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Trim Loss Optimization in Paper Production Using Reinforcement Artificial Bee Colony

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ABSTRACT In paper production, a jumbo reel is cut into multiple intermediate rolls, and each intermediate roll is then sheeted as finished goods. This problem is called a cutting stock problem and is proven to be NP-hard. The objective is to minimize material waste or trim loss from all the cuttings. In the case that any intermediate roll is not entirely used for its associated order, the intermediate roll itself could turn to be a dead stock. We use the concept of universal sizes of intermediate rolls to eliminate the dead stock. A pre-defined number of universal sizes of intermediate rolls is to be used to serve all the orders. The problem is solved using Reinforcement Artificial Bee Colony algorithm with Integer Linear Programming subroutine. This proposed approach is then tested with a set of 1,055 orders and 127 different sizes of sheet papers from a paper manufacturer. The results reveal that our method outperforms other algorithms. Our method offers the total trim loss of 3.51%, compared to the trim loss reported by the industry of at least 5%. This approach not only reduces the number of partially cut rolls, but also decreases the number of the jumbo reels needed to serve all the orders. Therefore, both the inventory cost and material cost can be saved.

INDEX TERMS Stock cutting, optimization, swarm intelligence, artificial bee colony algorithm, pulp and paper industry.

I. INTRODUCTION

To produce sheets of paper, the process typically starts from cutting large reels of paper stock—called *jumbo reels*—into smaller *intermediate rolls* of various widths on a cutting machine called winder. These rolls are then further cut into sheets on sheet cutters as shown in Fig. 1. A common objective of paper cutting operations is to determine cutting patterns that minimize material waste or *trim loss* at both the winders and the sheet cutters while satisfying various sizes of paper sheets demanded by the customers. This type of problem is called the cutting stock problem which is among the most extensively studied problems due to its wide range of applications and possible extensions [1], [2]. A huge number of possible patterns makes this problem prohibitive to solve to optimality [3], and the problem is known to be NP-hard [4], [5].

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In order to find the optimum solution of the cutting process, the following constraints must be satisfied during the production.

- The length and the width of jumbo reel are given according to the jumbo reels production.
- The width of intermediate rolls is determined by the various sheet sizes of the orders and their widths can be variety of sizes that they can cover all the orders.
- The specification of the cutter at the winder is the factors for determining the minimum trim width, maximum number of cutting sections, precision of cutting and range of sheet sizes.
- The specification of slitting knives of the cutters is the factors for determining the maximum number of cutting zones and its trim widths (or cutting tolerance).

In the paper production, the optimization process starts from finding the widths and the length of the intermediate rolls given the sizes and the number of sheets ordered. The optimization software such as linear programming and Solver is usually used to find cutting patterns which will provide

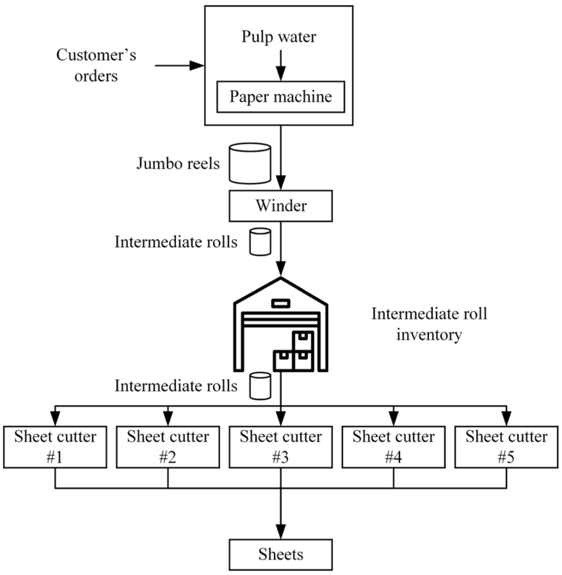

FIGURE 1. Paper production process.

the optimum trim loss. These intermediate rolls with various widths are kept in the stock for further cut into sheets.

The number of intermediate rolls is an issue for most paper production industry. Too many sizes of intermediate rolls are not desirable because they increase inventory (and hence inventory cost) and occupy valuable warehouse space. Certain sizes of the intermediate rolls are odd and unlikely to suit any future orders. They eventually become dead stock, or so called *leftovers*, and must be gotten rid of for an accounting reason or to make room for other inventories. A preferred solution is to have universally common sizes of the intermediate rolls for all the orders so that they are used repeatedly. These universal sizes could both alleviate the amount of the dead stock and increase the flow of the inventory which in turn reduces the storage space needed to keep the stock. Another benefit of having fast inventory-turn of intermediate rolls is to decrease jumbo reel stock. By having better utilization of the intermediate rolls, the need for additional jumbo reels to be cut into intermediate rolls is also minimized. A review of the cutting stock problem with leftover consideration can be found in [6].

This leftover stock could be reduced if the jumbo reels are cut into intermediate rolls with a limited number of sizes. Cui *et al.* [7] were interested in this problem but considered only one intermediate roll size.

In this study these intermediate roll sizes are permitted to be up to a pre-determined number and they are referred as *universal sizes*. Our goal is to determine a set of common intermediate roll sizes that serve all demanded sheet sizes, and yet minimize the total trim loss; whereby the "ballpark" number of the universal sizes is suggested by the management to ensure that the approach is practical to implement. The limited but sufficient number of universal sizes are necessary because too many universal sizes could result in stock buildup, but too few could reversely promote trim loss. To our knowledge, this approach has not yet been proposed in the literature.

The two-stage cutting stock problem is a mathematical program with discrete-value variables of valid paper sizes. The difficulty of this problem is in its extremely large problem space, i.e. there are too many possible solutions to evaluate by a brute-force search. Haessler and Sweeney [8] mentioned that heuristics were only practical approaches in solving this type of problem. Cherri *et al.* [6] added that heuristics allow flexibility to include additional problem-specific features of this problem.

Several methods are devised to solve the cutting stock problem. Chauhan *et al.* [9] proposed a binary nonlinear programming model with normally distributed demands. They applied a branch and price solution approach since the problem space was practically too large to be solved by commercial solvers. Chen *et al.* [10] studied two integer programming models for divisible and indivisible skiving and cutting stock problems. Kim *et al.* [11] formulated the two-stage cutting stock problem as a multiple-choice knapsack problem and proposed a multiple-choice knapsack-based heuristic with a mixed integer linear model to solve the problem. Sanchez *et al.* [12] applied a method that combined integer linear programming (ILP) with other metaheuristics for a binary cutting stock problem. They concluded that ILP with particle swarm optimization (PSO) yielded the best solution in high complexity problems. Kallrath *et al.* [13] used an improved column generation method to prohibit overproduction and tested it on some real-world cutting stock problems.

In order to find the optimum solution of the cutting process, the following constraints must be satisfied during production.

- The length and the width of jumbo reel are given according to the jumbo reels production.
- The width of intermediate rolls is determined by the various sheet sizes of the orders and their widths can be variety of sizes that they can cover all the orders.
- The specification of the cutter at the winder is the factors for determining the minimum trim width, maximum number of cutting sections, precision of cutting and range of sheet sizes.
- The specification of slitting knives of the cutters is the factors for determining the maximum number of cutting zones and its trim widths (or cutting tolerance).

The details are described in subsection *B* and *C* of the Problem Definition section.

Any solution approach of the cutting stock problem is greatly affected by the problem specifications [14]. Due to a large problem space required to be explored in the cutting stock problem, we applied a variant of the ABC algorithm in our proposed method. This is because ABC algorithm is good at exploration [15], [16].

