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Nonlinear Vibrations Analysis and Dynamic Responses of a Vertical Conveyor System Controlled by a Proportional Derivative Controller

Y. S. HAMED^{1,2}, HAMMAD ALOTAIBI¹, AND E. R. EL-ZAHAR^{3,4}

¹Department of Mathematics and Statistics, Faculty of Science, Taif University, Taif 21974, Saudi Arabia

²Department of Engineering Mathematics, Faculty of Electronic Engineering, Menoufia University, Menouf 32952, Egypt

³Department of Mathematics, College of Sciences and Humanities in Al-Kharj, Prince Sattam bin Abdulaziz University, Al-kharj 11942, Saudi Arabia

⁴Department of Basic Engineering Science, Faculty of Engineering, Menoufia University, Shebin El-Kom 32511, Egypt

Corresponding author: E. R. El-Zahar (er.elzahar@psau.edu.sa)

ABSTRACT In this paper, we introduce a nonlinear vibrations analysis and dynamic responses of a vertical conveyor system under multi excitation forces. By adding the nonlinear proportional derivative controller (NPD) to the vibrating motion of the vertical vibration conveyor, the energy was transferred between uncontrolled and controlled system. We calculate the approximate solutions of the vibrating system utilizing the method of multiple scales. In addition, we investigate the stability at worst resonance cases using phase plane technique, equations of frequency response, and averaging method. The vertical vibration conveyor behavior was studied numerically at the values of its different parameters. The results exhibit the efficiency of NPD control unit to avoid the oscillations of the vertical conveyor system. Numerical simulations have been carried out using MAPLE and MATLAB software's to ensure the fidelity of our results. Comparisons are made between analytical and numerical results. Also, findings of the present work are discussed in details and compared with published works.

INDEX TERMS Vertical shaking conveyor, vibrations, nonlinear proportional derivative control (NPD).

I. INTRODUCTION

In the field of electronics and mechanics, many researchers have examined oscillations of mechanical systems with periodic loads. Vertical conveyors are considered as efficient samples of control different parameter types for this problem. They have some features such as simple structure, energy used less, and low maintenance cost. The passive control technique is used to investigate the behavior of the system in the presence of multi types of excitation forces [1]–[3]. The dynamic behavior of the inclined cable resonance in the presence of harmonic excitation is discussed by [4]. Others have investigated the nonlinear behavior of the string beam system with multiple excitations at the case 1:1 internal resonance; they showed that there are jump phenomena in the curves of the frequency response [5]. The stability analyses and numerical response of a nonlinear-coupled pitch-roll ship system in the presences of parametric with harmonic excitation

forces were studied [6]. The straight vibrations of the vertical conveyor are checked in different stocking conditions [7]. The nonlinear analysis of the unbalanced mass of a vertical conveyor elevator, dynamic characteristics such as effects of excitations amplitude, nonlinearity, and damping are studied at primary resonance [8]. Shaking conveyers were investigated analytically and numerically applying the method of multiple scales at primary, super-harmonic and sub-harmonic resonances with cubic nonlinear spring and a vibration exciter (ideal and non-ideal). Numerical simulations showed that the important dynamic characteristics of the system and presented a periodic behavior for these conditions [9]–[11]. The mathematical study and nonlinear dynamic analysis of the vertical conveyor oscillations under different excitations were introduced [12]–[15]. The vertical conveyor analysis was investigated using unit control of positive position feedback and negative velocity feedback controls [16], [17]. The vibrations of the vertical conveyor are depressed using the nonlinear saturation controller (NSC), where the system subjected to external excitation [18]. The modified vertical

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conveyor behavior is studied at different simultaneous resonance cases using multiple scales technique [19]. For a Cartesian manipulator and micro electromechanical system, [20], [21] discussed transfer of energy, the stability, and bifurcation analyses using Poincarè maps and averaging method technique. The horizontally supported Jeffcott-rotor system oscillations was eliminated with nonlinear PD controller at primary resonance [22]. Study of nonlinear damping in large vibration amplitudes for the fractional viscoelastic standard solid model is studied analytically and experimentally [23]. The new vibratory conveyor transport possibilities, changes of direction transporting and velocity are analyzed with the angular velocity changes of the excitation vibrator [24]. The dynamics behavior, the influence of operative parameters, and the frictional properties of the cylindrical parts of the vibratory conveying are discussed [25]. The proportional derivative controller was applied to transfer the energy, and reduce large oscillations in the wind turbine system. In addition, the stability, bifurcation analysis, effect of different parameters were presented numerically in [26]. For more details about the nonlinear vibrations consequence of the geometrically nonlinear stiffness and damping one may refer to the references [27], [28]. Moreover, the analysis detailed of some dynamical systems with different forces was founded in the books [29]–[32]. This paper is organized as follows. In Section II, the nonlinear vibrations analysis and dynamic responses of a vertical conveyor system under multi excitation forces were introduced. Also, in this section we use multiple method to obtain the approximate solutions of the vibrating system. The stability of the system is analyzed by using averaging method. Section III presents numerically the stability and vertical vibration conveyor behavior at different parameters values of the system. In addition, this section shows the effects of NPD controller on system behaviors and presents a comparison between analytical and numerical results. The key results of the study are presented in Section IV, concluding and important remarks were presented.

II. MATHEMATICAL ANALYSIS

The model of vertical shaking conveyor with a spring of cubic nonlinearity and linear damping are shown in Figure 1, consist of an elastically cylinder having a helical track, four equal unbalanced masses P give torsional and vertical oscillations for the cylinder, and an electric motor to transfer the directed vibrations along and around the vertical axis leading the load moving upwards along the helix. The vertical and angular (torsional) vibrations in the conveyor are due to the vertical forces components P_z and horizontal components P_x , respectively.

Here, we modify and investigate the system [13] by increasing a tuned excitation force ($f_2 (\cos \Omega_2 t) (\sin \Omega_3 t)$) and linear and nonlinear coefficients of proportional-derivative controller ($-(px_1 + d\dot{x}_1) - (\alpha_3 x_1^2 \dot{x}_1 + \alpha_4 x_1 \dot{x}_1^2)$). The vertical shaking conveyor equations are obtained by using Lagrange's equation in two coordinates $q_1 = z = x_1$ and $q_2 = \psi = x_2$ (recognized in Appendix A).

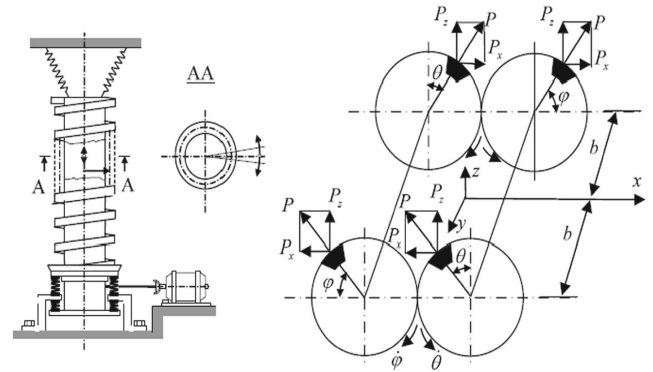


FIGURE 1. Vertical shaking conveyor system and its unbalanced masses [8], [13].

Proceeding as in Ref. [13], we modified the governing equation of the motion for vertical shaking conveyor as:

$$\begin{aligned} \ddot{x}_1 + 2\varepsilon\mu_1\dot{x}_1 + \omega_1^2x_1 + \varepsilon\alpha_1x_1^3 \\ = \varepsilon f_1 (\cos \Omega_1 t + \sin \Omega_1 t) + \varepsilon f_2 (\cos \Omega_2 t) (\sin \Omega_3 t) \\ - \varepsilon (px_1 + d\dot{x}_1) - \varepsilon (\alpha_3 x_1^2 \dot{x}_1 + \alpha_4 x_1 \dot{x}_1^2), \end{aligned} \quad (1)$$

$$\begin{aligned} \ddot{x}_2 + 2\varepsilon\mu_2\dot{x}_2 + \omega_2^2x_2 + \varepsilon\alpha_2x_2^3 \\ = \varepsilon f_3 (\cos \Omega_1 t + \sin \Omega_1 t) + \varepsilon f_4 (\cos \Omega_2 t) (\sin \Omega_3 t) \\ - \varepsilon (px_2 + d\dot{x}_2) - \varepsilon (\alpha_3 x_2^2 \dot{x}_2 + \alpha_4 x_2 \dot{x}_2^2). \end{aligned} \quad (2)$$

With the initial conditions $x_1(0) = 0.01$, $\dot{x}_1(0) = 0.01$, $x_2(0) = 0.01$ and $\dot{x}_2(0) = 0.01$, where x_1 and x_2 are the vertical and angular position of the trough of the conveyor system, ε is a small perturbation parameter, μ_1 and μ_2 are the damping coefficients of the vertical and angular springs, α_1 and α_2 are the nonlinear spring coefficients of the vertical and torsional spring, ω_1 , ω_2 , Ω_1 , Ω_2 , and Ω_3 are natural and excitation frequencies of the vertical shaking conveyor respectively, f_1, f_2, f_3 , and f_4 are excitation and tuned force amplitudes of the vertical shaking conveyor respectively, p, d, α_3 , and α_4 are linear and nonlinear coefficients of proportional derivative controller.

