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Nondominated Maneuver Strategy Set With Tactical Requirements for a Fighter Against Missiles in a Dogfight

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ABSTRACT Dogfight is often a continuous and multi-round process with missile attacks. If the fighter only considers the security when evading the incoming missile, it will easily lose the superiority in subsequent air combat. Therefore, it is necessary to maintain as much tactical superiority as possible while ensuring a successful evasion. The amalgamative tactical requirements of achieving multiple evasive objectives in a dogfight are taken into account in this paper. A method of generating a nondominated maneuver strategy set for evading missiles with tactical requirements is proposed. The tactical requirements include higher miss distance, less energy consumption, and higher terminal superiority. Then the evasion problem is defined and reformulated into a multi-objective optimization problem, which is solved by a redesigned multi-objective evolutionary algorithm based on decomposition (MOEA/D). Simulations are used to demonstrate the feasibility and effectiveness of the approach. A set of approximate Pareto-optimal solutions satisfying the tactical requirements are obtained. These solutions can not only guide the fighter to avoid being hit but also achieve the goal of relatively reducing energy consumption and improving terminal superiority.

INDEX TERMS Dogfight, decision-making, evasive maneuvers, multi-objective evolutionary algorithm, tactical requirements.

I. INTRODUCTION

The core of operational idea in air combat is "Shoot down the opponent and protect yourself". With the extensive equipment and application of advanced air-to-air missiles (AAM), fighters are facing the increasing threat of high-precision AAM. How to minimize the lethality of enemy AAM through evasive maneuvers is an essential skill for the fighter in a modern dogfight. Besides, active and passive jamming [1], even defending missiles [2] are usually carried out in this process. This paper only studies the problem of evasive maneuvers, which is of great significance to improve the survival probability of the fighter.

The literature on air combat deals with two concepts. The first is "beyond-visual-range (BVR) air combat". It deals with situations in which magnitude and rhythm of maneuvers are relatively moderate and the engagement distance is far [3]. That is, BVR air combat emphasizes the distance game and the key of evasive maneuvers in these situations is tactical planning. The second is "dogfight", in which the main features are relatively close distance, high-dynamic, and intense confrontation [4]. There is more of an emphasis on the game of space angle. Therefore, a fighter against missiles in a dogfight should use its dynamic advantage over that of the missile to perform a successful evasion rather than planned strategy [5], [6].

To complete various air combat missions, survivability of the fighter is the basis and prerequisite. Corresponding to the long-term research on the precision guidance capability of missiles [7], the decision of evasive maneuvers of the fighter against missiles in a dogfight has also received widespread attention. Some common approaches such as numerical simulation [8]–[12], optimum control [13]–[17], differential games [18]–[21], and intelligent algorithms [22]–[25] have

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been applied to solve the problem of evasive maneuvers in a dogfight. The qualitative or quantitative (optimal or approximate optimal) maneuvering strategies are obtained for the security objective of the fighter.

Numerical simulation was often used to calculate and analyze the characteristics advantageous region of several common evasive maneuvers. The features of high-g barrel rolls (HGB) for a fighter and the countermeasures for a proportional navigation guidance AAM against this maneuver were presented in [8]. This study was extended in [9], and it was shown that the HGB can be regarded as a kind of approximation of open-loop optimal solutions. Akdag and Altilar [10] analyzed and compared the penetration effects of fighters between the maneuver combinations of Immelmann followed by a HGB and split-s followed by a HGB. Furthermore, the maneuver of evading out the seeker cone and maximum-lateral-acceleration turn were analyzed in [11] and [12], respectively.

Different from the open-loop solutions obtained by numerical simulation, the evasion problem can be solved by optimum control to obtain the closed-loop analytical solutions. Carr *et al.* [13] solved a one-sided optimal control problem to rapidly estimate the co-states of the opponent, which were then used as the initial guess to obtain a solution using the semidirect nonlinear programming method. The evasion problem for a fighter against two missiles simultaneously was investigated in [14], which was solved by an optimization method based on the most rapid descent. Karelahti *et al.* [15], Karelahti and Virtanen [16] studied a receding horizon control (RHC) scheme for obtaining near-optimal controls in a feedback form, and the controls related to the current state were computed online. In [17], a similar problem was solved using nonlinear model predictive control (NMPC). In NMPC, the control problem is formulated as a cost minimisation problem with input and state constraints, whose principle of computing the control is similarly to RHC.

The evasion problem in a dogfight also could be formulated as a differential game [18], which will provide game optimal controls of one side against the optimal controls of the opponent. Imado and Kuroda [19] proposed a method to find many solutions of comparable values to the differential game, that is, ''a family of local solutions''. For the constrained optimization problem of closing the distance to the target, while avoiding the threatening missile, the problem is formulated as a two-team zero-sum differential-game model between the navigator and an adversary coalition of the target and the missile in [20]. Moreover, for optimal evasive maneuvers, the optimal support time of a missile and the dynamic-escape-zone of a fighter were solved based on a differential game in [21] and [5], respectively.

However, the differential game scheme is only suitable for highly simplified problems due to the curse of dimensionality, and this motivates other studied approaches. An evolutionary flight path planning algorithm capable of mapping evasive trajectories was developed in [22]. The remarkable advantage of evolutionary algorithms is that it can solve optimization problems with the presence of nonlinearity, parameter discontinuity and discrete input. In [23], a quantum-behaved particle swarm optimization algorithm based on rolling optimization concept was used to solve a similar problem. In recent years, machine learning methods such as neural networks [24] and reinforcement learning [25] have also begun to be used to generate evasive maneuvers of fighters. In addition, some scholars studied the condition where the motion parameters of the incoming missile are unknown [26], and there was also some literature generalized the evasion problem in a dogfight into a general pursuit-evasion problem (see [27], [28]).

The literature mentioned above has given very reasonable results for the evasion problem, but evasive maneuvers of the fighter only considered the objective of security (i.e., miss distance) and ignored the effect of maneuvers on the attack mission. It is well-known that dogfight is often a continuous and multi-round process with missile attacks. The fighter often needs to ensure the ability and superiority to complete the attack mission while ensuring a certain degree of survivability. Hence, evasive maneuvers should take the whole battle efficiency and tactical superiority into account.

The fighter usually has multiple tactical objectives when confronting incoming missiles in a dogfight. such as higher miss distance, less energy consumption [29] and higher terminal superiority. Therefore, this problem that involves several objectives has no unique optimal solution but rather a set of approximate Pareto optimal solutions, which can exhibit different tactical requirements of the pilot [30]. Despite its crucial position in a modern dogfight, this specific problem has received less attention than it should in the open literature. This paper considers the amalgamative tactical requirements of achieving multiple evasive objectives in a dogfight. The evasion problem is defined and reformulated into a multi-objective optimization problem. Then a redesigned multi-objective evolutionary algorithm based on decomposition (MOEA/D) [31] is proposed to solve the problem. An attempt is made to generate a nondominated and feasible solution set (i.e., maneuver strategies) with tactical requirements for a fighter against missiles in a dogfight.

The structure of this paper is organized as follows: Section II presents the problem analysis and formulation, which consist of the dynamics and constraints of the fighter and the missile. Section III designs the optimization model of evasive maneuvers and its solution algorithm. Section IV simulates the proposed method and discusses the experimental results. Finally, Section V summarizes the conclusions and future work.

II. PROBLEM ANALYSIS AND FORMULATION

A. DESCRIPTION OF THE PROBLEM

Ensuring both tactical superiority and survivability in air combat is crucial for a fighter. Evasive maneuvers of the fighter only consider the objective of miss distance in traditional dogfight evasion problems, which will lead to a lower tactical superiority in the subsequent dogfight. As stated in the introduction, the essential prerequisite of the fighter

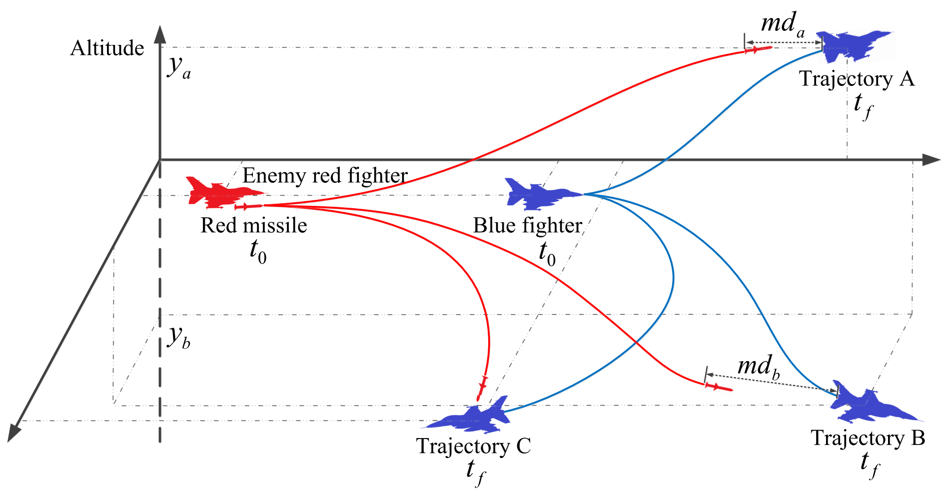

FIGURE 1. Scenario of an evasion problem in a dogfight.

threatened by AAM is to avoid being hit. On this basis, it is necessary to maintain as much tactical superiority as possible for the next round of attack mission.

