IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received June 16, 2020, accepted July 2, 2020, date of publication July 7, 2020, date of current version July 17, 2020.
Digital Object Identifier 10.1109/ACCESS.2020.3007731

Distributed Optimization for Multi-Agent Systems With Time Delay

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This work was supported by the National Natural Science Foundation of China under Grant 61573199 and Grant 61571441.

ABSTRACT The distributed optimization for multi-agent systems with time delay and first-order is investigated in this paper. The objective of the distributed optimization is to optimize the objective function composed of the sum of local objective functions, which can only be known by its corresponding agents. Firstly, a distributed algorithm for time-delay systems is proposed to solve the optimization problem that each agent depends on its own state and the state between itself and its neighbors. Secondly, Lyapunov-Krasovskii function is used to prove that the states of each agent can be asymptotically the same, and the states are optimal. Finally, an example is given for illustrating the analytical results and a comparison is also gave to illustrate the differences between the algorithm of this paper and other results.

INDEX TERMS Distributed optimization, multi-agent systems, time delay, Lyapunov-Krasovskii function, zero-gradient-sum algorithm.

I. INTRODUCTION

The problem of optimizing systems in a distributed way has been intensively investigated by more and more scientists in control area [1]–[4]. The purpose of the optimization is to minimize the sum of local objective functions in a distributed manner. A variety of algorithms in a decentralized manner have been proposed in [5]–[12] to find the solution of the optimization problem in various situations, including different objectives, different dynamic behavior, and so on. Consensus is a meaningful dynamic behavior in the multi-agent systems which is applied in formation control [13], [14]. Moreover, article [15] addressed the output consensus problem for multi-agent system with energy constraints. In recent years, some researchers have paid much attention to decentralized consensus optimization problems for multi-agent systems. For example, in [2], a combination of sub-gradient and consensus algorithm is used to solve the convex optimization problem. Moreover, the authors of [3] proposed a discrete-time projection method for decentralized convex optimization and that proved all agents converge to the intersection of their local sets. Then, the result in [3] was extended in [5] for multi-agent systems with continuous-time dynamics. In addition, A distributed optimization problem with bounded constraint was discussed under the general step-size in [6]. In particular, under the assumption of the undirected graph, authors in [7], [8] proposed a decentralized algorithm to deal with the optimization problem, and articles [8] proved that the algorithm's convergence rate has a lower bound. For balanced directed networks, it is shown in [9], [10] that the algorithm's convergence rate was derived.

Recently, by using the Hessian of local objective functions, a Zero-Gradient-Sum algorithm (ZGS) was introduced to solve the optimization problems for fixed undirected connected graphs when the objective functions are not constrained, detachable and convex [12]. For nonlinear networked dynamical systems, it was proved that the state converges asymptotically to the unknown minimizer at all times in which minimize the global objective function in a distributed way. Moreover, distributed optimization of ZGS with the event-triggered scheme for directed networks was studied [16]. A distributed optimization of ZGS for time-varying topology networks was studied in [17]. The article [18] extended the algorithm for the distributed optimization problem in finite time.

It is clear that the above algorithms are suitable for multi-agent systems without the communication time delay. But in real control systems, it is interesting that the optimization problem by a distributed manner for the systems with time-delay. Aiming at the distributed control

The associate editor coordinating the review of this manuscript and approving it for publication was Jianxiang Xi.

and optimization problems, the effects of communication delays were discussed in [11], [19]–[28]. The sub-gradient projection algorithm in [3] was extended to deal with the optimization problem with time-delays and constraints [11]. Reference [25] proposed a delay optimization algorithm in view of the push-sum way, which originates from [4]. Some results for the continuous-time distributed optimization problem with time delay were also obtained in [26]–[28].

