

Received July 2, 2020, accepted July 12, 2020, date of publication July 17, 2020, date of current version July 29, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3009975

Traffic Resource Allocation for Multi-Layer Networks

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This work was supported in part by the University Scientific Research Foundation of Hebei Province under Grant QN2019081, and in part by the Research on the Development of Social Science of Hebei Province under Grant 2019041201007.

ABSTRACT Traffic dynamics on multi-layer networks are of great importance to alleviate traffic congestion of real networks. In this work, we focus on information transport in multi-layer networks comprised of two coupled subnetworks. We investigate the relationship between correlated inter-layer couplings and resource allocation strategies on the traffic dynamics of multi-layer networks. Simulation results show that assortative coupling (AC) structure can improve the multi-layer networks traffic capacity more better than disassortative coupling (DC) and random coupling (RC) with degree-based (DE) efficient resources allocation strategy, and DC structure is better in enhancing the traffic capacity with the simplest average resources allocation (SA) strategy. Our work may shed some light on the traffic dynamics on multi-layer networks.

INDEX TERMS Multi-layer networks, traffic capacity, coupling correlation, betweenness centrality, resource allocation.

I. INTRODUCTION

In recent years, the complex network theory has become an important method for describing the structure and dynamics of traffic networks. Because of the growing demand for big data, a lot of networked infrastructures would confront the challenges of traffic congestion. It has been a more and more important issue to ensure uncongested traffic flows on such networked systems from the view of complex network framework in recent decades [1]–[4]. The study on the traffic capacity of complex network has gradually become a topic of theoretical interest in network science [5]–[7]. There are three ways to enhance the traffic capacity of complex network: adjusting the network topology, reallocating limited network resources, and developing effective routing strategies. The former two methods are considered as the “hard” strategies because they require changes the network structure or the resource allocation of the network. However, these strategies usually involve remarkable expense in either manpower or energy. Conversely, a heuristic routing strategy can be easily implemented through a software approach, which is considered a “soft” strategy [8]. Yang *et al.* [9] proposed a limited packet delivering capacity model for traffic dynamics in scale-free networks. With this model, the

total nodes packet delivering capacity was fixed, and each node packet delivering capacity was allocated proportional to its degree. He *et al.* [10] presented a node delivering capacity allocation strategy in which each node capacity was composed of uniform fraction and degree related proportion. Based on the node capacity allocation strategy, a new routing mechanism was proposed, called efficient weighted routing strategy, to improve network traffic capacity and transmission efficiency. Zhu *et al.* [11] proposed a heterogeneous node delivering capacity allocation strategy on a scale-free network to improve transport efficiency of packets. Zhao *et al.* [12] proposed a node delivering capacity allocation mechanism to control traffic congestion. Zhang *et al.* [13] proposed a dynamic queue resource allocation strategy for traffic in scale-free networks. In the case of limited queue resources, the queue length was reasonably distributed according to the betweenness of the nodes, and the performance of this strategy was better than the uniform queue length allocation strategy.

In fact, most actual networks are revealed to have a two-layer or multi-layer structure. For instance, the topologies of the Internet, the world wide web (WWW) and the peer to peer (P2P) networks are of two-layer structure [14]–[17]. The different layers of these networks do not exist independently, but interact with and depend on each other. The traffic capacity of multi-layer networks, which is determined

The associate editor coordinating the review of this manuscript and approving it for publication was Bohui Wang[✉].

by all of the layers, has become a hot research topic. For instance, in the communication network, the application layer network is usually mapped on the IP network, and the topology of each layer is different. The top layers of the Internet, WWW and P2P networks are logical sub-networks composed of virtual links. The network transmission tasks are mainly undertaken on the physical networks. In other words, the logical networks formed by applications are mapped on the underlying internet protocol (IP) networks. Then the application layer (logical layer) and the IP layer (physical layer) jointly restrict the traffic capacity of the multi-layer networks. Similarly, in a railway transportation network, each side of the logical layer is connected to the first and last stations of a particular train route [18]. Its transportation task is mapped on a specific physical railway network. Since the two-layer structure or the multi-layer structure can accurately reflect the topological characteristics of the actual networks, it is of great practical significance to study the traffic dynamic behaviour of multi-layer networks. Therefore, the research work in this field focuses on the analysis and optimization of traffic capacity of multi-layer networks to reduce network congestion. These overwhelming traffic demands have brought a heavy burden on network infrastructures [12]. In this paper, we focus on the resource allocation strategy with different inter-layer couplings to enhance the traffic capacity of two-layer network. In real networks, the total node delivering capacity may be finite and the resource allocation schemes have been designated when the networks are born. When the structure of the network model is determined, the nodes are allocated reasonable delivering capacity according to the correlation of the nodes betweenness, so that the traffic capacity of the network reaches the best. Generally, nodes with the larger degrees are often regarded as central nodes, and their delivering capacities should be set bigger [19]. At the same time, the coupling correlation between the upper and lower layers can be optimized to improve traffic efficiency of the two-layer network. Both the reasonable resource allocation strategies and coupling methods of two-layer network can increase the traffic capacity to avoid network congestions. Therefore, it is necessary to study the relationship between correlated inter-layer couplings and resource allocation strategies on the traffic dynamics of two-layer network.

The remainder of this paper is organized as follows. In Section II, we introduce relevant research work in recent years. In Section III, the network models, coup correlations and specific resource allocation strategies are described. In Section IV, the simulation experiment results and analysis are shown. Finally, we conclude our work in Section V. It is confirmed that the coupling characteristics between two-layer network have an impact on the traffic capacity in different resource allocation strategies.

