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On Shadowing the κ-μ Fading Model

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ABSTRACT In this paper, we extensively investigate the way in which κ-μ fading channels can be impacted by shadowing. Following from this, a family of shadowed κ-μ fading models are introduced and classified according to whether the underlying κ-μ fading undergoes single or double shadowing. In total, we discuss three types of *single* shadowed κ-μ model (denoted Type I to Type III) and three types of *double* shadowed κ-μ model (denoted Type I to Type III). The taxonomy of the *single* shadowed Type I - III models is dependent upon whether the fading model assumes that the dominant component, the scattered waves, or both experience shadowing. Although the physical definition of the examined models make no predetermination of the statistics of the shadowing process, for illustrative purposes, two example cases are provided for each type of *single* shadowed model by assuming that the shadowing is influenced by either a Nakagami-*m* random variable (RV) or an inverse Nakagami-*m* RV. It is worth noting that these RVs have been shown to provide an adequate characterization of shadowing in numerous communication scenarios of practical interest. The categorization of the *double* shadowed Type I - III models is dependent upon whether a) the envelope experiences shadowing of the dominant component, which is preceded (or succeeded) by a secondary round of (multiplicative) shadowing, or b) the dominant and scattered contributions are fluctuated by two independent shadowing processes, or c) the scattered waves of the envelope are subject to shadowing, which is also preceded (or succeeded) by a secondary round of multiplicative shadowing. Similar to the *single* shadowed models, we provide two example cases for each type of *double* shadowed model by assuming that the shadowing phenomena are shaped by a Nakagami-*m* RV, an inverse Nakagami-*m* RV or their mixture. It is worth highlighting that the *double* shadowed κ-μ models offer remarkable flexibility as they include the κ-μ, η-μ, and the various types of *single* shadowed κ-μ distribution as special cases. This property renders them particularly useful for the effective characterization and modeling of the diverse composite fading conditions encountered in communication scenarios in many emerging wireless applications.

INDEX TERMS Channel modeling, generalized fading, mobile to mobile communications, shadowed κ-μ fading, shadowing.

I. INTRODUCTION

The κ-μ fading model [1]–[3] is a generalized fading model which was developed to describe envelope fluctuations that arise due to the clustering of scattered multipath waves in addition to the presence of elective dominant components. It is characterized by two physical fading parameters, κ and μ. Here, κ denotes the ratio of the total power of the dominant component to the total power of the scattered waves and μ denotes the number of multipath clusters. It contains

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TABLE 1. Physical interpretation of the single shadowed κ-μ fading models.

Fading models	Shadowing type	Physical interpretation
Single shadowed κ-μ Type I	Shadowing of the dominant component	Physically, this situation may arise when the signal power delivered through the optical path between the transmitter and receiver is shadowed by objects moving within its locality. For example, blockages to the dominant path caused by cars, buildings and/or people present/moving within the locality of the transmitter or receiver.
Single shadowed κ-μ Type II	Shadowing of the scattered components	Physically, this situation may arise when the scattered signal components between the transmitter and receiver are shadowed by objects moving within their locality. For example, blockages to the scattered components due to cars, building and/or people present/moving within the locality of the transmitter or receiver.
Single shadowed κ-μ Type III	Shadowing of both the dominant component and scattered waves	Physically, this situation may arise when the dominant component and scattered signal components between the transmitter and receiver undergo shadowing caused by objects moving within the locality of the transmitter or receiver and/or large-scale effects.

other well-known fading models such as the Rice ($\kappa = K$, $\mu = 1$), Nakagami-m ($\kappa \rightarrow 0$, $\mu = m$), Rayleigh ($\kappa \rightarrow 0$, $\mu = 1$) and One-Sided Gaussian ($\kappa \rightarrow 0$, $\mu = 0.5$) as special cases. A κ-μ fading envelope can be affected by shadowing in many different ways. For instance, the dominant component, the scattered waves, or both can be impacted by this propagation phenomenon. It is also entirely possible that in addition to the dominant component being shadowed, further multiplicative shadowing[1] may occur which impacts the scattered signal, and also administers secondary shadowing to the already perturbed dominant component. Likewise, in addition to the scattered waves being shadowed, further shadowing may occur which impacts the dominant signal component, and administers secondary shadowing to the fluctuated scattered waves. As well as this, both the dominant component and scattered waves can be influenced by individual shadowing processes. Hence, a number of shadowing combinations give rise to a family of shadowed κ-μ fading models that can be classified depending on whether the underlying κ-μ fading undergoes single or double shadowing.

Traditionally, shadowing has been modeled using the lognormal distribution [4]. However, due to challenges which exist in relation to its tractability, the authors in [5] considered the use of gamma distribution. Similarly, the contributions in [6] and [7] considered the closely related Nakagami-m distribution due to its ability to exhibit semi-heavy tailed characteristics [7]. Recently, the authors in [8] and [9] effectively used the inverse Nakagami-m and inverse gamma distributions, respectively. Similar to the lognormal, gamma and Nakagami-m distributions, the inverse gamma and inverse Nakagami-m distributions have also shown to exhibit the necessary semi heavy-tailed behavior to accurately characterize shadowing. Moreover, they offer much of the analytical tractability available from using the gamma and Nakagami-m distributions.

[1] In this case, the total power of the dominant and scattered signal components are shadowed.

In this work, we discuss three types of single shadowed κ-μ fading model (denoted I to III) which assume that the multipath fading is manifested by the propagation mechanisms associated with κ-μ fading. In addition, these models consider that either the dominant component (Type I), the scattered waves (Type II), or both (Type III) suffer from a single shadowing process. We emphasize that these model frameworks are general and make no predetermination on the random variable (RV) that is responsible for characterizing the shadowing phenomena. For illustrative purposes, we provide two example cases for each type of single shadowed κ-μ fading model where it is assumed that the shadowing is influenced by either a Nakagami-m RV or an inverse Nakagami-m RV. We also introduce three types of double shadowed κ-μ fading model, denoted I to III. The Type I model assumes that in addition to the dominant component of a κ-μ signal being shadowed, further shadowing also occurs which impacts the scattered signal and also administers secondary shadowing to the already perturbed dominant component. Therefore, this model provides a convenient way to not only control the shadowing of the dominant component, but also any multiplicative shadowing which may be present in practical wireless channels. The Type II model considers that the dominant component and scattered waves of a κ-μ fading envelope are perturbed by two different shadowing processes. Lastly, the Type III model assumes that in addition to the scattered waves of a κ-μ signal being shadowed, the root mean square (rms) power of the dominant component and scattered waves also experience a secondary round of shadowing. Similar to the single shadowed models, two example cases for each of the three types of double shadowed model are discussed where it is assumed that the shadowing is shaped by a Nakagami-m RV, an inverse Nakagami-m RV or their mixture. Tables 1 and 2 summarize the various types of single shadowed and double shadowed κ-μ models introduced in this paper.

Due to the generality of the analysis presented here and under particular shadowing conditions, a number of the existing composite fading models found in the literature

TABLE 2. Physical interpretation of the double shadowed κ-μ fading models.

Fading models	Shadowing type	Physical interpretation
Double shadowed κ-μ Type I	Shadowing of the dominant component and secondary round of multiplicative shadowing	Physically, this situation may arise when the signal power delivered through the optical path between the transmitter and receiver is shadowed by objects moving within its locality (e.g., blockages to the dominant component due to cars, buildings and/or people), whilst further shadowing of the received power (combined multipath and dominant paths) may also occur due to obstacles moving in the vicinity of the transmitter or receiver and/or large-scale effects.
Double shadowed κ-μ Type II	Independent shadowing of the dominant component and scattered waves	Physically, this situation may arise when the dominant component and scattered signal components between the transmitter and receiver undergo independent shadowing caused by objects moving within their locality.
Double shadowed κ-μ Type III	Shadowing of the scattered components and secondary round of multiplicative shadowing	Physically, this situation may arise when the scattered signal components between the transmitter and receiver are shadowed by objects moving within their locality. Furthermore, additional shadowing of the received power (combined multipath and dominant paths) may occur due to obstacles in the vicinity of the transmitter or receiver and/or large-scale effects.

occur as special cases. For example, multiplicative composite fading models such as the κ-μ/inverse gamma and η-μ/inverse gamma fading models [9], which assume that a κ-μ or an η-μ RV is responsible for generating the multipath fading, and an inverse gamma RV for shaping the shadowing. Likewise, some line-of-sight (LOS) composite models[2] such as the κ-μ shadowed [6], [7][3] and shadowed Rician [10], [11] fading models are also found through the analysis conducted here. The κ-μ shadowed fading model presented in [6] and [7] assumes that the multipath fading is due to fluctuations brought about by a κ-μ RV, whilst the dominant signal component is fluctuated by a Nakagami-m RV. Moreover, it includes the κ-μ, η-μ and shadowed Rician fading models as special cases. It has shown to provide excellent agreement with field measurements obtained for body-centric fading channels [7] and land-mobile satellite channels [11]. Other notable composite fading models include the Nakagami-m/gamma [12], κ-μ/gamma [13]–[15], η-μ/gamma [16], [17], κ-μ/inverse Gaussian [18] and η-μ/inverse Gaussian [19], to name but a few. These models assume that the mean signal power of a Nakagami-m, κ-μ and an η-μ signal vary according to the gamma or the inverse Gaussian distributions.

The main contributions of this paper are now summarized as follows:

- Firstly, we perform a broad investigation of the way in which κ-μ fading can be affected by shadowing. We introduce a family of shadowed κ-μ models that are classified as either single or double shadowed models. Three types of single shadowed κ-μ fading model (Type I - III) and three types of double shadowed κ-μ fading model (Type I - III) are discussed.
- Secondly, a thorough physical interpretation for all three types of single and double shadowed κ-μ models is provided.
- Thirdly, we discuss two realistic example cases for each type of single and double shadowed κ-μ model by assuming that the shadowing is caused by a Nakagami-m RV, an inverse Nakagami-m RV or their mixture. Note that the model frameworks discussed in this paper are general and make no presumption on the RV that is responsible for shaping the shadowing characteristics. The examples discussed here are for illustrative purposes only.
- Finally, the generality of the double shadowed κ-μ fading models are highlighted through reduction to a number of well-known special cases.

The remainder of this paper is organized as follows. Section II and III describe and formulate the various types of single and double shadowed κ-μ model, respectively. Section IV presents some special cases of the double shadowed κ-μ models, corresponding numerical results and discussions. Lastly, Section V presents some useful concluding remarks.

II. SINGLE SHADOWED κ-μ MODELS

In this section, we investigate a number of different ways in which the κ-μ fading envelope can be impacted by a single shadowing process. This leads to three types of single shadowed fading model, denoted Type I to Type III, with their physical interpretation provided in Table 1.

A. SINGLE SHADOWED κ-μ TYPE I MODEL

Similar to the κ-μ fading model, the single shadowed κ-μ Type I fading model assumes that the received signal is composed of clusters of multipath waves propagating in non-homogeneous environments. Within each multipath cluster, the scattered waves have similar delay times and the delay spreads of different clusters are relatively large. The power of the scattered waves in each cluster is assumed to be identical whilst the power of the dominant component

[2]Many of the models presented in the literature for which the dominant signal component is subject to shadowing are often referred to as LOS composite fading models.

[3]It is noted that the κ-μ shadowed fading model presented in [6] and [7] is a type of single shadowed κ-μ model.

is assumed to be arbitrary. Unlike the κ-μ model, the single shadowed κ-μ Type I model assumes that the dominant component of each cluster can randomly fluctuate because of shadowing. Its signal envelope, R, can be expressed in terms of the in-phase and quadrature phase components as

$$R^2 = \sum_{i=1}^{\mu} (X_i + \xi p_i)^2 + (Y_i + \xi q_i)^2 \quad (1)$$

where ξ represents a RV which is responsible for introducing the shadowing, μ is a real-valued extension related to the number of multipath clusters, X_i and Y_i are mutually independent Gaussian random processes with mean $\mathbb{E}[X_i] = \mathbb{E}[Y_i] = 0$ and variance $\mathbb{E}\left[X_i^2\right] = \mathbb{E}\left[Y_i^2\right] = \sigma^2$, where $\mathbb{E}[\cdot]$ denotes the statistical expectation. Also, p_i and q_i are the mean values of the in-phase and quadrature phase components of the multipath cluster i. We now consider two example cases for the single shadowed Type I model, the details of which are discussed next.

Example 1: In our first example of the single shadowed κ-μ Type I model, we assume that the dominant component of a κ-μ signal undergoes variations induced by a Nakagami-m RV. Thus, in (1) ξ represents a Nakagami-m RV with shape parameter[4] m_d and $\mathbb{E}\left[\xi^2\right] = 1$. It is worth highlighting that this model was introduced as a generalization of the κ-μ fading model in [6][5] and [7]. Accordingly, the PDF of R is obtained as

$$f_R(r) = \frac{2m_d^{m_d}(1+\kappa)^{\mu}\mu^{\mu}}{\Gamma(\mu)(m_d+\kappa\mu)^{m_d}}\frac{r^{2\mu-1}}{\hat{r}^{2\mu}} e^{-\frac{r^2(1+\kappa)\mu}{\hat{r}^2}} \times {}_1F_1\left(m_d; \mu; \frac{\mu^2\kappa(1+\kappa)r^2}{\hat{r}^2(m_d+\kappa\mu)}\right) \quad (2)$$

where, $\kappa > 0$ is the ratio of the total power of the dominant component (d^2) to that of the scattered waves ($2\mu\sigma^2$), $\mu > 0$ is related to the number of clusters, $\hat{r} = \sqrt{\mathbb{E}[R^2]}$ represents the rms power of R, the mean signal power is given by $\mathbb{E}[R^2] = 2\mu\sigma^2 + d^2$, $\Gamma(\cdot)$ represents the gamma function and ${}_1F_1(\cdot;\cdot;\cdot)$ denotes the confluent hypergeometric function [20, eq. 9.210.1].

Now letting γ represent the instantaneous signal-to-noise ratio (SNR) of a single shadowed κ-μ Type I (example 1) fading channel, the corresponding PDF, $f_\gamma(\gamma)$, can be obtained from the envelope PDF given in (2) via a transformation of variables $\left(r = \sqrt{\gamma\,\hat{r}^2/\bar{\gamma}}\right)$ as follows.

$$f_\gamma(\gamma) = \frac{m_d^{m_d}(1+\kappa)^{\mu}\mu^{\mu}}{\Gamma(\mu)\bar{\gamma}(m_d+\kappa\mu)^{m_d}}\left(\frac{\gamma}{\bar{\gamma}}\right)^{\mu-1} e^{-\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}}} \times {}_1F_1\left(m_d; \mu; \frac{\mu^2\kappa(1+\kappa)\gamma}{\bar{\gamma}(m_d+\kappa\mu)}\right) \quad (3)$$

where $\bar{\gamma} = \mathbb{E}[\gamma]$ denotes the corresponding average SNR. Its CDF,[6] $F_\gamma(\gamma) \triangleq \int_0^{\gamma} f_\gamma(t)\,dt$, can be obtained from [6, eq. 6] as

$$F_\gamma(\gamma) = \frac{\mu^{\mu-1}m_d^{m_d}(1+\kappa)^{\mu}}{\Gamma(\mu)(\mu\kappa+m_d)^{m_d}}\left(\frac{1}{\bar{\gamma}}\right)^{\mu}\gamma^{\mu} \times \phi_2\left(\mu-m_d, m_d; \mu+1; -\frac{\mu(1+\kappa)\gamma}{\bar{\gamma}}, -\frac{\mu(1+\kappa)m_d\gamma}{\bar{\gamma}(\mu\kappa+m_d)}\right) \quad (4)$$

where $\phi_2(\cdot)$ is the bivariate confluent hypergeometric function [20].

Example 2: Our second example of the Type I model assumes that the dominant component of a κ-μ signal undergoes variations influenced by an inverse Nakagami-m RV. Thus, in (1) ξ represents an inverse Nakagami-m RV with shape parameter m_d and $\mathbb{E}\left[\xi^2\right] = 1$. The PDF of R for this example case can be obtained via Theorem 1 below.

Theorem 1: For κ, μ, r, $\hat{r} \in \mathbb{R}^+$ and $m_d > 1$, the PDF of the single shadowed κ-μ Type I fading model for the example case when ξ follows an inverse Nakagami-m RV is expressed as

$$f_R(r) = \sum_{i=0}^{\infty} \frac{4[(m_d-1)\kappa)]^{\frac{m_d+i}{2}} r^{2i+2\mu-1}(1+\kappa)^{i+\mu}}{\hat{r}^{2i+2\mu} i!\,\Gamma(m_d)\Gamma(i+\mu)} \times \mu^{\frac{1}{2}(3i+m_d)+\mu} e^{\frac{-r^2(1+\kappa)\mu}{\hat{r}^2}} K_{m_d-i}\left(2\sqrt{(m_d-1)\mu\kappa}\right) \quad (5)$$

where $K_\nu(\cdot)$ denotes the modified Bessel function of the second kind [21, eq. 9.6].

Proof: See Appendix A. ∎

The PDF of the instantaneous SNR, γ, of a single shadowed κ-μ Type I (example 2) fading channel, is obtained from the envelope PDF given in (5) via a transformation of variables as follows.

Corollary 1: For κ, μ, γ, $\bar{\gamma} \in \mathbb{R}^+$ and $m_d > 1$, the PDF of γ for the single shadowed κ-μ Type I fading model for the example case when ξ follows an inverse Nakagami-m RV is expressed as

$$f_\gamma(\gamma) = \sum_{i=0}^{\infty} \frac{2((m_d-1)\kappa\mu)^{\frac{1}{2}(m_d+i)}\gamma^{i+\mu-1}(\mu(1+\kappa))^{i+\mu}}{\bar{\gamma}^{i+\mu} i!\,\Gamma(m_d)\Gamma(\mu+i)} \times e^{-\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}}} K_{m_d-i}\left(2\sqrt{(m_d-1)\mu\kappa}\right). \quad (6)$$

Based on Corollary 1, its corresponding CDF, $F_\gamma(\gamma) = \int_0^{\gamma} f_\gamma(t)\,dt$, can be expressed via Lemma 1 as follows.

Lemma 1: For κ, μ, γ, $\bar{\gamma} \in \mathbb{R}^+$ and $m_d > 1$, the CDF of γ for the single shadowed κ-μ Type I fading model for the example case when ξ follows an inverse Nakagami-m RV is

[4] Note that throughout the manuscript we denote m_d, m_s and m_t as the shadowing parameters which are responsible for fluctuating the dominant, scattered or total (i.e. the combined dominant and scattered) components respectively.