This paper considers a typical scenario of an evasion problem in a dogfight as shown in Fig. 1. The opponent (i.e., enemy red fighter) launches the red missile toward the blue fighter (abbreviated as BF) at t_0. Then the BF starts evasive maneuvers against the incoming missile immediately after the launch. Fig. 1 shows three evasive trajectories performed by BF, trajectory A, B, and C. The strategy of A is mainly to climb, where the BF successfully evaded the missile with a miss distance md_a and a terminal altitude y_a at the terminal time t_f. And B also obtains a successful evasion by a dive strategy, where corresponding miss distance and terminal altitude are md_b and y_b, respectively. On the one hand, suppose that the velocity of BF at t_f in A is equal to that in B, then the terminal superiority (See details in Section III-B3) of A must be larger than that of B, due to $y_a > y_b$. But the process of A consumes much more energy than that of B, because the throttle of A needs to maintain a high thrust. On the other hand, under the same condition of energy consumption, the miss distance obtained by B must be larger than that of A, i.e., $md_b > md_a$, as a result of larger air drag for the missile at lower altitudes [6]. Of course, there are also many cases of failed evasion, such as trajectory C in which the BF performs a plane overloading turn and is finally hit by the missile.

It should be noted that Fig. 1 is just a schematic based on qualitative case descriptions, and there are innumerable analogous evasive trajectories, which have various results in respect of success or failure, miss distance, energy consumption, and terminal superiority. In terms of tactical significance, maximizing the miss distance means increasing the survival probability of the fighter. Decreasing energy consumption means more energy for subsequent multi-round combat, especially under the situation of jettisoning auxiliary fuel tank of the fighter. And maximizing the terminal superiority means a more superior state for the next round of missile duel.

According to the aforementioned analysis, the problem of evasive maneuvers in a dogfight that involves several tactical requirements has a set of Pareto optimal solutions. How to find these nondominated feasible strategies with different tactical requirements is the focus of this research. Firstly, dynamical models and constraints of the fighter and the missile in three-dimensional space are established in this paper, as well as simulation end conditions of the evasion problem. Then the details of the optimization model of evasive maneuvers strategy are presented, including objective function models based on tactical requirements. Finally, a redesigned MOEA/D is proposed to solve the model and obtain a nondominated maneuver strategy set.

To simplify the problem, the following assumptions without losing practicability are made.

1. The fighter can receive accurate state information of the missile;
2. The fighter can not evade the missile successfully without any maneuvers;
3. The damage range of the missile is a constant;
4. Ignore the change in the fighter weight and the earth's surface effect during the evasion.

B. ENGAGEMENT GEOMETRY

For unified modeling analysis on the confrontation process between the fighter and the missile, the engagement geometry relationship is established in three-dimensional space, as shown in Fig. 2.

The following coordinate systems are involved in Fig. 2, $Ox_gy_gz_g$, $O_mx_{gm}y_{gm}z_{gm}$, $O_mx_{tm}y_{tm}z_{tm}$, $O_fx_{gf}y_{gf}z_{gf}$, $O_fx_{tf}y_{tf}z_{tf}$, and $O_fx_by_bz_b$, which represent the geographic coordinate system, the concomitant inertial coordinate system of the missile, the trajectory coordinate system of the missile, the concomitant inertial coordinate system of the fighter, the trajectory coordinate system of the fighter, and the body coordinate system of the fighter, respectively. θ_f, ϕ_f, θ_m and ϕ_m are the flight path angle and the heading angle of the

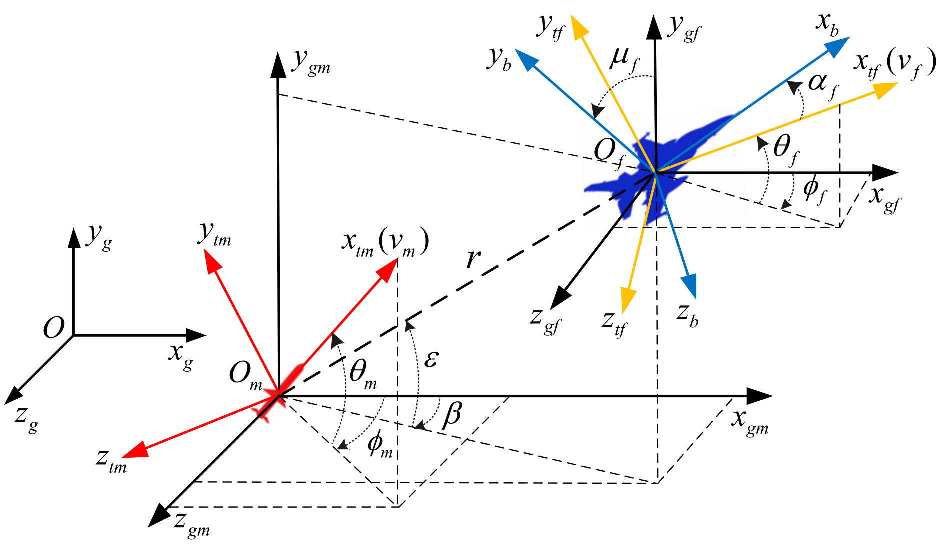

FIGURE 2. Engagement geometry relationship between the fighter and the missile.

fighter and the missile, respectively, where subscript f refers to the fighter and m to the missile (similarly hereinafter). The angle of attack α_f and the bank angle μ_f are used as the control of the fighter. Moreover, v_f is the fighter velocity vector. v_m is the missile velocity vector, which direction is assumed to be the same as that of the missile centerline. Relative motion state is usually set as $[r,\ \varepsilon,\ \beta]$, that is, the distance between the two sides, the inclination angle and the deflection angle of line-of-sight of the missile. Then the engagement geometry model could be described by

$$\dot{r} = \frac{x_r \dot{x}_r + y_r \dot{y}_r + z_r \dot{z}_r}{r} \tag{1}$$

$$\dot{\varepsilon} = \frac{(x_r^2 + z_r^2)\dot{y}_r - y_r(\dot{x}_r x_r + \dot{z}_r z_r)}{r^2\sqrt{x_r^2 + z_r^2}} \tag{2}$$

$$\dot{\beta} = \frac{\dot{z}_r x_r - \dot{x}_r z_r}{x_r^2 + z_r^2} \tag{3}$$

where

$$x_r = x_f - x_m \tag{4}$$

$$y_r = y_f - y_m \tag{5}$$

$$z_r = z_f - z_m \tag{6}$$

$$r = \sqrt{x_r^2 + y_r^2 + z_r^2} \tag{7}$$

and $x_f,\ y_f,\ z_f,\ x_m,\ y_m,\ z_m$ are corresponding position coordinates of the fighter and the missile in $Ox_g y_g z_g$.

C. FIGHTER MODEL

The fighter is modeled as a three-degree-of-freedom, point-mass model in three-dimensional, which constrained rotation kinematics are considered. The differential equations of motion are

$$\dot{x}_f = v_f \cos\theta_f \cos\phi_f \tag{8}$$

$$\dot{y}_f = v_f \sin\theta_f \tag{9}$$

$$\dot{z}_f = v_f \cos\theta_f \sin\phi_f \tag{10}$$

$$\dot{v}_f = \frac{1}{m_f}(\eta_f T_{f\,\max} \cos\alpha_f - D_f) - g\sin\theta_f \tag{11}$$

$$\dot{\theta}_f = \frac{\cos\mu_f(\eta_f T_{f\,\max}\sin\alpha_f + L_f)}{m_f v_f} - \frac{g}{v_f}\cos\theta_f \tag{12}$$

$$\dot{\phi}_f = \frac{(\eta_f T_{f\,\max}\sin\alpha_f + L_f)\sin\mu_f}{m_f v_f \cos\theta_f} \tag{13}$$

The lift force L_f and the drag force D_f are given by

$$L_f = 0.5\rho(y_f)v_f^2 S_f C_L(\alpha_f,\ M(y_f,\ v_f)) \tag{14}$$

$$D_f = 0.5\rho(y_f)v_f^2 S_f C_D(\alpha_f,\ M(y_f,\ v_f)) \tag{15}$$

where $\rho(y_f)$, S_f, $M(\cdot)$, $C_L(\cdot)$, and $C_D(\cdot)$ denote the atmospheric density, the reference wing area of the fighter, the Mach number, the lift coefficient, and the drag coefficient, respectively. Since the evasion time in a dogfight is generally brief, the gravitational acceleration g, the mass of the fighter m_f, and the maximum available thrust force $T_{f\,\max}$ are both assumed constant. Besides, the fighter is guided with α_f, μ_f and the thrust coefficient η_f. The thrust of the engine is therefore $\eta_f T_{f\,\max}$, which is directed parallel to the centerline vector of the fighter.