II. RELATED WORKS

The research of traffic capacity allocation strategy on two-layer network is essential, and the ideal allocation strategy can be helpful in optimizing traffic performance of real

networks. The shortest path routing (SPR) strategy is a traditional routing, and its the advantage is that packets can reach the destination as quickly as possible, but contestability these packets would quickly become congested on the central node [19], [20]. Kurant and Thiran [14] abstracted the actual physical layer structure and traffic information on the public transmission system into a two-layer network model. The logical layer structure was usually regarded as the upper layer, and the physical infrastructure was regarded as the lower layer. Wang *et al.* [21] designed a method to model a chinese train-station bipartite network into a weighted station network. A new metric was proposed to quantify the dependence between pairs of stations, which was shown to follow a shifted power-law distribution. Zou *et al.* [15] proposed a method of projecting a two-way network of chinese railway stations into a weighted station network. A new parameter was proposed to quantify the way in which the dependencies between site pairs show a shift following a power-law distribution. Pu *et al.* [18] studied the information transport in multi-layer networks, and found that the disassortative coupling was better than the random coupling and the assortative coupling. Gu *et al.* [22] proposed a model of traffic dynamics and revealed a transition at the onset of cooperation between layered networks. The cooperation strength, treated as an order parameter, changed from zero to positive at the transition point. Simulation results on artificial networks as well as two real chinese and european railway airline transportation networks, agreed with the analysed conclusions. Morris and Barthelemy [17] studied the transport processes on a two-layer network, and results indicated that coupled spatial systems could give rise to behavior that relies subtly on the interplay between the coupling and randomness in the source-sink distribution. Zhou *et al.* [23] studied an optimal routing strategy on the two-layer networks composed of two layers subnets, in which one subnet was a wireless network and the other was a wired network. The single-channel transmission mode and multi-channel transmission mode used in the wireless subnet were considered separately, and this method significantly could improve the traffic capacity. Du *et al.* [24] established a double-layered spatial network model that the low-speed lower layer was a regular lattice network, while the high-speed upper layer was a scale-free network embedded in the lattice network. The traffic dynamics on the established networks were investigated and found that the speed ratio of the two layers could affect the traffic flow via adjusting the time cost in the upper layer. Moreover, the cooperation between the two layers could result in the optimal network capacity. Tan *et al.* [25] explored the impact of interconnection methods on the traffic capacity in interconnected scale-free networks. They found that assortative coupling could increase delivering capacity more readily than disassortative and random coupling when the node delivering capacity was allocated based on node usage probability. Furthermore, the optimal coupling probability have been found for assortative coupling. Zhang *et al.* [26] removed edges between nodes with large betweenness to optimize the

structure of upper-layer network, and results showed that the traffic capacity of the two-layer network could be significantly improved with both the shortest path routing strategy and efficient routing strategy. Nian and Fu [27] compared three routing strategies, and investigated the variation of traffic capacity with coupling randomness. Zhang et al. [28] proposed a static weight routing strategy which considered the local information of the physical layer and the logical layer, and found that there exists an optimal value leading to the maximal network capacity. Ma et al. [29] proposed an improved global awareness routing (IGAR) strategy on two-layer network, which assigned appropriate weights to the physical links according to the betweenness. With the IGAR strategy, the traffic capacity of two-layer network can be increased more effectively.

In our model, we analyze the inter-layer couplings of multi-layer networks on traffic capacity, and investigate the relationship between resource allocation strategies and traffic capacity of the multi-layer networks. Furthermore, we determine the optimal resource allocation strategy parameters in the same coupling mode.

III. THE MODEL AND DEFINITIONS

A. COUPLED NETWORK MODEL

The degree distributions of many large-scale traffic networks, such as the Internet, phone networks, and airport networks, obey the power-law distribution $P(k) \sim k^{-\tau}$. The BA scale-free network is a widely used model that can construct networks with a power-law degree distribution. Therefore, we built a two-layer network topology on the BA model, which can be constructed through the two connection attachment of “growth” and “preferential” [2]. The lower layer of the multi-layer networks is called the physical layer $G_p = (V_p, E_p)$ while the upper layer can be named the logical layer $G_l = (V_l, E_l)$. The logical layer and the physical layer remain relatively independent, and the nodes of the two layers are coupled by one-to-one relationship according to the growth of the network model and the priority connection characteristics. Two important mechanisms are defined: growth attachment: start with m_0 fully connected nodes as seeds in the initial network, add a new node to the existing network at each time step, and then connect this node to m ($m \leq m_0$) existing nodes; preferential attachment: a new node will connect with an existing node i with probability $P(k_i) = k_i / \sum_{j=1} k_j$, where k_i is the degree of node i , k_j is the degree of j and i runs over all existing nodes. After t time step, this model will generate a scale-free network which consists of $t + m_0$ nodes and mt edges. The upper logical layer data packets are transmitted according to a given routing table, and each logical edge between two nodes is mapped to a physical layer routing path corresponding to the physical given routing table. The physical layer, which does not care about the routing table of the logical layer, provides stable transport channels to the logical layer, in which is only transmits the data packet from the source node to the destination node according to the physical routing table. As shown in Figure 1, we set the two-layer