[5] While the pioneering work presented in [6] refers to this model as κ-μ shadowed, to maintain consistency with the terminology adopted here we refer to it as an example of the single shadowed κ-μ Type I model.

[6] The outage probability (OP) of a communication system is defined as the probability that the instantaneous SNR drops below a given threshold, γ_{th}, i.e., $P_{\text{OP}}(\gamma_{\text{th}}) \triangleq P[0 \le \gamma \le \gamma_{th}] = F_\gamma(\gamma_{\text{th}})$. Therefore, the OP expressions for all of the models presented here can readily be obtained by replacing γ with γ_{th} in their respective CDF expression.

expressed as

$$F_{\gamma}(\gamma)=\sum_{i=0}^{\infty}\frac{2\left((m_d-1)\kappa\mu\right)^{\frac{1}{2}(m_d+i)}}{i!\Gamma(m_d)\Gamma(\mu+i)}\times \mathrm{K}_{m_d-i}\left(2\sqrt{(m_d-1)\mu\kappa}\right)\Gamma\left(i+\mu,\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}}\right). \tag{7}$$

Proof: See Appendix A. ■

B. SINGLE SHADOWED κ-μ TYPE II MODEL

The single shadowed κ-μ Type II fading model assumes that the scattered waves in each cluster can randomly fluctuate because of shadowing. Its signal envelope, R, can be formulated in terms of the in-phase and quadrature phase components as

$$R^2=\sum_{i=1}^{\mu}(\xi X_i+p_i)^2+(\xi Y_i+q_i)^2 \tag{8}$$

where ξ, X_i, Y_i, p_i, q_i and μ are as defined previously. We now consider two example cases for the single shadowed Type II model, the details of which are discussed next.

Example 1: In our first example of the single shadowed κ-μ Type II model, we assume that the scattered components of a κ-μ signal undergo variations induced by a Nakagami-m RV. Thus, in (8) ξ denotes a Nakagami-m RV with shape parameter m_s and $\mathbb{E}\left[\xi^2\right]=1$. The PDF of R for this example case can be obtained via Theorem 2 below.

Theorem 2: For κ, μ, m_s, r, $\hat{r}\in\mathbb{R}^+$, the PDF of the single shadowed κ-μ Type II fading model for the example case when ξ follows a Nakagami-m RV can be expressed as

$$f_R(r)=\sum_{i=0}^{\infty}\frac{4\,(m_s\mu)^{\frac{1}{2}(2i+m_s+\mu)}\,r^{2i+2\mu-1}\kappa^i\,(1+\kappa)^{i+\mu}}{i!\Gamma(m_s)\,\Gamma(i+\mu)\left(r^2(1+\kappa)+\hat{r}^2\kappa\right)^{\frac{1}{2}(2i-m_s+\mu)}}\times\frac{1}{\hat{r}^{m_s+\mu}}\mathrm{K}_{2i-m_s+\mu}\left(\frac{2\sqrt{m_s\mu\left(r^2(1+\kappa)+\hat{r}^2\kappa\right)}}{\hat{r}}\right). \tag{9}$$

Proof: See Appendix B. ■

The PDF of the instantaneous SNR, γ, of a single shadowed κ-μ Type II (example 1) fading channel, is obtained from the envelope PDF given in (9) via a transformation of variables as follows.

Corollary 2: For κ, μ, m_s, γ, $\bar{\gamma}\in\mathbb{R}^+$, the PDF of γ for the single shadowed κ-μ Type II fading model for the example case when ξ follows a Nakagami-m RV can be expressed as

$$f_{\gamma}(\gamma)=\sum_{i=0}^{\infty}\frac{2\,(m_s\mu)^{\frac{1}{2}(2i+m_s+\mu)}\,\kappa^i(1+\kappa)^{i+\mu}}{i!\Gamma(m_s)\Gamma(\mu+i)\left(\frac{\gamma(1+\kappa)}{\bar{\gamma}}+\kappa\right)^{\frac{1}{2}(2i-m_s+\mu)}}\times\frac{\gamma^{i+\mu-1}}{\bar{\gamma}^{i+\mu}}\mathrm{K}_{2i-m_s+\mu}\left(2\sqrt{m_s\mu\left(\frac{\gamma(1+\kappa)}{\bar{\gamma}}+\kappa\right)}\right). \tag{10}$$

Its CDF, $F_{\gamma}(\gamma)=\int_0^{\gamma}f_{\gamma}(t)\,dt$, can be expressed via Lemma 2 as follows.

Lemma 2: For κ, μ, m_s, γ, $\bar{\gamma}\in\mathbb{R}^+$, the CDF of γ for the single shadowed κ-μ Type II fading model for the example case when ξ follows a Nakagami-m RV can be expressed as[7]

$$F_{\gamma}(\gamma)=\frac{(1+\kappa)^{\mu}}{\kappa^{\mu}\Gamma(m_s)}\frac{\gamma^{\mu}}{\bar{\gamma}^{\mu}}\sum_{j=0}^{\infty}\frac{(\kappa\mu m_s)^j}{j!}\times\left[\frac{(-1)^j\Gamma(j+m_s)}{(\kappa\mu m_s)^{-m_s}}G_{2,2}^{1,1}\left(\frac{\gamma(1+\kappa)}{\kappa\bar{\gamma}}\Big|\begin{matrix}1+j-\mu+m_s,\,j+m_s\\0,\,-\mu\end{matrix}\right)-\frac{\pi\csc\left[\pi(\mu-m_s)\right]\,{}_2F_1\left(-j,1-j-\mu;1+\mu;\frac{\gamma(1+\kappa)}{\kappa\bar{\gamma}}\right)}{\Gamma(1+\mu)\Gamma(1+j+\mu-m_s)(\kappa\mu m_s)^{-\mu}}\right],\quad \mu-m_s\notin\mathbb{Z} \tag{11}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ denotes the Gauss hypergeometric function [20, eq. 9.100] and $G_{p',q'}^{m',n'}\left(z'\big|\cdot\cdot\right)$ represents the Meijer G function [22, eq. 07.34.02.0001.01].

Proof: See Appendix B. ■

Example 2: Our second example of the Type II model assumes that the scattered components of a κ-μ signal undergo variations induced by an inverse Nakagami-m RV. Thus, in (8) ξ denotes an inverse Nakagami-m RV with shape parameter m_s and $\mathbb{E}\left[\xi^2\right]=1$. The PDF of R for this example case can be obtained via Theorem 3 below.

Theorem 3: For κ, μ, r, $\hat{r}\in\mathbb{R}^+$ and $m_s>1$, the PDF of the single shadowed κ-μ Type II fading model for the example case when ξ follows an inverse Nakagami-m RV is expressed as

$$f_R(r)=\frac{2(m_s-1)^{m_s}(1+\kappa)^{\mu}\mu^{\mu}r^{2\mu-1}\hat{r}^{2m_s}}{\mathrm{B}(m_s,\mu)\left[r^2(1+\kappa)\mu+\hat{r}^2(m_s-1+\kappa\mu)\right]^{m_s+\mu}}\times{}_2F_1\left(\frac{m_s+\mu}{2},\frac{1+m_s+\mu}{2};\mu;\frac{4\mu^2\kappa(1+\kappa)r^2\hat{r}^2}{\left[r^2(1+\kappa)\mu+\hat{r}^2(m_s-1+\kappa\mu)\right]^2}\right) \tag{12}$$

where $\mathrm{B}(\cdot,\cdot)$ represents the Beta function [20, eq. 8.384].

Proof: See Appendix C. ■

The PDF of the instantaneous SNR, γ, of a single shadowed κ-μ Type II (example 2) fading channel, is obtained from the envelope PDF given in (12) via a transformation of variables as follows.

Corollary 3: For κ, μ, γ, $\bar{\gamma}\in\mathbb{R}^+$ and $m_s>1$, the PDF of γ for the single shadowed κ-μ Type II fading model for the example case when ξ follows an inverse Nakagami-m RV can

[7]Several equations throughout the paper will present conditions similar to those encountered in (11). These restrictions can easily be dealt with by applying an infinitesimally small shift to one of the parameters. For instance, in (11) the restriction $\mu-m_s\notin\mathbb{Z}$ can be straightforwardly overcome by introducing a perturbation term δ whose purpose is to shift m_s by a small amount so that the condition is satisfied.

be expressed as

$$f_\gamma(\gamma) = \frac{(m_s-1)^{m_s}(1+\kappa)^\mu \mu^\mu \gamma^{\mu-1}\bar{\gamma}^{m_s}}{B(m_s,\mu)\left(\gamma(1+\kappa)\mu+\bar{\gamma}(m_s-1+\kappa\mu)\right)^{m_s+\mu}} \times {}_2F_1\left(\frac{m_s+\mu}{2}, \frac{1+m_s+\mu}{2}; \mu; \frac{4\mu^2\kappa(1+\kappa)\gamma\bar{\gamma}}{(\gamma(1+\kappa)\mu+\bar{\gamma}(m_s-1+\kappa\mu))^2}\right). \quad (13)$$

Its CDF, $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, can be expressed via Lemma 3 as follows.

Lemma 3: For $\kappa, \mu, m_s, \gamma, \bar{\gamma} \in \mathbb{R}^+$, the CDF of γ for the single shadowed κ-μ Type II fading model for the example case when ξ follows an inverse Nakagami-m RV can be expressed as

$$F_\gamma(\gamma) = \frac{(m_s-1)^{m_s}\mu^{\mu-1}(1+\kappa)^\mu}{B(m_s,\mu)(-1+\kappa\mu+m_s)^{\mu+m_s}}\frac{\gamma^\mu}{\bar{\gamma}^\mu} \times \sum_{i=0}^{\infty}\frac{(m_s+\mu)_{2i}}{i!(\mu+1)_i}\left(\frac{\gamma\kappa(1+\kappa)\mu^2}{\bar{\gamma}(\kappa\mu+m_s-1)^2}\right)^i \times {}_2F_1\left(i+\mu, 2i+\mu+m_s; 1+i+\mu; -\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(-1+\kappa\mu+m_s)}\right) \quad (14)$$

where $(x)_{n'} \triangleq \frac{\Gamma(x+n')}{\Gamma(x)}$ denotes the Pochhammer symbol [20].

Proof: See Appendix C. ■

C. SINGLE SHADOWED κ-μ TYPE III MODEL

The single shadowed κ-μ Type III fading model assumes that the rms power of a κ-μ signal can randomly fluctuate because of shadowing. Its signal envelope, R, can be formulated in terms of the in-phase and quadrature phase components as

$$R^2 = \xi^2 \sum_{i=1}^{\mu}(X_i+p_i)^2 + (Y_i+q_i)^2 \quad (15)$$

where, ξ, X_i, Y_i, p_i, q_i and μ are as defined previously. We now consider two example cases for the single shadowed Type III model, the details of which are discussed next.

Example 1: In our first example of the single shadowed κ-μ Type III model, we assume that the multipath waves (both the dominant component and scattered waves) are subject to variations induced by a Nakagami-*m* RV. Thus, in (15) ξ represents a Nakagami-*m* RV with shape parameter m_t and $\mathbb{E}\left[\xi^2\right] = 1$. The PDF of R for this example case is given by Theorem 4.

Theorem 4: For $\kappa, \mu, m_t, r, \hat{r} \in \mathbb{R}^+$, the PDF of the single shadowed κ-μ Type III fading model for the example case when ξ follows a Nakagami-m RV can be expressed as

$$f_R(r) = \sum_{i=0}^{\infty}\frac{4(m_t\mu(1+\kappa))^{\frac{1}{2}(m_t+\mu+i)}(\kappa\mu)^i r^{m_t+\mu+i-1}}{e^{\kappa\mu}i!\Gamma(m_t)\Gamma(i+\mu)\hat{r}^{m_t+\mu+i}} \times K_{-m_t+\mu+n}\left(\frac{2r\sqrt{m_t\mu(1+\kappa)}}{\hat{r}}\right) \quad (16)$$

where, κ, μ, $\hat{r}$ are as defined previously.

Proof: See Appendix D. ■

Note that it is also possible to derive this PDF as a special case of the statistics of the product of κ-μ and Nakagami-*m* RVs as shown in [23] and [24]. The PDF of the instantaneous SNR, γ, of a single shadowed κ-μ Type III (example 1) fading channel, is obtained from the envelope PDF given in (16) via a transformation of variables as follows.

Corollary 4: For $\kappa, \mu, m_t, \gamma, \bar{\gamma} \in \mathbb{R}^+$, the PDF of γ for the single shadowed κ-μ Type III fading model for the example case when ξ follows a Nakagami-m RV can be expressed as

$$f_\gamma(\gamma) = \sum_{i=0}^{\infty}\frac{2(m_t\mu(1+\kappa))^{\frac{1}{2}(m_t+\mu+i)}(\kappa\mu)^i}{e^{\kappa\mu}i!\Gamma(m_t)\Gamma(i+\mu)} \times \frac{\gamma^{\frac{1}{2}(m_t+\mu+i)-1}}{\bar{\gamma}^{\frac{1}{2}(m_t+\mu+i)}} K_{-m_t+\mu+i}\left(2\sqrt{\frac{m_t\mu(1+\kappa)\gamma}{\bar{\gamma}}}\right). \quad (17)$$

Its CDF, $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, can be expressed via Lemma 4 as follows.

Lemma 4: For $\kappa, \mu, m_t, \gamma, \bar{\gamma} \in \mathbb{R}^+$, the CDF of γ for the single shadowed κ-μ Type III fading model for the example case when ξ follows a Nakagami-m RV can be expressed as

$$F_\gamma(\gamma) = \frac{1}{e^{\kappa\mu}\Gamma(m_t)}\sum_{i=0}^{\infty}\frac{(\kappa\mu)^i}{i!\Gamma(i+\mu)} \times G_{1,3}^{2,1}\left(\frac{\gamma(1+\kappa)\mu m_t}{\bar{\gamma}}\Big|_{m_t,\, i+\mu,\, 0}^{1}\right). \quad (18)$$

Proof: See Appendix D. ■

Example 2: Our second example of the Type III model assumes that the multipath waves are subject to variations induced by an inverse Nakagami-*m* RV. Thus, in (15) ξ represents an inverse Nakagami-*m* RV with shape parameter m_t and $\mathbb{E}\left[\xi^2\right] = 1$. This example of the single shadowed κ-μ Type III model was introduced in [9] as the κ-μ/inverse gamma fading model in which the mean power of the multipath waves were subject to fluctuations induced by an inverse gamma RV. Since the inverse Nakagami-*m* RV used for this analysis is assumed to have $\mathbb{E}\left[\xi^2\right] = 1$, the PDF of R for the single shadowed κ-μ Type III fading model for this example case can be obtained by substituting $\hat{r}^2 = (m_t-1)\hat{r}^2/m_t$ in [9], which yields

$$f_R(r) = \frac{2(1+\kappa)^\mu\mu^\mu e^{-\kappa\mu}\left((m_t-1)\hat{r}^2\right)^{m_t} r^{2\mu-1}}{B(m_t,\mu)\left(\hat{r}^2(m_t-1)+r^2(1+\kappa)\mu\right)^{m_t+\mu}} \times {}_1F_1\left(m_t+\mu; \mu; \frac{\mu^2\kappa(1+\kappa)r^2}{\hat{r}^2(m_t-1)+r^2(1+\kappa)\mu}\right) \quad (19)$$

where $m_t > 1$. The PDF of the instantaneous SNR, γ, of a single shadowed κ-μ Type III (example 2) fading channel, is obtained from the envelope PDF given in (19) via a transformation of variables as

$$f_\gamma(\gamma) = \frac{(1+\kappa)^\mu \mu^\mu e^{-\kappa\mu} ((m_t-1)\bar{\gamma})^{m_t} \gamma^{\mu-1}}{B(m_t,\mu)(\bar{\gamma}(m_t-1)+\gamma(1+\kappa)\mu)^{m_t+\mu}} \times {}_1F_1\left(m_t+\mu;\mu;\frac{\mu^2\kappa(1+\kappa)\gamma}{\bar{\gamma}(m_t-1)+\gamma(1+\kappa)\mu}\right). \quad (20)$$

Its CDF can be obtained from [9, eq. 5] as

$$F_\gamma(\gamma) = \sum_{i=0}^{\infty} \frac{e^{-\kappa\mu}(\mu\kappa)^i}{i!(\mu+i)B(m_t,\mu+i)}\left(\frac{\mu(\kappa+1)\gamma}{(m_t-1)\bar{\gamma}}\right)^{\mu+i} \times {}_2F_1\left(m_t+\mu+i,\mu+i;\mu+i+1;-\frac{\mu(\kappa+1)\gamma}{(m_t-1)\bar{\gamma}}\right). \quad (21)$$

III. DOUBLE SHADOWED κ-μ MODELS

In this section, we discuss three different ways in which the κ-μ fading envelope can be impacted by more than one shadowing process. To this end, we propose the double shadowed κ-μ Type I to Type III fading models with their physical interpretation provided in Table 2.

A. DOUBLE SHADOWED κ-μ TYPE I MODEL

The double shadowed κ-μ Type I model characterizes the propagation scenario in which the envelope experiences shadowing of the dominant component, which is preceded (or succeeded) by a secondary round of multiplicative shadowing. Its signal envelope, R, can be expressed in terms of the in-phase and quadrature phase components as

$$R^2 = A^2 \sum_{i=1}^{\mu} (X_i+\xi p_i)^2 + (Y_i+\xi q_i)^2 \quad (22)$$

where ξ, μ, X_i, Y_i, p_i and q_i are as defined previously and A represents a RV which introduces an additional degree of shadowing. As before, we now provide two example cases of the double shadowed κ-μ Type I model.