This maneuver control mechanism gets the fighter out of the constraints of the basic maneuver library, and the space of maneuver could be considered as a continuous space rather than restricted maneuver sequences. Furthermore, the command response process of the fighter is simplified as a first-order lag system, that is

$$\dot{\alpha}_f = (\alpha_{fc} - \alpha_f)/\tau_f \tag{16}$$

$$\dot{\mu}_f = (\mu_{fc} - \mu_f)/\tau_f \tag{17}$$

$$\dot{\eta}_f = (\eta_{fc} - \eta_f)/\tau_f \tag{18}$$

where τ_f is the time constant of the control system. The values of three flight control command parameters (i.e., decision variables), α_{fc}, μ_{fc}, and η_{fc}, are determined by the algorithm presented in this paper. Since the above model does not accurately describe the triaxial angular velocity motion of the fighter, the state and control parameters are constrained by following inequalities:

$$\alpha_{f\,\min} \leq \alpha_f \leq \alpha_{f\,\max} \tag{19}$$
$$\mu_{f\,\min} \leq \mu_f \leq \mu_{f\,\max} \tag{20}$$
$$0 \leq \eta_f \leq 1 \tag{21}$$
$$|\dot{\alpha}_f + \dot{\theta}_f \cos\mu_f + \dot{\phi}_f \cos\theta_f \sin\mu_f| \leq q_{\max} \tag{22}$$
$$|\dot{\mu}_f| \leq \dot{\mu}_{f\,\max} \tag{23}$$
$$|\ddot{\alpha}_f| \leq \ddot{\alpha}_{f\,\max} \tag{24}$$
$$|\ddot{\mu}_f| \leq \ddot{\mu}_{f\,\max} \tag{25}$$
$$y_{f\,\min} \leq y_f \tag{26}$$
$$\frac{L_f}{m_f g} \leq n_{f\,\max} \tag{27}$$

where the minimum angle of attack $\alpha_{f\,\min}$, the maximum angle of attack $\alpha_{f\,\max}$, the minimum bank angle $\mu_{f\,\min}$, the maximum bank angle $\mu_{f\,\max}$, the maximum pitch rate $q_{\max}$, the maximum roll rate $\dot{\mu}_{f\,\max}$, the maximum angular acceleration of the angle of attack $\ddot{\alpha}_{f\,\max}$, the maximum angular acceleration of bank angle $\ddot{\mu}_{f\,\max}$, the minimum altitude $y_{f\,\min}$, and the maximum available overload $n_{f\,\max}$ are assumed constant and depended on the characteristics of the fighter.

D. MISSILE MODEL

The main properties of the missile model are similar to that of the fighter. The dynamic equations of the missile are described by

$$\dot{x}_m = v_m \cos\theta_m \cos\phi_m \tag{28}$$
$$\dot{y}_m = v_m \sin\theta_m \tag{29}$$
$$\dot{z}_m = v_m \cos\theta_m \sin\phi_m \tag{30}$$
$$\dot{v}_m = \frac{1}{m_m(t)}(T_m(t) - D_m) - g\sin\theta_m \tag{31}$$
$$\dot{\theta}_m = \frac{g}{v_m}(n_{my} - cos\theta_m) \tag{32}$$
$$\dot{\phi}_m = \frac{n_{mz} g}{v_m \cos\theta_m} \tag{33}$$

where the mass of the missile $m_m(t)$ and the thrust $T_m(t)$ are functions of the missile's flight time t. $T_m(t)$ equals zero when t exceeds the maximum engine operating time of the missile t_p. The definition of the drag force D_m is the same as D_f, except that it will replace the corresponding aerodynamic parameters. n_{my} and n_{mz} are the overload of the missile in pitch and yaw channels, respectively, which are orthogonal to the velocity vector of the missile. The command response process is also designed as

$$\dot{n}_{my} = (n_{myc} - n_{my})/\tau_m \tag{34}$$
$$\dot{n}_{mz} = (n_{mzc} - n_{mz})/\tau_m \tag{35}$$

where τ_m is the time constant of the guidance system. According to the proportional navigation guidance scheme, which is stated as

$$n_{pny} = (N_m|\dot{r}|\dot{\varepsilon})/g + \cos\theta_m \tag{36}$$
$$n_{pnz} = (N_m|\dot{r}|\dot{\beta})/g \tag{37}$$

where n_{pny} and n_{pnz} refer to the required overload in pitch and yaw channels, respectively. N_m is the navigation constant and the term $\cos\theta_m$ is the compensation for gravity. Since n_{pny} and n_{pnz} cannot exceed the maximum achievable overload $n_{m\,\max}$, control commands n_{myc} and n_{mzc} are given by

$$n_{myc} = \frac{\min(n_{m\,\max}, n_m)}{n_m} n_{pny} \tag{38}$$
$$n_{mzc} = \frac{\min(n_{m\,\max}, n_m)}{n_m} n_{pnz} \tag{39}$$

where $n_m = \sqrt{{n_{pny}}^2 + {n_{pnz}}^2}$.

E. END CONDITIONS OF SIMULATION

Whether the evasive mission of the fighter in a dogfight is finished mainly depends on whether the incoming missile still has the capability of damage and guided flight. The actual damage space surface of the missile is very complicated due to its high-speed flight and explosive mechanics effect. Therefore, in this study, it is simply assumed that the damage range of the missile is an approximate sphere with a constant radius r_d and the explosive effect is maximized. In addition, the missile's guided flight capability is also limited by the maximum energy working time $t_{\max}$ and the minimum controllable flight velocity $v_{m\min}$. Under the condition that the initial states and parameters are determined, the evasive results of the fighter can be obtained through simulation. Besides, the differential dynamic equations of the fighter and the missile are integrated with a simulation time step Δt through the method of fourth-order Runge-Kutta. The results include the following possibilities.

1) THE FIGHTER IS HIT BY THE MISSILE

If there is a moment that satisfies the following conditions in the evasive process, then the missile is believed to have hit the fighter, that is, a failed evasion.

$$r(t) \leq r_d \quad and\ v_m(t) \geq v_{m\min}\ and\ t \leq t_{\max} \tag{40}$$

The fighter flying at altitudes exceed the altitude constraint can also result in a failed evasion, that is,

$$y_f < y_{f\,\min} \tag{41}$$

2) EVADE THE MISSILE SUCCESSFULLY

The missile will trigger the self-destruct sequence if the following conditions are satisfied before hitting the fighter.

$$t > t_{\max} \quad or\ v_m(t) < v_{m\min} \tag{42}$$

In the terminal phase of evasion, if

$$r(t_f) > r_d \tag{43}$$

then the fighter is also believed to have successfully evaded the missile, where the terminal time t_f is the time as the closing velocity equals zero, which is defined by

$$\dot{r}(t_f) = 0 \tag{44}$$

III. SOLUTION OF THE EVASION PROBLEM

A. OPTIMIZATION MODEL

The optimization model of evasive maneuvers for the fighter against the missile in a dogfight can be described by

$$J(u) = \varphi(x_f, t_f) + \int_{t_0}^{t_f} L(x, u, t)\, dt,$$

$$x = [x_f \;\; x_m]^T \tag{45}$$

$$\dot{x}_f = f_f(x_f, u), \quad x_f(t_0) = x_{f0} \tag{46}$$

$$\dot{x}_m = f_m(x_m, t), \quad x_m(t_0) = x_{m0} \tag{47}$$

$$g(x, u, \dot{u}, \ddot{u}) \le 0 \tag{48}$$

$$h(x_f) = 0 \tag{49}$$

where $u = [\alpha_f, \mu_f, \eta_f]^T$ is the control vector of the fighter, and the state vector of the system x consists of the state vector of the fighter $x_f = [x_f, y_f, z_f, v_f, \theta_f, \phi_f]^T$ and the state vector of the missile $x_m = [x_m, y_m, z_m, v_m, \theta_m, \phi_m, n_{my}, n_{mz}]^T$. x_{f0} and x_{m0} are the initial states of the fighter and the missile, respectively. t_0 denotes the initial time. Furthermore, the system models in (46) and (47) describing the dynamics of the aircrafts refer to (8)-(13) and (28)-(33). Constraints in (48) consists of (19)-(27). The terminal constraint is given in (43), and it should be noted that all end conditions in Section II-E will result in a terminal time.

