IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received June 23, 2020, accepted July 6, 2020, date of publication July 10, 2020, date of current version July 22, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3008519

Intelligent Adaptive Tracking Controller Design for Stochastic Switched Pure-Feedback Nonlinear Systems With Input Saturation and Non-Lower Triangular Structure

JI-DONG LIU[1], RUI-BING LI[1], XIAO-MEI LIU[2], BEN NIU[1], AND DONG YANG[3]
[1]School of Information Science and Engineering, Shandong Normal University, Jinan 250014, China
[2]School of Business, Shandong Normal University, Jinan 250014, China
[3]School of Engineering, Qufu Normal University, Rizhao 276826, China

Corresponding authors: Ji-Dong Liu (liujidong6@163.com) and Rui-Bing Li (sdnu_mrlrb@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61873151, Grant 61773192, Grant 61773246, Grant 61803225, and Grant 61803238, and in part by the Shandong Provincial Natural Science Foundation, China, under Grant ZR2019MF009.

ABSTRACT This paper concentrates on the design of intelligent adaptive tracking controller for stochastic switched nonlinear pure-feedback systems with input saturation and non-lower triangular structure. It needs to be emphasized that both the issues of pure-feedback structure and non-differential saturation nonlinearity are involved in the studied system. With the help of the mean-value theorem, a novel intelligent adaptive tracking controller is developed in this work to overcome the difficulty resulted from pure-feedback structure, and the inherent property of Gaussian functions is utilized to handle functions that are unknown and include all state variables. Moreover, through the universal intelligent approximation technology, a novel control strategy is constructed under the framework of backstepping, which guarantees that the tracking error can converge to a small neighborhood near the origin in the sense of mean quartic value and all signals of the nonlinear closed-loop system can be bounded in probability. Eventually, the effectiveness of the presented scheme is further illustrated by the simulation of two practical examples.

INDEX TERMS Stochastic switched nonlinear systems, non-lower triangular structure, pure-feedback structure, input saturation, intelligent adaptive tracking control.

I. INTRODUCTION

Since the nonlinear systems have important application value in practical engineering applications, for instance, power systems, computer network systems, aerospace systems, and multi-agent systems, etc., the control design of more complicated nonlinear dynamic systems has always become a research hotspot and difficulty in the field of control [1]. As we all know, the current research of nonlinear systems has gained abundant achievements [2]–[8], especially the rapid development of modern control theory, which provides broader development prospects and stronger technical support for the research of nonlinear systems. To name only a few, based on backstepping method, [9] proposed a new neural adaptive design approach for high-order nonlinear systems with the mismatched condition; [10] solved the stabilization difficulties for nonlinear stochastic systems with lower triangular structure by utilizing backstepping technique and adaptive control approach, and this novel scheme guarantees global asymptotic stability in probability. However, none of the above schemes is suitable for switched nonlinear systems. Since lots of practical systems can be modeled through common exchange frameworks, for instance, power systems, transportation systems, and intricate industrial processes, switched nonlinear systems are regarded as typical and important hybrid systems, which have attracted wide attention. At the moment, according to the main characteristics of the switched nonlinear system, the main methods that can be utilized to study and analyze the switched nonlinear systems are multiple Lyapunov functions method and common Lyapunov function method. In [11], by constructing the common Lyapunov function, the final controller can ensure

The associate editor coordinating the review of this manuscript and approving it for publication was Lifeng Ma.

the stability of the controlled system under any switched. Numerous practices, however, cannot keep the stability under the free switched signals, and may be stable under the limited switched signals. Although many control problems of the switched nonlinear system have been studied till now, and a great number of outstanding accomplishments have been published (see, for instance, [12]–[18] and the references therein), how to construct the appropriate switched signal is still a challenge in the procedure of controller design.

Furthermore, it is difficult to model the system in most cases due to many completely unknown nonlinearities existing in the actual switched nonlinear system. Fortunately, neural networks (NNs) and fuzzy logic systems (FLSs) are viewed to be powerful tools with the universal approximation performance for handling unknown nonlinear functions [19]–[31]. Whether Single Input Single Output(SISO) or Multiple Input Multiple Output(MIMO) nonlinear switched systems, a great number of works have been reported by combining adaptive neural control and backstepping method [32], [33]. However, the control strategies proposed in the previous work are only suitable for nonlinear switched systems with lower triangular structure. This means that the i-th subsystem function $f_i(\cdot)$ must include state variables $x_1, \ldots, x_i$ only. Fortunately, in order to break the limitation brought by the system structure, the researchers have made corresponding efforts. For example, the variable separation method proposed in [34] successfully solves the controller design difficulties result from the non-lower triangular structure. By employing the approximation capability of NNs or FLSs and variable separation method, a series of results about the design of controllers for nonlinear switched systems with non-lower triangular structures have been reported, for instance, [35], [36]. Unfortunately, the variable separation method needs to meet strict conditions, that is, the function of the system must be a strictly monotonically increasing bounded function. Hence, it is a very meaningful subject to improve the variable separation method or put forward a novel method to handle the non-lower triangular structure in future research.

In reality, input constraints, dead-zone inputs, and time delay are common phenomena in industrial production, aerospace, and other engineering fields. For nonlinear switched systems with time delay, [37] developed an adaptive tracking control algorithm. [38] solved the semi-global stability problem of nonlinear switched systems with dead-zone input. It must be noted that from the existing research results, the input saturation phenomenon in the system usually causes the performance of the system to decrease or become unstable, which seriously affects the system's normal operations. Fortunately, there are some design algorithms for nonlinear switched systems with input constraints and lower triangular structure, for instance, [39]–[44]. However, due to the lack of appropriate technical means to deal with the more complex structure of the system, compared with nonlinear systems with lower triangular structure, there are few research reports on nonlinear switched stochastic pure-feedback systems with input saturation and non-lower triangular structure. Although the input saturation and non-lower triangle structure are considered in [45], the method in [45] is not suitable for pure-feedback systems. Therefore, how to effectively solve this problem is the focus of this paper.

Inspired by the above observations, this work presents the neural network-based adaptive control method for stochastic switched nonlinear pure-feedback systems with input saturation and non-lower triangular structure. In the design scheme, the uncertain nonlinear functions in the system are handled by NNs. In the non-lower triangular structure, all states will appear in step i, while the virtual controller in step i can only contain the first i states, which makes the traditional backstepping method difficult to be used to deal with this structure. By employing the inherent property of Gaussian functions (i.e., ***Lemma 1***), the difficulty caused by the non-lower triangular structure is overcome. Additionally, the given reference signal is followed by the system output within the bounded error. Then, simulation results based on actual examples prove the effectiveness of the developed design algorithm. The contributions of this work can be outlined as follows: (1) This paper considers a more general switched nonlinear system, which extends the processing methods in the existing results to a switched nonlinear system framework with a wide range of applications. (2) The difficulty that the non-differential saturation nonlinearity is tackled by using a nonlinear smooth function of the input signal to approximate the saturation function. (3) To solve the problem of controller design for nonlinear switched systems with non-lower triangular structure, in this paper, the inherent property of Gaussian functions is utilized as a more effective tool than the variable separation method used in [35], [36].

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of stochastic switched nonlinear non-lower triangular pure-feedback systems described as follows:

$$\begin{cases} dx_i = h_{i,\sigma(t)}(\tilde{x}, x_{i+1})dt + \phi^T_{i,\sigma(t)}(x)d\omega, \quad 1 \le i \le n-1, \\ dx_n = h_{n,\sigma(t)}(x, u)dt + \phi^T_{n,\sigma(t)}(x)d\omega, \\ y = x_1, \end{cases} \tag{1}$$

where $y \in R$; $x = \tilde{x}_n$, $\tilde{x} = [x_1, x_2, \ldots x_i, x_{i+2}, \ldots, x_n]^T \in R^{n-1}$ with $\tilde{x}_i = [x_1, x_2, \ldots x_i]^T \in R^i (i = 1, 2, \ldots n)$ means the system output and state variable, respectively. $\omega \in R^r$ represents the standard Wiener process defined on space (Ω, F, P), which is called 'complete probability space'. $\sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, m\}$ denotes switched signal. $h_{i,\sigma(t)}(\cdot)$ and $\phi_{i,\sigma(t)}(\cdot) : R^n \to R^r (i = 1, 2, \ldots n)$ are the nonlinear functions which are smooth and unknown.

The signal u stands for system input affected by nonlinearity saturation that is nonsymmetric, as shown below:

$$u = sat(v) = \begin{cases} u_{\max}, & v \ge u_{\max} \\ v, & u_{\min} < v < u_{\max} \\ u_{\min}, & v \le u_{\min} \end{cases} \tag{2}$$

with unknown parameters $u_{\max} > 0$ and $u_{\min} < 0$, and v being input variable of u.

According to mean-value theorem in [46], functions $h_i(\cdot,\cdot)$ in (1) are converted as

$$h_{i,\sigma(t)}(\tilde{x}, x_{i+1}) = h_{i,\sigma(t)}(\tilde{x}, x_{i+1}^0) + f_{\mu_i,\sigma(t)}(x_{i+1} - x_{i+1}^0),\quad 1 \le i \le n-1,$$
$$h_{n,\sigma(t)}(x, u) = h_{n,\sigma(t)}(x, u^0) + f_{\mu_n,\sigma(t)}(u - u^0), \tag{3}$$

where the function $h_i(\cdot,\cdot)$ is distinctly analyzed between $h_i(\tilde{x}, x_{i+1})$ and $h_i(\tilde{x}, x_{i+1}^0)$, $f_{\mu_i} = f_i(\tilde{x}, x_{\mu_i}) = (\frac{\partial h_i(\tilde{x},x_{i+1})}{\partial x_{i+1}})\Big|_{x_{i+1}=x_{\mu_i}}$, $x_{n+1} = u$, and $x_{\mu_i} = \mu_i x_{i+1}^0 + (1-\mu_i)x_{i+1}$ with $0 < \mu_i < 1$, $i = 1, 2, \ldots, n$.