Example 1: In our first example of the double shadowed κ-μ Type I model, we assume that the shadowing of the dominant component is shaped by a Nakagami-m RV, whilst the second round of multiplicative shadowing is induced by an inverse Nakagami-m RV. Thus, in (22) ξ represents a Nakagami-m RV (with shape parameter m_d and $\mathbb{E}[\xi^2]=1$) whilst A denotes an inverse Nakagami-m RV (with shape parameter m_t and $\mathbb{E}[A^2]=1$). The PDF of the double shadowed κ-μ Type I fading model for this example case[8] can be obtained via [25, eq. 5] as

$$f_R(r) = \frac{2(m_t-1)^{m_t} m_d^{m_d} \mathcal{K}^\mu r^{2\mu-1}\hat{r}^{2m_t}}{(m_d+\mu\kappa)^{m_d} B(m_t,\mu)\left(\mathcal{K}r^2+(m_t-1)\hat{r}^2\right)^{m_t+\mu}} \times {}_2F_1\left(m_d, m_t+\mu;\mu;\frac{\mathcal{K}\mu\kappa r^2}{(m_d+\mu\kappa)\left(\mathcal{K}r^2+(m_t-1)\hat{r}^2\right)}\right) \quad (23)$$

[8]Note that this model was also introduced in [25] (as early results of this work) as a new fading model which is capable of characterizing both the shadowing of the dominant component and composite shadowing which may exist in wireless channels.

where $\mathcal{K}=\mu(1+\kappa)$. The PDF of the instantaneous SNR, γ, of a double shadowed κ-μ Type I (example 1) fading channel, is obtained from [25, eq. 6] as

$$f_\gamma(\gamma) = \frac{(m_t-1)^{m_t} m_d^{m_d} \mathcal{K}^\mu \gamma^{\mu-1}\bar{\gamma}^{m_t}}{(m_d+\mu\kappa)^{m_d} B(m_t,\mu)(\mathcal{K}\gamma+(m_t-1)\bar{\gamma})^{m_t+\mu}} \times {}_2F_1\left(m_d, m_t+\mu;\mu;\frac{\mathcal{K}_1\mu\kappa\gamma}{(\mathcal{K}\gamma+(m_t-1)\bar{\gamma})}\right) \quad (24)$$

where $\mathcal{K}_1 = \mathcal{K}/(m_d+\mu\kappa)$. Its CDF is obtained from [25, eq. 7 and eq. 8] as

$$F_\gamma(\gamma) = \left(\frac{m_d}{m_d+\kappa\mu}\right)^{m_d}\left(\frac{\mathcal{K}\gamma}{\bar{\gamma}(m_t-1)}\right)^{\mu}\sum_{i=0}^{\infty}\left(\frac{\mathcal{K}_1\mu\kappa\gamma}{\bar{\gamma}(m_t-1)}\right)^i \times \frac{(m_d)_i(i+\mu)_{m_t}}{i!\Gamma(m_t)(i+\mu)} {}_2F_1(i+\mu, i+\mu+m_t; i+\mu+1;\mathcal{T}) \quad (25)$$

where $\mathcal{T}=-\mathcal{K}\gamma/\bar{\gamma}(m_t-1)$. It is noted that for the case when $\bar{\gamma}(m_t-1)(m_d+\kappa\mu) > \kappa\mu^2(1+\kappa)\gamma$, (25) can be expressed in closed form as follows:

$$F_\gamma(\gamma) = \left(\frac{m_d}{m_d+\kappa\mu}\right)^{m_d}\left(\frac{\mathcal{K}\gamma}{\bar{\gamma}(m_t-1)}\right)^{\mu}\frac{\Gamma(m_t+\mu)}{\Gamma(m_t)\Gamma(\mu+1)} \times F^{2,1,0}_{1,1,0}\left(\begin{matrix} m_t+\mu,\mu; & m_d; & -; \\ 1+\mu; & \mu; & -; \end{matrix}\frac{\mathcal{K}_1\mu\kappa\gamma}{\bar{\gamma}(m_t-1)}, -\frac{\mathcal{K}\gamma}{\bar{\gamma}(m_t-1)}\right) \quad (26)$$

where $F^{2,1,0}_{1,1,0}\left(\begin{matrix}\cdot,\cdot; & \cdot; & \cdot; \\ \cdot; & \cdot; & \cdot;\end{matrix}\cdot,\cdot\right)$ denotes the Kampé de Fériet function [26].

Example 2: Our second example of the double shadowed Type I model assumes that the shadowing of the dominant component is brought about by an inverse Nakagami-m RV, whilst the second round of multiplicative shadowing is influenced by a Nakagami-m RV. Thus, in (22) ξ represents an inverse Nakagami-m RV (with shape parameter m_d and $\mathbb{E}[\xi^2]=1$) whilst A denotes a Nakagami-m RV (with shape parameter m_t and $\mathbb{E}[A^2]=1$). The PDF of the double shadowed κ-μ Type I fading model for this example case can be obtained via Theorem 5.

Theorem 5: For κ, μ, m_t, m_d, r, $\hat{r} \in \mathbb{R}^+$, the PDF of the double shadowed κ-μ Type I fading model when ξ represents an inverse Nakagami-m RV and A represents a Nakagami-m RV is

$$f_R(r) = \frac{8(\kappa\mu(m_d-1))^{\frac{m_d}{2}}(m_t\mathcal{K})^{\frac{\mu+m_t}{2}} r^{\mu+m_t-1}}{\Gamma(m_d)\Gamma(m_t)\hat{r}^{\mu+m_t}} \times \sum_{i=0}^{\infty}\frac{1}{i!\Gamma(i+\mu)}\left(\frac{r\mu\sqrt{\kappa(m_d-1)m_t(1+\kappa)}}{\hat{r}}\right)^i \times K_{m_d-i}\left(2\sqrt{(m_d-1)\mu\kappa}\right)K_{m_t-\mu-i}\left(\frac{2r\sqrt{\mathcal{K}m_t}}{\hat{r}}\right). \quad (27)$$

Proof: See Appendix E. ■

The PDF of the instantaneous SNR, γ, of a double shadowed κ-μ Type I (example 2) fading channel, is obtained from the envelope PDF given in (27) via a transformation of variables as follows.

Corollary 5: For κ, μ, m_t, m_d, γ, $\bar{\gamma} \in \mathbb{R}^+$, the PDF of γ for the double shadowed κ-μ Type I fading model for the example case when ξ represents an inverse Nakagami-m RV and A represents a Nakagami-m RV is

$$f_\gamma(\gamma) = \frac{4(\kappa\mu(m_d-1))^{\frac{m_d}{2}}(m_t\mathcal{K})^{\frac{1}{2}(\mu+m_t)}}{\Gamma(m_d)\Gamma(m_t)}\frac{\gamma^{\frac{1}{2}(\mu+m_t)-1}}{\bar{\gamma}^{\frac{1}{2}(\mu+m_t)}} \times \sum_{i=0}^{\infty}\frac{1}{i!\Gamma(i+\mu)}\left(\mu\sqrt{\frac{\gamma\kappa(m_d-1)m_t(1+\kappa)}{\bar{\gamma}}}\right)^i \times \mathrm{K}_{m_d-i}\left(2\sqrt{(m_d-1)\mu\kappa}\right)\mathrm{K}_{m_t-\mu-i}\left(2\sqrt{\frac{\gamma\mathcal{K}m_t}{\bar{\gamma}}}\right). \tag{28}$$

Its CDF, $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, can be expressed via Lemma 5 as follows.

Lemma 5: For κ, μ, m_t, m_d, γ, $\bar{\gamma} \in \mathbb{R}^+$, the CDF of γ for the double shadowed κ-μ Type I fading model for the example case when ξ represents an inverse Nakagami-m RV and A represents a Nakagami-m RV is

$$F_\gamma(\gamma) = \frac{2(\kappa\mu(m_d-1))^{\frac{m_d}{2}}}{\Gamma(m_d)\Gamma(m_t)}\sum_{i=0}^{\infty}\frac{\left(\sqrt{\mu\kappa(m_d-1)}\right)^i}{i!\Gamma(i+\mu)} \times \mathrm{K}_{m_d-i}\left(2\sqrt{(m_d-1)\mu\kappa}\right)G_{1,3}^{2,1}\left(\frac{\mathcal{K}\gamma m_t}{\bar{\gamma}}\Big|_{i+\mu,m_t,0}^{1}\right). \tag{29}$$

Proof: See Appendix E. ■

B. DOUBLE SHADOWED κ-μ TYPE II MODEL

The double shadowed κ-μ Type II model considers a κ-μ faded signal in which the dominant component and scattered waves experience two different shadowing processes. Its signal envelope, R, is given by

$$R^2 = \sum_{i=1}^{\mu}(AX_i + Bp_i)^2 + (AY_i + Bq_i)^2 \tag{30}$$

where μ, X_i, Y_i, p_i and q_i are as defined previously; A and B represent RVs that are responsible for introducing two different shadowing processes. We now consider two example cases for the double shadowed κ-μ Type II model, the details of which are discussed next.

Example 1: In our first example of the double shadowed Type II model, we assume that the dominant component of a κ-μ signal undergoes variations influenced by a Nakagami-m RV, whilst the scattered waves of a κ-μ signal are subject to variations induced by an inverse Nakagami-m RV. Thus, in (30) A denotes an inverse Nakagami-m RV with shape parameter m_s, and B represents a Nakagami-m RV with shape parameter m_d. Here, $\mathbb{E}\left[A^2\right]$ and $\mathbb{E}\left[B^2\right]$ are set equal to 1.

An analytical expression for the PDF of the double shadowed Type II fading model for this example case can be obtained via Theorem 6 below.

Theorem 6: For κ, μ, m_d, r, $\hat{r} \in \mathbb{R}^+$, and $m_s > 1$ the PDF of the double shadowed κ-μ Type II model when A denotes an inverse Nakagami-m RV and B denotes a Nakagami-m RV is

$$f_R(r) = \frac{2(m_s-1)^{m_s}m_d^{m_s+\mu}r^{2\mu-1}\Gamma(\mu+m_s)(1+\kappa)^\mu}{\kappa^{m_s+\mu}\Gamma(m_s)\Gamma(m_d)\mu^{m_s}\hat{r}^{2\mu}} \times \sum_{i=0}^{\infty}\frac{2^{2i}\left(\frac{\mu+m_s}{2}\right)_i(\theta_1)_i}{i!\Gamma(\mu+i)}\Gamma(i+m_d)\left(\frac{m_d(1+\kappa)r^2}{\hat{r}^2\kappa}\right)^i \times \mathrm{U}(2i+\mu+m_s, 1+i+\mu-m_d+m_s, \theta_2) \tag{31}$$

where $\theta_1 = \frac{1}{2}(1+m_s+\mu)$, $\theta_2 = \frac{m_d\left((m_s-1)\hat{r}^2+r^2(1+\kappa)\mu\right)}{\hat{r}^2\kappa\mu}$, and $\mathrm{U}(\cdot,\cdot,\cdot)$ is the confluent Tricomi hypergeometric function [21, eq. 13.1.3].

Proof: See Appendix F. ■

The PDF of the instantaneous SNR, γ, of a double shadowed κ-μ Type II (example 1) fading channel, is obtained from the envelope PDF given in (31) via a transformation of variables as follows.

Corollary 6: For κ, μ, m_d, γ, $\bar{\gamma} \in \mathbb{R}^+$ and $m_s > 1$, the PDF of γ for the double shadowed κ-μ Type II fading model when A denotes an inverse Nakagami-m RV and B denotes a Nakagami-m RV is

$$f_\gamma(\gamma) = \frac{(m_s-1)^{m_s}m_d^{m_s+\mu}\gamma^{\mu-1}\Gamma(\mu+m_s)(1+\kappa)^\mu}{\kappa^{m_s+\mu}\Gamma(m_s)\Gamma(m_d)\mu^{m_s}\bar{\gamma}^\mu} \times \sum_{i=0}^{\infty}\frac{2^{2i}\left(\frac{\mu+m_s}{2}\right)_i(\theta_1)_i\Gamma(i+m_d)}{i!\Gamma(\mu+i)}\left(\frac{m_d(1+\kappa)\gamma}{\kappa\bar{\gamma}}\right)^i \times \mathrm{U}\left(2i+\mu+m_s, 1+i+\mu-m_d+m_s, \theta_3''\right) \tag{32}$$

where $\theta_3'' = \frac{m_d(\bar{\gamma}(m_s-1)+\gamma(1+\kappa)\mu)}{\kappa\mu\bar{\gamma}}$.

Its CDF, $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, can be expressed via Lemma 6 as follows.

Lemma 6: For κ, μ, m_d, γ, $\bar{\gamma} \in \mathbb{R}^+$, and $m_s > 1$, the CDF of γ for the double shadowed κ-μ Type II fading model when A denotes an inverse Nakagami-m RV and B denotes a Nakagami-m RV is given by (33), shown at the bottom of the next page, where $\csc(\cdot) \triangleq 1/\sin(\cdot)$.

Proof: See Appendix F. ■

Example 2: Our second example of the double shadowed Type II model assumes that the dominant component of a κ-μ signal undergoes variations influenced by an inverse Nakagami-m RV whilst the scattered waves of a κ-μ signal are subject to variations induced by a Nakagami-m RV. Thus, in (30) A denotes a Nakagami-m RV (with shape parameter m_s and $\mathbb{E}\left[A^2\right] = 1$), and B represents an inverse Nakagami-m RV (with shape parameter m_d and $\mathbb{E}\left[B^2\right] = 1$). The PDF of the double shadowed Type II model for this example case[9] can be obtained via Theorem (7).

Theorem 7: For κ, μ, m_s, r, $\hat{r} \in \mathbb{R}^+$, and $m_d > 1$, the PDF of the double shadowed κ-μ Type II model when A denotes a Nakagami-m RV and B represents an inverse Nakagami-m RV is

$$f_R(r) = \frac{\csc(\pi m_d)\,2\pi}{r\Gamma(m_d)\Gamma(m_s)} \sum_{n=0}^{\infty} \Bigg[\frac{(-1)^n (\kappa\mu m_s(m_d-1))^n}{n!\Gamma(n+\mu)\Gamma(1+n-m_d)} G^{3,0}_{1,3}\left(\frac{r^2 m_s \mathcal{K}}{\hat{r}^2} \middle| \begin{matrix} 1-n \\ 1,\ \mu,\ -n+m_s \end{matrix} \right) - \frac{(\kappa\mu m_s(m_d-1))^{n+m_d}}{n!\Gamma(1+n+m_d)} \frac{1}{\Gamma(n+\mu+m_d)} G^{2,1}_{1,3}\left(\frac{r^2 m_s \mathcal{K}}{\hat{r}^2} \middle| \begin{matrix} 1-n-m_d \\ \mu,\ -n-m_d+m_s,\ 1 \end{matrix} \right) \Bigg], \quad m_d \notin \mathbb{Z} \tag{34}$$

where $\mathcal{K}$ is as defined previously.

Proof: See Appendix G. ■

The PDF of the instantaneous SNR, γ, of a double shadowed κ-μ Type II (example 2) fading channel, is obtained from the envelope PDF given in (34) via a transformation of variables as follows.

Corollary 7: For κ, μ, m_s, γ, $\bar{\gamma} \in \mathbb{R}^+$, and $m_d > 1$, the PDF of γ for the double shadowed κ-μ Type II fading model when A denotes a Nakagami-m RV and B represents an inverse Nakagami-m RV is

$$f_\gamma(\gamma) = \frac{\pi\csc(\pi m_d)}{\gamma\Gamma(m_d)\Gamma(m_s)} \sum_{n=0}^{\infty} \Bigg[\frac{(-1)^n (\kappa\mu m_s(m_d-1))^n}{n!\Gamma(n+\mu)\Gamma(1+n-m_d)} G^{3,0}_{1,3}\left(\frac{\gamma m_s \mathcal{K}}{\bar{\gamma}} \middle| \begin{matrix} 1-n \\ 1,\ \mu,\ -n+m_s \end{matrix} \right) - \frac{(\kappa\mu m_s(m_d-1))^{n+m_d}}{n!\Gamma(1+n+m_d)} \frac{1}{\Gamma(n+\mu+m_d)} G^{2,1}_{1,3}\left(\frac{\gamma m_s \mathcal{K}}{\bar{\gamma}} \middle| \begin{matrix} 1-n-m_d \\ \mu,\ -n-m_d+m_s,\ 1 \end{matrix} \right) \Bigg], \quad m_d \notin \mathbb{Z}. \tag{35}$$

[9]For conciseness, it is worth mentioning here that two further examples of the double shadowed model can readily be obtained from (30), which coincidentally lead to PDFs equivalent in form to (23) and (27). These can be found by letting $B = A\xi$, where A and ξ represent either a Nakagami-m and an inverse Nakagami-m RV or vice versa. As shown in [8], B^2 follows a Fisher-Snedecor $\mathcal{F}$ distribution [27]. Now, substituting for B in (30) we arrive at (22). Then letting A denote an inverse Nakagami-m RV and ξ represent a Nakagami-m RV and following the same statistical procedure highlighted in Section III.A, the PDF in (23) is deduced. If we let A denote a Nakagami-m RV and ξ represent an inverse Nakagami-m RV, we arrive at (27).

Its CDF, $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, can be expressed via Lemma 7 as follows.

Lemma 7: For κ, μ, m_s, γ, $\bar{\gamma} \in \mathbb{R}^+$, and $m_d > 1$ the CDF of γ for the double shadowed κ-μ Type II fading model when A denotes a Nakagami-m RV and B represents an inverse Nakagami-m RV is

$$F_\gamma(\gamma) = \frac{\pi\csc(\pi m_d)}{\Gamma(m_d)\Gamma(m_s)} \sum_{n=0}^{\infty} \Bigg[\frac{(-1)^n (\kappa\mu m_s(m_d-1))^n}{n!\Gamma(\mu+n)\Gamma(1+n-m_d)} \times G^{3,1}_{2,4}\left(\frac{\gamma m_s \mathcal{K}}{\bar{\gamma}} \middle| \begin{matrix} 1,\ 1-n \\ 1,\ \mu, -n+m_s,\ 0 \end{matrix} \right) - \frac{(\kappa\mu m_s(m_d-1))^{n+m_d}}{n!\Gamma(1+n+m_d)} \times \frac{1}{\Gamma(n+\mu+m_d)} G^{2,1}_{1,3}\left(\frac{\gamma m_s \mathcal{K}}{\bar{\gamma}} \middle| \begin{matrix} 1-n-m_d \\ \mu,\ -n-m_d+m_s,\ 0 \end{matrix} \right) \Bigg], \quad m_d \notin \mathbb{Z}. \tag{36}$$

Proof: See Appendix G. ■

C. DOUBLE SHADOWED κ-μ TYPE III MODEL

The double shadowed κ-μ Type III fading model considers a κ-μ faded signal in which the scattered waves in each cluster are subject to fluctuations caused by shadowing. As well as this, it assumes that the rms power of the dominant component and scattered waves may also be subject to random variations induced by shadowing. Its signal envelope, R is expressed as

$$R^2 = A^2 \sum_{i=1}^{\mu} (\xi X_i + p_i)^2 + (\xi Y_i + q_i)^2 \tag{37}$$

where ξ, A, μ, X_i, Y_i, p_i, and q_i are defined previously. As before, we now provide two example cases for the double shadowed Type III model.

Example 1: In our first example of the double shadowed κ-μ Type III model, we assume that the shadowing of the scattered components is influenced by an inverse Nakagami-m RV whilst the secondary round of multiplicative shadowing is induced by a Nakagami-m RV. Thus, in (37) A denotes a Nakagami-m RV (with shape parameter m_t and $\mathbb{E}\left[A^2\right] = 1$) whilst ξ represents an inverse Nakagami-m RV (with shape parameter m_s and $\mathbb{E}\left[\xi^2\right] = 1$). The PDF of the double shadowed Type III model for this example can be obtained via Theorem 8 below.

