IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received May 26, 2020, accepted July 1, 2020, date of publication July 15, 2020, date of current version July 28, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3008797

A Discrete Increment Model for Electricity Price Forecasting Based on Fractional Brownian Motion

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ABSTRACT Electricity price forecasting is essential for all participants of the power consumption market. However, electricity price series has complex properties such as high volatility and non-stationarity that make forecasting turn out to be very difficult. In this study, we aim to forecast electricity price series by the discrete increment model of fractional Brownian motion (fBm). A specific feature of the fBm is that it represents a typical non-stationary stochastic process with long-range dependent (LRD) characteristics. Analysis of electricity price series has LRD characteristics, and the Hurst exponent can be calculated. The Hurst exponent is the key parameter of fBm, and it is a measure of self-similarity. The stochastic differential equation driven by fBm is discretized into the discrete increment model for electricity price forecasting. Other parameters of the discrete increment model can be evaluated by the maximum likelihood estimation (MLE). The performance of the proposed method is demonstrated by using the data from the U.S. Energy Information Administration. The validity of the proposed model was compared with other methods.

INDEX TERMS Electricity price forecasting, fractional Brownian motion, long-range dependence, discrete increment model, maximum likelihood estimation.

I. INTRODUCTION

Variations of electricity price can affect economic benefits of all market participants [1]. From the perspective of market managers, the electricity price forecasting provides a scientific basis for promoting healthy, stable and orderly development [2]. In the electricity market, electricity price forecasting has become a very valuable tool for all market participants. Hence, it is necessary to study the problem of electricity price forecasting in depth [3].

In recent years, in order to achieve high-precision electricity price forecasting, some scholars began to put forward some methods. These methods can be roughly categorized into three groups: statistical methods, artificial intelligence methods and hybrid methods [4]. Statistical methods include controlled autoregressive moving average (CARMA) [5], autoregressive integrated moving average (ARIMA) [6] and a hybrid of autoregressive moving average (ARMA) and generalized autoregressive conditional heteroskedasticity (GARCH) [7]. Statistical methods generally provide good forecasting results, but its models are limited by linear assumptions [8]. Artificial intelligence methods are better than statistical methods because of the advantage to capture the nonlinear characteristics and rapid changes. Artificial intelligence methods include artificial neural network (ANN) [9], fuzzy neural network (FNN) [10], recurrent neural network (RNN) [11] and long short-term memory (LSTM) [12]. Although these methods can handle with multivariate processes and nonlinear problems, the network structure choice and its parameters mainly depend on the previous data. A single forecasting model cannot precisely analyze complex relations in non-stationary electricity price series. As a result, many researchers proposed hybrid models to improve the prediction accuracy [13], [14].

Present studies revealed that electricity price series is consistent with the long-range dependent (LRD) characteristics [15], which indicates that the past and present state of this series will have effect on future condition. The fractional Brownian motion (fBm) model is more applicable for practical applications when it is compared with a pure stochastic process [16]. The aim of this paper is to forecast electricity price series by using the fBm model.

The fBm is a non-stationary continuous stochastic process with stationary increments, which has well-known LRD

The associate editor coordinating the review of this manuscript and approving it for publication was Zhaojun Li.

properties [17] and governed by the Hurst exponent [18] ranging from 0 to 1. When H is closer to 1, then the greater degree of dependence. The fBm can be obtained as a stochastic integral from the Brownian motion (Bm) [19]. There are many methods to calculate the Hurst exponent. For example, the Hurst exponent can be calculated by the variance time method [20], absolute value estimation method [21] or rescaled range (R/S) analysis method [22]. We use R/S method in this paper, because this method is easy to apply when it is compared with the frequency domain algorithm.

We study the stochastic differential equation (SDE) driven by fBm, and an important example is the fractional version of the Black-Scholes model proposed by Cutland, Kopp and Willinger [23]. The discrete increment forecasting model is established by discretizing the SDE. The other two parameters μ and σ in discrete increment model are estimated by the maximum likelihood estimation (MLE) [24]. The data of case study is residential, commercial and industrial electricity price from U.S. Energy Information Administration. Several indicators of accuracy show that the proposed model obtains better accuracy in comparison with other models.

This paper is organized as follows. Section 2 describes fundamental theories of time series with LRD properties. Section 3 introduces the characteristics of the fBm and the generation of the fBm series. In section 4, discrete increment forecasting model is deduced, and the MLE is used for parameter estimation. In section 5, comparisons of error evaluation indicators in case study are given. Section 6 concludes the paper.

