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Multi-Axis Motion Control Based on Time-Varying Norm Optimal Cross-Coupled Iterative Learning

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ABSTRACT In the process of multi-axis contour tracking control, the traditional time-invariant method could lead to a significant error in contour tracking due to the existence of two different motion conditions, namely single-axis independent motion and multi-axis coupled motion. In order to tackle this issue, a time-varying weighting matrix has been developed considering the trajectory and time-varying random disturbance. In this paper, a time-varying control method for multi-axis motion based on norm optimal cross-coupling iterative learning is proposed. Compared to the time-invariant control method, the simulation and experiment results demonstrate that the proposed method can effectively reduce the contour error improving the multi-axis control precision.

INDEX TERMS Time-varying weighted matrix, iterative learning, norm optimal, cross coupling, multi-axis motion control.

I. INTRODUCTION

In multi-axis motion control, the contour error caused by discordant movement between the axes and the tracking error caused by each single axis in the machining process are the main errors in the machining of complex parts. Uniaxial tracking and contour control performances are both important factors affecting the overall multi-axis motion control performance. In this respect, coordinating the relationship between them is extremely important for multi-axis motion performance.

Multi-axis contour tracking control is a trend topic in academic literature. Cross-coupling control (CCC) method is used to reduce contour error by selecting appropriate coupling operators to coordinate the motions between axes. Iterative learning control (ILC) is a feed-forward control method which does not rely on the detailed model of the controlled system. Previous control information is used as current input signal. Based on the effort of repeated iterations, accurate tacking performance of the desired trajectory can be achieved. First, Koren [1], [2] put forward the cross-coupling controller and variable-gain cross-coupling controller to improve the multi-axis contour tracking performance. A common approach to reduce the control error, is the application of modern control theory. For example, Ouyang *et al.* [3] and [4], Brend *et al.* [5], Ge *et al.* [6], and de Rozario *et al.* [7] proposed a cross-coupling control method based on the location domain, which, compared with the time-domain cross-coupling controller, reduced the dependence on the precision of the coupling operator. Brend *et al.* [5] proposed an adaptive framework based on the norm optimized iterative learning control (ILC) method, which can adapt to the system uncertainty through a multi-model exchange. In this context, Ge *et al.* [6] analyzed Robust Monotonic Convergence (RMC) of norm optimization iterative learning control from a frequency domain perspective. This research highlighted the optimization algorithm effectiveness by analyzing the uncertainty range of the model. In addition, a filter has been introduced to optimize the iterative learning algorithm to solve the phase distortion problem in the frequency domain. Moreover, de Rozario *et al.* [7] designed an iterative learning method for linear variable parameter (LPV) system, improving the performance and convergence speed. Aschemann *et al.* [8] proposed an improved recursive least square method-based

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iterative learning, with a new norm optimal iterative learning control algorithm, which significantly improved the tracking control accuracy when meeting the safety requirements. Yu *et al.* [9] proposed a method integrating the normal optimal iterative learning control and linear quadratic error state feedback tracking method, which achieved fast error convergence and small residual error. In terms of linear time-invariant control system, Lin *et al.* [10] proposed a new biaxial cross-coupling synchronization control strategy based on the equivalent tangent contour error estimation model. It has effectively reduced the synchronization errors caused by servo parameter mismatch and external disturbance. Ge *et al.* [11] focused on the norm-optimal iterative learning control (NO-ILC) for single-input-single-output (SISO) linear time invariant (LTI) systems. and developed a NO-ILC weighted filter design method based on infinite time domain, demonstrating the optimality of NO-ILC in terms of tradeoff balancing between robustness, convergence speed, and steady-state error. Chen *et al.* [12] proposed a methodology that extends the recently developed point-to-point ILC framework to allow automatic via-point time allocation within a given point-to-point tracking task, thereby the energy consumption has been effectively reduced. Owens and Hatonen [13] proposed a parameter optimization approach based on ILC norm optimization theory, which improved convergence rates. Rong and Cheng [14] proposed an iterative learning identification algorithm based on time-varying neural network, which improved the convergence speed of the algorithm and the identification accuracy of nonlinear time-varying system. Lim and Barton [15] proposed the Pareto iterative learning control method to discuss the optimization of multiple performance objectives. In terms of the ILC norm optimization theory, Son *et al.* [16] developed an ILC optimization algorithm based on point-to-point robust monotone convergence. It effectively solved the problem of low iteration performance caused by the uncertainty of the model. McNab and Tsao [17] proposed a time stepping linear quadratic optimal control method for contour error, tracking error and control input over a future finite horizon, which improved the contour tracking control performance. Traditional optimization methods are based on the assumption of time invariance, focusing on the single axis tracking and contour tracking control. It is noted that different track changes in the whole movement process and time-varying random disturbance would generate larger error in trajectory tracing control process.

To solve the above-mentioned problems, a time-varying weighting matrix is designed to adjust the single-axis tracking weight and multi-axis contour weight in real time. Therefore, this research proposes a multi-axis motion time-varying control method based on norm optimal cross-coupling iterative learning [18], for an effective improvement of multi-axis contour tracking and single axis tracking accuracies.

II. CONTOUR ERROR MODEL

The contour error is defined as the distance between the actual position point and the nearest point of the reference curve. The linear contour trajectory is shown in Figure 1. At a certain moment, P is the reference position, P* is the actual position, |PP*| is the tracking error, |AP*| is the contour error, and θ is the angle between the X and Y axes.