Theorem 8: For κ, μ, m_t, r, $\hat{r} \in \mathbb{R}^+$, and $m_s > 1$ the PDF of the double shadowed κ-μ Type III fading model where A denotes a Nakagami-m RV and ξ denotes an inverse

$$F_\gamma(\gamma) = \frac{\pi\gamma^\mu(1+\kappa)^\mu \csc(\pi(\mu-m_d+m_s))\, m_d^\mu}{\kappa^\mu\Gamma(1+\mu)\Gamma(m_d)\Gamma(m_s)\bar{\gamma}^\mu} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{m_d(m_s-1)}{\kappa\mu} \right)^n \Bigg[\left(\frac{\kappa\mu}{m_d(m_s-1)} \right)^{\mu-m_d} \times \frac{\Gamma(n+m_d)}{\Gamma(1+n-\mu+m_d-m_s)}\, {}_2F_1\left(-n+\mu-m_d, -n+\mu-m_d+m_s; 1+\mu; -\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)} \right) - \frac{\Gamma(n+\mu+m_s)}{\Gamma(1+n+\mu-m_d+m_s)} \left(\frac{m_d(m_s-1)}{\kappa\mu} \right)^{m_s} {}_2F_1\left(-n, -n-m_s; 1+\mu; -\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)} \right) \Bigg], \quad \mu-m_d+m_s \notin \mathbb{Z} \tag{33}$$

Nakagami-m RV is

$$f_R(r) = \frac{2(m_s-1)^{m_s}(m_t\mathcal{K})^{\mu} r^{2\mu-1}}{\Gamma(m_t)\,\mathrm{B}(m_s,\mu)\,\hat{r}^{2\mu}(m_s-1+\kappa\mu)^{\mu+m_s}} \times \sum_{i=0}^{\infty} \frac{\left(\frac{1}{2}(m_s+\mu)\right)_i (\theta_1)_i \Gamma(i+m_s+m_t)\left(4r^2\kappa\mu\mathcal{K}m_t\right)^i}{i!(\mu)_i \hat{r}^{2i}(m_s-1+\kappa\mu)^{2i}} \times \mathrm{U}\left(2i+\mu+m_s, 1+i+\mu-m_t, \frac{r^2\mathcal{K}m_t}{\hat{r}^2(m_s-1+\kappa\mu)}\right) \quad (38)$$

in which $\mathcal{K}$ and θ_1 are defined previously.

Proof: See Appendix H. ■

The PDF of the instantaneous SNR, γ, of a double shadowed κ-μ Type III (example 1) fading channel, is obtained from the envelope PDF given in (38) via a transformation of variables as follows.

Corollary 8: For κ, μ, m_t, γ, $\bar{\gamma} \in \mathbb{R}^+$ and $m_s > 1$, the PDF of γ for the double shadowed κ-μ Type III fading model when A denotes a Nakagami-m RV and ξ denotes an inverse Nakagami-m RV is

$$f_\gamma(\gamma) = \frac{(m_s-1)^{m_s}(m_t\mathcal{K})^{\mu}\gamma^{\mu-1}}{\Gamma(m_t)\,\mathrm{B}(m_s,\mu)\,\bar{\gamma}^{\mu}(m_s-1+\kappa\mu)^{\mu+m_s}} \times \sum_{i=0}^{\infty} \frac{\left(\frac{1}{2}(m_s+\mu)\right)_i \left(\frac{1}{2}(\mu+m_s+1)\right)_i \Gamma(i+m_s+m_t)}{i!(\mu)_i\bar{\gamma}^i(m_s-1+\kappa\mu)^{2i}(4\gamma\kappa\mu\mathcal{K}m_t)^{-i}} \times \mathrm{U}\left(2i+\mu+m_s, 1+i+\mu-m_t, \frac{\gamma\mathcal{K}m_t}{\bar{\gamma}(m_s-1+\kappa\mu)}\right). \quad (39)$$

The CDF of this model, $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, can be expressed via Lemma 8 as follows.

Lemma 8: For κ, μ, m_t, γ, $\bar{\gamma} \in \mathbb{R}^+$ and $m_s > 1$, the CDF of γ for the double shadowed κ-μ Type III fading model when A denotes a Nakagami-m RV and ξ denotes an inverse Nakagami-m RV is

$$F_\gamma(\gamma) = \frac{(m_s-1)^{m_s}}{\Gamma(m_s)\Gamma(m_t)(m_s-1+\kappa\mu)^{m_s}} \sum_{i=0}^{\infty}\frac{1}{i!\Gamma(\mu+i)} \times \left(\frac{\kappa\mu}{m_s-1+\kappa\mu}\right)^i G_{2,3}^{2,2}\left(\frac{\mathcal{K}\gamma m_t}{\bar{\gamma}(-1+\kappa\mu+m_s)}\Big|^{1,1-i-m_s}_{i+\mu,m_t,0}\right). \quad (40)$$

Proof: See Appendix H. ■

Example 2: Our second example of the double shadowed κ-μ Type III model assumes that the shadowing of the scattered components is influenced by a Nakagami-m RV whilst the secondary round of multiplicative shadowing is induced by an inverse Nakagami-m RV. Thus, in (37) A denotes an inverse Nakagami-m RV (with shape parameter m_t and $\mathbb{E}\left[A^2\right] = 1$) whilst ξ represents a Nakagami-m RV (with shape parameter m_s and $\mathbb{E}\left[\xi^2\right] = 1$). The PDF of the double shadowed Type III model for this example can be obtained via Theorem 9 as follows.

Theorem 9: For κ, μ, m_s, r, $\hat{r} \in \mathbb{R}^+$, and $m_t > 1$ the PDF of the double shadowed κ-μ Type III fading model when A denotes an inverse Nakagami-m RV and ξ denotes a Nakagami-m RV is

$$f_R(r) = \frac{2\pi\,(\mathcal{K}m_s/(m_t-1))^{m_s} r^{2m_s-1}\hat{r}^{-2m_s}}{\Gamma(m_t)\Gamma(m_s)\sin(\pi(m_t+m_s))}\sum_{i=0}^{\infty}\frac{(\kappa\mu m_s)^i}{i!} \times \left(\mathcal{G}\left(\frac{\kappa\mu\hat{r}^2(m_t-1)}{r^2\mathcal{K}}\right)^{-i} - \mathcal{H}\left(\frac{\kappa\mu\hat{r}^2(m_t-1)}{r^2\mathcal{K}}\right)^{m_t+m_s} + \mathcal{J}\left(\frac{r^2\mathcal{K}m_s}{\hat{r}^2(m_t-1)}\right)^{i+\mu-m_s}(\kappa\mu m_s)^{-i}\right), \quad m_t+m_s \notin \mathbb{Z} \quad (41)$$

in which $\mathcal{K}$ is defined previously, and $\mathcal{G}$, $\mathcal{H}$ and $\mathcal{J}$ are given at the bottom of the next page where ${}_2\tilde{F}_2(a,b;c,d,z) = {}_2F_2(a,b;c,d,z)/(\Gamma(c)\Gamma(d))$ is a particular case of the generalized hypergeometric function [28, eq. 7.2.3.1].

Proof: See Appendix I. ■

The PDF of the instantaneous SNR, γ, of a double shadowed κ-μ Type III (example 2) fading channel, is obtained from the envelope PDF given in (41) via a transformation of variables as follows.

Corollary 9: For κ, μ, m_s, γ, $\bar{\gamma} \in \mathbb{R}^+$, and $m_t > 1$ the PDF of γ for the double shadowed κ-μ Type III fading model when A denotes an inverse Nakagami-m RV and ξ denotes a Nakagami-m RV is

$$f_\gamma(\gamma) = \frac{\pi\left(\frac{\mathcal{K}m_s}{m_t-1}\right)^{m_s}\gamma^{m_s-1}\bar{\gamma}^{-m_s}}{\Gamma(m_t)\Gamma(m_s)\sin(\pi(m_t+m_s))}\sum_{i=0}^{\infty}\frac{(\kappa\mu m_s)^i}{i!} \times \left[\mathcal{G}'\left(\frac{\kappa\mu\bar{\gamma}(m_t-1)}{\gamma\mathcal{K}}\right)^{-i} - \mathcal{H}'\left(\frac{\kappa\mu\bar{\gamma}(m_t-1)}{\gamma\mathcal{K}}\right)^{m_t+m_s} + \mathcal{J}'\left(\frac{\gamma\mathcal{K}m_s}{\bar{\gamma}(m_t-1)}\right)^{i+\mu-m_s}(\kappa\mu m_s)^{-i}\right], \quad m_t+m_s \notin \mathbb{Z}. \quad (45)$$

in which $\mathcal{K}$ is defined previously, and $\mathcal{G}'$, $\mathcal{H}'$ and $\mathcal{J}'$ are given at the bottom of the next page.

Its CDF, $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, can be expressed via Lemma 9 as follows.

Lemma 9: For κ, μ, m_s, γ, $\bar{\gamma} \in \mathbb{R}^+$ and $m_t > 1$, the CDF of γ for the double shadowed κ-μ Type III fading model when A denotes an inverse Nakagami-m RV and ξ denotes a Nakagami-m RV is given by (49)*, shown at the bottom of the next page.*

Proof: See Appendix I. ■

IV. SPECIAL CASES OF THE DOUBLE SHADOWED κ-μ FADING MODELS AND NUMERICAL RESULTS

A. SOME SPECIAL CASES

The PDFs given in (23), (27), (31), (34), (38) and (41) represent an extremely versatile set of fading models as they inherit the generalities of the various types of single shadowed κ-μ fading model. Recall that in the double shadowed κ-μ Type I model the m_d parameter denotes the intensity of shadowing that the dominant signal component undergoes, whilst the m_t parameter represents the degree of fluctuations that both the dominant and scattered signal components undergo as a result of the secondary shadowing process. Letting $m_t \to \infty$ in (23), we obtain the PDF of the single shadowed κ-μ Type I (example 1) model, whilst letting $m_d \to \infty$, we obtain the PDF of the single shadowed κ-μ Type III (example 1) fading model. Allowing, $m_d \to \infty$ and $\hat{r}^2 = m_t\hat{r}^2/(m_t-1)$ yields the κ-μ/inverse gamma fading model. Hence, letting $m_t \to \infty$ and $m_d \to \infty$, we obtain the PDF of the κ-μ fading model. These results are illustrated in Fig. 1 and are in agreement with the corresponding Monte Carlo (MC) simulations. The PDF of the η-μ/inverse gamma model can also be obtained from the double shadowed κ-μ Type I (example 1) model by setting $m_d \to \mu$, $\kappa = (1-\eta)/2\eta$, $\mu = 2\mu$ and $\hat{r}^2 = m_t\hat{r}^2/(m_t-1)$. Thus, letting $m_t \to \infty$, $m_d \to \mu$, $\kappa = (1-\eta)/2\eta$ and $\mu = 2\mu$ we obtain the PDF of the η-μ fading model. Likewise, the PDFs of the double shadowed Rician Type I (example 1) [30], shadowed Rician, and Rician fading models can be obtained from (23) by first

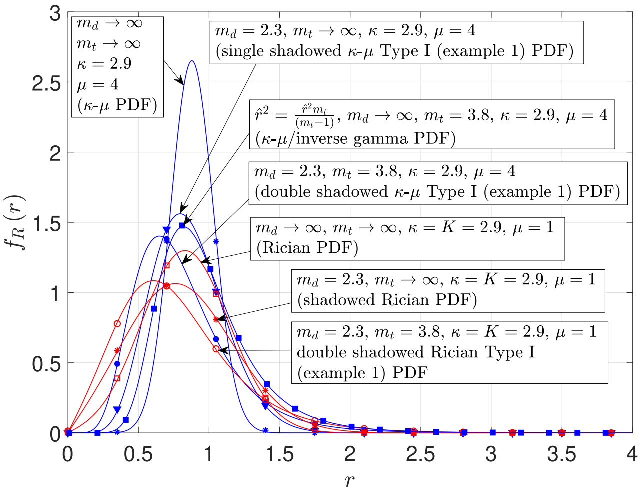

FIGURE 1. The PDF of the double shadowed κ-μ Type I (example 1) model reduced to some of its special cases: κ-μ (blue asterisk markers), single shadowed κ-μ Type I (example 1) (blue triangle markers), κ-μ/inverse gamma (blue square markers), Rician (red square markers), shadowed Rician (red asterisk markers), double shadowed Rician Type I (example 1) (red circle markers). Here, $\hat{r} = 0.8$, lines represent analytical results, and the markers represent simulation results.

$$\mathcal{G} = \frac{(-1)^i \pi \csc\left(\pi\left(\mu - m_s\right)\right)}{\Gamma\left(1 + i - \mu + m_s\right)} \, {}_2\tilde{F}_2\left(1 - i - m_s, -i + \mu - m_s; \mu, 1 - i - m_t - m_s; -\frac{\kappa\mu\hat{r}^2(m_t-1)}{r^2\mathcal{K}}\right), \quad \mu - m_s \notin \mathbb{Z} \tag{42}$$

$$\mathcal{H} = \frac{\Gamma\left(\mu + m_t\right)\Gamma\left(i + m_s\right)}{\Gamma\left(-m_t\right)} \, {}_2\tilde{F}_2\left(1 + m_t, \mu + m_t; 1 + i + m_t + m_s, i + \mu + m_t + m_s; -\frac{\kappa\mu\hat{r}^2(m_t-1)}{r^2\mathcal{K}}\right) \tag{43}$$

$$\mathcal{J} = \frac{\Gamma\left(-i - \mu + m_s\right)\sin\left(\pi\left(m_t + m_s\right)\right)}{\sin\left(\pi\left(\mu + m_t\right)\right)} \, {}_2\tilde{F}_2\left(-i, 1 - i - \mu; \mu, 1 - i - \mu - m_t; -\frac{\kappa\hat{r}^2(m_t-1)}{r^2(1+\kappa)}\right), \quad m_s - \mu \notin \mathbb{Z};\ \mu + m_t \notin \mathbb{Z} \tag{44}$$

$$\mathcal{G}' = \frac{(-1)^i \pi \csc\left(\pi\left(\mu - m_s\right)\right)}{\Gamma\left(1 + i - \mu + m_s\right)} \, {}_2\tilde{F}_2\left(1 - i - m_s, -i + \mu - m_s; \mu, 1 - i - m_t - m_s; -\frac{\kappa\mu\bar{\gamma}\left(m_t-1\right)}{\gamma\mathcal{K}}\right), \quad \mu - m_s \notin \mathbb{Z} \tag{46}$$

$$\mathcal{H}' = \frac{\Gamma\left(\mu + m_t\right)\Gamma\left(i + m_s\right)}{\Gamma\left(-m_t\right)} \, {}_2\tilde{F}_2\left(1 + m_t, \mu + m_t; 1 + i + m_t + m_s, i + \mu + m_t + m_s; -\frac{\kappa\mu\bar{\gamma}\left(m_t-1\right)}{\gamma\mathcal{K}}\right) \tag{47}$$

$$\mathcal{J}' = \frac{\Gamma\left(-i - \mu + m_s\right)\sin\left(\pi\left(m_t + m_s\right)\right)}{\sin\left(\pi\left(\mu + m_t\right)\right)} \, {}_2\tilde{F}_2\left(-i, 1 - i - \mu; \mu, 1 - i - \mu - m_t; -\frac{\kappa\mu\bar{\gamma}\left(m_t-1\right)}{\gamma\mathcal{K}}\right), \quad m_s - \mu \notin \mathbb{Z};\ \mu + m_t \notin \mathbb{Z} \tag{48}$$

$$\begin{aligned}
F_\gamma(\gamma) = {} & \frac{\pi\left(\frac{\mathcal{K}m_s}{m_t-1}\right)^{m_s}}{\Gamma\left(m_t\right)\Gamma\left(m_s\right)\sin\left(\pi\left(m_t + m_s\right)\right)} \frac{\gamma^{m_s}}{\bar{\gamma}^{m_s}} \sum_{i=0}^{\infty} \frac{\left(\kappa\mu m_s\right)^i}{i!} \left(\frac{\Gamma\left(m_s - i - \mu\right)\sin\left(\pi\left(m_s + m_t\right)\right)}{\left(i + \mu\right)\sin\left(\pi\left(\mu + m_t\right)\right)\left(\kappa\mu m_s\right)^i} \left(\frac{\gamma\mathcal{K}m_s}{\bar{\gamma}\left(m_t-1\right)}\right)^{i+\mu-m_s}\right. \\
& \times {}_2\tilde{F}_2\left(-i, -i - \mu; \mu, 1 - i - \mu - m_t; \frac{\kappa\mu\bar{\gamma}\left(1 - m_t\right)}{\mathcal{K}\gamma}\right) + \frac{\Gamma\left(\mu - m_s - i\right)}{i + m_s}\left(\frac{\gamma\mathcal{K}}{\kappa\mu\bar{\gamma}\left(m_t-1\right)}\right)^i \\
& \times {}_2\tilde{F}_2\left(-i - m_s, \mu - i - m_s; \mu, 1 - i - m_s - m_t; \frac{\kappa\mu\bar{\gamma}\left(1 - m_t\right)}{\mathcal{K}\gamma}\right) - \frac{\Gamma\left(i + m_s\right)\Gamma\left(\mu + m_t\right)}{\Gamma\left(1 - m_t\right)}\left(\frac{\kappa\mu\bar{\gamma}\left(m_t-1\right)}{\gamma\mathcal{K}}\right)^{m_s+m_t} \\
& \left.\times {}_2\tilde{F}_2\left(m_t, \mu + m_t; 1 + i + m_s + m_t, i + \mu + m_s + m_t; \frac{\kappa\mu\bar{\gamma}\left(1 - m_t\right)}{\mathcal{K}\gamma}\right)\right), \quad m_t + m_s \notin \mathbb{Z};\ \mu - m_s \notin \mathbb{Z}
\end{aligned} \tag{49}$$

TABLE 3. Special cases of the double shadowed κ-μ Type I (example 1), Type I (example 2) and Type II (example 1) fading models.

