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Extended Periodic Inspection Policies for a Single Unit System Subject to Shocks

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ABSTRACT In this paper, two types of failures are taken into account to extend the classical periodic inspection policy for a single unit system when it has failed, in which type I failure can be rectified by a minimal repair and type II failure should be removed by a corrective replacement. More specifically, we investigate three extended periodic inspection models for a system subject to two kinds of distinctive shocks, i.e., a general periodic inspection model where the system is checked at periodic time epochs over an infinite time span (Policy A), a periodic inspection model with the consideration of quality warranty where the system is periodically checked within a maximal inspection number (Policy B), and a random periodic inspection model where the system is either periodically checked or randomly checked, whichever takes place first (Policy C). For each extended model, the average maintenance cost in one renewal cycle under special conditions is minimized to seek the optimal inspection interval theoretically and the numerical example is arranged to authenticate it analytically. Last but not least, comparisons are made among the extended models, indicating that the optimum solution varies from policy to policy.

INDEX TERMS Average cost rate, periodic inspection, quality warranty, random inspection, two types of failures.

I. INTRODUCTION

For the purpose of retaining systems with a high reliability and decreasing the huge losses caused by catastrophic failures, the significance of maintenance has been increasing greatly with the progressive innovation of modern technology in the past few decades [1]. Maintenance theories have been actually applied not only to manufacturing, industrial, and mechanical engineering but also to information, communication, and software engineering [2]. For advanced countries, maintenance will be more important even than production, manufacture, and construction on account that public infrastructure there has been finished and been rushed into an intensive maintenance period.

Maintenance can be classified generally into two major categories, i.e., preventive maintenance (PM) and corrective maintenance (CM) in terms of MIL-STD-721B [3]. PM means that all maintenance behaviors are performed in advance in an attempt to keep an item in a specified condition by providing systematic inspection, detection, and prevention of incipient failures, whereas CM is the maintenance which is implemented when the system has failed. According to Wang [4], maintenance can also be classified into the following five types based on the degree to which the operating condition of a system is restored by maintenance implementation: (1) perfect maintenance, where system operating condition is restored to "as good as new" (AGAN); (2) minimal maintenance, in which system failure rate is not altered by any maintenance actions and system operating condition after maintenance is referred to "as bad as old" (ABAO); (3) imperfect maintenance, where system operating state is restored to somewhere between AGAN and ABAO; (4) worse maintenance, where system condition becomes worse than that just prior to its failure; and (5) worst maintenance, in which maintenance makes system break down. It should be acknowledged that both PM and CM could be a perfect, minimal, imperfect, worse or worst one.

Replacement, which is often regarded as a perfect maintenance, plays an important role in maintenance theories [5], [6]. The most distinguished replacement policy

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for a system is based on whose age, which is called age replacement, where the system is always replaced with a new one at failure or at a constant age T $(0 < T \leq \infty)$ if it has not failed up to T, whichever takes place first [7], [8]. For large and complex systems comprising many kinds of units such as computers and airplanes, it is reasonable to make the planed replacement at periodic times $T, 2T, \cdots$ with minimal repair at failures, and the policy of which is referred to as periodic replacement [9], [10]. If a system consists of a block or group of units, their ages are not readily observed and only failures are known. Under this case all units are periodically replaced independently of their ages in use and this policy is called block replacement, which is commonly employed for complex electronic systems and electrical parts [11], [12].

Other than that, inspection is commonly applied as an advanced technology to maintain a high system availability for the reason that inspection arrangement enables to monitor system defective states [13]. Through inspection, potential defects are identified and preventive maintenance policies can be carried out if needed [14], [15]. Consequently, severe system failures are avoided in advance, impelling inspection to be an effective measure to improve system performance. Inspections can be conducted periodically at $T, 2T, \cdots$ or non-periodically at successive times $T_1, T_2, \cdots$. Any failure is detected at the next checking time epoch and the defective system is replaced immediately. Periodic inspection is widely adopted in practice owing to its sufficient convenient implementation and adequate effectiveness [16], [17]. In the literature, two general inspection policies have been considered, i.e., a hidden failure based inspection model, where system failures are always detected only through inspections, and a delay time based inspection model, where system failure is regarded as a two-stage process, in which the first stage is from the new installation to an initial point of a defect's arrival and the second stage is from that point to a revealed failure if the defect is unattended [18], [19].

From the actual engineering point of view, not each random shock arriving on a system has a traumatic influence and correspondingly, the concept of two types of failures is proposed [20]. It is always assumed that the system is subject to two types of failures in terms of shocks, i.e., type I failure (minimal failure) which can be rectified by a minimal repair and type II failure (catastrophic failure) which should be removed by a corrective replacement [21]–[23]. This modeling framework was firstly proposed by Brown and Proschan [20], in which an item is returned to be the "good as new" state with probability p and returned to be a functioning state with probability $q = 1 - p$, but it is only as good as a device of age equal to its age at failure. Later in [24], Sheu considered a general age replacement which incorporates minimal repair, planned and unplanned replacements, and general random repair costs. He assumed that an operating unit is completely replaced at a planned age T $(T > 0)$, and it is either replaced by a new one with probability $p(t)$ or undergoes minimal repair with probability $q(t) = 1 - p(t)$ if the unit fails at $t < T$. More researches on two types of failures are seen [25]–[28].

To the best of our knowledge, inspection policies for systems which are subject to two types of failures in terms of shocks have not been addressed yet. In this paper, three extended periodic inspection models are investigated for a single unit system suffering from two types of failures, in which type I failure is minimal failure and can be rectified by a minimal repair, where system failure rate is undisturbed, and type II failure is catastrophic failure and should be removed by a corrective replacement. The expected long-run maintenance cost rate in every model is designed as an objective function, and the optimal solution is obtained theoretically and verified numerically. Contributions of this paper lie in three aspects:

- Two types of failures are incorporated into periodic inspection policies when a single unit system subject to a non-homogeneous Poisson shock process has failed.
- Three advanced periodic inspection models are developed, in which the average maintenance cost rate is minimized analytically to seek the optimal check interval.
- Comparisons among the three models are addressed in order to illustrate which policy is better under rational assumptions.

The outline of the remainder of this paper is organized as follows. In Section 2, we state the basic problem of two types of failures and present some notations. From Section 3 to Section 5, three extended periodic inspection policies are proposed, and in which model two types of failures are considered. More specifically, we investigate a general periodic inspection model (Policy A) in Section 3 and extend it into a periodic inspection policy considering quality warranty (Policy B) in Section 4. Section 5 proposes a random inspection model for a single unit system, where the system is randomly checked at successive time epochs, independent of its failure time and also periodically checked (Policy C). System failure is detected by either random or periodic inspection, whichever occurs first. Comparisons among the three models are made in Section 6 and we draw conclusions in Section 7, as well as some future research directions.

II. PROBLEM STATEMENT AND NOTATIONS

A new system starts to operate from its installation and its failure time X has a general distribution $F(t) = P(X \leq t)$ with a finite mean $\mu \equiv \int_0^\infty \bar{F}(t)\mathrm{d}t < \infty$ where $\bar{\Phi}(t) \equiv 1 - \Phi(t)$ for any function $\Phi(t)$. When $F(t)$ has a density function $f(t) \equiv \mathrm{d}F(t)/\mathrm{d}t$, the failure rate function is $r(t) \equiv f(t)/\bar{F}(t)$. When the system fails at t, it is subject to one of two types of failures. One is type I failure with probability $p(t)$ which can be rectified by a minimal repair and the other is type II failure with probability $q(t) = 1 - p(t)$ which must be removed with a new one. It is noted that $r(t)$ is undisturbed by any minimal repair.

