensation were illustrated in Fig. 2, and the compensation currents are shown in Fig. 7.

TABLE 4. Spreadsheet of inputs with harmonic components and conventional calculations.

A1	B	C	D	E	F	G	H	I
2	**Wire m**	**Fund. freq h1**		**DC comp h0**	**Harmonic h3**		**Harmonic h5**	
3	**Inputs at PCC**	*U*m,h [Vrms], α [deg]		*U*m,h [Vrms]	*U*m,h [Vrms], α [deg]		*U*m,h [Vrms], α [deg]	
4	CRMS voltages 1	240.00	0.00	2.00	70.00	0.00	48.00	0.00
5	2	238.00	-120.00	2.00	70.00	-120.00	48.00	-120.00
6	3	242.00	-240.00	1.50	70.00	-240.00	48.00	-240.00
7	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	CRMS currents	***I*s** m,h [A], α [deg]		***I*s** m,h [A]	***I*s** m,h [A], α [deg]		***I*s** m,h [A], α [deg]	
9	1	120.00	-10.00	0.50	40.00	0.00	24.00	0.00
10	2	125.00	-128.00	0.50	40.00	-120.00	24.00	-120.00
11	3	115.00	-252.00	0.40	40.00	-240.00	24.00	-240.00
12	4	-11.47	-60.31	-1.40	0.00	-126.87	0.00	-126.87
13	Sum I	0.000000		0.00	0.00		0.00	
14	R, X	r(m,1)	x(m,1)	r(m,0)	r(m,0)	x(m,3)	r(m,5)	x(m,5)
15	1	0.020	0.040	0.020	0.010	0.035	0.010	0.045
16	2	0.030	0.060	0.020	0.010	0.035	0.010	0.045
17	3	0.010	0.020	0.030	0.010	0.035	0.010	0.045
18	4	0.030	0.010	0.030	0.030	0.010	0.030	0.010
19	Ppcc(h) before opt reassgt	85044.79		2.60	8400.00		3456.00	
20	Tot. Ppcc *bef* opt reassgt	96903.39						
21	Loss ||Is'h||^2 (h)	892.95		0.07	48.00		17.28	
22	Tot. loss||Is'||^2 bef opt reass	958.30						
23	||Is'||	30.96						
24	Pth(h) before comp	85937.74		2.67	8448.00		3473.28	
25	Pth before comp	97861.69						
26	||***I*s**||	225.38						
27	||***U***|| before	444.92						
28	Conv. AP: S = ||***I*s**|| ||***U***(bef)||	100274.60						
29	**PF** conventional	0.9664						

Notes:

Blue values are data input.

Red calculations and values are part of the minimum requirement to calculate PF, AP and the reduced loss possible with compensation.

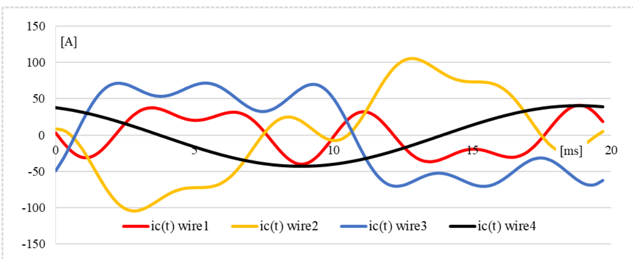

FIGURE 7. Example 2 compensation current waveforms in each wire at the PCC.

The steps of the whole calculation, carried out in the full spreadsheet, are laid out in Tables 4 and 5. Table 4 gives the input parameters and the calculation of conventional measurements of the power at the PCC (P_{PCC}), the delivery loss, $||\boldsymbol{I}'_S||^2$, the power P_{Th} at the Thévenin point without compensation, and the conventional value of S and PF as measured at the PCC. Table 5 presents the calculation of the measurements following the steps of Table 1 and the results are discussed in Section VII.

C. EXAMPLE 3: FROM LEV-ARI AND STANKOVIĆ

Unable to find a standard model with which to compare our approach, we considered several power system models used by other authors. The approach of Lev-Ari *et al.* [38]–[40] is like ours in several respects. It is developed in the frequency domain in the Hilbert space of projecting orthogonally one vector onto another and both their approach and ours do not consider line impedances to be negligible, though their approach relies on the Hilbert transform. Their approach is adaptive and includes load admittances, while ours arrives directly at the measured quantities of loss, PF and AP, and is independent of load admittances.

This third example is a three-phase system without a neutral [39]. The input parameters are the same for both calculations. The currents have fundamental and fifth harmonic components and are unequal. The Thévenin and PCC voltages are unequal. The resistances and inductances are the same in all three wires, so our correction of the reference used for resistance weighting the Thévenin point voltages has no effect. This detail, not mentioned in their example, might be significant were the resistances unequal. Further, their example includes adaptation of the load power when the voltage at the PCC changes with compensation; it is not based only on the analysis of the losses for delivering the original load power P_{PCC}. Although using nominally the same example, there are small differences between the results in [39] and a smaller set of results in [40].

The key results of the two approaches are compared in Table 6.

TABLE 5. Spreadsheet of rigorous calculation of PF and AP.