A. PERTURBATION ANALYSIS

Using the method of multiple scales [31], [32] to obtain the solutions for (1)–(2) in the form:

$$x_1(t; \varepsilon) = x_{10}(T_0, T_1) + \varepsilon x_{11}(T_0, T_1) + O(\varepsilon^2), \quad (3)$$

$$x_2(t; \varepsilon) = x_{20}(T_0, T_1) + \varepsilon x_{21}(T_0, T_1) + O(\varepsilon^2). \quad (4)$$

The derivatives can be written in the following:

$$\left. \begin{aligned} \frac{d}{dt} &= D_0 + \varepsilon D_1 + \dots \\ \frac{d^2}{dt^2} &= D_0^2 + 2\varepsilon D_0 D_1 + \dots \end{aligned} \right\}. \quad (5)$$

For the first-order approximation, let us introduce the two time scales and the derivatives, where $T_n = \varepsilon^n t$ and $D_n = \partial / \partial T_n$, (for $n = 0, 1$). We substitute equations (3)-5 into

(1) and (2) and equate the coefficients of equal powers of (ε) leads to

$O(\varepsilon^0)$:

$$(D_0^2 + \omega_1^2)x_{10} = 0, \tag{6a}$$

$$(D_0^2 + \omega_2^2)x_{20} = 0, \tag{6b}$$

$O(\varepsilon^1)$:

$$\begin{aligned} &(D_0^2 + \omega_1^2)x_{11} \\ &= -2D_0D_1x_{10} - 2\mu_1D_0x_{10} - \alpha_1x_{10}^3 \\ &\quad + f_1(\cos \Omega_1t + \sin \Omega_1t) + f_2(\cos \Omega_2t)(\sin \Omega_3t) \\ &\quad - px_{10} - dD_0x_{10} - \alpha_3x_{10}^2(D_0x_{10}) - \alpha_4x_{10}(D_0x_{10})^2, \end{aligned} \tag{7a}$$

$$\begin{aligned} &(D_0^2 + \omega_2^2)x_{21} \\ &= -2D_0D_1x_{20} - 2\mu_2D_0x_{20} - \alpha_2x_{20}^3 \\ &\quad + f_3(\cos \Omega_1t + \sin \Omega_1t) + f_4(\cos \Omega_2t)(\sin \Omega_3t) \\ &\quad - px_{20} - dD_0x_{20} - \alpha_3x_{20}^2(D_0x_{20}) - \alpha_4x_{20}(D_0x_{20})^2. \end{aligned} \tag{7b}$$

We have written the solutions of equation (6) in the form:

$$x_{10} = A_1 \exp(i\omega_1T_0) + \bar{A}_1 \exp(-i\omega_1T_0), \tag{8a}$$

$$x_{20} = A_2 \exp(i\omega_2T_0) + \bar{A}_2 \exp(-i\omega_2T_0). \tag{8b}$$

where A_1, A_2 are complex functions in T_1 . Inserting equation (8) into equation (7) with eliminating the secular terms $e^{\pm i\omega_1T_0}$ and $e^{\pm i\omega_2T_0}$, the solutions of equation (7) will be in the form:

$$\begin{aligned} x_{11} = &\left(\alpha_1 + i\alpha_3\omega_1 - \alpha_4\omega_1^2\right) \frac{A_1^3}{8\omega_1^2} \exp(3i\omega_1T_0) \\ &+ \left(\frac{1}{2} - \frac{i}{2}\right) \left(\frac{f_1}{(\omega_1^2 - \Omega_1^2)}\right) \exp(i\Omega_1T_0) \\ &- \left(\frac{if_2}{4(\omega_1^2 - (\Omega_2 + \Omega_3)^2)}\right) \exp(i(\Omega_2 + \Omega_3)T_0) \\ &+ \left(\frac{if_2}{4(\omega_1^2 - (\Omega_2 - \Omega_3)^2)}\right) \exp(i(\Omega_2 - \Omega_3)T_0) + cc, \end{aligned} \tag{9a}$$

$$\begin{aligned} x_{21} = &\left(\alpha_2 + i\alpha_3\omega_2 - \alpha_4\omega_2^2\right) \frac{A_2^3}{8\omega_2^2} \exp(3i\omega_2T_0) \\ &+ \left(\frac{1}{2} - \frac{i}{2}\right) \left(\frac{f_3}{(\omega_2^2 - \Omega_1^2)}\right) \exp(i\Omega_1T_0) \\ &- \left(\frac{if_4}{4(\omega_2^2 - (\Omega_2 + \Omega_3)^2)}\right) \exp(i(\Omega_2 + \Omega_3)T_0) \\ &+ \left(\frac{if_4}{4(\omega_2^2 - (\Omega_2 - \Omega_3)^2)}\right) \exp(i(\Omega_2 - \Omega_3)T_0) + cc. \end{aligned} \tag{9b}$$

where cc are complex conjugates. The resonance cases are classified into:

(A) Primary resonance: $\Omega_1 \cong \omega_1, \Omega_1 \cong \omega_2$.

(B) Sub-harmonic resonance: $\Omega_1 \cong 3\omega_1, \Omega_1 \cong 3\omega_2$.

(C) Super-harmonic resonance: $\Omega_1 \cong \frac{\omega_1}{3}, \Omega_1 \cong \frac{\omega_2}{3}$.

(D) Combined resonance: $(\Omega_2 \pm \Omega_3) \cong \omega_1, (\Omega_2 \pm \Omega_3) \cong \omega_2$.

(E) Simultaneous resonance:

we consider any combination of the above resonance cases as simultaneous resonance.

B. THE AVERAGING METHOD

Utilizing the averaging method [31], [32] for equations (1) and (2) to get the frequency response equations, when $\varepsilon = 0$, the equations (1) and (2) are written as:

$$\ddot{x}_1 + \omega_1^2x_1 = 0, \tag{10a}$$

$$\ddot{x}_2 + \omega_2^2x_2 = 0. \tag{10b}$$

The equations (10a) and (10b) have solutions in the form:

$$x_1 = a_1 \cos(\omega_1t + \varphi_1), \tag{11a}$$

$$x_2 = a_2 \cos(\omega_2t + \varphi_2). \tag{11b}$$

where a_i, φ_i and ω_i (for $i = 1, 2$) are constants. From equation (10), we get:

$$\dot{x}_1 = -\omega_1a_1 \sin(\omega_1t + \varphi_1), \tag{12a}$$

$$\dot{x}_2 = -\omega_2a_2 \sin(\omega_2t + \varphi_2). \tag{12b}$$

When $\varepsilon \neq 0$ is a small enough, we take a_i and φ_i , (for $i = 1, 2$) are functions in time t for equations (1) and (2) and equations (11a) and (11b) are differentiated with time t yields:

$$\begin{aligned} \dot{x}_1 = &\dot{a}_1 \cos(\omega_1t + \varphi_1) - \omega_1a_1 \sin(\omega_1t + \varphi_1) \\ &- a_1\dot{\varphi}_1 \sin(\omega_1t + \varphi_1), \end{aligned} \tag{13a}$$

$$\begin{aligned} \dot{x}_2 = &\dot{a}_2 \cos(\omega_2t + \varphi_2) - \omega_2a_2 \sin(\omega_2t + \varphi_2) \\ &- a_2\dot{\varphi}_2 \sin(\omega_2t + \varphi_2). \end{aligned} \tag{13b}$$

Comparing equations (12a), (13a) and equations (12b), (13b), we get:

$$\dot{a}_1 \cos(\omega_1t + \varphi_1) - a_1\dot{\varphi}_1 \sin(\omega_1t + \varphi_1) = 0, \tag{14a}$$

$$\dot{a}_2 \cos(\omega_2t + \varphi_2) - a_2\dot{\varphi}_2 \sin(\omega_2t + \varphi_2) = 0. \tag{14b}$$

Differentiating equations (12a) and (12b) with respect to t , we obtain:

$$\begin{aligned} \ddot{x}_1 = &-\omega_1\dot{a}_1 \sin(\omega_1t + \varphi_1) - \omega_1^2a_1 \cos(\omega_1t + \varphi_1) \\ &- \omega_1a_1\dot{\varphi}_1 \cos(\omega_1t + \varphi_1), \end{aligned} \tag{15a}$$

$$\begin{aligned} \ddot{x}_2 = &-\omega_2\dot{a}_2 \sin(\omega_2t + \varphi_2) - \omega_2^2a_2 \cos(\omega_2t + \varphi_2) \\ &- \omega_2a_2\dot{\varphi}_2 \cos(\omega_2t + \varphi_2). \end{aligned} \tag{15b}$$