Let $\{N(t), t \geq 0\}$ be a non-homogeneous Poisson process (NHPP) with an intensity function $r(t)$, $\{N_1(t), t \geq 0\}$

and $\{N_2(t), t \geq 0\}$ be the counting processes describing the number of type I failures and the number of type II failures in $(0, t]$, respectively. Then, $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ are two independent non-homogeneous Poisson processes with intensities $p(t)r(t)$ and $q(t)r(t)$, respectively [29]. Denoting Z be the waiting time of the first occurrence of type II failure, we have $Z = \inf\{t|N_2(t) = 1, t \geq 0\}$. Thus, the survival function of Z is

$$\bar{F}_Z(t) = P(Z > t) = P(N_2(t) = 0) = \exp\left\{-\int_0^t q(x)r(x)dx\right\}. \tag{1}$$

It is also noted that the mean number of type I failures in $(0, t]$ is $E[N_1(t)] = \int_0^t p(x)r(x)dx$. For a clear exposition in this paper, a list of notations is provided.

X	System failure time
Z	Waiting time of the first occurrence of type I failure
$T, 2T, \cdots$	Periodic inspection time epochs
$Y_1, Y_2, \cdots$	Random inspection time epochs
$F(t)$	System failure distribution
$r(t)$	System failure rate function
$\bar{F}_Z(t)$	Survival function of Z
$G(t)$	Distribution of Y_j $(j = 1, 2, \cdots)$
$\{N(t), t \geq 0\}$	A NHPP with intensity
$\{N_1(t), t \geq 0\}$	Number of type I failures in $(0, t]$
$\{N_2(t), t \geq 0\}$	Number of type II failures in $(0, t]$
$p(t)$	Probability of type I failure at system failure
$q(t)$	Probability of type II failure at system failure
c_m	Cost of each minimal repair
c_i	Cost of each inspection
c_c	Cost of each number of random jobs
c_d	Downtime cost per unit of time
c_r	Replacement cost
$C(T)$	Expected long-run maintenance cost rate

III. GENERAL PERIODIC INSPECTION MODEL (POLICY A)

A. MODELLING FRAMEWORK

For the general periodic inspection model, the system is periodically checked at $T, 2T, \cdots$ with a cost c_i. Any type I failure is minimally repaired with a cost c_m and any type II failure is detected at the next checking time point, then the system is replaced by a new one with a cost c_r upon the first occurrence of type II failure immediately. It is assumed that $c_r > c_i$ and $c_r > c_m$. The preparation times for minimal repair, periodic inspection, and replacement are negligible. The process of general periodic inspection with checking times $T, 2T, \cdots$ is shown as that in Fig.1.

Let $D(t)$ be the expected maintenance cost of the system over time interval $(0, t]$. Denoting K_i $(i = 1, 2, \cdots)$ and C_i $(i = 1, 2, \cdots)$ be the length of the successive replacement cycles and the operational cost over the renewal interval K_i,

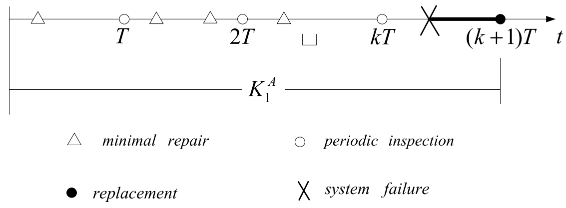

FIGURE 1. Process of general periodic inspection policy.

respectively, we have a renewal reward process $\{(K_i, C_i)\}$. According to the renewal reward theorem [30], we obtain

$$C(T) = \lim_{t\to\infty} \frac{D(t)}{t} = \frac{E[C_1]}{E[K_1]}. \tag{2}$$

The costs of minimal repairs in the first replacement interval K_1 are

$$\begin{aligned} c_{m1} &= c_m \sum_{k=0}^{\infty} \int_0^{(k+1)T} p(t)r(t)dtP(kT < Z \leq (k+1)T) \\ &= c_m \sum_{k=0}^{\infty} \int_0^{(k+1)T} \int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}t\mathrm{d}F_Z(t). \end{aligned} \tag{3}$$

The costs of inspections in the first replacement interval K_1 are

$$\begin{aligned} c_{i1} &= c_i \sum_{k=0}^{\infty} (k+1)P(kT < Z \leq (k+1)T) \\ &= c_i \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} (k+1)\mathrm{d}F_Z(t) \\ &= c_i \sum_{k=0}^{\infty} \bar{F}_Z(kT). \end{aligned} \tag{4}$$

The downtime costs in the first replacement interval K_1 are

$$\begin{aligned} c_{d1} &= c_d \sum_{k=0}^{\infty} ((k+1)T - t)P(kT < Z \leq (k+1)T) \\ &= c_d \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} ((k+1)T - t)\mathrm{d}F_Z(t) \\ &= c_d \sum_{k=0}^{\infty} T\bar{F}_Z(kT) - c_d \int_0^{\infty} \bar{F}_Z(t)\mathrm{d}t. \end{aligned} \tag{5}$$

The mean length of the first replacement interval K_1 is

$$\begin{aligned} E[K_1] &= \sum_{k=0}^{\infty} ((k+1)T)P(kT < Z \leq (k+1)T) \\ &= \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} ((k+1)T)\mathrm{d}F_Z(t) \\ &= \sum_{k=0}^{\infty} T\bar{F}_Z(kT). \end{aligned} \tag{6}$$

Thus, the expected long-run maintenance cost rate becomes

$$C(T) = \frac{c_{m1} + c_{i1} + c_{d1} + c_r}{E[K_1]} = \frac{c_m \sum_{k=0}^{\infty} \int_0^{(k+1)T} \int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}t\mathrm{d}F_Z(t) + c_i \sum_{k=0}^{\infty} \bar{F}_Z(kT) - c_d \int_0^{\infty} \bar{F}_Z(t)\mathrm{d}t + c_r}{\sum_{k=0}^{\infty} T\bar{F}_Z(kT)} + c_d. \quad (7)$$

B. OPTIMIZATION

Note that $C(T)$ in Eq.(7) is a function of T and the aim is to find an optimal T^* which minimizes $C(T)$. Differentiating $C(T)$ with T and setting it equal to zero, we have

$$\left[\sum_{k=0}^{\infty}\left\{(k+1)p\left[(k+1)T\right]r\left[(k+1)T\right]\int_{kT}^{(k+1)T}\mathrm{d}F_Z(t) + \int_0^{(k+1)T} p(t)r(t)\mathrm{d}t\left[(k+1)f((k+1)T) - kf(kT)\right] - c_i\sum_{k=0}^{\infty} kf_Z(kT)\right\}\right]\sum_{k=0}^{\infty} T\bar{F}_Z(kT) - \left[c_m\sum_{k=0}^{\infty}\int_0^{(k+1)T}\int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}t\mathrm{d}F_Z(t) + c_i\sum_{k=0}^{\infty}\bar{F}_Z(kT) - c_d\int_0^{\infty}\bar{F}_Z(t)\mathrm{d}t + c_r\right]\sum_{k=0}^{\infty}\left[\bar{F}_Z(kT) - kTf_Z(kT)\right] = 0, \quad (8)$$

in which $f_Z(t) = \mathrm{d}F_Z(t)/\mathrm{d}t$.