Then, substituting (3) into (1) and defining $x_{i+1}^0 = 0$, $u^0 = 0$, one has

$$\begin{cases} dx_i = (h_{i,\sigma(t)}(\tilde{x}, 0) + f_{\mu_i,\sigma(t)}x_{i+1})dt + \phi_{i,\sigma(t)}^T(x)dw, \\ dx_n = (h_{n,\sigma(t)}(x, 0) + f_{\mu_n,\sigma(t)}u)dt + \phi_{n,\sigma(t)}^T(x)dw, \\ y = x_1. \end{cases} \tag{4}$$

It is easy to find the two sharp corners $u_{\max}$ and $u_{\min}$ in the saturation nonlinearity existing in (2). In order to dispose this difficult, with the help of the method proposed in [47], $f(v)$ can be changed into

$$f(v) = \begin{cases} u_{\max} * \tanh(\frac{v}{u_{\max}}), & v \ge 0, \\ u_{\min} * \tanh(\frac{v}{u_{\min}}), & v < 0, \end{cases}$$
$$= \begin{cases} u_{\max} * \frac{e^{\frac{v}{u_{\max}}} - e^{-\frac{v}{u_{\max}}}}{e^{\frac{v}{u_{\max}}} + e^{-\frac{v}{u_{\max}}}}, & v \ge 0, \\ u_{\min} * \frac{e^{\frac{v}{u_{\min}}} - e^{-\frac{v}{u_{\min}}}}{e^{\frac{v}{u_{\min}}} + e^{-\frac{v}{u_{\min}}}}, & v < 0. \end{cases} \tag{5}$$

What's more, we have

$$u = sat(v) = f(v) + d(v) \tag{6}$$

with the bound of $d(v)$ being calculated as

$$\begin{aligned} |d(v)| &= |sat(v) - f(v)| \\ &\le \max\{u_{\max}(1-\tanh(1)), u_{\min}(\tanh(1)-1)\} \\ &= D. \end{aligned} \tag{7}$$

By utilizing the mean-value theorem [46], we can obtain

$$f(v) = f(0) + f_{v_\mu}v, \tag{8}$$

where $\mu(0 < \mu < 1)$ stands for a constant, $f_{v_\mu} = (\frac{\partial f(v)}{\partial v})\,|_{v=v_\mu}$, and $v_\mu = \mu v + (1-\mu)v_0$. Because of $f_0 = 0$, (8) can be converted into

$$f(v) = f_{v_\mu}v. \tag{9}$$

Considering (4), (8) and (9), yields

$$\begin{cases} dx_i = (h_{i,\sigma}(\tilde{x}, 0) + f_{\mu_i,\sigma}x_{i+1})dt + \phi_{i,\sigma}^T(x)d\omega, \\ dx_n = (h_{n,\sigma}(x, 0) + f_{\mu_n,\sigma}(f_{v_\mu}v + d(v)))dt \\ \qquad + \phi_{n,\sigma}^T(x)d\omega \\ y = x_1. \end{cases} \tag{10}$$

The purpose of this paper is to design a neural adaptive tracking controller v so that the output trajectory y can follow a signal y_d, and all signals of the nonlinear closed-loop system can be bounded in probability. In the mean time, in order to stabilize the system (10), the following necessary assumptions are performed.

Assumption 1: Assuming that the sign of smooth $f_{\mu_i,\sigma(t)}$, $i = 1, 2, \ldots, n$ is known, and there are unknown constants b_m and b_M so that, for $1 \le i \le n$,

$$0 < b_m \le \left|f_{\mu_i,\sigma(t)}\right| \le b_M < \infty, \quad \forall(\tilde{x}_i, x_{i+1}) \in R^i \times R. \tag{11}$$

Assumption 2: There exists an unknown constant $f_m > 0$, which makes the function f_{v_μ} in (11) satisfy

$$0 < f_m \le f_{v_\mu} \le 1. \tag{12}$$

Then, according to Assumption 1 and (12), we have

$$0 < b \le f_{u_i,\sigma(t)}, \quad i = 1, 2, \ldots, n-1, \ 0 < b \le f_{\mu_n}f_{v_\mu} \tag{13}$$

with constant $b = \min\{b_m, b_m f_m\}$ being unknown.

Assumption 3: The time-varying signal y_d has not only nth-order derivative, but it is smooth and bounded.

In this work, the radial basis function (RBF) NNs can be utilized to online approximate a continuous functions $h(Z)$ that is unknown over a compact set Ω_Z. The RBF NNs is expressed as follows:

$$h_{nn}(Z) = W^T S(Z), \tag{14}$$

where $W = [w_1, \ldots, w_l]^T \in R^l$ denotes weight vector and $l > 1$ is the number of NNs node. $S(Z) = [s_1(z), \ldots, s_l(z)]^T$ stands for the basis function vector with $Z \in \Omega_Z \subset R^q$ being the input vector and q being the input dimensions of NNs. In general, $s_i(Z)$ is selected as the Gaussian function that can be shown as

$$s_i(Z) = \exp[-\frac{(Z-\mu_i)^T(Z-\mu_i)}{\eta^2}]. \tag{15}$$

Furthermore, the function $h(Z)$ can be estimated by (14) in a bounded closed set $\Omega_Z \in R^q$ with an arbitrary accuracy $\varepsilon > 0$ as $h(Z) = W^{*T}S(Z) + \delta(Z)$, in which W^* stands for the ideal constant weight vector, which is described as follows:

$$W^* = \arg\min_{W \in \bar{R}^l}\left\{\sup_{Z\in\Omega_Z}\left|h(Z) - W^T S(Z)\right|\right\} \tag{16}$$

and $\delta(Z)$ stands for the approximation error and $|\delta(Z)| < \varepsilon$.

Lemma 1 [48]: Define $S(\bar{x}_q) = [S_1(\bar{x}_q), \ldots, S_l(\bar{x}_q)]^T$ as the basis function vector of RBF NN and $\bar{x}_q = [x_1, \ldots, x_q]^T$. Then, we have

$$\left\|S(\bar{x}_q)\right\|^2 \le \|S(\bar{x}_k)\|^2 \tag{17}$$

with integer $0 < k \le q$.

III. CONTROLLER DESIGN PROCEDURE AND STABILITY ANALYSIS

In this section, a novel design scheme will be introduced for system (10) by resorting to RBF NNs and Backstepping method. Correspondingly, the controller design framework for system (10) is as follows:

$$\alpha_i(Z_i) = -(k_i + \frac{3}{4})z_i - \frac{z_i^3}{2a_i^2}\hat{\theta}S_i^T(Z_i)S_i(Z_i),$$
$$1 \leq i \leq n-1, \tag{18}$$
$$v(Z_n) = -(k_n + \frac{3}{4\eta^2})z_n - \frac{z_n^3}{2a_n^2}\hat{\theta}S_n^T(Z_n)S_n(Z_n), \tag{19}$$

where positive constants k_i, $a_i(i = 1, 2, \ldots, n)$ and η should be designed, $Z_1 = [x_1, y_d, \dot{y}_d]^T \in \Omega_{Z_1} \subset R^3$, $Z_i = \left[\tilde{x}_i^T, \hat{\theta}, \tilde{y}_d^{(i)T}\right]^T \in \Omega_{Z_i} \subset R^{2i+2}$ $(i = 2, \ldots, n-1)$, and z_i satisfies

$$z_i = x_i - \alpha_{i-1}, \tag{20}$$

where $\alpha_0 = y_d$ and α_i is a virtual controller. θ is estimated by $\hat{\theta}$, and

$$\theta = \max\left\{\frac{1}{b}\left\|W_{i,k}^*\right\|^2; i = 1, 2, \ldots, n\right\} \tag{21}$$

The adaptive law is described by

$$\dot{\hat{\theta}} = \sum_{i=1}^{n}\frac{\lambda}{2a_i^2}z_i^6S_i^T(Z_i)S_i(Z_i) - \gamma\hat{\theta}, \tag{22}$$

where $\gamma > 0$ and $\lambda > 0$.

Step 1: Based on $z_1 = x_1 - y_d$, we have

$$dz_1 = (h_{1,k}(\tilde{x}, 0) + f_{\mu_1,k}x_2 - \dot{y}_d)dt + \phi_{1,k}^T(x)d\omega. \tag{23}$$

Select the Lyapunov function candidate as

$$V_1 = \frac{1}{4}z_1^4 + \frac{b}{2\lambda}\tilde{\theta}^2, \tag{24}$$

where $\tilde{\theta} = \theta - \hat{\theta}$.

Considering (23) and $It\hat{o}$ formula, the following inequality is obtained:

$$LV_1 \leq z_1^3\left(h_{1,k}\left(\tilde{x}, 0\right) + f_{\mu_1,k}x_2 - \dot{y}_d + \frac{3}{4}l_1^{-2}z_1\left\|\phi_{1,k}(x)\right\|^4\right) + \frac{3}{4}l_1^2 - \frac{b}{\lambda}\tilde{\theta}\dot{\hat{\theta}} \tag{25}$$

with $l_1 > 0$.

Defining $\bar{h}_{1,k} = h_{1,k}(\tilde{x}, 0) - \dot{y}_d + (3/4)l_1^{-2}z_1\left\|\phi_{1,k}(x)\right\|^4 + (3/4)z_1$, then (25) is rewritten as

$$LV_1 \leq z_1^3\left(f_{\mu_1,k}x_2 + \bar{h}_{1,k}\right) - \frac{3}{4}z_1^4 + \frac{3}{4}l_1^2 - \frac{b}{\lambda}\tilde{\theta}\dot{\hat{\theta}}. \tag{26}$$

Then, a RBF NN $W_{1,k}^TS(Z_1)$, $X_1 \in \Omega_{X_1} \subset R^{n+2}$ is used to approximate $\bar{h}_{1,k}$, so that

$$\bar{h}_{1,k} = W_{1,k}^{*T}S_1\left(X_1\right) + \delta_1\left(X_1\right), \left|\delta_1\left(X_1\right)\right| \leq \varepsilon_1 \tag{27}$$

with $X_1 = [x, y_d, \dot{y}_d]^T$, $\varepsilon_1 > 0$, and $\delta_1\left(X_1\right)$ being approximation error.