In order to extend the continuous-time ZGS optimization algorithm, the effects of communication time delay are considered in this paper. A delay-independent distributed optimization algorithm of multi-agent systems is derived. Our algorithm builds on the work of Guo *et al.* [27]. It is well known that the delay-dependent or delay-independent conditions for time delay systems are often studied. The aforementioned results in [26]–[28] require establishing the maximum delay bound that guarantees stability for distributed optimization. But, in some practical applications, delay-independent distributed optimization laws are preferable over delay-dependent laws as the robustness in the delay. In the light of few results on the construction of delay-independent output feedback laws for distributed optimization with input delay, we presented a delay-independent distributed optimization algorithm in this paper. The main contributions of this paper are as follows: Firstly, a delay-independent distributed optimization algorithm is presented, which is more convenient in practical application. Secondly, according to the method of constructing the Lyapunov-Krasovskii function, the consensus and convergence of the algorithm are proved.

We organize the rest of this article as follows. In Section 2, we introduce the basic notation, statement of graph theory, useful lemma and definite. In Section 3, a new optimization algorithm for the multi-agent system with time delay caused by communication is presented and we proved that all agents would asymptotically track to the optimal which minimizes the total objective function and all agents will reach the same state. Some numerical simulation results are given in Section 4. The conclusions of this paper are given in Section 5.

II. PRELIMINARIES AND NOTATIONS

In this section, the following notations will be used in this paper. Let f: $R^n \rightarrow R$ is a continuously differentiable function, the gradient of function f is denoted by ∇f, the Hessian of function f is defined as $\nabla^2 f$ and its inverse is defined as $\nabla^2 f^{-1}$. $\frac{d}{dt}$ represents the differential quotient with respect to time t, R^n is the real vector in n dimension space, $R^{n\times n}$ represents a set of $n \times n$ real matrices. The unit matrix is denoted by $I_n \in R^{n\times n}$. Given $s \in R^n$, $\|s\|_1$ is the 1-norm of vector s, $\|s\|$ is the 2-norm of vector s, which is called the Euclidean norm. The transpose of a vector or a matrix is denoted by "T".

Here is some basic graph theory in this subsection. The graph $G = (\mathcal{V}, \mathcal{E})$ consists of node set $\mathcal{V} = \{1, 2, \ldots N\}$ and edges set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is called undirected graph. If $(i, j) \in \mathcal{E}$, then there exist an edge between agents i and j, which means that they are neighbors. $N_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$ is the set of neighbors representing the vertex i. $A = [a_{ij}] \in R^{N\times N}$ is adjacent matrix of the graph G, which will satisfy the conditions (1)$a_{ii} = 0$; (2)$a_{ij} = a_{ji} = 1$; if $(i, j) \in \mathcal{E}$; (3)$a_{ij} = 0$; if $(i, j) \notin \mathcal{E}$.

In this paper, the system that consists of N agents is considered. The dynamics of each agent can be expressed by a single integrator as follows:

$$\dot{x}_i(t) = u_i(t) \quad i = 1, 2, \cdots, N. \tag{1}$$

where $x_i(t) \in R^m$ represents the state vector of agent i, and $u_i(t) \in R^m$ is the control input acting on agent i. Let f_i :$R^m \times R^+ \longrightarrow R$ is a local cost function of agent i with respect to time t, which is only known by the agent i. Our goal is to make a control law for (1) through the exchange of local information such that each agent can achieve the same optimal state $x^*(t)$ which is the optimal point of the optimization problem in this paper.

$$x^*(t) = arg\min f(x(t), t), \quad x(t) \in R^m \tag{2}$$

where f :$R^m \times R^+ \longrightarrow R$ is the global objective function and denoted by $f(x(t)) \triangleq \sum_{i=1}^{N} f_i(x(t))$. Then, the above distributed convex optimization problem be rewritten as follows

$$minimize \sum_{i=1}^{N} f_i(x_i(t)), \quad subject\ to\ x_i(t) = x_j(t). \tag{3}$$

Obviously, the problem can be converted to a synchronization problem and a minimization problem on the global objective function $f(x(t), t) = \sum_{i=1}^{N} f_i(x_i(t), t)$ in a distributed manner.

Next, some definitions and lemmas concerning strongly convex functions are presented, which will be used in this paper.