II. LONG-RANGE DEPENDENT PROCESS

The autocorrelation function (ACF) for the time series $x(t)$ has the form

$$R(\tau) = E[X(t)X(t+\tau)] \tag{1}$$

where τ is the time lag. If $\int_{-\infty}^{\infty} R(\tau)d\tau = \infty$, this time series can be considered as long-range dependent [16]. When τ is large enough, the LRD autocorrelation process takes a form of the power-law distribution:

$$R(\tau) \sim c\tau^{-\beta}, (\tau \to \infty) \tag{2}$$

where c is the constant and β ranges within the interval $0 < \beta < 1$. The description of LRD can be replaced by the Hurst exponent, which is related with β as follows: $\beta = 2 - H$.

To determine whether the time series has the LRD properties, the Hurst exponent [18] must to be calculated. Time series with exponents larger than 0.5 are shown to be LRD, exponents smaller than 0.5 are called anti-persistent time series. In this study we use the R/S analysis method to calculate the Hurst exponent. It is a method to describe the degree of self-similarity of a time series. In addition, the algorithm is clear and easy to apply in comparison with the frequency domain algorithm [22]. The basic steps of the R/S analysis method are the following:

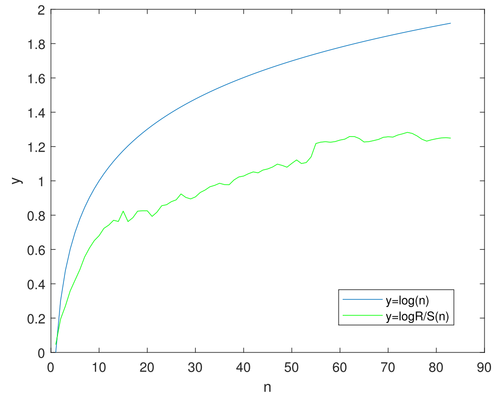
Step 1: The time series $\{X_t : t = 1, 2, \ldots, N\}$ is divided into integer sub-intervals, the total number of samples is N, the length of each sub-interval is $n\,(2 \le n \le N)$. Corresponding to the mean $P(n)$ and the standard deviation $S(n)$ for each sub-interval are calculated, respectively.

Step 2: The corresponding cumulative deviation $X(t, i)$ between each sub-interval and its range of variation $R(n)$ of each sub-interval are calculated.

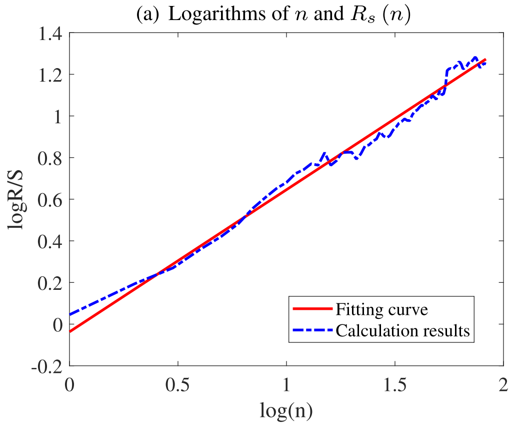
Step 3: The ratio of each range to the standard deviation is $R_s(n) = R(n)/S(n)$, $R_s(n)$ with different interval lengths is obtained by taking different $n\,(2 \le n \le N)$ values.

Step 4: Logarithms of n and $R_s(n)$ are taken respectively $\lg(R_s(n)) = \lg c + H\lg n$, c is a statistic constant and the slope obtained by using least squares fitting is the value of Hurst parameter.

For example, the R/S analysis method is used to calculate the Hurst exponent of the electricity price series sample, and the results are shown in Fig. 1 with H=0.6826.



(a) Logarithms of n and $R_s(n)$



(b) Slope fitted by least squares regression

FIGURE 1. R/S method to calculate the Hurst exponent of electricity price series sample.

III. SERIES RECONSTRUCTION OF FRACTIONAL BROWNIAN MOTION

A. CHARACTERISTICS OF FRACTIONAL BROWNIAN MOTION

The stochastic process $\{B_t^H, t \ge 0\}$ called the fBm with Hurst exponent $H\,(0 < H < 1)$ [25] is given by

$$B_t^H = B_0^H + \frac{1}{\Gamma(H+0.5)} \int_{-\infty}^{t} K(t-s)dB(s) \tag{3}$$

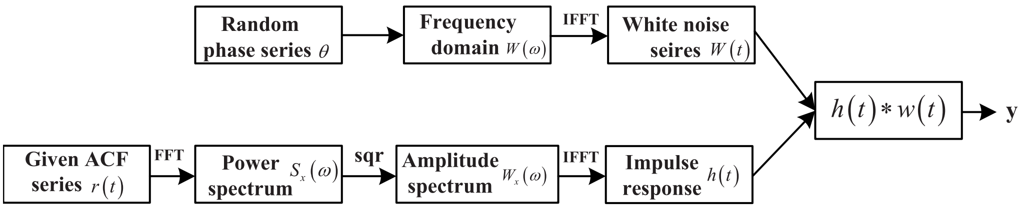


FIGURE 2. The flowchart of generating LRD series.

where $B(s)$ is Brownian motion, and the $\Gamma(x)$ is the Gamma function:

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt \tag{4}$$

The integral kernel in (3) is

$$K(t-s) = \begin{cases} (t-s)^\alpha, & (0 \le s \le t) \\ (t-s)^\alpha - (-s)^\alpha, & (s < 0) \end{cases} \tag{5}$$

where $\alpha = H - 0.5$.

