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Reduced-Order Filtering for Singular Markovian Jump Systems With Incomplete Transition Rates

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ABSTRACT This paper considers the reduced-order H_∞ filtering problem for singular Markovian jump systems (SMJSs) with incomplete transition rates (ITRs) by using augmented system method. The considered conditions in this paper are necessary and sufficient (NS), whereas the existing conditions are mainly sufficient. To be concrete, by extracting system matrices in the considered system from augmented system, NS condition for the existence of the full-order H_∞ filtering is provided in terms of linear matrix inequalities (LMIs). However, it is hard to extend the condition to the existence of the reduced-order H_∞ filtering. Thus, by fixing augmented system matrices, NS condition for the existence of the reduced-order one is presented to guarantee the desired filtering error system to be stochastically admissible with H_∞ performance level. Furthermore, there are neither complicated matrix transformation nor equality/rank constraints in this paper. One numerical and one practical examples are illustrated to demonstrate the effectiveness of the achieved results.

INDEX TERMS Singular Markovian jump system, incomplete transition rates, sufficient and necessary conditions, reduced-order filtering.

I. INTRODUCTION

Singular systems, also referred to descriptor systems, implicit systems and generalized state-space systems [1], [2], which are formed by a set of coupled algebraic and differential equations. It is a generalized representation of the state-space system. Thus, singular systems can model various kinds of practical systems, such as networks, power systems, flexible robots and so on [3]–[6].

On the other hand, Markovian jump systems (MJSs) represent a convenient mathematical model to describe system dynamics in a situation when the system experiences frequent unpredictable parameter variations. MJSs have been studied both in many practical systems such as chemical process, manufacturing systems, flight systems and so on [7], [8] and in theoretical researches [9]. In the past decades, transition rates (TRs) in the jumping process are usually assumed to be completely known. However, it is difficult to implement the practical control systems to accurately estimate the TRs. Therefore, study on the MJSs with ITRs receives the attention of researchers [10]. When singular systems experience abrupt changes, which lead to famous singular MJSs (SMJSs) [11], [12]. Note that the research on SMJSs are even more difficult than the regular MJSs since the properties of stability, regularity and causality (discrete-time) or non-impulsiveness (continuous-time) should be taken into account simultaneously. Thus, research on SMJSs is of significance, and majority of theoretical and applied results have been widely researched. To name a few, in [13], the problem of asynchronous H_∞ control for SMJSs with redundant channels under the dynamic event-triggered scheme is studied. To save the resource of bandwidth limited network, a dynamic event-triggered scheme has been proposed. The design of finite-time mixed H_∞ and passive asynchronous filter for T-S fuzzy SMJSs with uncertain transition rates under the dynamic event-based scheme has been discussed in [14]. An asynchronous filter is considered such that the phenomena of asynchronous modes between the original SMJSs and the considered filter is modelled as a hidden Markov model. In [15]–[19], H_∞ filtering has respectively reported for SMJSs, and some sufficient conditions on full- and reduced-order H_∞ filtering have been derived. Especillay, the NS full-order H_∞ filter condition for SMJSs has been achieved in [19]. However, the information of all TRs and estimated state are required. It is hard to extend the condition in [19] to the reduced-order filtering problems for SMJSs with ITRs. Thus, how to derive an NS condition on reduced-order filtering with ITRs constitutes this paper.

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In this paper, the reduced-order H_∞ filtering problem is considered for SMJSs with ITRs. Note that the achieved H_∞ filter matrices in [19] depend on all information of TRs and the estimated state. Thus, its method cannot be extended to the reduced-order ones for SMJSs with ITRs. In this paper, with the aid of augmented system method, system matrices (such as A_i, B_{wi}) are extracted to construct an augmented system ($\bar{A}_i = diag\{A_i, 0\}$, $\tilde{B}_{wi} = [B_{wi}^T, 0]^T$). Then, by using elimination method, the necessary and sufficient (NS) full-order H_∞ filtering is received for SMJSs with ITRs. The filter matrices can be computed by a set of LMIs. However, similar with [19], the full-order NS conditions cannot be extended to the reduced-order ones due to some special matrix structure (such as Λ_i in [19]). In this case, tuning the order of filter matrices and without separating the augmented system matrices (i.e. $\bar{A}_i$, $\tilde{B}_{wi}$), the NS reduced-order H_∞ filtering is proposed for SMJSs with ITRs. Compared with some existing works, there are neither complicated matrix transformation nor equality/rank constraints in proposed conditions.

Notation: Throughout this paper, $\mathcal{R}^n$ represents the n-dimensional Euclidean space; X^T denotes the transpose of X; (Ω, F, P) is a probability space with Ω is the sample F is the algebra of subsets of sample space and P is the probability measure on F; '*' in LMIs represents the symmetric term of the matrix; $X > 0 (< 0)$ means X is a symmetric positive(negative) definite matrix; $He[X]$ means that $X + X^T$; $\lambda_{\min}(X)$ respects the minimum eigenvalue of X; $\mathcal{E}(X)$ denotes the mathematical expectation operator of X; $\mathcal{L}_2[0, \infty)$ refers to the space of square-integrable vector functions over $[0, \infty)$; $|X|$ denotes the Euclidean norm for vectors of X; $col[X, Y]$ denotes $[X^T, Y^T]^T$; $diag\{\ldots\}$ represent a block diagonal matrix.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider a class of singular Markovian jump systems (SMJSs), which are defined on a probability space (Ω, F, P)

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + B_w(r_t)w(t), \\ y(t) = C(r_t)x(t), \\ z(t) = L(r_r)x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the system state, $y(t) \in \mathcal{R}^p$ is the measured output, $z(t) \in \mathcal{R}^q$ is the signal to be estimated and $w(t) \in \mathcal{R}^m$ is the disturbance input that belongs to $\mathcal{L}_2[0, \infty)$, $\phi(t)$ is a compatible vector valued initial function. The matrix $E \in \mathcal{R}^{n\times n}$ may be singular and $rank(E) = r \le n$. $A(r_t)$, $B(r_t)$, $B_w(r_t)$, $C(r_t)$, and $L(r_t)$ are known real constant matrices with appropriate dimensions for each $r_t \in \mathbb{S}$. $r_t, t \ge 0$ is a continuous-time Markovian process with right continuous trajectories and take values in a finite set $\mathbb{S} = \{1, 2, \ldots, N\}$ with transition rate matrix $\Pi \triangleq \{\pi_{ij}\}$ given by

$$Pr\{r_{t+\sigma} = j | r_t = i\} = \begin{cases} \pi_{ij}\sigma + o(\sigma), & j \ne i \\ 1 + \pi_{ii}\sigma + o(\sigma), & j = i \end{cases}$$

where $\sigma > 0$, $\lim_{\sigma\to 0} o(\sigma)/\sigma = 0$, and $\pi_{ij} \ge 0$, for $j \ne i$, is the transition rate from mode i at time t to mode j at time $t + \sigma$ and $\pi_{ii} = -\sum_{j=1, j\ne i}^{N} \pi_{ij}$. Furthermore, this paper is concerned with the H_∞ filtering for SMJSs with ITRs. That is, some elements in Π are unknown. Take a 3 operation modes for example, $\Pi = \begin{bmatrix} ? & ? & \pi_{13} \\ \pi_{21} & ? & ? \\ ? & ? & ? \end{bmatrix}$, where "$?$" represents unknown element. For convenience, $i \in \mathbb{S}$, we denote

$S_k^i \triangleq \{j : \pi_{ij} \text{ is known for } j \in \mathbb{S}\}$,

$S_{uk}^i \triangleq \{j : \pi_{ij} \text{ is unknown } j \in \mathbb{S}\}$.