By employing Young's inequality and Lemma 1, we have

$$\begin{aligned} z_1^3\bar{h}_{1,k} &\leq \frac{bz_1^6}{2a_1^2}\frac{\left\|W_{1,k}^*\right\|^2}{b}S_1^T(X_1)S_1(X_1) + \frac{1}{2}a_1^2 + \frac{3}{4}z_1^4 + \frac{1}{4}\varepsilon_1^4 \\ &\leq \frac{bz_1^6}{2a_1^2}\frac{\left\|W_{1,k}^*\right\|^2}{b}S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}a_1^2 + \frac{3}{4}z_1^4 + \frac{1}{4}\varepsilon_1^4 \\ &\leq \frac{b}{2a_1^2}z_1^6\theta S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}a_1^2 + \frac{3}{4}z_1^4 + \frac{1}{4}\varepsilon_1^4 \end{aligned} \tag{28}$$

with $Z_1 = [x_1, y_d, \dot{y}_d]^T \in \Omega_{Z_1} \subset R^3$ and a_1 being a positive constant.

Combining (27) and (28), yields

$$LV_1 \leq z_1^3f_{\mu_1,k}x_2 + \frac{b}{2a_1^2}z_1^6\theta S_1^T(Z_1)S_1(Z_1) + \frac{1}{2}a_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}l_1^2 - \frac{b}{\lambda}\tilde{\theta}\dot{\hat{\theta}}. \tag{29}$$

Employing the formula (20) with $i = 1$, one has

$$\begin{aligned} LV_1 &\leq z_1^3f_{\mu_1,k}\left(x_2 - \alpha_1 + \alpha_1\right) + \frac{b}{2a_1^2}z_1^6\theta S_1^T(Z_1)S_1(Z_1) \\ &\quad + \frac{1}{2}a_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}l_1^2 - \frac{b}{\lambda}\tilde{\theta}\dot{\hat{\theta}} \\ &\leq z_1^3f_{\mu_1,k}z_2 + z_1^3f_{\mu_1,k}\alpha_1 + \frac{b}{2a_1^2}z_1^6\theta S_1^T(Z_1)S_1(Z_1) \\ &\quad + \frac{1}{2}a_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}l_1^2 - \frac{b}{\lambda}\tilde{\theta}\dot{\hat{\theta}}. \end{aligned} \tag{30}$$

Furthermore, by applying Assumption 1 and the virtual controller in (18) with $i = 1$, it is easy to show

$$\begin{aligned} z_1^3f_{\mu_1,k}\alpha_1 &\leq -k_1f_{\mu_1,k}z_1^4 - \frac{3}{4}f_{\mu_1,k}z_1^4 \\ &\quad - \frac{b}{2a_1^2}z_1^6\hat{\theta}S_1^T(Z_1)S_1(Z_1) \\ &\leq -k_1f_{\mu_1,k}z_1^4 - \frac{3}{4}f_{\mu_1,k}z_1^4 \\ &\quad - \frac{b}{2a_1^2}z_1^6\hat{\theta}S_1^T(Z_1)S_1(Z_1). \end{aligned} \tag{31}$$

For the term $z_1^3g_{\mu_1,k}z_2$, it is true that

$$\begin{aligned} LV_1 &\leq -k_1f_{\mu_1,k}z_1^4 - \frac{3}{4}f_{\mu_1,k}z_1^4 + z_1^3f_{\mu_1,k}z_2 + \frac{1}{2}a_1^2 + \frac{1}{4}\varepsilon_1^4 \\ &\quad + \frac{3}{4}l_1^2 + \frac{b}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2a_1^2}z_1^6S_1^T(Z_1)S_1(Z_1) - \dot{\hat{\theta}}\right) \\ &\leq -c_1z_1^4 + \frac{b_M}{4}z_2^4 + \varrho_1 \\ &\quad + \frac{b}{\lambda}\tilde{\theta}\left(\frac{\lambda}{2a_1^2}z_1^6S_1^T(Z_1)S_1(Z_1) - \dot{\hat{\theta}}\right), \end{aligned} \tag{32}$$

where $c_1 = k_1f_{\mu_1,k} > 0$ and $\varrho_1 = (1/2)a_1^2 + (3/4)l_1^2 + (1/4)\varepsilon_1^4$. It is worth pointing out that the term $(1/4)b_Mz_2^4$ will be handled later.

Step $i\,(2 \le i \le n-1)$***:*** On the basis of $z_i = x_i - \alpha_{i-1}$ and *Itô* formula, it is easy to obtain

$$dz_i = (h_{i,k}(\tilde{x},0) + f_{u_i,k}x_{i+1} - L\alpha_{i-1})dt + \left(\phi_{i,k}(x) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\phi_{j,k}(x)\right)^T d\omega, \quad (33)$$

where

$$L\alpha_{i-1} = \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}h_{j,k}\left(\tilde{x}, x_{j+1}\right) + \sum_{j=0}^{i-1}\frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2 \alpha_{i-1}}{\partial x_p \partial x_q}\phi_{p,k}^T(x)\phi_{q,k}(x). \quad (34)$$

Select the Lyapunov function candidate as

$$V_i = V_{i-1} + \frac{1}{4}z_i^4. \quad (35)$$

Then, we have

$$LV_i = LV_{i-1} + z_i^3\left(h_{i,k}\left(\tilde{x},0\right) + f_{\mu_i,k}x_{i+1} - L\alpha_{i-1}\right) + \frac{3}{2}z_i^2\left(\phi_{i,k}(x) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\phi_{j,k}(x)\right)^T \times \left(\phi_{i,k}(x) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\phi_{j,k}(x)\right). \quad (36)$$

Similar to step 1, the following inequality holds

$$LV_{i-1} \le \sum_{j=1}^{i-1}\left(-c_j z_j^4 + \varrho_j\right) + \frac{b_M}{4}z_i^4 - \sum_{m=1}^{i-2}\frac{\partial \alpha_m}{\partial \hat{\theta}}z_{m+1}^3\sum_{j=i}^{n}\frac{\lambda}{2a_j^2}z_j^6 S_j^T(Z_j)S_j(Z_j) + \frac{b}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{i-1}\frac{\lambda}{2a_j^2}z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right), \quad (37)$$

where $c_j = k_j b_m > 0$, $\varrho_j = (1/2)\,a_j^2 + (1/4)\,\varepsilon_j^4 + (3/4)\,l_j^2$, and $j = 1, 2, \ldots, i-1$.

Taking the completion of squares into account, we have

$$\frac{3}{2}z_i^2\left\|\phi_{i,k}(x) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\phi_{j,k}(x)\right\|^2 \le \frac{3l_i^2}{4} + \frac{3z_i^4}{4l_i^2}\left\|\phi_{i,k}(x) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\phi_{j,k}(x)\right\|^4, \quad (38)$$

where l_i represents a positive parameter.

Combining (34), (37) and (38), and utilizing (36) yield that

$$LV_i \le \sum_{j=1}^{i-1}\left(-c_j z_j^4 + \varrho_j\right) + z_i^3(f_{\mu_i,k}x_{i+1} + h_{i,k}\left(\tilde{x},0\right) - \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2 \alpha_{i-1}}{\partial x_p \partial x_q}\phi_{p,k}^T(x)\phi(x)_{q,k} + \frac{1}{4}b_M z_i + \frac{3}{4}l_i^{-2}z_i\left\|\phi_{i,k}(x) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\phi_{j,k}(x)\right\|^4 - \sum_{j=0}^{i-1}\frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}h_{j,k}\left(\tilde{x}, x_{j+1}\right)) + \frac{3}{4}l_i^2 + \frac{b}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{i-1}\frac{\lambda}{2a_j^2}z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) - \sum_{m=1}^{i-2}\frac{\partial \alpha_m}{\partial \hat{\theta}}z_{m+1}^3\sum_{j=i}^{n}\frac{\lambda}{2a_j^2}z_j^6 S_j^T(Z_j)S_j(Z_j). \quad (39)$$

Considering the adaptive law in (22), the following equality can be obtained

$$\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} = \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\left(\sum_{j=1}^{i}\frac{\lambda}{2a_j^2}z_j^6 S_j^T(Z_j)S_j(Z_j) - \gamma\hat{\theta}\right) + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\sum_{j=i+1}^{n}\frac{\lambda}{2a_j^2}z_j^6 S_j^T(Z_j)S_j(Z_j). \quad (40)$$

Then, (39) can be converted to

$$LV_i \le \sum_{j=1}^{i-1}\left(-c_j z_j^4 + \varrho_j\right) + z_i^3\left(f_{\mu_i,k}x_{i+1} + \bar{h}_{i,k}\right) + \frac{b}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{i-1}\frac{\lambda}{2a_j^2}z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) - \sum_{m=1}^{i-1}\frac{\partial \alpha_m}{\partial \hat{\theta}}z_{m+1}^3\sum_{j=i+1}^{n}\frac{\lambda}{2a_j^2}z_j^6 S_j^T(Z_j)S_j(Z_j) - \frac{3}{4}z_i^4 + \frac{3}{4}l_i^2, \quad (41)$$

where

$$\bar{h}_{i,k} = h_{i,k}\left(\tilde{x},0\right) + \frac{1}{4}b_M z_i - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}h_{j,k}\left(\tilde{x}, x_{j+1}\right) + \frac{3}{4}z_i - \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2 \alpha_{i-1}}{\partial x_p \partial x_q}\phi(x)_{p,k}^T\phi(x)_{q,k} + \frac{3}{4}l_i^{-2}z_i\left\|\phi_{i,k}(x) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\phi_{j,k}(x)\right\|^4$$

$$
\begin{aligned}
&-\frac{\lambda}{2a_i^2} z_i^3 S_i^T(Z_j) S_i(Z_j) \sum_{m=1}^{i-2} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3 \\
&-\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \left(\sum_{j=1}^{i} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j) S_j(Z_j) - \gamma \hat{\theta} \right) \\
&-\sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)}. \qquad (42)
\end{aligned}
$$

Again, a RBF NN $W_{i,k}^T S_i(X_i)$ is utilized to model $\bar{h}_{i,k}$, then, one can derive that

$$\bar{h}_{i,k} = W_{i,k}^{*T} S_i(X_i) + \delta_i(X_i) \qquad (43)$$

with $X_i = [x, \hat{\theta}, \bar{y}_d^{(i)T}]^T$, and $|\delta(X_i)| \le \varepsilon_i$.