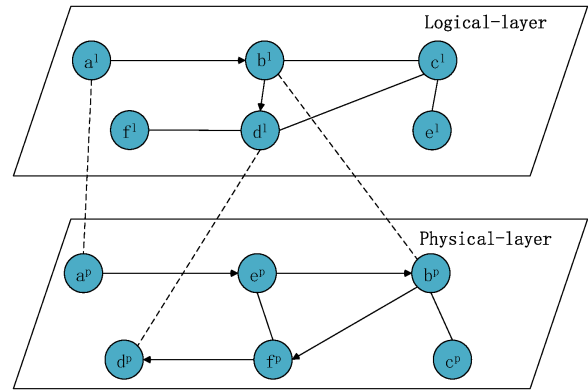


FIGURE 1. Legend of multiplex networks.

network to be adopt the same SPR routing policy. The traffic routing between a^l and d^l in the logical layer is $RP_{a^l, d^l} = \{a^l, b^l, d^l\}$ the corresponding logical edge is l_{a^l, b^l} and l_{b^l, d^l} . The logical edge maps to the physical layer corresponding to the path $\{a^p, e^p, b^p\}$ and $\{b^p, f^p, d^p\}$. The entire traffic routing of the physical layer packet from the source node to the destination node is $RP'_{a^p, d^p} = \{a^p, e^p, b^p, f^p, d^p\}$.

The topology of the multi-layer networks plays a crucial role in the information traffic dynamics process. When structural correlation exists between the two layers, the traffic capacity of the whole network system may be enhanced or weakened [30], [31]. According to the degree correlation, the correlation between the layers can be quantified by the spearman rank correlation coefficient as [20], [28], [32], [33]

$$P_r = 1 - 6 \frac{\sum_{i=0}^N \Delta_i^2}{N(N^2 - 1)}, \quad (1)$$

where N denotes network size, Δ_i^2 indicates the difference in node degrees between the multiplex networks, P_r indicates the degree of network coupling. The range of P_r is $[-1, 1]$, as $P_r \approx -1$ in this case, there is a maximum negative correlation of the network, that is disassortative coupling. When all nodes are randomly linked, the layers of the network are completely uncorrelated, at this time $P_r \approx 0$, in this case we have

$$6 \frac{\sum_{i=0}^N \Delta_i^2}{N(N^2 - 1)} \approx 1. \quad (2)$$

In a two-layer network with the greatest positive correlation, any pair of nodes with the same level in each layer are matched, which is $\Delta_i^2 = 0$, indicates that any pair of matching nodes have the smallest difference. So we have $P_r \approx 1$, according to equation (1) after random rematching, one pair of nodes $\Delta_i^2 = 0$, then it is possible that the difference between $1 - q$ and Δ'_i may be q . Equation (1) can be written as

$$P_r = 1 - 6 \frac{q \sum_{i=0}^N \Delta_i^2}{N(N^2 - 1)}, \quad (3)$$

combining equations (2) and (3) can get the correlation coefficient after rematching

$$P_r \approx 1 - q. \quad (4)$$

In the previous work, three kinds of coupling patterns based on the degrees of nodes have been considered [34].

1) ASSORTATIVE COUPLING (AC)

Positive correlation, the nodes in the two-layer network are coupled according to the degree, random order if there are points with the same degree of existence. According to $L_i \rightarrow P_i$, the internal one-to-one relationship between multi-layer networks, the node degree values of the logical layer and the physical layer are sorted from large to small. The logical layer and the physical layer are coupled with the nodes by large to large degree, and low to low degree, which are one-to-one coupling type. In the AC mode, a node with a large moderate value is connected to a node with a large preference, and it is easier to transmit quickly under certain transmission routing conditions.

2) DISASSORTATIVE COUPLING (DC)

Negative correlation, in contrast to AC, the logical layer nodes are sorted according to the degree value from large to small, while the physical layer nodes are sorted according to the degree value from small to large. If there are nodes with the same degree of value, they are randomly ordered. According to $L_i \rightarrow P_i (j = N - i + 1)$, a one-to-one coupling between the multi-layer networks are established, that is, the nodes of logical layer with larger degrees are coupled with those nodes of with the physical layer with smaller degrees, forming two networks with one-to-one coupling type. In the DC mode, a node with a large degree of value is connected to a node with a small preference. Under certain transmission routing conditions, the greater the node difference, the easier it is to transmit quickly.

3) RANDOM COUPLING (RC)

Random coupling, the nodes of the logical layer and the physical layer are randomly coupling with one-to-one coupling type. In the RC mode, the connection between nodes is random, showing different transmission characteristics under different routing strategies.

B. TRAFFIC MODEL

In this model, each node of the upper and lower layers can be considered as both host and router, and its queue buffer is assumed infinite. The data packets transport in the entire network is a dynamic iterative process. Each time step, the entire network generates R packets (packet generation rate). The source and destination addresses of each data packet are randomly selected differently. Set the data packet delivering capacity of each node in the network to C , that is, each node can deal with C data packets at each time step [30]–[32]. When data packet is generated, it is placed at the end of

the queue of the node and transmitted by the FIFO (first-in-first-out) rule. Packets on the logical layer are transmitted by the established routing strategy to find a corresponding path for transmission. If there are more than one paths, randomly select one, and when they reach the destination node, they are removed [8], [35]. In this brief, packets are considered being sent in discrete time steps. When the network is in a free flow state, there is few data packet congestion at each node. With the increase of data packet generation rate R , the network will go from free state to congestion state. There is a critical generation rate R_c which equals the average number of the newly generated packets per node each time step when the phase transition occurs in the network. When the data packet generation rate $R < R_c$, the generated data packet and the removed data packet is in equilibrium, so the network is in self-contained state. By the state of circulation, when $R = R_c$, the network is in a transition state from free state to congestion state. As $R > R_c$, the number of generated packets is larger than that removed, which makes the network congested. An ordered parameter $H(R)$ is usually used to describe as

$$H(R) = \lim_{t \rightarrow \infty} \frac{C < \Delta W >}{R < \Delta t >}, \quad (5)$$

among them, ΔW denotes the amount of data packets in the network from time t to $t + \Delta t$ and the growth rate of data packets at each time when the rate of data packet generation is R [36]–[38]. As $R < R_c$, $H(R) = 0$, the network is in free flow state. As $R > R_c$, $H(R) > 0$, a number of data packets are in queue, and the network is in a congested state. Therefore, R_c is the largest data packet generation rate in the network under the condition of maintaining free flow.