Fading models	double shadowed κ-μ Type I (example 1)	double shadowed κ-μ Type I (example 2)	double shadowed κ-μ Type II (example 1)
Single shadowed κ-μ Type I (example 1) [6]	$\underline{m_t} \rightarrow \infty, \underline{m_d} = m_d,$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	-	$\underline{m_s} \rightarrow \infty, \underline{m_d} = m_d,$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$
Single shadowed κ-μ Type I (example 2)	-	$\underline{m_t} \rightarrow \infty, \underline{m_d} = m_d,$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	-
Single shadowed κ-μ Type II (example 1)	-	-	-
Single shadowed κ-μ Type II (example 2)	-	-	$\underline{m_s} \rightarrow m_s, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$
Single shadowed κ-μ Type III (example 1) [23]	-	$\underline{m_t} \rightarrow m_t, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	-
Single shadowed κ-μ Type III (example 2)	$\underline{m_t} = m_t, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	-	-
$\eta - \mu$/inverse gamma [9]	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \mu$ $\underline{\kappa} \rightarrow \frac{(1-\eta)}{2\eta}, \underline{\mu} = 2\mu$ $\underline{\hat{r}}^2 = \frac{m_t \hat{r}^2}{(m_t - 1)}$	-	$\underline{m_s} \rightarrow \infty, \underline{m_d} \rightarrow \mu$ $\underline{\kappa} \rightarrow \frac{(1-\eta)}{2\eta}, \underline{\mu} = 2\mu$ $\underline{\hat{r}}^2 = \frac{m_s \hat{r}^2}{(m_s - 1)}$
κ-μ	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	$\underline{m_s} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} = \kappa, \underline{\mu} = \mu$
η-μ	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \mu,$ $\underline{\kappa} = \frac{(1-\eta)}{2\eta}, \underline{\mu} = 2\mu$	-	$\underline{m_s} \rightarrow \infty, \underline{m_d} \rightarrow \mu,$ $\underline{\kappa} = \frac{(1-\eta)}{2\eta}, \underline{\mu} = 2\mu$
Shadowed Rician [11]	$\underline{m_t} \rightarrow \infty, \underline{m_d} = m_d,$ $\underline{\kappa} = K, \underline{\mu} = 1$	-	$\underline{m_s} \rightarrow \infty, \underline{m_d} = m_d,$ $\underline{\kappa} = K, \underline{\mu} = 1$
Rician	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} = K, \underline{\mu} = 1$	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} = K, \underline{\mu} = 1$	$\underline{m_s} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} = K, \underline{\mu} = 1$
Nakagami-q (Hoyt) [29]	$\underline{m_t} \rightarrow \infty, \underline{m_d} = 0.5,$ $\underline{\kappa} = \frac{(1-q^2)}{2q^2}, \underline{\mu} = 1$	-	$\underline{m_s} \rightarrow \infty, \underline{m_d} = 0.5,$ $\underline{\kappa} = \frac{(1-q^2)}{2q^2}, \underline{\mu} = 1$
Nakagami-m	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} \rightarrow 0, \underline{\mu} = m$	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} \rightarrow 0, \underline{\mu} = m$	$\underline{m_s} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} \rightarrow 0, \underline{\mu} = m$
Rayleigh	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} \rightarrow 0, \underline{\mu} = 1$	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} \rightarrow 0, \underline{\mu} = 1$	$\underline{m_s} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} \rightarrow 0, \underline{\mu} = 1$
One-sided Gaussian	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} \rightarrow 0, \underline{\mu} = 0.5$	$\underline{m_t} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} \rightarrow 0, \underline{\mu} = 0.5$	$\underline{m_s} \rightarrow \infty, \underline{m_d} \rightarrow \infty,$ $\underline{\kappa} \rightarrow 0, \underline{\mu} = 0.5$

setting $\mu = 1$, $\kappa = K$ (the Rician K-factor), followed by appropriate substitutions for m_d and m_t. Fig. 1 shows the shape of the PDF for these special cases in red.

Likewise, the double shadowed κ-μ Type I (example 2) model contains the single shadowed κ-μ Type I (example 2) and Type III (example 1) models as special cases. Letting $m_t \rightarrow \infty$ in (27), we obtain the PDF of the single shadowed κ-μ Type I (example 2) model, and letting $m_d \rightarrow \infty$ we obtain the PDF of the single shadowed κ-μ Type III (example 1) model. Allowing both $m_t \rightarrow \infty$ and $m_d \rightarrow \infty$, the PDF of the κ-μ fading model is deduced. The PDF given in (31) (double shadowed Type II (example 1)) also represents a flexible fading model as it contains the single shadowed κ-μ Type I (example 1), Type II (example 2), η-μ/inverse gamma, κ-μ and η-μ fading models as special cases. Letting $m_s \rightarrow \infty$ in (31), we obtain the PDF of the single shadowed κ-μ Type I (example 1) model, whilst letting $m_d \rightarrow \infty$ we obtain the PDF of the single shadowed κ-μ Type II (example 2) model. Allowing both $m_s \rightarrow \infty$ and $m_d \rightarrow \infty$ in (31), the double shadowed κ-μ Type II (example 1) fading model coincides with the κ-μ fading model. Table 3 summarizes the special cases of the double shadowed κ-μ Type I (example 1), Type I (example 2) and Type II (example 1) models whilst Table 4 summarizes the special cases of the double shadowed κ-μ Type II (example 2), Type III (example 1) and Type III (example 2) models.

B. NUMERICAL RESULTS

Fig. 2 a) and b) show some plots of the PDF of the single shadowed κ-μ Type II (example 2), and double shadowed Type I (example 1) models for different values of κ, μ, m_s, m_d, m_t, and $\hat{r}$. Note that the values of the parameters are chosen to illustrate the wide range of shapes that the new shadowed fading models can exhibit, and thus their capability to model accurately the versatile fading conditions encountered in numerous communication scenarios relating to emerging wireless applications. Fig. 2 a)

TABLE 4. Special cases of the double shadowed κ-μ Type II (example 2), Type III (example 1) and Type III (example 2) fading models.

Fading models	double shadowed κ-μ Type II (example 2)	double shadowed κ-μ Type III (example 1)	double shadowed κ-μ Type III (example 2)
Single shadowed κ-μ Type I (example 1) [6]	-	-	-
Single shadowed κ-μ Type I (example 2)	$\underline{m_s} \to \infty, \underline{m_d} = m_d$, $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	-	-
Single shadowed κ-μ Type II (example 1)	$\underline{m_s} \to m_s, \underline{m_d} \to \infty$, $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	-	$\underline{m_s} \to m_s, \underline{m_t} \to \infty$, $\underline{\kappa} = \kappa, \underline{\mu} = \mu$
Single shadowed κ-μ Type II (example 2)	-	$\underline{m_s} \to m_s, \underline{m_t} \to \infty$, $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	-
Single shadowed κ-μ Type III (example 1) [23]	-	$\underline{m_s} \to \infty, \underline{m_t} \to m_t$, $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	-
Single shadowed κ-μ Type III (example 2)	-	-	$\underline{m_s} = \infty, \underline{m_t} \to m_t$, $\underline{\kappa} = \kappa, \underline{\mu} = \mu$
η-μ/inverse gamma	-	-	-
κ-μ	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$, $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} = \kappa, \underline{\mu} = \mu$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} = \kappa, \underline{\mu} = \mu$
η-μ	-	-	-
Shadowed Rician [11]	-	-	-
Rician	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$, $\underline{\kappa} = K, \underline{\mu} = 1$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} = K, \underline{\mu} = 1$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} = K, \underline{\mu} = 1$
Nakagami-q (Hoyt) [29]	-	-	-
Nakagami-m	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$, $\underline{\kappa} \to 0, \underline{\mu} = m$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} \to 0, \underline{\mu} = m$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} \to 0, \underline{\mu} = m$
Rayleigh	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$, $\underline{\kappa} \to 0, \underline{\mu} = 1$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} \to 0, \underline{\mu} = 1$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} \to 0, \underline{\mu} = 1$
One-sided Gaussian	$\underline{m_s} \to \infty, \underline{m_d} \to \infty$, $\underline{\kappa} \to 0, \underline{\mu} = 0.5$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} \to 0, \underline{\mu} = 0.5$	$\underline{m_s} \to \infty, \underline{m_t} \to \infty$, $\underline{\kappa} \to 0, \underline{\mu} = 0.5$

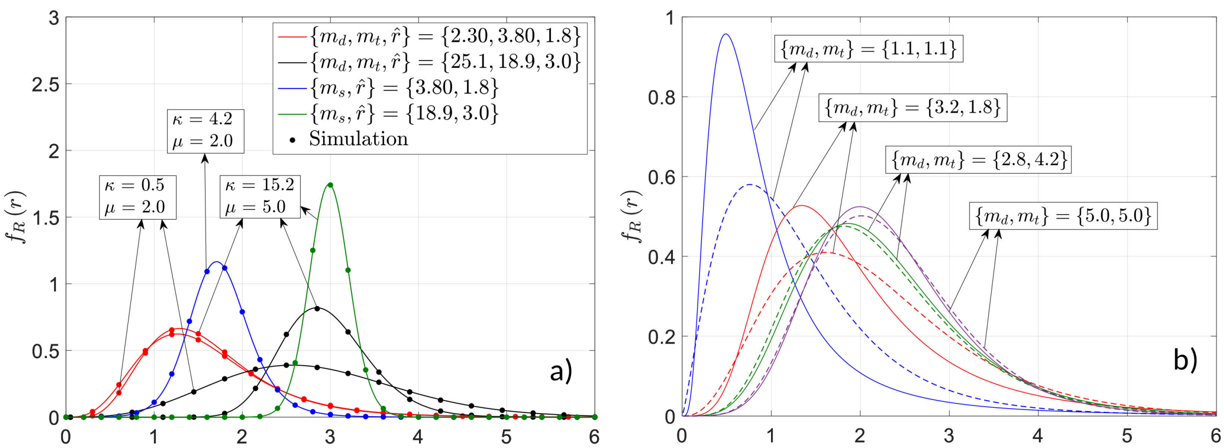

FIGURE 2. a) PDF of the single shadowed κ-μ Type II (example 2) (blue and green lines) and double shadowed κ-μ Type I (example 1) (red and black lines) models. Lines denote analytical results; circle markers denote simulation results. b) PDF of the double shadowed κ-μ Type I (example 1) and (example 2) models for different values of m_d and m_t. $\kappa = 3.9$, $\mu = 2.4$, and $\hat{r} = 2.5$. Solid and dashed lines denote Type I (example 1) and Type I (example 2) models, respectively.

shows the PDF of the single shadowed κ-μ Type II (example 2) and double shadowed Type I (example 1) fading models for $\{\kappa, \mu\} = \{0.5, 2.0\}, \{4.2, 2.0\}, \{15.1, 5.0\}$, $\{m_d, m_t, \hat{r}\} = \{2.3, 3.8, 1.8\}, \{25.1, 18.9, 3.0\}$ and $\{m_s, \hat{r}\} = \{3.8, 1.8\}, \{18.9, 3.0\}$. In all cases, the analytical results agree with the corresponding MC simulations.

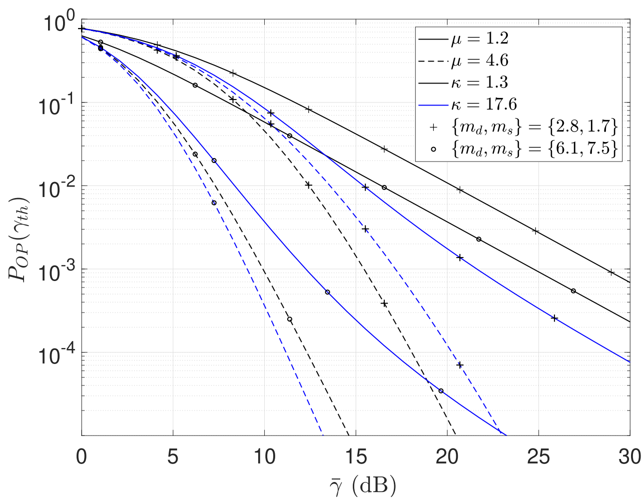

FIGURE 3. Outage probability of the double shadowed Type 1 (example 1) model versus $\bar{\gamma}$ for different values of κ, μ, m_d and m_s. Here $\gamma_{th} = 0$ dB.

Fig. 3 shows the outage probability of the double shadowed Type 1 (example 1) model versus $\bar{\gamma}$ for different multipath and shadowing conditions. As expected, we observe that the outage probability increases for lower values of κ, μ, m_d and m_s parameters. Moreover, the rate at which the outage probability decreases is faster as these parameters grow large. Furthermore, Fig. 4 shows the outage probability of the double shadowed Type 1 (example 2) model versus $\bar{\gamma}$ for different values of γ_{th} when κ, μ, m_t and m_d are fixed. We observe that for a fixed $\bar{\gamma}$, the outage probability increases, as expected, as γ_{th} increases. In all cases, the analytical results agree with the MC simulations.

V. CONCLUSION

For the first time, this paper has discussed the various ways in which a κ-μ fading envelope can be affected by shadowing. A family of shadowed κ-μ fading models were proposed and classified based on whether the underlying κ-μ envelope undergoes single or double shadowing. Three types of single and double shadowed κ-μ model were introduced. These model frameworks are general and do not depend on pre-defined RVs that are responsible for shaping the shadowing characteristics. However, for illustrative purposes, two example cases for each type of single and double shadowed model were discussed where it was assumed that the shadowing is shaped by a Nakagami-*m* RV, an inverse Nakagami-*m* RV or their mixture. Finally, it is worth remarking that the proposed double shadowed models are very general and have been reduced to a number of well-known special cases. This property renders them useful both theoretically and practically as they can provide accurate modeling of the versatile composite fading conditions encountered in emerging wireless applications with stringent quality of service requirements.

APPENDIX A
PROOF OF THEOREM 1 AND LEMMA 1

Considering the signal model given in (1) where ξ is assumed to be an inverse Nakagami-*m* RV with shape parameter m_d and $\mathbb{E}\left[\xi^2\right] = 1$, its PDF is given by

$$f_{\xi}\left(\xi\right) = \frac{2(m_d - 1)^{m_d}}{\Gamma\left(m_d\right)\xi^{2m_d+1}} e^{-\frac{m_d-1}{\xi^2}}. \tag{50}$$

To determine the envelope distribution of the single shadowed κ-μ Type I (example 2) fading model we average the conditional PDF, $f_{R|\xi}\left(r|\xi\right)$, with the PDF of ξ given in (50)

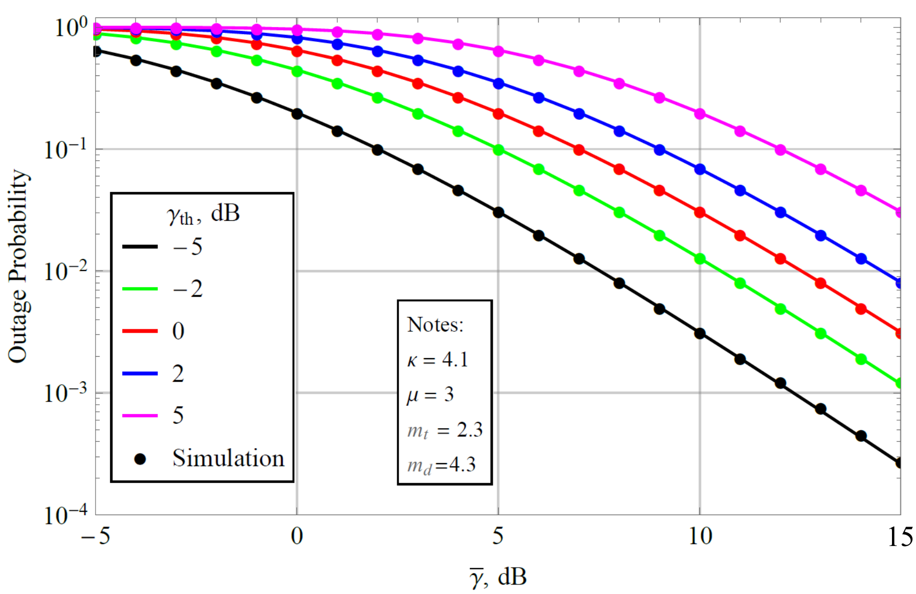

FIGURE 4. Outage probability of the double shadowed Type 1 (example 2) model versus $\bar{\gamma}$ for different γ_{th}. Lines denote analytical results and circle markers denote simulation results.

i.e.

$$f_R(r) \triangleq \int_0^\infty f_{R|\xi}(r|\xi) f_\xi(\xi)\, d\xi. \tag{51}$$

The signal model for the single shadowed κ-μ Type I (example 2) fading model, insinuates that the conditional probability, $f_{R|\xi}(r|\xi)$, follows a κ-μ distribution with PDF [1]

$$f_{R|\xi}(r|\xi) = \frac{r^\mu}{\sigma^2(\xi d)^{\mu-1}} \mathrm{e}^{\frac{-r^2-\xi^2 d^2}{2\sigma^2}} \mathrm{I}_{\mu-1}\left(\frac{\xi d r}{\sigma^2}\right) \tag{52}$$

where d^2 and σ^2 are as defined in section II.A (also see [1]), and $\mathrm{I}_\nu(\cdot)$ is the modified Bessel function of the first kind and order ν.

An analytical expression for the PDF of the single shadowed κ-μ Type I (example 2) fading model can be obtained by substituting (52) and (50) into (51) as follows:

$$f_R(r) = \int_0^\infty \frac{2r^\mu (m_d-1)^{m_d}\, \mathrm{e}^{-\frac{(m_d-1)}{\xi^2} - \frac{r^2+\xi^2 d^2}{2\sigma^2}}}{\sigma^2(\xi d)^{\mu-1}\Gamma(m_d)\,\xi^{2m_d+1}} \mathrm{I}_{\mu-1}\left(\frac{\xi d r}{\sigma^2}\right) d\xi. \tag{53}$$

Now replacing the modified Bessel function of the first kind with [22, 03.02.02.0001.01] in (53), followed by solving the integral using [20, eq. 3.471.9], and finally substituting $d = \sqrt{2\mu\sigma^2\kappa}$; $\sigma = \sqrt{\frac{\hat{r}^2}{2\mu(1+\kappa)}}$ in the resultant expression, we obtain the PDF of the single shadowed κ-μ Type I (example 2) fading model shown in (5).

Substituting the SNR PDF (see (6)) of the single shadowed κ-μ Type I (example 2) fading model in $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\, dt$, changing the order of integration and summation, solving the integral using [21, eq. 6.5.1] and finally performing some algebraic manipulations, we obtain the CDF of the single shadowed κ-μ Type I (example 2) fading model shown in (7). This completes the proof.