30	Calculate Vth(m,n) line voltages with respect to original PCCC side common reference							
31	1	243.24	1.02	2.01	70.41	1.13	48.25	1.28
32	2	242.86	-118.37	2.01	70.41	-118.87	48.25	-118.73
33	3	243.61	120.47	1.51	70.41	121.13	48.25	121.27
34	4	0.36	138.12	-0.04	0.00	71.57	0.00	71.57
35	Weighted null point necessary offset to substract from above to obtain Vthnull(m,h)							
36	$e_{\mathrm{ref\,h}}$	67.69	106.03	1.50	0.00	0.22	0.00	90.00
37	Calculate Thevenin's line voltages Vth(m,n)(null) with respect to weighted null point, giving PF and AP							
38	1	268.84	-13.06	0.51	70.41	1.13	48.25	1.28
39	2	295.05	-109.13	0.51	70.41	-118.87	48.25	-118.73
40	3	178.86	125.89	0.01	70.41	121.13	48.25	121.27
41	4	67.39	-74.13	-1.54	0.00	177.96	0.00	-90.00
42	weighted sum	0.00		0.00	0.00		0.00	
43	||Vth(h)(null)'||^2	9865907		105	1487424		698475.06	
44	||*V*th(null)'||^2	12051911						
45	Calculate min. loss, PF and AP							
46	||*V*th'||	3472						
47	||$\boldsymbol{I}_A$'||^2 (min loss)	791.94						
48	**PF** bef comp (by losses)	0.9091						
49	**AP**=||Is'|| ||Vth'||	107468.05	max P transmittable from source with orig loss					
50	Loss reduction poss.by comp.	166.36						
51	Calculate Pth(opt), K_A, and optimal currents $I_{A\,(m,n)}$ for each wire m and frequency h							
52	Pth(opt)	97695.32						
53	K_A =	0.00811						
54	Optimal current components m,h 1	108.97	-13.06	0.21	57.08	1.13	39.11	1.28
55	2	79.72	-109.13	0.21	57.08	-118.87	39.11	-118.73
56	3	144.98	125.89	0.00	57.08	121.13	39.11	121.27
57	4	18.2085	-74.13	-0.42	0.00	177.96	0.00	-90.00
58	sum $\boldsymbol{I}_A$ (check)	0.00		0.00	0.00		0.00	
59	||$\boldsymbol{I}_A$'h||^2 (per h)	648.30		0.0069	97.74		45.90	
60	||$\boldsymbol{I}_A$'||^2 (sum all h)	791.94	Check c47					
61	Calculate Ppcc voltages U(m,h) from original PCC measurement reference after opt reassgt							
62	1	240.68	-0.04	2.04	69.87	-0.49	47.89	-0.82
63	2	241.65	-119.38	2.04	69.87	-120.49	47.89	-120.82
64	3	241.54	119.72	1.54	69.87	119.51	47.89	119.18
65	Check sum 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00
66	Ppcc(h) after opt reassgt	79326.82		0.85	11959.63		5616.09	
67	Total PpccC *after* opt reassgt	96903.39	check c20					
68	Pth(h) opt	79975.11		0.85	12057.37		5661.99	
69	Total Pth opt (all h)	97695.32	check c52					
70	***Compensation*** approach: calculate currents I_C(m,h):							
71	1	12.61	17.47	0.29	17.10	-176.23	15.13	-176.70
72	2	55.86	-155.48	0.29	17.10	63.77	15.13	63.30
73	3	50.11	-9.29	0.40	17.10	-56.23	15.13	-56.70
74	4	29.48	111.20	-0.98	0.00	44.56	0.00	90.00
75	Check sum $\boldsymbol{I}_C$	0.00		0.00	0.00		0.00	
76	***Reconciliation:*** Loss reduction components and alternative PF ratios							
77	||$\boldsymbol{I}_C$ h'||^2	147.99		0.03717	8.77706		6.86697	
78	||$\boldsymbol{I}_C$'||^2	163.67						
79	2x <$\boldsymbol{I}_A$'h, $\boldsymbol{I}c$'h>	96.67		0.03	-58.52		-35.48	
80	2x <$\boldsymbol{I}_A$', $\boldsymbol{I}_C$'>	2.70						
81	Loss reduction	166.36	check=c50					
82	**PF** = P/AP	0.9091	by P and AP at Thevenin point					
83	||$\boldsymbol{I}_A$'||	28.14						
84	**PF**=||$\boldsymbol{I}_A$'||/ ||$\boldsymbol{I}_S$'||	0.9091	by current ratio					

Note: Red calculations and values are part of the minimum requirement to calculate PF, AP and the reduced loss possible with compensation.

There is close agreement on values before compensation and the minimum possible loss for the same P_{PCC} delivered.

In our approach, the AP is identified as the maximum power $||\boldsymbol{V}'_{\mathrm{Th(null)}}||||\boldsymbol{I}'_S||$ that could be sent out from the Thévenin point for the same losses as before compensation. It is not the same as the optimal power shown as (b) in Table 6, needing to be sent out after compensation to deliver the original power to the PCC. An avoidable loss of (a)-(c) = 988 W before compensation corresponds to the "wattless" power that does not reach the PCC and can be eliminated by compensation.

TABLE 6. Comparison of two approaches in Hilbert space.