Inserting equations (11), (13) and (15) into equations (1a) and (1b), we obtain:

$$-\dot{a}_1\omega_1 \sin(\omega_1t + \varphi_1) - a_1\dot{\varphi}_1\omega_1 \cos(\omega_1t + \varphi_1)$$

$$\begin{aligned}
 & -2\varepsilon\mu_1\omega_1a_1 \sin(\omega_1t + \varphi_1) + \frac{3\varepsilon\alpha_1}{4}a_1^3 \cos(\omega_1t + \varphi_1) \\
 & + \frac{3\varepsilon\alpha_1}{4}a_1^3 \cos(3\omega_1t + 3\varphi_1) - \varepsilon f_1 (\cos(\Omega_1t) + \sin(\Omega_1t)) \\
 & - \frac{\varepsilon f_2}{2} \sin((\Omega_2 + \Omega_3)t) + \frac{\varepsilon f_2}{2} \sin((\Omega_2 - \Omega_3)t) \\
 & + \varepsilon pa_1 \cos(\omega_1t + \varphi_1) \\
 & - \varepsilon da_1 \sin(\omega_1t + \varphi_1) - \frac{\varepsilon\alpha_3\omega_1}{4}a_1^3 \sin(\omega_1t + \varphi_1) \\
 & - \frac{\varepsilon\alpha_3\omega_1}{4}a_1^3 \sin(3\omega_1t + 3\varphi_1) + \frac{\varepsilon\alpha_4\omega_1^2}{4}a_1^3 \cos(\omega_1t + \varphi_1) \\
 & - \frac{\varepsilon\alpha_4\omega_1^2}{4}a_1^3 \cos(3\omega_1t + 3\varphi_1), \tag{16a}
 \end{aligned}$$

$$\begin{aligned}
 & -\dot{a}_2\omega_2 \sin(\omega_2t + \varphi_2) - a_2\dot{\varphi}_2\omega_2 \cos(\omega_2t + \varphi_2) \\
 & - 2\varepsilon\mu_2\omega_2a_2 \sin(\omega_2t + \varphi_2) + \frac{3\varepsilon\alpha_2}{4}a_2^3 \cos(\omega_2t + \varphi_2) \\
 & + \frac{3\varepsilon\alpha_2}{4}a_2^3 \cos(3\omega_2t + 3\varphi_2) - \varepsilon f_3 (\cos(\Omega_1t) + \sin(\Omega_1t)) \\
 & - \frac{\varepsilon f_4}{2} \sin((\Omega_2 + \Omega_3)t) + \frac{\varepsilon f_4}{2} \sin((\Omega_2 - \Omega_3)t) \\
 & + \varepsilon pa_2 \cos(\omega_2t + \varphi_2) - \varepsilon da_2 \sin(\omega_2t + \varphi_2) \\
 & - \frac{\varepsilon\alpha_3\omega_2}{4}a_2^3 \sin(\omega_2t + \varphi_2) \\
 & - \frac{\varepsilon\alpha_3\omega_2}{4}a_2^3 \sin(3\omega_2t + 3\varphi_2) + \frac{\varepsilon\alpha_4\omega_2^2}{4}a_2^3 \cos(\omega_2t + \varphi_2) \\
 & - \frac{\varepsilon\alpha_4\omega_2^2}{4}a_2^3 \cos(3\omega_2t + 3\varphi_2). \tag{16b}
 \end{aligned}$$

Substituting equations (14a)-(14b) into equations (16a)-(16b) and then solving them for \dot{a}_1 , \dot{a}_2 , $\dot{\varphi}_1$ and $\dot{\varphi}_2$ yields

$$\begin{aligned}
 \dot{a}_1 = & -\varepsilon\mu_1a_1 \{1 - \cos(2\omega_1t + 2\varphi_1)\} \\
 & + \frac{\varepsilon\alpha_1a_1^3}{8\omega_1} \{2 \sin(2\omega_1t + 2\varphi_1) + \sin(4\omega_1t + 4\varphi_1)\} \\
 & - \frac{\varepsilon f_1}{2\omega_1} \{\cos((\Omega_1 + \omega_1)t + \varphi_1) - \sin((\Omega_1 + \omega_1)t + \varphi_1) \\
 & - \cos((\Omega_1 - \omega_1)t - \varphi_1) + \sin((\Omega_1 - \omega_1)t - \varphi_1)\} \\
 & + \frac{\varepsilon f_2}{4\omega_1} \{\cos((\Omega_2 + \Omega_3 + \omega_1)t + \varphi_1) \\
 & - \cos((\Omega_2 + \Omega_3 - \omega_1)t - \varphi_1) \\
 & - \cos((\Omega_2 - \Omega_3 + \omega_1)t + \varphi_1) \\
 & + \cos((\Omega_2 - \Omega_3 - \omega_1)t - \varphi_1)\} \\
 & + \frac{\varepsilon pa_1}{4\omega_1} \sin(2\omega_1t + 2\varphi_1) \\
 & - \frac{\varepsilon da_1}{2} \{1 - \cos(2\omega_1t + 2\varphi_1)\} \\
 & - \frac{\varepsilon\alpha_3a_1^3}{8} \{1 - \cos(4\omega_1t + 4\varphi_1)\} - \frac{\varepsilon\alpha_4\omega_1a_1^3}{8} \\
 & \times \{\sin(4\omega_1t + 4\varphi_1) - \sin(2\omega_1t + 2\varphi_1)\}, \tag{17a}
 \end{aligned}$$

$$\begin{aligned}
 a_1\dot{\varphi}_1 = & -\varepsilon\mu_1a_1 \sin(2\omega_1t + 2\varphi_1) \\
 & + \frac{\varepsilon\alpha_1a_1^3}{8\omega_1} \{3 + 4 \sin(2\omega_1t + 2\varphi_1) + \sin(4\omega_1t + 4\varphi_1)\} \\
 & - \frac{\varepsilon f_1}{2\omega_1} \{\cos((\Omega_1 + \omega_1)t + \varphi_1) + \sin((\Omega_1 + \omega_1)t + \varphi_1) \\
 & + \cos((\Omega_1 - \omega_1)t - \varphi_1) + \sin((\Omega_1 - \omega_1)t - \varphi_1)\}
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{\varepsilon f_2}{4\omega_1} \{\sin((\Omega_2 + \Omega_3 + \omega_1)t + \varphi_1) \\
 & + \sin((\Omega_2 + \Omega_3 - \omega_1)t - \varphi_1) \\
 & - \sin((\Omega_2 - \Omega_3 + \omega_1)t + \varphi_1) \\
 & - \sin((\Omega_2 - \Omega_3 - \omega_1)t - \varphi_1)\} \\
 & + \frac{\varepsilon pa_1}{4\omega_1} \{1 + \cos(2\omega_1t + 2\varphi_1)\} - \frac{\varepsilon da_1}{2} \sin(2\omega_1t + 2\varphi_1) \\
 & - \frac{\varepsilon\alpha_3a_1^3}{8} \{2 \sin(2\omega_1t + 2\varphi_1) + \sin(4\omega_1t + 4\varphi_1)\} \\
 & + \frac{\varepsilon\alpha_4\omega_1a_1^3}{8} \{1 - \cos(4\omega_1t + 4\varphi_1)\}, \tag{17b}
 \end{aligned}$$

$$\begin{aligned}
 \dot{a}_2 = & -\varepsilon\mu_2a_2 \{1 - \cos(2\omega_2t + 2\varphi_2)\} \\
 & + \frac{\varepsilon\alpha_2a_2^3}{8\omega_2} \{2 \sin(2\omega_2t + 2\varphi_2) \\
 & + \sin(4\omega_2t + 4\varphi_2)\} - \frac{\varepsilon f_3}{2\omega_2} \\
 & \times \left\{ \cos((\Omega_1 + \omega_2)t + \varphi_2) - \sin((\Omega_1 + \omega_2)t + \varphi_2) \right\} \\
 & \times \left\{ -\cos((\Omega_1 - \omega_2)t - \varphi_2) + \sin((\Omega_1 - \omega_2)t - \varphi_2) \right\} \\
 & + \frac{\varepsilon f_4}{4\omega_2} \{\cos((\Omega_2 + \Omega_3 + \omega_2)t + \varphi_2) \\
 & - \cos((\Omega_2 + \Omega_3 - \omega_2)t - \varphi_2) \\
 & - \cos((\Omega_2 - \Omega_3 + \omega_2)t + \varphi_2) \\
 & + \cos((\Omega_2 - \Omega_3 - \omega_2)t - \varphi_2)\} \\
 & + \frac{\varepsilon pa_2}{4\omega_2} \sin(2\omega_2t + 2\varphi_2) - \frac{\varepsilon da_2}{2} \{1 - \cos(2\omega_2t + 2\varphi_2)\} \\
 & - \frac{\varepsilon\alpha_3a_2^3}{8} \{1 - \cos(4\omega_2t + 4\varphi_2)\} - \frac{\varepsilon\alpha_4\omega_2a_2^3}{8} \\
 & \times \{\sin(4\omega_2t + 4\varphi_2) - \sin(2\omega_2t + 2\varphi_2)\}, \tag{18a}
 \end{aligned}$$

$$\begin{aligned}
 a_2\dot{\varphi}_2 = & -\varepsilon\mu_2a_2 \sin(2\omega_2t + 2\varphi_2) \\
 & + \frac{\varepsilon\alpha_2a_2^3}{8\omega_2} \{3 + 4 \sin(2\omega_2t + 2\varphi_2) \\
 & + \sin(4\omega_2t + 4\varphi_2)\} - \frac{\varepsilon f_3}{2\omega_2} \\
 & \times \left\{ \cos((\Omega_1 + \omega_2)t + \varphi_2) + \sin((\Omega_1 + \omega_2)t + \varphi_2) \right\} \\
 & \times \left\{ +\cos((\Omega_1 - \omega_2)t - \varphi_2) + \sin((\Omega_1 - \omega_2)t - \varphi_2) \right\} \\
 & - \frac{\varepsilon f_4}{4\omega_2} \{\sin((\Omega_2 + \Omega_3 + \omega_2)t + \varphi_2) \\
 & + \sin((\Omega_2 + \Omega_3 - \omega_2)t - \varphi_2) \\
 & - \sin((\Omega_2 - \Omega_3 + \omega_2)t + \varphi_2) \\
 & - \sin((\Omega_2 - \Omega_3 - \omega_2)t - \varphi_2)\} \\
 & + \frac{\varepsilon pa_2}{4\omega_2} \{1 + \cos(2\omega_2t + 2\varphi_2)\} - \frac{\varepsilon da_2}{2} \sin(2\omega_2t + 2\varphi_2) \\
 & - \frac{\varepsilon\alpha_3a_2^3}{8} \{2 \sin(2\omega_2t + 2\varphi_2) + \sin(4\omega_2t + 4\varphi_2)\} \\
 & + \frac{\varepsilon\alpha_4\omega_2a_2^3}{8} \{1 - \cos(4\omega_2t + 4\varphi_2)\}. \tag{18b}
 \end{aligned}$$