The fBm exhibits LRD properties for 0<H<1, but for H= 0.5 this process becomes the Brownian motion, and the ACF is defined as follows:

$$E[B_t^H B_s^H] = \frac{\sigma^2}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right) \tag{6}$$

B. GENERATION OF FRACTIONAL BROWNIAN MOTION SERIES

By performing the convolution between the Gaussian noise and the impulse response of a linear system with LRD properties, the output response is a fractional stochastic series with LRD properties. On the basis of the Wiener-Khinchine theorem, the power spectral density of a time series is the Fourier transformation of ACF:

$$S_x(\omega) = F[r_x(t)] \tag{7}$$

We can further derive the relationship between impulse response and ACF [17]:

$$h(t) = F^{-1}\left\{F(r_x)^{1/2}\right\} \tag{8}$$

The white Gaussian noise can be defined by

$$w(t) = F^{-1}[W(\omega)] \tag{9}$$

where $W(\omega) = e^{j\theta(\omega)}$ and $\theta(\omega)$ is a real function with random distribution. In view of (8) and (9), the expression for simulation of LRD data $y(t)$ is given by

$$y(t) = w(t) * h(t) = \omega(t) * F^{-1}\left\{F(r_x)^{1/2}\right\} \tag{10}$$

Finally, the flowchart of generating LRD series is depicted in Fig. 2.

IV. DISCRETE INCREMENT MODEL OF FRACTIONAL BROWNIAN MOTION

A. THE DISCRETE INCREMENT MODEL

For stochastic process S_t, consider the fractional Black-Scholes [23] SDE driven by fBm is

$$dS_t = \mu S_t dt + \sigma S_t dB_t^H \tag{11}$$

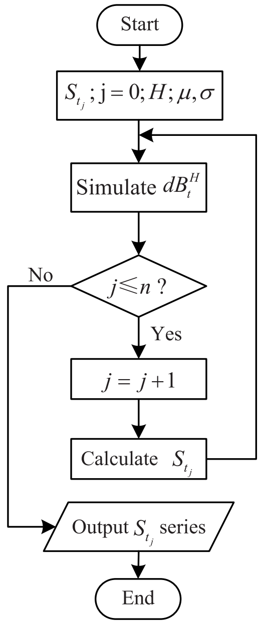


FIGURE 3. The flowchart of fBm simulation.

where μ and σ represent the drift coefficient and diffusion coefficient, respectively, B_t^H is fBm with Hurst exponent H.

From (11), we should simulate increments first before simulate this process. A simple approach is taken in this article, which is to simulate with the Monte Carlo method using the extended form $dB_t^H = \omega(t)(dt)^H$ of the Maruyama [26]. And use it to represent the increment of fBm, thus obtaining the SDE

$$dS_t = \mu S_t dt + \sigma S_t \omega(t)(dt)^H \tag{12}$$

where $\omega(t)$ is a zero-mean standardized normal distribution. The time period is segmented into N equal intervals, time interval is Δt, and the discrete SDE is

$$\Delta S_t = \mu S_t \Delta t + \sigma S_t \omega(t)(\Delta t)^H \tag{13}$$

where $\Delta S_t = S_{t+1} - S_t$. Then, we obtained the discrete increment model of fBm [27], [28] as follows:

$$S_{t+1} = S_t + \mu S_t \Delta t + \sigma S_t \omega(t)(\Delta t)^H \tag{14}$$

The fBm simulation using by the Monte Carlo method. The idea of this method is to perform many simulations based on experimental samples, and the average value of all simulation results obtained at each point is corresponding approximation. Select the initial value, the fBm simulation flowchart is presented in Fig. 3. The flowchart of electricity

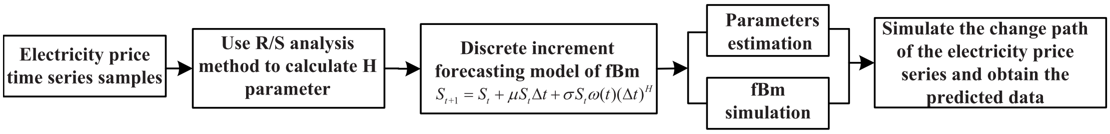


FIGURE 4. The flowchart of electricity price forecasting.