Based on the trigonometric relationship, the contour error equations are reported below:

$$\varepsilon = E_y \cos\theta - E_x \sin\theta \tag{1}$$

$$C_x = \sin\theta \tag{2}$$

$$C_y = \cos\theta \tag{3}$$

where C_x and C_y are the coupling coefficients in the cross-coupling control. Then the Eq.(1) can be written as follows:

$$\varepsilon = -C_x e_x + C_y e_y \tag{4}$$

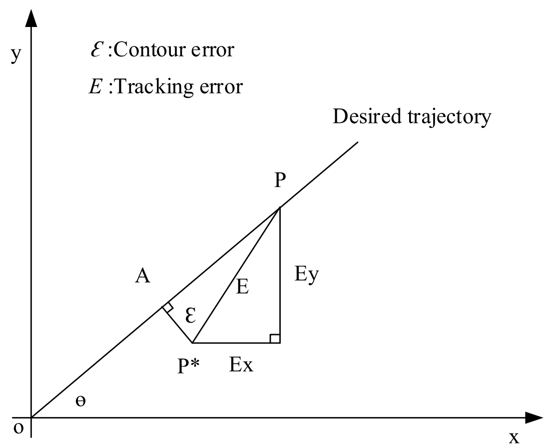

FIGURE 1. Contour error model.

III. NORM-OPTIMAL TIME-VARYING CROSS-COUPLING ITERATIVE LEARNING CONTROL

A. NORM OPTIMAL CROSS-COUPLED ITERATIVE LEARNING CONTROL

The norm optimal iterative learning controller [19], [20] applies norm optimization and iterative learning to the multi-axis motion control. Iterative learning control can lead to an improved tracking performance in system with a repetitive motion because the previous cycle control signal $\mathrm{U_{j-1}}$ and the previous error signal $\mathrm{E_{j-1}}$ are used to form the current control signal $\mathrm{U_j}$. The block diagram of iterative learning control system is shown in Figure 2. In the figure, Xr and Yr are input signals of X and Y axes, and Xj and Yj are output signals of X and Y axes

The norm optimal iterative learning controller is derived from the quadratic optimal problem. The quadratic performance objective function to evaluate the expected trajectory can be expressed as per Eq. (5):

$$\begin{aligned} J &= \|e_{j+1}\|_Q^2 + \|u_{j+1}\|_S^2 + \|u_{j+1} - u_j\|_R^2 \\ &= e_{j+1}^T Q e_{j+1} + u_{j+1}^T S u_{j+1} + (u_{j+1} - u_j)^T R (u_{j+1} - u_j) \end{aligned} \tag{5}$$

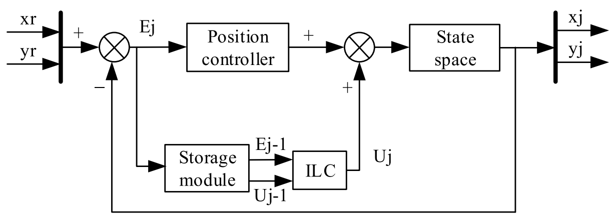

FIGURE 2. Iterative learning control block diagram.

where, $\|e_{j+1}\|_Q^2$ is the norm of error signal, $\|u_{j+1}\|_S^2$ is the norm of control signal, and $\|u_{j+1}-u_j\|_R^2$ is the norm of control signal change rate. (Q, S, R) are respectively the error signal, the weight matrix of the control signal and the control signal change rate. They are all positive definite symmetric matrices whose general form is.

$$(Q, S, R) \triangleq (qI, sI, rI) \tag{6}$$

The norm iterative learning control is usually expressed as:

$$u_{j+1} = L_u u_j + L_e e_j \tag{7}$$

where, L_u and L_e is the optimal learning gain matrix. P is the system matrix.

$$L_u = \left(P^T QP + S + R\right)^{-1} \left(P^T QP + R\right) \tag{8}$$

$$L_e = \left(P^T QP + S + R\right)^{-1} P^T Q \tag{9}$$

B. NORM-OPTIMAL TIME-VARYING CROSS-COUPLING ITERATIVE LEARNING CONTROL

The norm optimization iterative learning control is based on state space in a discrete form. In order to minimize the large error of contour trajectory caused by trajectory and external disturbance, the concept of weight matrix is introduced. The design of the weight matrix aims at combining the optimal iterative learning control with the contour error calculation model for the dynamic adjustment of uniaxial tracking and multi-axial coupling between multi-axis motions.

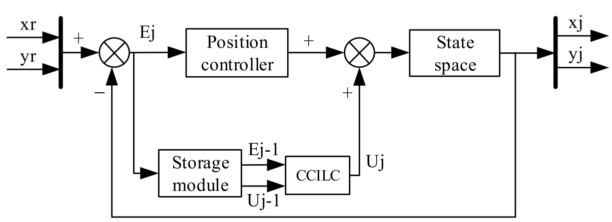

FIGURE 3. Block diagram of norm cross-coupling time-varying iterative learning control.

In order to design a control method that meets the norm optimal iterative learning control requirements, the quadratic performance objective function J is introduced to evaluate the tracking of multi-axis contour trajectory, and its expression is shown in Eq. (10):

$$J = e_{j+1}^T Q_e^{tv} e_{j+1} + u_{j+1}^T S^{tv} u_{j+1} + \Delta u_{j+1}^T R^{tv} \Delta u_{j+1} \tag{10}$$

where, $\left(Q^{tv}, S^{tv}, R^{tv}\right)$ are respectively the time-varying weight matrix of contour error signal, control signal and control signal change rate respectively. They are all symmetric positive definite matrices, and their general form is reported in Eqs. (11-13):

$$Q^{tv} = \sum_Q \cdot \left[\Gamma 1_Q + \Gamma 2_Q \cdot C_Q^T C_Q\right] \tag{11}$$

$$S^{tv} = \sum_S \cdot \left[\Gamma 1_S + \Gamma 2_S \cdot C_S^T C_S\right] \tag{12}$$

$$R^{tv} = \sum_R \cdot \left[\Gamma 1_R + \Gamma 2_R \cdot C_R^T C_R\right] \tag{13}$$

C(·) matrix represents the coupling relationship between signals in MIMO system. C_Q is the coupling coefficient matrix defining contour error in multi-axis system; C_S is the coupling coefficient matrix defining control signal in multi-axis system; and C_R is the coupling coefficient matrix defining control signal change rate in multi-axis system; $\Gamma 1_Q$ matrix and $\Gamma 2_Q$ are the weight coefficients applying to uniaxial tracking error signal and coupling error signal respectively. $\Gamma 1_S$ and $\Gamma 2_S$ are weight coefficients applied to uniaxial tracking control signal and coupling control signal respectively. $\Gamma 1_R$ and $\Gamma 2_R$ are the weight coefficients applied to the change rate of uniaxial tracking control signal and coupling control signal respectively.