In view of the multiple tactical requirements of the fighter against the missile in a dogfight, higher miss distance, less energy consumption, and higher terminal superiority, the optimization objective vector J in (45) is defined as

$$\min J(u) = (J_m(u), J_e(u), J_s(u)) \tag{50}$$

where $J_m(u)$, $J_e(u)$, and $J_s(u)$ are objective functions of miss distance, energy consumption, and terminal superiority, respectively, which are all designed as a minimized form in the next section.

B. EVASIVE OBJECTIVES BASED ON TACTICAL REQUIREMENTS

1) MISS DISTANCE

Miss distance is the critical performance measure to ensure the security of the fighter, which is defined as the distance between the fighter and the missile at the time of the closest approach, that is, $\dot{r}(t_f)= 0$. Therefore, the objective function of miss distance is of the form

$$J_m(u) = 1/r(t_f) \tag{51}$$

Note that if the fighter is hit by the missile, i.e., $r(t_f) \le r_d$, the calculation of miss distance is meaningless. Hence, $J_m(u)$ will be set as a large constant for punishment in this case.

2) ENERGY CONSUMPTION

The energy consumption of the fighter is proportional to the magnitude of the engine thrust, which is defined as $\eta_f T_{f\,\max}$ according to the dynamic equations of the fighter. Since $T_{f\,\max}$ is assumed as a constant, η_f directly reflects the energy consumption of the fighter. Therefore, the objective function of the energy consumption is designed as the average of η_f in whole process of the evasion, that is,

$$J_e(u) = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} \eta_f(t)\, dt \tag{52}$$

3) TERMINAL SUPERIORITY

Dogfight is often a process of continuous switching between attacking and evading missions. If the situation of the fighter is dominant at the terminal phase of the evasion, it will have a more superior state for the next round of missile duel. Hence, this is essentially a problem of situation assessment. As we all know, AAM will have a higher probability of successful attack under conditions of higher altitude. Besides, AAM launched by the fighter at a higher velocity will pose a greater threat to the enemy fighter. It can be seen that the terminal velocity and altitude of the fighter are the important effect factors of the terminal superiority. Therefore, this paper introduces the specific energy to measure superiority, which is expressed as

$$E_f = y_f + 0.5v_f^2/g \tag{53}$$

Based on this, the objective function of the terminal superiority is designed as

$$J_s(u) = \begin{cases} 0.1, & 2E_t \le E_f \\ 0.5E_t/(3E_f - E_t), & 0.5E_t \le E_f \le 2E_t \\ 1, & E_f \le 0.5E_t \end{cases} \tag{54}$$

where E_t is the specific energy of the enemy fighter, which calculation is based on the assumption that the enemy fighter flies horizontally at a constant velocity by its initial states.

C. REDESIGNED MOEA/D

As mentioned previously, the problem of evasive maneuvers with tactical requirements is defined and reformulated into a multi-objective optimization problem in this study. There is no prior definite preference (i.e., the definition of objective weights) for these requirements, and the choice of maneuver strategies needs to be based on real-time battlefield conditions and tactical preferences of the pilot, so a set of nondominated solutions is expected. Finding the complete Pareto optimal set and front of this problem is complicated and time-consuming. Even more significant is the generation of approximate and feasible nondominated maneuver strategy set. To solve the problem, this section redesigns the MOEA/D from the aspects of encoding mechanism, normalized aggregate function, offspring generation strategy and external population (EP) update.

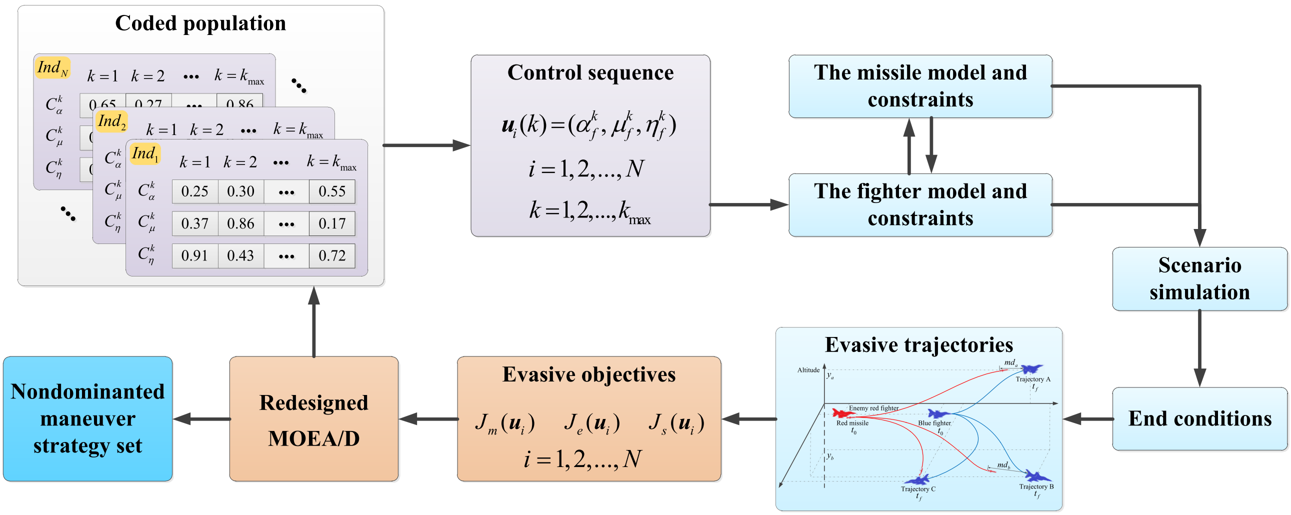

FIGURE 3. The schematic diagram of encoding and its evolutionary mechanism.

1) ENCODING MECHANISM

The search for a nondominated maneuver strategy set begins by randomly generating an initial population of feasible strategies, and an individual in the population reflects a set of the fighter control history over span of $t_{\max}$. In other words, the upper limit of the simulation time is set as the maximum energy working time of the missile, and that is also the maximum possible time for the missile to hit the fighter. Therefore, the decision vector of the fighter in the kth decision-making period can be expressed as $u(k) = (\alpha_f^k,\ \mu_f^k,\ \eta_f^k)$, $k = 1, 2, \ldots, k_{\max}$, $k_{\max} = ceil(t_{\max}/\Delta t_d)$, where Δt_d denotes the length of the period and $ceil(\cdot)$ is the function of rounding toward positive infinity. Because the real number coding has better performance in searchability and operability, the following transformations are used for coding of the angle of attack, the bank angle, and the thrust coefficient of the fighter.

$$\alpha_f^k = \alpha_{f\,\min} + C_\alpha^k \cdot (\alpha_{f\,\max} - \alpha_{f\,\min}),\quad C_\alpha^k \in [0,\ 1] \tag{55}$$
$$\mu_f^k = \mu_{f\,\min} + C_\mu^k \cdot (\mu_{f\,\max} - \mu_{f\,\min}),\quad C_\mu^k \in [0,\ 1] \tag{56}$$
$$\eta_f^k = C_\eta^k,\quad C_\eta^k \in [0,\ 1] \tag{57}$$

where C_α^k, C_μ^k, and C_η^k ($k = 1, 2, \ldots, k_{\max}$) are random number evenly distributed within [0, 1], that is, the corresponding encoded values for decision variables. Based on this, the schematic diagram of encoding and its evolutionary mechanism is shown in Fig. 3.

N individuals Ind_i ($i = 1, 2, \ldots, N$) encoding according to the above methods constitute the initial encoded population, and each individual Ind_i is decoded to obtain a set of maneuver control sequences of the fighter $u_i(k) = (\alpha_f^k,\ \mu_f^k,\ \eta_f^k)$. Based on the models and constraints of the fighter and the missile, as well as the end conditions of simulation, the corresponding evasive trajectories can be obtained by digital scenario simulation in three-dimensional space. Then the individuals will be evaluated to return the fitness results, i.e., $J_m(u_i)$, $J_e(u_i)$, and $J_s(u_i)$ ($i = 1, 2, \ldots, N$). The following redesigned MOEA/D is used to carry out the evolutionary operation on the population, and the cycles will be repeated until the maximum generation has been reached. Finally, the remaining excellent individuals will constitute the required nondominated maneuver strategy set with tactical requirements.