Definition 1 ([29]): Let a set $K \subseteq \mathbb{R}^m$, K is said to be convex if $ax + (1 - a)y \in K$, for all $x, y \in K$ and $0 \leq a \leq 1$. Let a set K is convex, a function $f(.)$:$K \rightarrow \mathbb{R}$ is convex if $f(ax+(1-a)y) \leq af(x)+(1-a)f(y), \forall x, y \in K, 0 \leq a \leq 1$.

Definition 2 ([30]): $f(x)$ is called m-strongly($m > 0$) convex if and only if for all $x, y \in R^n$

$$(y - x)^T(\nabla f(y) - \nabla f(x)) \geq m\|y - x\|^2, \quad x \neq y. \tag{4}$$

If the above inequality(4) is satisfied and f is twice differentiable with respect to x, then $\nabla^2 f(x) \geq mI_n$.

Definition 3: A set $D \subset R^n$ is compact if and only if D is bounded and closed.

Lemma 1 ([30]): Let a function $f(x)$: $\mathbb{R}^m \rightarrow \mathbb{R}$ is a continuously differentiable and convex, $f(x)$ is said to be minimized if and only if $\nabla f = 0$.

Lemma 2 Given a convex and compact set $D \subset R^m$, f: $R^m \rightarrow R$ is a twice continuously differentiable function. f is said to be locally strongly convex on any D if there exists a constant $\alpha > 0$ such that the following three equivalent conditions are obtained [31], [32]

$$\begin{cases} f(y)-f(x)-\nabla f(x)^T(y-x) \geq \frac{\alpha}{2}\|y-x\|^2, & \forall x,y \in D \\ (\nabla f(y)-\nabla f(x))^T(y-x) \geq \alpha\|y-x\|^2, & \forall x,y \in D \\ \nabla^2 f(x) \geq \alpha I_m, & \forall x,y \in D \end{cases} \tag{5}$$

Then, for any positive constant $\beta > 0$, the equivalent conditions are given ([30], [32])

$$\begin{cases} f(y)-f(x)-\nabla f(x)^T(y-x) \leq \frac{\beta}{2}\|y-x\|^2, & \forall x,y \in D \\ (\nabla f(y)-\nabla f(x))^T(y-x) \leq \beta\|y-x\|^2, & \forall x,y \in D \\ \nabla^2 f(x) \leq \beta I_m, & \forall x,y \in D \end{cases} \tag{6}$$

Lemma 3 (LaSalle's Invariance Principle ([33])): Consider the differential equations as follows:

$$\frac{dx}{dt} = f(x,t) \tag{7}$$

where x, f represent the state vector and the function vector of n dimension, respectively. Let $V : R^n \to R$ is a non-negative and continuously differentiable function, $D \subseteq R^n$ is a compact set, if there exists $V(x) \in C(D,R)$ such that $\frac{dV}{dt}|_{(7)} \leq 0$. Therefore, let $E = \{x|\frac{dV}{dt}|_{(7)} = 0, x \in D\}$, $M \subset E$ is the largest invariant set of E. If as $t \to \infty$, each solution starting from D will asymptotically converge to M. In particular, if $M = \{0\}$, then the zero solution of the equations (7) is asymptotically stable.

To facilitate the later analyses, the following assumptions are necessary.

Assumption 1: The communication topology among agents regard as undirected graph G which is connected for all time.

Assumption 2: The each local cost function $f_i(x_i(t))$ is twice continuously differentiable and strongly convex.

Problem 1: For the system (1), design the controller $u_i(t)$ of agent i, such that $x_i(t)$ converges to x^*, and $x_i(t) = x_j(t)$, $(i,j = 1,2,\ldots N)$, where x^* is the optimal which minimizes the global objective function.