According to the contour error modeling method reported in Eq. (4), the contour error can be expressed as $\varepsilon = C_y E_y - C_x E_x$, or in the matrix form as per Eq. (14):

$$\varepsilon(k) = C(k)\, e(k) \tag{14}$$

where, $C(k)$ is the coupling gain coefficient of contour error, which can be expressed as:

$$C(k) = \left[-C_x(k)\ C_y(k)\right] \tag{15}$$

Eq. (14) can be replaced into Eq. (10) yielding Eq. (16):

$$J = e_{j+1}^T \left(\Gamma 2_Q C^T C + \Gamma 1_Q I\right) e_{j+1} + u_{j+1}^T S u_{j+1} + \Delta u_{j+1}^T R \Delta u_{j+1} \tag{16}$$

where, the multi-axis coupling weight matrix is as follows:

$$C^T C = \begin{bmatrix} C^T(0)\, C(0) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C^T(N-1)\, C(N-1) \end{bmatrix} \tag{17}$$

From the expression of the contour error gain coefficient, it is possible to write:

$$C^T(k)\, C(k) = \begin{bmatrix} c_x(k)\, c_x(k) & -c_x(k)\, c_y(k) \\ -c_y(k)\, c_x(k) & c_y(k)\, c_y(k) \end{bmatrix} \tag{18}$$

Matrices $\Gamma 1_Q$ and $\Gamma 2_Q$ take the form of (19) and (20), as shown at the bottom of the next page, where they are diagonal matrices for a 2-dof system. where, $\Gamma 1_{(\cdot)} + \Gamma 2_{(\cdot)} = I$,Gain $\gamma_{(\cdot)}(k)$ determines the weight of uniaxial or contour tracking.

When $\gamma_{(\cdot)}(k) = 1$, the whole system focus on uniaxial tracking, while when $\gamma_{(\cdot)}(k) = 0$, the whole system focus on contour tracking.

Gain weight matrix $\sum_{(\cdot)}$ determines the overall weight of system error signal, control signal and control signal change variable. When the motion type is switched, the position will have a big mutation. Then the design of gain weight matrix is applied to reduce the error caused by the movement path switching. For a 2-axis system, the general form of $\sum_{(\cdot)}$ is shown in formula (21).

$$\sum_{(\cdot)} = \begin{bmatrix} \begin{bmatrix} \sigma_{(\cdot)}(1) & \\ & \sigma_{(\cdot)}(1) \end{bmatrix} & & 0 \\ & \ddots & \\ 0 & & \begin{bmatrix} \sigma_{(\cdot)}(N) & 0 \\ 0 & \sigma_{(\cdot)}(N) \end{bmatrix} \end{bmatrix} \tag{21}$$

Equation (11), (12) and (13) are substituted into the objective function, and the optimality condition $\frac{1}{2}\frac{\partial J_{j+1}}{\partial u_{j+1}} = 0$ can be used to get:

$$\left[J^T Q^{tv} J + S^{tv} + R^{tv}\right] u_{j+1} = \left[J^T Q^{tv} J + R^{tv}\right] u_j + J^T Q^{tv} e_j \tag{22}$$

Then, the optimal iterative learning formula of time-varying multiaxial coupling can be obtained from formula (22).

$$u_{j+1} = L_u^{tv} u_j + L_e^{tv} e_j \tag{23}$$

$$L_u^{tv} = \left(P^T Q^{tv} P + S^{tv} + R^{tv}\right)^{-1} \left(P^T Q^{tv} P + R^{tv}\right) \tag{24}$$

$$L_e^{tv} = \left(P^T Q^{tv} P + S^{tv} + R^{tv}\right)^{-1} P^T Q^{tv} \tag{25}$$

C. PERFORMANCE EVALUATION

Convergence is a necessary condition for iterative learning control algorithm. Given the ILC controller and the system dynamics $y_j = Pu_j$ (with $y_0 = 0, d_j = 0$), the trial domain dynamics is $u_{j+1} = (L_u - L_e P) u_j + L_e y_r$. For linear time-invariant systems, general control systems are asymptotically stable, so the convergence condition must be satisfied:

$$\|L_u - L_e P_t\|_{i2} < 1 \tag{26}$$

Based on Eq. (9) and Eq. (10), it is possible to write: $L_u - L_e P = \left(P^T Q^{tv} P + S^{tv} + R^{tv}\right)^{-1} R^{tv}$. Since $\left(Q^{tv}, S^{tv}, R^{tv}\right)$ is symmetric positive semi-definite, $P^T Q^{tv} P + S^{tv} + R^{tv}$ is positive definite, the spectral radius of $L_u - L_e P$ is less than 1. Therefore, the system convergence is achieved [21].

Robustness is an index to measure the ability of a control system to keep a performance unchanged under uncertain disturbance, and it is the key to judge whether the designed control system can be applied in practice. In general, the actual control system P_t is composed of the nominal model P and the uncertainty Δp, namely $P_t = P(I + \Delta p)$. with the multiplicative uncertainty $\Delta p = W\Delta$ and $\|\Delta\|_{i2} \leq 1$.

Therefore, the robust convergence condition of the actual control system can be expressed as:

$$\begin{aligned} &\|L_u - L_e P_t\|_{i2} < 1 \\ &\Rightarrow \max_{\Delta} \left\|(P^T Q^{tv} P + S^{tv} + R^{tv})^{-1}(R^{tv} - P^T Q^{tv} P W\Delta)\right\|_{i2} < 1 \end{aligned} \tag{27}$$

Consider (27) with $\|R\| = 0$. Then a sufficient condition for robust convergence is given by

$$\left\|\left(P^T Q^{tv} P + S^{tv}\right)^{-1} P^T Q^{tv} P W\Delta\right\|_{i2} < 1. \tag{28}$$

with $P^T Q^{tv} P + S^{tv}$ a symmetric positive definite matrix. $P^T Q^{tv} P + S^{tv} = \mathrm{X}\sum \mathrm{X}^T$ with X a unitary matrix and Σ diagonal and of full rank with diagonal elements λ_i.