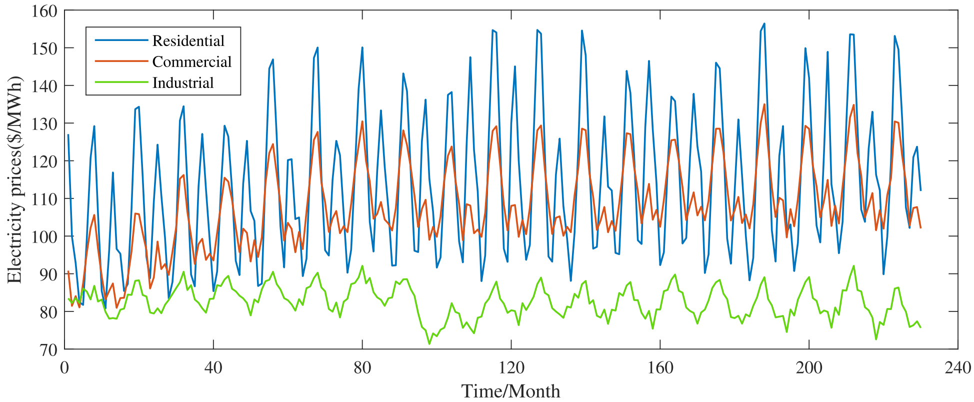


FIGURE 5. Electricity price series from January 2001 to February 2020.

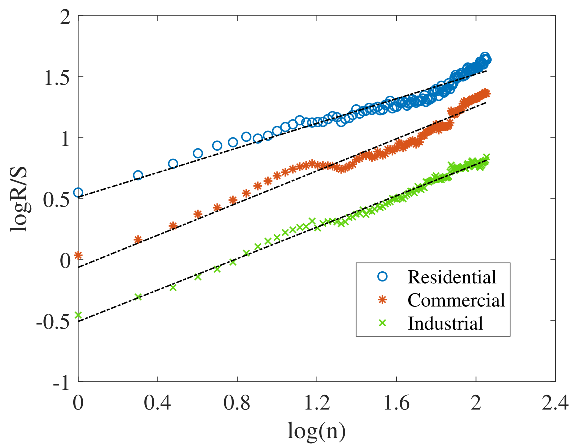


FIGURE 6. R/S method to calculate the Hurst exponent of electricity price time series.

price forecasting by the discrete increment model of fBm is shown in Fig. 4.

B. PARAMETERS ESTIMATION OF DISCRETE INCREMENT MODEL

For the completeness, two unknown parameters μ and σ in SDE also are required. The general solution of (11) is

$$S_t = S_0 \cdot \exp\left[-\int -\left(\mu dt + \sigma dB_t^H\right)\right] = S \cdot \exp(\mu t + \sigma B_t^H) \tag{15}$$

By taking the logarithm of (15), one obtains

$$\ln(S_t) = S(\mu t + \sigma B_t^H) \tag{16}$$

Accordingly, the parameter estimation of (15) is actually equivalent to the parameter estimation:

$$Y_t = \mu t + \sigma B_t^H, t \geq 0 \tag{17}$$

TABLE 1. Hurst exponent of electricity price time series.

	Residential	Commercial	Industrial
Hurst parameters	0.5041	0.6586	0.6437

TABLE 2. Evaluation indicators of forecasting results.

Type	Time	MAE	MAPE (%)	RMSE	Max relative error (%)
Residential electricity price	6 months	2.91	2.54	3.03	3.45
	12 months	1.74	2.75	3.74	6.49
	18 months	5.35	4.59	6.33	10.21
	24 months	6.54	5.72	7.73	13.01
Commercial electricity price	6 months	1.82	1.67	1.91	2.35
	12 months	2.01	3.00	3.74	6.86
	18 months	4.84	4.40	5.93	12.20
	24 months	5.68	5.05	6.68	12.37
Industrial electricity price	6 months	2.37	3.05	2.43	4.03
	12 months	2.18	4.42	3.80	7.42
	18 months	4.24	5.35	4.83	11.69
	24 months	6.06	7.53	7.12	17.45

In this paper, we use MLE to perform the parameter estimation of the discrete increment model of fBm with discrete observations. Given a certain time series, the specific parameters can be estimated. Assuming interval of the acquired time series is Δt, and the observation vector consist of $Y = (Y_0, Y_{\Delta t}, \ldots, Y_{n\Delta t})^T$, with corresponding time vector $t = (0, \Delta t, \ldots, n\Delta t)^T$ for the $n+1$ observation data. Assuming the fBm vector is $B_t^H = \left(B_0^H, B_{\Delta t}^H, \ldots, B_{n\Delta t}^H\right)^T$, the maximum likelihood estimators of the μ and σ are [24]

$$\hat{\mu} = \frac{t^T \Gamma_{H,i,j}^{-1} Y}{t^T \Gamma_{H,i,j}^{-1} t} \tag{18}$$

$$\hat{\sigma}^2 = \frac{1}{n}\left[Y^T \Gamma_{H,i,j}^{-1} Y - \frac{\left(t^T \Gamma_{H,i,j}^{-1} Y\right)^2}{t^T \Gamma_{H,i,j}^{-1} t}\right] \tag{19}$$

where $\Gamma_{H,i,j} = \frac{1}{2}(\Delta t)^{2H}\left(i^{2H} + j^{2H} - |i-j|^{2H}\right)_{i,j=01,2,\ldots,n}$.