Recently, many variants of the ABC algorithm have been developed. Karaboga and Kaya [17] proposed a variant of the ABC algorithm called Adaptive and Hybrid ABC algorithm (aABC). An arithmetic crossover operation was applied in the solution updating equation. Updating a solution using crossover operation with the global best solution generated promising solution and accelerated the convergence speed. Zhang *et al.* [18] proposed a two-archive multi-objective artificial bee colony algorithm (TMABC-FS) for solving a multi-objective feature selection. In the TMABC-FS algorithm, two operators are introduced, which are convergence-guiding search for employed bees and diversity-guiding search for onlooker bees. Lu *et al.* [19] proposed two updating equations for the onlooker bee phase to increase the convergence speed. A Cauchy mutation operator is integrated into the solution updating to balance the global and local searches. Wang *et al.* [16] improved the ABC algorithm using a neighborhood selection mechanism (NS-ABC). Instead of the probability selection, the NS-ABC algorithm chooses the best solution in the neighborhood radius to generate a candidate solution. The scout bee phase was also improved by using the opposition-based learning and the neighborhood radius.

Although the ABC algorithm was originally developed for solving numerical optimization problems, some variants have been successfully developed for solving combinatorial problems, e.g. job shop scheduling problem [20], travelling salesman problems [21]–[25], colormap quantization [26], quadratic assignment problem [27], and multi-attribute decision-making problem [28].

We apply Artificial Bee Colony algorithm with reinforcement solution updating (R-ABC) in [29] and [30] to solve this two-stage cutting stock problem with universal sizes. R-ABC is a swarm algorithm and is a variant of the Artificial Bee Colony (ABC) algorithm [31]. It is used in conjunction with an ILP based subroutine and tested with an actual demand data set.

The main contributions are summarized as follows.

- Instead of traditional two-stage paper cutting, we applied the concept of universal sizes of intermediate rolls. Rather than various sizes of intermediate rolls, our proposed method guarantees that the number of sizes of intermediate rolls in the stock does not exceed a predetermined number.
- Finding the universal sizes of intermediate rolls is a combinatorial NP-hard problem. Therefore, we modified the R-ABC algorithm for solving this problem, which is combinatorial. The R-ABC algorithm was adopted to generate candidate solutions.
- A feasibility matrix is developed for checking whether a solution is feasible.
- A loss matrix is defined to calculate the trim loss at the cutters.
- For a given number of universal sizes, ILP is applied to obtain the cutting patterns at the winders.

The organization of this paper is as follows. The next section gives definition of the problem being studied in more detail. Section 3 introduces the proposed approach to solve the problem. Section 4 presents the experiment and its results. Finally, Section 5 concludes the paper.

II. PROBLEM DEFINITION

This section provides more details of the problem and the concept of the universal sizes. Different types of trim loss are described, and the mathematical model is formulated.

A. PROBLEM DESCRIPTION

As earlier mentioned, the jumbo reels are cut into smaller intermediate rolls prior to be sheeted. The planner must determine the most appropriate patterns that best fit the width of the reels so that the trim loss at the two edges are minimized. The intermediate rolls are then cut into sheets. Again, there is another trim loss at the two edges of the intermediate rolls when they are being cut at the sheet cutters as shown in Fig. 1. There is natural material loss threshold at both winders and cutters because there must be certain space for the cutting machines to hold the paper in place. The best cutting patterns are those that cut exactly at this threshold. However, the chance that the various sheet sizes from the orders would suit the threshold exactly is rather rare. On the contrary, it is possible that when a jumbo reel is cut it results in intermediate rolls with widths that are too large or too small to suit other remaining orders (i.e. give small trim loss). These intermediate rolls are considered as leftovers and will be kept as inventory. They will be used only when sheet sizes of future orders suit the widths of the leftovers by wasting merely small trim loss.

Another possibility is that order quantities do not need the entire length of the roll or conversely just exceed that length. The case study manufacturer resolves this difficulty by having an agreement with the customers that the manufacturer may adjust the order quantity within $+/-$ 5% range to cope with this particular mismatch.

Although the physical cutting of the jumbo reels and intermediate rolls occur independently, their cutting pattern decisions are not. This is because the cutting of the jumbo reels determines the width of the intermediate rolls. Therefore, the cutting decisions must balance between the trim loss at the winders and the sheet cutters to reduce the total trim loss.

B. TRIM LOSS

In two-stage cutting stock problem, the cutting at the winders and another at the sheet cutters generate trim loss. Fig. 2 shows a jumbo reel is cut into intermediate rolls (sections) and then further to sheets (zones).

A winder winds and cuts a jumbo reel at the same time. Each jumbo reel is slit into multiple sections (or intermediate rolls). Intermediate rolls from the same jumbo reel could have different widths. The remainder after cutting a jumbo reel into intermediate rolls is the trim loss at the winder. Each intermediate roll is then sheeted by a sheet cutter. The number of sheets cut by the sheet cutter is called the number of zones. It is required by the sheet cutter specification that all the zones must have equal width and all sheets of that intermediate roll must be in the same size. Again, the excess after cutting an intermediate roll into sheets is the trim loss at the sheet cutter.

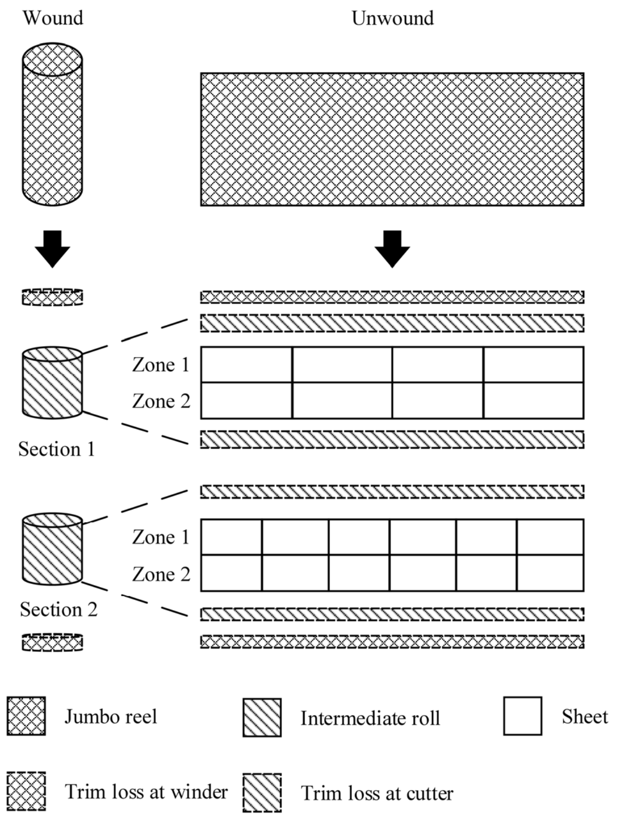

FIGURE 2. Parts of a jumbo reel.

There are minimum trim loss thresholds at both the winders and the sheet cutters due to the machine specifications as earlier mentioned. To prevent a large amount of material loss from trimming the jumbo reels and the intermediate rolls, the manufacturer sets maximum widths of the trim at the winders and the sheet cutters. There are also limits for the maximum number of sections and zones that the machines can perform. These values are shown in Table 1.

There are four possible cases that may happen when an intermediate roll is sheeted. Fig. 3 illustrates these cases. Fig. 3(a) shows a rare case where the number of sheets ordered can perfectly use the entire length of the roll without any material loss at the end. Fig. 3(b) is a case when an order does not completely need the entire length of the roll, and so there is some small remaining at the end of the roll. This remaining is too small to produce any product, so it is considered as loss. Fig. 3(c) shows a case that the order is already fulfilled, but there is still considerable amount of paper at the end of the roll. If the remaining is within a certain percentage of the order, this order will be prolonged so that the entire length of the roll is used. Fig. 3(d) is a case that the order is already fulfilled, but there is a large amount of remaining paper in the roll. This roll will be kept as stock to match future orders.

The case in Fig. 3(d) is undesirable since it increases the inventory. The agreement between the manufacturer and the customers that an order may be shrunk or extended to a certain percentages (Δ_{shrunk} and $\Delta_{extended}$) to resolve this particular problem. This order quantity adjustment is to change from a 3(d) case to other cases.