III. DISTRIBUTED OPTIMIZATION ALGORITHM WITH TIME-DELAY

In this section, to solve the problem in this article, we propose a new controller to find an optimal value that minimizes the sum of the local function. The new distributed controller is shown as follows:

$$u_i(t) = -2[\nabla^2 f_i(x_i(t))]^{-1}\sum_{j\in N_i} a_{ij}(x_i(t)-x_j(t-\tau)) \tag{8}$$

$$x_i(0) = x_i^*, \quad i = 1,2,\ldots,N. \tag{9}$$

where $x_i(t)$, $(i = 1,2,\ldots N)$ and $u_i(t) \in R^m$ represent the state vector and the control input vector of agent i, respectively. τ is a positive constant and represents the communication delays between agents, $x_j(t-\tau)$ $(j = 1,2,\ldots N)$ represents the parameter estimate of agent i, a_{ij} represents the connection weight relates to the undirected graph G, $[\nabla^2 f_i(x_i(t))]^{-1}$ is the inverse of the Hessian matrix of f_i on $x_i(t)$. x_i^* is the initial position of the agent $i(i = 1,2,\ldots,N)$ and satisfies $\sum_{i=1}^N \nabla f_i(x_i^*) = 0$, implying that x_i^*, $(i = 1,2,\ldots,N)$ is the local optimal values. Meanwhile, we will give the results of this article as follows.

Theorem 1: For the system (1) with the control algorithm (8), all agents asymptotically converge to the same position, which is an optimal solution of the problem in this paper.

Proof 1: Let the positive semi-definite Lyapunov candidate function of the system (1) be defined as

$$V(t) = \sum_{i=1}^N (f_i(x^*) - f_i(x_i(t)) - \nabla f_i(x_i(t))^T(x^*-x_i(t))) + \sum_{i=1}^N\sum_{j=1}^N a_{ij}\int_{t-\tau}^t (x_i(s)-x^*)^T(x_i(s)-x^*)ds. \tag{10}$$

The derivative with respect to t along the system (1) is

$$\begin{aligned}\dot{V}(t) &= -\sum_{i=1}^N \dot{x}_i(t)^T(\nabla^2 f_i(x_i(t)))^T(x^*-x_i(t)) \\ &\quad + \sum_{i=1}^N\sum_{j=1}^N a_{ij}\|x_i(t)-x^*\|^2 \\ &\quad - \sum_{i=1}^N\sum_{j=1}^N a_{ij}\|x_i(t-\tau)-x^*\|^2 \\ &= 2\sum_{i=1}^N\sum_{j=1}^N a_{ij}(x_i(t)-x_j(t-\tau))^T(x^*-x_i(t)) \\ &\quad + \sum_{i=1}^N\sum_{j=1}^N a_{ij}\|x_i(t)-x^*\|^2 \\ &\quad - \sum_{i=1}^N\sum_{j=1}^N a_{ij}\|x_i(t-\tau)-x^*\|^2 \\ &= -2\sum_{i=1}^N\sum_{j=1}^N a_{ij}(x_i(t)-x^*)^T(x_i(t)-x^*) \\ &\quad + 2\sum_{i=1}^N\sum_{j=1}^N a_{ij}(x_j(t-\tau)-x^*)^T(x_i(t)-x^*) \\ &\quad + \sum_{i=1}^N\sum_{j=1}^N a_{ij}\|x_i(t)-x^*\|^2 \\ &\quad - \sum_{i=1}^N\sum_{j=1}^N a_{ij}\|x_i(t-\tau)-x^*\|^2 \\ &= -2\sum_{i=1}^N\sum_{j=1}^N a_{ij}\|(x_i(t)-x^*)\|^2 \\ &\quad + 2\sum_{i=1}^N\sum_{j=1}^N a_{ij}(x_j(t-\tau)-x^*)^T(x_i(t)-x^*)\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|x_i(t)-x^*\|^2\\
&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|x_i(t-\tau)-x^*\|^2\\
=&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|(x_i(t)-x^*)\|^2\\
&+2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(x_j(t-\tau)-x^*)^T(x_i(t)-x^*)\\
&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|x_i(t-\tau)-x^*\|^2 \qquad (11)
\end{aligned}
$$