Similar to (28), one has

$$
\begin{aligned}
z_i^3 \bar{h}_{i,k} &\le \frac{b z_i^6}{2a_i^2} \frac{\left\| W_{i,k}^* \right\|^2}{b} S_i^T(X_i) S_i(X_i) + \frac{1}{2} a_i^2 + \frac{3}{4} z_i^4 + \frac{1}{4} \varepsilon_i^4 \\
&\le \frac{b z_i^6}{2a_i^2} \frac{\left\| W_{i,k}^* \right\|^2}{b} S_i^T(Z_i) S_i(Z_i) + \frac{1}{2} a_i^2 + \frac{3}{4} z_i^4 + \frac{1}{4} \varepsilon_i^4 \\
&\le \frac{b}{2a_i^2} z_i^6 \theta S_i^T(Z_i) S_i(Z_i) + \frac{1}{2} a_i^2 + \frac{3}{4} z_i^4 + \frac{1}{4} \varepsilon_i^4, \qquad (44)
\end{aligned}
$$

where $a_i > 0$.

Additionally, based on (41) and (44), one has

$$
\begin{aligned}
LV_i \le &-\sum_{j=1}^{i-1} c_j z_j^4 + \sum_{j=1}^{i} \varrho_j + z_i^3 f_{\mu_i,k} z_{i+1} + z_i^3 f_{\mu_i,k} \alpha_i \\
&+ \frac{b}{\lambda} \tilde{\theta} \left(\sum_{j=1}^{i-1} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j) S_j(Z_j) - \dot{\hat{\theta}} \right) \\
&- \sum_{m=1}^{i-1} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3 \sum_{j=i+1}^{n} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j) S_j(Z_j) \\
&+ \frac{b}{2a_i^2} z_i^6 \theta S_i^T(Z_i) S_i(Z_i). \qquad (45)
\end{aligned}
$$

Using the virtual control signal a_i in (18), we have

$$
\begin{aligned}
LV_i \le &\frac{b}{\lambda} \tilde{\theta} \left(\sum_{j=1}^{i} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j) S_j(Z_j) - \dot{\hat{\theta}} \right) \\
&- \sum_{m=1}^{i-1} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3 \sum_{j=i+1}^{n} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j) S_j(Z_j) \\
&- \sum_{j=1}^{i} c_j z_j^4 + \sum_{j=1}^{i} \varrho_j + \frac{z_{i+1}^4}{4} b_M, \qquad (46)
\end{aligned}
$$

where $c_j = k_j b_m > 0$, $\varrho_j = (1/2)\, a_j^2 + (3/4)\, l_j^2 + (1/4)\, \varepsilon_j^4$, and $j = 1, 2, \ldots, i$.

Step n: Based on (20) and *Itô* formula, we have

$$
\begin{aligned}
dz_n = &\left(h_{n,k}(x, 0) + f_{\mu_n,k} \left(f_{\nu_{\mu,k}} v + d(v) \right) - L\alpha_{n-1} \right) dt \\
&+ \left(\phi_{n,k}(x) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \phi_{j,k}(x) \right) d\omega, \qquad (47)
\end{aligned}
$$

where $L\alpha_{n-1}$ is displayed in (34) with $i = n$.

Selecting

$$V_n = V_{n-1} + \frac{1}{4} z_n^4, \qquad (48)$$

and adopting (41) with $i = n - 1$, one has

$$
\begin{aligned}
LV_n \le &-\sum_{j=1}^{n-1} c_j z_j^4 + \sum_{j=1}^{n-1} \varrho_j - \frac{3}{4} z_n^4 \\
&+ \frac{b}{\lambda} \tilde{\theta} \left(\sum_{j=1}^{n-1} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j) S_j(Z_j) - \dot{\hat{\theta}} \right) \\
&+ \frac{3}{4} l_n^2 + z_n^3 (f_{\mu_n,k} \left(f_{\nu_{\mu},k} v + d(v) \right) + \bar{h}_{n,k}), \qquad (49)
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{h}_{n,k} = &\, h_{n,k}(\tilde{x}, 0) - L\alpha_{n-1} + \left(\frac{b_M}{4} + \frac{3}{4} \right) z_n \\
&- \frac{\lambda}{2a_n^2} z_j^3 S_j^T(Z_j) S_j(Z_j) \sum_{m=1}^{n-2} \frac{\partial \alpha_m}{\partial \alpha_j} z_{m+1}^3 \\
&+ \frac{3}{4 l_n^2} z_n \left\| \phi_{n,k}(x) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \alpha_j} \phi_{j,k}(x) \right\|^4. \qquad (50)
\end{aligned}
$$

Similarly, a RBF NN $W_{n,k}^T S_n(Z_n)$ is employed to approximate $\bar{h}_{i,k}$ so that $\bar{h}_{n,k} = W_{n,k}^{*T} S_n(X_n) + \delta_n(X_n)$ with $X_n = \left[\tilde{x}, \hat{\theta}, \bar{y}_d^{(n)T} \right]^T$ and the approximation error $\delta_n(X_n)$ satisfies $|\delta_n(X_n)| \le \varepsilon_n$. From (28), we have

$$
\begin{aligned}
z_n^3 \bar{h}_{n,k} \le &\frac{b z_n^6}{2a_n^2} \frac{\left\| W_{n,k}^* \right\|^2}{b} S_n^T(X_n) S_n(X_n) + \frac{1}{2} a_n^2 \\
&+ \frac{3}{4} z_n^4 + \frac{1}{4} \varepsilon_n^4 \\
\le &\frac{b z_n^6}{2a_n^2} \frac{\left\| W_{n,k}^* \right\|^2}{b} S_n^T(Z_n) S_n(Z_n) + \frac{1}{2} a_n^2 \\
&+ \frac{3}{4} z_n^4 + \frac{1}{4} \varepsilon_n^4 \\
\le &\frac{b}{2a_n^2} z_n^6 \theta S_n^T(Z_n) S_n(Z_n) + \frac{1}{2} a_n^2 \\
&+ \frac{3}{4} z_n^4 + \frac{1}{4} \varepsilon_n^4, \qquad (51)
\end{aligned}
$$

where a_n is a design parameter.

Then, using (50) and (51), one has

$$\begin{aligned} LV_n \le & -\sum_{j=1}^{n-1} c_j z_j^4 + \sum_{j=1}^{n-1} \varrho_j + \frac{1}{2}a_n^2 + \frac{3}{4}l_n^4 + \frac{1}{4}\varepsilon_n^4 \\ & + \frac{b}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{n-1} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) \\ & + z_n^3 f_{\mu_n,k}\left(f_{v_{\mu,k}} v + d(v)\right) \\ & + \frac{b}{2a_n^2} z_n^6 \theta S_n^T(Z_n)S_n(Z_n). \end{aligned} \tag{52}$$

What's more, by considering the real controller v in (19), utilizing (14), Assumptions 1 and 2, it yields

$$\begin{aligned} z_n^3 f_{\mu_n,k} f_{v_{\mu,k}} v \le & -k_n f_{\mu_n,k} f_m z_n^4 - \frac{3}{4\eta^2} f_{\mu_n,k} f_m z_n^4 \\ & - \frac{z_n^6}{2a_n^2} b\hat{\theta} S_n^T(Z_n)S_n(Z_n) \\ \le & -k_n f_{\mu_n,k} f_m z_n^4 - \frac{3}{4\eta^2} f_{\mu_n,k} f_m z_n^4 \\ & - \frac{z_n^6}{2a_n^2} b\hat{\theta} S_n^T(Z_n)S_n(Z_n), \end{aligned} \tag{53}$$

$$z_n^3 f_{\mu_n,k} d(v) \le \frac{3}{4\eta^2} f_{\mu_n,k} f_m z_n^4 + \frac{1}{4f_m}\eta^2 b_M D^4. \tag{54}$$

From (53) and (54), (52) becomes

$$\begin{aligned} LV_n \le & -\sum_{j=1}^{n-1} c_j z_j^4 - k_n f_{\mu_n,k} f_m z_n^4 + \sum_{j=1}^{n-1} \varrho_j + \frac{1}{2}a_n^2 + \frac{3}{4}l_n^4 \\ & + \frac{b}{\lambda}\tilde{\theta}\left(\sum_{j=1}^{n} \frac{\lambda}{2a_j^2} z_j^6 S_j^T(Z_j)S_j(Z_j) - \dot{\hat{\theta}}\right) \\ & + \frac{1}{4}\varepsilon_n^4 + \frac{1}{4f_m}\eta^2 b_M D^4. \end{aligned} \tag{55}$$

Using the adaptive law $\dot{\hat{\theta}}$ in (22), one has

$$\begin{aligned} LV_n \le & -\sum_{j=1}^{n-1} c_j z_j^4 - k_n f_{\mu_n,k} f_m z_n^4 + \sum_{j=1}^{n-1} \varrho_j + \frac{1}{2}a_n^2 + \frac{3}{4}l_n^4 \\ & + \frac{1}{4}\varepsilon_n^4 + \frac{1}{4f_m}\eta^2 b_M D^4 + \frac{b\gamma}{\lambda}\tilde{\theta}\hat{\theta}. \end{aligned} \tag{56}$$

Notice that $\frac{b\gamma}{\lambda}\tilde{\theta}\hat{\theta} \le -\left(\frac{b\gamma}{2\lambda}\right)\tilde{\theta}^2 + \left(\frac{b\gamma}{2\lambda}\right)\theta^2$, we have

$$LV_n \le -\sum_{j=1}^{n} c_j z_j^4 - \left(\frac{b\gamma}{2\lambda}\right)\tilde{\theta}^2 + \sum_{j=1}^{n} \varrho_j, \tag{57}$$

where $c_j = k_j b_m > 0$, $\varrho_j = \frac{1}{2}a_j^2 + \frac{3}{4}l_j^2 + \frac{1}{4}\varepsilon_j^4, j = 1, 2, \ldots, n-1$, $c_n = k_n b_m f_m > 0$, and $\varrho_n = \frac{b\gamma}{2\lambda}\theta^2 + \frac{1}{2}a_n^2 + \frac{3}{4}l_n^2 + \frac{1}{4}\varepsilon_n^4 + \frac{1}{4f_m}\eta^2 b_M D^4$.