Furthermore, with $\mathrm{N} \triangleq (\mathrm{X}\Sigma\mathrm{X}^T)^{-1} P^T Q^{tv} PW$ and $\|(\mathrm{X}\Sigma\mathrm{X}^T)^{-1} P^T Q^{tv} PW\|_{i2} \triangleq \gamma < 1$ we have $\|\mathrm{N}\|_{i2} = \gamma < 1$. Therefore:

$$\begin{aligned} &\max_{\Delta} \|(P^T Q^{tv} P + S^{tv} + R^{tv})^{-1}(R^{tv} - P^T Q^{tv} P W\Delta)\|_{i2} \\ &\leq \max_{\Delta} \left\|\left(\mathrm{X}\Sigma\mathrm{X}^T + rI\right)^{-1} \left(rI + \mathrm{X}\Sigma\mathrm{X}^T N\Delta\right)\right\|_{i2} \\ &\leq \max_{\Delta} \left\|(\Sigma + rI)^{-1} \left(r\mathrm{X}^T + \Sigma\mathrm{X}^T N\Delta\right)\right\|_{i2} \\ &\leq \left\|(\Sigma + rI)^{-1} (rI + \gamma\Sigma)\right\|_{i2} \\ &= \max_{i} \frac{\gamma\lambda_i + r}{\lambda_i + r} < 1, \forall r \in \mathbb{R} \geq 0 \end{aligned} \tag{29}$$

Therefore, the parameter $R^{tv} = rI$ would not affect the robust convergence of the system. The parameter S^{tv} should

$$\Gamma 1_{(\cdot)} = \begin{bmatrix} \begin{bmatrix} \gamma_{(\cdot)}(1) & \\ & \gamma_{(\cdot)}(1) \end{bmatrix} & & 0 \\ & \ddots & \\ 0 & & \begin{bmatrix} \gamma_{(\cdot)}(N) & 0 \\ 0 & \gamma_{(\cdot)}(N) \end{bmatrix} \end{bmatrix} \tag{19}$$

$$\Gamma 2_{(\cdot)} = \begin{bmatrix} \begin{bmatrix} 1-\gamma_{(\cdot)}(1) & \\ & 1-\gamma_{(\cdot)}(1) \end{bmatrix} & & 0 \\ & \ddots & \\ 0 & & \begin{bmatrix} 1-\gamma_{(\cdot)}(N) & 0 \\ 0 & 1-\gamma_{(\cdot)}(N) \end{bmatrix} \end{bmatrix} \tag{20}$$

be designed to meet the condition of robust convergence in Eq. (28). Similar statements and conlusions are also provided in the research conducted by Donkers *et al.* [22].

The steady state error e_{ss} is indicated as below:

$$e_{ss} \triangleq \lim_{j\to\infty} e_j \triangleq e_\infty \tag{30}$$

The steady state error is derived from the steady state command signal u_{ss}:

$$u_{ss} \triangleq \lim_{j\to\infty} \triangleq u_j u_\infty = (I - L_u + L_e P)^{-1} L_e y_r \tag{31}$$

With $e_\infty = y_r - Pu_\infty - d_j$, the steady state error is given by,

$$\begin{aligned} e_\infty &= [I - P(P^T Q^{tv} P + S^{tv})^{-1} P^T Q^{tv}] y_r \\ &\quad + [I + P(P^T Q^{tv} P + S^{tv} + 2R^{tv})^{-1} P^T Q^{tv}] d_j \end{aligned} \tag{32}$$

According to formula (32), the external random disturbance signal d_j will cause continuous fluctuations in the stable error of the system. In addition, increasing the value of $\|R\|_{i2}$ will reduce the influence of d_j on the steady-state error.

D. WEIGHT MATRIX DESIGN

In the norm optimal iterative learning control, the design of the weight matrix plays a significant role since it affects the convergence, robustness and control performance of the algorithms. There is no fixed theoretical method for adjusting the parameters of the weight matrix, therefore the selection needs to be determined through an iterative debugging prior to a practical application.

In order to avoid the instability of the actual system and the influence of the control performance of the algorithm, the first step is to analyze the performance of the algorithm and identify the appropriate parameter debugging method based on practical experience. This paper summarizes the design method of a set of weight matrices:

1) Design of the weight matrix Q: The weight matrix Q is related to the expected error weight. In the design of time-varying matrix Q, the coupling condition and contour error e of the system should be determined firstly, then the gain matrix $\Gamma 1_Q$ and $\Gamma 2_Q$ of single or multi axis motion should be allocated according to the contour track, then it is necessary to determine the weight of control signal error in the overall system performance. The design is completed upon the successful performance. Alternatively, repeat if the performance evaluation is not satisfied. A flow chart for the design of time-varying weight matrix Q is shown in Figure 4. In general, the single-axis tracking error decreases as Q increases.

2) Design of the weight matrix S: It is noted that the design of the parameters has a close relationship with the system model. However, the control system has difficulty in recognize its model in practices. Thus, S must be designed such that the system is robustly monotonically convergent. Start with an S yielding $\|S\|_{i2} \approx 0.01\,\|P\|_{i2}$.