Because $a_{ij}=a_{ji}=1$(when $(i,j)\in\xi$), we have

$$
\begin{aligned}
&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|(x_i(t)-x^*)\|^2\\
&+2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(x_j(t-\tau)-x^*)^T(x_i(t)-x^*)\\
&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|x_i(t-\tau)-x^*\|^2\\
=&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|(x_i(t)-x^*)\|^2\\
&+2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(x_j(t-\tau)-x^*)^T(x_i(t)-x^*)\\
&-\sum_{j=1}^{N}\sum_{i=1}^{N}a_{ji}\|x_j(t-\tau)-x^*\|^2\\
=&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|(x_i(t)-x^*)\|^2\\
&+2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(x_j(t-\tau)-x^*)^T(x_i(t)-x^*)\\
&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|x_j(t-\tau)-x^*\|^2\\
=&-(\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|(x_i(t)-x^*)\|^2\\
&-2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(x_j(t-\tau)-x^*)^T(x_i(t)-x^*)\\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\|x_j(t-\tau)-x^*\|^2)\\
=&-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(x_i(t)-x_j(t-\tau))^T(x_i(t)-x_j(t-\tau)) \qquad (12)
\end{aligned}
$$

The equation (11) can be rewritten by

$$
\begin{aligned}
\dot{V}(t)&=-\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(x_i(t)-x_j(t-\tau))^T(x_i(t)-x_j(t-\tau))\\
&\le 0 \qquad (13)
\end{aligned}
$$

It is clear that $\dot{V}(t)\le 0$, $\forall t\ge 0$ and $V(t)\ge 0$. Thus, $V(t)$ is a non-increasing function for all time. This implies that the exist $c>0$ for all $t\ge 0$, $V(t)\le V(0)<c<\infty$. In addition, $\dot{V}(t)\le 0$ means that $\dot{V}(t)=0$ or $\dot{V}(t)<0$. On the one hand, if $\dot{V}(t)=0$, then $V(t)$ is positive constant and $x_i(t)=x_j(t-\tau)$, $(i,j=1,2,\ldots N)$, $\forall t\ge 0$, because of the fact that $x_i(0)=x_i^*$, $(i=1,2,\ldots,N)$. Thus, for all $t\ge 0$, $x_1(t)=x_2(t)=\ldots=x_N(t)=x_1^*=x_2^*=\ldots=x_N^*$. This implies x_i^* is the global optimal point which minimizes $f(x(t),t)=\sum_{i=1}^{N}f_i(x_i(t))$, which is a contradiction. Therefore, $\dot{V}(t)\ne 0$. Moreover, if $\dot{V}(t)<0$, then $V(t)$ is monotonically decreasing. $V(t)\ge\frac{\alpha}{2}\|x_i(t)-x^*\|^2\ge 0$ by Lemma 2 and $V(x^*)=0$, we have $\lim V(t)=0$, as $t\to\infty$. That is, $x_1(t)=x_2(t)=\ldots=x_N(t)=x^*$, for $t\to\infty$. Let the level set $W_1=\{x_i|V(t)\le c, c>0\}$, it is clear that the W_1 is bounded closed set, thus W_1 is compact set by Definition 3. According to Lemma 3, the each solution starting from W_1 will asymptotically converge to the set $W_2=\{x_i|\dot{V}(t)=0\}$. This occurs only when $x_1=x_2=\ldots=x_N$, which means that the all agents' state in the system (1) can reach the same state. In summary, $x_1=x_2=\ldots=x_N=x^*$, which implies that the each agent asymptotically tend to the optimal state x^*. Therefore, it can be seen that as $t\to\infty$, $f(x(t),t)=\sum_{i=1}^{N}f_i(x_i(t),t)$ will be minimized and $x_i=x_j$, for all $i,j\in\mathbb{N}$. In other words, the problems in this paper can be solved.

Remark 1: In this paper, a delay-independent distributed optimization algorithm is proposed. The results in [26]–[28] require establishing the maximum delay bound.

Remark 2: In (8), we consider the case that only the state of neighbor contains time-invariant communication delay.