In addition, if $S_k^i = \emptyset$, $\pi_k^i \triangleq \sum_{j\in S_k^i} \pi_{ij}$, and when $i \in S_{uk}^i$, it is necessary to provide a lower bound π_d^i for it and we have $\pi_d^i \le -\pi_k^i$. And for each possible $r_t = i$, $i \in \mathbb{S}$, a matrix $M(r_t)$ will be denoted by M_i, $A(r_t)$ by A_i, $A_d(r_t)$ by A_{di} and so on.

The following preconditions are essential for main results.

Lemma 1: [23] Given a symmetric matrix $\Omega \in \mathcal{R}^{n\times n}$, two matrices $\Psi \in \mathcal{R}^{n\times m}$ and $\Phi \in \mathcal{R}^{k\times n}$ with $rank(\Psi) < n$ and $rank(\Phi) < n$. Consider the problem of finding some matrices G such that

$$\Omega + \Psi G \Phi + (\Psi G \Phi)^T < 0. \tag{2}$$

Then (2) is solvable for G if and only if

$$\Psi^\perp \Omega \Psi^{\perp T} < 0, \quad \Phi^\perp \Omega \Phi^{\perp T} < 0. \tag{3}$$

Lemma 2: [20] Let P be symmetric such that $E_L^T P E_L > 0$ and Q to be non-singular. Then, $PE + U^T Q V^T$ is non-singular and its inverse is expressed as $(PE + U^T Q V^T)^{-1} = \bar{P}E^T + V\bar{\Phi}U$, where $\bar{P} = \bar{P}^T$ and $\bar{Q}$ is nonsingular such that $E_R^T \bar{P} E_R = (E_L^T P E_L)^{-1}$, $\bar{Q} = (V^T V)^{-1} Q^{-1} (UU^T)^{-1}$.

Lemma 3: System (1) with ITRs is stochastically admissible with H_∞ performance if and only if there exist symmetric matrices P_{1i}, nonsingular matrices Q_{1i} for $i \in \mathbb{S}$ and $j \in S_{uk}^i$ such that

$$E_L^T P_{1i} E_L > 0, \tag{4}$$

$$\begin{bmatrix} \delta_{i1} & \Lambda_{1i}^T B_{wi} & C_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad if\ i \in S_k^i, \tag{5}$$

$$\begin{bmatrix} \bar{\delta}_{i1} & \Lambda_{1i}^T B_{wi} & C_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad if\ i \in S_{uk}^i, \tag{6}$$

where

$$\begin{aligned} \delta_{i1} &= He[A_i^T \Lambda_{1i}] + \sum_{j\in S_k^i} \pi_{ij} E^T P_{1j} E - \pi_k^i E^T P_{1j} E, \\ \bar{\delta}_{i1} &= He[A_i^T \Lambda_{1i}] + \sum_{j\in S_k^i} \pi_{ij} E^T P_{1j} E - \pi_k^i E^T P_{1j} E \\ &\quad + \pi_d^i E^T P_{1i} E - \pi_d^i E^T P_{1j} E, \\ \Lambda_{1i} &= P_{1i} E + U^T Q_{1i} V^T. \end{aligned}$$

and π_d^i is a given lower bound for the unknown diagonal element.

Proof: Two steps are given as follows.

Step (I): Connecting with the proof of Lemma 3 in [19], the sufficiency and necessity of the following inequalities have been finished.

$$E_L^T P_{1i} E_L > 0, \quad (7)$$

$$\begin{bmatrix} \delta_i & \Lambda_{1i}^T B_{wi} & C_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (8)$$

where $\delta_i = He[A_i^T \Lambda_{1i}] + \sum_{j=1}^N \pi_{ij} E^T P_{1j} E$.

Step (II): Connecting with the proof of Theorem 1 in [10], it yields the following two cases.

Case 1: if $i \in S_k^i$, then $\sum_{j=1}^N \pi_{ij} E^T P_{1j} E$ in (8) is equivalent to $\sum_{j\in S_k^i} \pi_{ij} E^T P_{1j} E - \pi_k^i E^T P_{1j} E$.

Case 1: if $i \in S_k^i$, then $\sum_{j=1}^N \pi_{ij} E^T P_{1j} E$ in (8) is equivalent to $\sum_{j\in S_k^i} \pi_{ij} E^T P_{1j} E - \pi_k^i E^T P_{1j} E + \pi_d^i E^T P_{1i} E - \pi_d^i E^T P_{1j} E$. Rearranging (8), it yields Lemma 3 of present paper. This is completed the proof.

III. MAIN RESULTS

In this section, the reduced-order H_∞ filter existence condition for SMJSs (1) with ITRs will be presented.

Firstly, consider the following filter for the estimation of $z(t)$:

$$\begin{cases} E_f \dot{x}_f(t) = A_{fi} x_f(t) + B_{fi} y(t), \\ z_f(t) = C_{fi} x_f(t) + D_{fi} y(t), \end{cases} \quad (9)$$

where $x_f(t) \in \mathcal{R}^{\hat{n}} (\hat{n} \leq n)$, $z_f(t) \in \mathcal{R}^q$, $E_f, A_{fi} \in \mathcal{R}^{\hat{n}\times\hat{n}}$, $B_{fi} \in \mathcal{R}^{\hat{n}\times p}$, $C_{fi} \in \mathcal{R}^{q\times\hat{n}}$, $D_{fi} \in \mathcal{R}^{q\times p}$ are to be determined. Let $\tilde{x}(t) = col[x(t), x_f(t)]$, $\tilde{z}(t) = z(t) - z_f(t)$. Then, the filtering error system can be represented as

$$\begin{cases} \tilde{E}\dot{\tilde{x}}(t) = \tilde{A}_i \tilde{x}(t) + \tilde{B}_{wi} w(t), \\ \tilde{z}(t) = \tilde{C}_i \tilde{x}(t), \end{cases} \quad (10)$$

where

$$\tilde{E}_i = \begin{bmatrix} E & 0 \\ 0 & E_f \end{bmatrix}, \tilde{A}_i = \begin{bmatrix} A_i & 0 \\ B_{fi} C_i & A_{fi} \end{bmatrix},$$

$$\tilde{B}_i = \begin{bmatrix} B_{wi} \\ 0 \end{bmatrix}, \tilde{C}_i = \begin{bmatrix} L_i - D_{fi} C_i & -C_{fi} \end{bmatrix}.$$

Furthermore, define R, S satisfying $R\tilde{E} = 0$ and $\tilde{E}S = 0$ are both satisfied. Note that the matrices $\tilde{A}_i$ and $\tilde{C}_i$ in (10), which can be written as

$$\tilde{A}_i = \bar{A}_i + F G_i H_i, \tilde{C}_i = \bar{C}_i + J G_i H_i. \quad (11)$$

Associated with Lemma 3, we have the following proposition:

Proposition 1.: System (10) with ITRs is stochastically admissible with H_∞ performance if and only if there exist symmetric matrix P_i, nonsingular matrix Q_i for $i \in \mathbb{S}$ and $j \in S_{uk}^i$ such that

$$\tilde{E}_L^T P_i \tilde{E}_L > 0, \quad (12)$$

$$\Omega_{1i} + \Psi_i G_i \Phi_i + (\Psi_i G_i \Phi_i)^T < 0, \quad i \in S_k^i, \quad (13)$$

$$\Omega_{2i} + \Psi_i G_i \Phi_i + (\Psi_i G_i \Phi_i)^T < 0, \quad i \in S_{uk}^i, \quad (14)$$

where

$$\Omega_{si} = \begin{bmatrix} \Delta_{si} & \Lambda_i^T \tilde{B}_{wi} & \bar{C}_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix}, \ s = 1, 2,$$

$$\Delta_{i1} = He[\bar{A}_i^T \Lambda_i] + \sum_{j\in S_k^i} \pi_{ij} \tilde{E}^T P_j \tilde{E} - \pi_k^i \tilde{E}^T P_j \tilde{E},$$

$$\Delta_{i2} = He[\bar{A}_i^T \Lambda_{1i}] + \sum_{j\in S_k^i} \pi_{ij} \tilde{E}^T P_j \tilde{E} - \pi_k^i \tilde{E}^T P_j \tilde{E} + \pi_d^i \tilde{E}^T P_i \tilde{E} - \pi_d^i \tilde{E}^T P_j \tilde{E},$$

$$G_i = \begin{bmatrix} D_{fi} & C_{fi} \\ B_{fi} & A_{fi} \end{bmatrix}, \Lambda_i = P_i \tilde{E} + R^T Q_i S^T,$$