C. PERFORMANCE PARAMETER

1) THRESHOLD OF PACKET GENERATION RATE

As shown in equations (5), the change of the order parameter is used to represent the change of the network state. When $H(R)$ increase rapidly from 0, a network phase change occurs, and the value of R is the critical value of the packet generation rate R_c . R_c is the maximum packet generation rate when the network can process data packets normally. As long as the packet generation rate R does not exceed this threshold, the network packet transport can be free and unimpeded. When the packet generation rate R increases to R_c , the network changes from free state to congested state, and the data packets start to stay in the network. R_c is an important parameter to measure network traffic capacity and an important indicator to describe the dynamic behaviour of network information flow.

2) AVERAGE PACKET TRANSMISSION TIME

To improve traffic efficiency, the transmission time of data packets should be shortened as much as possible in general. The average transmission time is also an important factor for measuring the traffic dynamics of multi-layer networks. Average transmission time $\langle T \rangle$ of a data packet is defined as

the average time it takes in the network, that is the time in which the data packet is transmitted from the source node to destination node. Its evolution equations are given as

$$\langle T \rangle = \lim_{x \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n T_i, \quad (6)$$

where n is the number of packets arrival at the destination node within a specified time. $\langle T \rangle$ is the transmission time of the data packet i , which includes the transmission time in the data packet network and the waiting time at the congested node. Information transmission will encounter the problem of delay [39], [40]. The data packet will be blocked during the transmission process and cause delay, which has been taken into account in the simulation experiment. The average transmission time $\langle T \rangle$ at this time includes the normal transmission time of the transmitted data packet and the time of waiting on the link.

When R is less than R_c , $\langle T \rangle$ depends only on the transmission time, which is relatively small. However, when $R > R_c$, $\langle T \rangle$ increases with R rapidly.

3) AVERAGE AMOUNT OF INFORMATION

Another important parameter to measure traffic efficiency is the average amount of information. Define the average amount of information $\langle N_t \rangle$ of the packets as: the average number of data packets reach the destination node in a time step. The evolution equations are given as

$$\langle N_t \rangle = \lim_{t \rightarrow \infty} \frac{1}{\Delta T} \sum_{i=1}^N m_i(t), \quad (7)$$

where t is the simulation time, ΔT is the time window and $\langle N_t \rangle$ is the number of network nodes, $m_i(t)$ is the number of data packets reach the destination node i at time t .

4) AVERAGE PATH LENGTH

Average path length $\langle L \rangle$ is the average length between any two nodes, and its evolution equations are given as

$$\langle L \rangle = \frac{1}{\frac{1}{2}N(N+1)} \sum_{i \geq j} d_{ij}, \quad (8)$$

where N is the total number of nodes in the network, and d_{ij} is the length between node i and node j . If the SPR strategy is adopted, $\langle L \rangle$ is the average shortest path length. As shown in the equation (8), a smaller $\langle L \rangle$ means that the average data packet reaches the destination node faster, and indicates that the network transmission performance is better. In the process of studying the traffic dynamics, $\langle L \rangle$ can be used to measure the traffic efficiency of multi-layer networks.

D. STRATEGY ANALYSIS

In this study, the SPR strategy is adopted in multi-layer networks. The nodes with high degree may be the nodes with heavy load, and should be allocated more delivering capacity. In order to make the traffic flow of the whole network

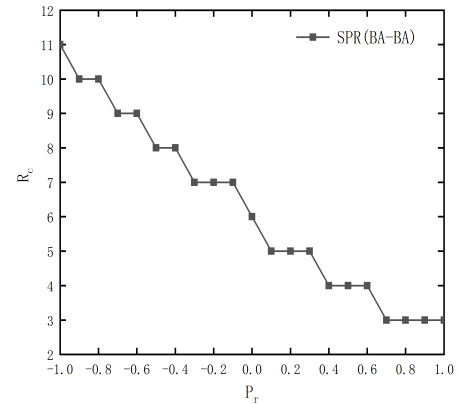


FIGURE 2. The relationship between the coupling correlation coefficient P_r and the critical value R_c of the packet generation rate, when the SPR strategy is adopted.