APPENDIX B
PROOF OF THEOREM 2 AND LEMMA 2

If ξ is a Nakagami-m RV with shape parameter m_s and $\mathbb{E}\left[\xi^2\right] = 1$, its PDF is given by

$$f_\xi(\xi) = \frac{2m_s^{m_s}\xi^{2m_s-1}}{\Gamma(m_s)} \mathrm{e}^{-m_s\xi^2}. \tag{54}$$

The signal model presented in (8) insinuates that the conditional probability, $f_{R|\xi}(r|\xi)$, follows a κ-μ distribution with PDF [1]

$$f_{R|\xi}(r|\xi) = \frac{r^\mu}{\sigma^2\xi^2 d^{\mu-1}} \mathrm{e}^{\frac{-r^2-d^2}{2\sigma^2\xi^2}} \mathrm{I}_{\mu-1}\left(\frac{dr}{\sigma^2\xi^2}\right). \tag{55}$$

An analytical expression for the PDF of the single shadowed κ-μ Type II (example 1) fading model can be obtained by substituting (55) and (54) into (51) as

$$f_R(r) = \int_0^\infty \frac{2r^\mu m_s^{m_s}\xi^{2m_s-3}}{\sigma^2 d^{\mu-1}\Gamma(m_s)} \mathrm{e}^{-m_s\xi^2 - \frac{r^2+d^2}{2\sigma^2\xi^2}} \mathrm{I}_{\mu-1}\left(\frac{dr}{\sigma^2\xi^2}\right) d\xi. \tag{56}$$

Now replacing the modified Bessel function of the first kind with its series representation [22, 03.02.02.0001.01] in (56), solving the integral using [20, eq. 3.471.9], and finally substituting $d = \sqrt{2\mu\sigma^2\kappa}$; $\sigma = \sqrt{\frac{\hat{r}^2}{2\mu(1+\kappa)}}$ in the resultant expression, we obtain (9).

Substituting the SNR PDF (see (10)) of the single shadowed κ-μ Type II (example 1) fading model in $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\, dt$, changing the order of integration and summation, expressing the modified Bessel function of the second kind using its power series representation [21, eq. 9.6.2 and 9.6.10], and again changing the order of integration and summation, we obtain (57), shown at the bottom of the next page. The inner integral can be solved using [31, eq. 2.2.6.1]. Now performing some algebraic manipulations the CDF can be expressed as (58), shown at the bottom of the next page. Next, let us define $\tilde{S}_1$ and $\tilde{S}_2$ as follows.

$$\tilde{S}_1 = \sum_{i=0}^\infty\sum_{j=0}^\infty \frac{1}{i!\Gamma(i+\mu+1)} \frac{(\kappa\mu m_s)^j}{j!\Gamma(-2i+j+m_s-\mu+1)} \times \left(\frac{\gamma(1+\kappa)}{\bar{\gamma}\kappa}\right)^i {}_2F_1\left[i+\mu, 2i-j+\mu-m_s, 1+i+\mu, -\frac{\gamma(1+\kappa)}{\bar{\gamma}\kappa}\right], \tag{59}$$

and

$$\tilde{S}_2 = \sum_{i=0}^\infty\sum_{j=0}^\infty \frac{1}{i!\Gamma(i+\mu+1)} \frac{(\kappa\mu m_s)^j}{j!\Gamma(2i+j-m_s+\mu+1)} \times \left(\frac{\gamma\kappa(1+\kappa)(m_s\mu)^2}{\bar{\gamma}}\right)^i {}_2F_1\left[-j, i+\mu, 1+i+\mu, -\frac{\gamma(1+\kappa)}{\bar{\gamma}\kappa}\right]. \tag{60}$$

Then the corresponding CDF can be rewritten as:

$$F_\gamma(\gamma) = \frac{(m_s\mu)^{\frac{m_s+\mu}{2}}(1+\kappa)^\mu\pi}{\Gamma(m_s)\bar{\gamma}^\mu \sin[(\mu-m_s)\pi]} \times \left[\gamma^\mu\left(\kappa\sqrt{\mu m_s}\right)^{-\mu+m_s}\tilde{S}_1 - \left(\sqrt{\mu m_s}\right)^{\mu-m_s}\gamma^\mu\tilde{S}_2\right]. \tag{61}$$

Considering $\tilde{S}_1$, we rewrite the Gauss hypergeometric function in terms of its contour integral[10] representation i.e,

$$\begin{aligned}
&{}_2F_1\left[i+\mu, 2i-j+\mu-m_s, 1+i+\mu, -\frac{\gamma(1+\kappa)}{\bar{\gamma}\kappa}\right] \\
&= \frac{\Gamma(1+i+\mu)}{\Gamma(i+\mu)\Gamma(2i-j+\mu-m_s)} \frac{1}{2\pi\mathrm{j}} \oint_{\mathcal{L}} \frac{\Gamma(t_1)\Gamma(i+\mu-t_1)}{\Gamma(1+i+\mu-t_1)} \\
&\quad\times \Gamma(2i-j+\mu-m_s-t_1)\left(\frac{\gamma(1+\kappa)}{\kappa\bar{\gamma}}\right)^{-t_1} \mathrm{d}t_1
\end{aligned} \tag{62}$$

[10]Note that in some equations, the gamma function in the contour integral kernel may render an indeterminate value. However, this does not present a problem here as the appropriate choice of contours required to solve these integrals must exclude any indeterminate results. The interested reader is directed to [32] for a comprehensive discussion on the appropriate contour choice.

where $\mathbb{j} = \sqrt{-1}$ is the imaginary unit and $\mathcal{L}$ is a suitable contour in the complex space. Now simplifying $\tilde{S}_1$, we obtain

$$\tilde{S}_1 = \sum_{j=0}^{\infty} \frac{(\kappa\mu m_s)^j}{j!} \sum_{i=0}^{\infty} \frac{1}{i!\Gamma(i+\mu)\Gamma(2i-j+\mu-m_s)} \times \frac{1}{\Gamma(1-2i+j-\mu+m_s)} \left(\frac{\gamma(1+\kappa)}{\kappa\bar{\gamma}}\right)^i \frac{1}{2\pi\mathbb{j}} \oint_{\mathcal{L}} \Gamma(t_1) \times \frac{\Gamma(-t_1+\mu+i)\Gamma(2i-j+\mu-m_s-t_1)}{\Gamma(1+i+\mu-t_1)} \times \left(\frac{\gamma(1+\kappa)}{\kappa\bar{\gamma}}\right)^{-t_1} \mathrm{d}t_1. \quad (63)$$

Furthermore, using the Residue Theorem, we represent the summation over the index i as another contour integral, keeping in mind the following relationship: $\underset{t\to -i}{Res}\,\Gamma(t) f(t) = \frac{(-1)^i}{i!} f(-i)$, to obtain

$$\tilde{S}_1 = \sum_{j=0}^{\infty} \frac{(\kappa\mu m_s)^j}{j!} \left(\frac{1}{2\pi\mathbb{j}}\right)^2 \oint_{\mathcal{L}} \frac{\Gamma(t_1)\Gamma(t_2)\Gamma(-t_1-t_2+\mu)}{\Gamma(-t_2+\mu)\Gamma(1-t_1-t_2+\mu)} \times \frac{\Gamma(-j-t_1-2t_2+\mu-m_s)}{\Gamma(-j-2t_2+\mu-m_s)\Gamma(1+j+2t_2-\mu+m_s)} \times \left(\frac{\gamma(1+\kappa)}{\kappa\bar{\gamma}}\right)^{-t_1} \left(-\frac{\gamma(1+\kappa)}{\kappa\bar{\gamma}}\right)^{-t_2} \mathrm{d}t_1 \mathrm{d}t_2. \quad (64)$$

Applying transformation of integration variables such that $t_1 \to u_1 - u_2$ and $t_2 \to u_2$, using the sum of residues to evaluate the integral on variable u_2, followed by using [28, eq. 7.3.5.2] and simplifying, we obtain

$$\tilde{S}_1 = \frac{\sin[\pi(\mu-m_s)]}{\pi} \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(j+m_s)(\kappa\mu m_s)^j}{j!} \frac{1}{2\pi\mathbb{j}} \times \oint_{\mathcal{L}} \frac{\Gamma(u_1)\Gamma(-j-u_1+\mu-m_s)}{\Gamma(1-u_1+\mu)\Gamma(j+u_1+m_s)} \times \left(\frac{\gamma(1+\kappa)}{\kappa\bar{\gamma}}\right)^{-u_1} \mathrm{d}u_1, \quad \mu - m_s \notin \mathbb{Z} \quad (65)$$

where the integral over u_1 can be interpreted as a Meijer G function resulting in

$$\tilde{S}_1 = \frac{\sin[\pi(\mu-m_s)]}{\pi} \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma(j+m_s)(\kappa\mu m_s)^j}{j!} \times G_{2,2}^{1,1}\left(\frac{\gamma(1+\kappa)}{\kappa\bar{\gamma}} \middle| \begin{matrix} 1+j-\mu+m_s, j+m_s \\ 0, -\mu \end{matrix}\right), \quad \mu - m_s \notin \mathbb{Z}. \quad (66)$$

To simplify $\tilde{S}_2$, we first change the order of summation by summing over the infinite triangle $2i + j = n$ or $j = n - 2i$, then express the Gauss hypergeometric function in terms of its power series representation [22, eq. 07.23.02.0001.01], again change the order of summation ($i + k = j$ or $k = j - i$), followed by using [28, eq. 7.3.5.2] and finally simplifying, we obtain

$$\tilde{S}_2 = \sum_{n=0}^{\infty} \frac{(\kappa\mu m_s)^n}{n!\Gamma(1+n-m_s+\mu)\Gamma(\mu+1)} \times {}_2F_1\left[-n, 1-n-\mu, 1+\mu, \frac{\gamma(1+\kappa)}{\bar{\gamma}\kappa}\right]. \quad (67)$$

$$F_\gamma(\gamma) = \sum_{i=0}^{\infty} \frac{2(m_s\mu)^{\frac{2i+m_s+\mu}{2}} \kappa^i (1+\kappa)^{i+\mu}}{i!\Gamma(m_s)\Gamma(i+\mu)} \frac{1}{\bar{\gamma}^{i+\mu}} \frac{\pi}{2\sin[(2i-m_s+\mu)\pi]} \times \left[\sum_{j=0}^{\infty} \frac{1}{j!\Gamma(-2i+j+m_s-\mu+1)} \int_0^\gamma \frac{t^{i+\mu-1}}{\left(\frac{t}{\bar{\gamma}}(1+\kappa)+\kappa\right)^{\frac{2i-m_s+\mu}{2}}} \left(\sqrt{m_s\mu\left(\frac{t}{\bar{\gamma}}(1+\kappa)+\kappa\right)}\right)^{2j-2i+m_s-\mu} \mathrm{d}t - \sum_{j=0}^{\infty} \frac{1}{j!\Gamma(2i+j-m_s+\mu+1)} \int_0^\gamma \frac{t^{i+\mu-1}}{\left(\frac{t}{\bar{\gamma}}(1+\kappa)+\kappa\right)^{\frac{2i-m_s+\mu}{2}}} \left(\sqrt{m_s\mu\left(\frac{t}{\bar{\gamma}}(1+\kappa)+\kappa\right)}\right)^{2j+2i-m_s+\mu} \mathrm{d}t\right], \quad \mu - m_s \notin \mathbb{Z} \quad (57)$$

$$F_\gamma(\gamma) = \frac{(m_s\mu)^{\frac{m_s+\mu}{2}}(1+\kappa)^\mu \pi}{\Gamma(m_s)\bar{\gamma}^\mu \sin[(\mu-m_s)\pi]} \left[\frac{\gamma^\mu}{(\kappa\sqrt{\mu m_s})^{\mu-m_s}} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{1}{i!\Gamma(i+\mu+1)} \frac{(\kappa\mu m_s)^j}{j!\Gamma(m_s-\mu+1-2i+j)} \left(\frac{\gamma(1+\kappa)}{\bar{\gamma}\kappa}\right)^i \times {}_2F_1\left[i+\mu, 2i-j+\mu-m_s, 1+i+\mu, -\frac{\gamma(1+\kappa)}{\bar{\gamma}\kappa}\right] - \frac{\gamma^\mu}{(\sqrt{\mu m_s})^{m_s-\mu}} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{1}{i!\Gamma(i+\mu+1)} \times \frac{(\kappa\mu m_s)^j}{j!\Gamma(2i+j-m_s+\mu+1)} \left(\frac{\gamma\kappa(1+\kappa)(m_s\mu)^2}{\bar{\gamma}}\right)^i {}_2F_1\left[-j, i+\mu, 1+i+\mu, -\frac{\gamma(1+\kappa)}{\bar{\gamma}\kappa}\right]\right], \quad \mu - m_s \notin \mathbb{Z} \quad (58)$$

Now substituting $\tilde{S}_1$ and $\tilde{S}_2$ back into (61) and performing some algebraic manipulations, we obtain the CDF of the single shadowed κ-μ Type II (example 1) fading model shown in (11). This completes the proof.

APPENDIX C
PROOF OF THEOREM 3 AND LEMMA 3

A closed form expression for the PDF of the single shadowed κ-μ Type II (example 2) model, is obtained by substituting (55) and (50) (after replacing m_d with m_s) into (51),

$$f_R(r) = \int_0^\infty \frac{2r^\mu (m_s-1)^{m_s} \mathrm{e}^{-\frac{(m_s-1)}{\xi^2} - \frac{r^2+d^2}{2\sigma^2\xi^2}}}{\sigma^2 d^{\mu-1}\Gamma(m_s)\,\xi^{2m_s+3}} \mathrm{I}_{\mu-1}\left(\frac{dr}{\sigma^2\xi^2}\right) d\xi. \tag{68}$$

The above integral is identical to [33, eq. 2.15.3.2]. Now, substituting for d and σ in the resultant expression and performing some algebraic manipulations, we obtain (12).

Substituting the SNR PDF (see (13)) of the single shadowed κ-μ Type II (example 2) fading model in $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, expressing the Gauss hypergeometric function in terms of its power series representation [22, eq. 07.23.02.0001.01] i.e,

$$\begin{aligned} &{}_2F_1\left[\frac{m_s+\mu}{2}, \frac{m_s+\mu+1}{2}, \mu, \frac{4\mu^2\kappa(1+\kappa)t\bar{\gamma}}{(t(1+\kappa)\mu+\bar{\gamma}(m_s-1+\kappa\mu))^2}\right] \\ &\quad = \sum_{k=0}^\infty \frac{\left(\frac{m_s+\mu}{2}\right)_k \left(\frac{m_s+\mu+1}{2}\right)_k \left(4\mu^2\kappa(1+\kappa)t\bar{\gamma}\right)^k}{k!\,(\mu)_k\,(t(1+\kappa)\mu+\bar{\gamma}(m_s-1+\kappa\mu))^{2k}}, \end{aligned} \tag{69}$$

changing the order of integration and summation, followed by solving the integral using [31, eq. 2.2.6.1] and finally performing some algebraic manipulations, we obtain the CDF of the single shadowed κ-μ Type II (example 2) fading model shown in (14). This completes the proof.

APPENDIX D
PROOF OF THEOREM 4 AND LEMMA 4

We determine the envelope distribution of the single shadowed κ-μ Type III (example 1) fading model using (51). Here, the conditional probability, $f_{R|\xi}(r|\xi)$ is given by

$$f_{R|\xi}(r|\xi) = \frac{2\mu(1+\kappa)^{\frac{\mu+1}{2}} r^\mu \mathrm{e}^{\frac{-\mu(1+\kappa)r^2}{\xi^2\hat{r}^2}}}{\kappa^{\frac{\mu-1}{2}} \mathrm{e}^{\kappa\mu} \left(\xi^2\hat{r}^2\right)^{\frac{\mu+1}{2}}} \mathrm{I}_{\mu-1}\left(\frac{2\mu\sqrt{\kappa(1+\kappa)}r}{\sqrt{\xi^2\hat{r}^2}}\right). \tag{70}$$

Substituting (70) and (54) (after replacing m_s with m_t) into (51), followed by replacing the modified Bessel function of the first kind with its series representation [22, 03.02.02.0001.01], and solving the resulting integral using [20, eq. 3.471.9], we obtain (16).

Substituting the SNR PDF (see (17)) of the single shadowed κ-μ Type III (example 1) fading model in $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, changing the order of integration and summation, and expressing the modified Bessel function of the second kind in terms of the Meijer G function [28, eq. 8.4.23.1], we obtain

$$\begin{aligned} F_\gamma(\gamma) = &\sum_{i=0}^\infty \frac{(m_t\mu(1+\kappa))^{\frac{m_t+\mu+i}{2}} (\kappa\mu)^i}{\mathrm{e}^{\kappa\mu} i!\,\Gamma(m_t)\,\Gamma(i+\mu)\,\bar{\gamma}^{\frac{m_t+\mu+i}{2}}} \int_0^\gamma t^{\frac{m_t+\mu+i}{2}-1} \\ &\times G_{0,2}^{2,0}\left(\frac{(m_t\mu(1+\kappa))t}{\bar{\gamma}}\middle| \frac{-m_t+\mu+i}{2}, -\frac{-m_t+\mu+i}{2}\right) \mathrm{d}t. \end{aligned} \tag{71}$$

Expressing the Meijer G function in terms of its contour integral representation i.e,

$$\begin{aligned} &G_{0,2}^{2,0}\left(\frac{(m_t\mu(1+\kappa))t}{\bar{\gamma}}\middle| \frac{-m_t+\mu+i}{2}, -\frac{-m_t+\mu+i}{2}\right) \\ &= \frac{1}{2\pi\mathrm{j}} \oint_{\mathcal{L}} \Gamma\left(\frac{-m_t+\mu+i}{2}+x\right)\Gamma\left(-\frac{-m_t+\mu+i}{2}+x\right) \\ &\quad \times \left(\frac{(m_t\mu(1+\kappa))t}{\bar{\gamma}}\right)^{-x} \mathrm{d}x, \end{aligned} \tag{72}$$

changing the order of integration, simplifying, and interpreting the contour integral as a Meijer G function, we obtain

$$\begin{aligned} &F_\gamma(\gamma) \\ &= \sum_{i=0}^\infty \frac{(m_t\mu(1+\kappa))^{\frac{m_t+\mu+i}{2}} (\kappa\mu)^i \gamma^{\frac{m_t+\mu+i}{2}}}{\mathrm{e}^{\kappa\mu} i!\,\Gamma(m_t)\,\Gamma(i+\mu)\,\bar{\gamma}^{\frac{m_t+\mu+i}{2}}} \\ &\quad \times G_{1,3}^{2,1}\left(\frac{(m_t\mu(1+\kappa))\gamma}{\bar{\gamma}}\middle| \begin{matrix} 1-\frac{m_t+\mu+i}{2} \\ \frac{-m_t+\mu+i}{2}, -\frac{-m_t+\mu+i}{2}, -\frac{m_t+\mu+i}{2} \end{matrix}\right) \mathrm{d}t. \end{aligned} \tag{73}$$

Now performing some algebraic manipulations and using [32, eq. 1.60], we obtain the CDF of the single shadowed κ-μ Type III (example 1) fading model shown in (18). This completes the proof.