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix}, \tilde{B}_{wi} = \begin{bmatrix} B_{wi} \\ 0 \end{bmatrix}, \bar{C}_i = \begin{bmatrix} L_i & 0 \end{bmatrix},$$

$$\Psi_i = \begin{bmatrix} \Lambda_i^T F \\ 0 \\ J \end{bmatrix}, \Phi_i = \begin{bmatrix} H_i & 0 \end{bmatrix},$$

$$F = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, J = [-I, 0], H_i = \begin{bmatrix} C_i & 0 \\ 0 & I \end{bmatrix}.$$

Then, by Lemma 1, (13) $\Leftrightarrow$

$$\Phi_i^{\perp} \Omega_{1i} \Phi_i^{T\perp} < 0, \quad (15)$$

$$\Psi_i^{\perp} \Omega_{1i} \Psi_i^{T\perp} < 0, \quad (16)$$

and (14) $\Leftrightarrow$

$$\Phi_i^{\perp} \Omega_{2i} \Phi_i^{T\perp} < 0, \quad (17)$$

$$\Psi_i^{\perp} \Omega_{2i} \Psi_i^{T\perp} < 0. \quad (18)$$

Now, we will present a full-order H_∞ filtering such that system (10) is stochastically admissible with H_∞ performance.

Proposition 2: There exists a full-order H_∞ filtering in (9) such that system (10) is stochastically admissible with H_∞ performance γ if and only if there exist symmetric matrices P_{1i}, $\bar{P}_{1i}$, nonsingular matrices Q_{1i}, $\bar{Q}_{1i}$ for $i \in \mathbb{S}$ and $j \in S_{uk}^i$ such that

$$\begin{bmatrix} E_L^T P_{1i} E_L & I \\ I & E_R^T \bar{P}_{1i} E_R \end{bmatrix} > 0,$$

$$\text{if } i \in S_k^i, \quad (19)$$

$$\begin{bmatrix} C_i^{\perp} \delta_{i1} C_i^{\perp T} & C_i^{\perp} \Lambda_{1i} B_{wi} & C_i^{\perp} C_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (20)$$

$$\begin{bmatrix} \delta_{i2} & B_{wi} & R_{11}(x) & R_{21}(x) \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -S_{11}(x) & 0 \\ * & * & * & -S_{21}(x) \end{bmatrix} < 0,$$

$$\text{if } i \in S_{uk}^i, \quad (21)$$

$$\begin{bmatrix} C_i^{\perp} \bar{\delta}_{i1} C_i^{\perp T} & C_i^{\perp} \Lambda_{1i}^T B_{wi} & C_i^{\perp} C_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (22)$$

$$\begin{bmatrix} \bar{\delta}_{i2} & B_{wi} & R_{11}(x) & \bar{R}_{21}(s) \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -S_{11}(x) & 0 \\ * & * & * & -S_{21}(x) \end{bmatrix} < 0, \tag{23}$$

where

$$\begin{aligned} \delta_{i2} &= He[A_i\bar{\Lambda}_{1i}] + \pi_{ii}E\bar{P}_{1i}E^T, \\ \bar{\delta}_{i2} &= He[A_i\bar{\Lambda}_{1i}] + \pi_d^i E\bar{P}_{1i}E^T, \\ R_{11}(x) &= [\sqrt{\pi_{ij}}E\bar{P}_{1i}E_R]_{j\in S_k^i}/\{i\}, \\ R_{21}(x) &= \sqrt{-\pi_k^i}E\bar{P}_{1i}E_R, \\ \bar{R}_{21}(x) &= \sqrt{-\pi_d^i - \pi_k^i}E\bar{P}_{1i}E_R, \\ S_{11}(x) &= diag\{E_R^T\bar{P}_{1j}E_R\}_{j\in S_k^i}/\{i\}, \\ S_{21}(x) &= E_R^T\bar{P}_{1j}E_R. \end{aligned}$$

Furthermore, if P_{1i}, $\bar{P}_{1i}$, Q_{1i}, $\bar{Q}_{1i}$ are the solutions of (19)-(23), then H_∞ filter matrices G_i can be given by substituting the solutions of (19)-(23) into (13), (14), where

$$\Lambda_i = \begin{bmatrix} \Lambda_{1i} & \bar{\Lambda}_{1i}^{-1} - \Lambda_{1i} \\ \bar{\Lambda}_{1i}^{-1} - \Gamma_{1i} & \Lambda_{1i} - \bar{\Lambda}_{1i}^{-1} \end{bmatrix}, P_i = \begin{bmatrix} P_{1i} & P_{12i} \\ P_{12i}^T & P_{3i} \end{bmatrix},$$

the other notations are defined in Lemma 3 and Proposition 1.

Proof: Recalling Ψ_i and Φ_i in (13), (14), by a simple calculation, we can obtain

$$\Phi_i^\perp = \begin{bmatrix} \begin{bmatrix} C_i^\perp & 0 \end{bmatrix} & 0 & 0 \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & I & 0 \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & 0 & I \end{bmatrix}, \tag{24}$$

$$\Psi_i^\perp = \begin{bmatrix} \begin{bmatrix} I & 0 \end{bmatrix} & 0 & 0 \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & I & 0 \end{bmatrix} \begin{bmatrix} \Lambda_i^{-T} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}. \tag{25}$$

Next, we will probe the NS condition for $i \in S_k^i$ as follows. (The proof process for $i \in S_{uk}^i$ is similar, which is omitted).

(Necessity). For symmetric matrices P_i, $\bar{P}_i$ and nonsingular matrices Q_i and $\bar{Q}_i$, we have the following partition

$$P_i = \begin{bmatrix} P_{1i} & P_{12i} \\ P_{12i}^T & P_{3i} \end{bmatrix}, \bar{P}_i = \begin{bmatrix} \bar{P}_{1i} & \bar{P}_{12i} \\ \bar{P}_{12i}^T & \bar{P}_{3i} \end{bmatrix}, \tag{26}$$

$$Q_i = \begin{bmatrix} Q_{1i} & Q_{12i} \\ Q_{21i} & Q_{3i} \end{bmatrix}, \bar{Q}_i = \begin{bmatrix} \bar{Q}_{1i} & \bar{Q}_{12i} \\ \bar{Q}_{21i} & \bar{Q}_{3i} \end{bmatrix}. \tag{27}$$

Then, substituting (24)-(27) into conditions (15) and (16), which respectively lead to (20) and

$$\begin{bmatrix} \bar{\delta}_{21i} & B_{wi} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{28}$$

where

$$\begin{aligned} \bar{\delta}_{21i} &= He[A_i\bar{\Lambda}_{1i}] + \sum_{j\in\mathbb{S}}\pi_{ij}\mathfrak{P}_{ij} - \pi_k^i\mathfrak{P}_{ij}, \\ \mathfrak{P}_{ij} &= E\bar{P}_{1i}E^TP_{1j}E\bar{P}_{1i}E^T \\ &\quad + He[E\bar{P}_{1i}E^TP_{12j}E\bar{P}_{12i}^TE^T] \\ &\quad + E\bar{P}_{12i}E^TP_{3j}E\bar{P}_{12i}^TE^T. \end{aligned}$$