C. PERIODIC SOLUTIONS

This section defines two detuning parameters σ_1 and σ_2 in terms of $(\Omega_1 = \omega_1 + \varepsilon\sigma_1$ and $\Omega_2 - \Omega_3 = \omega_2 + \varepsilon\sigma_2)$ to investigate the stability of the system at the resonances

(primary $\Omega_1 \cong \omega_1$ and combined $\Omega_2 - \Omega_3 \cong \omega_2$), and the slowly varying parts and constant terms are only in equations (17)–(18), so we get:

$$\dot{a}_1 = -\mu_1 a_1 - \frac{da_1}{2} - \frac{\alpha_3 a_1^3}{8} + \frac{f_1}{2\omega_1} \sin \theta_1 - \frac{f_1}{2\omega_1} \cos \theta_1, \quad (19a)$$

$$\begin{aligned} \dot{\theta}_1 = & \sigma_1 - \frac{p}{2\omega_1} - \frac{3\alpha_1 a_1^2}{8\omega_1} - \frac{\alpha_4 \omega_1 a_1^2}{8} + \frac{f_1}{2\omega_1 a_1} \sin \theta_1 \\ & + \frac{f_1}{2\omega_1 a_1} \cos \theta_1, \end{aligned} \quad (19b)$$

$$\dot{a}_2 = -\mu_2 a_2 - \frac{da_2}{2} - \frac{\alpha_3 a_2^3}{8} + \frac{f_4}{4\omega_2} \cos \theta_2, \quad (20a)$$

$$\dot{\theta}_2 = \sigma_2 - \frac{p}{2\omega_2} - \frac{3\alpha_1 a_2^2}{8\omega_2} - \frac{\alpha_4 \omega_2 a_2^2}{8} - \frac{f_4}{2\omega_2 a_2} \sin \theta_2. \quad (20b)$$

where $\theta_1 = \sigma_1 T_1 - \varphi_1$ and $\theta_2 = \sigma_2 T_1 - \varphi_2$

D. STABILITY ANALYSES AND EQUILIBRIUM SOLUTIONS

The steady-state solution occurs when, $\dot{a}_1 = \dot{a}_2 = \dot{\theta}_1 = \dot{\theta}_2 = 0$ and, the steady state solutions are given by:

$$\mu_1 a_1 + \frac{da_1}{2} + \frac{\alpha_3 a_1^3}{8} = \frac{f_1}{2\omega_1} \sin \theta_1 - \frac{f_1}{2\omega_1} \cos \theta_1, \quad (21a)$$

$$\begin{aligned} \sigma_1 - \frac{p}{2\omega_1} - \frac{3\alpha_1 a_1^2}{8\omega_1} - \frac{\alpha_4 \omega_1 a_1^2}{8} \\ = -\frac{f_1}{2\omega_1 a_1} \sin \theta_1 - \frac{f_1}{2\omega_1 a_1} \cos \theta_1, \end{aligned} \quad (21b)$$

$$\mu_2 a_2 + \frac{da_2}{2} + \frac{\alpha_3 a_2^3}{8} = \frac{f_4}{4\omega_2} \cos \theta_2, \quad (22a)$$

$$\sigma_2 - \frac{p}{2\omega_2} - \frac{3\alpha_1 a_2^2}{8\omega_2} - \frac{\alpha_4 \omega_2 a_2^2}{8} = \frac{f_4}{4\omega_2 a_2} \sin \theta_2. \quad (22b)$$

By solving equations (21)-(22) for the fixed points, we obtained

$$\begin{aligned} \sigma_1^2 - \left(\frac{p}{\omega_1} + \frac{3\alpha_1 a_1^2}{4\omega_1} + \frac{\alpha_4 \omega_1 a_1^2}{4} \right) \sigma_1 + \left(\mu_1^2 + \mu_1 d + \frac{p^2}{4\omega_1^2} + \frac{d^2}{4} \right. \\ \left. + \frac{9\alpha_1^2 a_1^4}{64\omega_1^2} + \frac{3\alpha_1 p a_1^2}{8\omega_1^2} + \frac{3\alpha_1 \alpha_4 a_1^4}{32} + \frac{\alpha_4 p a_1^2}{8} + \frac{\alpha_4^2 \omega_1^2 a_1^4}{64} \right. \\ \left. + \frac{\alpha_3 \mu_1 a_1^2}{4} + \frac{\alpha_3 d a_1^2}{8} + \frac{\alpha_3^2 a_1^4}{64} - \frac{f_1^2}{2a_1^2 \omega_1^2} \right) = 0, \end{aligned} \quad (23)$$

$$\begin{aligned} \sigma_2^2 - \left(\frac{p}{\omega_2} + \frac{3\alpha_2 a_2^2}{4\omega_2} + \frac{\alpha_4 \omega_2 a_2^2}{4} \right) \sigma_2 + \left(\mu_2^2 + \mu_2 d + \frac{p^2}{4\omega_2^2} + \frac{d^2}{4} \right. \\ \left. + \frac{9\alpha_2^2 a_2^4}{64\omega_2^2} + \frac{3\alpha_2 p a_2^2}{8\omega_2^2} + \frac{3\alpha_2 \alpha_4 a_2^4}{32} + \frac{\alpha_4 p a_2^2}{8} + \frac{\alpha_4^2 \omega_2^2 a_2^4}{64} \right. \\ \left. + \frac{\alpha_3 \mu_2 a_2^2}{4} + \frac{\alpha_3 d a_2^2}{8} + \frac{\alpha_3^2 a_2^4}{64} - \frac{f_4^2}{16a_2^2 \omega_2^2} \right) = 0. \end{aligned} \quad (24)$$

The stability of the nonlinear solution is analyzed by considering the following:

$$\begin{aligned} a_1 = a_{10} + a_{11}, \quad a_2 = a_{20} + a_{21}, \\ \theta_1 = \theta_{10} + \theta_{11}, \quad \theta_2 = \theta_{20} + \theta_{21} \end{aligned} \quad (25)$$

where $a_{10}, a_{20}, \theta_{10}$, and θ_{20} are the solutions of (19)–(20). Inserting (25) into (19)–(20) and linearizing equations in $a_{11}, a_{21}, \theta_{11}$ and θ_{21} , we get

$$\begin{aligned} \dot{a}_{11} = & \left(-\mu_1 - \frac{d}{2} - \frac{3\alpha_3 a_{10}^2}{8} \right) a_{11} \\ & + \left(\frac{f_1}{2\omega_1} (\cos(\theta_{10}) + \sin(\theta_{10})) \right) \theta_{11}, \end{aligned} \quad (26a)$$

$$\begin{aligned} \dot{\theta}_{11} = & \left(-\frac{3\alpha_1 a_{10}}{4\omega_1} - \frac{\omega_1 \alpha_4 a_{10}}{4} \right. \\ & \left. - \frac{f_1}{2\omega_1 a_{10}^2} (\cos(\theta_{10}) + \sin(\theta_{10})) \right) a_{11} \\ & + \left(\frac{f_1}{2\omega_1 a_{10}} (\cos(\theta_{10}) - \sin(\theta_{10})) \right) \theta_{11}. \end{aligned} \quad (26b)$$

$$\begin{aligned} \dot{a}_{21} = & \left(-\mu_2 - \frac{d}{2} - \frac{3\alpha_3 a_{20}^2}{8} \right) a_{21} - \left(\frac{f_4}{4\omega_2} \sin(\theta_{20}) \right) \theta_{21}, \end{aligned} \quad (27a)$$

$$\begin{aligned} \dot{\theta}_{21} = & \left(\frac{3\alpha_2 a_{20}}{4\omega_2} + \frac{\omega_2 \alpha_4 a_{20}}{4} - \frac{f_4}{4\omega_2 a_{20}^2} \sin(\theta_{20}) \right) a_{21} \\ & + \left(\frac{f_4}{4\omega_2 a_{20}} \cos(\theta_{20}) \right) \theta_{21}. \end{aligned} \quad (27b)$$

The equations (26)-(27) can be written in the matrix form as:

$$\begin{bmatrix} \dot{a}_{11} & \dot{\theta}_{11} & \dot{a}_{21} & \dot{\theta}_{21} \end{bmatrix}^T = [J] \begin{bmatrix} a_{11} & \theta_{11} & a_{21} & \theta_{21} \end{bmatrix}^T. \quad (28)$$

where $[J]$ is the Jacobian matrix of equations (26)-(27).

$$[J] = \begin{bmatrix} l_1 & l_2 & 0 & 0 \\ l_3 & l_4 & 0 & 0 \\ 0 & 0 & l_5 & l_6 \\ 0 & 0 & l_7 & l_8 \end{bmatrix}. \quad (29)$$

where l_j ($j = 1, 2, \dots, 8$) are constants.