more uniform, the delivering capacity of the nodes should be reasonably allocated. Both the frequently used simplest average resource allocation (SA) strategy and degree-based efficient (DE) resource allocation strategy are used as comparison, and the effects of the two different resource allocation strategies on delivering capacity are analyzed through experiments under the optimal condition of continuous coupling of multi-layer networks. The corresponding delivering capacity of physical layer node i^p is set to C_i , which means that node i^p can deliver C packets per time step. The average delivering capacity of each node in the physical layer is as

$$\langle C \rangle = \frac{1}{N} \sum_{i=1}^N C_i, \quad (9)$$

where N is the network size, and C_i is the delivering capacity of the node i^p . In order to reasonably allocate the node delivering capacity, the total delivering capacity of the nodes in the network $N\langle C \rangle$ are reallocated according to the DE allocation strategy, and the delivering capacity allocated to physical layer node i^p is as

$$C_i = \frac{(k_{i^l} + k_{i^p})^\alpha}{\sum_{j=1}^N (k_{j^l} + k_{j^p})^\alpha} N\langle C \rangle, \quad (10)$$

where k_{i^l} is the degree of the logical layer node i^l , k_{i^p} is the degree of the physical layer node i^p , k_{j^l} and k_{j^p} are the pair of the upper and lower layer nodes, and α is a tunable parameter, especially when $\alpha = 0$, it is the traditional SA allocation strategy. Assigning node delivering capacity accorded to equation (10) may cause C_i to have decimals, which can be handled according to the following principles that the delivering capacity of the node i^p is C_i , and the node i^p can process $\lfloor C_i \rfloor$ data packets every time step. Then $(C_i - \lfloor C_i \rfloor)$ is compared with a random number r from 0 to 1. If $r < (C_i - \lfloor C_i \rfloor)$, the node i^p can process one more packet, that is, the node i^p can processed $(\lfloor C_i \rfloor + 1)$ packets; if $r \geq (C_i - \lfloor C_i \rfloor)$, node i^p can only process $(\lfloor C_i \rfloor)$ packets at each time step.

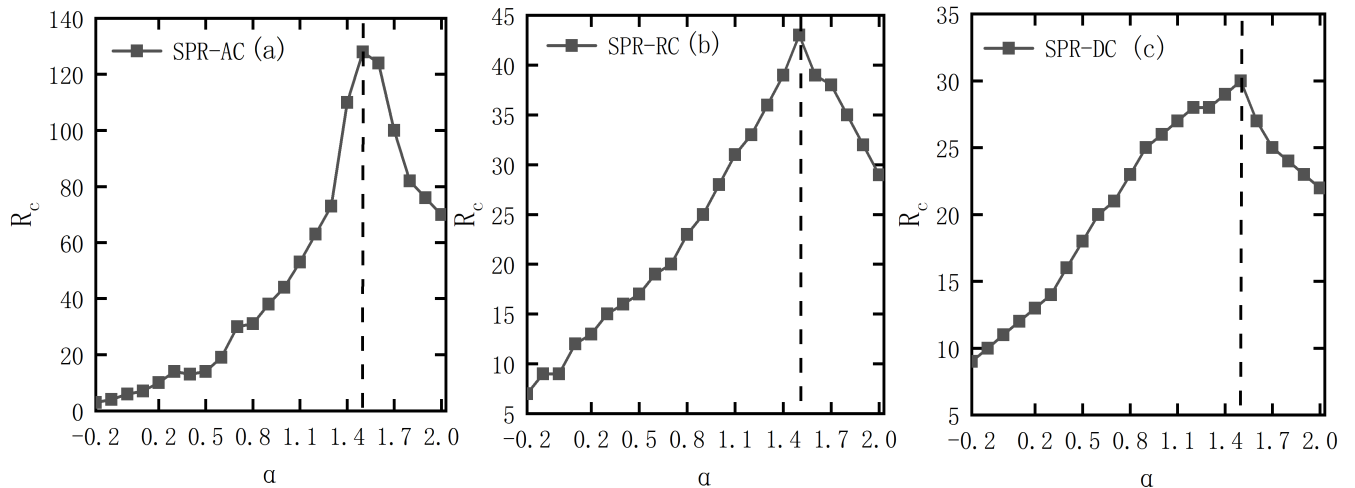


FIGURE 3. (a) AC Coupling. (b) RC Coupling. (c) DC Coupling. In a double-layer BA-BA network, set the network size $N = 400$, the average processing capacity (C) = 4, and the average degree (k) = 8. When the SPR strategy and DE distribution strategy are adopted, the relationship between the critical value R_c of the packet generation rate and the control parameter α .