Quantity		Lev-Ari & Stanković [39]	Malengret & Gaunt
P_{PCC} (before compensation)		16080 W	16083 W
P_{PCC} (after compensation)		Not given	16083 W
Loss before compensation	(a)	1944 W	1941 W
Conventional AP, S = $\|\|v\|\|\ \|\|i_S\|\|$		20.6 kVA	20545 VA
Conventional pf by P/S at PCC		(0.78)	0.7828
Power sent out before compensation		18024 W	18024 W
Opt. power sent out to deliver P_{PCC}	(b)	Not given	17036 W
Loss after compensation for same delivered P_{PCC}	(c)	1528 W Theoretical min. 950 W	953 W
P_{Th} (max) power possible to send from source for same loss as before comp.		Not given	24316 W
Max. power possible at PCC for same loss as before compensation		With zero power shunt compensator 21.4 kVA	With zero power PPD 22375 W
AP by authors' definition	(d)	22.4 kVA	24316 W
PF before comp. given by b/d or √(c/a)		Not given	0.7006

AP is defined in [39] as the power that can be delivered to the PCC for the same losses, given as:

$$||\boldsymbol{V}'_{\text{Th(null)}}||\,||\boldsymbol{I}'_{\text{S}}|| - ||\boldsymbol{I}'_{\text{S}}||^2.$$

A PF based on this AP would not return a quantity consistent with the Cauchy-Schwarz Inequality notion that the angle between two vectors (namely $\boldsymbol{V}'_{\text{Th(null)}}, \boldsymbol{I}'_{\text{S}}$) in inner product space [1] is:

$$\cos(\emptyset) = <\boldsymbol{I}'_{\text{S}}, \boldsymbol{V}'_{\text{Th(null)}}>/||\boldsymbol{I}'_{\text{S}}||\,||\boldsymbol{V}'_{\text{Th(null)}}|| \leq 1$$

Only in the case where $\boldsymbol{I}'_{\text{S}}$ is in the direction of $\boldsymbol{V}'_{\text{Th(null)}}$ would the PF be optimal and equal to 1.

In our opinion, PF can be defined only at the Thévenin point and not at the PCC side, unless the wire impedances are negligible.

VII. DISCUSSION

A. CONTRIBUTION OF TRANSPORTATION APPROACH

In 1927, Budeanu proposed that the AP in one phase of a circuit or load was $\text{S}^2 = \text{P}^2 + \text{Q}_\text{B}^2 + \text{D}_\text{B}^2$ where Q_B = reactive power and D_B = distortion power [42]. The relation, derived from the fundamental and harmonic voltages measured at the PCC, was reproduced in many publications, extended to three-phase loads and adopted in IEEE definitions and standards [2], [42] without referring to the delivery system impedances.

Our objective is different: to characterize the loss and the relative efficiency of the delivery system to the PCC. The PF and AP depend on the parameters of the delivery system and the distribution of currents in the lines as well as the 'circuit' load at the PCC.

The formulation of the general power theory in the frequency domain adds frequency-dependent line parameters to our earlier time-domain formulations and identifies the active elements of current and power, avoidable losses and possible compensation. This allows the PF and AP of a system/load to be defined at the PCC, and the avoidable loss to be compensated if desired.

The analysis is consistent with our earlier findings [24], [25] that a physically-realizable model of the system requires only two components of the 'non-active' power: the power/energy transferred instantaneously between wires and the energy requiring local storage at the PCC during the period of a wavelength.

The analysis defines all the active CRMS active current components $I_{\text{A m,h}}$ for all M wires and all H+1 orders of frequency. $\boldsymbol{I}_\text{A} = \text{K}_\text{A}\boldsymbol{V}_{\text{Th(null)}}R^{-1}$ is a generalized formula in which the components of active current are:

- proportional to the *common factor* K_A, the magnitude of which scales to the Thévenin lines the distribution of the current components, the delivered power, and AP;
- proportional to each of their respective weighted Thévenin voltage CRMS components $V_{\text{Th(null)(m,h)}}$ measured from their *respective harmonic weighted null point reference*;
- inversely proportional to the *respective line resistances*.

Further, K_A reduces to G_e as used by Fryze and many following his approach, provided all line resistances are equal and line impedances negligible.

A practical advantage of the frequency domain approach is that it condenses the measurement data input to frequency components, making them easier to transfer to a processor to measure PF and AP or derive the compensation currents for controlling a PPD.

B. COMPENSATION EFFECT

Losses incur costs, and the benefits of loss reduction lead to the popular concept of PF compensation, such as by shunt capacitors to reduce the phase displacement between voltages and currents of loads with inductive components. PPDs provide alternative approaches to all compensation, even of distortion and unbalance.

Many approaches to passive or active compensators have been proposed, with various degrees of success in minimizing losses, generally arising from assumptions that compromise rigorous power theory. For example, a general assumption that the voltages at the PCC do *not* change with the introduction of compensation currents is inconsistent with physical laws, and results in invalid active currents. Current compensation that removes all non-active components from the current delivered from the source will change the voltages at the PCC. For this reason, definitions of AP assuming invariant voltages at the PCC after compensation cannot be valid.

The basic theory does not depend on introducing compensating currents at the PCC; instead it identifies the PF and AP associated with the load at the PCC of the power system without compensation. It retains the initial network structure (represented by the Thévenin equivalent circuit) to calculate compensation where required.