Thus, to investigate the stability of the steady-state solutions, we must find the eigenvalues of the Jacobian matrix, which can be obtained from the following:

$$\begin{vmatrix} l_1 - \lambda & l_2 & 0 & 0 \\ l_3 & l_4 - \lambda & 0 & 0 \\ 0 & 0 & l_5 - \lambda & l_6 \\ 0 & 0 & l_7 & l_8 - \lambda \end{vmatrix} = 0. \quad (30)$$

Expanding this determinant, yields

$$\lambda^4 + r_1 \lambda^3 + r_2 \lambda^2 + r_3 \lambda + r_4 = 0. \quad (31)$$

where r_1, r_2, r_3 , and r_4 are constants (given in Appendix B). The solution is stable if the real parts of the eigenvalue are negative; otherwise, the solution is unstable. The sufficient conditions for all the roots of (31) to contain negative real parts, we apply the Routh–Hurwitz criterion, so that:

$$r_1 > 0, \quad r_1 r_2 - r_3 > 0, \quad r_3 (r_1 r_2 - r_3) - r_1^2 r_4 > 0, \quad r_4 > 0 \quad (32)$$

III. RESULTS AND DISCUSSIONS

To investigate the numerical results of system equations (1) and (2), the algorithm of Runge-Kutta of the fourth-order is applied. Also, we studied the stability of the vertical conveyor system with the averaging method and frequency response function, and the different parameters effects on the controlled system behavior were examined. Finally, we investigated the comparison between the analytical results with the numerical ones.

A. SYSTEM BEHAVIOR BEFORE CONTROL

The system behavior was studied numerically at the worst resonance cases by considering the following parameters:

$$\begin{aligned} \mu_1 &= 0.00825, \mu_2 = 0.01875, \alpha_1 = 0.005, \alpha_2 = 0.0083, \\ f_1 &= 0.1, f_2 = 0.005, f_3 = 0.15, f_4 = 0.003, \omega_1 = 2.25, \\ \omega_2 &= 2.292, p = 1, d = 0.05, \alpha_3 = -0.05, \alpha_4 = -0.0025. \end{aligned}$$

In figure 2, we introduce the phase plane and time histories of the two modes of vertical conveyor system before control at primary and combined condition $\Omega_1 \cong \omega_1$ and $\Omega_2 - \Omega_3 \cong \omega_2$. With this figure, the responses of the two modes of the conveyor system x_1 and x_2 are nearly about **3** and **1** respectively, and phase planes show multi-limit cycle.

B. SYSTEM BEHAVIOR AFTER CONTROL

Figure 3 simulates the time histories for the two modes of vertical conveyor system after applying the linear and nonlinear proportional-derivative controller at primary and combined resonance $\Omega_1 \cong \omega_1, \Omega_2 - \Omega_3 \cong \omega_2$. In this figure, we suppressed the output steady amplitudes from about **3**, and **1** to about **0.14**, and **0.1755**, respectively and the controller reduced the vibrations of the two modes of the controlled system by about **95.33%** and **82.45%** from its value before controllers, respectively and, the efficiency of the controllers E_a are nearly about **22** for x_1 and **6** for x_2 .

C. ENERGY TRANSFER IN THE VERTICAL CONVEYOR SYSTEM

Figures 4(a, b) show the transfer of energy between the two modes of the vertical conveyor system at primary and combined resonance case $\Omega_1 \cong \omega_1, \Omega_2 - \Omega_3 \cong \omega_2$. These figures show that the energy is transferred from uncontrolled system to the system after applying the PD controller.

D. CURVES OF FREQUENCY RESPONSE FOR THE CONTROLLED SYSTEM

This section investigates the stability zone, frequency response curves, and several parameters effect of the controlled system. The solid and dot lines refer to the stable and unstable curves respectively, but the yellow region refers to stability zone regions.

The output of the controlled system amplitudes decreased with increasing the values of damping μ_1 , control parameters α_3 , and d as shown in figures 5(a, c, e). Furthermore, for

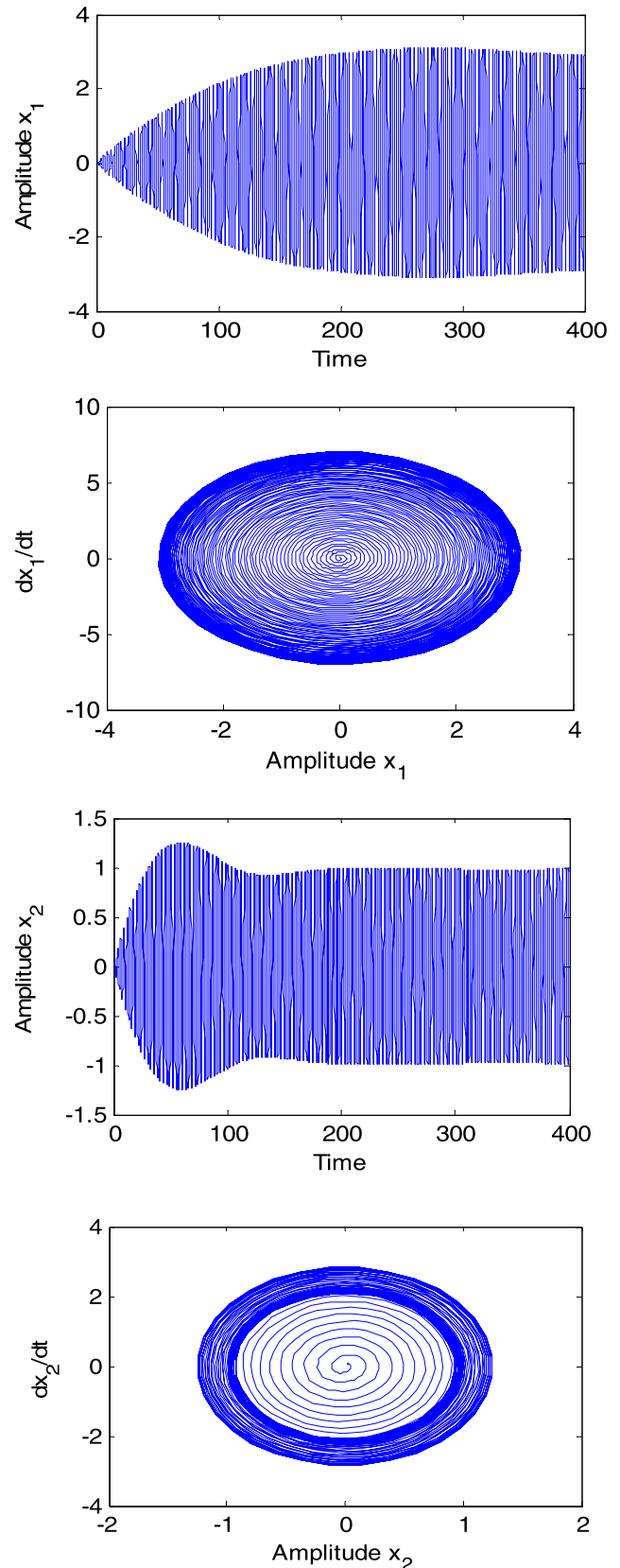


FIGURE 2. The amplitude of conveyor system and its phase plane without PD controller at primary and combined resonance case $\Omega_1 \cong \omega_1, \Omega_2 - \Omega_3 \cong \omega_2$ and $p = 0, d = 0, \alpha_3 = 0, \alpha_4 = 0$.