3) Design of the weight matrix R: in case of an external random disturbance in the control system, the steady-state

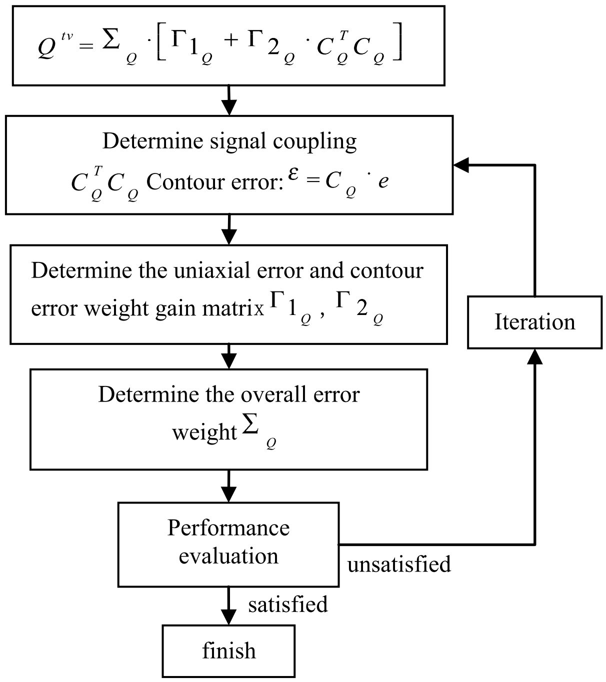

FIGURE 4. Time-varying weighting matrix design block diagram.

error of the system would fluctuate. Therefore, when adjusting the weight matrix R, it is necessary to start from R = 0 and gradually increase the value of R until the steady-state error of the system fluctuates within the expected range (or until the root mean square error RMS of the system stop increasing).

4) Repeating the process: If the performance process of the optimal iterative learning control cannot meet the design requirements, the system identification needs to be reestablished. Therefore, the theoretical model established by the system is closer to the actual system while the influence of the model uncertainty is reduced. Steps 1)-3) would be repeated until the designed weight matrix meets the desired performance requirements and converges within the required number of iterations.

Weight matrix design steps 1)-4) provide a general method to design a standard optimal learning controller.

IV. SIMULATION AND EXPERIMENT

In order to verify the feasibility of the time-variant norm iterative learning-cross coupling control method, the time-variant and time-invariant weights of arc linear trajectories are simulated. In addition, the contour errors are compared by time-variant and time-invariant trajectory simulations and experiments.

The experimental equipment used in this research includes three units in Figure 5: upper machine, under machine and motion control platform.

The upper computer integrates the hardware and software systems, including a PC and LabVIEW. It establishes a local area network communicate with the lower computer and based on TCP/IP protocol. The lower unit computer integrates

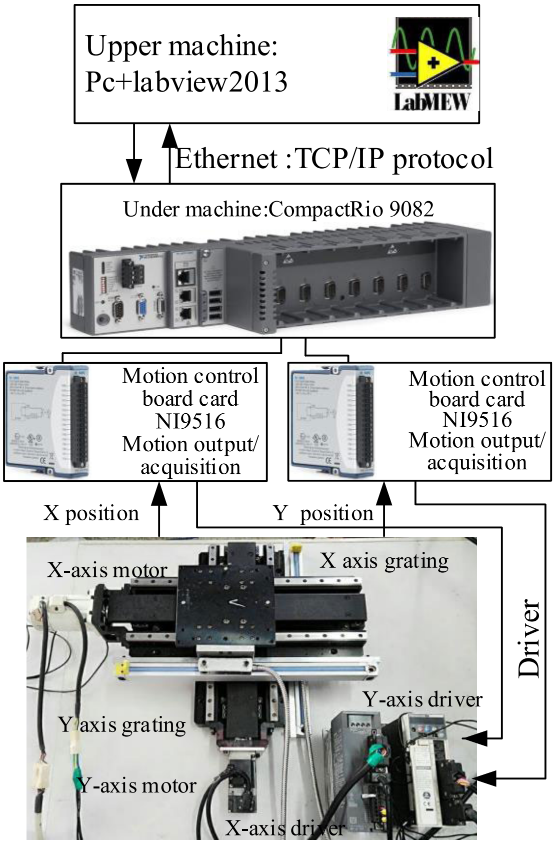

FIGURE 5. Experimental control platform.

the CompactRio 9082 rapid prototype control hardware produced by national instruments (NI) and the NI9516 motion control board card. The motion control platform is an x-y working platform composed of the servo motor and the-lead screw guide rail(5mm). Two permanent magnet synchronous motors with different parameters are used in this experimental work. Specifically, the X-axis is driven by Mitsubishi's MR-JE-10A series driver and HF-KN13J-S100 servo motor while the Y-axis is driven by Panasonic MSDA-023A1A series driver and MSMA022A1C servo motor. The position information is collected by closed grating ruler with a resolution of 5um.

A. MATLAB/SIMULINK SIMULATION RESULTS

The first step is to develop the system model and compute its state space. The model identification is performed on the controlled objects (i.e. the driver, the motor and the NI motion platform) to obtain input and output signals collected by NI9516 control board card. Combined with system identification tools in MATLAB toolbox, the state matrix of the controlled object on X axis and Y axis can be obtained as follows:

$$A_x = \begin{bmatrix} 1 & 0.00108 \\ -0.0214 & 0.8394 \end{bmatrix} \quad B_x = \begin{bmatrix} -0.00001432 \\ 0.01778 \end{bmatrix}$$

$$C_x = \begin{bmatrix} 97.72 & 0.03964 \end{bmatrix} \quad D_x = [0]$$

$$A_y = \begin{bmatrix} 1 & -0.0009441 \\ -0.01842 & -0.9989 \end{bmatrix} \quad B_y = \begin{bmatrix} 0.0009379 \\ -0.8658 \end{bmatrix}$$

$$C_y = \begin{bmatrix} 77.45 & -0.002833 \end{bmatrix} \quad D_y = [0]$$

In the simulation experiment of circular linear trajectory motion, the simulation time and the sampling period are set to be 4s and 1ms, respectively. Then the parameters are adjusted, i.e. control signal coefficient s=0.01 and control signal change coefficient r=0.02. In order to facilitate analysis, S^{TV} and R^{TV} are set to be sI and rI, respectively. In addition, the X-axis and Y-axis position controller proportional coefficients are 35 and 30, respectively.