APPENDIX E
PROOF OF THEOREM 5 AND LEMMA 5

The signal envelope, R, of the double shadowed κ-μ Type I (example 2) fading model is given by (22). Here, A follows a Nakagami-m distribution with shape parameter m_t, and ξ follows an inverse Nakagami-m distribution with shape parameter m_d. It is noted that the model in (22) may be viewed as a product of a Nakagami-m RV and a single shadowed κ-μ Type I (example 2) RV. According to standard probability procedure, this PDF can be obtained as

$$f_R(r) = \int_0^\infty \frac{1}{a} f_T\left(\frac{r}{a}\right) f_A(a)\,\mathrm{d}a \tag{74}$$

where $f_T(t)$ is given in (5). Replacing the respective PDFs in (74) and changing the order of integration and summation, yields

$$\begin{aligned} &f_R(r) \\ &= \sum_{i=0}^\infty \frac{8\left[(m_d-1)\kappa\right]^{\frac{m_d+i}{2}} \mathcal{K}^{i+\mu} \mu^{\frac{i+m_d}{2}} r^{2i+2\mu-1} m_t^{m_t}}{\hat{r}^{2i+2\mu} i!\,\Gamma(m_d)\,\Gamma(i+\mu)\Gamma(m_t)} \\ &\quad \times \mathrm{K}_{m_d-i}\left(2\sqrt{(m_d-1)\mu\kappa}\right) \int_0^\infty a^{2(m_t-i-\mu)-1} \mathrm{e}^{-a^2 m_t - \frac{r^2\mathcal{K}}{a^2\hat{r}^2}}\,\mathrm{d}a. \end{aligned} \tag{75}$$

Solving this integral using [31, eq. 2.3.16.1] and simplifying, we obtain (27).

Following the same procedure used for deriving the CDF of the single shadowed κ-μ Type II (example 1) fading model (see Appendix B), the SNR CDF for the double shadowed κ-μ Type I (example 2) fading model is obtained as shown in (29). This completes the proof.

APPENDIX F
PROOF OF THEOREM 6 AND LEMMA 6

We determine the envelope distribution, R, of the double shadowed κ-μ Type II (example 1) fading model when A and B vary according to the inverse Nakagami-m and Nakagami-m distributions, respectively, from the following integral

$$f_R(r) = \int_0^\infty \int_0^\infty f_{R|\alpha,\beta}(r|\alpha,\beta) f_\alpha(\alpha) f_\beta(\beta)\, d\alpha d\beta \quad (76)$$

where

$$f_{R|\beta}(r|\beta) = \int_0^\infty f_{R|\alpha,\beta}(r|\alpha,\beta) f_\alpha(\alpha)\, d\alpha \quad (77)$$

and the double shadowed κ-μ Type II (example 1) signal model insinuates that $f_{R|\alpha,\beta}(r|\alpha,\beta)$ follows a κ-μ distribution with PDF [1]

$$f_{R|\alpha,\beta}(r|\alpha,\beta) = \frac{r^\mu}{\sigma^2\alpha^2(\beta d)^{\mu-1}} \mathrm{e}^{\frac{-r^2-(\beta d)^2}{2\sigma^2\alpha^2}} \mathrm{I}_{\mu-1}\left(\frac{\beta d r}{\sigma^2\alpha^2}\right) \quad (78)$$

whilst $f_\alpha(\alpha)$ is similar to (50) where ξ and m_d are replaced with α and m_s, respectively. Likewise, $f_\beta(\beta)$ is similar to (54) where ξ is replaced with β, and m_s is replaced with m_d. Integrating with respect to α, we obtain an expression similar to (12) conditioned on β,

$$\begin{aligned} f_R(r) &= \frac{2^{m_s+2}(m_s-1)^{m_s} r^{2\mu-1}\sigma^{2m_s} m_d^{m_d}\Gamma(m_s+\mu)}{\Gamma(m_s)\Gamma(m_d)\left(r^2+2(m_s-1)\sigma^2\right)^{m_s+\mu}} \\ &\times \sum_{i=0}^{\infty} \frac{\left(\frac{m_s+\mu}{2}\right)_i \left(\frac{m_s+\mu+1}{2}\right)_i \left(4d^2r^2\right)^i}{i!\Gamma(\mu+i)\left(r^2+2(m_s-1)\sigma^2\right)^{2i}} \int_0^\infty \beta^{2i+2m_d-1} \\ &\times \left(\frac{d^2\beta^2}{r^2+2(m_s-1)\sigma^2}+1\right)^{-2i-\mu-m_s} \mathrm{e}^{-m_d\beta^2} d\beta. \end{aligned} \quad (79)$$

Now solving the integral in (79) using [21, eq. 13.2.5], followed by substituting [22, 07.33.17.0007.01] for the hypergeometric U function (Tricomi confluent hypergeometric function), $d = \sqrt{2\mu\sigma^2\kappa}$ and $\sigma = \sqrt{\frac{\hat{r}^2}{2\mu(1+\kappa)}}$, and finally simplifying the resultant expression we obtain (31).

Substituting the SNR PDF (see (32)) of the double shadowed κ-μ Type II (example 1) fading model in $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\, dt$, changing the order of integration and summation, expressing the Tricomi hypergeometric function in terms of its power series representation [21, eq. 13.1.2 and 13.1.3], we obtain (80), shown at the bottom of the next page. Again, changing the order of integration and summation, using [31, eq. 2.2.6.1] and performing some algebraic manipulations, we obtain (81), shown at the bottom of the next page. Now, let us define $\tilde{S}_3$ and $\tilde{S}_4$ as follows:

$$\begin{aligned} \tilde{S}_3 &= \sum_{i=0}^{\infty}\sum_{k=0}^{\infty} \frac{(-1)^i\Gamma(2i+k+\mu+m_s)}{i!k!\Gamma(i+\mu+1)\Gamma(1+i+k+\mu+m_s-m_d)} \\ &\times \left(\frac{m_d(m_s-1)}{\kappa\mu}\right)^k \left(\frac{m_d\gamma(1+\kappa)}{\kappa\bar{\gamma}}\right)^i \\ &\times {}_2F_1\left[-k, i+\mu, 1+i+\mu, -\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right] \end{aligned} \quad (82)$$

and

$$\begin{aligned} \tilde{S}_4 &= \sum_{i=0}^{\infty}\sum_{k=0}^{\infty} \frac{(-1)^i\Gamma(i+k+m_d)}{i!k!\Gamma(i+\mu+1)\Gamma(1-i+k-\mu+m_d-m_s)} \\ &\times \left(\frac{m_d(m_s-1)}{\kappa\mu}\right)^k \left(\frac{\mu\gamma(1+\kappa)}{\bar{\gamma}(m_s-1)}\right)^i \\ &\times {}_2F_1\left[i+\mu, i-k+\mu-m_d+m_s, 1+i+\mu, -\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right] \end{aligned} \quad (83)$$

then the CDF can be rewritten as

$$\begin{aligned} F_\gamma(\gamma) &= \frac{(m_s-1)^{m_s} m_d^{m_s+\mu}\Gamma(\mu+m_s)(1+\kappa)^\mu\gamma^\mu}{\kappa^{m_s+\mu}\Gamma(m_s)\Gamma(m_d)\Gamma(\mu+m_s)\mu^{m_s}\bar{\gamma}^\mu} \\ &\times \frac{\pi}{\sin[\pi(\mu-m_d+m_s)]}\Bigg[-\tilde{S}_3 + m_d^{-\mu+m_d-m_s} \\ &\times \left(\frac{\kappa\mu}{m_s-1}\right)^{\mu-m_d+m_s} \tilde{S}_4\Bigg], \quad \mu-m_d+m_s \notin \mathbb{Z}. \end{aligned} \quad (84)$$

To simplify $\tilde{S}_3$, we first change the order of summation using the index transformation $i+k=n$ or $k=n-i$, then we express the Gauss hypergeometric function in terms of its power series representation [22, 07.23.02.0001.01], i.e,

$$\begin{aligned} &{}_2F_1\left[i-n, i+\mu, 1+i+\mu, -\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right] \\ &= \sum_{j=0}^{\infty} \frac{(i-n)_j(i+\mu)_j}{j!(1+i+\mu)_j}\left(-\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right)^j, \end{aligned} \quad (85)$$

again changing the order of summation using the index transformation $j=k-i$ and simplifying, we obtain

$$\begin{aligned} \tilde{S}_3 &= -\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{\Gamma(k-n)\Gamma(n+\mu+m_s)\sin(n\pi)}{\pi(k+\mu)\Gamma(1+k)\Gamma(1+n+\mu-m_d+m_s)\Gamma(\mu)} \\ &\times \left(\frac{m_d(m_s-1)}{\kappa\mu}\right)^n \left(-\frac{\gamma\mu(1+\kappa)}{\bar{\gamma}(m_s-1)}\right)^k \\ &\times {}_2F_1[-k, n+\mu+m_s, \mu, 1]. \end{aligned} \quad (86)$$

Now using [28, eq. 7.3.5.4] and performing the sum over index k, followed by some algebraic manipulations, we obtain

$$\tilde{S}_3 = \sum_{n=0}^{\infty} \frac{\Gamma(n+\mu+m_s)(m_d(m_s-1))^n}{n!\Gamma(\mu+1)\Gamma(1+n+\mu+m_s-m_d)(\kappa\mu)^n} \times {}_2F_1\left[-n,-n-m_s,1+\mu,-\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right]. \quad (87)$$

To simplify $\tilde{S}_4$, we first express the Gauss hypergeometric function in terms of its power series representation [22, eq. 07.23.02.0001.01] i.e,

$$ {}_2F_1\left[i+\mu, i-k+\mu-m_d+m_s, 1+i+\mu, -\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right] = \sum_{j=0}^{\infty} \frac{(i+\mu)_j(i-k+\mu-m_d+m_s)_j}{j!(1+i+\mu)_j}\left(-\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right)^j. \quad (88)$$

This is followed by performing the sum over the infinite triangle $i+j=n$ or $j=n-i$, simplifying, again performing the sum over index n, followed by some algebraic manipulations to obtain

$$\tilde{S}_4 = \sum_{k=0}^{\infty} \frac{\Gamma(k+m_d)(m_d(m_s-1))^k}{k!(\mu+1)\Gamma(1+k-\mu+m_d-m_s)(\kappa\mu)^k} \times {}_2F_1\left[-k+\mu-m_d, -k+\mu-m_d+m_s, 1+\mu, -\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right]. \quad (89)$$

Finally, substituting $\tilde{S}_3$ and $\tilde{S}_4$ back into (84) and simplifying, we obtain the CDF of the double shadowed κ-μ Type II (example 1) fading model shown in (33). This completes the proof.

APPENDIX G
PROOF OF THEOREM 7 AND LEMMA 7

The envelope distribution, R, of the double shadowed κ-μ Type II (example 2) fading model when A and B vary according to Nakagami-m and inverse Nakagami-m distributions, respectively, can be obtained through (76). The double shadowed κ-μ Type II (example 2) signal model presented in (30) insinuates that $f_{R|\alpha,\beta}(r|\alpha,\beta)$ follows a κ-μ distribution with PDF given in (78). Also, $f_\alpha(\alpha)$ is similar to (54) with ξ replaced by α, and $f_\beta(\beta)$ is similar to (50) with ξ replaced by β. Integrating with respect to β, we obtain

$$f_R(r) = \frac{8\mathcal{K}^\mu(\kappa\mu(m_d-1))^{\frac{m_d}{2}} m_s^{m_s} r^{2\mu-1}}{\Gamma(m_d)\Gamma(m_s)\hat{r}^{2\mu}} \sum_{i=0}^{\infty}\frac{1}{i!\Gamma(\mu+i)} \times \left(\frac{r^2\mathcal{K}\sqrt{\kappa\mu(m_d-1)}}{\hat{r}^2}\right)^i \int_0^\infty \alpha^{-1-3i-2\mu-m_d+2m_s} \times e^{-\left(\alpha^2 m_s+\frac{r^2\mathcal{K}}{\alpha^2\hat{r}^2}\right)} \mathrm{K}_{i-m_d}\left(\frac{2\sqrt{\kappa\mu(m_d-1)}}{\alpha}\right) d\alpha. \quad (90)$$

$$F_\gamma(\gamma) = \frac{(m_s-1)^{m_s} m_d^{m_s+\mu}\Gamma(\mu+m_s)(1+\kappa)^\mu}{\kappa^{m_s+\mu}\Gamma(m_s)\Gamma(m_d)\mu^{m_s}\bar{\gamma}^\mu} \sum_{i=0}^{\infty}\frac{\Gamma(2i+\mu+m_s)\Gamma(i+m_d)}{i!\Gamma(i+\mu)\Gamma(\mu+m_s)} \int_0^\gamma t^{\mu-1}\left(\frac{m_d(1+\kappa)t}{\kappa\bar{\gamma}}\right)^i \times \frac{\pi(-1)^{i+1}}{\sin[\pi(\mu-m_d+m_s)]}\left(\frac{1}{\Gamma(i+m_d)\Gamma(1+i+\mu-m_d+m_s)}\sum_{k=0}^{\infty}\frac{(2i+\mu+m_s)_k}{k!(1+i+\mu-m_d+m_s)_k}\left(\frac{m_d(\bar{\gamma}(m_s-1)+t(1+\kappa)\mu)}{\kappa\mu\bar{\gamma}}\right)^k - \left(\frac{m_d(\bar{\gamma}(m_s-1)+t(1+\kappa)\mu)}{\kappa\mu\bar{\gamma}}\right)^{-i-\mu+m_d-m_s}\frac{1}{\Gamma(1-i-\mu+m_d-m_s)\Gamma(2i+\mu+m_s)}\sum_{k=0}^{\infty}\frac{(i+m_d)_k}{k!(1-i-\mu+m_d-m_s)_k} \times \left(\frac{m_d(\bar{\gamma}(m_s-1)+t(1+\kappa)\mu)}{\kappa\mu\bar{\gamma}}\right)^k\right)dt, \quad \mu-m_d+m_s\notin\mathbb{Z} \quad (80)$$

$$F_\gamma(\gamma) = \frac{(m_s-1)^{m_s} m_d^{m_s+\mu}\Gamma(\mu+m_s)(1+\kappa)^\mu\gamma^\mu}{\kappa^{m_s+\mu}\Gamma(m_s)\Gamma(m_d)\Gamma(\mu+m_s)\mu^{m_s}\bar{\gamma}^\mu}\frac{\pi}{\sin[\pi(\mu-m_d+m_s)]}\left(-\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^i\Gamma(2i+k+\mu+m_s)}{i!k!\Gamma(1+i+\mu)} \times \frac{1}{\Gamma(1+i+k+\mu-m_d+m_s)}\left(\frac{m_d(m_s-1)}{\kappa\mu}\right)^k\left(\frac{\gamma m_d(1+\kappa)}{\kappa\bar{\gamma}}\right)^i {}_2F_1\left[-k,i+\mu,1+i+\mu,-\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right] + m_d^{-\mu+m_d-m_s}\left(\frac{\kappa\mu}{m_s-1}\right)^{\mu-m_d+m_s}\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^i\Gamma(i+k+m_d)}{i!k!\Gamma(1+i+\mu)\Gamma(1-i+k-\mu+m_d-m_s)}\left(\frac{m_d(m_s-1)}{\kappa\mu}\right)^k \times \left(\frac{\gamma\mu(1+\kappa)}{\bar{\gamma}(m_s-1)}\right)^i {}_2F_1\left[i+\mu,i-k+\mu-m_d+m_s,1+i+\mu,-\frac{\gamma(1+\kappa)\mu}{\bar{\gamma}(m_s-1)}\right]\right), \quad \mu-m_d+m_s\notin\mathbb{Z} \quad (81)$$

This integral is solved by replacing the Bessel function with its power series [22, 03.04.06.0002.01] and changing the order of integration and summation. Now using [31, eq. 2.3.16.1] we obtain (91), shown at the bottom of the page. Representing the Bessel function in (91) using the Meijer G function [22, 03.04.26.0008.01] and performing some algebraic manipulations we obtain

$$f_R(r) = \frac{\csc(\pi m_d)\, 2\pi \mathcal{K}^{m_s} r^{2m_s-1}}{\Gamma(m_d)\,\Gamma(m_s)\, m_s^{-m_s}\hat{r}^{2m_s}}\Bigg[S_1' - (\kappa\mu(m_d-1))^{m_d} \times \left(\frac{r^2\mathcal{K}}{\hat{r}^2}\right)^{-m_d} S_2'\Bigg], \quad m_d \notin \mathbb{Z} \tag{92}$$

where

$$S_1' = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{(-1)^i}{i!j!\Gamma(i+\mu)} \frac{(\kappa\mu m_s(m_d-1))^{i+j}}{\Gamma(1+i+j-m_d)} \times \left(\frac{\hat{r}^2}{r^2 m_s\mathcal{K}}\right)^{i+j} G_{0,2}^{2,0}\left(\frac{r^2 m_s\mathcal{K}}{\hat{r}^2}\middle|\, 0,\ 2i+j+\mu-m_s\right) \tag{93}$$

and

$$S_2' = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{(-1)^i}{i!j!\Gamma(i+\mu)} \frac{(\kappa\mu m_s(m_d-1))^{j}}{\Gamma(1-i+j+m_d)} \times \left(\frac{\hat{r}^2}{r^2 m_s\mathcal{K}}\right)^{j} G_{0,2}^{2,0}\left(\frac{r^2 m_s\mathcal{K}}{\hat{r}^2}\middle|\, 0,\ i+j+\mu+m_d-m_s\right). \tag{94}$$

The first double summation, S_1', can be reduced by using the index transformation $j = n - i$ and the identity [32, eq. 1.60] as follows

$$S_1' = \sum_{n=0}^{\infty} \frac{(\kappa\mu m_s(m_d-1))^n}{\Gamma(1+n-m_d)}\left(\frac{\hat{r}^2}{r^2 m_s\mathcal{K}}\right)^n \times \sum_{i=0}^{n}\frac{(-1)^i}{i!(n-i)!\Gamma(i+\mu)} \times G_{0,2}^{2,0}\left(\frac{r^2 m_s\mathcal{K}}{\hat{r}^2}\middle|\, 0,\ i+n+\mu-m_s\right). \tag{95}$$