In particular, when $F(t) = 1 - \mathrm{e}^{-\lambda t}$, $p(t) = p$ $(0 \le p < 1)$, Eq.(7) becomes

$$C(T_A) = \frac{c_m\lambda(1-q)T_A\frac{1}{1-\mathrm{e}^{-\lambda q T_A}} + c_i\frac{1}{1-\mathrm{e}^{-\lambda q T_A}} - \frac{c_d}{\lambda q} + c_r}{\frac{T_A}{1-\mathrm{e}^{-\lambda q T_A}}} + c_d = \frac{c_m\lambda(1-q)T_A + c_i - \left[\frac{c_d}{\lambda q} - c_r\right]\left(1 - \mathrm{e}^{-\lambda q T_A}\right)}{T_A} + c_d, \quad (9)$$

and Eq.(8) becomes

$$1 - (1 + \lambda q T_A)\,\mathrm{e}^{-\lambda q T_A} = \frac{c_i}{\frac{c_d}{\lambda q} - c_r}. \quad (10)$$

Let the left-hand side of Eq.(10) be $Q(T_A)$, i.e., $Q(T_A) = 1 - (1 + \lambda q T_A)\,\mathrm{e}^{-\lambda q T_A}$. It is clear that $Q(0) = 0$, $Q(\infty) = \lim_{T_A\to\infty} Q(T_A) = \infty$, and $Q(T_A)$ increases strictly with T_A. Hence, when $c_d/(\lambda q) - c_r > 0$, there exists an optimal T_A^* satisfying Eq.(10) which minimizes $C(T_A)$ in Eq.(9). Fig.2 shows the expected long-run maintenance cost rate for different λ and q given that $c_m = 2$, $c_i = 5$, $c_d = 20$, $c_r = 10$.

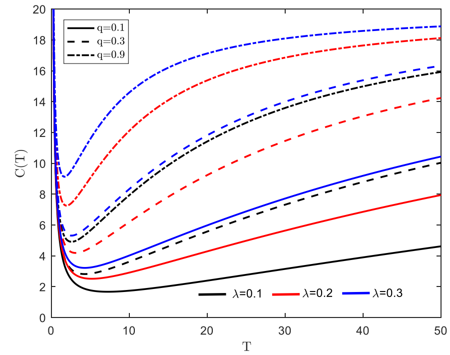

FIGURE 2. Comparisons of $C(T_A)$ given that $c_m = 2$, $c_i = 5$, $c_d = 20$, $c_r = 10$.

TABLE 1. Comparisons of T_A^* and $C(T_A^*)$ given $c_m = 2$, $c_i = 5$, $c_d = 20$, $c_r = 10$.

$q(t)$	$\lambda=0.1$		$\lambda=0.2$		$\lambda=0.3$	
	T_A^*	$C(T_A^*)$	T_A^*	$C(T_A^*)$	T_A^*	$C(T_A^*)$
0.1	7.262	1.674	5.202	2.516	4.293	3.22
0.2	5.202	2.316	3.754	3.453	3.118	4.39
0.3	4.293	2.821	3.118	4.19	2.606	5.313
0.4	3.754	3.253	2.743	4.823	2.306	6.105
0.5	3.387	3.638	2.491	5.389	2.106	6.811
0.6	3.118	3.99	2.306	5.905	1.962	7.455
0.7	2.91	4.317	2.165	6.384	1.853	8.05
0.8	2.743	4.623	2.053	6.832	1.768	8.606
0.9	2.606	4.913	1.962	7.255	1.699	9.128
1.0	2.491	5.189	1.886	7.656	1.645	9.622

Table 1 compares the optimal T_A^* and its corresponding $C(T_A^*)$ for different $q(t)$ given that $c_m = 2$, $c_i = 5$, $c_d = 20$, $c_r = 10$.

From Fig.2 and Table 1, it is clear that the optimal periodic inspection period T_A^* decreases with λ and q while the corresponding expected long run maintenance cost rate $C(T_A^*)$ increases with λ and q.

IV. PERIODIC INSPECTION MODEL WITH QUALITY WARRANTY (POLICY B)

A. MODELLING FRAMEWORK

In actual engineering, the operating time of most units would be finite, especially for those mission-oriented products [31], [32]. For example, missiles are composed of various kinds of electric, electronic, and mechanical parts. They are stored in a storage system and are ready to generate high power in a very short time, under which condition the missiles should be exchanged after the total inspection times have exceeded a predetermined time or the total inspection numbers have reached a foreordained number in terms of quality warranty. Hence, it is assumed that a pre-specified inspection number due to quality warranty is N, i.e., the system is correctively replaced at time NT even if it has not failed. The preparation times for minimal repair, periodic inspection, and replacement are still negligible. The process of periodic inspection considering quality warranty with checking times kT $(k = 1, 2, \cdots, N)$ is shown as that in Fig.3, in which the notations are the same with those in Section 3.

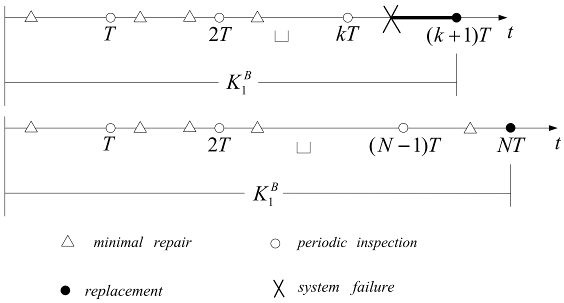

FIGURE 3. Process of periodic inspection policy with quality warranty.

Then, the costs of minimal repairs in the first replacement interval K_1 are

$$\begin{aligned} c_{m2} &= c_m \left[\sum_{k=0}^{N-1} \int_0^{(k+1)T} \int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}t\mathrm{d}F_Z(t) \right. \\ &\quad \left. + \int_0^{NT} p(t)r(t)\mathrm{d}t\bar{F}_Z(NT) \right] \\ &= c_m \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}t\bar{F}_Z(kT). \end{aligned} \tag{11}$$

The costs of inspections in the first replacement interval K_1 are

$$\begin{aligned} c_{i2} &= c_i \left[\sum_{k=0}^{N-1} (k+1)P(kT < Z \le (k+1)T) + NP(Z > NT) \right] \\ &= c_i \left[\sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} (k+1)\mathrm{d}F_Z(t) + N\bar{F}_Z(NT) \right] \\ &= c_i \sum_{k=0}^{N-1} \bar{F}_Z(kT). \end{aligned} \tag{12}$$

The downtime costs in the first replacement interval K_1 are

$$\begin{aligned} c_{d2} &= c_d \sum_{k=0}^{N-1} ((k+1)T - t)P(kT < Z \le (k+1)T) \\ &= c_d \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} [(k+1)T - t]\mathrm{d}F_Z(t) \\ &= c_d \sum_{k=0}^{N-1} T\bar{F}_Z(kT) - c_d \int_0^{NT} \bar{F}_Z(t)\mathrm{d}t. \end{aligned} \tag{13}$$