The time-varying trajectory strategy used in this paper is shown in Figure 6. Two trajectory tracking strategies are marked in the figure, where t1-t2 and t3-t4 represent uniaxial independent tracking segments. Additionally, for the axes coordinate motions, t2-t3 and t4-t1 segments adopt the principle of contour tracking control. In order to reduce the influence of the arc and the line switching, the overall weight of the error is shown in Figure 7, where t1, t2, t3, and t4 represent the turning points of the arc and the straight lines.

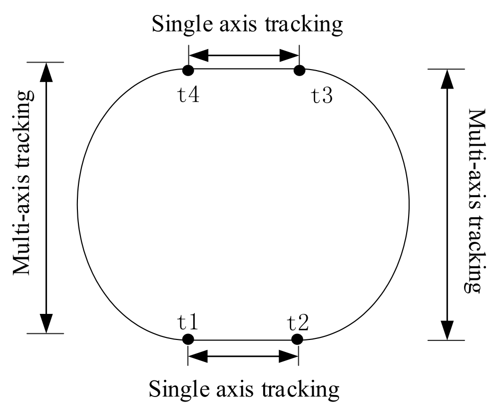

FIGURE 6. Time-varying trajectory assignment strategy.

After 21 iterations, the time-varying and time-invariant arc linear simulation motion track is shown in Figure 8. It demonstrates the contour track after the convergence of two kinds of contour errors. In Figure 9 (a, b) show partially enlarged views of the two parts indicated as a and b in Figure 8, respectively. It can be noticed that compared to the time-invariant norm optimal cross-coupling Iterative learning control, the contour trajectory after convergence in the time-variant norm optimal cross-coupling iterative learning control is closer and smoother than the command contour trajectory.

The simulation diagram of the relationship between RMS contour error and iteration times of time-varying and time-invariant control is shown in Figure 10. It shows that the state RMS contour error can be steady over 5 times iteration, which demonstrates that the control system can converge quickly. Therefore, compared with the traditional method, the time-varying cross-coupling iterative learning control

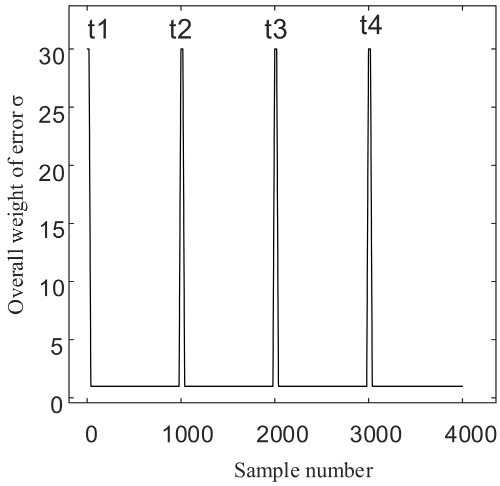

FIGURE 7. Overall weight of error σ.

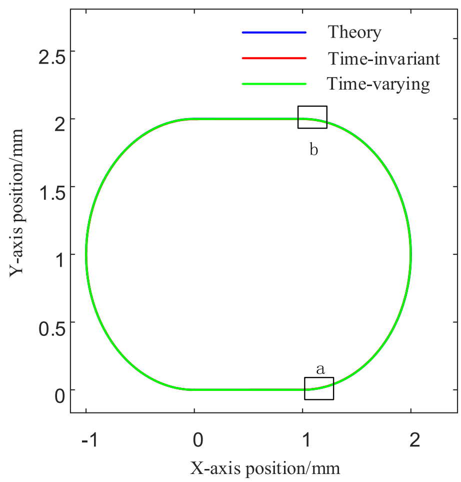

FIGURE 8. Time-varying and time-invariant arc linear motion trajectory (simulation).

method based on the optimal norm has better tracking performance.

TABLE 1. Simulation diagram of time-varying and time-invariant RMS errors (um).

Control method	X axis RMS	Y axis RMS	RMS contour error
Time-invariant	4.71	4.21	1.62
Time-varying	1.03	1.25	0.64

In the motion simulation, the error values of the two different control methods are shown in Table 1. The RMS error values of the time-invariant norm iterative learning control and the time-varying norm iterative learning control in the uniaxial tracking error and the contour error convergence are listed. According to Table 1, compared with the time-invariant control method, the X-axis error, Y-axis error and contour error of the time-varying control method were reduced by 78.01%, 70.24% and 60.49%, respectively.

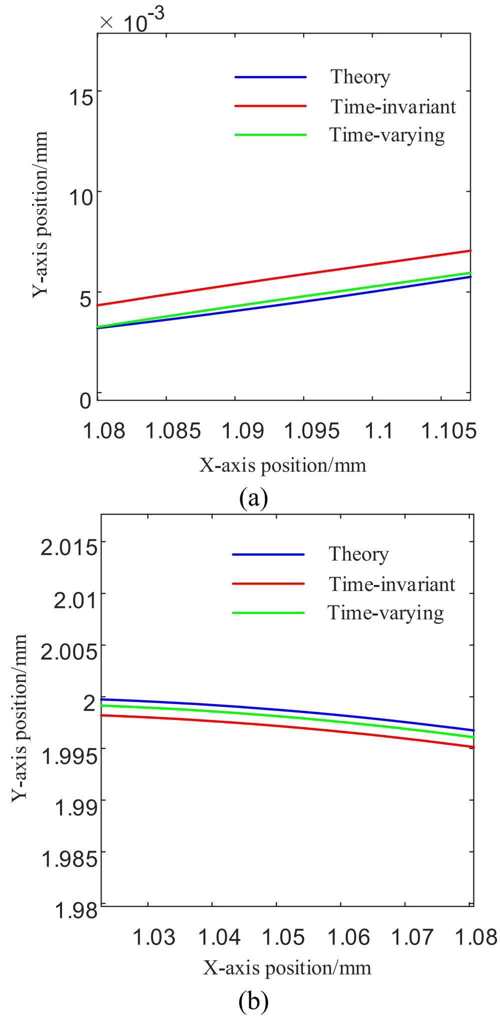

FIGURE 9. Local enlarged view of two parts of arc linear contour track a and b (simulation).