TABLE 1. Parameter values in paper cutting process.

Parameter	Description	Value
W	Width of jumbo reels (inches)	104
L	Length of jumbo reels (inches)	98,425.2
$x_{\min}$	Minimum width of intermediate rolls (inches)	25
$x_{\max}$	Maximum width of intermediate rolls (inches)	76
D	Maximum number of universal sizes	20
δ	Precision of cutting (inch)	0.25
$numSections_{\max}$	Maximum number of sections	4
$numZones_{\max}$	Maximum number of zones	3
$winderTrim_{\min}$	Minimum trim at winder (inches)	3
$winderTrim_{\max}$	Maximum trim at winder (inches)	50
$cutterTrim_{\min}$	Minimum trim at sheet cutter (inches)	1.5
$cutterTrim_{\max}$	Maximum trim at sheet cutter (inches)	6
Δ_{shrunk}	Percentage that an order is allowed to be shrunk	5
$\Delta_{extended}$	Percentage that an order is allowed to be extended	5

C. PROBLEM FORMULATION

The cutting machines can only cut with certain precision, δ. Let $x_{\min}$ and $x_{\max}$ be the minimum and the maximum sizes of the intermediate rolls. The set of valid (producible) universal sizes is:

$$validX = \{x_{\min}, x_{\min} + \delta, x_{\min} + 2\delta, \ldots, x_{\max}\} \tag{1}$$

The proposed method utilizes the R-ABC algorithm which is a population-based search. Let SN be the number of population and $i \in \{1, 2, 3, \ldots, SN\}$ be the index of the population set. Each population, $\vec{x}_i$, is a vector with D dimensions where D is the number of universal sizes. Each of the dimensions in $\vec{x}_i$ is to be selected from the set $validX$. The population $\vec{x}_i$ vector can be written as:

$$\vec{x}_i = \langle x_{i,1}, x_{i,2}, x_{i,3}, \ldots, x_{i,d}, \ldots, x_{i,D}\rangle \tag{2}$$

where d is the index of the dimension. Table 2 summarizes the notation used in problem formulation.

Fig. 4 and Fig. 5 depict cutting at a winder and a sheet cutter, respectively. For example, let us assume that the 4$^{\text{th}}$ jumbo reel ($p = 4$) is being cut into two sections. The first section ($q = 1$) may have a size $x_{i,3}$, and the second section ($q = 2$) has a size $x_{i,6}$. These trigger the variables $y_{i,3,4,1}$ and $y_{i,6,4,2}$ to be 1.

The intermediate roll with the size of $x_{i,6}$ from section 2 of jumbo reel 4 is to serve order 21, Ord_{21}. The number of sheets to be produced for order Ord_{21} is $n_{21,4,2}$. The number of zones of this intermediate roll can be calculated by (3).

$$numZones_{s,d} = \left\lfloor \frac{x_{i,d}}{w_{s,p,q}} \right\rfloor; \quad if\ y_{i,d,p,q} = 1 \tag{3}$$

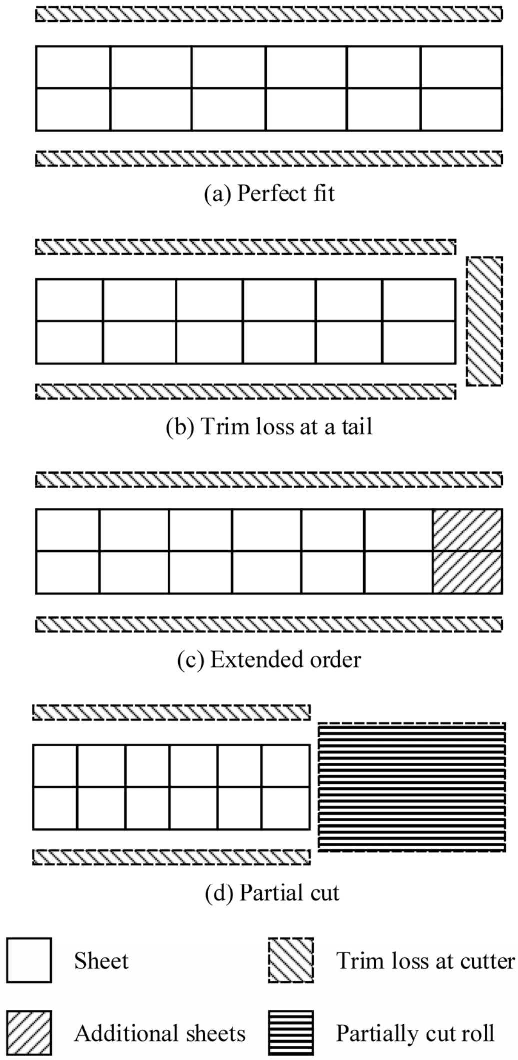

FIGURE 3. Four cases of trim loss at the sheet cutters.

where $numZones_{s,d}$ denotes the number of zones to be cut from the intermediate roll, and each zone is cut with the size $x_{i,d}$ for order Ord_s.

TABLE 2. Notation used in the trim loss optimization problem.

Symbol	Category	Description
SN	Solution	Number of solutions in a population
i	Solution	Solution index; $i \in \{1,2,3,\ldots,SN\}$
t	Solution	Iteration index
$numOrder$	Order	Number of orders
s	Order	Order index; $s \in \{1,2,3,\ldots,numOrders\}$
Ord_s	Order	An order; $Ord_s = \langle W_s, L_s, N_s \rangle$
W_s	Order	Width of sheets for order Ord_s
L_s	Order	Length of sheets for order Ord_s
N_s	Order	Number of sheets for order Ord_s
$Orders$	Order	A set of orders placed by customers; $Orders = \{Ord_1, Ord_2, Ord_3, \ldots, Ord_{numOrders}\}$
W	Jumbo reel	Width of jumbo reels
L	Jumbo reel	Length of jumbo reels
P	Jumbo reel	Number of used jumbo reels
p	Jumbo reel	Jumbo reel index; $p \in \{1,2,3,\ldots,P\}$
Q_p	Jumbo reel	Number of sections cut from jumbo reel p; $Q_p \le numSections_{\max}$
q	Jumbo reel	Section index; $q \in \{1,2,3,\ldots,Q_p\}$
D	Intermediate roll	Maximum number of universal sizes
d	Intermediate roll	Universal size index; $d \in \{1,2,3,\ldots,D\}$
K	Intermediate roll	Number of possible sizes
$validX$	Intermediate roll	A set of all valid universal sizes
k	Intermediate roll	Possible size index; $k \in \{1,2,3,\ldots,K\}$
$x_{i,d}$	Intermediate roll	A universal size of solution $\vec{x}_i$; $x_{i,d} \in validX$
$y_{i,d,p,q}$	Intermediate roll	$y_{i,d,p,q} \in \{0,1\}$; $y_{i,d,p,q} = 1$ if universal size $x_{i,d}$ is cut from section q of jumbo reel p; otherwise, $y_{i,d,p,q} = 0$.
$z_{i,d}$	Intermediate roll	Number of intermediate rolls with the sizes of $x_{i,d}$; $z_{i,d} = \sum_{p=1}^{P}\sum_{q=1}^{Q_p} y_{i,d,p,q}$
$w_{s,p,q}$	Sheet	Width of sheets cut from section q of jumbo reel p for order Ord_s
$l_{s,p,q}$	Sheet	Length of sheets cut from section q of jumbo reel p for order Ord_s
$n_{s,p,q}$	Sheet	Number of sheets cut from section q of jumbo reel p for order Ord_s

III. PROPOSED ALGORITHM

This section gives an overview of the proposed method, then discusses considerations of the algorithm regarding actual practice. The mathematical model and the procedure of the algorithm are then explained.