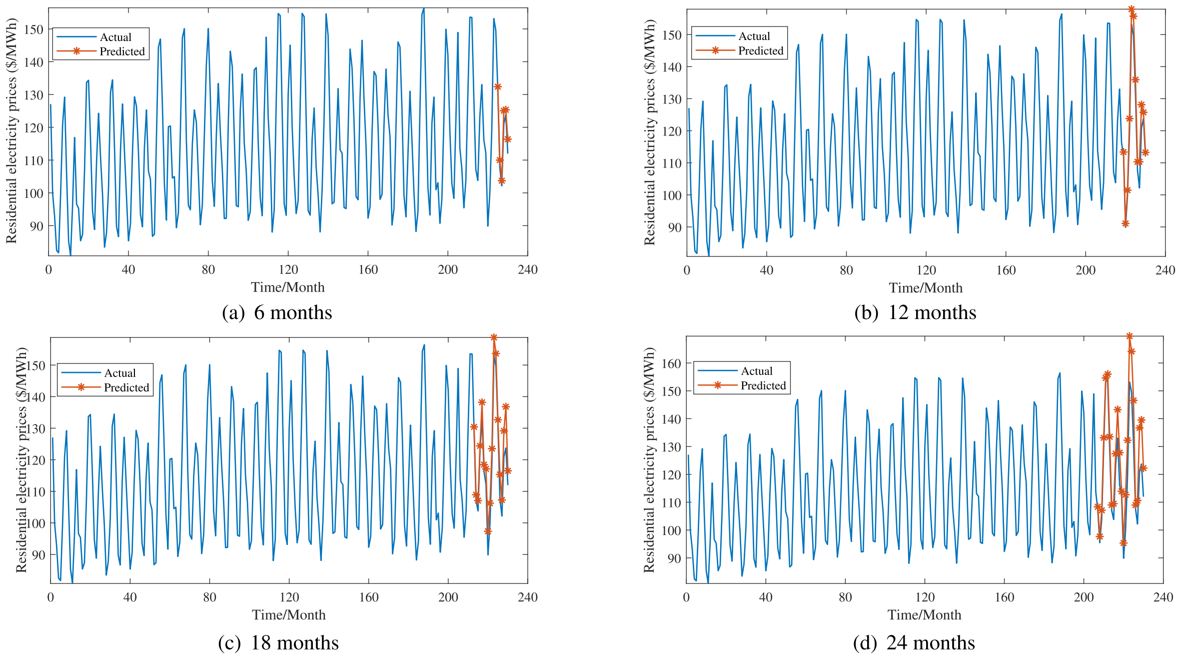


FIGURE 7. 24-month residential electricity price forecasting by fBm.