So far, the main results of the work are provided as ***Theorem 1*** .

Theorem 1: Taking into account the pure-feedback stochastic closed-loop nonlinear switched system with non-lower triangular structure and input saturation, under the virtual controller (18), the actual controller (19), and the adaptive law (22), the neural adaptive tracking scheme can ensure that all the signals in the closed-loop system are bounded in probability. Particularly, Giving any initial conditions $z_j(0)$ and $\hat{\theta}(0) \ge 0$ belonging to Ω_0 (where Ω_0 is a compact set, which is selected suitably), the error z_j $(j = 1, 2, \cdots, n)$ and $\tilde{\theta}$ remain in a bounded closed set Ω_Z shown as

$$\begin{aligned} \Omega_Z = \{z_j, \tilde{\theta} & \left| \sum_{j=1}^{n} E\left[\left|z_j\right|^4\right] \le 4V(0) + 4\frac{\gamma_0}{v_0}, \left|\tilde{\theta}\right| \right. \\ & \le \sqrt{\frac{2\lambda}{b}\left(V(0) + \frac{\gamma_0}{v_0}\right)}, j = 1, 2, \cdots, n\}, \end{aligned} \tag{58}$$

and finally can converge to bounded closed set Ω_s showen as

$$\begin{aligned} \Omega_s = \{z_j, \tilde{\theta} & \left| \sum_{j=1}^{n} E\left[\left|z_j\right|^4\right] \le 4\frac{\gamma_0}{v_0}, \left|\tilde{\theta}\right| \right. \\ & \le \sqrt{\frac{2\lambda}{b}\frac{\gamma_0}{v_0}}., j = 1, 2, \cdots, n\} \end{aligned} \tag{59}$$

Proof: Selecting $V = V_n$ and defining $v_0 = \min\left\{4c_j, \gamma, j = 1, 2, \cdots, n\right\}$, $\gamma_0 = \sum_{j=1}^{n} \varrho_j$.

Then, (57) can be transformed as:

$$LV \le -v_0 V + \gamma_0, \quad t \ge 0. \tag{60}$$

What's more, according to [49], one has

$$\frac{dE[V(t)]}{dt} \le -v_0 E[V(t)] + \gamma_0. \tag{61}$$

So, we can further obtain

$$0 \le E[V(t)] \le \left(V(0) - \frac{\gamma_0}{v_0}\right) e^{-v_0 t} + \frac{\gamma_0}{v_0}. \tag{62}$$

It yields

$$E[V(t)] < V(0) + \frac{\gamma_0}{v_0}, \quad \forall t > 0. \tag{63}$$

Then, it can be inferred from [50] that all signals remain bounded in probability.

Based on (62), one can infer

$$E[V(t)] \le e^{-v_0 t}[V(0)] + \frac{\gamma_0}{v_0}, \quad \forall t > 0. \tag{64}$$

As $t \to \infty$, we get

$$E[V(t)] \le \frac{\gamma_0}{v_0}. \tag{65}$$

Therefore, it can be inferred that all signals of the closed-loop system remain uniformly ultimately boundedness in probability, and it is easy to infer that error $z_j(j = 1, 2, \ldots, n)$ and $\tilde{\theta}$ can converge to a set Ω_s, which is a bounded closed set.

Remark 1: Notice that nonlinear multilayer NNs can also be used to estimate the unknown functions, and their approximation accuracy is often better than that of linear two-layer NNs such as RBF NNs. However, our main concern is not to improve the accuracy of estimating the unknown functions, but just to deal with the unknown functions through some kind of NNs. Fortunately, RBF NNs can achieve this goal and thus they are adopted in this paper.

Remark 2: Although the proposed algorithm in this paper guarantees that the tracking error can converge to a small neighborhood near the origin in the sense of mean quartic value, and all signals of the closed-loop system (1) can be bounded in probability, our algorithm can not achieve the asymptotic tracking. Therefore, it is necessary to further improve our algorithm to achieve the asymptotic tracking in the future.

IV. SIMULATION EXAMPLES

Example 1: So as to prove the applicability of the proposed control algorithm, a practical example-Brusselator model is considered. The Brusselator model describes a specific chemical reaction, which was proposed by Turing in an article published in 1952 [51], and it was studied in detail by Prigogine and colleagues [52]. This model is called "Brusselator" because its founder worked in Brussels. It has became one of the paradigms of chaos research and one of the most famous nonlinear oscillation models in chemical kinetics. The model is given below:

$$\begin{cases} \dot{x}_1 = C - (D+1)x_1 + x_1^2 x_2, \\ \dot{x}_2 = Dx_1 + (2+\cos(x_1))u - x_1^2 x_2, \\ y = x_1, \end{cases} \tag{66}$$

where x_1 and x_2 represent the concentration of the chemical reaction intermediate, and positive parameters C, D describe the supply of "storage" chemicals. In [53], the below expression was written: "As a simplified model for describing chemical reactions, the Brusselsator model is derived from a series of approximated partial differential equations." Therefore, there are errors and unknown nonlinearities in actual chemical reactions. Meanwhile, the presence of stochastic perturbations and input saturation is unavoidable in Brusselsator model, since it is a actual chemical reaction. Therefore, the controlled Brusselsator model assumes that

$$\begin{cases} dx_1 = (C_{1,\sigma} - (D_{1,\sigma}+1)x_1 + x_1^2 x_2)dt + f_{1,\sigma}(X)dw, \\ dx_2 = (D_{2,\sigma}x_1 + (2+\cos(x_1))u - x_1^2 x_2)dt \\ \qquad + f_{2,\sigma}(X)dw, \\ y = x_1, \end{cases} \tag{67}$$

where ω is a random perturbation, $f(X)$ is an uncertain nonlinear function. The saturation limits are chosen as $u_{\max} = 80$ and $u_{\min} = -50$, respectively. Select $y_d = \sin(t) + 1$ as reference signal. Then, $f_{1,1} = A\sin(x_1 x_2)$, $f_{2,1} = Ax_2\cos x_1$, $f_{1,2} = A\sin(x_1^2 + x_2^2)$, $f_{2,2} = Ax_2^2$.

According to Theorem 1, for the system (67), α_1, v and $\dot{\hat{\theta}}$ are constructed as

$$\begin{aligned} \alpha_1 &= -(k_1 + \frac{3}{4})z_1 - \frac{1}{2a_1^2} z_1^3 \hat{\theta} S_1^T(Z_1)S_1(Z_1), \\ v &= -(k_2 + \frac{3}{4\eta^2})z_2 - \frac{1}{2a_2^2} z_2^3 \hat{\theta} S_2^T(Z_2)S_2(Z_2), \\ \dot{\hat{\theta}} &= \sum_{i=1}^{2} \frac{\lambda}{2a_i^2} z_i^6 S_i^T(Z_i)S_i(Z_i) - \gamma\hat{\theta}. \end{aligned} \tag{68}$$

The system parameters are designed as $k_1 = k_2 = 100$, $a_1 = a_2 = 3$, $\gamma = 0.1$, $\lambda = 1$, $A = 10^{-8}$, $C_{1,1} = D_{1,1} = D_{2,1} = 0.1$, $C_{1,2} = D_{1,2} = D_{2,2} = 0.2$, $\eta = 1$, and the initial condition $[x_1(0), x_2(0), \hat{\theta}(0)] = [0.1, 0.1, 0]$.

Finally, from the simulation results in Figure 1-7, we can see that the validity of the theoretical derivation of the paper has been verified. Fig. 1 is the trajectories of $y(t)$ and $y_d(t)$ in this example. Fig. 2 describes the responses of tracking error z_1. Fig. 3-6 show the responses of control law v, signal u, the adaptive parameter $\hat{\theta}$, and the evolution of switched signal

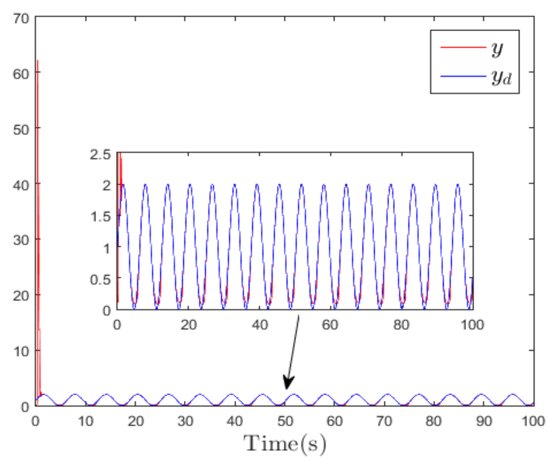

FIGURE 1. The responses of the system out $y(t)$ and reference signal $y_d(t)$.