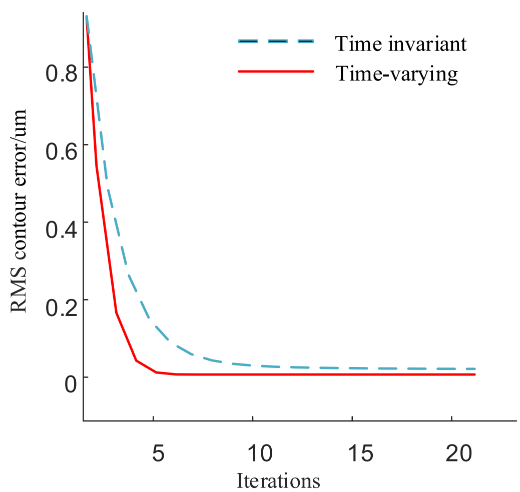

FIGURE 10. Relationship between RMS contour error and iteration number of time-varying and time-invariant control (simulation).

Compared with the time-invariant norm cross-coupling iterative learning control method, the time-varying norm cross-coupling iterative learning control method has significant advantages in improving contour control accuracy.

In order to analyze the effect of sampling period on control performance, the comparative simulation experiments were carried out whose sampling periods are set to be 10ms, 5ms and 1ms, respectively. Based on the results presented in Figure 11 and Figure 12, the smaller the sampling period is, the better tracking performance is achieved. Moreover, the benefit of reducing sampling period from 5ms to 1ms is less than reducing sampling period from 10ms to 5ms. In addition, the reduction of the sampling period increases the amount of computation, the memory sources, and the requirements on the controller. Therefore, there is a trade-off between resource consumption and tracking performance in selecting the sampling period.

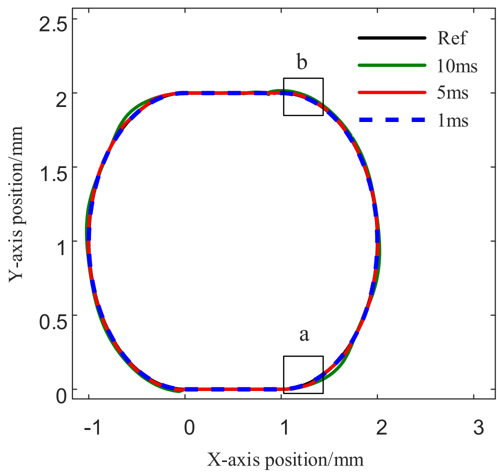

FIGURE 11. Simulation trajectory of 1ms, 5ms and 10ms(simulation).

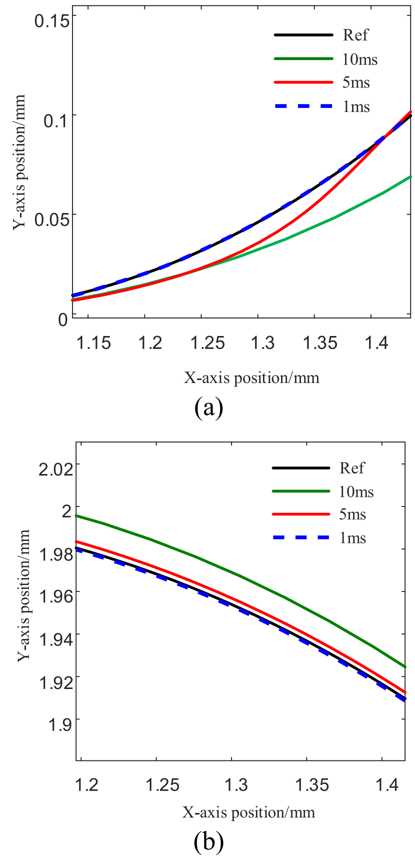

FIGURE 12. Partial enlargement of parts a and b (simulation).

B. EXPERIMENT

The parameter setting method of the arc linear experiment can refer to the parameter setting of the simulated contour trajectory. In the experiment of circular linear trajectory motion, the experiment time is set to be 16s. Due to the size limit of the controller memory, the sampling period is set to be 5ms.

Then the parameters are adjusted. The control signal coefficient and control signal change coefficient are set to be s=0.01, r=0.02. In order to facilitate analysis, the S^{TV} and R^{TV} are set to be sI and rI, respectively. The X-axis and Y-axis position controller proportional coefficients kx and ky are 30.

The motion trajectory of the time-invariant and time-invariant arc straight line experiment is shown in Figure 13. It shows the contour trajectory of the experiment after the convergence of the two kinds of contour errors by the time-invariant and time-invariant norm optimal cross-coupling iterative learning control. In Figure 14, a and b are the local enlarged pictures of part a and part b in Figure 13, respectively. Compared with the time-invariant norm optimal cross-coupled iterative learning control, the converged contour trajectory is also closer to the instruction contour trajectory.

The experimental diagram of the relationship between RMS contour error and iteration times of time-varying and time-invariant control is shown in Figure 14. After iteration, the RMS contour error of time-varying control method is stable at about 13μm, and the RMS contour error of time-invariant control method is stable at about 5μm. After 4 times iteration, the system converges to a steady state value, and the RMS contour error of the time-varying control method is significantly less than that of the time-invariant control method.

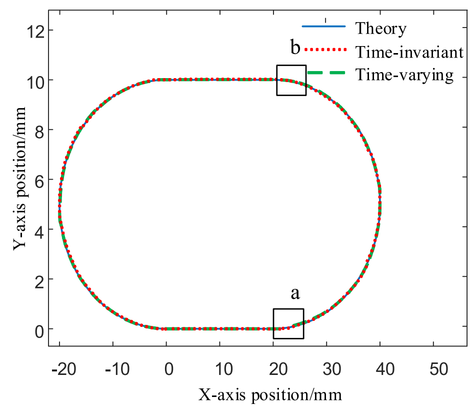

FIGURE 13. Time-varying and time-invariant arc linear motion trajectory (experiment).