When compensation is implemented to optimize delivery by reducing the avoidable loss component, the voltages at the PCC will change on some or all wires. The change will affect any intermediate voltage-dependent loads and require the Thévenin equivalent parameters to be re-determined. Similarly, a PPD can improve the voltage magnitude and/or balance the voltages supplying the load at the PCC and, by changing the load, might adjust the power drawn from the Thévenin side of the system.

The term $2\langle \boldsymbol{I}'_{\mathrm{A}}, \boldsymbol{I}'_{\mathrm{C}} \rangle$, introduced by compensation is part of the change in losses and is typically small. (If the vectors were orthogonal, the inner product would be zero. This occurs if the delivery system is assumed to have no impedance and is not valid in practical systems.) In most cases, its non-zero value indicates the 'compensation' current component $\boldsymbol{I}_{\mathrm{C}}$ is not orthogonal to the active current $\boldsymbol{I}_{\mathrm{A}}$ but has two components, one in direction of $\boldsymbol{I}'_{\mathrm{A}}$ and one perpendicular to $\boldsymbol{I}'_{\mathrm{A}}$, arising from the change of the PCC voltages with compensation, and challenging further the concept of reactive power and the geometric power model.

Where harmonic currents in a load are driven by harmonic voltages, and are not detrimental to the load, then the exclusion of the harmonic frequencies from delivering power will increase delivery losses without a compensating benefit. In some circumstances, current waveform distortion is undesirable or unacceptable, and only a measurement of the power quantity at the fundamental frequency power delivery is considered relevant. Where power transfer only at the fundamental frequency current is required, the power no longer delivered at the harmonic frequencies must be delivered by the fundamental current instead, increasing the associated losses and the voltage drop. The scaling parameter $\mathrm{K_A}$ decreases and is no longer optimal in terms of losses, although it may be desirable in terms of power quality. However, a suitable PPD can provide optimum power delivery as well as load balancing and harmonics removal.

Clearly, there are many opportunities for power conditioning, and the details of the many forms of PPDs are beyond the scope of this paper.

The topic of compensation would be incomplete without reference to safety. Measurement of PF and AP has no effect on a system, simply characterizing its performance in terms of two parameters. However, as soon as current components are reassigned in the wires of the delivery system and the total losses are reduced, the loss in one or more wires might increase, and the distribution of voltages in the wires at the PCC will change too. In most cases, the changes will be beneficial. However, in systems operating at their voltage and thermal limits, compensation might cause a limit to be violated, as might occur also in traditionally compensated systems.

C. APPLICABILITY

Most variables of practical power delivery systems are accommodated in the formulation, and no engineering principles are violated. The model and analysis approach the conditions for a representational measurement in power system steady-state conditions.

The measurement is based on the fundamental frequency, which might differ from the rated power frequency. The measurement of power, PF, AP and energy, though valid, will return different quantities according to the window of measurement, such as for a chosen time period or several fundamental frequency wavelengths, as might be defined for a standard or declared differently for a particular purpose. The time period of measurement is an operational measurement constraint.

Measurement is based also on the Thévenin equivalent circuit parameters, assumed constant during the measurement.

The processor time for the actual AP and PF measurement calculation is negligible (<1 ms using a typical industrial processor). However, a first measurement is delayed by the period needed to extract the Thévenin parameters, and the CRMS components of the currents and voltages. Further, where operational measurement standards require measurement of average values over several cycles, this will extend the minimum duration of a first measurement. Thereafter, a sliding window can be utilized for rapid re-calculation if required. Other delays must be considered when compensation is implemented, based on the parameters of the preceding wavelength. Compensation causes the load currents and voltages to change dynamically, locally at the PCC and at intermediate voltage-dependent loads on the system, requiring re-measurement.

D. INTERPRETING THE RESULTS

A 'conventional' approach to PF defined by $\mathrm{S} = ||\boldsymbol{v}||\ ||\boldsymbol{i}_{\mathrm{S}}||$ at the PCC neglects the resistance and inductance of the delivery system. Example 1 introduces a delivery system with unbalanced impedances and unbalanced sinusoidal voltages and currents; and Example 2 adds extra power with harmonic components. The results are compared in Table 7.

TABLE 7. Comparison of results of Examples 1 and 2.

	Example 1		Example 2	
	Convention'l	GPT	Convention'l	GPT
$\mathrm{P_{PCC}}$ delivered	85.0 kW	85.0 kW	96.9 kW	96.9 kW
PF	0.982	0.914	0.966	0.909
AP as defined	86.6 kVA	93.9 kW	100.3 kVA	107.5 kW
$\mathrm{P_{loss}}$ before comp.	893 W	893 W	958 W	958 W
Gen. capacity needed, $\mathrm{P_{Th\ bef}}$		85.9 kW		97.9 kW
Opt gen. capacity, same $\mathrm{P_{PCC}}$, $\mathrm{P_{Th\ aft}}$		85.8 kW		97.7 kW
Opt. loss $\mathrm{P_{loss\ aft}}$		746 W		792 W
$\mathrm{P_{loss}}$ avoided		147 W		166 W

Rigorously including unbalance (in Example 1) and unbalance and harmonics (Example 2) in the system models and calculations leads to lower measurements of the PF of the load on the system, compared with a conventional approach. Or, the other way around, violating assumptions of sinusoidal waveforms and balanced voltages, currents, and wire impedances, invalidates the measurement of PF based on a hypothetical ideal power system.