2) NORMALIZED AGGREGATE FUNCTION

According to the MOEA/D [31], the number of the subproblems is set as N, and $\lambda^1, \lambda^2, \ldots, \lambda^N$ are N different weight vectors, where each vector $\lambda^i = (\lambda_1^i,\ \lambda_2^i,\ \lambda_3^i)$ ($\sum_{j=1}^{3} \lambda_j^i = 1$, $\lambda_j^i \geq 0$, $i = 1, \ldots, N$) in this research. To convert the multi-objective optimization problem into a number of scalar optimization problems $\min g^i(u)$ ($i = 1, 2, \ldots, N$), the objective function of each subproblem is designed as an aggregate function of all objective components (i.e., $J_m(u)$, $J_e(u)$, and $J_s(u)$). The Tchebycheff decomposition approach is used to build the aggregate function in this paper. It can be seen from (51), (52), and (54) that there is a certain difference in the value range of the three objective functions. In order to avoid the influence of the difference on the algorithm performance, it is necessary to normalize the objective function value. Therefore, the scalar optimization problem is of the form

$$\min g^{te}(u|\lambda^i, z^*) = \max_{j=m,e,s} \left\{ \lambda_j^i \left| \frac{J_j(u) - z_j}{z_j^{\max} - z_j} \right| \right\} \tag{58}$$

where $z^* = (z_m^*, z_e^*, z_s^*)$ is the optimal reference point, $z_j^* = \min J_j(u)$ ($j = m, e, s$) is usually used as the approximate optimal reference point, $z_j = \min \{J_j(u) | u \in pop\}$, $z_j^{\max} = \max \{J_j(u) | u \in pop\}$, and pop refers to the population of current generation.

3) OFFSPRING GENERATION STRATEGY

The strategy of offspring generation has an important influence on the search performance of the evolutionary algorithm. Differential evolution (DE) and polynomial mutation were used to generate better quality new individuals in [32]. It should be noted that the difference information in DE operator is similar to the gradient, and it cannot effectively guide the algorithm search as the difference between randomly selected individuals is large. Therefore, the optimal individual is added to the differential evolution process in this paper, which will make the population approach towards the direction of the Pareto front more efficiently. Besides, the proposed strategy selects more parent individuals in the subproblem neighborhood individuals to recombine, which is conducive to the generation of diverse individuals.

Firstly, let Ind_i^{best} be the best individual in the current neighborhood $B(i)$ of the subproblem $ming^i(u)$, and Ind_i^j, Ind_i^k, Ind_i^l, Ind_i^m $(j, k, l, m \in [1, 2, \ldots N])$ are randomly selected individuals in the $B(i)$ (different from Ind_i^{best}). Then, each element $Ind'_{i,d}$ in the generated new individual $Ind'_i = (Ind'_{i,1}, Ind'_{i,2}, \ldots, Ind'_{i,3k_{\max}})$ can be expressed as

$$Ind'_{i,d} = \begin{cases} Ind_i^{best,d} + F_1(Ind_i^{j,d} - Ind_i^{k,d}) \\ \quad + F_2(Ind_i^{l,d} - Ind_i^{m,d}) \ \ with\ probability\ CR \\ Ind_i^{object,d} \qquad\qquad with\ probability\ 1 - CR \end{cases} \tag{59}$$

where $Ind_i^{object,d}$ is the object individual and CR, F_1, F_2 are control parameters of DE [32].

On this basis, the method of generating new offspring individual Ind_i^{new} through polynomial mutation is referred to [32], where the distribution index and the mutation rate are denoted as η and p_m, respectively. Furthermore, when the coded value in a new individual exceeds their corresponding boundary, it will be replaced by the boundary.

4) EP UPDATE

An EP is designed in the MOEA/D for retaining the nondominated solutions. However, the size of EP is limited by computing resources, the too large size of EP will greatly affect the calculation efficiency, so EP needs to be pruned and updated to maintain the distributivity of the nondominated solutions. EP will be pruned by the value of the crowding distance [33] between the nondominated solutions in this paper. The algorithm of EP update is illustrated as Algorithm 1, where Ind_i^{new} and N_e refer to the new offspring individual and the maximum number of individuals in EP, respectively.

5) PSEUDOCODE OF THE REDESIGNED MOEA/D

On the basis of the above algorithm description, the pseudocode of the redesigned MOEA/D is illustrated as Algorithm 2, where $G_{\max}$, N, $\lambda^1, \lambda^2, \ldots, \lambda^N$, N_b, N_r, N_e, δ, CR, F_1, F_2, η, p_m, $\{u_1, u_2, \ldots, u_{N_e}\}$, and $\{J(u_1), J(u_2), \ldots, J(u_{N_e})\}$ refer to the maximum generation, a set of weight vectors, the size of the neighborhood of each weight vector, the maximum number of parent individuals replaced by offspring individuals in the neighborhood, the maximum number of individuals, the probability of selecting parents from the neighborhood in EP, the control parameters in reproduction, the approximate nondominanted maneuver strategy set, and the approximate Pareto front, respectively.

Algorithm 1 EP Update

```
Input: Ind_i^new, N_e.
Output: EP.
if the size of EP does not exceed N_e then
    Add the Ind_i^new into the EP;
    Update the EP by the Pareto dominance relation, and
    remove all the individuals dominated by Ind_i^new from
    the EP;
else
    if Ind_i^new dominates some individuals in the EP then
        Remove all the individuals dominated by Ind_i^new
        from the EP and add the Ind_i^new into the EP;
    else if Ind_i^new is in a nondominated relationship with
    all members of the EP then
        Ind_i^new and all members of the EP are both
        performed for crowding distance calculation
        [33], and delete the individual with the smallest
        distance value;
    else
        Ind_i^new is dominated by all members of the EP,
        then discard Ind_i^new;
    end
end
Output the updated EP.
```

IV. SIMULATION AND ANALYSIS

In this section, simulations are used to demonstrate the feasibility and effectiveness of the proposed method in this paper. The first experiment simulates the evasive situations of the fighter controlled by an invariable maneuver strategy, and also verifies the models and performance parameters of the fighter and the missile. Then, based on the first experimental simulation scenario, the second experiment uses the algorithm proposed in this paper to control evasive maneuvers of the fighter against the missile, and a satisfactory set of maneuver strategies, as mentioned earlier, is obtained. Finally, the third experiment further validates the effectiveness of the proposed algorithm by comparing it with the other two classical algorithms.

A. SIMULATION SETTINGS

The model parameters of the fighter and the missile used in the following simulations correspond to a generic fighter and a generic short-range AAM.

The performance parameters of the fighter model are set as: $T_{f\max} = 54597N$, $S_f = 27.88m^2$, $m_f = 9298kg$, $\tau_f = 0.2s$, $n_{f\max} = 8.5$, $\alpha_{f\min} = 0°$, $\alpha_{f\max} = 60°$, $\mu_{f\min} = -180°$,

Algorithm 2 Redesigned MOEA/D

Input: $G_{\max}, N, \lambda^1, \lambda^2, \ldots, \lambda^N, N_b, N_r, N_e, \delta,$ CR, F_1, F_2, η, p_m.

Output: $\{u_1,\ u_2,\ \ldots,\ u_{N_e}\}$, $\{J(u_1),\ J(u_2),\ \ldots,\ J(u_{N_e})\}$.