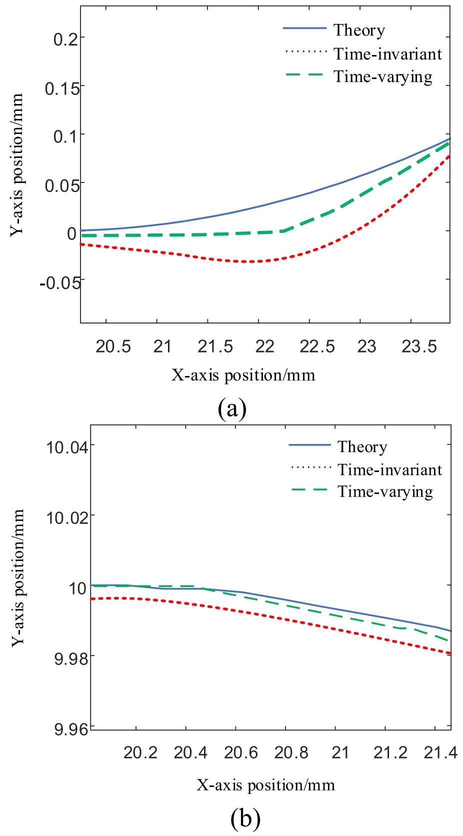

FIGURE 14. Local enlarged view of two parts of arc linear contour track a and b (experiment).

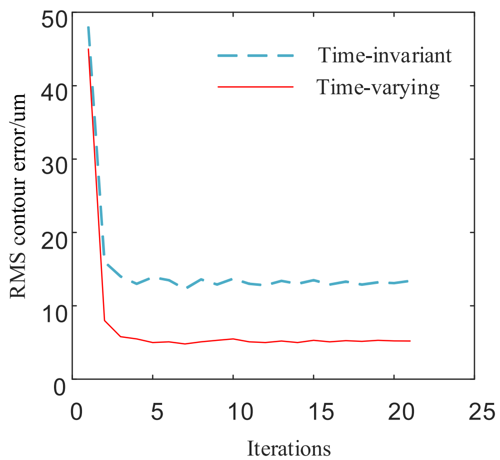

FIGURE 15. Relationship between RMS contour error and iteration number of time-varying and time-invariant control (experiment).

TABLE 2. The time-varying and time-invariant rms error table with a control period of 5ms (um).

Control method	X axis RMS	Y axis RMS	RMS contour error
Time-invariant	34.63	31.67	13.18
Time-varying	16.34	13.25	5.21

In addition, the track tracking performance of time-varying control method is significantly improved (see Figure 14 and Figure15). In the trajectory movement experiment, the error values of the two different control methods are shown in Table 2. Compared with the time-invariant control method, the X-axis error, Y-axis error and contour error of the time-varying control method are reduced by 52.82%, 58.16% and 60.47%, respectively.

TABLE 3. The time-varying rms error table with control cycles of 10ms and 5ms (um).

Control period	X axis RMS	X axis RMS	RMS contour error
10ms	32.75	20.68	9.33
5ms	16.34	13.25	5.21

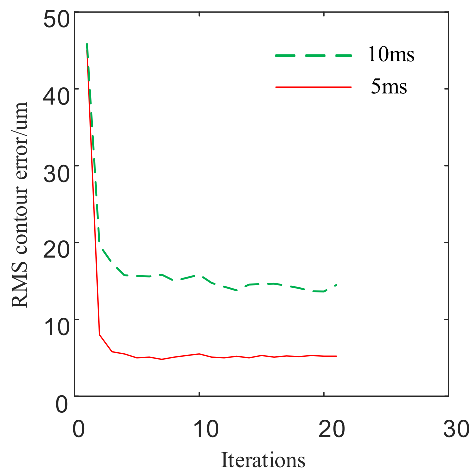

FIGURE 16. Comparison of time-varying RMS errors with control periods of 10ms and 5ms (experiment).

The results of time-varying algorithms under different sample periods are demonstrated in Table 3 and Figure 16. It is identified that EMS errors would become stable after 5 iterations in the sampling periods of 10ms and 5ms. In addition, compared to the results in sample period of 10ms, the errors in X-axis, Y-axis and EMS contour error in sampling period of 5ms were reduced by 54.88%, 64.72% and 49.89%, respectively.

Simulation and experimental results indicate that the time-varying norm optimal cross-coupling iterative learning control method proposed in this paper can effectively reduce the multi-axis contour tracking error and significantly improve the precision of contour control.

V. CONCLUSION

In this paper, a time-varying norm based iterative learning control method has been proposed. In addition, the convergence, stability conditions and time-varying weights of the controller are provided. Compared with the time-invariant control method, the proposed method reduces the x-axis error, y-axis error, and contour error by 78.01%, 70.24% and 60.49% respectively in simulation, and decreases by 52.82%, 58.16% and 60.47% in experiments. The results indicate that the time-varying control method proposed can effectively reduce the single-axis tracking error of the system, increase the compatibility between the axes, and significantly reduce the contour error of the moving system. The paper also provides the four-step tuning guidelines for the decision-making of the controller coefficients for specific tracking improvements.

Future research would focus on the optimization of the algorithm, i.e. reduce the complexity of the computation and lower the requirements on controller performance. In addition, the approach proposed in this research can be adjust and applied in coordinated control of multi-axis systems.

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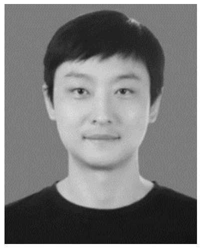

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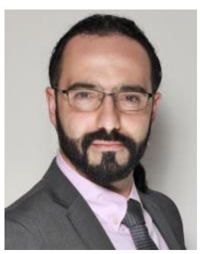

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