With a constraint that only fundamental frequency power is permitted, practically implemented by harmonic blocking filters, then the power P_{PCC} required by the load must be delivered by $\boldsymbol{I}_{\mathrm{A}}$ with only fundamental frequency components. Effectively, the 'benefit' of the parallel impedances of the extra lines at harmonic frequencies is lost, and the minimum loss in Example 2 increases to 971 W. A full study of power quality and optimal power delivery at high power factor is beyond the scope of this paper.

E. DEFINING PF AND AP

The PF of the system supplying the load at the PCC is an index of the *relative* delivery efficiency, given by the square root of (minimum possible loss by redistribution of the transmitted current components at the Thévenin source divided by the actual losses incurred in delivering to the PCC the original power without redistribution or compensation). Thus, the original statement in Section I needs to be clarified: "unity power factor means minimum possible line loss for a given total active power transmitted *from a source*."

The PF is also equal to the ratio P/AP, described in IEEE Std 1459-2010 [2] as a utilization factor indicator. Therefore, the AP (with units of Watts) is the maximum power that could be dispatched from the source by the optimum distribution of the components of the active current $\boldsymbol{I}_{\mathrm{A}}$ and with the same Thévenin source voltages, and delivered through the same system for the same delivery loss as the original uncompensated power P_{PCC} received at the PCC.

The maximum power received at the PCC for the same losses and after compensation is less than the AP by the quantity of the minimum loss. This differs from many interpretations of AP as being the power supplied to a load, such as in [2], which applies only when the delivery system has no impedance.

F. REVERSE POWER FLOW: DISTRIBUTED GENERATION

There are four approaches to controlling reactive power operation of DG. One is at a fixed specified level of PF operation. With voltage-var or active power-reactive power control, when voltage is high, the generator must be under-excited and absorb vars on a sliding scale up to the rated limits. Only under a voltage-active power characteristic must generation be reduced when voltages are high. Clearly, the three PF or reactive power approaches do not consider loss optimization and the potential of flexible control to reduce the loss to a minimum for the power delivered.

Especially as distortion and unbalance tend to be higher towards the lower voltage ends of a network, a more appropriate control of DG can be achieved by turning around the power flow of Fig. 1 and controlling the currents to minimize the loss in delivering power/energy to the network, thereby reducing the avoidable loss of generated energy on extra heating of the wires. The detailed approach to answering the third question in Section II will be described in a separate paper.

G. OTHER POTENTIAL APPLICATIONS

We conceive that in smart grids it will be economic to transport only the useful energy and supply locally the components of power for which no net energy is required. Power electronics provides this capability and will become even more significant with lower losses. Since battery systems (BESS) and most DG already include power electronic controllers, the adoption of more appropriate control algorithms using the losses-based definition of PF has the potential to reduce total energy losses, consistent with economic cost allocation.

Scott counselled in 1898 "While it is desirable, it is not as essential to have absolute scientific accuracy in all of the definitions as it is to have definite definitions of capacity and performance, and definite methods of testing which are mutually understood" [43]. Building on the research of many engineers during the subsequent 120 years, a representational, accurate, measurable definition of the minimum possible and avoidable losses attributable to a load at any point (PCC) is now available. This changes the perspective. The continued empirical approximation of PF, AP and reactive power in technical and economic studies where model assumptions are likely to be violated is no longer necessary. AP as an indicator of the utilization of the delivery system might be of interest to wires operators. Regulators can now take an evidence-based approach to rate-making that incorporates PF penalties or avoidable losses according to measurements that are uniquely defined. The NEMA meter study [12] and some preliminary research in energy trading [44] indicate the new definition could have significant financial implications when applied to metering and tariffs.

Other potential applications include economically justified load compensators, including STATCOMs, for loss reduction in both delivery systems and motor loads, and the injection of renewable energy into delivery systems. The definition opens new approaches to power quality assessment and voltage stability analysis (such as where distortion by geomagnetically induced currents compromises conventional approaches [7]), and wherever conventional approximations are inadequate.

H. EVALUATION AGAINST CRITERIA OF IEEE STD 1459

According to IEEE Std 1459- 2010 [2] a generalized power theory providing simultaneously for energy billing; evaluating electric energy quality; detecting the major sources of waveform distortion; and calculations for designing mitigation equipment was not available at that time. As proposed here, the new theory, calculations and measurement process can be applied:

- at a PCC anywhere in a power system, to measuring power, energy, AP, and avoidable loss in consistent units of W and Wh, and the PF as the relative loss efficiency of the power delivery, and these can be used in various combinations for electricity trading; and
- in calculations of the intra- and inter-wire energy transfer required in mitigation equipment, including the worst case conditions that might be defined as the basis of design of the equipment, and in operation.

The approach allows measurement of the actual delivery loss without optimal distribution of the current components and the loss avoidable by compensation. The PF before compensation measures the ratio of minimum possible to actual loss for an unchanged delivery of P_{PCC}. The AP measures the potential utilization of the delivery system for the original level of delivery loss. These parameters can be used for system and compensator design, and energy billing.