Recalling (12), we have $E_L^TP_{3j}E_L > 0$ and

$$\begin{aligned} E_L^TP_{1j}E_L &= (E_R^T\bar{P}_{1j}E_R)^{-1} + E_L^TP_{12j}E_L \\ &\quad \times (E_L^TP_{3j}E_L)^{-1}(E_L^TP_{12j}^TE_L). \end{aligned} \tag{29}$$

Further, we get

$$\begin{aligned} \mathfrak{P}_{ij} &= E\bar{P}_{1i}E_R(E_R^T\bar{P}_{1j}E_R)^{-1}(E\bar{P}_{1i}E_R)^T \\ &\quad + \mathfrak{E}_{ij}^T(E_L^TP_{3j}E_L)^{-1}\mathfrak{E}_{ij}, \end{aligned} \tag{30}$$

$$\mathfrak{E}_{ij} = E_L^TP_{3j}E\bar{P}_{12i}^TE^T + E_L^TP_{12j}^TE\bar{P}_{1i}E^T. \tag{31}$$

Thus, condition (28) gives

$$\begin{bmatrix} \tilde{\delta}_{21i} & B_{wi} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{32}$$

where

$$\begin{aligned} \tilde{\delta}_{21i} &= He[A_i\bar{\Lambda}_{1i}] + \sum_{j\in\mathbb{S}}\pi_{ij}E\bar{P}_{1i}E_R \\ &\quad \times (E_R^T\bar{P}_{1j}E_R)^{-1}(E\bar{P}_{1i}E_R)^T - \pi_k^iE\bar{P}_{1i}E_R \\ &\quad \times (E_R^T\bar{P}_{1j}E_R)^{-1}(E\bar{P}_{1i}E_R)^T. \end{aligned}$$

By Shur complement, we get (15) $\Rightarrow$ (21).

Next, we shall probe (12) is equivalent to condition (19). Condition (12) satisfies the following relations:

$$\begin{aligned} 0 &< \begin{bmatrix} E_L^TP_{1i}E_L & E_L^TP_{12i}E_L \\ E_L^TP_{12i}^TE_L & E_L^TP_{3i}E_L \end{bmatrix}, \\ &\Leftrightarrow \\ 0 &< E_L^TP_{3i}E_L, 0 < E_L^TP_{1i}E_L \\ &\quad - E_L^TP_{12i}E_L(E_L^TP_{3i}E_L)^{-1}E_L^TP_{12i}^TE_L \\ &= (E_R^T\bar{P}_{1i}E_R)^{-1}, \\ &\Leftrightarrow \\ 0 &< E_L^TP_{12i}E_L(E_L^TP_{3i}E_L)^{-1}E_L^TP_{12i}^TE_L \\ &= E_L^TP_{1i}E_L - (E_R^T\bar{P}_{1i}E_R)^{-1}, 0 < E_R^T\bar{P}_{1i}E_R, \end{aligned}$$

which implies condition (19). Summarizing the above discussions, the proof of necessity is completed.

(Sufficiency). Construct P_i and Q_i as follows

$$P_i = \begin{bmatrix} P_{1i} & P_{12i} \\ * & P_{3i} \end{bmatrix}, \tag{33}$$

$$Q_i = \begin{bmatrix} Q_{1i} & Q_{12i} \\ Q_{12i} & Q_{3i} \end{bmatrix}, \tag{34}$$

where

$$\begin{aligned} P_{12i} &= -P_{3i}, Q_{12i} = -Q_{3i}, \\ \tilde{P}_{3i} &= P_{1i} - E_L(E_L^TE_L)^{-1} \\ &\quad \times (E_R^T\bar{P}_{1i}E_R)^{-1}(E_L^TE_L)^{-1}E_L^T, \\ \tilde{Q}_{12i} &= Q_{1i} - (V^TV\bar{Q}_{1i}UU^T)^{-1}, \end{aligned}$$

and P_{1i}, $\bar{P}_{1i}$ Q_{1i}, $\bar{Q}_{1i}$ are the same as those Lemma 3. Then, from (33) and (34), Λ_i in (15) and (16) can be constructed as

$$\Lambda_i = P_i\tilde{E}_i + R^TQ_iS^T = \begin{bmatrix} \Lambda_{1i} & \Lambda_{12i} \\ \Lambda_{12i} & -\Lambda_{12i} \end{bmatrix}, \tag{35}$$

where $\Lambda_{12i} = \bar{\Lambda}_{1i}^{-1} - \Lambda_{1i}$ and Λ_{1i}, $\bar{\Lambda}_{1i}$ have been defined in Proposition 2.

In view of Lemma 2, one has $\bar{\Lambda}_i = \bar{P}_i\tilde{E}^T + S\bar{Q}_iR$, where $R = diag\{U, U\}$, $S = diag\{S, S\}$ and

$$\bar{P}_i = \begin{bmatrix} \bar{P}_{1i} & \bar{P}_{1i} \\ \bar{P}_{1i} & \bar{P}_{3i} \end{bmatrix}, \bar{Q}_i = \begin{bmatrix} \bar{Q}_{1i} & \bar{Q}_{2i} \\ \bar{Q}_{2i} & \bar{Q}_{3i} \end{bmatrix}. \quad (36)$$

Therefore, we have

$$\bar{\Lambda}_i^T\tilde{E}^T P_j\tilde{E}\bar{\Lambda}_i = \tilde{E}\bar{P}_i\tilde{E}^T P_j\tilde{E}\bar{P}_i\tilde{E}^T = \begin{bmatrix} \mathfrak{F}_{ij} & \star \\ \star & \star \end{bmatrix}, \quad (37)$$

where $\mathfrak{F}_{ij} = E\bar{P}_{1i}E_R(E_R^T\bar{P}_{1j}E_R)^{-1}E_R^T\bar{P}_{1i}E^T$ and $\star$ indicates irrelevant matrices. Then, applying (33), (36) and (37) to conditions (15) and (16), respectively, we have (20) and

$$\begin{bmatrix} \delta_{211i} & B_{wi} \\ * & -\gamma^2 I \end{bmatrix} < 0, \quad (38)$$

where $\delta_{211i} = He[A_i\bar{\Lambda}_{1i}] + \sum_{j\in S_k^i} \pi_{ij}\mathfrak{F}_{ij} - \pi_k^i\mathfrak{F}_{ij}$. Thus, we can conclude that (20) and (21) hold if (15) and (16) hold, respectively.

Further, set $P_{1L} = E_L^T P_{1i}E_L$, $\bar{P}_{1R} = (E_R^T\bar{P}_{1i}E_R)^{-1}$, in this case, condition (19) can be expressed as

$$\begin{aligned} 0 &< P_{1L} - \bar{P}_{1R}, 0 < \bar{P}_{1R}, \\ &\Leftrightarrow \\ 0 &< \begin{bmatrix} P_{1L} & \bar{P}_{1R} - P_{1L} \\ \bar{P}_{1R} - P_{1L} & P_{1L} - \bar{P}_{1R} \end{bmatrix}, \end{aligned}$$

which implies condition (12). Summarizing the above statements, the proof of Proposition 2 is completed.

Based on Theorem 1, we can directly derive the following NS condition for SMJSs with CTRs.