The mean length of the first replacement interval K_1 is

$$\begin{aligned} E[K_1] &= \sum_{k=0}^{N-1} [(k+1)T]P(kT < Z \le (k+1)T) + NTP(Z > NT) \\ &= \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} [(k+1)T]\mathrm{d}F_Z(t) + NT\bar{F}_Z(NT) \\ &= \sum_{k=0}^{N-1} T\bar{F}_Z(kT). \end{aligned} \tag{14}$$

Thus, the expected long-run maintenance cost rate becomes

$$\begin{aligned} C(N,T) &= \frac{c_{m2} + c_{i2} + c_{d2} + c_r}{E[K_1]} \\ &= \frac{c_m \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}t\bar{F}_Z(kT) + c_i \sum_{k=0}^{N-1} \bar{F}_Z(kT) - c_d \int_0^{NT} \bar{F}_Z(t)\mathrm{d}t + c_r}{\sum_{k=0}^{N-1} T\bar{F}_Z(kT)} + c_d. \end{aligned} \tag{15}$$

B. OPTIMIZATION

If $C(N,T)$ is jointly convex in (N,T), there exists an optimal (N^*, T^*) which minimizes $C(N,T)$ in Eq.(15). First, we consider the optimization problem with respect to T for a given N. It is clearly seen that $\lim_{T\to 0} C(N,T) = \infty$ and $\lim_{T\to\infty} C(N,T) = c_d$. Thus, there exists a positive T^* $(0 < T^* \le \infty)$ which minimizes $C(N,T)$ for a specified $N \ge 1$. Differentiating $C(N,T)$ with respect to T, we judge that the optimal T^* satisfies

$$\begin{aligned} &\left[c_m \sum_{k=0}^{N-1} \left\{ (k+1)p((k+1)T)r((k+1)T)\bar{F}_Z(kT) - kp(kT) \right. \right. \\ &\quad \left. \times r(kT)\bar{F}_Z(kT) - \int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}\,t kf(kT) \right\} - c_d N\bar{F}_Z(NT) \\ &\quad \left. - c_i \sum_{k=0}^{N-1} kf_Z(kT) \right] \sum_{k=0}^{N-1} T\bar{F}_Z(kT) - \left[c_m \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}t \right. \\ &\quad \left. \times \bar{F}_Z(kT) + c_i \sum_{k=0}^{N-1} \bar{F}_Z(kT) - c_d \int_0^{NT} \bar{F}_Z(t)\mathrm{d}t + c_r \right] \\ &\quad \times \left[\sum_{k=0}^{N-1} \bar{F}_Z(kT) - \sum_{k=0}^{N-1} kTf_Z(kT) \right] = 0. \end{aligned} \tag{16}$$

Then for a given T, forming the inequalities $C(N+1,T) - C(N,T) \ge 0$ and $C(N-1,T) - C(N,T) > 0$, we have

$$L(N,T) \ge c_r \quad \text{and } L(N-1,T) < c_r, \tag{17}$$

in which,

$$\begin{aligned} L(N,T) &= \frac{c_m \left[\sum_{k=0}^{N-1} \bar{F}_Z(kT) \sum_{k=0}^{N} \int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}t\bar{F}_Z(kT) \right]}{\bar{F}_Z(NT)} \\ &\quad - \frac{c_m \left[\sum_{k=0}^{N} \bar{F}_Z(kT) \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} p(t)r(t)\mathrm{d}t\bar{F}_Z(kT) \right]}{\bar{F}_Z(NT)} \end{aligned}$$

$$-\frac{c_d}{\bar{F}_Z(NT)}\times\left[\sum_{k=0}^{N-1}\bar{F}_Z(kT)\int_0^{(N+1)T}\bar{F}_Z(t)\mathrm{d}t\right.$$
$$\left.-\sum_{k=0}^{N}\bar{F}_Z(kT)\int_0^{NT}\bar{F}_Z(t)\mathrm{d}t\right].$$

In particular, when $F(t)=1-\mathrm{e}^{-\lambda t}$, $p(t)=p$ $(0\le p<1)$, Eq.(15) becomes

$$C(N_B,T_B)$$
$$=\frac{c_i\frac{1-\mathrm{e}^{-N_B\lambda qT_B}}{1-\mathrm{e}^{-\lambda qT_B}}+c_m\lambda(1-q)T_B\frac{1-\mathrm{e}^{-N_B\lambda qT_B}}{1-\mathrm{e}^{-\lambda qT_B}}-c_d\frac{1-\mathrm{e}^{-N_B\lambda qT_B}}{\lambda q}+c_r}{\frac{1-\mathrm{e}^{-N_B\lambda qT_B}}{1-\mathrm{e}^{-\lambda qT_B}}T_B}+c_d$$
$$=\frac{c_i}{T_B}+c_m\lambda(1-q)+c_d-\frac{1-\mathrm{e}^{-\lambda qT_B}}{\lambda qT_B}\times\left[c_d-\frac{\lambda qc_r}{1-\mathrm{e}^{-N_B\lambda qT_B}}\right].\tag{18}$$

Differentiating $C(N_B,T_B)$ in Eq.(18) with respect to T_B, we have

$$\left[\frac{c_d}{\lambda q}-\frac{c_r}{1-\mathrm{e}^{-N_B\lambda qT_B}}\right]\left[1-(1+\lambda qT_B)\mathrm{e}^{-\lambda qT_B}\right]-\frac{c_rN_B\lambda qT_Be^{-N_B\lambda qT_B}\left(1-\mathrm{e}^{-\lambda T_B}\right)}{(1-\mathrm{e}^{-N_B\lambda qT_B})^2}=c_i.\tag{19}$$

Denoting the left-hand side of Eq.(19) by $Q_{N_B}(T_B)$, we have $\lim_{T_B\to 0}Q_{N_B}(T_B)<0$ and $\lim_{T_B\to\infty}Q_{N_B}(T_B)=c_d/(\lambda q)-c_r$. Hence, there exists an optimal T_B^* which satisfies Eq.(19) as long as $c_d/(\lambda q)-c_r>c_i$. Furthermore, $Q_{N_B+1}(T_B)-Q_{N_B}(T_B)$, as shown at the bottom of the page.

It is clear that $Q_{N_B+1}(T_B)>Q_{N_B}(T_B)$, i.e., $Q_{N_B}(T_B)$ increases strictly with N_B and the optimal T_B^* decreases with N_B. When $N_B=1$, according to Eq.(19), T_1^* satisfies

$$1-(1+\lambda qT_B)\mathrm{e}^{-\lambda qT_B}=\frac{\lambda q(c_i+c_r)}{c_d}.\tag{20}$$

When $N_B\to\infty$, according to Eq.(19), T_∞^* satisfies

$$1-(1+\lambda qT_B)\mathrm{e}^{-\lambda qT_B}=\frac{\lambda qc_i}{c_d-\lambda qc_r}.\tag{21}$$

Thus, the optimal T_B^* satisfies $T_\infty^*\le T_B^*\le T_1^*$. Fig.4 and Fig.5 show the expected long-run maintenance cost rate for different λ and q given that $N_B=1$ and $N_B=5$, respectively, in which $c_m=2$, $c_i=5$, $c_d=20$, $c_r=10$.