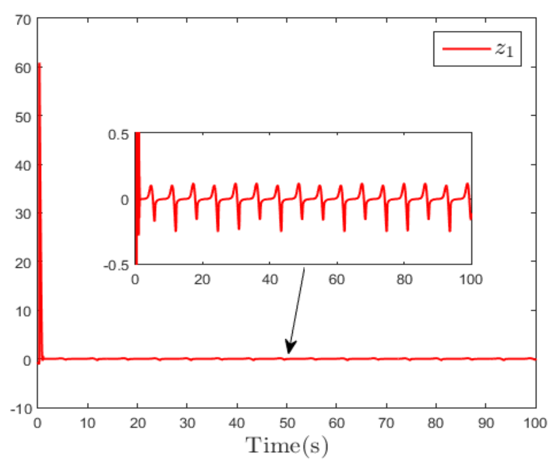

FIGURE 2. The responses of the tracking error z_1.

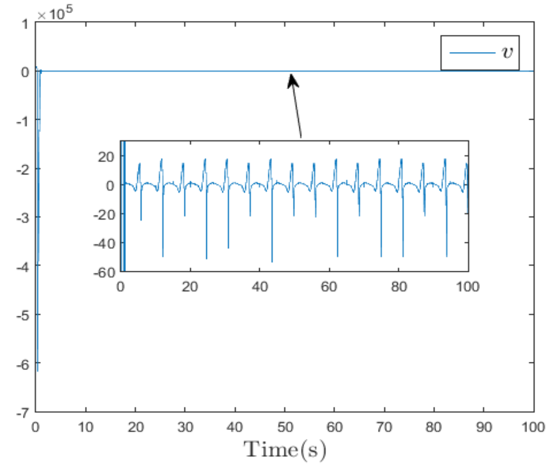

FIGURE 3. **The responses of the control law v.**

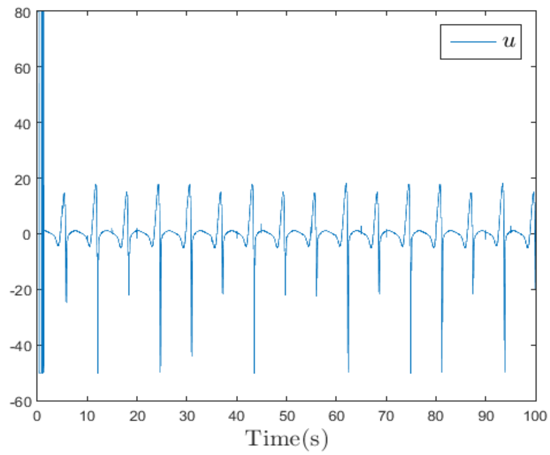

FIGURE 4. **The responses of the signal u.**

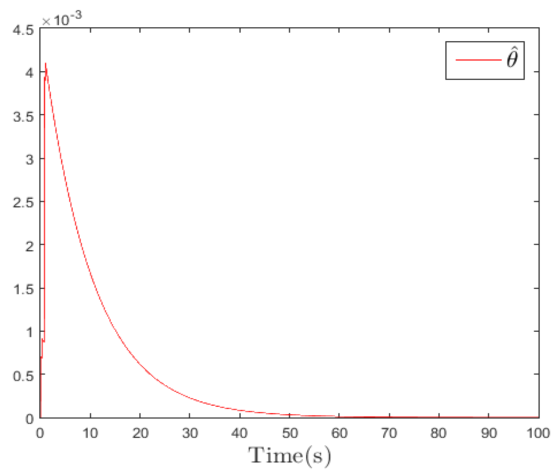

FIGURE 5. **The responses of the adaptive law $\hat{\theta}$.**

$\sigma(t)$ respectively. So, from the simulation results, the proposed controller guarantees the tracking performance and the boundedness of the closed-loop system signals.

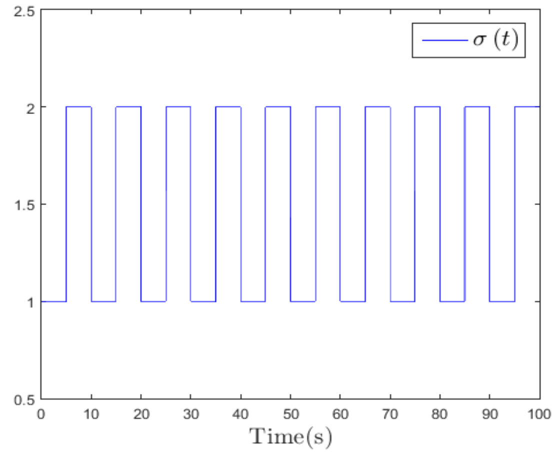

FIGURE 6. **The responses of the switched signal $\sigma(t)$.**

Example 2: The model of the one-link manipulator [54] is as follows:

$$\begin{cases} D\ddot{q} + B\dot{q} + N\sin(q) = \tau + \tau_d, \\ M\dot{\tau} + H\tau = u - K_m\dot{q}. \end{cases} \tag{69}$$

Let $x_1 = q, x_2 = \dot{q}, x_3 = \tau$, and consider the stochastic effect, (69) can be changed to

$$\begin{cases} dx_1 = x_2 dt + \phi_{1,\sigma}^T(x)\, d\omega \\ dx_2 = \left(\dfrac{1}{D_{2,\sigma}}x_3 - \dfrac{B_{2,\sigma}}{D_{2,\sigma}}x_2 - \dfrac{N_{2,\sigma}}{D_{2,\sigma}}\sin(x_1) + \dfrac{\tau_{d_{2,\sigma}}}{D_{2,\sigma}}\right) dt \\ + \phi_{2,\sigma}^T(x)\, d\omega \\ dx_3 = \left(\dfrac{1}{M_{3,\sigma}}u - \dfrac{K_{m_{3,\sigma}}}{M_{3,\sigma}}x_2 - \dfrac{H_{3,\sigma}}{M_{3,\sigma}}x_3\right) dt + \phi_{3,\sigma}^T(x)\, d\omega \\ y = x_1 \end{cases} \tag{70}$$

Select $y_d = \sin(0.1t)$ as reference signal. Then $\phi_{1,1} = A\sin(x_1x_2x_3)$, $\phi_{1,2} = A\cos(x_1x_2x_3)$, $\phi_{2,1} = Ax_3\cos(x_1x_2)$, $\phi_{2,2} = A\sin\left(x_1^2 + x_2^2 + x_3^2\right)$, $\phi_{3,1} = Ax_1\sin(x_2x_3)$, $\phi_{3,2} = A\cos\left(x_1^2 + x_2^2 + x_3^2\right)$

According to Theorem 1, for the system (70), α_1, α_2, v are constructed as

$$\begin{aligned} \alpha_i &= -(k_i + \frac{3}{4})z_i - \frac{1}{2a_i^2}z_i^3\hat{\theta}S_i^T(Z_i)S_i(Z_i), \quad i = 1, 2, \\ v &= -(k_3 + \frac{3}{4\eta^2})z_3 - \frac{1}{2a_3^2}z_3^3\hat{\theta}S_3^T(Z_3)S_3(Z_3), \\ \dot{\hat{\theta}} &= \sum_{i=1}^{2}\frac{\lambda}{2a_i^2}z_i^6S_i^T(Z_i)S_i(Z_i) - \gamma\hat{\theta}. \end{aligned} \tag{71}$$

The system parameters are designed as $k_1 = k_2 = k_3 = 50$, $a_1 = a_2 = a_3 = 0.3$, $\gamma = 0.1$, $\lambda = 1$, $u_{\max} = 800$, $u_{\min} = -500$, $\eta = 1$, $A = 10^{-8}$, $D_{2,1} = 500$, $D_{2,2} = 550$, $B_{2,1} = 0.1$, $B_{2,2} = 0.2$, $N_{2,1} = 0.1$, $N_{2,2} = 0.15$, $\tau_{d_{2,1}} = 1$, $\tau_{d_{2,2}} = 1.5$, $M_{3,1} = 0.4$, $M_{3,2} = 0.5$, $K_{m_{3,1}} = 0.02$, $K_{m_{3,2}} = 0.01$, $H_{3,1} = 10$,

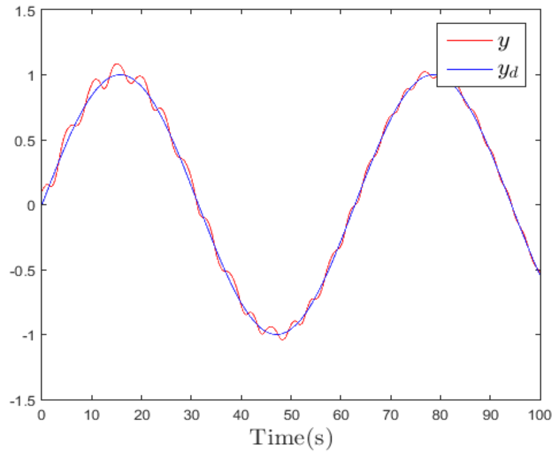

FIGURE 7. The responses of the system out $y(t)$ and reference signal $y_d(t)$.

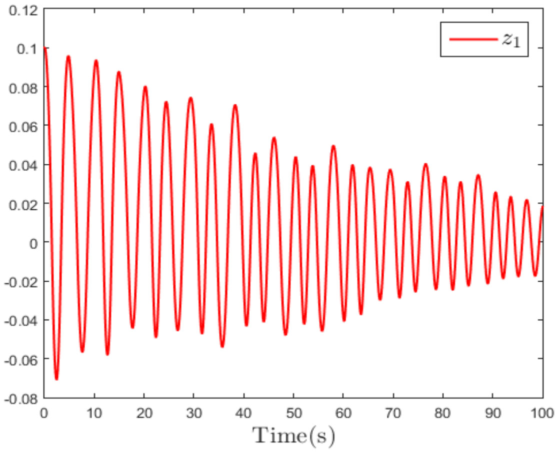

FIGURE 8. The responses of the tracking error z_1.