Corollary 1: There exists a full-order H_∞ filter in (9) such that system (10) is stochastically admissible with H_∞ performance γ if and only if there exist symmetric matrices P_{1i}, $\bar{P}_{1i}$, nonsingular matrices Q_{1i}, $\bar{Q}_{1i}$ for $i \in \mathbb{S}$ such that (19) and

$$\begin{bmatrix} C_i^{\perp}\delta_{11i}C_i^{\perp T} & C_i^{\perp}\Lambda_{1i}^T B_{wi} & C_i^{\perp}L_i^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (39)$$

$$\begin{bmatrix} \delta_{21i} & B_{wi} & [\sqrt{\pi_{ij}}E\bar{P}_iE_R]_{j\in\mathbb{S}}/\{i\} \\ * & -\gamma^2 I & 0 \\ * & * & -diag\{E_R^T\bar{P}_jE_R\}_{j\in\mathbb{S}}/\{i\} \end{bmatrix} < 0, \quad (40)$$

where

$$\begin{aligned} \delta_{11i} &= He[A_i^T\Lambda_{1i}] + \sum_{j\in\mathbb{S}} \pi_{ij}E^T P_{1j}E, \\ \delta_{21i} &= He[A_i\bar{\Lambda}_{1i}] + \pi_{ii}E\bar{P}_{1i}E^T, \\ \Lambda_{1i} &= P_{1i}E + U^T Q_{1i}V^T, \bar{\Lambda}_{1i} = \bar{P}_{1i}E^T + V\bar{Q}_{1i}U. \end{aligned}$$

Furthermore, if P_{1i}, $\bar{P}_{1i}$, Q_{1i}, $\bar{Q}_{1i}$ are the solutions of (19), (39) and (40), then H_∞ filter matrices G_i can be given by substituting the solutions of (19), (39) and (40) into (13), where Ω_{1i}, Λ_i and $\bar{\Lambda}_i$ are defined in Proposition 2.

Remark 1: It is noted that NS full-order H_∞ filtering has been proposed for SMJSs with CTRs in [19]. Since the filter matrices in [19] are constructed with the TRs, the TRs should be completely known. Fortunately, the filter matrices can be directly given by solving Proposition 2 if the TRs is incomplete. Thus, the method of present paper is more general than the result proposed in [19].

Remark 2: It is noted that the methods in Proposition 2 and [19] cannot be extended to derive the reduced-order one since some special matrix structure, such as, E_f, Λ_i, and $\bar{\Lambda}_i$ in Proposition 2 and [19]. To be concrete, i) in [19], filter parameters A_{fi}, B_{fi}, C_{fi} is dependent on full-order n. ii) In [19] and Proposition 2, Λ_i must be completely known for the solvability of filter parameters. iii) $E_f = E$ is determined in [19] and Proposition 2 due to the utilization of Lemma 2. Thus, without constraints, we will give the following NS reduced-order H_∞ filtering in another insight.

Theorem 1: There exists a reduced-order H_∞ filtering in (9) such that system (10) is stochastically admissible with H_∞ performance γ if and only if there exist symmetric matrices P_i, $\bar{P}_i$, nonsingular matrices Q_i, $\bar{Q}_i$ for $i \in \mathbb{S}$ and $j \in S_{uk}^i$ such that (12) and

$$\begin{aligned} &\text{if } i \in S_k^i, \\ &\mathcal{N}_i\Omega_{1i}\mathcal{N}_i^T < 0, \quad (41) \\ &\begin{bmatrix} F\Delta_{i2}F^T & F\tilde{B}_{wi} & FR_1(x) & FR_2(x) \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -S_1(x) & 0 \\ * & * & * & -S_2(x) \end{bmatrix} < 0, \quad (42) \\ &\text{if } i \in S_{uk}^i, \\ &\mathcal{N}_i\Omega_{2i}\mathcal{N}_i^T < 0, \quad (43) \\ &\begin{bmatrix} F\bar{\Delta}_{i2}F^T & F\tilde{B}_{wi} & FR_1(x) & F\bar{R}_2(s) \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -S_1(x) & 0 \\ * & * & * & -S_2(x) \end{bmatrix} < 0, \quad (44) \end{aligned}$$

where

$$\begin{aligned} \Delta_{i2} &= He[\bar{A}_i\bar{\Lambda}_i] + \pi_{ii}\tilde{E}\bar{P}_i\tilde{E}^T, \\ \bar{\Delta}_{i2} &= He[\bar{A}_i\bar{\Lambda}_i] + \pi_d^i\tilde{E}\bar{P}_i\tilde{E}^T, \\ R_1(x) &= [\sqrt{\pi_{ij}}\tilde{E}\bar{P}_i\tilde{E}_R]_{j\in S_k^i}/\{i\}, \\ R_2(x) &= \sqrt{-\pi_k^i}\tilde{E}\bar{P}_i\tilde{E}_R, \bar{R}_2(s) = \sqrt{-\pi_d^i - \pi_k^i}\tilde{E}\bar{P}_i\tilde{E}_R, \\ S_1(x) &= diag\{\tilde{E}_R^T\bar{P}_j\tilde{E}_R\}_{j\in S_k^i}/\{i\}, S_2(x) = \tilde{E}_R^T\bar{P}_j\tilde{E}_R, \\ \Lambda_i &= P_i\tilde{E} + R^T Q_iS^T, \bar{\Lambda}_i = \bar{P}_i\tilde{E}^T + S\bar{Q}_iR, \end{aligned}$$

and the other notations are defined in Proposition 1.

In this case, inspired by [21], [22], for given any appropriately dimensional matrices $R_i > 0$, Z_i and Υ_i satisfying $||\Upsilon_i|| \leq 1$. Φ_i, Ψ_i satisfying $\Phi_{Li}\Phi_{Ri} = \Phi_i$, $\Psi_i\Psi_{Li}\Psi_{Ri} = \Psi_i$, where Φ_{Li}, Φ_{Ri} and Ψ_{Li}, Ψ_{Ri} are any full rank factors of Φ_i and Ψ_i. Then, if P_i, $\bar{P}_i$, Q_i, $\bar{Q}_i$ are the solutions of (12), (41)-(44). Then reduced-order H_∞ filter parameter matrices G_i can be given by

$$G_i = \Psi_{Ri}^+K_{ki}\Phi_{Li}^+ + Z_i - \Psi_{Ri}^+\Psi_{Ri}Z_i\Phi_{Li}\Phi_{Li}^+, \quad i \in S_k^i, \quad (45)$$
$$G_i = \Psi_{Ri}^+K_{uki}\Phi_{Li}^+ + Z_i - \Psi_{Ri}^+\Psi_{Ri}Z_i\Phi_{Li}\Phi_{Li}^+, \quad i \in S_{uk}^i, \quad (46)$$

where

$$
\begin{aligned}
W_{ki} &= (\Psi_{Li}^T R_i^{-1} \Psi_{Li}^T - \Omega_{1i})^{-1} > 0,\\
S_{ki} &= R_i - \Psi_{Li}^T [W_{ki} - W_{ki}\Phi_{Ri}^T(\Phi_{Ri} W_{ki} \Phi_{Ri}^T)^{-1}]\Psi_{Li},\\
K_{ki} &= -R_i^{-1}\Psi_{Li}^T W_{ki} \Psi_{Ri}^T (\Phi_{Ri} W_{ki}\Phi_{Ri}^T)^{-1}\\
&\quad + R_i^{-1} S_i^{1/2} \Upsilon_i (\Phi_{Ri} W_{ki} \Phi_{Ri}^T)^{-1/2},\\
W_{uki} &= (\Psi_{Li}^T R_i^{-1} \Psi_{Li}^T - \Omega_{2i})^{-1} > 0,\\
S_{uki} &= R_i - \Psi_{Li}^T W_{uki} [I - \Phi_{Ri}^T (\Phi_{Ri} W_{uki} \Phi_{Ri}^T)^{-1}]\Psi_{Li},\\
K_{uki} &= -R_i^{-1}\Psi_{Li}^T W_{uki}\Psi_{Ri}^T(\Phi_{Ri} W_{uki}\Phi_{Ri}^T)^{-1}\\
&\quad + R_i^{-1} S_i^{1/2} L_i (\Phi_{Ri} W_{uki} \Phi_{Ri}^T)^{-1/2}.
\end{aligned}
$$