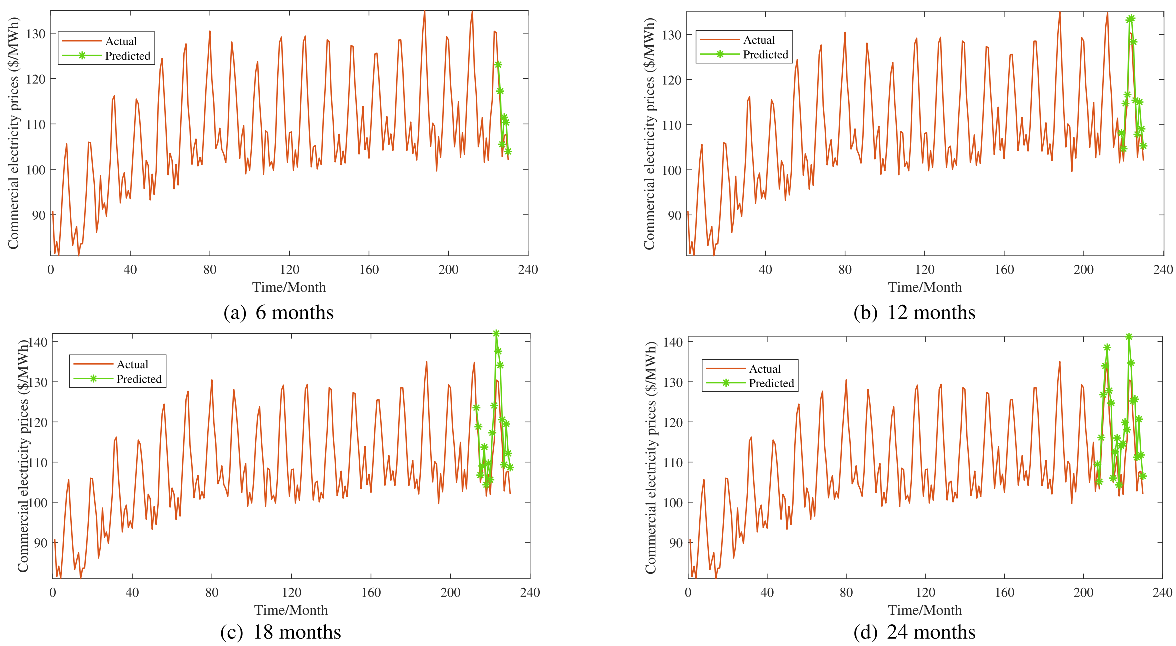


FIGURE 8. 24-month commercial electricity price forecasting by fBm.

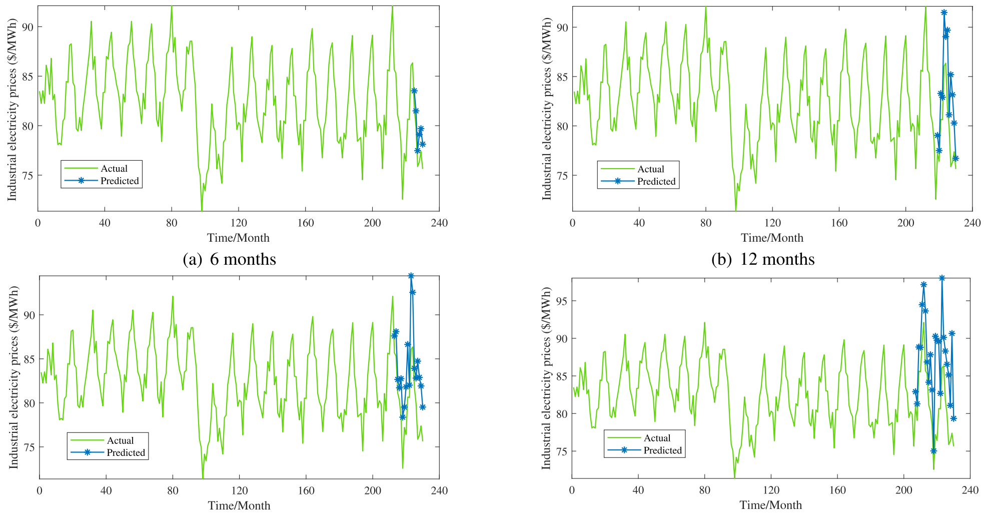
V. CASE STUDY

A. DATA DESCRIPTION

To verify the effectiveness of the developed forecasting model, the monthly eletricity price datasets collected from U.S. Energy Information Administration [29], including residential, commercial and industrial monthly electricity price. The total number of each dataset is 230 from January 2001 to February 2020, it presents a significant difference between different datasets according to the collected samples in Fig. 5. The stochastic nature of the electricity price series can be reflected. The forecasting results can better verify the applicability of the discrete increment model of fBm.