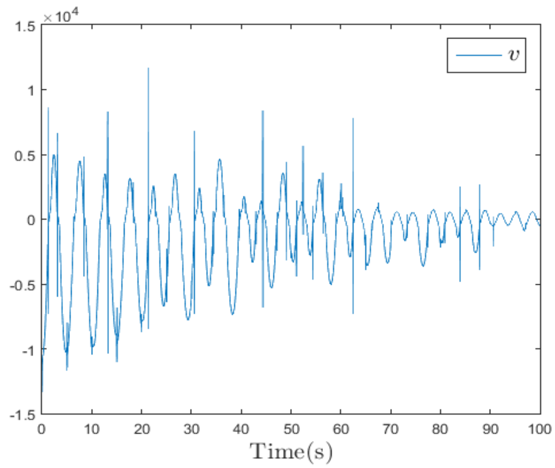

FIGURE 9. The responses of the control law v.

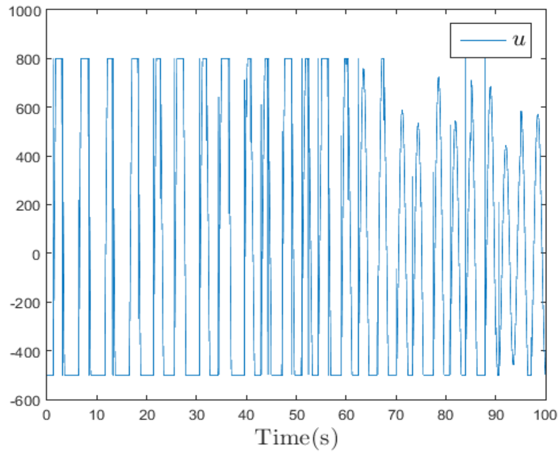

FIGURE 10. The responses of the signal u.

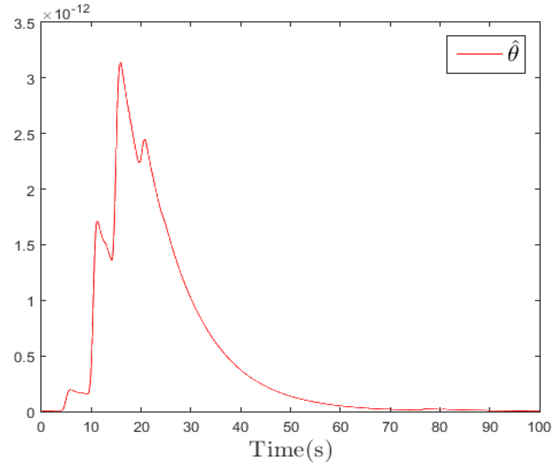

FIGURE 11. The responses of the adaptive law $\hat{\theta}$.

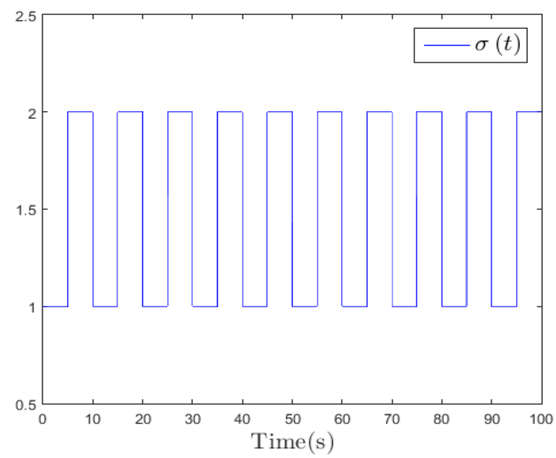

FIGURE 12. The responses of the switched signal $\sigma(t)$.

$H_{3,2} = 15$, and the initial condition $[x_1(0), x_2(0), x_3(0), \hat{\theta}(0)] = [0.1, 0.1, 0.1, 0]$.

Finally, from the simulation results in Figure 7-12, we can see that the validity of the theoretical derivation of the paper has been verified again. Fig. 7 is the trajectories of $y(t)$ and $y_d(t)$ in this example. Fig. 8 describes the responses of tracking error z_1. Fig. 9-12 show the responses of control law v, signal u, the adaptive parameter $\hat{\theta}$, and the evolution of switched signal $\sigma(t)$ respectively. So, from the simulation results, the proposed controller guarantees the tracking performance and the boundedness of the closed-loop system signals.

V. CONCLUSION

In this work, an intelligent adaptive tracking control algorithm is designed for pure-feedback stochastic switched nonlinear systems which are subject to input saturation and non-lower triangular structure. In the design process, not only the backstepping technology and universal intelligent approximation technology are successfully applied to the more general intelligent control for a class of uncertain stochastic nonlinear switched systems, but also the design problems resulted from the pure-feedback and non-lower triangular structures are solved by simple scaling methods. The designed controllers ensure that all signals of the closed-loop system remain bounded in probability. It is worth noting that our work only consider SISO stochastic nonlinear switched systems. Hence, the control of the MIMO stochastic switched nonlinear systems are the focus of our attention in the future work.

REFERENCES

[1] E. A. Sisbot, L. F. Marin-Urias, R. Alami, and T. Simeon, "A human aware mobile robot motion planner," *IEEE Trans. Robot.*, vol. 23, no. 5, pp. 874–883, Oct. 2007.
[2] F. Xu, L. Dong, D. Wang, X. Li, and R. Rakkiyappan, "Globally exponential stability of nonlinear impulsive switched systems," *Math. Notes*, vol. 97, nos. 5–6, pp. 803–810, May 2015.
[3] L. Ma, Z. Wang, Y. Liu, and F. E. Alsaadi, "A note on guaranteed cost control for nonlinear stochastic systems with input saturation and mixed time-delays," *Int. J. Robust Nonlinear Control*, vol. 27, no. 18, pp. 4443–4456, Dec. 2017.
[4] X. Li, X. Fu, and R. Rakkiyappan, "Delay-dependent stability analysis for a class of dynamical systems with leakage delay and nonlinear perturbations," *Appl. Math. Comput.*, vol. 226, pp. 10–19, Jan. 2014.
[5] X. Li, J. Shen, H. Akca, and R. Rakkiyappan, "LMI-based stability for singularly perturbed nonlinear impulsive differential systems with delays of small parameter," *Appl. Math. Comput.*, vol. 250, pp. 798–804, Jan. 2015.
[6] L. Ma, X. Fang, Y. Yuan, J. Zhang, and Y. Bo, "Dissipative control for nonlinear Markovian jump systems with mixed time-delays: The discrete-time case," *Int. J. Robust Nonlinear Control*, vol. 30, no. 7, pp. 2871–2888, May 2020.
[7] X. Lv, R. Rakkiyappan, and X. Li, "M-stability criteria for nonlinear differential systems with additive leakage and transmission time-varying delays," *Nonlinear Anal., Model. Control*, vol. 23, no. 3, pp. 380–400, Jun. 2018.
[8] L. Ma, Z. Wang, Q.-L. Han, and H.-K. Lam, "Envelope-constrained H_∞ filtering for nonlinear systems with quantization effects: The finite horizon case," *Automatica*, vol. 93, pp. 527–534, Jul. 2018.
[9] X.-J. Xie and J. Tian, "Adaptive state-feedback stabilization of high-order stochastic systems with nonlinear parameterization," *Automatica*, vol. 45, no. 1, pp. 126–133, Jan. 2009.
[10] S.-J. Liu, S. Z. Ge, and J.-F. Zhang, "Adaptive output-feedback control for a class of uncertain stochastic non-linear systems with time delays," *Int. J. Control*, vol. 81, no. 8, pp. 1210–1220, 2008.
[11] M.-L. Chiang and L.-C. Fu, "Adaptive stabilization of a class of uncertain switched nonlinear systems with backstepping control," *Automatica*, vol. 50, no. 8, pp. 2128–2135, Aug. 2014.
[12] D. Yang, X. Li, and J. Qiu, "Output tracking control of delayed switched systems via state-dependent switching and dynamic output feedback," *Nonlinear Anal., Hybrid Syst.*, vol. 32, pp. 294–305, May 2019.
[13] B. Niu, P. Zhao, J.-D. Liu, H.-J. Ma, and Y.-J. Liu, "Global adaptive control of switched uncertain nonlinear systems: An improved MDADT method," *Automatica*, vol. 115, May 2020, Art. no. 108872, doi: 10.1016/j.automatica.2020.108872.
[14] D. Yang, X. Li, J. Shen, and Z. Zhou, "State-dependent switching control of delayed switched systems with stable and unstable modes," *Math. Methods Appl. Sci.*, vol. 41, no. 16, pp. 6968–6983, Nov. 2018.
[15] B. Niu, M. Liu, and A. Li, "Global adaptive stabilization of stochastic high-order switched nonlinear non-lower triangular systems," *Syst. Control Lett.*, vol. 136, Feb. 2020, Art. no. 104596, doi: 10.1016/j.sysconle.2019.104596.
[16] B. Niu, Y. J. Liu, W. L. Zhou, H. T. Li, P. Y. Duan, and J. Q. Li, "Multiple Lyapunov functions for adaptive neural tracking control of switched nonlinear nonlower-triangular systems," *IEEE Trans. Cybern.*, vol. 50, no. 5, pp. 1877–1886, May 2019.
[17] L. Ma, X. Huo, X. Zhao, B. Niu, and G. Zong, "Adaptive neural control for switched nonlinear systems with unknown backlash-like hysteresis and output dead-zone," *Neurocomputing*, vol. 357, pp. 203–214, Sep. 2019.
[18] B. Niu, D. Wang, M. Liu, X. M. Song, H. Q. Wang, and P. Y. Duan, "Adaptive neural output feedback controller design of switched non-lower triangular nonlinear systems with time-delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 4, pp. 1076–1087, Dec. 2019.
[19] X. Zheng, B. Hu, and D. Lu, "A multi-objective virtual network embedding algorithm in cloud computing," *J. Internet Technol.*, vol. 17, no. 4, pp. 633–642, 2016.
[20] H. Liu, B. Hu, and P. Moore, "HCI model with learning mechanism for cooperative design in pervasive computing environment," *J. Internet Technol.*, vol. 16, no. 2, pp. 201–210, 2015.
[21] W. Yan, H. Liu, C. Zanni-Merk, and D. Cavallucci, "IngeniousTRIZ: An automatic ontology-based system for solving inventive problems," *Knowl.-Based Syst.*, vol. 75, pp. 52–65, Feb. 2015.
[22] S. Xing, F. Liu, Q. Wang, X. Zhao, and T. Li, "A hierarchical attention model for rating prediction by leveraging user and product reviews," *Neurocomputing*, vol. 332, pp. 417–427, Mar. 2019.
[23] Z. Yao, J. Yao, and W. Sun, "Adaptive RISE control of hydraulic systems with multilayer neural-networks," *IEEE Trans. Ind. Electron.*, vol. 66, no. 11, pp. 8638–8647, Nov. 2019.
[24] M. Zhang, H. Zhang, J. Li, L. Wang, Y. Fang, and J. Sun, "Supervised graph regularization based cross media retrieval with intra and inter-class correlation," *J. Vis. Commun. Image Represent.*, vol. 58, pp. 1–11, Jan. 2019.
[25] B. Hu, H. Wang, X. Yu, W. Yuan, and T. He, "Sparse network embedding for community detection and sign prediction in signed social networks," *J. Ambient Intell. Hum. Comput.*, vol. 10, no. 1, pp. 175–186, Jan. 2019.
[26] X. Dong, H. X. Zhang, L. Zhu, W. B. Wan, Z. H. Wang, Q. Wang, P. L. Guo, H. Ji, and J. D. Sun, "Weighted locality collaborative representation based on sparse subspace," *J. Vis. Commun. Image Represent.*, vol. 58, pp. 1–11, Jan. 2019.
[27] L. Zhu, Z. Huang, Z. Li, L. Xie, and H. T. Shen, "Exploring auxiliary context: Discrete semantic transfer hashing for scalable image retrieval," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 11, pp. 5264–5276, Nov. 2018.
[28] J. Sun, Y. Wang, J. Li, W. Wan, D. Cheng, and H. Zhang, "View-invariant gait recognition based on kinect skeleton feature," *Multimedia Tools Appl.*, vol. 77, no. 19, pp. 24909–24935, Oct. 2018.
[29] D. Shi, L. Zhu, Z. Cheng, Z. Li, and H. Zhang, "Unsupervised multi-view feature extraction with dynamic graph learning," *J. Vis. Commun. Image Represent.*, vol. 56, pp. 256–264, Oct. 2018.
[30] H. Wang, B. Chen, X. Liu, K. Liu, and C. Lin, "Robust adaptive fuzzy tracking control for pure-feedback stochastic nonlinear systems with input constraints," *IEEE Trans. Cybern.*, vol. 43, no. 6, pp. 2093–2104, Dec. 2013.
[31] Y. Zhang, Y. Chen, H. Yu, X. Yang, W. Lu, and H. Liu, "Wearing-independent hand gesture recognition method based on EMG armband," *Pers. Ubiquitous Comput.*, vol. 22, no. 3, pp. 511–524, Jun. 2018.
[32] B. Niu, D. Wang, N. D. Alotaibi, and F. E. Alsaadi, "Adaptive neural state-feedback tracking control of stochastic nonlinear switched systems: An average dwell-time method," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 4, pp. 1076–1087, Apr. 2019.
[33] H. Ma, H. Li, H. Liang, and G. Dong, "Adaptive fuzzy event-triggered control for stochastic nonlinear systems with full state constraints and actuator faults," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 11, pp. 2242–2254, Nov. 2019.