Proof: Recalling Ψ_i and Φ_i in (13) associated with reduced order $\hat{n}$, we can obtain

$$
\Phi_i^{\perp} = \begin{bmatrix} N_i & 0_{n,m} & 0_{n,q} \\ 0_{m,n+\hat{n}} & I_m & 0_{m,q} \\ 0_{q,n+\hat{n}} & 0_{q,m} & I_q \end{bmatrix}, \tag{47}
$$

$$
\begin{aligned}
\Psi_i^{\perp} &= \begin{bmatrix} F & 0_{n,m} & 0_{n,q} \\ 0_{m,n+\hat{n}} & I_m & 0_{m,q} \end{bmatrix}\\
&\quad \times \begin{bmatrix} \Lambda_i^{-T} & 0_{n+\hat{n},m} & 0_{n+\hat{n},q} \\ 0_{m,n+\hat{n}} & I_m & 0_{m,q} \\ 0_{q,n+\hat{n}} & 0_{q,m} & I_q \end{bmatrix}.
\end{aligned} \tag{48}
$$

Then we shall will the NS condition for $i \in S_k^i$ as follow, the proof process for $i \in S_{uk}^i$ is similar, which is omitted.

Associated with Lemma 3, condition (15) is equivalent to (41). Next, we need probe the (16) is equivalent to (42).

As the proof of Proposition 2, condition (16) can be written as (32), connecting with Lemma 3, condition (32) can be rewritten as

$$
\begin{bmatrix} \tilde{\Delta}_{i21} & B_{wi} \\ * & -\gamma^2 I \end{bmatrix} < 0, \quad i \in S_k^i, \tag{49}
$$

where

$$
\begin{aligned}
\tilde{\Delta}_{i21} &= He[\bar{A}_i \bar{\Lambda}_i] + \sum_{j \in S_k^i} \pi_{ij}\Pi_{ij} - \pi_k^i \Pi_{ij},\\
\Pi_{ij} &= \tilde{E}\bar{P}_i \tilde{E}_R (\tilde{E}_R^T \bar{P}_j \tilde{E}_R)^{-1} (\tilde{E}\bar{P}_i \tilde{E}_R)^T.
\end{aligned}
$$

Further, by Shur complement to (49), we get (42). Summarizing the above statements, the proof of Theorem 1 is completed.

Corollary 2: There exists a reduced-order H_∞ filtering in (9) such that system (10) stochastically admissible with H_∞ performance γ if and only if there exist symmetric matrices P_i, $\bar{P}_i$, nonsingular matrices Q_i, $\bar{Q}_i$ for each $i \in \mathbb{S}$ such that (12) and

$$
\mathcal{N}_i \Omega_{1i} \mathcal{N}_i^T < 0, \tag{50}
$$

$$
\begin{bmatrix} F\Delta_{i2}F^T & F\tilde{B}_{wi} & F[\sqrt{\pi_{ij}}\tilde{E}\bar{P}_i\tilde{E}_R]_{j\in\mathbb{S}/\{i\}} \\ * & -\gamma^2 I & 0 \\ * & * & -diag\{\tilde{E}_R^T\bar{P}_j\tilde{E}_R\}_{j\in\mathbb{S}/\{i\}} \end{bmatrix} < 0 \tag{51}
$$

where

$$
F = [I_n, 0_{n,n+\hat{n}}], \mathcal{N}_i = diag\{N_i, I\},\ N_i = [C_i^{\perp}, 0_{n,\hat{n}+q}],
$$

$$
\begin{aligned}
\Delta_{i2} &= He[\bar{A}_i \bar{\Lambda}_i] + \pi_{ii}\tilde{E}\bar{P}_i\tilde{E}^T,\\
\Lambda_i &= P_i\tilde{E} + R^T Q_i S^T,\ \bar{\Lambda}_i = \bar{P}_i \tilde{E}^T + S\bar{Q}_i R,
\end{aligned}
$$

and the other notations are defined in Proposition 1.

In this case, inspired by [21], [22], for given any appropriately dimensional matrices $R_i > 0$, Z_i and Υ_i satisfying $||\Upsilon_i|| \leq 1$. Φ_i, Ψ_i satisfying $\Phi_{Li}\Phi_{Ri} = \Phi_i$, $\Psi_i\Psi_{Li}\Psi_{Ri} = \Psi_i$, where Φ_{Li}, Φ_{Ri} and Ψ_{Li}, Ψ_{Ri} are any full rank factors of Φ_i and Ψ_i. Then, if P_i, $\bar{P}_i$, Q_i, $\bar{Q}_i$ are the solutions of (12), (50) and (51). Then reduced-order H_∞ filter matrices G_i can be given by

$$
G_i = \Psi_{Ri}^{+} K_i \Phi_{Li}^{+} + Z_i - \Psi_{Ri}^{+}\Psi_{Ri} Z_i \Phi_{Li}\Phi_{Li}^{+}, \tag{52}
$$

where

$$
\begin{aligned}
K_i &= -R_i^{-1}\Psi_{Li}^T W_i \Psi_{Ri}^T (\Phi_{Ri} W_i \Phi_{Ri}^T)^{-1}\\
&\quad + R_i^{-1} S_i^{1/2} \Upsilon_i (\Phi_{Ri} W_i \Phi_{Ri}^T)^{-1/2},\\
W_i &= (\Psi_{Li}^T R_i^{-1} \Psi_{Li}^T - \Omega_i)^{-1},\\
S_i &= R_i - \Psi_{Li}^T [W_i - W_i \Phi_{Ri}^T (\Phi_{Ri} W_i \Phi_{Ri}^T)^{-1}]\Psi_{Li}.
\end{aligned}
$$

Remark 3: Note that Theorem 1 in [19] and Proposition 2 of present paper, the attention is focused on system matrices of the considered system (1). And the first aim is to obtain $\Lambda_{1i} = P_{1i}E + U^T Q_{1i} V^T$ by building matrices Λ_i. However, the order $2n$ cannot be changed in [19] and Proposition 2. Thus, in Theorem 1 of present paper, we directly focus on system (10), the Lyapunov decision order is $n + \hat{n}$, we can first get matrices Λ_i, the order is also $n+\hat{n}$. Thus, the reduced-order H_∞ filtering is obtained.

Remark 4: It is well known that reduced-order filter design is a very important issue in many applications, especially when fast data processing is necessary with a process of limited power. Therefore, considerable attention has been devoted to the study of reduced-order filter design over the past few years. In some existing works [7], [23], sufficient conditions for reduced-order H_∞ filtering are derived on equality/rank constraints, which are hard to find a solution to perfectly satisfy the equality constraints due to its round-off errors in computation. Furthermore, complicated matrix transformation and matrix structures inverse the mathematical derivation. In this paper, the obtained results have neither complicated matrix transformation nor equality/rank constraint, which make the conditions easier to find numerical solutions than the existing works [7], [23].

Remark 5: In this paper, only the switching probabilities were considered. However, in practice, there is usually a restriction to the switching frequency. In this case, by combining with method in [24], [25]. The present synthesis method can be extended to deal with singular systems with average dwell time (ADT) switching.

IV. EXAMPLES

In this section, we will give one numerical and one practical examples to demonstrate the applicability of the proposed approaches.