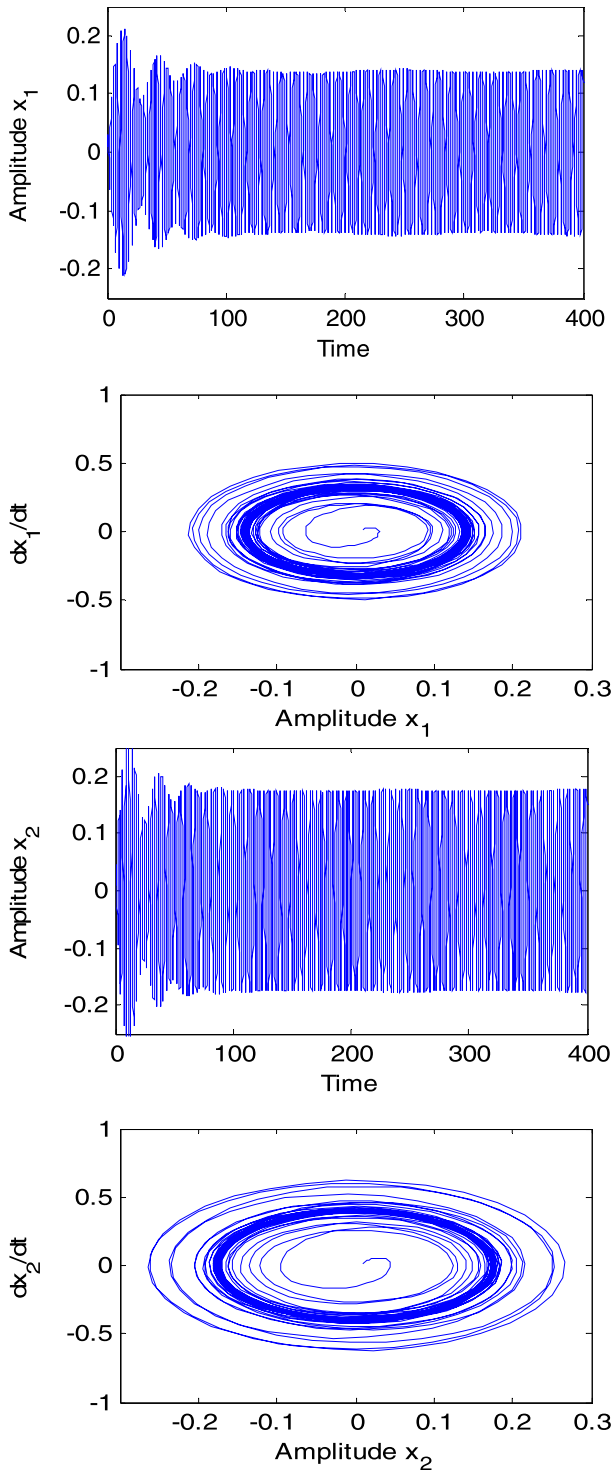


FIGURE 3. The amplitude of conveyor system and its phase plane with PD controller at primary and combined resonance case $\Omega_1 \cong \omega_1$, $\Omega_2 - \Omega_3 \cong \omega_2$ and $p = 1$, $d = 0.05$, $\alpha_3 = -0.05$, $\alpha_4 = -0.0025$.

negative and positive values of α_1 and α_4 , figures 5(b, d) depict the jump phenomena, multi solutions, soft and hard spring due to bent the curves to the right and left respectively. As shown in figure 5(f), increasing the value p shifts the curves of the controlled system to the right, which is useful in the controller performance. In addition, the controlled

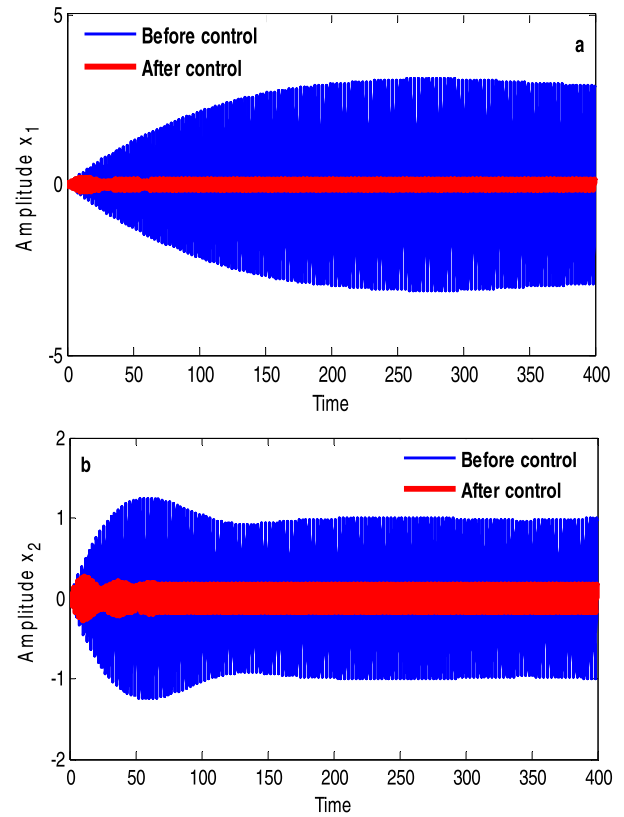


FIGURE 4. Transfer of energy between the two modes of system before and after control at primary and combined resonance case $\Omega_1 \cong \omega_1$, $\Omega_2 - \Omega_3 \cong \omega_2$.

system response increased with increasing the values of the amplitude force f_1 as shown in figures 5(g).

The controlled system behavior decreased with the increase of the damping μ_2 and the control parameter d as shown in figures 6(a, b). Also, with the increased values of the control parameter p , the curves are shifted to the right as showed in figure 6(c). Also, the controlled system amplitudes are directly proportional with increasing of the amplitude force f_4 as shown in figures 6(d).

F. COMPARISON OF ANALYTICAL AND NUMERICAL SIMULATION

In this section, we investigated the validation between the numerical simulation for the system equations (1), (2) with perturbation solution of equations (23) and (24) at different values of system parameters p, d, α_3 and α_4 at primary and combined resonance case $\Omega_1 \cong \omega_1, \Omega_2 - \Omega_3 \cong \omega_2$ as shown in figure 7. The red line indicates the solution of perturbation, while the blue line refers to numerical simulations. In these figures, we observe a good agreement between the analytical results with the numerical ones.

G. COMPARISON WITH PUBLISHED WORK

This section presents a comparison between our work and previous publish works.

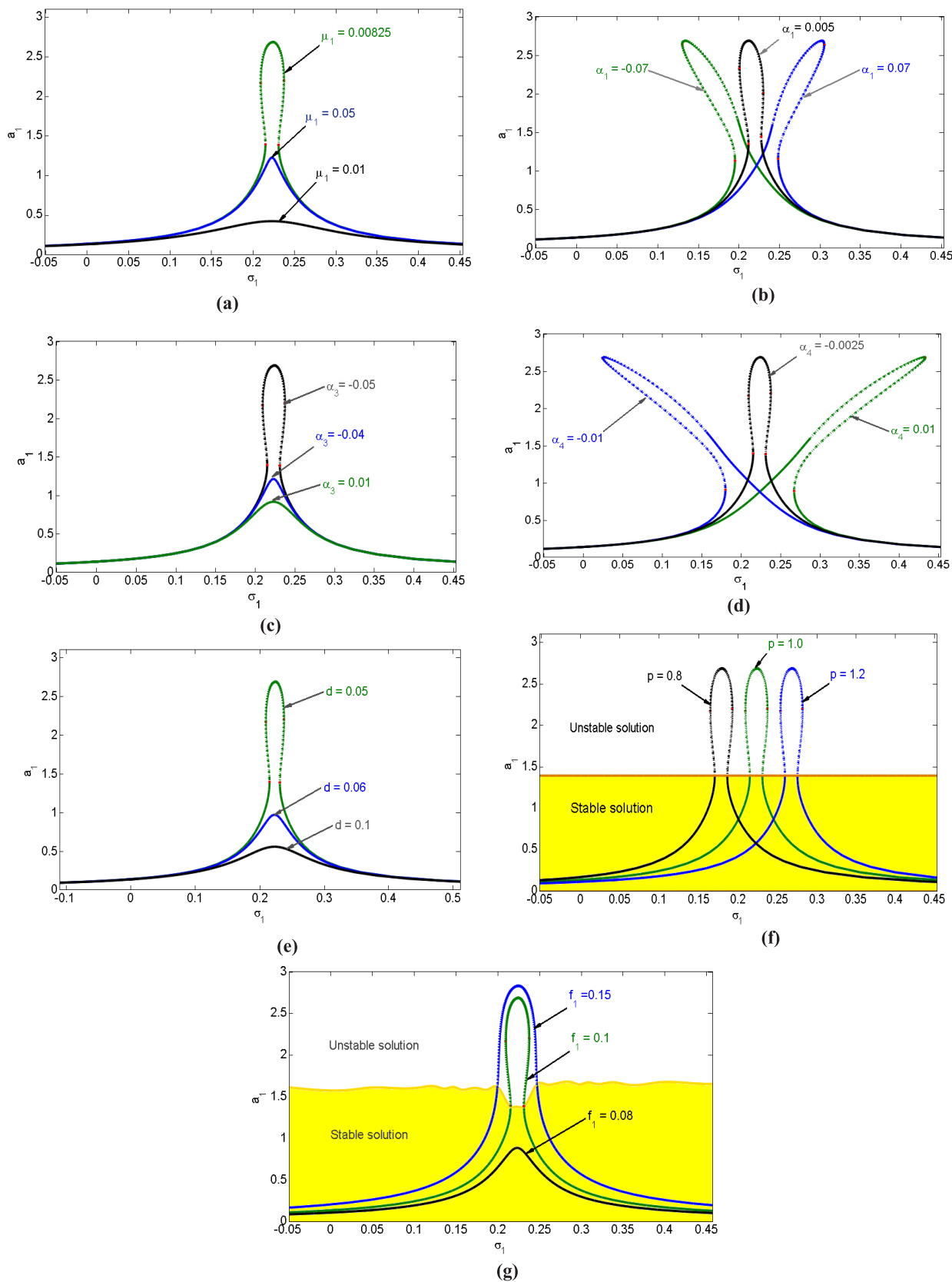


FIGURE 5. Effect of different controlled system parameters on frequency response curves at some values of (a) The damping coefficient μ_1 (b) The nonlinear parameter α_1 (c) The control parameter α_3 (d) The control parameter α_4 (e) The control parameter d (f) The control parameter p (g) The amplitude force f_1 .