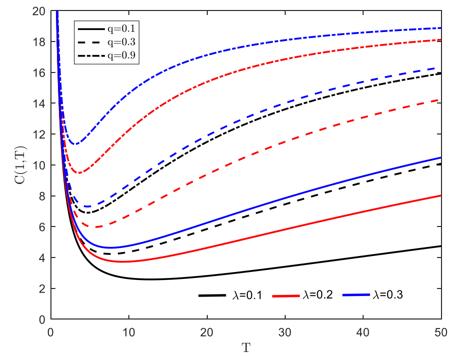

FIGURE 4. Comparisons of $C(1,T_B)$ given that $c_m=2$, $c_i=5$, $c_d=20$, $c_r=10$.

Table 2 and Table 3 compare the optimal T_B^* and its corresponding $C(N_B,T_B^*)$ given that $N_B=1$ and $N_B=5$, respectively, in which $c_m=2$, $c_i=5$, $c_d=20$, $c_r=10$.

From Table 2 and Table 3, it is clear that the optimal periodic inspection period T_B^* decreases with λ and q while the corresponding expected long run maintenance cost rate $C(N_B,T_B^*)$ increases with λ and q for a given N_B. In addition, the optimal T_B^* decreases with N_B, satisfying that $T_\infty^*\le T_B^*\le T_1^*$.

V. RANDOM PERIODIC INSPECTION MODEL (POLICY C)

A. MODELLING FRAMEWORK

Most systems need to execute successive jobs in actual engineering, leading to that it is impossible or impractical

$$Q_{N_B+1}(T_B)-Q_{N_B}(T_B)i=c_r\left(1-\mathrm{e}^{-\lambda qT_B}\right)\mathrm{e}^{-N_B\lambda qT_B}\frac{1-(1+\lambda qT_B)\mathrm{e}^{-\lambda qT_B}}{\left(1-\mathrm{e}^{-N_B\lambda qT_B}\right)\left(1-\mathrm{e}^{-(N_B+1)\lambda qT_B}\right)}$$
$$+c_r\lambda qT_B\left(1-\mathrm{e}^{-\lambda qT_B}\right)\mathrm{e}^{-N_B\lambda qT_B}\left[\frac{N_B}{\left(1-\mathrm{e}^{-N_B\lambda qT_B}\right)^2}-\frac{(N_B+1)\mathrm{e}^{-\lambda qT_B}}{\left(1-\mathrm{e}^{-(N_B+1)\lambda qT_B}\right)^2}\right]$$
$$=c_r\left(1-\mathrm{e}^{-\lambda qT_B}\right)\mathrm{e}^{-N_B\lambda qT_B}\frac{1-(1+\lambda qT_B)\mathrm{e}^{-\lambda qT_B}}{\left(1-\mathrm{e}^{-N_B\lambda qT_B}\right)\left(1-\mathrm{e}^{-(N_B+1)\lambda qT_B}\right)}$$
$$+\frac{N_B\left(1-\mathrm{e}^{-(N_B+1)\lambda qT_B}\right)^2-(N_B+1)\mathrm{e}^{-\lambda qT_B}\left(1-\mathrm{e}^{-N_B\lambda qT_B}\right)^2}{\left(1-\mathrm{e}^{-N_B\lambda qT_B}\right)^2\left(1-\mathrm{e}^{-(N_B+1)\lambda qT_B}\right)^2}$$
$$=c_r\left(1-\mathrm{e}^{-\lambda qT_B}\right)\mathrm{e}^{-N_B\lambda qT_B}\frac{1-(1+\lambda qT_B)\mathrm{e}^{-\lambda qT_B}}{\left(1-\mathrm{e}^{-N_B\lambda qT_B}\right)\left(1-\mathrm{e}^{-(N_B+1)\lambda qT_B}\right)}$$
$$+\frac{\mathrm{e}^{-\lambda qT_B}\left[N_B(\mathrm{e}^{-\lambda qT_B}-1)(1-\mathrm{e}^{-(2N_B+1)\lambda qT_B})-\left(1-\mathrm{e}^{-N_B\lambda qT_B}\right)^2\right]}{\left(1-\mathrm{e}^{-N_B\lambda qT_B}\right)^2\left(1-\mathrm{e}^{-(N_B+1)\lambda qT_B}\right)^2}$$

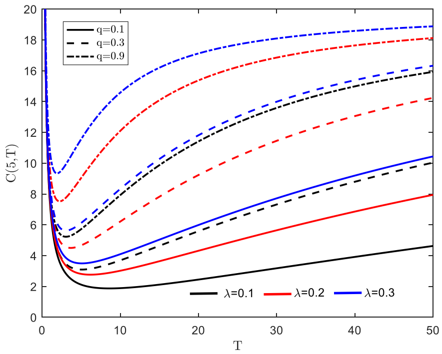

FIGURE 5. Comparisons of $C(5, T_B)$ given that $c_m = 2$, $c_i = 5$, $c_d = 20$, $c_r = 10$.

TABLE 2. Comparisons of T_B^* and $C(1, T_B^*)$ given $c_m = 2$, $c_i = 5$, $c_d = 20$, $c_r = 10$.

$q(t)$	$\lambda = 0.1$		$\lambda = 0.2$		$\lambda = 0.3$	
	T_B^*	$C(1,T_B^*)$	T_B^*	$C(1,T_B^*)$	T_B^*	$C(1,T_B^*)$
0.1	12.777	2.579	9.204	3.723	7.626	4.629
0.2	9.204	3.523	6.688	5.015	5.582	6.172
0.3	7.626	4.229	5.582	5.972	4.687	7.303
0.4	6.688	4.815	4.927	6.755	4.161	8.221
0.5	6.051	5.321	4.485	7.428	3.808	9.003
0.6	5.582	5.772	4.161	8.021	3.552	9.687
0.7	5.219	6.18	3.912	8.554	3.357	10.297
0.8	4.927	6.555	3.714	9.04	3.203	10.848
0.9	4.687	6.903	3.552	9.487	3.079	11.35
1.0	4.485	7.228	3.416	9.90	2.977	11.812

TABLE 3. Comparisons of T_A^* and $C(5, T_B^*)$ given $c_m = 2$, $c_i = 5$, $c_d = 20$, $c_r = 10$.

$q(t)$	$\lambda = 0.1$		$\lambda = 0.2$		$\lambda = 0.3$	
	T_B^*	$C(1,T_B^*)$	T_B^*	$C(1,T_B^*)$	T_B^*	$C(1,T_B^*)$
0.1	8.601	1.871	6.152	2.764	5.067	3.495
0.2	6.152	2.564	4.421	3.744	3.656	4.698
0.3	5.067	3.095	3.656	4.498	3.034	5.624
0.4	4.421	3.544	3.202	5.135	2.666	6.406
0.5	3.981	3.939	2.893	5.697	2.416	7.097
0.6	3.656	4.298	2.666	6.206	2.235	7.722
0.7	3.404	4.628	2.49	6.675	2.095	8.297
0.8	3.202	4.935	2.349	7.111	1.985	8.832
0.9	3.034	5.224	2.235	7.522	1.896	9.334
1.0	2.893	5.497	2.138	7.909	1.822	9.809

to maintain them in a strictly periodic fashion [33], [34]. Suppose that a single unit system is checked at periodic time epochs $T, 2T, \cdots$ and also checked at successive times $Y_1, Y_2, \cdots$, where $Y_0 = 0$ and $Z_j = Y_{j+1} - Y_j$ $(j = 0, 1 \cdots)$ are independently and identically distributed with a distribution $G(x)$. The distribution of Y_j is represented by the $j-th$ fold convolution of $Y_0 = 0$ with itself, i.e. $G^{(j)}(x) = P(Y_j \leq x)$ and $G^{(0)}(x) \equiv 1$ for $x \geq 0$. The first occurrence of type II failure is detected by either random or periodic inspection, whichever comes first and then, the system is replaced immediately, which is shown in Fig.6.