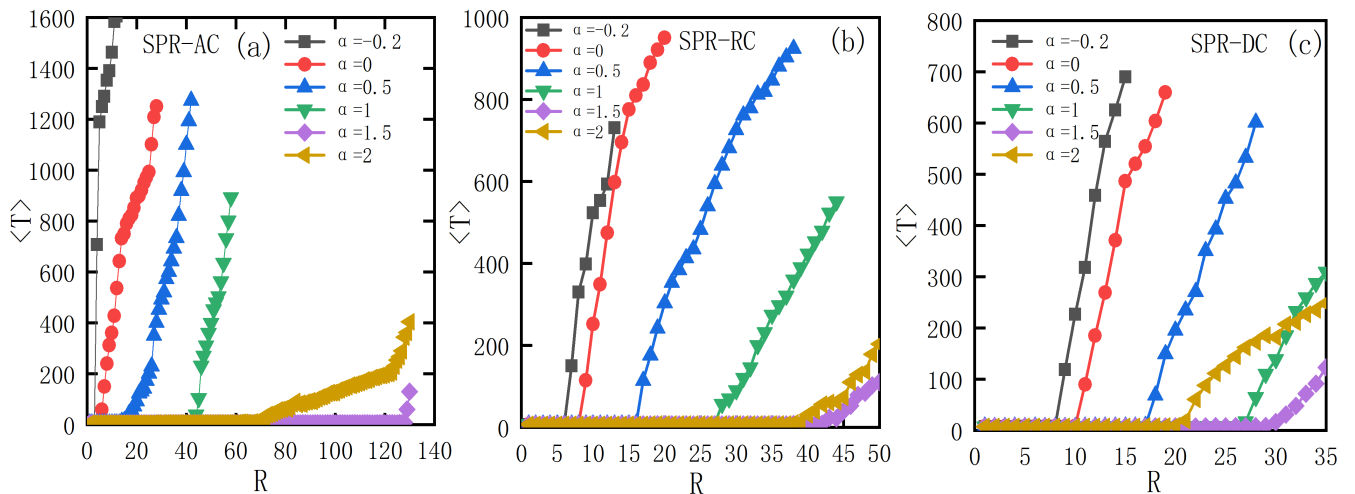


FIGURE 4. (a) AC Coupling. (b) RC Coupling. (c) DC Coupling. In a double-layer BA-BA network, set the network size $N = 400$, the average processing capacity (C) = 4, control parameter α value range $[-0.2, 2]$, and the average degree (k) = 8. When the SPR strategy and DE distribution strategy are adopted, the relationship between average transmission time (T) and the packet generation rate R .

IV. SIMULATION RESULTS

In this paper, in order to study the influence of the degree coupling between logical layer and physical layer on the delivering capacity allocation strategies, simulation experiments are carried out. Both the two-layer are set to be the BA scale-free network with the characteristics of the actual network, with each physical layer physical layer physical layer node delivering capacity $C = 4$, the cache length at the node is infinitely long. Set the network size to medium size $N = 400$. If the size is too large, there may be some limitations. The average degree $\langle k \rangle = 8$, which can ensure that the network model is a fully connected network. In the following experimental simulations are set to a unified experimental conditions. In order to improve the accuracy, the running time steps for each packet generation rate H

is 10000 in our simulations. As shown in Figure 2, with the network size $N = 400$ and average degree (k) = 8, study the influence of the degree coupling on the delivering capacity allocation strategies when P_r changes continuously from -1 to 1. As the degree of coupling correlation P_r increases, the values of the key packet generation rate R_c descend. When the SPR strategy is adopted, it can be seen that when $P_r = -1$, the critical generation rate $R_c = 11$ is the largest value, that is, with the DC mode, the traffic capacity could be most greatly enhanced. When $P_r = 0$, the degree coupling is in RC mode with $R_c = 7$. When $P_r = -1$, it is in AC coupling mode with $R_c = 3$, which is at the minimum value.

To investigate the relationship between control parameter α and the critical generation rate R_c adopted different coupling modes. As shown in Figure 3, the network models are set

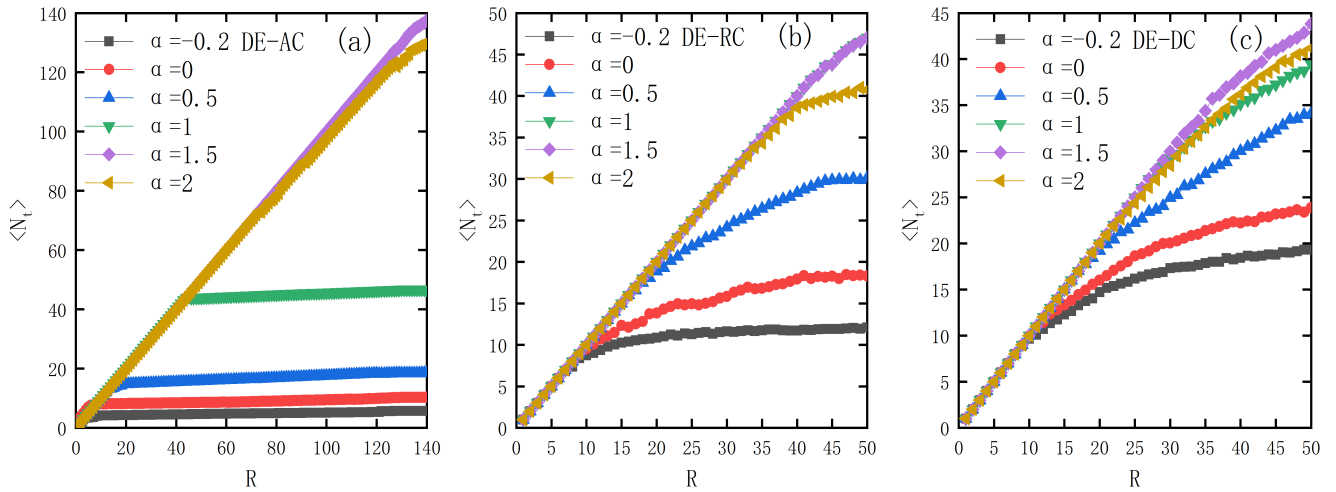


FIGURE 5. (a) AC Coupling. (b) RC Coupling. (c) DC Coupling. In a double-layer BA-BA network, set the network size $N = 400$, the average processing capacity $\langle C \rangle = 4$, control parameter α value range $[-0.2, 2]$, and the average degree $\langle k \rangle = 8$. When the SPR strategy and DE distribution strategy are adopted, the relationship between the average information flow N_t and the packet generation rate R .