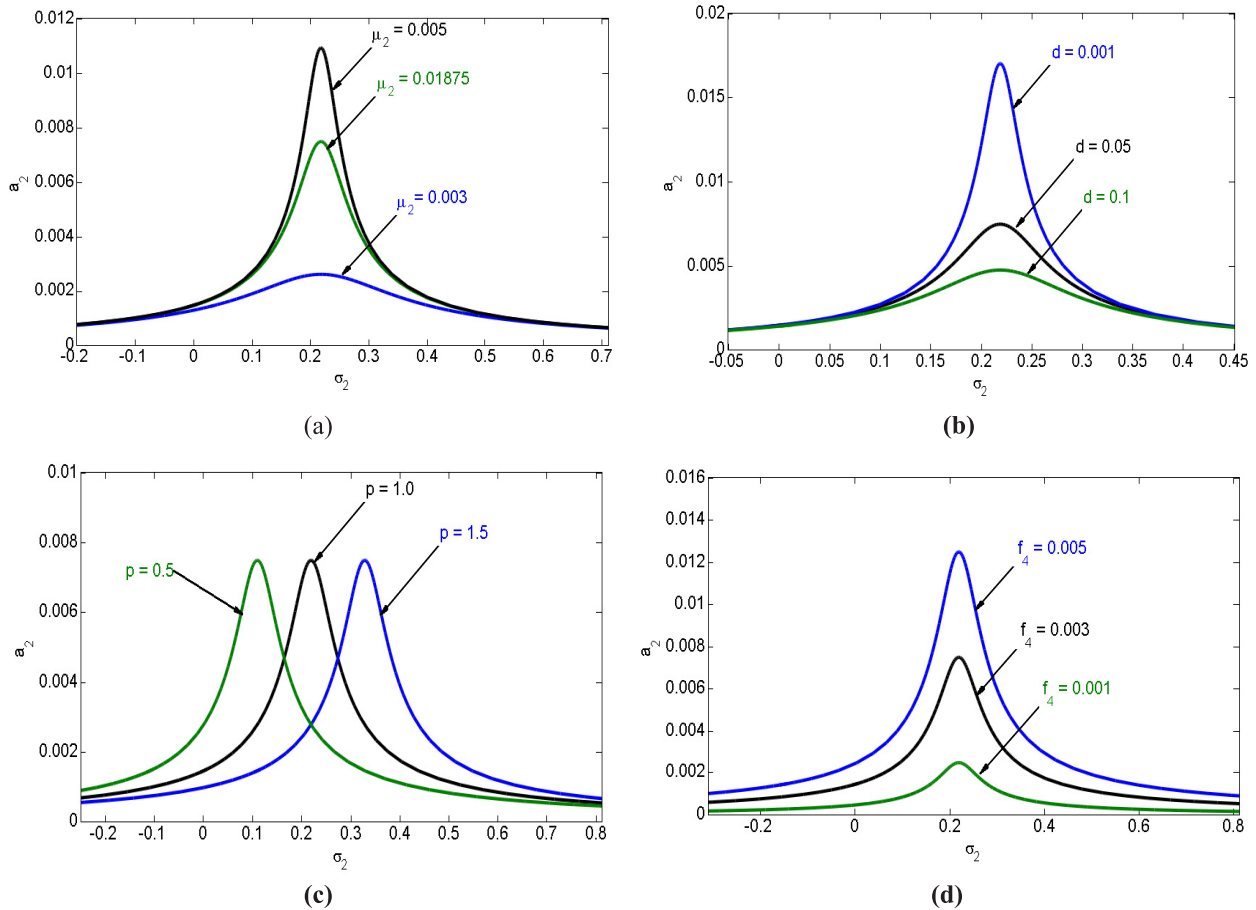


FIGURE 6. Effect of different controlled system parameters on frequency response curves at some values of (a) The damping coefficient μ_2 (b) The control parameter d (c) The control parameter p (d) The amplitude force f_4 .

- a. Bayirođlu [8] studied the vertical conveyor vibration without any controller at primary resonance and harmonic excitation force via multiple scales method.
- b. Bayirođlu [13] presented a mathematical study and nonlinear dynamic analysis of the vertical conveyor oscillations presented in Ref. [8] at primary, sub-harmonic, and super-harmonic responses.
- c. EL-Sayed, and Bauomy [16] investigated the vertical conveyor analysis for Ref. [13] with adding PPF controllers at simultaneous primary and internal resonances. The controller reduced the vibrations of both modes by about **99.88%** and **99.97%** and the efficiency of the controller E_a are about **850** and **3400**, respectively.
- d. Hamed *et al.* [17] investigated the vertical conveyor analysis for Ref. [13] with adding a unit control of negative velocity feedback and parametric excitation force at simultaneous primary and principle parametric resonances. The controller reduced the vibrations of both modes by about **99.83%** and **99.73%** and the efficiency of the controller E_a are about **600** and **400**, respectively.
- e. Amer *et al.* [18] investigated the vertical conveyor analysis for Ref. [13] with adding NSC controllers at simultaneous primary and internal resonances. The controller reduced the vibrations of both modes by about **58.33%** and **75%** and the efficiency of the controller E_a are about **2.4** and **4**, respectively.
- f. Bauomy and EL-Sayed [19] investigated the vertical conveyor analysis for Ref. [13] with adding multi parametric excitation forces at two different simultaneous sub-harmonic, and combined resonances without any control.
- g. In our work, we examined nonlinear vibrations analysis and dynamic responses of a vertical conveyor system for Ref. [13] with adding multi tuned excitation forces using the nonlinear proportional derivative (NPD) controller. Also, we investigate how the energy transfers between uncontrolled and controlled system as shown in **figure 4**. The stability is analyzed by applying the method of averaging. In the numerical results, the controller reduced the vibrations of the two modes of controlled system by about **95.33%** and **82.45%** from its value before controllers, respectively and, the

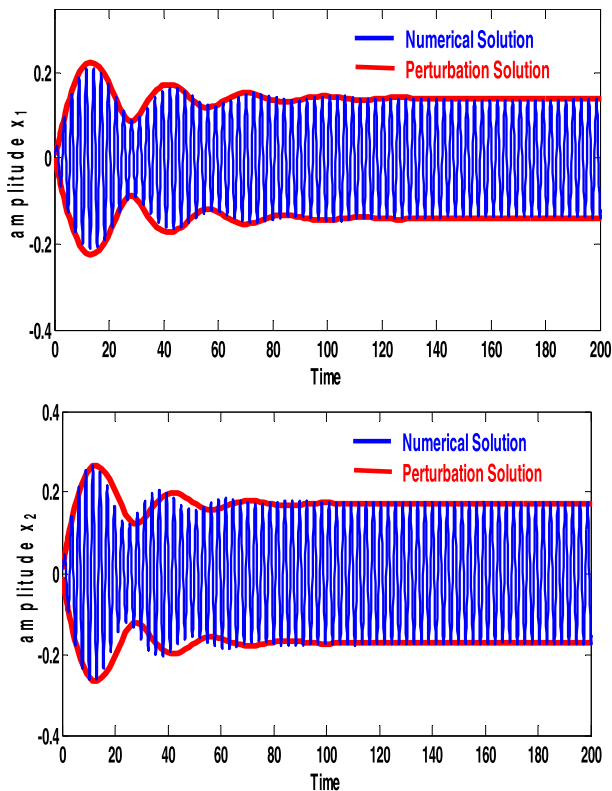


FIGURE 7. Comparison of analytical and numerical simulation of the system at $p = 1, d = 0.05, \alpha_3 = -0.05, \alpha_4 = -0.0025, \Omega_1 = \omega_1, \Omega_2 - \Omega_3 = \omega_2$.

efficiency of the controllers E_a are nearly about **22** for x_1 and **6** for x_2 as shown in **figure 3**.

IV. CONCLUSION

Vibrating conveyors are widely used in elevators, iron and steel industry, metallurgy industry, chemical plants, feedstock, small parts for processing equipment and production lines to transport a wide range of bulk materials and particles. The nonlinear vibrations analysis and dynamic responses of a vertical conveyor system under multi excitation forces were studied. The approximate solutions and resonance cases of vibrating system were calculated utilizing the method of multiple scales, the approximate solutions of higher orders are very complicated due to the nonlinearity and its degree and therefore software’s algorithms should be used. The stability, vertical vibration conveyor behavior was achieved numerically at different parameter values of the system. From the overall study, we concluded:

1. The output steady amplitudes for the two modes of controlled system are nearly about **95.33%** and **82.45%** from its value before controllers, respectively.
2. The efficiency of the controller E_a for the two modes of controlled system is about **22** and **6**.
3. The output steady amplitudes of controlled system are monotonous decreasing function in the damping μ_1, μ_2 , the controller parameters α_3, d and

monotonous increasing function in the amplitude force f_1, f_4 .

4. The Hopf bifurcations, saddle-node, and jump phenomena were appeared for varying the controller nonlinearity α_1 and α_4 .
5. For the best performance of the controller, the natural frequency ω_1 must be adjusted to the measured Ω_1 value and the natural frequency ω_2 must be adjusted to the measured $\Omega_2 - \Omega_3$ value for the two modes of controlled system respectively.
6. Increasing the values μ_1, μ_2, d , and α_3 have tightened the energy channel between the controller and the system modes specifically which is useful in the controller performance.
7. Increasing the controller value p , shifts the curves to the right which is a problem for the controller performance.
8. The analytical results are well agreement with the numerical simulations.

In future work, we can study the proposed system via multi controller such as, modified positive position feedback (MPPF), nonlinear saturation controller (NSC) and nonlinear proportional integral derivative (NPID) controller. Moreover, the suggested controlled system encourages the experimental studies to present a design algorithm, analysis, and the computational complexity of the controller design.