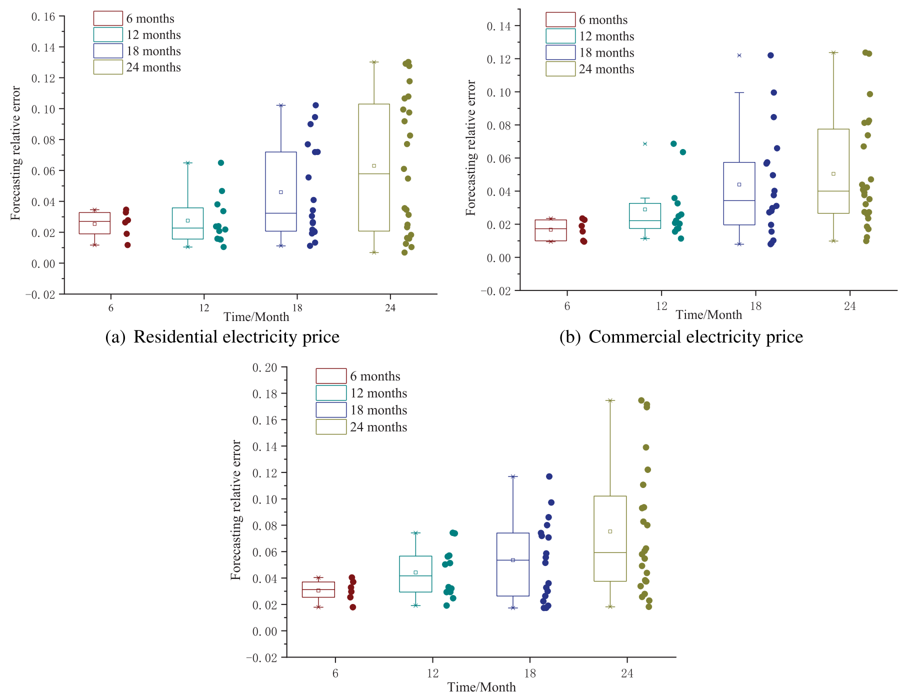
B. FORECASTING PROCESS

This experiment based on the discrete increment model of fBm to forecast the electricity price series of the next 6, 12, 18 and 24 months for historical data. The R/S analysis method is used to calculate the Hurst exponent of the electricity price series, and the results are shown in Fig. 6. Residential, commercial and industrial data are translated to distinguish



(a) 6 months (b) 12 months (c) 18 months (d) 24 months

FIGURE 9. 24-month commercial electricity price forecasting by fBm.



(a) Residential electricity price (b) Commercial electricity price (c) Industrial electricity price

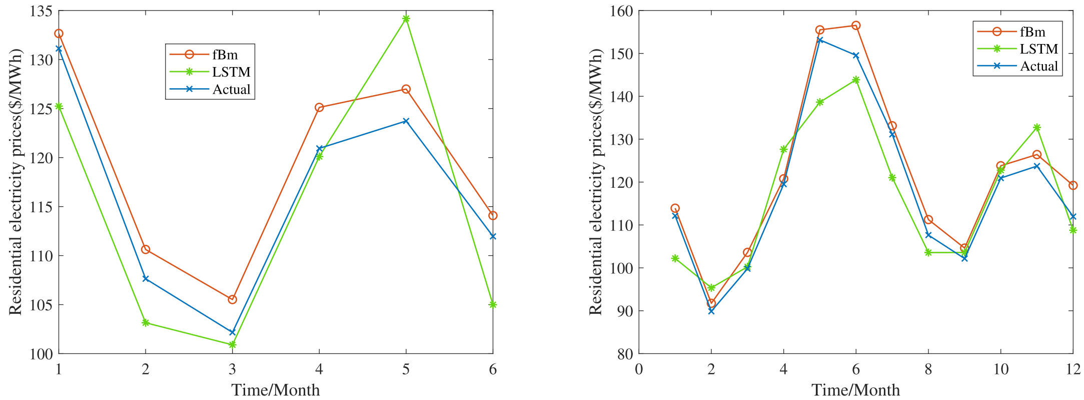
FIGURE 10. Relative error of forecasting results.

the plots. In addition, Table 1 shows the Hurst exponent of the electricity price series. The results reveal that electricity price series fulfill the LRD characteristics.

The simulated forecasting trends are shown in Figs. 7-9, respectively. Error analysis of forecasting results, such as MAE (Mean Absolute Error), MAPE (Mean Absolute Percentage Error, %) RMSE (Root Mean Squares Error) and max relative error (%) are shown in Table 2. And the box plots of the relative error are shown in Fig. 10, which shows the maximum value of the relative error clearly. As demonstrated by our results, the longer the forecasting time series, the lager the forecasting error. Our forecasting results are consistent with LRD characteristics, correlation and prediction accuracy gradually reduced as the distance increases. Furthermore, the purpose of this study is to determine the forecasting time for residential, commercial, and industrial electricity prices based on discrete increment model of fBm. To guarantee the accuracy of forecasting, it is necessary to choose 6 or 12 months as the forecasting time points.

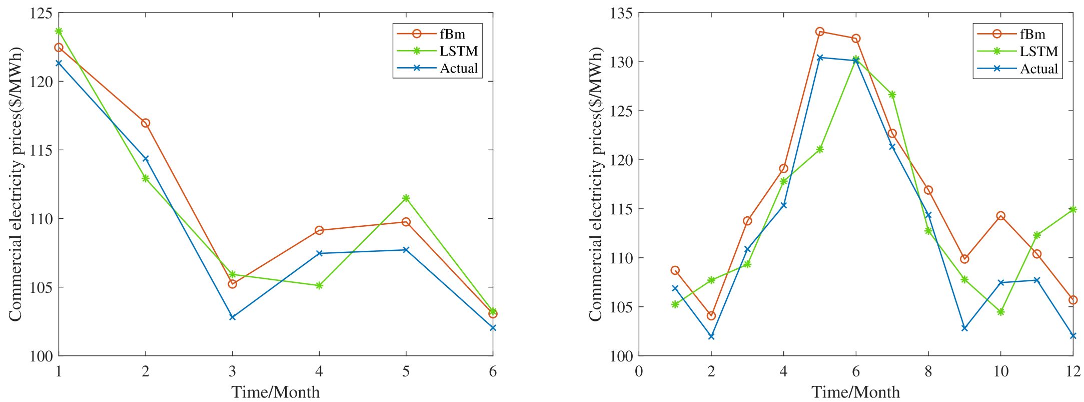
C. COMPARATIVE ANALYSIS OF DIFFERENT MODELS

To verify the proposed forecasting performance, we have performed electricity price forecasting in comparison with different models. Comparison results obtained from different