Because of the large problem space as discussed in section B, a swarm-based algorithm with a reinforcement learning mechanism, called R-ABC algorithm, is applied to solve the problem. The R-ABC algorithm puts an emphasis on dimensions that previously find better solutions. Specifically, universal sizes close to the dimensions that frequently give smaller trim loss have higher chances to be sampled through the reinforcement learning mechanism of the algorithm. This mechanism tends to perform well in high-dimensional problems [29]. Every newly generated solution must be checked for its feasibility. If the solution is a feasible one, the trim loss at the sheet cutters is calculated. The output of the R-ABC algorithm is a set of universal sizes that can serve all the orders and becomes constraints for the ILP subroutine [32]–[34]. The outputs from the ILP subroutine are cutting patterns at the winders, and the total number of jumbo reels required. Fig. 6 recapitulates this optimization procedure.

A. NUMBER OF UNIVERSAL SIZES SELECTION

To illustrate how large the problem space is, let K be the number of possible width sizes of the intermediate rolls, and D be the number of universal sizes selected from these possible sizes. Then there are $C(K, D)$ possible combinations

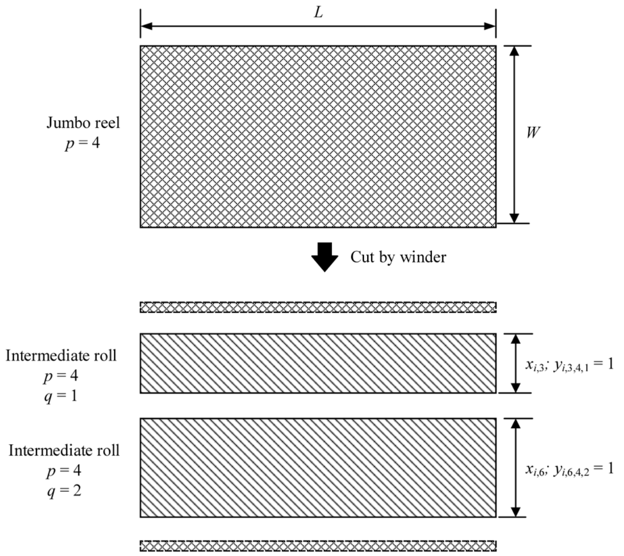

FIGURE 4. An example of cutting a jumbo reel by the winder.

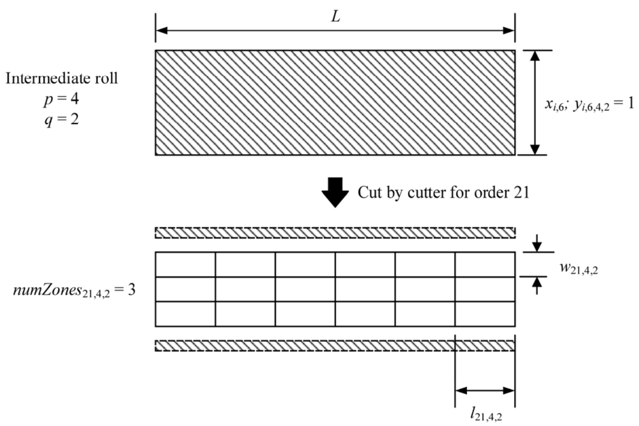

FIGURE 5. An example of cutting an intermediate roll by the sheet cutter.

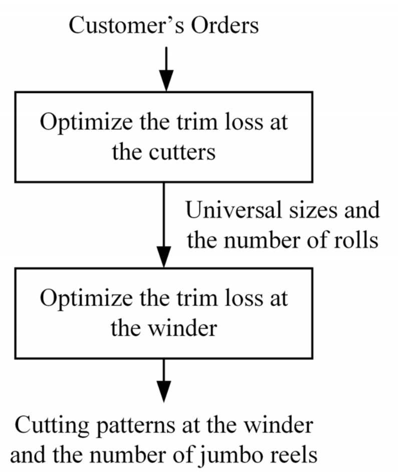

FIGURE 6. Overview of the trim loss optimization procedure.

of the solutions. Using the data from the numerical example in section IV, the value K is 205 and suppose that D is 20. Thus, there are $C(205, 20)$ or 2.71E+27 possible solutions.

In practice, K is rarely changed because it is the result of the cutting precision of the winders, so the problem space or the number of possible solutions of $C(K, D)$ usually depends solely on the value D. Fig. 7 plots the number of possible solutions as a function of D when $K = 205$.

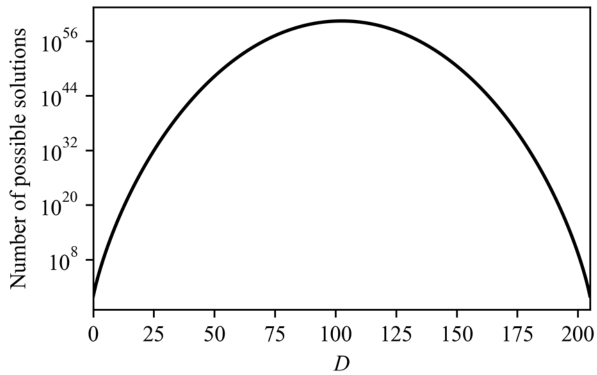

FIGURE 7. The number of possible solution when $K = 205$.

From the computational point of view, the two ends of the range D are preferred because their search spaces are smaller. For the purpose of merely minimizing the trim loss, the higher number of universal sizes is more desirable because there are more cutting patterns that could be generated from these universal sizes. Hence, there are more possibilities that some of these patterns may better fit the width of the jumbo reels and reduce the trim loss. From the inventory perspective however, the planner wants only adequate number of universal sizes to reduce the leftover inventory. A solution to this caveat is to have a sufficient number of universal sizes to accommodate different sheet sizes in the orders and keep the leftover inventory low, and yet be computationally tractable.

B. TOTAL TRIM LOSS CALCULATION

The objective function is to minimize the total trim loss as shown in (4). The first term in (4) is the trim loss occurred at the winders, and the other term from the sheet cutters.

$$\begin{aligned} totalTrimLoss_i = & \sum_{p=1}^{P} \sum_{q=1}^{Q_p} \Big(\sum_{j=1}^{D} (x_{i,d} \times L \times y_{i,d,p,q}) \\ & - (w_{s,p,q} \times l_{s,p,q} \times n_{s,p,q}) \Big) \\ & + \sum_{p=1}^{P} \Big((W \times L) - \sum_{q=1}^{Q_p} \sum_{d=1}^{D} \\ & \times (x_{i,d} \times L \times y_{i,d,p,q}) \Big) \end{aligned} \tag{4}$$

where $totalTrimLoss_i$ denotes the trim loss of solution $\vec{x}_i$, and $x_{i,d}$ refers to the d^{th} universal size in the solution $\vec{x}_i$ and $d \in \{1, 2, 3, \ldots, D\}$. W and L are the width and length of the jumbo reel. For jumbo reel q, let Q_p be the number of sections to be cut from this reel, and $w_{s,p,q}$, $l_{s,p,q}$, and $n_{s,p,q}$ be the width, length, and number of sheets to be produced from section p for order Ord_s. $y_{i,d,p,q}$ is 1 if the universal size $x_{i,d}$ is cut from section p of this jumbo reel q; otherwise $y_{i,d,p,q}$ is 0. The index i is the solution index and $i \in \{1, 2, 3, \ldots, SN\}$.