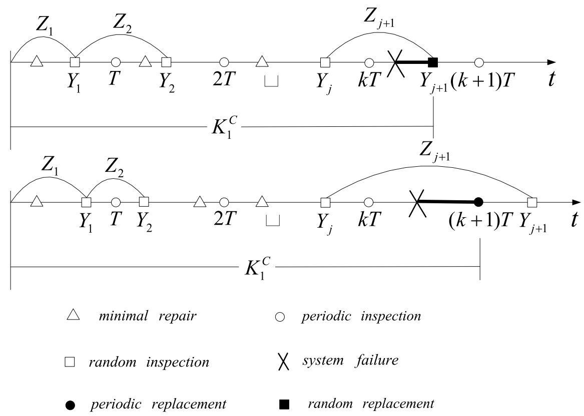

FIGURE 6. Process of random periodic inspection policy.

The probability that the first type II failure is detected by periodic check is

$$p_1 = \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} \left\{ \sum_{j=0}^{\infty} \int_0^t \bar{G}[(k+1)T - x] \mathrm{d}G^{(j)}(x) \right\} \mathrm{d}F_Z(t)$$
$$= \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t \bar{G}[(k+1)T - x] \mathrm{d}G^{(j)}(x) \mathrm{d}F_Z(t), \tag{22}$$

and the probability that it is checked by random check is

$$p_2 = \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} \left\{ \sum_{j=0}^{\infty} \int_0^t [G((k+1)T - x) - G(t - x)] \times \mathrm{d}G^{(j)}(x) \right\} \mathrm{d}F_Z(t)$$
$$= \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t \int_{t-x}^{(k+1)T-x} \mathrm{d}G(v) \mathrm{d}G^{(j)}(x) \mathrm{d}F_Z(t), \tag{23}$$

where should note that $p_1 + p_2 \equiv 1$.

Let c_c be the cost of each random check at $Y_1, Y_2, \cdots$ and the other parameters be the same with them in Section 3. Thus, the costs of minimal repairs in the first replacement interval K_1 are

$$c_{m3} = c_m \left\{ \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t \int_0^{(k+1)T} \bar{G}[(k+1)T - x] \times p(v)\lambda(v) \mathrm{d}v \mathrm{d}G^{(j)}(x) \mathrm{d}F_Z(t) \right\}$$
$$+ c_m \left\{ \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t \int_{t-x}^{(k+1)T-x} [G((k+1)T - x) - G(t - x)] p(v)\lambda(v) \mathrm{d}v \mathrm{d}G^{(j)}(x) \mathrm{d}F_Z(t) \right\}. \tag{24}$$

The costs of periodic inspections in the first replacement interval K_1 are

$$c_{i3} = c_i \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t (k+1)\bar{G}[(k+1)T-x] \times \mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t) \right\} + c_i \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t k\,[G((k+1)T-x)-G(t-x)] \mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t) \right\}. \tag{25}$$

The costs of random inspections in the first replacement interval K_1 are

$$c_{c3} = c_c \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t j\bar{G}[(k+1)T-x]\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t) \right\} + c_c \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t (j+1)\,[G((k+1)T-x) - G(t-x)]\,\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t) \right\}. \tag{26}$$

The downtime costs in the first replacement interval K_1 are

$$c_{d3} = c_d \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t [(k+1)T-x]\,\bar{G} \times [(k+1)T-x]\,\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t) \right\} + c_d \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t \int_{t-x}^{(k+1)T-x} (x+v-t)\mathrm{d}G(v)\, \mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t) \right\}. \tag{27}$$

The mean length of the first replacement interval K_1 is

$$E[K_1] = \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t (k+1)T\bar{G} \times [(k+1)T-x]\,\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t) + \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^t \int_{t-x}^{(k+1)T-x} (x+v) \times \mathrm{d}G(v)\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t). \tag{28}$$

Thus, the expected long-run maintenance cost rate becomes

$$\widetilde{C}(T) = \frac{c_{m3}+c_{i3}+c_{c3}+c_{d3}+c_r}{E[K_1]}$$

$$= \frac{\begin{array}{l} c_m \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t\int_0^{(k+1)T} \bar{G}[(k+1)T-x]\,p(v)\lambda(v)\mathrm{d}v\, \mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t)\right\} + c_m \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t\int_{t-x}^{(k+1)T-x} p(v)\lambda(v)\,[G((k+1)T-x)-G(t-x)]\,\mathrm{d}v\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t)\right\} \\ + c_i \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t (k+1)\bar{G}[(k+1)T-x]\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t)\right\} + c_i \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t k\,[G((k+1)T-x)-G(t-x)]\,\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t)\right\} + c_c \\ \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t j\bar{G}[(k+1)T-x]\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t)\right\} + c_c \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t (j+1)\,[G((k+1)T-x)-G(t-x)]\,\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t)\right\} \\ + c_d \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t [(k+1)T-x]\,\bar{G}[(k+1)T-x]\,\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t)\right\} + c_d \left\{ \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t\int_{t-x}^{(k+1)T-x} (x+v-t)\mathrm{d}G(v)\, \mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t)\right\} + c_r \end{array}}{\begin{array}{l}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t (k+1)T\bar{G}[(k+1)T-x]\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t) \\ + \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \int_{kT}^{(k+1)T}\int_0^t\int_{t-x}^{(k+1)T-x} (x+v)\mathrm{d}G(v)\mathrm{d}G^{(j)}(x)\mathrm{d}F_Z(t).\end{array}} \tag{29}$$

B. OPTIMIZATION

Actually, it is rather troublesome to optimize $\widetilde{C}(T)$ in Eq.(29) due to its constructional complexity. Hence, we tend to carry out another random periodic inspection policy, i.e., the system is only checked at periodic time epochs $T, 2T, \cdots$. Eq.(29) becomes

$$\hat{C}(T) = \frac{\hat{c}_{m3}+\hat{c}_{i3}+\hat{c}_{c3}+\hat{c}_{d3}+c_r}{E[K_1]}$$

$$= \frac{\begin{array}{l} c_m \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} \int_0^{(k+1)T} p(t)r(t)\mathrm{d}t\mathrm{d}F_Z(t) \\ + c_i \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} (k+1)\mathrm{d}F_Z(t) \\ + c_c \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} [M(k+1)T - M(t)]\mathrm{d}F_Z(t) \\ + c_d \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} [(k+1)T-t]\mathrm{d}F_Z(t) + c_r \end{array}}{\sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} ((k+1)T)\mathrm{d}F_Z(t)}, \tag{30}$$

in which $M(t) = \sum_{j=1}^{\infty} G^{(j)}(t)$ represents the expected number of random jobs during $(0, t]$. Differentiating $\hat{C}(T)$ in Eq.(30) with respect to T and setting it equal to zero, we have