IV. SIMULATION RESULTS

Example 1: In this section, we take a example to illustrate the effectiveness of the results of this article. Simulation is performed with six agents moving in a 1D plane. In simulation, a connected undirected topology is considered, the adjacency matrix $A=[a_{ij}]$ and the local objective functions $f_i(x_i(t))$ are assumed as follows [27]:

$$
A=\begin{bmatrix}
0&1&0&1&0&1\\
1&0&1&0&1&1\\
0&1&0&1&1&0\\
1&0&1&0&1&0\\
0&1&1&1&0&1\\
1&1&0&0&1&0
\end{bmatrix}
$$

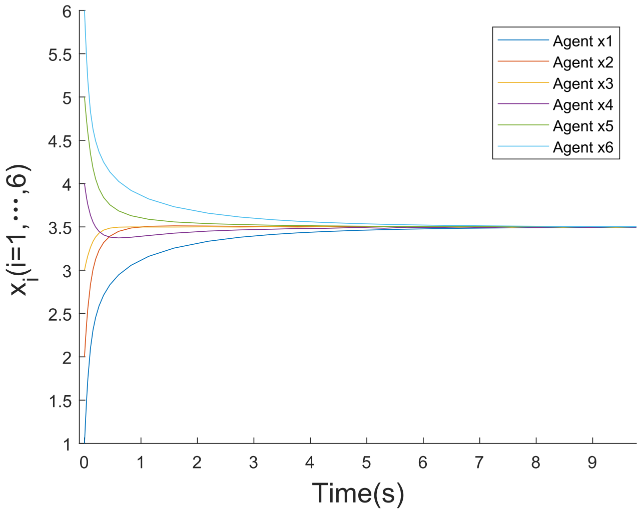

FIGURE 1. The states of 6 agents evolving.

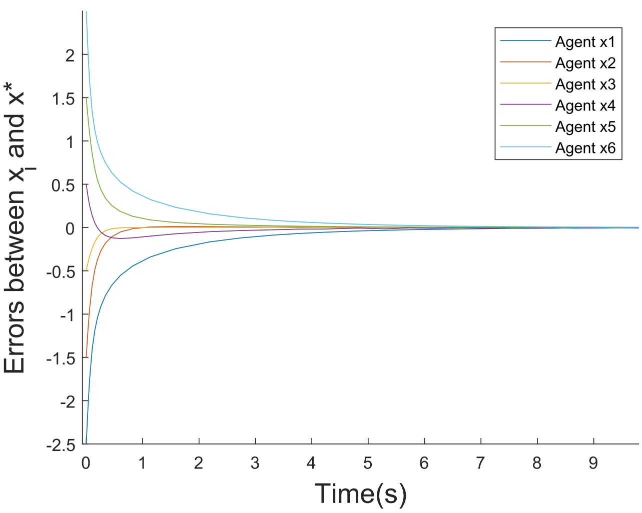

FIGURE 2. The errors between agents' and the optimal state.

$$f_i(x_i) = \frac{1}{8}(x_i - i)^6 + 6(x_i - i)^2, \quad i = 1, 2, 3, 4, 5, 6 \quad (14)$$

Our objective is to minimize the function $f(x(t), t) = \sum_{i=1}^{N} f_i(x_i(t), t)$ in distributed method. Note that $f_1, f_2, f_3, f_4, f_5, f_6$ are twice differentiable and strongly convex, i.e., we can get the Hessian of $f_1, f_2, f_3, f_4, f_5, f_6$ and unique minimizer of the global objective function $\sum_{i=1}^{6} f_i(x_i(t))$. According to Lemma 1 and by simple calculations, we have that the minimum value of the total cost function $\sum_{i=1}^{6} f_i(x_i(t))$ is 168.887 and the optimal solution is $x^* = 3.5$. The initial states are chosen as $x_1(0) = 1, x_2(0) = 2, x_3(0) = 3, x_4(0) = 4, x_5(0) = 5, x_6(0) = 6$. In the algorithm (8), choosing the time-invariant delay τ is taken as 0.2s. The numerical simulations are given in Figure. 1-2. Obviously, we can see that the state of six agents converge to the same state, and the error between the state and the optimal value converges to zero asymptotically. The simulation shows that our algorithm can achieve consistency and the global objective function to obtain the optimal. In a word, the algorithm (8) can do with the problem of (3).