Example 1: To demonstrate the efficiency of full-order H_∞ filtering in Corollary 1 and Corollary 2 for system (1) with CTRs, the following parameters are given:

$$A_1 = \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -2.5 & 1 \\ -0.1 & 0.3 & -3.8 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.8 & 0.3 & 0 \end{bmatrix}, L_1 = \begin{bmatrix} 0.5 \\ -0.1 \\ 1 \end{bmatrix}^T,$$
$$A_2 = \begin{bmatrix} -2.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -2 \end{bmatrix}, B_2 = \begin{bmatrix} -0.6 \\ 0.5 \\ 0 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} -0.5 & 0.2 & 0.3 \end{bmatrix}, L_2 = \begin{bmatrix} 0 \\ 1 \\ 0.4 \end{bmatrix}^T,$$
$$\Pi = \begin{bmatrix} -0.6 & 0.6 \\ 0.9 & -0.9 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

TABLE 1. The minimum allowable γ for different methods.

methods	Theorem 1, [15]	Corollary 1	Corollary 2
γ_{min}	0.4087	0.2632	0.2632

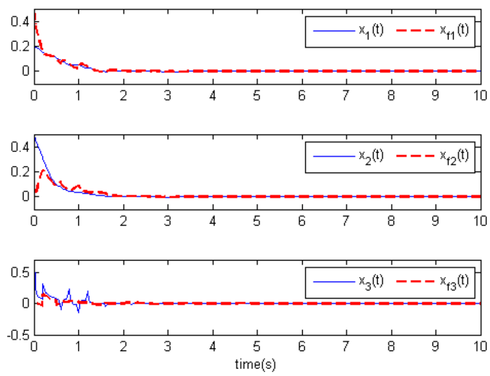


FIGURE 1. Responses of $x(t)$ their estimations.

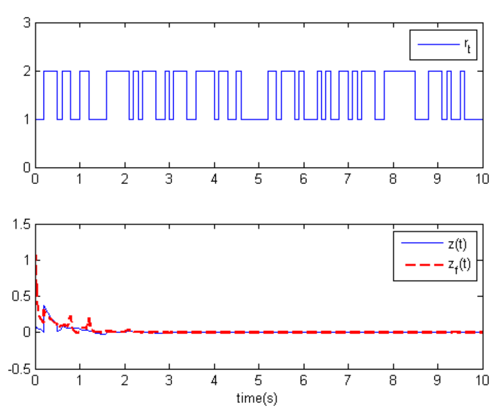


FIGURE 2. One possible switching signals and responses of $z(t)$ and their estimations.

A comparison among [15], Corollary 1 and Corollary 2 of present paper is depicted in Table 1. Obviously, Corollary 1 and Corollary 2 are equivalent for full-order H_∞ filtering, which are superior than the method in [15]. Furthermore, Corollary 2 can be also used to reduced-order filtering while Corollary 1 can not be. If $\hat{n} = 2$ in Corollary 2, the minimum H_∞ performance level γ is 0.2754. It concludes that Corollary 2 is more general than Corollary 1. Next, supposing $\gamma = 2$, by using Corollary 1, the full-order H_∞filter matrices $G_i = \left[\begin{array}{c|c} D_{fi} & C_{fi} \\ \hline B_{fi} & A_{fi} \end{array}\right]$ can be given by

$$G_1 = \left[\begin{array}{c|ccc} 0.0005 & 0.0001 & -0.0002 & 0.0010 \\ \hline -1.9011 & 1.5078 & 0.6197 & -1.1988 \\ -0.7221 & 0.5685 & 0.2266 & -0.0740 \\ 0.7099 & -0.5585 & -0.2269 & 0.0687 \end{array}\right] \times 10^3$$

$$G_2 = \left[\begin{array}{c|ccc} 0.8299 & 0.4125 & 0.8310 & 0.1473 \\ \hline 59.7086 & 29.3448 & -11.1787 & -18.9317 \\ -32.2933 & -14.1969 & 3.1590 & 10.5842 \\ -14.7964 & -7.4667 & 3.8307 & 2.5805 \end{array}\right].$$

With the initial states of $x(0) = col[0.2, 0.5, 0]$, $x_f(0) = col[0, 0, 0]$, $r(0) = 1$ and $w(t) = -0.3sin(5t)e^{-0.8t}$. From Figure 1 and Figure 2, it is seen that proposed full-order H_∞ filtering is efficient.

Example 2: To demonstrate the efficiency of reduced-order H_∞ filtering in Theorem 1 for system (1) with ITRs. Consider a DC motor driving a load that changes randomly [19].

$$\begin{cases} J\dot{v}_i = K_t c(t) - b_i v(t), \\ u(t) = R_i c(t) + K_v v(t), \end{cases} \tag{53}$$

where $v(t)$, $u(t)$, K_t, K_v, R_i mean the current, the speed of the shaft, the input voltage, the torque constant, the electromotive force and the electric resistor, respectively. The coefficient relations such as $J_i = J_m + \frac{J_{ci}}{n^2}$, $b_i = b_m + \frac{b_{ci}}{n^2}$ hold, where J_m and J_{ci} are the moments of the motor and the load; b_m and b_{ci} are the damping ratios with gear ratio n. In this case that $J_m = 0.5kgm$, $J_{c1} = 50kgm$, $J_{c2} = 150\ kgm$, $b_{c1} = 100$, $b_{c2} = 240$, $R_1 = R_2 = 1\Omega$, $b_m = 1$, $k_t = 2Nm/A$, $k_v = 1Vs/rad$, $b_{v1} = b_{v2} = 0.4$ and $n = 10$, the stabilized DC motor model with disturbance can be modeled as SMJS (1) with state $x(t) = [v(t), c(t)]^T$ which has the following system matrices:

$$A_1 = \begin{bmatrix} -2 & 3 \\ -0.52 & -0.908 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1.7 & 1.5 \\ -1.03 & -0.64 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

For the system output, we set the following coefficients:

$$C_1 = \begin{bmatrix} 10 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 11 & 0 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, L_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Singular matrix is given as $E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and the transition rate matrix is given as

$$\Pi = \begin{bmatrix} -0.0193 & 0.0193 \\ ? & ? \end{bmatrix}.$$

Now, a first-order H_∞ filter will be designed. Suppose the required H_∞ norm bound is $\gamma = 0.5$ and the following parameters are given as $\pi_d^2 = -1$, $E_f = 1$, $R = S^T = [0, 1, 0]$, $\mathcal{N}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, $\mathcal{N}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Then, by solving Theorem 1 of present paper, a first-order filter matrices can be given as

$$G_1 = \left[\begin{array}{c|c} 0.1580 & -0.2322 \\ 0.5000 & -1.4643 \end{array}\right], G_2 = \left[\begin{array}{c|c} 0.1192 & 0.6262 \\ 0.4000 & -0.8393 \end{array}\right].$$

With initial state $x(0) = col[0.2, 0]$, $x_f(0) = 0$, $r_0 = 1$, $w(t) = 0.1sin(5t)e^{-0.2t}$. From Figure 3, it can be observed that the first-order filter design method is efficient.