$$\begin{aligned}&\left\{\sum_{k=0}^{\infty}(k+1)p\left[(k+1)T\right]r\left[(k+1)T\right]\int_{kT}^{(k+1)T}\mathrm{d}F_Z(t)\right.\\&+\sum_{k=0}^{\infty}\left[(k+1)f((k+1)T)-kf(kT)\right]\int_{0}^{(k+1)T}p(t)r(t)\mathrm{d}t\\&+\sum_{k=0}^{\infty}\left[(k+1)f((k+1)T)-kf(kT)\right]\left[M((k+1)T)-M(t)\right]\\&+\sum_{k=0}^{\infty}\sum_{j=1}^{\infty}(k+1)g^{(j)}((k+1)T)\int_{kT}^{(k+1)T}\mathrm{d}F_Z(t)\\&\left.-\,c_i\sum_{k=0}^{\infty}kf_Z(kT)\right\}\\&\times\sum_{k=0}^{\infty}T\bar{F}_Z(kT)-\left\{\sum_{k=0}^{\infty}(k+1)p\left[(k+1)T\right]\right.\\&\times\, r\left[(k+1)T\right]\int_{kT}^{(k+1)T}\mathrm{d}F_Z(t)\\&\left.+\sum_{k=0}^{\infty}\left[(k+1)f((k+1)T)-kf(kT)\right]\int_{0}^{(k+1)T}p(t)r(t)\mathrm{d}t\right\}\\&\times\sum_{k=0}^{\infty}\left[\bar{F}_Z(kT)-kTf_Z(kT)\right]=0,\end{aligned}\qquad(31)$$

in which $g^{(j)}(t) = \mathrm{d}G^{(j)}(t)/\mathrm{d}t$.

In particular, when $F(t) = 1 - \mathrm{e}^{-\lambda t}$, $p(t) = p$ $(0 \le p < 1)$, and $G(t) = 1 - \mathrm{e}^{-\theta t}$, Eq.(30) is simplified as

$$\begin{aligned}\hat{C}(T_C) &= \frac{c_m\lambda(1-q)T_C\frac{1}{1-\mathrm{e}^{-\lambda qT_C}}+c_i\frac{1}{1-\mathrm{e}^{-\lambda qT_C}}+c_c\left[\frac{\theta T_C}{1-\mathrm{e}^{-\lambda qT_C}}-\frac{\theta}{\lambda q}\right]-\frac{c_d}{\lambda q}+c_r}{\frac{T_C}{1-\mathrm{e}^{-\lambda qT_C}}}+c_d\\&=\frac{\left[c_m\lambda(1-q)+c_c\theta\right]T_C+c_i-\left[\frac{c_c\theta+c_d}{\lambda q}-c_r\right]\left(1-\mathrm{e}^{-\lambda qT_C}\right)}{T_C}+c_d,\end{aligned}\qquad(32)$$

and Eq.(31) becomes

$$1-(1+\lambda qT_C)\,\mathrm{e}^{-\lambda qT_C}=\frac{c_i}{\frac{c_c\theta+c_d}{\lambda q}-c_r}.\qquad(33)$$

It has been proven that the left-hand side of Eq.(33) increases strictly with T_C from $Q(0) = 0$ to $Q(\infty) = \lim_{T_C\to\infty} Q(T_C) = \infty$ from Eq.(10) in Section 3. Hence, when $(c_c\theta + c_d)/\lambda q - c_r > 0$, there exists an optimal T_C^* satisfying Eq.(33) which minimizes $C(T_C)$ in Eq.(32). Fig.7 and Fig.8 show the expected long-run maintenance cost rate for different λ and q given that $\theta = 0.1$ and $\theta = 0.5$, respectively, in which $c_m = 2$, $c_i = 5$, $c_c = 5$, $c_d = 20$, $c_r = 10$.

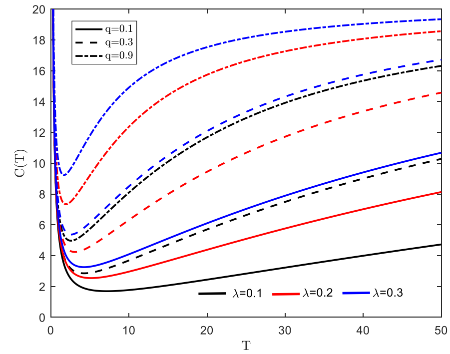

FIGURE 7. Comparisons of $\hat{C}(T_C)$ given that $c_m = 2$, $c_i = 5$, $c_c = 5$, $c_d = 20$, $c_r = 10$ and $\theta = 0.1$.

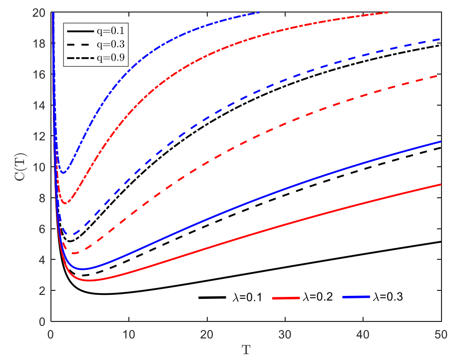

FIGURE 8. Comparisons of $\hat{C}(T_C)$ given that $c_m = 2$, $c_i = 5$, $c_c = 5$, $c_d = 20$, $c_r = 10$ and $\theta = 0.5$.

TABLE 4. Comparisons of T_C^* and $\hat{C}(T_C^*)$ given $c_m = 2$, $c_i = 5$, $c_c = 5$, $c_d = 20$, $c_r = 10$, and $\theta = 0.1$.

$q(t)$	$\lambda=0.1$		$\lambda=0.2$		$\lambda=0.3$	
	T_C^*	$C(T_C^*)$	T_C^*	$C(T_C^*)$	T_C^*	$C(T_C^*)$
0.1	7.17	1.692	5.135	2.541	4.237	3.251
0.2	5.135	2.341	3.704	3.488	3.076	4.434
0.3	4.237	2.851	3.076	4.234	2.569	5.367
0.4	3.704	3.288	2.706	4.874	2.273	6.168
0.5	3.342	3.678	2.456	5.446	2.075	6.882
0.6	3.076	4.034	2.273	5.968	1.932	7.533
0.7	2.871	4.364	2.134	6.452	1.824	8.135
0.8	2.706	4.674	2.023	6.905	1.739	8.697
0.9	2.569	4.967	1.932	7.333	1.672	9.226
1.0	2.456	5.246	1.857	7.739	1.617	9.726

Table 4 and Table 5 compare the optimal T_C^* and its corresponding $C(T_C^*)$ given that $\theta = 0.1$ and $\theta = 0.5$, respectively, in which $c_m = 2$, $c_i = 5$, $c_c = 5$, $c_d = 20$, and $c_r = 10$.

From Table 4 and Table 5, it is clear that the optimal periodic inspection period T_C^* decreases with λ and q while the corresponding expected long run maintenance cost rate $C(T_C^*)$ increases with λ and q for a given θ. In addition, the optimal T_C^* decreases with θ.

VI. COMPARISONS

A. COMPARISONS BETWEEN POLICY A AND POLICY B

In this part, we compare the optimal T_A^* in the general periodic inspection model with T_B^* in the periodic inspection model

TABLE 5. Comparisons of T_C^* and $\widehat{C}(T_C^*)$ given $c_m = 2$, $c_i = 5$, $c_c = 5$, $c_d = 20$, $c_r = 10$, and $\theta = 0.5$.