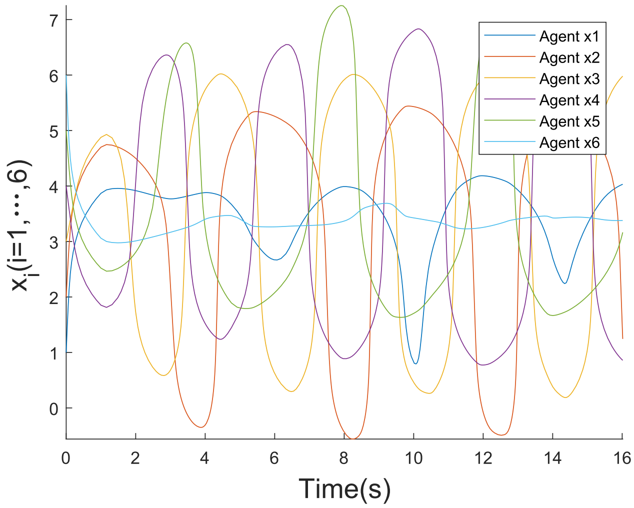

FIGURE 3. The states of 6 agents evolving by using the algorithm in [27] when $\tau = 10$s.

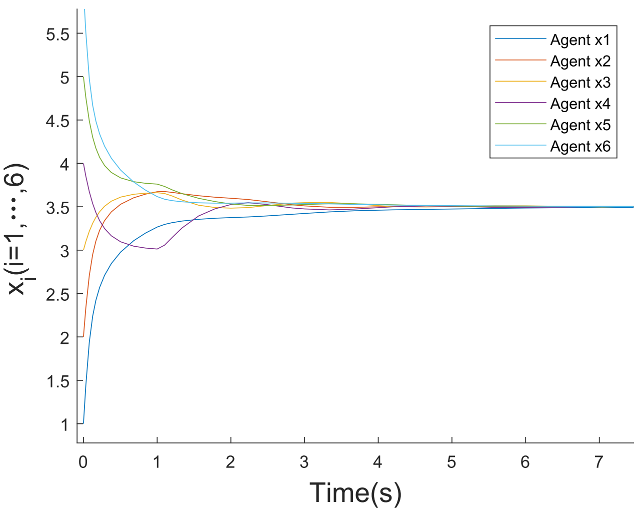

FIGURE 4. The states of 6 agents evolving by using (8) when $\tau = 10$s.

Comparisons 1: Until now, many algorithms based on the ZGS method are given for solving (3). By comparison, the innovation of our paper is that it does not require the communication delays to be bounded. To verify this point, we compare our controller with the ZGS algorithm in [27]. Let's consider a large time delay, which is chosen as $\tau = 10$s. The network topology and initial conditions are the same as those in the above example. Similar to the above calculations, we get the same results as above, such as $x^* = 3.5$, $\sum_{i=1}^{5} f_i(x_i(t)) = 168.887$. By using the algorithm in [27] and our control scheme (8), the numerical simulations are given in Figure. 3- 4. The simulation shows that our algorithm can achieve consistency and the global objective function to obtain the optimal, but the algorithm in [27] can not achieve. In summary, the above simulation results verify our theorems.

V. CONCLUSION

This paper investigate the distributed optimization problem for multi-agent system. The systems with the communication delays and first-order dynamics are considered. We propose a control protocol that rests on the local information between each agent and its neighbors. By constructing the Lyapunov-Krasovskii function approach, it is proved that all agents can track the optimal state and reach the same state. Finally, we illustrate the results by a numerical simulation. Our future works are concerned on the convergence speed and implementation complexity of the proposed algorithm, distributed optimization for multi-agent systems with time-varying communication delay and directed networks.

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