The approach does not differentiate current harmonics and unbalance, conventionally considered as being generated by the load, from the voltage harmonics and unbalance, because their effects are all combined in the measurement of one PF and one AP quantity that incorporate all the many degrees of freedom inherent in the measurement. It is possible, though, to identify the contribution of non-fundamental-frequency power components without compensation, and after compensation including or excluding all non-fundamental-frequency components. These do not evaluate an undefined 'energy quality' beyond these quantities and the conventional concepts of power quality may need to be re-visited.

VIII. CONCLUSION

In a 'perfect' balanced system with sinusoidal waveforms and all phase wires of equal and negligible impedance, the conventional approach to apparent and reactive power is valid, but it is impractical. Power factor used to describe the impedance of a load or appliance in the form of R/|Z|, independent of the delivery system, has only a weak relationship to the delivery losses or voltage drop in practical power systems.

Power systems with any number of wires, unbalance, dc components, periodic waveform distortion, and unequal and/or frequency-dependent wire resistances can only be represented adequately by a model incorporating all the variables, which is the approach taken in this paper.

The definitions of power and apparent power are derived in vector space linear algebra and are representational to the extent that the practical conditions do not violate the properties of the model. Power factor is a measure of the relative efficiency of delivering power to the PCC.

The approach is based on minimizing the loss associated with the delivery of power to the point of measurement. The two assumptions in the approach are that the supply system can be modelled by a Thévenin equivalent circuit and the active (or real) power delivered is constant.

We have shown that the calculation of the quantities of apparent power, power factor and delivery loss is relatively simple, and requires relatively little data transfer (as frequency components derived from measured voltages and currents) to an inexpensive processor where the calculation is made. The accuracy of the derived parameters obviously depends on the accuracy of measurement of voltages, currents and the Thévenin equivalent parameters of the system, and there are many methods for making such measurements, including by the compensator.

This formulation produces unambiguous and repeatable results, resolving the problems of all non-sinusoidal periodical and bandwidth-limited waveforms that can be resolved into a limited set of frequency components.

Steinmetz, Bell and Silsbee [4], [5] would have known the loss associated with a dc load at the PCC. A power factor rigorously defined in the presence of unbalance and distortion would have given them the avoidable "wattless" loss through PF^2 and enabled them to calculate the power needing to be sent out for each load. The AP would have them enabled them to calculate how much more power could have been sent out to optimally compensated loads for the same losses and thereby increase the utilization of the delivery system. These same concepts are still useful for power system design and operation and are embedded in many tariffs, though not always with scientific accuracy.

Power factor, apparent power, reactive power, and non-active power as defined in IEEE Standard 1459-2010 and other standards clearly do not lead to representational measurement and expose users to the uncertainty inherent in operational measurements in practical systems.

The scientific accuracy of this proposed novel, rigorous and practical (general) definition of power factor and apparent power avoids the assumptions made in all other definitions we have found, approaches representative measurement, questions fundamentally the validity of the concept of reactive power and its non-power units, enables evidence-based decisions on matters such as cost allocation and whether mitigation of relatively inefficient power delivery is justifiable, and has profound implications for the definitions of power quantities in engineering standards, mitigation equipment design, power system analysis, electricity regulation, and education and further research.

APPENDIX

The contents of selected cells from Examples 1 and 2 are listed, identified by the row and column in Tables 4 and 5. These Excel formulae in rows 19 to 84 of the spreadsheet are the arithmetic symbol representation of the equations in the linear algebra.

$$\begin{aligned}
C19 &= C4{*}C9{*}COS(RADIANS(D4\text{-}D9)) \\
&\quad + C5{*}C10{*}COS(RADIANS(D5\text{-}D10)) \\
&\quad + C6{*}C11{*}COS(RADIANS(D6\text{-}D11)) \\
&\quad + C7{*}C12{*}COS(RADIANS(D7\text{-}D12)) \\
C20 &= C19 + E19 + F19 + H19 \\
C21 &= C9\text{^}2{*}C15 + C10\text{^}2{*}C16 + C11\text{^}2{*}C17 \\
&\quad + C12\text{^}2{*}C18 \\
C22 &= C21 + E21 + F21 + H21 \\
C23 &= SQRT(C22) \\
C24 &= C19 + C21 \\
C25 &= C24 + E24 + F24 + H24
\end{aligned}$$

$$\begin{aligned}
C26 = {} & ((C9{\wedge}2 + C10{\wedge}2 + C11{\wedge}2 + C12{\wedge}2) \\
& + (E9{\wedge}2 + E10{\wedge}2 + E11{\wedge}2 + E12{\wedge}2) \\
& + (F9{\wedge}2 + F10{\wedge}2 + F11{\wedge}2 + F12{\wedge}2) \\
& + (H9{\wedge}2 + H10{\wedge}2 + H11{\wedge}2 + H12{\wedge}2) \\
& + (K9{\wedge}2 + K10{\wedge}2 + K11{\wedge}2 + K12{\wedge}2)){\wedge}0.5
\end{aligned}$$

$$\begin{aligned}
C27 = {} & ((C4{\wedge}2 + C5{\wedge}2 + C6{\wedge}2 + C7{\wedge}2) \\
& + (E4{\wedge}2 + E5{\wedge}2 + E6{\wedge}2 + E7{\wedge}2) \\
& + (F4{\wedge}2 + F5{\wedge}2 + F6{\wedge}2 + F7{\wedge}2) \\
& + (H4{\wedge}2 + H5{\wedge}2 + H6{\wedge}2 + H7{\wedge}2) \\
& + (K4{\wedge}2 + K5{\wedge}2 + K6{\wedge}2 + K7{\wedge}2)){\wedge}0.5
\end{aligned}$$