The above objective function is to be minimized under constraints (5)-(10).

$$\sum_{d=1}^{D}\sum_{q=1}^{Q_p} y_{i,d,p,q} \leq Q_p \leq numSections_{max}, \quad p \in \{1,2,3,\ldots,P\} \tag{5}$$

$$1 \leq numZones_{s,d} \leq numZones_{max}, \quad p \in \{1,2,3,\ldots,P\},\ q \in \{1,2,3,\ldots,Q_p\} \tag{6}$$

$$\sum_{q}^{Q_p}\sum_{j=1}^{D} \left(x_{i,d} \times y_{i,d,p,q}\right) + winderTrim_{min} \leq W, \quad p \in \{1,2,3,\ldots,P\} \tag{7}$$

$$cutterTrim_{min} \leq x_{i,d} - \left(w_{s,p,q} \times numZones_{s,d}\right) \leq cutterTrim_{max}, y_{i,d,p,q} = 1, \quad p \in \{1,2,3,\ldots,P\}, q \in \{1,2,3,\ldots,Q_p\} \tag{8}$$

$$\sum\nolimits_{d=1}^{D} y_{i,d,p,q} = 1, \quad p \in \{1,2,3,\ldots,P\},\ q \in \{1,2,3,\ldots,Q_p\} \tag{9}$$

$$N_s \times (1-\Delta_{shrunk}) \leq \sum\nolimits_{p=1}^{P}\sum\nolimits_{q=1}^{Q_p} n_{s,p,q} \leq N_s \times (1+\Delta_{extended}) \tag{10}$$

Equation (5) restricts the number of sections to be cut from any jumbo reel to be only integers from 1 to $numSections_{\max}$. Equation (6) limits the number of zones in any section to be an integer between 1 to $numZones_{\max}$. Equation (7) specifies that the sum of selected universal sizes and the minimum trim at the winder must not exceed the width of the jumbo reel. Equation (8) indicates that the difference between the universal size and the total width summed from multiple zones must be within an acceptable range of $[cutterTrim_{\min}, cutterTrim_{\max}]$. Equation (9) allows only one universal size to be cut in any given zone. Equation (10) permits that the total number of sheets could be shrunk or extended up a certain percentage of the original order.

C. FEASIBILITY CHECKING

A solution $\vec{x}_i$ is considered feasible if each of the orders can be served by at least one of the universal sizes. To check if a solution is feasible, a feasibility matrix is introduced. An order Ord_s contains information of the width, length, and number of sheets, which are denoted by W_s, L_s, and N_s, respectively.

The feasibility matrix $\mathbf{FM}(\vec{x}_i)$ is a function of solution $\vec{x}_i$ and has $numOrders \times D$ dimensions as shown in (11).

$$\mathbf{FM}(\vec{x}_i) = \begin{bmatrix} f_{1,x_{i,1}} & f_{1,x_{i,2}} & \cdots & f_{1,x_{i,D}} \\ f_{2,x_{i,1}} & f_{2,x_{i,2}} & \cdots & f_{2,x_{i,D}} \\ \vdots & \vdots & \ddots & \vdots \\ f_{numOrders,x_{i,1}} & f_{numOrders,x_{i,2}} & \cdots & f_{numOrders,x_{i,D}} \end{bmatrix} \tag{11}$$

where $f_{s,x_{i,d},}$ is a flag indicating whether intermediate rolls of the size of $x_{i,d}$ can serve order Ord_s.

The values of $f_{s,x_{i,d},}$ depend on two conditions. Let P_1 in (12) be true if the number of zones is between 1 and $numZones_{\max}$, and P_2 in (13) be true if the trim loss at the sheet cutter is within $cutterTrim_{\min}$ to $cutterTrim_{\max}$ range, given universal size $x_{i,d}$ and order Ord_s. Then the value of $f_{s,x_{i,d},}$ can be determined from (14).

$$P_1 : 1 \leq numZones_{s,d} \leq numZones_{\max} \tag{12}$$

$$P_2 : cutterTrim_{\min} \leq x_{i,d} - W_s \times numZones_{s,d} \leq cutterTrim_{\max} \tag{13}$$

$$f_{s,x_{i,d}} = \begin{cases} 1; & P_1 \wedge P_2 \equiv TRUE \\ 0; & \text{Otherwise} \end{cases} \tag{14}$$

A solution $\vec{x}_i$ is said to be feasible if (15) is true. On the contrary, if the value of $\sum_{d=1}^{D} f_{s,x_{i,d}}$ for any order Ord_s equals to zero, then it means that no universal size in solution $\vec{x}_i$ can serve order Ord_s, or the solution is infeasible.

$$\forall s \left(s \in \{1,2,3,\ldots,numOrders\} \rightarrow \sum\nolimits_{d=1}^{D} f_{s,x_{i,d}} \geq 1\right) \tag{15}$$

D. LOSS MATRIX

To calculate trimming loss at the sheet cutters, which is the first term in (4), a loss matrix $\mathbf{LM}(\vec{x}_i)$ is introduced. The matrix is a function of solution $\vec{x}_i$ with $numOrders \times D$ dimensions. The loss matrix is defined as:

$$\mathbf{LM}(\vec{x}_i) = \begin{bmatrix} m_{1,x_{i,1}} & m_{1,x_{i,2}} & \cdots & m_{1,x_{i,D}} \\ m_{2,x_{i,1}} & m_{2,x_{i,2}} & \cdots & m_{2,x_{i,D}} \\ \vdots & \vdots & \ddots & \vdots \\ m_{numOrders,x_{i,1}} & m_{numOrders,x_{i,2}} & \cdots & m_{numOrders,x_{i,D}} \end{bmatrix} \tag{16}$$

The entry $m_{s,x_{i,d}}$ denotes the trim loss at the sheet cutter when intermediate rolls with universal size $x_{i,d}$ are cut for order Ord_s. Equation (17), as shown at the bottom of the next page, is used to calculate each entry of the loss matrix, where U_s refers to the number of sheets that are actually cut for order Ord_s, and $\left\lceil \frac{U_s}{numZones_{s,d}} \right\rceil \times L_s$ denotes the total length of intermediate rolls with the size $x_{i,d}$ needed for this particular order.

The value of U_s depends on two conditions described by (18) and (19). Equation (18) shows a condition (condition I) in which order Ord_s is already satisfied, but there is enough remaining material to produce additional sheets that do not exceed $\Delta_{extended}$ percent of the order. Equation (19) is another condition (condition II) in which order Ord_s is not fully fulfilled, but the rest of the order is less than Δ_{shrunk} percent of the original order. If neither condition is satisfied, the actual number of sheets is set to be equal to the number of sheets ordered. Table 3 shows the conditions under which the value

TABLE 3. Actual values of number of sheets in different conditions.

Condition I	Condition II	Values of U_s
True	True	$\left\lceil \frac{\left\lceil \frac{N_s}{numZones_{s,d}} \right\rceil \times L_s}{L} \right\rceil \times numZones_{s,d} \times \left\lfloor \frac{L}{L_s} \right\rfloor$
True	False	$\left\lceil \frac{\left\lfloor \frac{N_s}{numZones_{s,d}} \right\rfloor \times L_s}{L} \right\rceil \times numZones_{s,d} \times \left\lfloor \frac{L}{L_s} \right\rfloor$
False	True	$\left\lfloor \frac{\left\lceil \frac{N_s}{numZones_{s,d}} \right\rceil \times L_s}{L} \right\rfloor \times numZones_{s,d} \times \left\lfloor \frac{L}{L_s} \right\rfloor$
False	False	N_s

of U_s is to be calculated.

$$N_s \leq \left\lceil \frac{\left\lceil \frac{N_s}{numZones_{s,d}} \right\rceil \times L_s}{L} \right\rceil \times numZones_{s,d} \times \left\lfloor \frac{L}{L_s} \right\rfloor \leq N_s \times \left(1 + \frac{\Delta_{extended}}{100}\right) \quad (18)$$

$$N_s \geq \left\lfloor \frac{\left\lceil \frac{N_s}{numZones_{s,d}} \right\rceil \times L_s}{L} \right\rfloor \times numZones_{s,d} \times \left\lfloor \frac{L}{L_s} \right\rfloor \geq N_s \times \left(1 - \frac{\Delta_{shrunk}}{100}\right) \quad (19)$$

If the solution $\vec{x}_i$ is feasible, then each order Ord_s is assigned with the universal size that best fits the order. The universal size $x_{i,fit(s)}$ is considered as best-fit for order Ord_s if for all $x_{i,j} \in \vec{x}_i$ we have $m_{s,x_{i,fit(s)}} \leq m_{s,x_{i,j}}$ where the value $m_{s,x_{i,fit(s)}}$ is the trim loss at the sheet cutter.