For each weight vector $\lambda^i (i = 1, 2, \ldots, N)$, select the N_b nearest weight vector as its neighborhood $B(i) = \{i_1,\ i_2, \ldots, i_{N_b}\}$ based on the Euclidean distances between the other weight vectors, where neighborhood weight vectors of λ^i are denoted as $\lambda^{i_1},\ \lambda^{i_2},\ \ldots, \lambda^{i_{N_b}}$;

Generate an initial population $\{Ind_1,\ Ind_2, \ldots,\ Ind_N\}$ randomly, which will be decoded to obtain $pop = \{u_1,\ u_2,\ \ldots,\ u_N\}$ by (55)-(57). Set $F^i = J(u_i)$ $(i = 1, 2, \ldots, N)$, where $J(u_i) = (J_m(u_i),\ J_e(u_i),\ J_s(u_i))$;

Initialize $z = (z_m, z_e, z_s)$ by setting $z_j = \min\left\{J_j(u)|u \in pop\right\} (j = m, e, s)$;

Set EP as an empty set, and initialize the iteration counter $g = 0$;

for $g = 1$ *to* $G_{\max}$ **do**

  for $i = 1$ *to* N **do**

    Set $P_s = \begin{cases} B(i) & \textit{if rand} < \delta \\ \{1, 2, \ldots, N\} & \textit{otherwise}, \end{cases}$ where *rand* is a random number uniformly distributed in [0, 1];

    Randomly select four individuals from P_s to generate a new individual Ind_i' based on (59), then perform polynomial mutation on Ind_i' to generate the new individual Ind_i^{new};

    Use the boundary value to replace the new individual beyond its boundary;

    for $j = m, e, s$ **do**

      if $z_j > J_j(u_{new})$ **then**

        set $z_j = J_j(u_{new})$, where u_{new} is decoded from Ind_i^{new};

      end

    end

    Set $c = 0$, then perform the following steps;

    1) **if** $c = N_r$ *or* P_s *is empty* **then**

      Get out of this loop;

    else

      Select an index q randomly from P_s;

    end

    2) **if** $g^{te}(u_{new}|\lambda^q, z) \le g^{te}(u_q|\lambda^q, z)$ **then**

      Set $Ind_q = Ind_i^{new}$, $F^q = J(u_{new})$, and $c = c + 1$, where the normalized aggregate function is based on (58);

    end

    3) Remove q from P_s, and go to 1);

    Update the EP based on the Algorithm 1;

  end

end

Output the updated $\{u_1,\ u_2,\ \ldots,\ u_{N_e}\}$ and $\{J(u_1),\ J(u_2),\ \ldots,\ J(u_{N_e})\}$.

TABLE 1. The initial situation of the dogfight scenario.

	$x(km)$	$y(km)$	$z(km)$	$z(km)$	$\theta(rad)$	$\phi(rad)$
Fighter	10	9	20	250	0	$\pi/3$
Missile	0	9	20	250	0	$\pi/6$

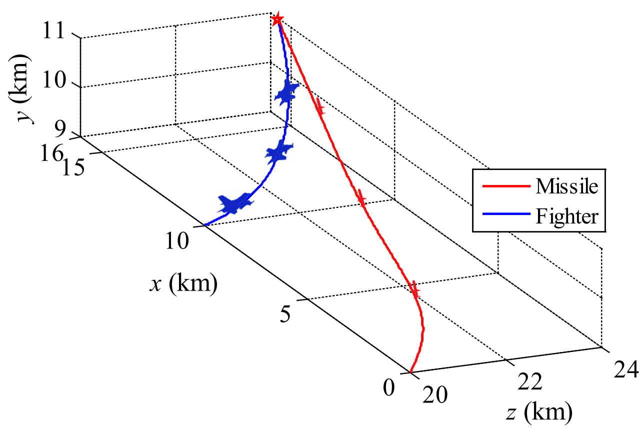

FIGURE 4. Three-dimensional trajectories under invariable maneuver strategy.

$\mu_{f\max} = 180°$, $q_{\max} = 25°/s$, $\ddot{\alpha}_{f\max} = 40°/s^2$, $\ddot{\mu}_{f\max} = 120°/s^2$, and $y_{f\min} = 0.5km$. The performance parameters of the missile model are set as: $m_m(0) = 100kg$, $T_m(0) = 15.6kN$, $t_p = 5.2s$, $\tau_m = 0.2s$, $N_m = 4$, $n_{m\max} = 50$, $t_{\max} = 30s$, $v_{m\min} = 400m/s$, $r_d = 12m$, and $\Delta t = 0.02s$. The parameters of the algorithm are set as: $\Delta t_d = 1s$, $N = 400$, $CR = 1.0$, $F_1 = 0.5$, $F_2 = 0.5$, $\eta = 20$, $p_m = 0.3$, $N_e = 200$, $G_{\max} = 2000$, $N_b = 40$, $N_r = 8$, and $\delta = 0.9$.

All simulation experiments were performed in MATLAB R2012a environment on a PC with Intel Core i7-2.5GHz CPU and 4GB memory.

B. SIMULATION EXPERIMENT 1

Firstly, setting a typical scenario of a dogfight, where the enemy fighter launches a short-range AAM to intercept the fighter and the fighter immediately begins to perform evasive maneuvers. The initial situation of the scenario consists of the initial states of the fighter and the missile, i.e., x_{f0} and x_{m0}, which are set as shown in Table 1.

According to the status in Table 1, the fighter and the missile have a relative distance $r = 10km$ and form an approximate tail-chase geometry. In this simulation experiment, the fighter is controlled by an invariable maneuver strategy in the whole process of evading, and the control vector is set as $u = [\alpha_f,\ \mu_f,\ \eta_f]^T = [5°,\ -30°,\ 1]^T$. The simulation results are shown in Fig. 4-Fig. 7.

The simulation experiment results show that the fighter is hit by the missile at 22.6s. As can be seen from Fig. 4 and Fig. 7, the evasive maneuver strategy of the fighter is to perform a slow climb and turn to the left to maneuver away from the incoming missile. The trajectory also conforms to the characteristic of the preset constant control

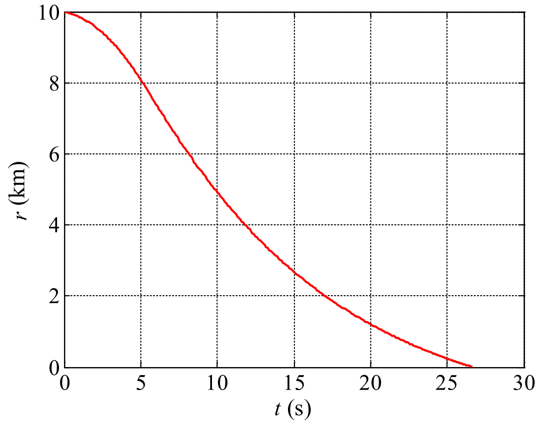

FIGURE 5. Time history of the relative distance.

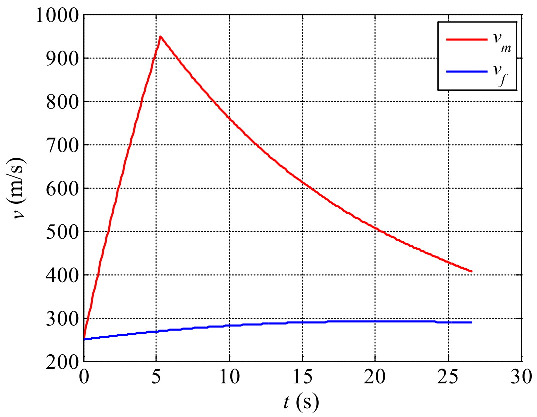

FIGURE 6. Time history of velocity of aircrafts.

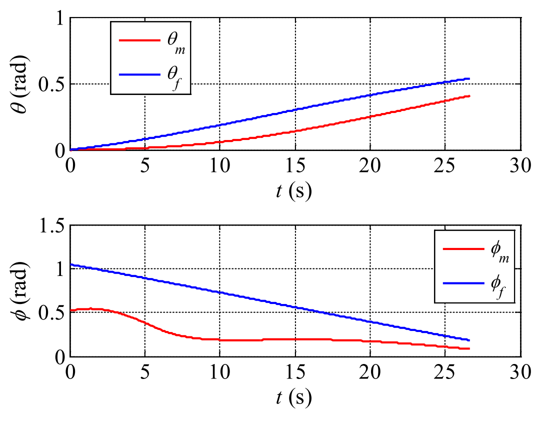

FIGURE 7. Time history of track angle of aircrafts.

vector. According to the preset thrust coefficient (maintain the maximum thrust), the fighter also accelerates to escape within aerodynamic constraints (see Fig. 6). The missile does not trigger the end conditions of $v_{m\min}$ or $t_{\max}$ during the process. Furthermore, the relative distance gradually decreases from $10km$ to $r < r_d$ (see Fig. 5).