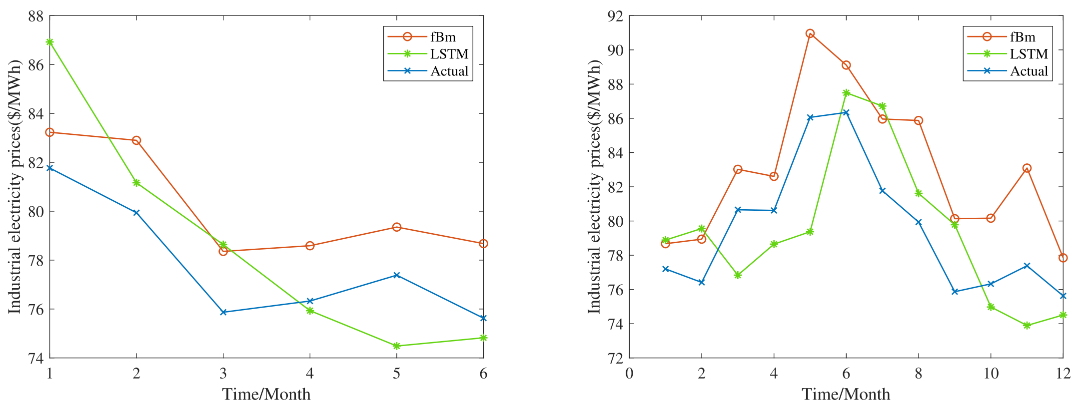
(a) 6 months (b) 12 months

FIGURE 11. Comparison of different models for residential electricity price forecasting.



(a) 6 months (b) 12 months

FIGURE 12. Comparison of different models for commercial electricity price forecasting.



(a) 6 months (b) 12 months

FIGURE 13. Comparison of different models for industrial electricity price forecasting.

forecasting models for test samples, including residential, commercial, and industrial electricity prices are shown in Figs. 11-13, respectively. It clearly illustrates that the forecasting curve obtained by the proposed model is in close agreement with the actual values. The results show that the proposed model can well capture the change trend of electricity price series. In addition, error evaluation indicators comparison of different forecasting models based on the samples is displayed in Table 3. As shown in table, the max relative error (%) of the proposed model are smaller than other model in all testing samples. Thus, the proposed model can obtain a more accurate forecasting result.

TABLE 3. Comparison of different models for evaluation indicators of forecasting results.

Type	Model	Time	MAE	MAPE (%)	RMSE	Max relative error (%)
Residential	fBm	6 months	2.91	2.54	3.03	3.45
		12 months	1.74	2.75	3.74	6.49
	LSTM	6 months	4.98	4.21	5.99	8.45
		12 months	2.47	4.99	7.37	9.47
Commercial	fBm	6 months	1.82	1.67	1.91	2.35
		12 months	2.01	3.00	3.74	6.86
	LSTM	6 months	2.36	2.18	2.53	3.49
		12 months	2.70	4.03	5.66	12.61
Industrial	fBm	6 months	2.37	3.05	2.43	4.03
		12 months	2.18	4.42	3.80	7.42
	LSTM	6 months	2.21	2.80	2.74	6.31
		12 months	2.34	5.06	4.34	8.78

VI. CONCLUSION

Precise electricity price forecasting can help consumers and producers in their production plans to maximize benefits of both sides. Firstly, this paper has verified that the electricity price series has LRD characteristics by R/S analysis method. We put forward to forecast electricity price series based on discrete increment model of fBm from a novel perspective. The fBm is a non-stationary stochastic process, which is consistent with the LRD characteristics of the electricity price series. The validity of the discrete increment model of fBm was comprehensively verified by using three types of electricity price data including residential, commercial and industrial sample. Our results agree with the LRD characteristics, correlation and forecasting accuracy gradually reduced as the forecasting time increases. In addition, the proposed model has achieved good forecasting accuracy as a single mathematical model by comparison with another model. Hence, future work can further improve the accuracy by developing the hybrid model.

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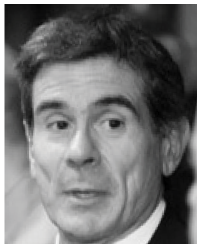

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