The required number of intermediate rolls $z_{i,d}$ of universal size $x_{i,d}$ for order Ord_s is calculated by:

$$z_{i,d} = \sum_{s=1}^{S} \frac{U_s}{\left\lfloor \frac{x_{i,d}}{W_s} \right\rfloor \times \left\lfloor \frac{L}{L_s} \right\rfloor} \quad (20)$$

To calculate the trim loss at the winder, intermediate rolls (with only sizes within the set of universal sizes) are first arranged in order to be cut from jumbo reels. Cutting various sizes of intermediate rolls from a jumbo reel using the universal sizes from solution $\vec{x}_i$ is a one-dimensional cutting stock problem. This problem can be formulated as an ILP as:

$$\text{Min} \sum_{p=1}^{P} \left(W - \sum_{q=1}^{Q_p} \sum_{d=1}^{D} \left(x_{i,d} \times y_{i,d,p,q} \right) \right) \quad (21)$$

subject to (5), (7), (9), and :

$$\sum_{p=1}^{P} \sum_{q=1}^{Q_p} y_{i,d,p,q} = z_{i,d}; \quad d \in \{1, 2, 3, \ldots, D\}; \; y_{i,d,p,q} \in \{0, 1\} \quad (22)$$

The objective function (21) is to minimize the total trim loss at the winder from all the jumbo reels under a set of cutting condition constraints.

Finally, using the total trim loss of solution $\vec{x}_i$ calculated in (4), the fitness value of this solution is:

$$Fit\,(\vec{x}_i) = \frac{1}{1 + totalTrimLoss_i} \quad (23)$$

E. OPTIMIZATION APPROACH

Fig. 8 shows the flowchart of the proposed trim loss optimization procedure upon which the R-ABC algorithm is based.

In the initialization phase, SN random initial solutions and SN initial reinforcement vectors are generated. For a D-dimension problem, each initial solution $\vec{x}_i^0$ in (24) comprises a set of D random distinct universal sizes. Each initial universal size $x_{i,d}^0$ in (25) must be a member of $validX$ as defined in (1).

$$\vec{x}_i^0 = \langle x_{i,1}^0, x_{i,2}^0, x_{i,3}^0, \ldots, x_{i,d}^0, \ldots, x_{i,D}^0 \rangle \quad (24)$$

$$x_{i,d}^0 \in validX; \;\; d \in \{1, 2, 3, \ldots, D\} \quad (25)$$

The trim losses of this initial solution are then calculated. The initial reinforcement vector, $\vec{r}^0$, is associated with this initial solution. The initial reinforcement values are set by (26).

$$\vec{r}^0 = \langle r_{i,1}^0, r_{i,2}^0, r_{i,3}^0, \ldots, r_{i,D}^0 \rangle; \, r_d^0 = \frac{1}{D} \quad (26)$$

The optimization process starts with the employed bee phase. To generate a new candidate solution, an employed bee selects a random dimension and updates the value of universal size in the selected dimension using (27). The value of the universal size after updating must differ from the values in other dimensions.

$$v_{i,d}^t = x_{i,d}^t + rand\,[-1, 1] \cdot \left(x_{i,d}^t - x_{k,d}^t\right) \quad (27)$$

where $v_{i,d}^t$ denotes the d^{th} dimension of candidate solution $\vec{v}_i^t$ in iteration t. The value of d is a random integer in the interval of $[1, D]$. The index of solution k for $k \neq i$ is randomly selected in the interval of $[1, SN]$. If $k = i$ then the last part of (27) would be zero, and the value of $v_{i,d}^t$ would be the same as the value of $x_{i,d}^t$.

However, the values of universal sizes $x_{i,d}$ must be members of the set $validX$. The value in the selected dimension after updating is rounded to the nearest possible width.

$$m_{s,x_{i,d}} = \begin{cases} \left(x_{i,d} - W_s \times numZones_{s,d}\right) \times \left(\left\lceil \frac{U_s}{numZones_{s,d}} \right\rceil \times L_s \right), & cutterTrim_{\min} \leq x_{i,d} - W_s \times numZones_{s,d} \leq cutterTrim_{\max} \\ invalid, & Otherwise \end{cases} \quad (17)$$

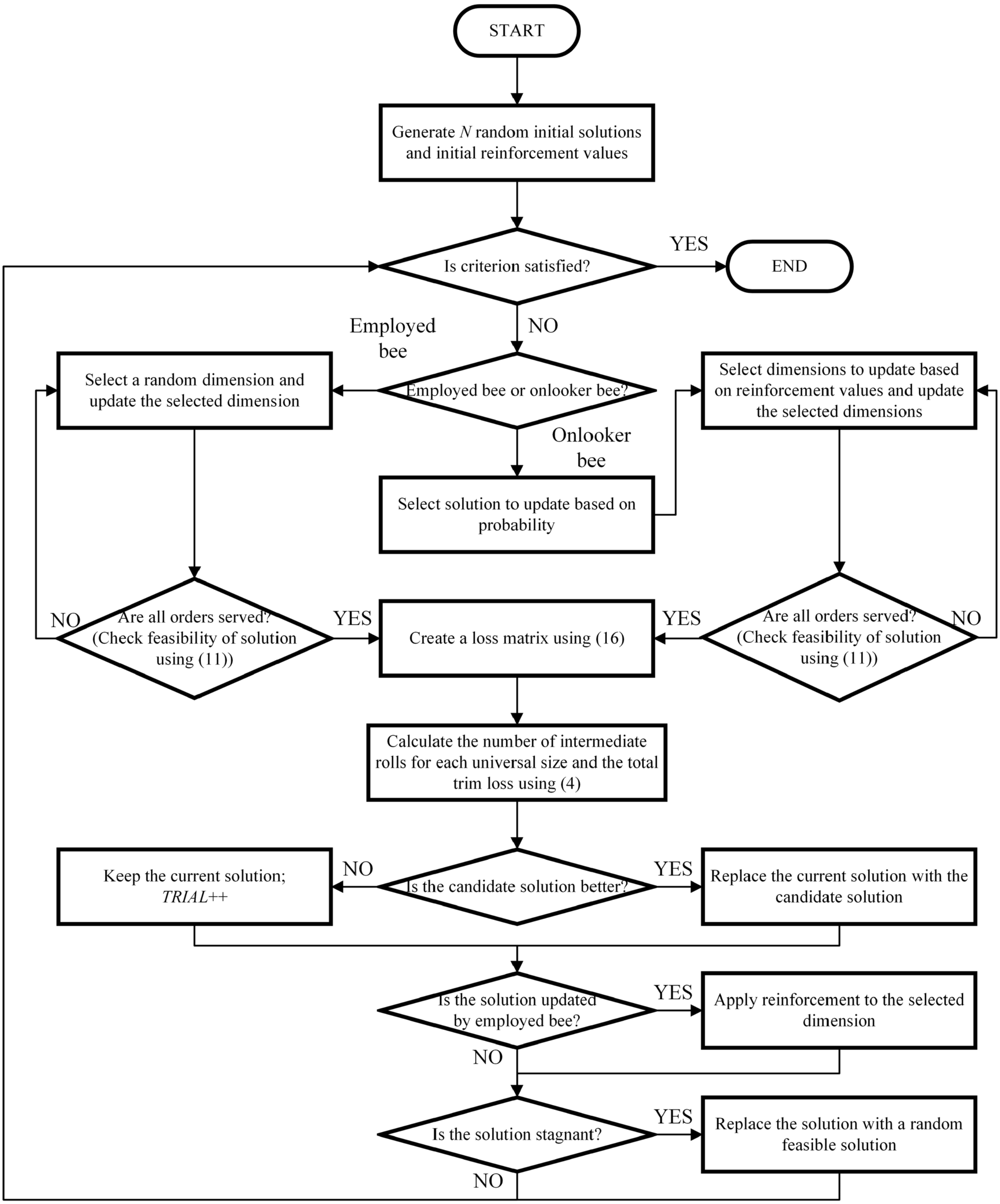

FIGURE 8. **Flowchart of the proposed trim loss optimization procedure.**

After solution updating by an employed bee, a feasibility matrix in (11) is generated to check whether the candidate solution ($\vec{v}_i^{\,t}$) is feasible. A feasible solution must be able to serve all the orders, and all constraints regarding the cutters must be satisfied. If the candidate solution passes the feasibility check, a loss matrix corresponding to the candidate solution is computed. Otherwise, the employed bee keeps updating the value in the selected dimension until reaching

100 times. If there is no feasible solution found, the employed bee randomly selects another dimension and updates the value in the selected dimension. If there is no feasible solution found by the employed bee after selecting a random dimension 100 times, the current solution is kept.