$N = 400$ and $\langle k \rangle = 8$, the SPR strategy is used, and the range of α is $[-0.2, 2]$. It can be seen that the optimal value of α adopted the SPR strategy is 1.5 in the three coupling modes, so the value of α is set to be 1.5 in the following simulations. When $\alpha = 0$, the delivering capacity of each physical layer node is the same, that is, the SA strategy. It can be seen that with the AC coupling mode, when $\alpha = 0$, $R_c = 6$, and when $\alpha = 1.5$, $R_c = 128$, which is larger than R_c when α takes $-0.2, 0.7, 2$. It is clear that R_c is not monotonic of α . Similarly, with the RC coupling mode, $R_c = 43$ when $\alpha = 1.5$, and with the DC coupling mode, $R_c = 30$ when $\alpha = 1.5$. adopted the SPR strategy, with the AC coupling mode of these three couplings, the capacity of two-layer network can reach the largest value $R_c = 135$ when $\alpha = 1.5$, which makes the data packet flow more uniform and network traffic capability can be greatly improved.

For traffic efficiency, the transmission time of data packets in the network should be as small as possible, and the traffic capacity of the two-layer network should be as large as possible. The relationships between average transmission time $\langle T \rangle$ and R are shown in Figure 4. Adopted the DE allocation strategy, the average transmission time $\langle T \rangle$ changes with the packet generation rate R . As $R < R_c$, the network system is in a free state, and the value of $\langle T \rangle$ is small, almost zero. When $R > R_c$, the network begins to enter a congested state and the value of $\langle T \rangle$ suddenly increases. By comparing the conditions adopted different control parameters, it can be seen that the optimal network parameter $\alpha_c = 1.5$, that is, when $\alpha = 1.5$, the critical value of R_c is the largest.

As shown in Figure 5, in order to measure the network transmission performance, it is necessary to study the variation trend of average information amount $\langle N_t \rangle$ with the DE resource allocation strategy. Adopted different coupling modes, the average information amount $\langle N_t \rangle$ increases with the packet generation rate R , and the allocation strategy is set

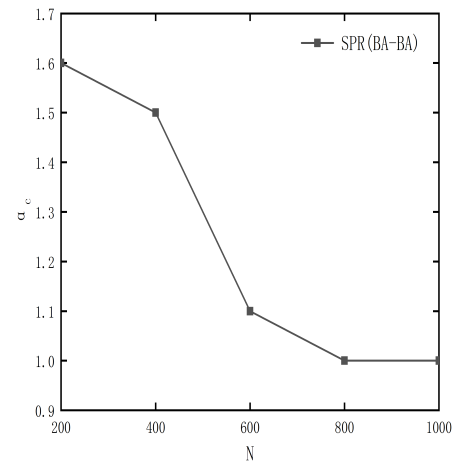


FIGURE 6. The relationship between the optimal value of the control parameter α_c and the network size N , when the SPR strategy and DE distribution strategy are adopted.

to the DE strategy. It can be seen that the optimal control parameter of the three-coupling mode is $\alpha_c = 1.5$. The average information amount is the largest, then the $\langle N_t \rangle$ value in the AC coupling mode is significantly larger than the $\langle N_t \rangle$ value of the RC and DC. It can be seen that with the AC coupling, the network traffic capability can be more improved adopted the DE allocation strategy.

In order to study the stability of the DE allocation strategy adopted different network scales, it is necessary to investigate the relationship between the optimal control parameter α_c and the network size N . As shown in Figure 6, as the network size N increases (N is the number of network nodes per layer, from 200 to 1000), the optimal control parameter α_c continues to decrease. The discovery of this trend is conducive to the design of large-scale networks. When the network requires a relatively large traffic capacity, the DE allocation strategy can

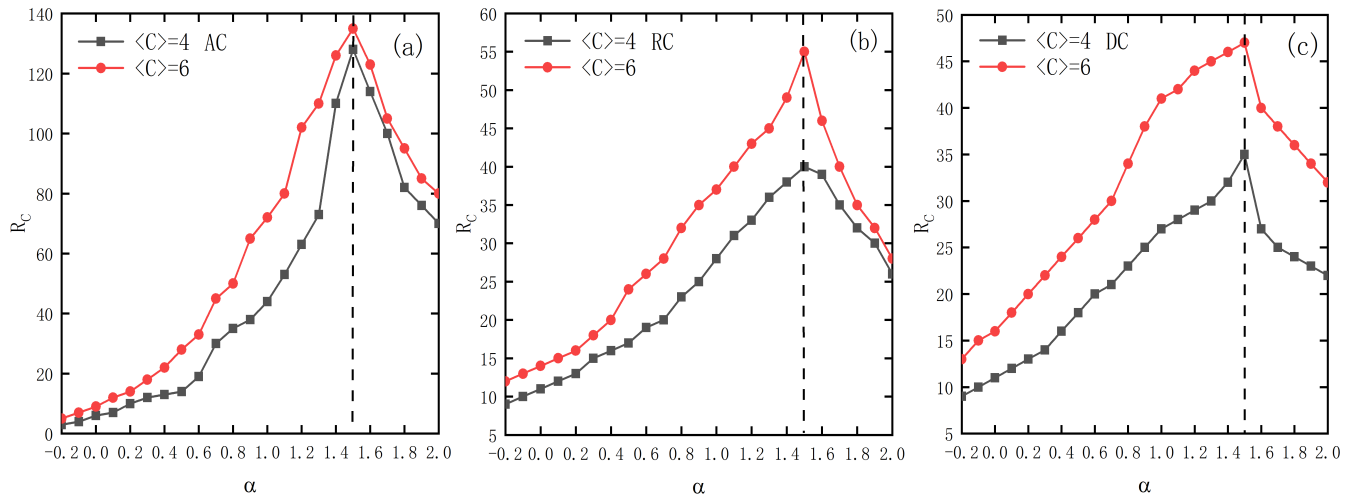


FIGURE 7. (a) AC Coupling. (b) RC Coupling. (c) DC Coupling. In a double-layer BA-BA network, set the network size $N = 400$, the control parameter α value range $[-0.2, 2]$, and the average degree $\langle k \rangle = 8$. When the SPR strategy and the DE allocation strategy are adopted, the relationship between the critical value R_c of the two average processing capacity packet generation rates and the control parameter α .