Note that the sum on the index i can be expressed in the form of [28, eq. 5.3.8.5]. Following this and performing some algebraic manipulations, we obtain

$$S_1' = \left(\frac{r^2 m_s\mathcal{K}}{\hat{r}^2}\right)^{-m_s} \sum_{n=0}^{\infty}\frac{(-1)^n(\kappa\mu m_s(m_d-1))^n}{n!\Gamma(n+\mu)\Gamma(1+n-m_d)} \times G_{1,3}^{3,0}\left(\frac{r^2 m_s\mathcal{K}}{\hat{r}^2}\middle|\begin{matrix}1-n\\ 1,\ \mu,\ -n+m_s\end{matrix}\right). \tag{96}$$

The second double summation, S_2', is reduced by rewriting the Meijer G function in terms of its contour integral representation [22, eq. 07.34.07.0001.01] and shifting the order of integration and summation as follows

$$S_2' = \sum_{j=0}^{\infty}\frac{(\kappa\mu m_s(m_d-1))^j}{j!}\left(\frac{\hat{r}^2}{r^2 m_s\mathcal{K}}\right)^j \frac{1}{2\pi\mathbb{j}} \times \oint_{\mathcal{L}} \frac{\Gamma(x)\,\hat{r}^{2x}}{(r^2 m_s\mathcal{K})^x}\sum_{i=0}^{\infty}\frac{(-1)^i\Gamma(i+j+\mu+m_d-m_s+x)}{i!\Gamma(i+\mu)\Gamma(1-i+j+m_d)}\,dx. \tag{97}$$

The inner sum is solved in closed form as

$$S_2' = \sum_{j=0}^{\infty}\frac{(\kappa\mu\hat{r}^2 m_s(m_d-1))^j}{j!\,(r^2 m_s\mathcal{K})^j}\frac{1}{2\pi\mathbb{j}}\oint_{\mathcal{L}}\frac{\Gamma(x)\,\hat{r}^{2x}}{(r^2 m_s\mathcal{K})^x} \times \frac{\Gamma(j+x+\mu+m_d-m_s)\,\Gamma(m_s-x)}{\Gamma(1+j+m_d)\,\Gamma(j+\mu+m_d)\,\Gamma(-j-x-m_d+m_s)}dx \tag{98}$$

and the contour integral is interpreted as a Meijer G function [22, eq. 07.34.07.0001.01] as shown below

$$S_2' = \sum_{j=0}^{\infty}\frac{(\kappa\mu m_s(m_d-1))^j}{j!\Gamma(1+j+m_d)\Gamma(j+\mu+m_d)}\left(\frac{\hat{r}^2}{r^2 m_s\mathcal{K}}\right)^j \times G_{1,3}^{2,1}\left(\frac{r^2 m_s\mathcal{K}}{\hat{r}^2}\middle|\begin{matrix}1-m_s\\ 0,\ j+\mu+m_d-m_s,\ 1+j+m_d-m_s\end{matrix}\right). \tag{99}$$

Now, performing some algebraic manipulations we obtain

$$S_2' = \sum_{j=0}^{\infty}\frac{(\kappa\mu m_s(m_d-1))^j}{j!\Gamma(1+j+m_d)\Gamma(j+\mu+m_d)} \times G_{1,3}^{2,1}\left(\frac{r^2 m_s\mathcal{K}}{\hat{r}^2}\middle|\begin{matrix}1-m_s-j\\ -j,\ \mu+m_d-m_s,\ 1+m_d-m_s\end{matrix}\right). \tag{100}$$

Substituting S_1' and S_2' obtained in (96) and (100) into (92) and simplifying, we obtain the double shadowed κ-μ Type II (example 2) PDF shown in (34).

Substituting the SNR PDF (see (35)) of the double shadowed κ-μ Type II (example 2) fading model in $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, changing the order of integration and summation, expressing the Meijer G functions in terms of their contour integral representations, changing the order of integration and finally, reinterpreting the contour integrals as Meijer G functions, we obtain the CDF of the double shadowed

$$f_R(r) = \frac{4\pi\mathcal{K}^\mu m_s^{m_s} r^{2\mu-1}\hat{r}^{-2\mu}}{\sin(\pi m_d)\,\Gamma(m_d)\,\Gamma(m_s)}\Bigg[\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^i m_s^i}{i!j!\Gamma(i+\mu)}\frac{(\kappa\mu(m_d-1))^{j+i}}{\Gamma(1+i+j-m_d)}\left(\frac{r^2\mathcal{K}}{\hat{r}^2 m_s}\right)^{\frac{m_s-j-\mu}{2}} \mathrm{K}_{-2i-j-\mu+m_s}\left(\frac{2r\sqrt{\mathcal{K}m_s}}{\hat{r}}\right) - \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^i m_s^i}{i!j!\Gamma(i+\mu)}\frac{(\kappa\mu(m_d-1))^{j+m_d}}{\Gamma(1-i+j+m_d)}\left(\frac{r^2\mathcal{K}}{\hat{r}^2 m_s}\right)^{\frac{i-j-\mu-m_d+m_s}{2}} \mathrm{K}_{-i-j-\mu-m_d+m_s}\left(\frac{2r\sqrt{\mathcal{K}m_s}}{\hat{r}}\right)\Bigg], \quad m_d \notin \mathbb{Z} \tag{91}$$

κ-μ Type II (example 2) fading model shown in (36). This completes the proof.

APPENDIX H
PROOF OF THEOREM 8 AND LEMMA 8

The double shadowed κ-μ Type III (example 1) model can be viewed as a product of a Nakagami-m RV and a single shadowed κ-μ Type II (example 1) RV. Accordingly, its PDF can be obtained by first replacing (12) with the hypergeometric function expressed in terms of its power series expression [21, eq. 15.1.1], then substituting the resultant expression and (54) (after replacing ξ and m_s with α and m_t) in (74), changing the order of integration and summation, and finally followed by some algebraic manipulations as

$$f_R(r) = \frac{4\,(m_s-1)^{m_s}\, m_t^{m_t}\,\hat{r}^{2\,m_s}}{\mathcal{K}^{m_s}\Gamma\,(m_t)\,\mathrm{B}\,(m_s,\mu)\,r^{2\,m_s+1}} \sum_{i=0}^{\infty} \frac{\left(\frac{m_s+\mu}{2}\right)_i (\theta_1)_i}{i!(\mu)_i} \times \left(\frac{4\kappa\hat{r}^2}{r^2(1+\kappa)}\right)^i \int_0^\infty \alpha^{-1+2\,i+2\,m_s+2\,m_t} \times \left(1+\frac{\alpha^2\hat{r}^2\,(m_s-1+\kappa\mu)}{r^2(1+\kappa)\mu}\right)^{-2\,i-\mu-m_s} \mathrm{e}^{-m_t\alpha^2}\mathrm{d}\alpha. \tag{101}$$

Solving this integral using [21, eq. 13.2.5] followed by some algebraic manipulations yields (38).

Substituting the SNR PDF (see (39)) of the double shadowed κ-μ Type III (example 1) fading model in $F_\gamma\,(\gamma) = \int_0^\gamma f_\gamma\,(t)\,dt$, changing the order of integration and summation, expressing the Tricomi confluent hypergeometric function in terms of its Meijer G representation [22, eq. 07.33.26.0004.01], using [28, eq. 1.16.2.1], and simplifying, we obtain the CDF of the double shadowed κ-μ Type III (example 1) fading model shown in (40). This completes the proof.

APPENDIX I
PROOF OF THEOREM 9 AND LEMMA 9

The double shadowed κ-μ Type III (example 2) model can be viewed as a single shadowed κ-μ Type III (example 2) model in which the variation of the scattered waves is influenced by a Nakagami-m RV. The PDF of the envelope R can be obtained via

$$f_R(r) = \int_0^\infty f_{R|\alpha}(r|\alpha) f_\alpha(\alpha)\,d\alpha \tag{102}$$

where $f_\alpha(\alpha)$ is similar to (54) such that ξ is replaced by α. $f_{R|\alpha}(r|\alpha)$ can be obtained from (19) by first replacing $\kappa = d^2/(2\mu\sigma^2)$ and $\hat{r} = 2\mu\sigma^2 + d^2$, then multiplying σ by α and finally using $d = \sqrt{2\mu\sigma^2\kappa}$; $\sigma = \sqrt{\frac{\hat{r}^2}{2\mu(1+\kappa)}}$ as follows:

$$f_{R|\alpha}(r|\alpha) = \frac{2(m_t-1)^{m_t}\hat{r}^{2m_t}\alpha^{2m_t}}{((1+\kappa)\mu)^{m_t} B\,(m_t,\mu)\,r^{1+2m_t}} \times \left(1+\frac{\alpha^2\hat{r}^2(m_t-1)}{r^2(1+\kappa)\mu}\right)^{-\mu-m_t} \exp\left(-\frac{\kappa\mu}{\alpha^2}\right) \times {}_1F_1\left(\mu+m_t;\mu;\frac{\kappa\mu}{\alpha^2}\left(1+\frac{\alpha^2\hat{r}^2(m_t-1)}{r^2(1+\kappa)\mu}\right)^{-1}\right). \tag{103}$$

Now substituting (103) into (102) we obtain

$$f_R(r) = \frac{4((1+\kappa)\mu)^{-m_t}(m_t-1)^{m_t} m_s^{m_s} r^{-1-2m_t}}{B\,(m_t,\mu)\,\Gamma\,(m_s)\,\hat{r}^{-2m_t}} \times \int_0^\infty \frac{\alpha^{-1+2m_t+2m_s}}{\exp\left(\alpha^2 m_s + \frac{\kappa\mu}{\alpha^2}\right)} \left(1+\frac{\alpha^2\hat{r}^2(m_t-1)}{r^2(1+\kappa)\mu}\right)^{-\mu-m_t} \times {}_1F_1\left(\mu+m_t;\mu;\frac{\kappa\mu}{\alpha^2}\left(1+\frac{\alpha^2\hat{r}^2(m_t-1)}{r^2(1+\kappa)\mu}\right)^{-1}\right) d\alpha. \tag{104}$$

It is possible to rewrite the hypergeometric function in terms of its Mellin-Barnes contour integral representation using [28, eq. 7.2.3.12], whilst the exponential function can be written as a product of two contour integrals using [28, eq. 8.4.3.1 and eq. 8.4.3.2] and [28, eq. 8.2.1.1]. Now performing some algebraic manipulations we obtain

$$f_R(r) = \frac{4\mathcal{K}^{-m_t}(m_t-1)^{m_t}\hat{r}^{2\,m_t}}{\Gamma\,(m_t)\,\Gamma\,(m_s)\,r^{1+2\,m_t}} \left(\frac{1}{2\pi\mathbb{j}}\right)^3 \int_0^\infty \oint_{\mathcal{L}} \Gamma\,(t_1) \times \frac{\Gamma\,(-t_2)\,\Gamma\,(-t_3)\,\Gamma\,(t_3+\mu+m_t)\,(\kappa\mu)^{t_2+t_3}}{\Gamma\,(t_3+\mu)\,(-1)^{t_3} m_s^{t_1-m_s}} \times \alpha^{2\theta_3-1}\left(1+\frac{\alpha^2\hat{r}^2(m_t-1)}{r^2\mathcal{K}}\right)^{-t_3-\mu-m_t} \mathrm{d}t_1\mathrm{d}t_2\mathrm{d}t_3\mathrm{d}\alpha \tag{105}$$

where $\theta_3 = m_t + m_s - t_1 - t_2 - t_3$, $\mathcal{K}$, $\mathbb{j}$ and $\mathcal{L}$ are as defined previously. Now changing the order of integration, the inner integral can be solved using [31, eq. 2.2.5.24], which results in the triple contour integral,

$$f_R(r) = \frac{2(m_s\mathcal{K})^{m_s} r^{2\,m_s-1}}{\Gamma\,(m_t)\,\Gamma\,(m_s)\,(m_t-1)^{m_s}\hat{r}^{2\,m_s}} \left(\frac{1}{2\pi\mathbb{j}}\right)^3 \times \oint_{\mathcal{L}} \Theta(t_1,t_2,t_3)\left(\frac{r^2\mathcal{K}m_s}{\hat{r}^2(m_t-1)}\right)^{-t_1} \times \left(\frac{r^2\mathcal{K}}{\kappa\mu\hat{r}^2(m_t-1)}\right)^{-t_2}\left(\frac{-r^2\mathcal{K}}{\kappa\mu\hat{r}^2(m_t-1)}\right)^{-t_3} \mathrm{d}t_1\mathrm{d}t_2\mathrm{d}t_3 \tag{106}$$

where

$$\Theta(t_1,t_2,t_3) = \frac{\Gamma\,(t_1)\,\Gamma\,(-t_2)\,\Gamma\,(-t_3)\,\Gamma\,(\theta_3)\,\Gamma\,(\theta_4)}{\Gamma\,(t_3+\mu)} \tag{107}$$

and $\theta_4 = t_1 + t_2 + 2\,t_3 + \mu - m_s$. It is possible to obtain a multi-fold series representation from the above contour integral using the sum of residues theorem. The residues for the variable t_1 are taken around the poles of $\Gamma(t_1)$ and $\Gamma(\theta_4)$; the residues for t_2 are taken from $\Gamma(-t_2)$ and $\Gamma(\theta_3)$; and the residues for t_3 are taken from $\Gamma(-t_3)$. This results in

$$f_R(r) = \frac{2\,(\mathcal{K}m_s/(m_t-1))^{m_s}\, r^{2m_s-1}}{\Gamma\,(m_t)\,\Gamma\,(m_s)\,\hat{r}^{2m_s}}(S_1+S_2+S_3) \tag{108}$$

$$S_1 = \sum_{i,j,k'=0}^{\infty} \frac{(-1)^{i+j}\Gamma\left(-i+j+2\,k'+\mu-m_s\right)\Gamma\left(i-j-k'+m_t+m_s\right)}{i!j!k'!\Gamma(k'+\mu)} \left(\frac{r^2\mathcal{K}m_s}{\hat{r}^2(m_t-1)}\right)^i \left(\frac{\kappa\mu\hat{r}^2(m_t-1)}{r^2\mathcal{K}}\right)^{j+k'}, \quad \mu-m_s \notin \mathbb{Z} \tag{109}$$

$$S_2 = \sum_{i,j,k'=0}^{\infty} \frac{(-1)^{i+j}\Gamma\left(j+k'+\mu+m_t\right)\Gamma\left(-i-j+k'-m_t-m_s\right)}{i!j!k'!\Gamma(k'+\mu)} (\kappa\mu m_s)^i \left(\frac{\kappa\hat{r}^2(m_t-1)}{r^2(1+\kappa)}\right)^{j+m_t+m_s}, \quad m_t+m_s \notin \mathbb{Z} \tag{110}$$

$$S_3 = \sum_{i,j,k'=0}^{\infty} \frac{(-1)^{i+j}\Gamma\left(i+k'+\mu+m_t\right)\Gamma\left(-i-j-2\,k'-\mu+m_s\right)}{i!j!k'!\Gamma(k'+\mu)(\kappa\mu m_s)^{-j}} \left(\frac{r^2\mathcal{K}m_s}{\hat{r}^2(m_t-1)}\right)^{i+2\,k'+\mu-m_s} \left(\frac{\kappa\mu\hat{r}^2(m_t-1)}{r^2\mathcal{K}}\right)^{k'}, \quad m_s-\mu \notin \mathbb{Z} \tag{111}$$

where S_1, S_2 and S_3 are given at the top of the page. The first triple summation S_1 can be reduced by summing over the infinite triangle $j = n - k$ and using [31, eq. 4.2.5.55] as

$$S_1 = \sum_{i,n=0}^{\infty} \frac{\pi\,\csc\left(\pi\left(\mu-m_s\right)\right)\Gamma\left(i-n+m_t+m_s\right)}{i!n!\Gamma(n+\mu)\Gamma\left(1+i-n-\mu+m_s\right)} \times \frac{\Gamma\left(i+m_s\right)}{\Gamma\left(i-n+m_s\right)}\left(\frac{\kappa\mu\hat{r}^2(m_t-1)}{r^2\mathcal{K}}\right)^n \times \left(\frac{r^2\mathcal{K}m_s}{\hat{r}^2(m_t-1)}\right)^i, \quad \mu-m_s \notin \mathbb{Z}. \tag{112}$$

The triple summation S_2 can be simplified by summing it over index k'. This results in a Gauss hypergeometric function whose argument is one. Using [28, eq. 7.3.5.2], we obtain

$$S_2 = \sum_{i,j=0}^{\infty} \frac{(-1)^{i+j}\Gamma\left(-i-j-m_t-m_s\right)\Gamma\left(i+m_s\right)}{i!j!\Gamma\left(i+j+\mu+m_t+m_s\right)} \times \frac{\Gamma\left(j+\mu+m_t\right)}{\Gamma\left(-j-m_t\right)}\left(\frac{\kappa\mu\hat{r}^2(m_t-1)}{r^2\mathcal{K}}\right)^{j+m_t+m_s} \times \left(\kappa\mu m_s\right)^i, \quad m_t+m_s \notin \mathbb{Z}. \tag{113}$$

To reduce S_3, it is required to first perform the variable transformation $j = n - k'$ followed by $i = j - k'$, then performing some algebraic manipulations the inner sum on the index k' is solved using [33, eq. 4.2.5.25] as follows:

$$\sum_{k'=0}^{n}\frac{1}{(j-k')!k'!(n-k')!\Gamma(k'+\mu)} = \sum_{k'=0}^{n}\frac{\binom{n}{k'}\binom{j+\mu-1}{k'+\mu-1}}{n!\Gamma(j+\mu)} = \frac{\Gamma(j+n+\mu)}{j!n!\Gamma(j+\mu)\Gamma(n+\mu)}. \tag{114}$$

Now performing some algebraic manipulations, we obtain

$$S_3 = \sum_{j,n=0}^{\infty} \frac{(-1)^{j+n}\Gamma(j+n+\mu)\Gamma\left(j+\mu+m_t\right)}{j!n!\Gamma(j+\mu)\Gamma(n+\mu)} \times \Gamma\left(m_s-\mu-j-n\right)\left(\kappa\mu m_s\right)^n\left(\frac{r^2\mathcal{K}m_s}{\hat{r}^2(m_t-1)}\right)^{j+\mu-m_s}. \tag{115}$$

Summing (112) over index n, (113) over index j, (115) over the infinite triangle $j = i - n$, the double shadowed κ-μ Type III (example 2) PDF simplifies to (41).

Substituting the SNR PDF (see (45)) of the double shadowed κ-μ Type III (example 1) fading model in $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(t)\,dt$, changing the order of integration and summation, solving the integrals using [28, eq. 1.16.1.1], and finally performing some algebraic manipulations, we obtain the CDF of the double shadowed κ-μ Type III (example 2) fading model shown in (49). This completes the proof.

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