When the employed bee updates and finds a feasible candidate solution, a loss matrix is generated to calculate the trim loss at the cutters from the candidate solution as well as the number of intermediate rolls needed. The cutting pattern and the trim loss at the winder are calculated using the ILP in (21), and all constraints regarding the winder, i.e. (5), (7) and (9), must be satisfied. The total trim loss is then calculated using (4).

If the trim loss of the candidate solution is less than that of the current solution, the employed bee replaces the current solution with the candidate one, and a positive reinforcement is applied to the selected dimension using (28). Then the value of *TRIAL* is reset. Otherwise, the employed bee keeps the current solution, and *TRIAL* of the current solution is increased by 1. Consequently, a negative reinforcement is applied to the selected dimension using (29). The values of reward factor α and penalty factor β are calculated as shown in (30).

$$r_j^{t+1} = \begin{cases} r_j^t + \alpha\left(1 - r_j^t\right), & j = d \\ r_j^t \times (1-\alpha), & j \neq d \end{cases} \quad (28)$$

$$r_j^{t+1} = \begin{cases} r_j^t + (1-\beta), & j = d \\ \dfrac{\beta}{D-1} + r_j^t \times (1-\beta), & j \neq d \end{cases} \quad (29)$$

$$\alpha = \beta = \frac{Fit\left(\vec{v}_i^{\,t}\right)}{\sum_{n=1}^{SN} Fit\left(\vec{v}_n^{\,t}\right)} \quad (30)$$

where r_j^t denotes the reinforcement value of the j^{th} dimension in iteration t, and $j \in \{1, 2, 3, \ldots, D\}$.

After updating by all employed bees, solutions are further updated by onlooker bees. Each onlooker bee selects some dimensions to update based on the reinforcement values. For each dimension, an onlooker bee generates a real random number in [0, $1/D$]. If the random number is smaller than the reinforcement value of that dimension, the onlooker bee selects the dimension to update. Therefore, the dimension with a high reinforcement value has a higher probability to be selected than that with a low reinforcement value. The onlooker bee updates the values in the selected dimensions using (31).

$$v_{i,d}^t = x_{i,d}^t + rand\,[-1, 1] \cdot r_d^t \cdot \left(x_{i,k}^t - x_{BSF,k}^t\right) \quad (31)$$

where $v_{i,d}^t$ denotes the optimization parameter of a candidate solution $\vec{v}_i^{\,t}$ for the dimension $d \in \{1, 2, 3, \ldots, D\}$ in iteration t. The index $k \in \{1, 2, 3, \ldots, D\}$ is a random dimension for all dimension d. The $rand\,[-1, 1]$ is a random real number in the range of $[-1, 1]$. The $x_{BSF,k}^t$ is the optimization parameter of the best solution found so far in dimension k.

After solution updating, the onlooker bees work in a similar fashion as the employed bees. The values in the selected dimension after updating are rounded to the nearest possible widths. Each onlooker bee generates a feasibility matrix to check whether the candidate solution is feasible. If a candidate solution provides less trim loss than the current solution, the current solution is replaced by the candidate solution, and the stagnant counter is reset. Otherwise, the stagnant counter of the current solution is increased by 1.

In the scout bee phase, if the stagnant counter of any solution reaches a stagnant limit which is counted by the number of function evaluations (FEs), the solution is abandoned. The associated employed bee turns itself into a scout bee and discovers a set of new random distinct universal sizes using (32).

$$x_{i,d}^t \in validX; \quad d \in \{1, 2, 3, \ldots, D\} \quad (32)$$

IV. EXPERIMENTS AND RESULTS

The method of using universal sizes is unique. Hence, there is no existing benchmark problem in the literature. To evaluate the performance of the proposed method, we compare the quality of the solutions obtained from the R-ABC algorithm [29], [30] with those from the ABC algorithm [35] and two state-of-the-art ABC algorithms, which are aABC [17] and TMABC [18], as well as the randomly generating solutions (referred as Random).

The code for ABC and R-ABC was provided by the original authors, whereas that of aABC and TMABC was rewritten based on published literature. The code for aABC was validated against the numerical results of benchmark functions in [36]. Because the TMABC-FS algorithm in the original paper [18] was designed for solving a feature section problem, some modifications were required. The followings are the key differences between the TMABC-FS and the TMABC in our experiment.

- Two objective values of the TMABC-FS algorithm which were the classification accuracy and the feature cost were replaced with only one objective value which was the trim loss at the cutters.
- Because the crowding distance values of the solution cannot be calculated in this problem, the probability function of the TMABC-FS algorithm was replaced with that of the ABC algorithm.

All the algorithms listed in the previous paragraph were applied to a data set with slight adjustment from a paper manufacturer in Thailand. The data contained 1,055 orders with different quantities of sheet sizes. The widths of these sheets are used to create cutting patterns. From the demand data, certain sheet sizes were ordered repeatedly and there were only 127 distinct sheet widths ranging from 15.5 to 80 inches. Fig. 9 shows the distribution of the sheet widths and the total sum of all sheet lengths from the orders.

The values of the parameters used in the experiment are shown in Table 4. The number of solutions equals to half of the colony size. The initial solutions of each algorithm

TABLE 4. Values of parameters.

Parameter	Symbol	ABC	aABC	TMABC	R-ABC
Number of solutions or food sources	SN	25	25	25	25
Stagnant limit	$LIMIT$	100	100	100	100
Adaptivity coefficient	α_{aABC}	-	0.5	-	-
Crossover rate	γ_{aABC}	-	0.5	-	-
Maximum size of leader archive	N_a	-	-	30	-
Maximum size of external archive	N'_a	-	-	30	-

TABLE 5. Results obtained for a Set of 1,055 orders.

Description	Random	ABC	aABC	TMABC	R-ABC
Average %loss	7.33	4.80	4.28	5.07	**3.51**
Standard Deviation	1.61	1.22	**0.34**	1.79	0.44
Minimum %loss	4.71	3.23	3.66	2.90	**2.84**
Maximum %loss	10.26	7.54	4.88	7.52	**4.36**
Average number of used jumbo reels (rolls)	6,740.25	6,567.60	6,548.55	6,599.15	**6,483.30**
Average area of partially cut rolls (sq.in.)	1.58E9	1.44E9	1.43E9	1.52E9	**1.32E9**
Average execution time per run (minutes)	**94.95**	170.29	207.41	154.68	168.30
Shapiro-Wilk test of normality	0.9697	0.9269	0.9543	0.8675	0.9255
	(p = 0.9374)	(p = 0.4367)	(p = 0.7716)	(p = 0.0962)	(p = 0.4222)
Degree of freedom	10.3126	11.2579	17.1468	10.0724	
t-test for equality of means with unequal variances (two-tailed)	7.0529	3.2217	4.7049	2.7189	
	(p = 0.0000)	(p = 0.0081)	(p = 0.0002)	(p = 0.0216)	
Ranking by the Friedman's test	4.7	2.9	2.9	3.0	**1.5**