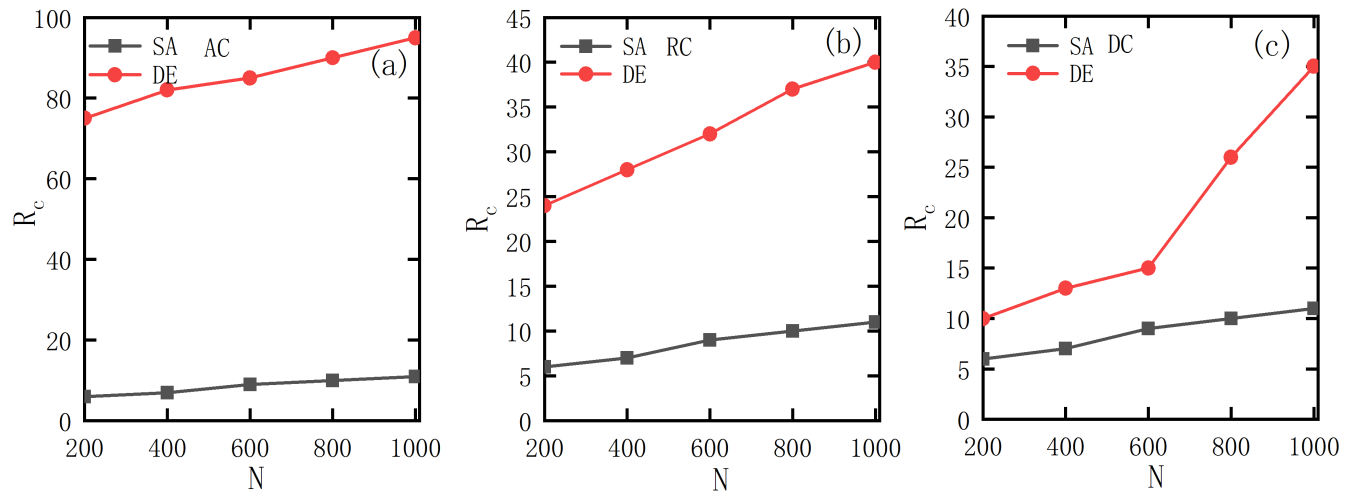


FIGURE 8. (a) AC Coupling. (b) RC Coupling. (c) DC Coupling. The relationship between packet generation rate R_c and network size N , when DE and SA strategies are adopted.

be used to select the appropriate α_c according to the network size.

Then, the optimal value of the control parameter α of the two-layer coupled network adopted the DE strategy is discussed below. When the average node delivering capacity (C) of the physical layer network takes different values 4 and 6, the number of network nodes is set to $N = 400$ with average node degree $\langle k \rangle = 8$, and α has an optimal value α_c such that R_c takes the maximum value. As shown in Figure 7, the optimal α_c value is $\alpha = 1.5$. Adopted the DE allocation strategy, the value of R_c is larger when $\langle C \rangle = 6$ than $\langle C \rangle = 4$. When $\langle C \rangle = 6$, R_c can reach 135 with AC coupling mode, the maximum value R_c can reach 55 with in RC coupling mode, and a maximum value R_c can reach 47 with DC coupling mode. When the control parameter $\alpha_c = 1.5$, the critical packet generating rate R_c obtain the optimum value with the

DE allocation strategy. With the DE allocation strategy, the AC coupling is the best mode to improve the network traffic capacity.

In different scales, the R_c values of the two-layer network are presented under the SA and DE strategies. As shown in Figure 8, the critical packet generation rate R_c gradually increases as the network size N increases with these two resource allocation strategies. That is to say, when the network size is larger, the traffic capacity of the two-layer network can be improved more. As the network size increases, the traffic capacity R_c with the SA allocation strategy changes less the DE allocation strategy. By comparison, the DE allocation strategy which allocates the delivering capacity of the physical layer nodes according to the degree of the logical and the physical layers, is more effective to enhance network traffic capacity with AC, RC and DC coupling modes.

V. CONCLUSION

At present, most of the researches on traffic dynamics focus on monolayer networks. In fact, the actual network does not exist independently but has a hierarchical or interdependence structure. Therefore, it is necessary to study the coupling relationship of this layered and dependent networks. The interaction mechanism restricts traffic dynamics of the two-layer network. In this paper, we have studied three coupling modes between the two layers of two-layer network and two resource allocation strategies of the physical layer, and tried to improve the network traffic capacity based on the relationship between the coupling modes and allocation strategies. Through simulation experiments, it is found that the AC coupling mode are used to obtain the largest traffic capacity R_c with both the DE and SA resource allocation strategies on the BA-BA two-layer network. The AC coupling mode makes the traffic flow more uniform in the both the logical and physical layers, and is beneficial to transmit data packets. The suitable coupling mode and its corresponding resource allocation strategy of two-layer network is an important method to improve the traffic capacity. The real world presents a two-layer or two-layer complex structure, and their coupling modes and resource allocation strategies are multifarious. Studying the coupling interaction mechanism of two-layer network is helpful for real network design and optimization.

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