[34] B. Chen, X. P. Liu, S. S. Ge, and C. Lin, "Adaptive fuzzy control of a class of nonlinear systems by fuzzy approximation approach," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 6, pp. 1012–1021, Dec. 2012.
[35] H. Wang, X. Liu, and K. Liu, "Adaptive fuzzy tracking control for a class of pure-feedback stochastic nonlinear systems with non-lower triangular structure," *Fuzzy Sets Syst.*, vol. 302, pp. 101–120, Nov. 2016.
[36] B. Chen, C. Lin, X. Liu, and K. Liu, "Adaptive fuzzy tracking control for a class of MIMO nonlinear systems in nonstrict-feedback form," *IEEE Trans. Cybern.*, vol. 45, no. 12, pp. 2744–2755, Dec. 2015.
[37] S. S. Ge and K. P. Tee, "Approximation-based control of nonlinear MIMO time-delay systems," *Automatica*, vol. 43, no. 1, pp. 31–43, Jan. 2007.
[38] R. B. Li, X. M. Wang, X. M. Liu, B. Niu, and D. Yang, "Adaptive neural tracking control for uncertain switched nonlinear non-lower triangular system with disturbances and dead-zone input," *Int. J. Control, Automat. Syst.*, vol. 18, pp. 1–8, Dec. 2019.
[39] X. Zhao, P. Shi, X. Zheng, and L. Zhang, "Adaptive tracking control for switched stochastic nonlinear systems with unknown actuator dead-zone," *Automatica*, vol. 60, pp. 193–200, Oct. 2015.
[40] M. Chen, S. S. Ge, and B. Ren, "Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints," *Automatica*, vol. 47, no. 3, pp. 452–465, Mar. 2011.
[41] Z. Lin, A. Saberi, and A. A. Stoorvogel, "Semiglobal stabilization of linear discrete-time systems subject to input saturation, via linear feedback—An ARE-based approach," *IEEE Trans. Autom. Control*, vol. 41, no. 8, pp. 1203–1207, Aug. 1996.
[42] W. Lan and J. Huang, "Semiglobal stabilization and output regulation of singular linear systems with input saturation," *IEEE Trans. Autom. Control*, vol. 48, no. 7, pp. 1274–1280, Jul. 2003.
[43] H. Wang, B. Chen, X. Liu, K. Liu, and C. Lin, "Adaptive neural tracking control for stochastic nonlinear strict-feedback systems with unknown input saturation," *Inf. Sci.*, vol. 269, pp. 300–315, Jun. 2014.
[44] H. Li, B. Lu, Z. Qi, R. Lu, and L. Wang, "Adaptive fuzzy control of stochastic nonstrict-feedback nonlinear systems with input saturation," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 8, pp. 2185–2197, Aug. 2017.
[45] D. Cui, B. Niu, H. Wang, and D. Yang, "Adaptive neural tracking control of nonlinear stochastic switched non-lower triangular systems with input saturation," *Neurocomputing*, vol. 364, pp. 192–202, Oct. 2019.
[46] T. M. Apostol, *Mathematical Analysis*. Reading, MA, USA: Addison-Wesley, 1963.
[47] C. Wen, J. Zhou, Z. Liu, and H. Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Trans. Autom. Control*, vol. 56, no. 7, pp. 1672–1678, Jul. 2011.
[48] Y. Sun, B. Chen, C. Lin, H. Wang, and S. Zhou, "Adaptive neural control for a class of stochastic nonlinear systems by backstepping approach," *Inf. Sci.*, vol. 369, pp. 748–764, Nov. 2016.
[49] H. Deng, M. Krstic, and R. J. Williams, "Stabilization of stochastic nonlinear systems driven by noise of unknown covariance," *IEEE Trans. Autom. Control*, vol. 46, no. 8, pp. 1237–1253, Aug. 2001.
[50] Y.-J. Liu and S. Tong, "Adaptive NN tracking control of uncertain nonlinear discrete-time systems with nonaffine dead-zone input," *IEEE Trans. Cybern.*, vol. 45, no. 3, pp. 497–505, Mar. 2015.
[51] A. Turing, "The chemical basis of morphogenesis," *Bull. Math. Biol.*, vol. 52, nos. 1–2, pp. 153–197, 1990.
[52] G. Nicolis and I. Prigogine, *Self-Organization in Nonequilibrium Systems*. New York, NY, USA: Wiley, 1977.
[53] S. S. Ge and C. Wang, "Uncertain chaotic system control via adaptive neural design," *Int. J. Bifurcation Chaos*, vol. 12, no. 5, pp. 1097–1109, May 2002.
[54] B. Niu, C. K. Ahn, H. Li, and M. Liu, "Adaptive control for stochastic switched nonlower triangular nonlinear systems and its application to a one-link manipulator," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 10, pp. 1701–1714, Oct. 2018.

JI-DONG LIU received the B.S. degree in computer science and technology from Qufu Normal University, Rizhao, China, in 2017. He is currently pursuing the M.S. degree in computer application technology with the School of Information Science and Engineering, Shandong Normal University, Jinan, China. His current research interests include nonlinear systems, neural networks, and adaptive control.

RUI-BING LI was born in Shandong, China, in 1995. He received the B.S. degree in computer science and technology from the Taishan College, Taian, China, in 2018. He is currently pursuing the M.S. degree in computer technology with Shandong Normal University, Jinan, China. His current research interests include stochastic nonlinear systems, neural networks, and adaptive control.

XIAO-MEI LIU received the M.S. degree in operational research and cybernetics from Ludong University, China, in 2010, and the Ph.D. degree in control theory and control engineering from Southeast University, China, in 2014. She is currently a Lecturer with the Business School, Shandong Normal University. Her research interests include switched systems theory, optimal switch-time control theory of switched stochastic systems, and robust control of systems with time-delay.

BEN NIU received the Ph.D. degree in control theory and applications from Northeastern University, in 2013. He is currently a Professor with the College of Information Science and Engineering, Shandong Normal University. His research interests include switched systems, stochastic systems, robust control, intelligent control, and their applications.

DONG YANG was born in Weifang, Shandong, China, in 1985. He received the M.S. degree from Qufu Normal University, Rizhao, China, in 2010, and the Ph.D. degree in control theory and control engineering from Northeastern University, in 2017. He is currently with the School of Engineering, Qufu Normal University. His research interests include switched systems, LPV systems, Markovian jump systems, and distributed control.