This experiment designs and validates an initial scenario of a dogfight in which the fighter is located inside the guaranteed kinetic-capture zone [5] of the short-range AAM with general performance. In this case, the fighter controlled by

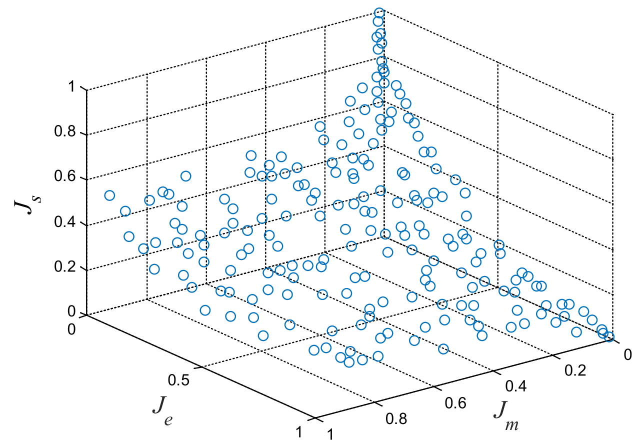

FIGURE 8. Approximate Pareto front of the evasion problem with tactical requirements.

an invariable maneuver strategy has a certain risk of being hit. Apparently, it makes no sense to consider tactical requirements such as energy consumption and terminal superiority if the fighter is hit. Besides, the models and performance parameters of the fighter and the missile are also validated primitively through this simulation experiment.

C. SIMULATION EXPERIMENT 2

To verify the feasibility and effectiveness of the proposed method in this paper, the initial scenario of a dogfight in this simulation experiment is the same with that of the simulation experiment 1, but the evasive maneuvers of the fighter are controlled by the proposed algorithm. Then a series of satisfactory results are obtained through this simulation.

1) NONDOMINATED MANEUVER STRATEGY SET

Based on the above simulation settings and the initial scenario, an approximate Pareto front of this problem is obtained, as shown in Fig. 8. It should be noted that the objective function values in Fig. 8 have all been normalized and mapped to the interval [0, 1], where the original values are $J_m \in [4.564 \times 10^{-4},\ 0.0827]$, $J_e \in [6.90 \times 10^{-3},\ 0.9659]$, and $J_s \in [0.1446,\ 1]$.

Each point of the approximate Pareto front in Fig. 8 represents a set of maneuver control sequences and its feasible evasive maneuver trajectory. These maneuver strategies have a nondominated relationship in the tactical requirements such as miss distance, energy consumption, and terminal superiority. The pilot can choose the appropriate maneuver strategy according to the tactical requirements in the current situation of a dogfight. The strategy can not only control the fighter to avoid being hit by the missile but also relatively reduce the energy consumption of the fighter and improve the terminal superiority. Furthermore, the experimental results also prove that there is no absolute optimal solution for evasive maneuvers in a dogfight, but a set of Pareto optimal solutions (i.e., maneuver strategies) under the actual multiple tactical requirements.

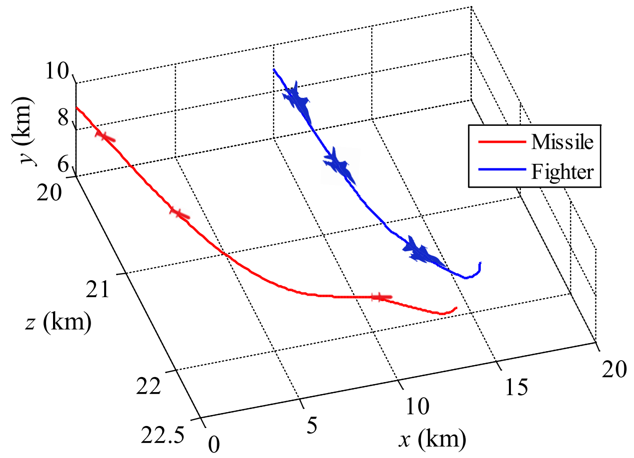

FIGURE 9. Three-dimensional trajectories under the strategy of optimal miss distance.

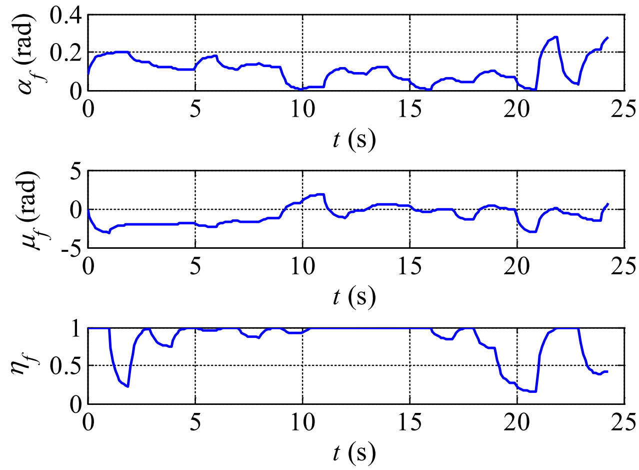

FIGURE 10. Maneuver control sequences for the strategy of optimal miss distance.

In order to further verify the effectiveness of the proposed method, the simulation results and analyses under the optimal conditions of each tactical requirements are presented below.

2) THE OPTIMAL MISS DISTANCE

To achieve the objective of optimal miss distance, the evasive maneuver strategy obtained by the algorithm in this research is shown in Fig. 9, which maneuver control sequences (i.e., the angle of attack, the bank angle, and the thrust coefficient) are shown in Fig. 10.

As can be seen from Fig. 9, the fighter mainly performs a dive strategy, using the large drag force at low altitude to effectively reduce the velocity of the missile, and making a sharp turn at the terminal phase to maximize the miss distance. According to the simulation results, since the incoming missile's flight velocity is less than the minimum controllable flight velocity at 24.3s (i.e., $v_m(t) < v_{m\min}$), the fighter obtains a successful evasion (see Fig. 15). Furthermore, the objective function values under this strategy are $J_m = 4.564 \times 10^{-4}$, $J_e = 0.6790$, and $J_s = 0.3057$, respectively.

3) THE OPTIMAL ENERGY CONSUMPTION

To achieve the objective of optimal energy consumption, the evasive maneuver strategy obtained by the algorithm is shown in Fig. 11, which maneuver control sequences are shown in Fig. 12.

As can be seen from Fig. 11, the maneuver trajectory of the fighter under this strategy is roughly similar to that under the optimal condition of miss distance, that is, the fighter also mainly performs dive maneuver. But this strategy controls the fighter to dive to a lower altitude, and the maneuver of the sharp turn is also cancelled. This is mainly for the purpose of minimizing energy consumption. Fig. 12 shows the approximate zero thrust state of the fighter over a long period of time. Moreover, the simulation results show that the fighter successfully evades the missile as it exceeds the constraint of velocity $v_m(t) < v_{m\min}$ at 24.6s (see Fig. 15). The objective function values under this strategy are J_m= 2.3048×10^{-3}, J_e= 6.90×10^{-3}, and $J_s = 0.5602$, respectively.

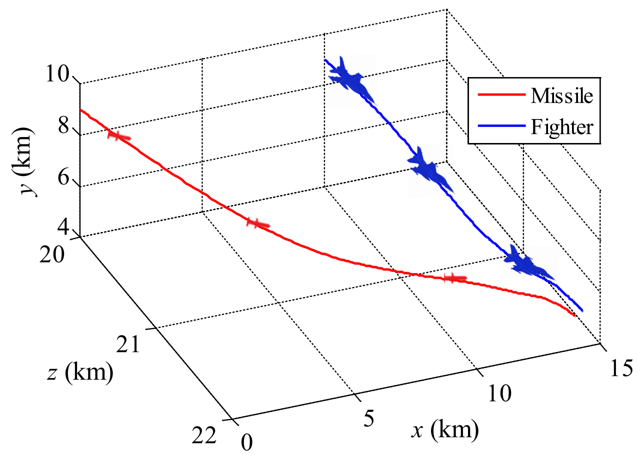

FIGURE 11. Three-dimensional trajectories under the strategy of optimal energy consumption.

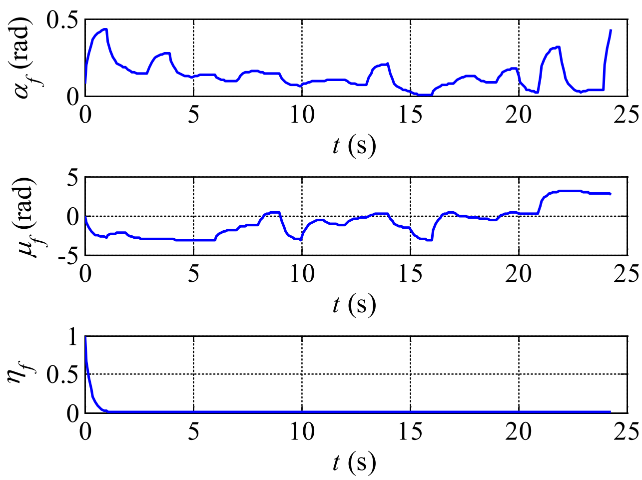

FIGURE 12. Maneuver control sequences for the strategy of optimal energy consumption.