$$C28 = C26^{*}C27$$

$$C29 = C20/C28$$

$$\begin{aligned}
C31 = {} & \mathrm{IMABS(COMPLEX}((C4^{*}\mathrm{COS(RADIANS}(D4)) \\
& + C9^{*}\mathrm{COS(RADIANS}(D9))^{*}C15 \\
& - C9^{*}\mathrm{SIN(RADIANS}(D9))^{*}D15), \\
& (C4^{*}\mathrm{SIN(RADIANS}(D4)) \\
& + C9^{*}\mathrm{SIN(RADIANS}(D9))^{*}C15 \\
& + C9^{*}\mathrm{COS(RADIANS}(D9))^{*}D15)))
\end{aligned}$$

$$\begin{aligned}
D31 = {} & \mathrm{IMARGUMENT(COMPLEX} \\
& ((C4^{*}\mathrm{COS(RADIANS}(D4)) \\
& + C9^{*}\mathrm{COS(RADIANS}(D9))^{*}C15 \\
& - C9^{*}\mathrm{SIN(RADIANS}(D9))^{*}D15), \\
& (C4^{*}\mathrm{SIN(RADIANS}(D4)) \\
& + C9^{*}\mathrm{SIN(RADIANS}(D9))^{*}C15 \\
& + C9^{*}\mathrm{COS(RADIANS}(D9))^{*}D15)))^{*}180/\mathrm{PI}()
\end{aligned}$$

$$\begin{aligned}
C36 = {} & \mathrm{IMABS(COMPLEX}((C31/C15)^{*} \\
& \mathrm{COS(RADIANS}(D31)) \\
& + (C32/C16)^{*}\mathrm{COS(RADIANS}(D32)) \\
& + (C33/C17)^{*}\mathrm{COS(RADIANS}(D33)) \\
& + (C34/C18)^{*}\mathrm{COS(RADIANS}(D34)), \\
& (C31/C15)^{*}\mathrm{SIN(RADIANS}(D31)) \\
& + (C32/C16)^{*}\mathrm{SIN(RADIANS}(D32)) \\
& + (C33/C17)^{*}\mathrm{SIN(RADIANS}(D33)) \\
& + (C34/C18)^{*}\mathrm{SIN(RADIANS}(D34))))/ \\
& (1/C15 + 1/C16 + 1/C17 + 1/C18)
\end{aligned}$$

$$\begin{aligned}
D36 = {} & \mathrm{IMARGUMENT(COMPLEX}((C31/C15)^{*} \\
& \mathrm{COS(RADIANS}(D31)) \\
& + (C32/C16)^{*}\mathrm{COS(RADIANS}(D32)) \\
& + (C33/C17)^{*}\mathrm{COS(RADIANS}(D33)) \\
& + (C34/C18)^{*}\mathrm{COS(RADIANS}(D34)), \\
& (C31/C15)^{*}\mathrm{SIN(RADIANS}(D31)) \\
& + (C32/C16)^{*}\mathrm{SIN(RADIANS}(D32)) \\
& + (C33/C17)^{*}\mathrm{SIN(RADIANS}(D33)) \\
& + (C34/C18)^{*}\mathrm{SIN(RADIANS}(D34))))^{*}180/\mathrm{PI}()
\end{aligned}$$

$$\begin{aligned}
C38 = {} & \mathrm{IMABS(COMPLEX}(C31^{*}\mathrm{COS(RADIANS}(D31)) \\
& - C36^{*}\mathrm{COS(RADIANS}(D36)), \\
& C31^{*}\mathrm{SIN(RADIANS}(D31)) \\
& - C36^{*}\mathrm{SIN(RADIANS}(D36))))
\end{aligned}$$

$$\begin{aligned}
D38 = {} & \mathrm{IMARGUMENT(COMPLEX}(C31^{*} \\
& \mathrm{COS(RADIANS}(D31)) \\
& - C36^{*}\mathrm{COS(RADIANS}(D36)), \\
& C31^{*}\mathrm{SIN(RADIANS}(D31)) \\
& - C36^{*}\mathrm{SIN(RADIANS}(D36))))^{*}180/\mathrm{PI}()
\end{aligned}$$

$$\begin{aligned}
C43 = {} & C38{\wedge}2/C15 + C39{\wedge}2/C16 \\
& + C40{\wedge}2/C17 + C41{\wedge}2/C18
\end{aligned}$$

$$C44 = C43 + E44 + F44 + H44$$

$$C46 = C44{\wedge}0.5$$

$$C47 = (1/2^{*}(\mathrm{SQRT}(C44) - \mathrm{SQRT}(C44 - 4^{*}C20))){\wedge}2$$

$$C48 = \mathrm{SQRT}(C47/C22)$$

$$C49 = C23^{*}C46$$

$$C50 = C22 - C47$$

$$C52 = C20 + C47$$

$$C53 = C52/C44$$

$$C54 = C53^{*}C38/C15$$

$$D54 = D38$$

$$\begin{aligned}
C59 = {} & C54{\wedge}2^{*}C15 + C55{\wedge}2^{*}C16 + C56{\wedge}2^{*}C17 \\
& + C57{\wedge}2^{*}C18
\end{aligned}$$