APPENDIX A

As in Ref. [13]:

Apply subsequently Lagrange’s equation to obtain action equations for vertical conveyor:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) + \frac{\partial D}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i. \quad (A.1)$$

where q_i and Q_i are respectively the coordinates and forces. The kinetic energy is denoted by T which is given as:

$$T = \frac{1}{2}m\dot{z}^2 + \frac{1}{2}I_z\dot{\psi}^2. \quad (A.2)$$

where m and I_z are respectively the trough mass and inertia moment of the conveyor. The potential energy (V) is given by:

$$V = \frac{1}{2}k_1z^2 + \frac{1}{4}k_2z^4 + \frac{1}{2}k_{t1}\psi^2 + \frac{1}{3}k_{t2}\psi^4. \quad (A.3)$$

where the vertical and angular positions are denoted by z and ψ respectively. The nonlinear constants of the vertical and angular springs are given by k_1, k_2, k_{t1}, k_{t2} . The Rayleigh dissipation function D is

$$D = \frac{1}{2}c\dot{z}^2 + \frac{1}{2}c_t\dot{\psi}^2. \quad (A.4)$$

where the vertical and angular springs damping constants are denoted by c and c_t , respectively. By using the Lagrange’s equation for two coordinates $q_1 = z$ and $q_2 = \psi$, we obtain the following equations of motion as:

$$m\ddot{z} + c\dot{z} + k_1z + k_2z^3 = \sum P_z, \quad (A.5)$$

$$I_z \ddot{\psi} + c_B \dot{\psi} + k_{t1} \psi + k_{t2} \psi^3 = \sum M_z. \quad (A.6)$$

where $\sum P_z$ and $\sum M_z$ are respectively the total forces and its moment of the unbalanced masses, and represented by the following equations:

$$\sum P_z = 2P(\cos \theta + \sin \varphi) = 2m_0 r_0 \Omega^2 (\cos \Omega_1 t + \sin \Omega_1 t), \quad (A.7)$$

$$\sum M_z = 2Pb(\cos \theta + \sin \varphi) = 2m_0 r_0 \Omega^2 b(\cos \Omega_1 t + \sin \Omega_1 t). \quad (A.8)$$

where the force of unbalanced masses is $P = 2m_0 r_0 \Omega^2$, and the unbalanced mass, radius and angular frequency of the unbalanced mass are respectively denoted by m_0 , r_0 , Ω . Also, $\theta = \varphi = \Omega_1$ are the constant angular velocities of unbalanced masses.

To obtain the vertical and angular movement equations of the vertical conveyor, we substitute equations (A.7) and (A.8) into equations (A.5) and (A.6), gives

$$m \ddot{z} + c \dot{z} + k_1 z + k_2 z^3 = 2m_0 r_0 \Omega^2 (\cos \Omega_1 t + \sin \Omega_1 t) \quad (A.9)$$

$$I_z \ddot{\psi} + c_B \dot{\psi} + k_{t1} \psi + k_{t2} \psi^3 = 2m_0 r_0 \Omega^2 b (\cos \Omega_1 t + \sin \Omega_1 t) \quad (A.10)$$

Dividing (A.9) by m and (A.10) by I_z , we obtain:

$$\ddot{z} + \frac{c}{m} \dot{z} + \frac{k_1}{m} z + \frac{k_2}{m} z^3 = \frac{2m_0 r_0 \Omega^2}{m} (\cos \Omega_1 t + \sin \Omega_1 t), \quad (A.11)$$

$$\ddot{\psi} + \frac{c_B}{I_z} \dot{\psi} + \frac{k_{t1}}{I_z} \psi + \frac{k_{t2}}{I_z} \psi^3 = \frac{2m_0 r_0 \Omega^2 b}{I_z} (\cos \Omega_1 t + \sin \Omega_1 t). \quad (A.12)$$

Equations (A.11) and (A.12) can be written as

$$\ddot{z} + 2\mu_1 \dot{z} + \omega_1^2 z + \alpha_1 z^3 = f_1 (\cos \Omega_1 t + \sin \Omega_1 t) \quad (A.13)$$

$$\ddot{\psi} + 2\mu_2 \dot{\psi} + \omega_2^2 \psi + \alpha_2 \psi^3 = f_3 (\cos \Omega_1 t + \sin \Omega_1 t) \quad (A.14)$$

where

$$\omega_1^2 = \frac{k_1}{m}, \quad \varepsilon = \frac{1}{m}, \quad \mu_1 = \frac{c}{2}, \quad \alpha_1 = k_2, \quad f_1 = 2m_0 r_0 \Omega^2$$

$$\omega_2^2 = \frac{k_{t1}}{I_z}, \quad \varepsilon = \frac{1}{I_z}, \quad \mu_2 = \frac{c_B}{2}, \quad \alpha_2 = k_{t2}, \quad f_3 = 2m_0 r_0 \Omega^2 b$$

The vertical shaking conveyor equations for (A.13) and (A.14) with taking $z = x_1$ and $\psi = x_2$ as:

$$\ddot{x}_1 + 2\varepsilon \mu_1 \dot{x}_1 + \omega_1^2 x_1 + \varepsilon \alpha_1 x_1^3 = \varepsilon f_1 (\cos \Omega_1 t + \sin \Omega_1 t), \quad (A.15)$$

$$\ddot{x}_2 + 2\varepsilon \mu_2 \dot{x}_2 + \omega_2^2 x_2 + \varepsilon \alpha_2 x_2^3 = \varepsilon f_3 (\cos \Omega_1 t + \sin \Omega_1 t). \quad (A.16)$$

Then we modified equations (A.15) and (A.16) into equations (1) and (2) as presented inside this manuscript.

APPENDIX B

$$l_1 = \left(-\mu_1 - \frac{d}{2} - \frac{3\alpha_3 a_{10}^2}{8} \right),$$

$$l_2 = \left(\frac{f_1}{2\omega_1} (\cos(\theta_{10}) + \sin(\theta_{10})) \right),$$

$$l_3 = \left(-\frac{3\alpha_1 a_{10}}{4\omega_1} - \frac{\omega_1 \alpha_4 a_{10}}{4} - \frac{f_1}{2\omega_1 a_{10}^2} (\cos(\theta_{10}) + \sin(\theta_{10})) \right),$$

$$l_4 = \left(\frac{f_1}{2\omega_1 a_{10}} (\cos(\theta_{10}) - \sin(\theta_{10})) \right),$$

$$l_5 = \left(-\mu_2 - \frac{d}{2} - \frac{3\alpha_3 a_{20}^2}{8} \right), \quad l_6 = - \left(\frac{f_4}{4\omega_2} \sin(\theta_{20}) \right),$$

$$l_7 = \left(\frac{3\alpha_2 a_{20}}{4\omega_2} + \frac{\omega_2 \alpha_4 a_{20}}{4} - \frac{f_4}{4\omega_2 a_{20}^2} \sin(\theta_{20}) \right),$$

$$l_8 = \left(\frac{f_4}{4\omega_2 a_{20}} \cos(\theta_{20}) \right) \theta_{21},$$

$$r_1 = (-l_1 - l_4 - l_5 - l_8),$$

$$r_2 = (-l_1 l_4 + l_1 l_5 + l_1 l_8 - l_2 l_3 + l_4 l_5 + l_4 l_8 + l_5 l_8 - l_6 l_7),$$

$$r_3 = (-l_1 l_4 l_5 - l_1 l_4 l_8 - l_1 l_5 l_8 + l_1 l_6 l_7 + l_2 l_3 l_5 + l_2 l_3 l_8 - l_4 l_5 l_8 + l_4 l_6 l_7),$$

$$r_4 = (l_1 l_4 l_5 l_8 - l_1 l_4 l_6 l_7 - l_2 l_3 l_5 l_8 + l_2 l_3 l_6 l_7).$$

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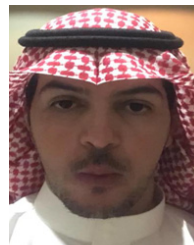
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Y. S. HAMED received the M.Sc. and Ph.D. degrees in mathematics from the Faculty of Science, Menoufia University, Egypt, in 2005 and 2009, respectively. Since 2009, he has been an Associate Professor of mathematics with the Mathematics and Statistics Department, Taif University, Saudi Arabia. He is currently an Associate Professor of engineering mathematics with the Department of Physics and Engineering Mathematics, Faculty of Electronic Engineering, Menoufia University. He supervised and examined some of M.Sc. and Ph.D. degree students. His research interests include theory of differential equations and its application, numerical Analysis, modeling, dynamical systems control, chaotic systems, renewable energy systems, and vibration control. He is an Editor of the *International Journal of Control, Automation and Systems (IJCAS)*.



HAMMAD ALOTAIBI received the Ph.D. degree in mathematics from Adelaide University, Australia, in 2017. His areas of research interests include modeling of complex multi scale dynamical systems, atomic simulation, and the numerical and analytical computational methods for solving differential equations.



E. R. EL-ZAHAR received the M.Sc. and Ph.D. degrees in engineering mathematics from Menoufia University, Egypt, in 2004 and 2008, respectively. He is currently a Full Professor of engineering mathematics with the Department of Mathematics, Prince Sattam Bin Abdulaziz University. He is the author of more than 70 scientific articles and two textbooks in refereed journals and international conferences. He has served as an editorial board member and a referee for many reputed international journals. His research interests include theory of differential equations and its application, numerical analysis, modeling, numerical, semi-analytical, and computational methods for solving differential equations and engineering systems.

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