$q(t)$	$\lambda = 0.1$		$\lambda = 0.2$		$\lambda = 0.3$	
	T_C^*	$C(T_C^*)$	T_C^*	$C(T_C^*)$	T_C^*	$C(T_C^*)$
0.1	6.835	1.759	4.891	2.638	4.034	3.37
0.2	4.891	2.438	3.524	3.626	2.924	4.603
0.3	4.034	2.97	2.924	4.403	2.438	5.576
0.4	3.524	3.426	2.569	5.071	2.154	6.411
0.5	3.178	3.832	2.329	5.667	1.963	7.156
0.6	2.924	4.203	2.154	6.211	1.825	7.836
0.7	2.727	4.548	2.019	6.716	1.72	8.465
0.8	2.569	4.871	1.912	7.189	1.638	9.053
0.9	2.439	5.176	1.825	7.636	1.571	9.606
1.0	2.329	5.467	1.752	8.06	1.517	10.13

with the consideration of quality warranty and their corresponding $C(T_A^*)$ and $C(T_B^*)$ under rational simplifications. By comparing Eq.(10) and Eq.(20), we have

$$L_{AB} = \frac{c_i}{\frac{c_d}{\lambda q} - c_r} - \frac{\lambda q(c_i + c_r)}{c_d} = \frac{\lambda q c_r (c_i + c_r - \frac{c_d}{\lambda q})}{c_d \left[\frac{c_d}{\lambda q} - c_r\right]} < 0. \tag{34}$$

Hence, for $0 < \lambda < \infty$ and $c_i + c_r - c_d/(\lambda q) < 0$, $T_A^* < T_B^*$. In addition, comparing Eq.(9) and Eq.(18), we have

$$\begin{aligned} &C(T_A^*) - C(T_B^*) \\ &= \frac{c_m\lambda(1-q)T_A^* + c_i - \left[\frac{c_d}{\lambda q} - c_r\right]\left(1 - e^{-\lambda q T_A^*}\right)}{T_A^*} + c_d - \frac{c_i}{T_B^*} \\ &\quad - c_m\lambda(1-q) - c_d + \frac{1}{\lambda q T_B^*}\left(1 - e^{-\lambda q T_B^*}\right) \\ &\quad \times \left[c_d - \frac{\lambda q c_r}{1 - e^{-N_B \lambda q T_B^*}}\right] \\ &= c_i\left[\frac{1}{T_A^*} - \frac{1}{T_B^*}\right] + \frac{c_d}{\lambda q}\left[\frac{1 - e^{-\lambda q T_B^*}}{T_B^*} - \frac{1 - e^{-\lambda q T_A^*}}{T_A^*}\right] \\ &\quad + c_r\left[\frac{1 - e^{-\lambda q T_A^*}}{T_A^*} - \frac{1 - e^{-\lambda q T_B^*}}{(1 - e^{-N_B \lambda q T_B^*})T_B^*}\right] \\ &< -c_i\left[\frac{1}{T_B^*} - \frac{1}{T_A^*}\right] - \left[\frac{c_d}{\lambda q} - c_r\right]\left[\frac{1 - e^{-\lambda q T_A^*}}{T_A^*} - \frac{1 - e^{-\lambda q T_B^*}}{T_B^*}\right] \\ &< 0. \end{aligned} \tag{35}$$

Which means that $C(T_A^*) < C(T_B^*)$ for the condition $c_d/(\lambda q) - c_r > 0$. From Table 1, Table 2 and Table 3, we see that $T_A^* < T_B^*$ and $C(T_A^*) < C(T_B^*)$ hold, which illustrates that both the optimal inspection interval and its corresponding maintenance cost rate increase with the consideration of quality warranty. That is, Policy A is better than Policy B.

B. COMPARISONS BETWEEN POLICY A AND POLICY C

In this part, we compare the optimal T_A^* in the general periodic inspection model with T_C^* in the random periodic inspection model and their corresponding $C(T_A^*)$ and $C(T_C^*)$ under rational simplifications. By comparing Eq.(10) and Eq.(33), we have

$$L_{AC} = \frac{c_i}{\frac{c_d}{\lambda q} - c_r} - \frac{c_i}{\frac{c_c\theta + c_d}{\lambda q} - c_r} = \frac{c_c c_i \theta}{\lambda q\left[\frac{c_d}{\lambda q} - c_r\right]\left[\frac{c_c\theta}{\lambda q} + \frac{c_d}{\lambda q} - c_r\right]} > 0. \tag{36}$$

Hence, for $0 < \lambda < \infty$ and $0 < \theta < \infty$, $T_A^* > T_C^*$. In addition, comparing Eq.(9) and Eq.(32), we have

$$\begin{aligned} &C(T_A^*) - C(T_C^*) \\ &= \frac{c_m\lambda(1-q)T_A^* + c_i - \left[\frac{c_d}{\lambda q} - c_r\right]\left(1 - e^{-\lambda q T_A^*}\right)}{T_A^*} + c_d \\ &\quad - \frac{[c_m\lambda(1-q) + c_c\theta]T_C^* + c_i - \left[\frac{c_c\theta + c_d}{\lambda q} - c_r\right]\left(1 - e^{-\lambda q T_C^*}\right)}{T_C^*} \\ &\quad - c_d = -c_i\left[\frac{1}{T_C^*} - \frac{1}{T_A^*}\right] - \left[\frac{c_d}{\lambda q} - c_r\right] \\ &\quad \times \left[\frac{1 - e^{-\lambda q T_A^*}}{T_A^*} - \frac{1 - e^{-\lambda q T_C^*}}{T_C^*}\right] \\ &\quad - c_c\theta\left[1 - \frac{1 - e^{-\lambda q T_C^*}}{T_C^*}\right] < 0. \end{aligned} \tag{37}$$

Which means that $C(T_A^*) < C(T_C^*)$ for the condition $c_d/(\lambda q) - c_r > 0$. From Table 1, Table 4 and Table 5, we see that $T_A^* > T_C^*$ and $C(T_A^*) < C(T_C^*)$ hold, which illustrates that the optimal inspection interval increases while its corresponding maintenance cost decreases with the consideration of random jobs. That is, Policy A is also better than Policy C.

VII. CONCLUSION AND FUTURE WORKS

Shocks on a single unit system are categorized into two distinct types. One type brings minor damage to the system and can be rectified by a minimal repair, and the other causes catastrophic damage and can only be removed by a corrective replacement. In this paper, we extend the classical periodic inspection policy into three advanced models, i.e., a general periodic inspection in which the system is checked at periodic time epochs over an infinite time span, a periodic inspection model with quality warranty in which the system is checked at periodic time epochs with an allowable inspection number, and a random periodic inspection model where the system is checked either at periodic time epochs or at random working times, whichever occurs first. The long run maintenance cost rate is minimized to seek the optimum replacement interval in each model analytically and examples are presented numerically to validate the theoretical results. Comparisons are made among the three models, showing that the optimal policy is $T_C^* < T_A^* < T_B^*$ under the same assumptions.

It is obviously evident that our models are not only applied to periodic inspections, but to aperiodic inspections where the system is non-periodically checked at $T_1, T_2, \cdots, T_k$ [35].

The future research emphasis is to extend the assumption to that where inspection may be imperfect. In addition, for degradation systems whose failure distribution is complex in terms of degradation-threshold-shock (DTS) theory [36]–[38], seeking a proper inspection policy analytically is another significant direction.

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