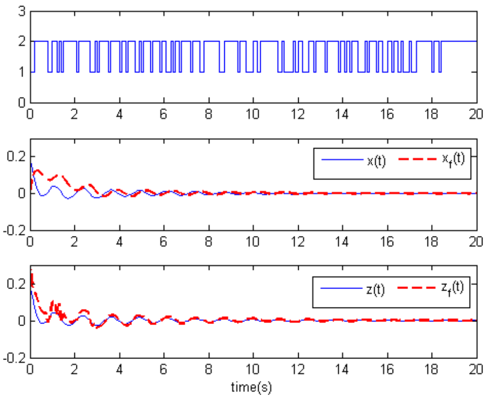


FIGURE 3. **One possible switching signals, responses of $x(t)$, $z(t)$ and their estimations.**

V. CONCLUSION

In this paper, the reduced-order H_∞ filtering problem is considered for SMJSs with ITRs. By separating augmented system matrices, the existence of NS full-order H_∞ filtering was formulated in terms of strict LMIs. To gap the existence of NS reduced-order H_∞ filtering for SMJSs, without separating the augmented system matrices, the NS reduced-order H_∞ filtering was successfully derived. The complicated matrix transformation and equality/rank constraints are avoided in this paper. Two examples are illustrated to demonstrate the effectiveness of the achieved results. In the future, extending the methods in this paper to more complicated situations, such as for SMJSs with ADP switching in [24], [25] and time delay [26] deserves further exploration.

REFERENCES

[1] L. Dai, *Singular Control Systems*. London, U.K.: Springer-Verlag, 1989.

[2] Y. Tian and Z. Wang, "Extended dissipativity analysis of singular Takagi–Sugeno fuzzy systems with time delay via two improved techniques," *Int. J. Syst. Sci.*, vol. 2, pp. 1–11, Jun. 2020, doi: 10.1080/00207721.2020.1784494.

[3] Y. Tian and Z. Wang, "H$_\infty$ performance state estimation for static neural networks with time-varying delays via two improved inequalities," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, early access, May 19, 2020, doi: 10.1109/TCSII.2020.2995604.

[4] Y. Tian and Z. Wang, "Stability analysis for delayed neural networks based on the augmented Lyapunov-krasovskii functional with delay-product-type and multiple integral terms," *Neurocomputing*, vol. 410, pp. 295–303, Oct. 2020, doi: 10.1016/j.neucom.2020.05.045.

[5] Y. Wang, T. Zhang, J. Ren, and J. Li, "Network–based integral sliding mode control for descriptor systems with event–triggered sampling scheme," *Int. J. Robust Nonlinear Control*, vol. 29, no. 10, pp. 2757–2776, Jul. 2019, doi: 10.1002/rnc.4545.

[6] Y. Tian and Z. Wang, "A new integral inequality approach for extended dissipative filters design of singular Markovian jump systems with discrete and distributed delays," *Circuits, Syst., Signal Process.*, vol. 39, no. 6, pp. 2900–2921, Jun. 2020.

[7] Y. Tian and Z. Wang, "Extended dissipativity analysis for Markovian jump neural networks via double-integral-based delay-product-type Lyapunov functional," *IEEE Trans. Neural Netw. Learn. Syst.*, early access, Jul. 23, 2020, doi: 10.1109/TNNLS.2020.3008691.

[8] J. Ren, Y. Tian, and Q. Zhang, "Stability analysis and controller synthesis of continuous-time nonhomogeneous Markovian jump systems with state and input delays," *J. Franklin Inst.*, May 2020, doi: 10.1016/j.jfranklin.2020.05.015.

[9] Y. Tian, Y. Wang, and J. Ren, "Stability analysis and control design of singular Markovian jump systems via a parameter-dependent reciprocally convex matrix inequality," *Appl. Math. Comput.*, vol. 386, Dec. 2020, Art. no. 125471, doi: 10.1016/j.amc.2020.125471.

[10] L. Zhang and J. Lam, "Necessary and sufficient conditions for analysis and synthesis of Markov jump linear systems with incomplete transition descriptions," *IEEE Trans. Autom. Control*, vol. 55, no. 7, pp. 1695–1701, Jul. 2010.

[11] Z. Wu, H. Su, and J. Chu, "Delay-dependent filtering for singular Markovian jump time-delay systems," *Signal Process.*, vol. 90, no. 6, pp. 1815–1824, Jun. 2010.

[12] Z. Wu, H. Su, and J. Chu, "Delay-dependent H_∞ control for singular Markovian jump systems with time delay," *Optim. Contr. Appl. Met.*, vol. 30, no. 5, pp. 443–461, Mar. 2010.

[13] Y. Wang, G. Zhuang, and F. Chen, "A dynamic event-triggered H_∞ control for singular Markov jump systems with redundant channels," *Int. J. Syst. Sci.*, vol. 51, no. 1, pp. 158–179, Jan. 2020.

[14] Y. Wang, G. Zhuang, X. Chen, Z. Wang, and F. Chen, "Dynamic event-based finite-time mixed H_∞ and passive asynchronous filtering for T–S fuzzy singular Markov jump systems with general transition rates," *Nonlinear Anal., Hybrid Syst.*, vol. 36, May 2020, Art. no. 100874, doi: 10.1016/j.nahs.2020.100874.

[15] G. Wang, P. Zhang, and Q. Zhang, "A generalized Robust H_∞ filtering for singular Markovian jump systems and its applications," *Int. J. Robust Nonlinear Control*, vol. 24, no. 18, pp. 3491–3507, Dec. 2014.

[16] C. F. Morais, J. M. Palma, P. L. D. Peres, and R. C. L. F. Oliveira, "An LMI approach for H_∞ and H_∞ reduced-order filtering of uncertain discrete-time Markov and Bernoulli jump linear systems," *Automatica*, vol. 95, pp. 463–471, Sep. 2018.

[17] L. Rong, X. Peng, L. Liu, and B. Zhang, "Improved reduced-order fault detection filter design for polytopic uncertain discrete-time Markovian jump systems with time-varying delays," *Complexity*, vol. 2018, pp. 1–15, Oct. 2018, doi: 10.1155/2018/9489620.

[18] H. Peng, R. Q. Lu, P. Shi, and Y. Xu, "Reduced-order filtering for networks with Markovian jumping parameters and missing measurements," *Int. J. Control*, vol. 92, no. 12, pp. 2737–2749, Dec. 2019.

[19] C. Park, N. Kwon, and P. Park, "Optimal H_∞ filtering for singular Markovian jump systems," *Syst. Control Lett.*, vol. 118, pp. 22–28, Mar. 2018.

[20] Y. Xia, E.-K. Boukas, P. Shi, and J. Zhang, "Stability and stabilization of continuous-time singular hybrid systems," *Automatica*, vol. 45, no. 6, pp. 1504–1509, Jun. 2009.

[21] T. Iwasaki and R. E. Skelton, "All controllers for the general H_∞ control problem: LMI existence conditions and state space formulas," *Automatica*, vol. 30, no. 8, pp. 1307–1317, Aug. 1994.

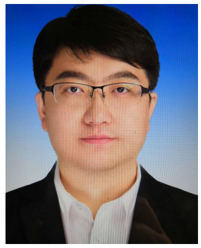
[22] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H_∞ control," *Int. J. Robust Nonlinear Control*, vol. 4, no. 4, pp. 421–448, 1994.

[23] S. Xu and J. Lam, "Reduced-order filtering for singular systems," *Syst. Control Lett.*, vol. 56, no. 1, pp. 48–57, Jan. 2007.

[24] J. Zhou, Y. Wang, X. Zheng, Z. Wang, and H. Shen, "Weighted H_∞ consensus design for stochastic multi-agent systems subject to external disturbances and ADT switching topologies," *Nonlinear Dyn.*, vol. 96, no. 2, pp. 853–868, Feb. 2019.

[25] J. Zhou, Y. Liu, J. Park, Q. Kong, and Z. Wang, "Fault-tolerant anti-synchronization control for chaotic switched neural networks with time delay and reaction diffusion," *Discrete Continuous Dyn. Syst.-S*, to be published, doi: 10.3934/dcdss.2020357.

[26] Y. Tian and Z. Wang, "A new multiple integral inequality and its application to stability analysis of time-delay systems," *Appl. Math. Lett.*, vol. 105, Jul. 2020, Art. no. 106325, doi: 10.1016/j.aml.2020.106325.



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