$$C60 = C59 + E59 + F59 + H59$$

$$\begin{aligned}
C62 = {} & \mathrm{IMABS(COMPLEX}(C31^{*}\mathrm{COS(RADIANS}(D31)) \\
& - C54^{*}C15^{*}\mathrm{COS(RADIANS}(D54)) \\
& + C54^{*}D15^{*}\mathrm{SIN(RADIANS}(D54)) \\
& - (C34^{*}\mathrm{COS(RADIANS}(D34)) \\
& - C57^{*}C18^{*}\mathrm{COS(RADIANS}(D57)) \\
& + C57^{*}D18^{*}\mathrm{SIN(RADIANS}(D57))), \\
& C31^{*}\mathrm{SIN(RADIANS}(D31)) \\
& - C54^{*}C15^{*}\mathrm{SIN(RADIANS}(D54)) \\
& - C54^{*}D15^{*}\mathrm{COS(RADIANS}(D54)) \\
& - (C34^{*}\mathrm{SIN(RADIANS}(D34)) \\
& - C57^{*}C18^{*}\mathrm{SIN(RADIANS}(D57)) \\
& - C57^{*}D18^{*}\mathrm{COS(RADIANS}(D57)))))
\end{aligned}$$

$$\begin{aligned}
D62 = {} & \mathrm{IMARGUMENT(COMPLEX}(0.00000001 \\
& + C31^{*}\mathrm{COS(RADIANS}(D31)) \\
& - C54^{*}C15^{*}\mathrm{COS(RADIANS}(D54)) \\
& + C54^{*}D15^{*}\mathrm{SIN(RADIANS}(D54)) \\
& - (C34^{*}\mathrm{COS(RADIANS}(D34)) \\
& - C57^{*}C18^{*}\mathrm{COS(RADIANS}(D57)) \\
& + C57^{*}D18^{*}\mathrm{SIN(RADIANS}(D57))), \\
& C31^{*}\mathrm{SIN(RADIANS}(D31)) \\
& - C54^{*}C15^{*}\mathrm{SIN(RADIANS}(D54))
\end{aligned}$$

$$
\begin{aligned}
&- \text{C54*D15*COS(RADIANS(D54))}\\
&- \text{(C34*SIN(RADIANS(D34))}\\
&- \text{C57*C18*SIN(RADIANS(D57))}\\
&- \text{C57*D18*COS(RADIANS(D57)))))*180/PI()}
\end{aligned}
$$

$$
\begin{aligned}
\text{C66} &= \text{C62*C54*COS(RADIANS(D62} - \text{D54))}\\
&\quad + \text{C63*C55*COS(RADIANS(D63} - \text{D55))}\\
&\quad + \text{C64*C56*COS(RADIANS(D64} - \text{D56))}\\
&\quad + \text{C65*C57*COS(RADIANS(D65} - \text{D57))}\\
\text{C67} &= \text{C66} + \text{E66} + \text{F66} + \text{H66}\\
\text{C68} &= \text{C38*C54*COS(RADIANS(D38} - \text{D54))}\\
&\quad + \text{C39*C55*COS(RADIANS(D39} - \text{D55))}\\
&\quad + \text{C40*C56*COS(RADIANS(D40} - \text{D56))}\\
&\quad + \text{C41*C57*COS(RADIANS(D41} - \text{D57))}\\
\text{C69} &= \text{C68} + \text{E68} + \text{F68} + \text{H68}\\
\text{C71} &= \text{IMABS(COMPLEX(C9*COS(RADIANS(D9))}\\
&\quad - \text{C54*COS(RADIANS(D54)),}\\
&\quad \text{C9*SIN(RADIANS(D9))}\\
&\quad - \text{C54*SIN(RADIANS(D54))))}\\
\text{D71} &= \text{IMARGUMENT(COMPLEX(C9*}\\
&\quad \text{COS(RADIANS(D9))}\\
&\quad - \text{C54*COS(RADIANS(D54)),}\\
&\quad \text{C9*SIN(RADIANS(D9))}\\
&\quad - \text{C54*SIN(RADIANS(D54))))*180/PI()}\\
\text{C77} &= \text{C71\^{}2*C15} + \text{C72\^{}2*C16} + \text{C73\^{}2*C17}\\
&\quad + \text{C74\^{}2*C18}\\
\text{C78} &= \text{C77} + \text{E77} + \text{F77} + \text{H77}\\
\text{C79} &= \text{2*(C54*C71*COS(RADIANS(D54} - \text{D71))*C15}\\
&\quad + \text{C55*C72*COS(RADIANS(D55} - \text{D72))*C16}\\
&\quad + \text{C56*C73*COS(RADIANS(D56} - \text{D73))*C17}\\
&\quad + \text{C57*C74*COS(RADIANS(D57} - \text{D74))*C18)}\\
\text{C80} &= \text{C79} + \text{E79} + \text{F79} + \text{H79}\\
\text{C81} &= \text{C78} + \text{C80}\\
\text{C82} &= \text{C52/C49}\\
\text{C83} &= \text{SQRT(C47)}\\
\text{C84} &= \text{C83/C23}
\end{aligned}
$$

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