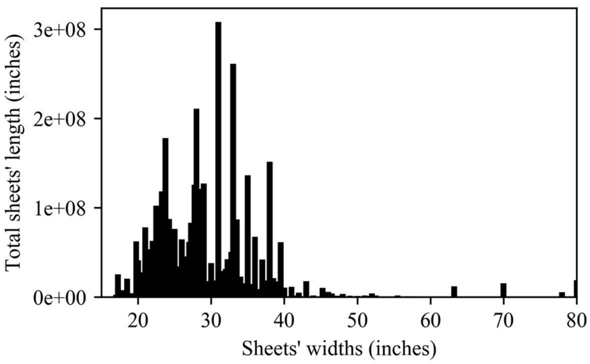

FIGURE 9. The distribution of the sheet width and their total length.

were independently uniform-randomly generated. For each algorithm, the experiment was independently run 10 times on a computer with Intel®Core™i7-7500U 2.70GHz CPU and 8GB physical memory. The maximum number of FEs was 10,000. The percentage of total trim loss is calculated by (33).

$$\%loss = \frac{totalTrimLoss - minLoss}{areaOfPaper - minLoss} \times 100 \quad (33)$$

where *totalTrimLoss* denotes the trim loss was calculated from (4), and *minLoss* is the minimum trim loss. The notation *areaOfPaper* refers to the total areas of jumbo reels used to cut the sheets for all orders.

Table 5 shows the results obtained from the Random, ABC, aABC, TMABC and R-ABC algorithms. The R-ABC algorithm provides the best of average, minimum, and maximum percentage of loss. The aABC algorithm provides the smallest value of the standard deviation. The number of jumbo reels used by the R-ABC algorithm is less than that of the other algorithms. This means that the solutions provided by the R-ABC algorithm require the least raw material to serve the same orders when compared to the other algorithms. Since leftover rolls are in different sizes, we measure how algorithms perform in this regard using average area of partially cut rolls. The average area of partially cut rolls provided by the R-ABC algorithm is also the least among the results obtained from all the algorithms.

The Random consumes the least average execution time, but it provides the worst of average, minimum, and maximum percentage of loss. The R-ABC algorithm consumes the average execution time of 168.30 minutes, which is practical.

Table 5 also shows the results from the normality tests, two-samples t-tests, and ranking by the Friedman's test. The Shapiro-Wilk test for normality is chosen because it is recommended when the sample size is fewer than 50 [37], [38]. The null hypothesis of the Shapiro-Wilk test is that the data is normally distributed. The Shapiro-Wilk test was performed by using the code from [39]. The p values of the normality test for all algorithms are greater than 0.05. Therefore, we may conclude that the data of each algorithm is normally distributed at the 95% level of confidence. The two-samples t-tests were conducted to compare the results from the R-ABC and those of the other algorithms. The null hypothesis of the t-test was to check if the percentage of total trim loss (indicated as t-tests for equality of means with unequal variances (two-tailed)) from the R-ABC was the same as that of the other algorithms. The t-tests was executed in a pairwise fashion where one of the algorithms must be R-ABC. The p value from each of the pair that the R-ABC was being contested against was lower than 0.05. This indicates that the mean of the total trim loss obtained from the R-ABC algorithm is statistically significantly different from those from the other algorithms at the 95% level of confidence. The Friedman's test was conducted by using KEEL [40], [41] to rank the performance of these algorithms in term of the percentage of the total trim loss. The ranking results by the Friedman's test revealed that R-ABC was the leading algorithm among all the candidates with the rank of 1.5. The second-best algorithms were tie between ABC and aABC with the rank of 2.9. The TMABC and Random were ranked at 3.0 and 4.7, orderly.

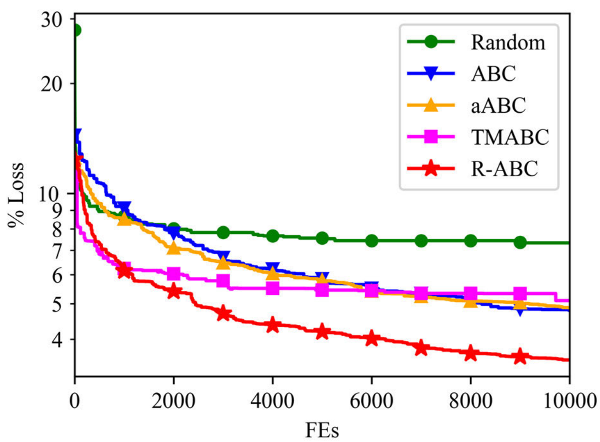

FIGURE 10. Convergence performance on the trim loss optimization.

Fig. 10 shows the convergence speed of the algorithms. The horizonal axis is the number of FEs, and the vertical axis is the average percentage of total trim loss calculated by (33) in a logarithmic scale. Each line shows the average of the best solutions found so far of an individual algorithm. The Random initially converges more quickly than the other algorithms, but it stagnates after 2,000 FEs. The R-ABC algorithm on the other hand outperforms the other algorithms after about 1,000 FEs.

V. CONCLUSION

The two-stage one-dimensional cutting stock problem is addressed in this paper. In addition to total trim loss minimization as the objective to the problem, the leftover stock which incurs inventory holding cost is also being minimized by having a limited but sufficient number of universal intermediate roll sizes. This leftover inventory although persists in the cutting stock problem, but has rarely been resolved in the literature.

Due to a large problem space of the problem, a metaheuristic, called R-ABC algorithm, is applied to cope with the issue. Conceptually, the algorithm is used to sample new feasible solutions, while an ILP subroutine is called to evaluate the solutions.

There are five main steps in the proposed method; sampling a new solution and check its feasibility through the feasibility matrix, calculating the trim loss at the sheet cutters using the loss matrix, determining the optimal cutting patterns at the winder by the ILP subroutine, computing the total trim loss, and updating the reinforcement vector and solutions. The new solutions are sampled based on the R-ABC approach which focuses its computational effort more on feasible solutions which in turn reduces the problem space. All steps are repeated until the termination condition is satisfied.

The R-ABC approach is evaluated with a revised data set provided by a paper manufacturer in Thailand. The data contains 1,055 orders of 127 distinct sheet sizes. The results from the experiments show that the proposed method with the R-ABC algorithm provides average, minimum, and maximum loss percentages less than those of other algorithms. Several statistic tests were performed, and the results showed that the R-ABC approach statistically significantly gave the best performance in term of the average percentage total trim loss. Moreover, compared with the current total trim loss of approximately 5% reported by the manufacturer, the R-ABC algorithm yields a better loss at 3.51%. The convergence plot shows that the R-ABC algorithm outperforms the other algorithms. The average execution time required by the R-ABC approach is at least comparable to that of the other candidate algorithms. Thus, the proposed method with the R-ABC algorithm is effective in respect to the solution quality, convergence speed, and practical execution time.

Furthermore, when compare the results of the ABC, aABC, and TMABC algorithms, the R-ABC algorithm offers a smaller percentage of total trim loss of $4.80 - 3.51 = 1.29\%$, $4.28 - 3.51 = 0.77\%$, and $5.07 - 3.51 = 1.56\%$, or a saving of more than 84, 65, and 115 jumbo reels, respectively. This jumbo reels requirement reduction is equivalent to a saving of material cost over 3.9, 3.0, and 5.3 million in Thai Baht or 123,000, 95,000, 170,000 USD, respectively, for this particular set of data (a jumbo reel costs about 45,800 Thai Baht or 1,470 USD). Additional potential saving by this approach could be obtained if it is applied to other existing gram-grade variety of papers currently produced by the manufacturer. The reduction in the number of partially cut rolls is also another potential saving in the inventory holding cost. The partially cut rolls is reduced as a result of limiting the pre-determined number of allowable universal sizes of the intermediate rolls. This limitation facilitates faster inventory turns of the universal-size rolls. If the distribution of future orders remains similar to that of the test data set, the D universal sizes can still serve those orders, and the leftover inventory should be depleted.

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