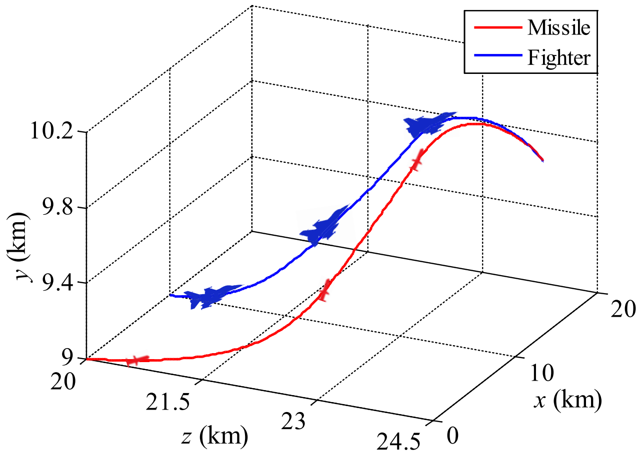

FIGURE 13. Three-dimensional trajectories under the strategy of optimal terminal superiority.

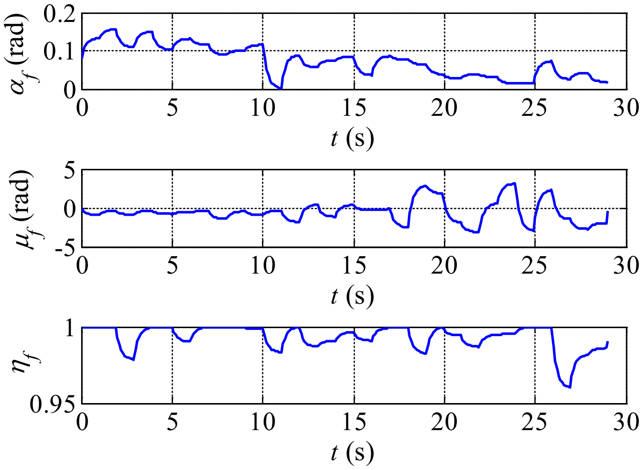

FIGURE 14. Maneuver control sequences for the strategy of optimal terminal superiority.

4) THE OPTIMAL TERMINAL SUPERIORITY

To achieve the objective of optimal terminal superiority, the evasive maneuver strategy obtained by the algorithm is shown in Fig. 13, which maneuver control sequences are shown in Fig. 14.

Similarly, the simulation results show that the incoming missile triggers the self-destruct sequence due to $v_m(t) < v_{m\min}$ at 29s (see Fig. 15). Fig. 13 shows that the fighter obtains a successful evasion mainly by the maneuver combinations of climbing followed by diving. The deceleration effect on AAM is less obvious due to the small drag force at high altitude, so the evasive time of the fighter under this strategy is longer than that of the above two strategies. But this strategy gives the fighter a larger terminal velocity and altitude, which means a larger terminal superiority. Besides, the objective function values under this strategy are $J_m = 6.1876 \times 10^{-3}$, $J_e = 0.9599$, and $J_s = 0.1446$, respectively.

The following is a comparative analysis of the velocity curves and the relative distance curves under the above three optimal strategies, as shown in Fig. 15 and Fig. 16, respectively.

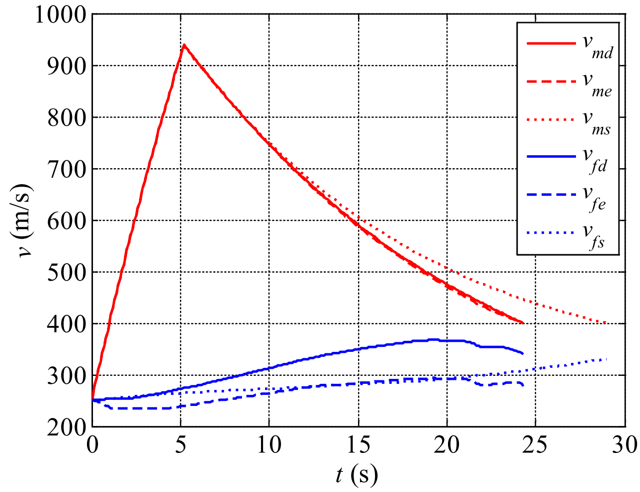

FIGURE 15. Comparison of velocity curves under the three optimal strategies.

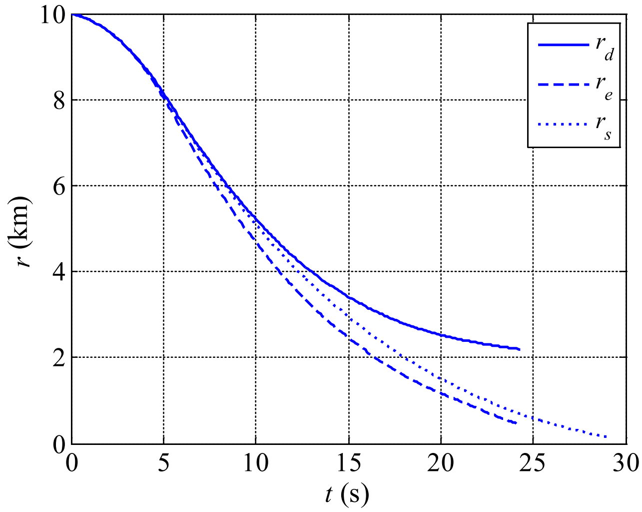

FIGURE 16. Comparison of relative distance curves under the three optimal strategies.

In Fig. 15, v_{md}, v_{me}, v_{ms}, v_{fd}, v_{fe}, v_{fs} refer to the velocity curves of the missile and the fighter under the conditions of optimal miss distance, optimal energy consumption and optimal terminal superiority, respectively. As can be seen from Fig. 15, the missile in v_{ms} has the smallest deceleration effect and the longest flight time (i.e., evasive time). The terminal velocities of v_{fd} and v_{fs} are both large and close to each other in value. The energy of the fighter in v_{fs} is mainly used for climbing and increasing velocity, while the energy of the fighter in v_{fd} is mainly used for maneuvering and increasing miss distance. Although their terminal velocities are roughly the same, the altitude superiority of the fighter in v_{fs} is more obvious (see Fig. 9 and Fig. 13). Besides, since the terminal velocity of v_{fd} is partly due to the contribution of gravity, its average thrust coefficient is less than that in v_{fs} (see Fig. 10 and Fig. 14). In comparison, the fighter in v_{fe} mainly relies on gravity to dive, and its terminal velocity and altitude are both low, but its energy consumption is also minimal.

In Fig. 16, r_d, r_e, r_s refer to the relative distance curves under the conditions of optimal miss distance, optimal energy

TABLE 2. Statistical data of CPU time (in hours) by the three algorithms.

	Max	Min	Mean	Std
R	2.06	1.99	2.03	0.0206
M	1.88	1.83	1.86	0.0193
N	2.96	2.87	2.92	0.0417

consumption and optimal terminal superiority, respectively. The curves show that the decrease rate of r_e is the fastest, which means the missile can approach the fighter faster. Furthermore, according to the simulation results, the miss distance of the three optimal strategies are $r_d(t_f) = 2191.06m$, $r_e(t_f) = 433.88m$, and $r_s(t_f) = 161.61m$, respectively. Hence, the fighter in r_d obtains the optimal miss distance, followed by r_e and r_s. In other words, a better objective function value for one tactical requirement will cause the other two objectives to be relatively poor. These results also further verify the contradictoriness of multiple actual tactical requirements and the non-dominance of evasive maneuver strategies, as well as the effectiveness of the proposed method in this research.

D. SIMULATION EXPERIMENT 3

In order to further validate the effectiveness of the proposed redesigned MOEA/D (abbreviated as R), the classical MOEA/D [31] (abbreviated as M) and NSGA-II [33] (abbreviated as N) are selected as comparison algorithms for test and analysis in this experiment. Where the distribution index of the simulated binary crossover and polynomial mutation are set as $\eta_c = \eta_m = 20$, and the crossover rate and mutation rate are set as $p_c= 0.9$, $p_m= 0.3$, respectively. The other relevant algorithm parameters are the same as those in Section IV-A. In this experiment, the three algorithms are compared and tested from two aspects of computation times and the quality of solutions. To reduce the impact of random errors on the statistical results, each algorithm experiment is independently executed 10 times. Furthermore, due to the complexity of the actual issues presented in this paper, the real Pareto optimal solution set is difficult to be obtained, so the performance indicators of spacing [34] and coverage [35] are used to evaluate the quality of solutions in this experiment.

Firstly, when solving the problem of the evasive maneuver strategy set in this paper, the statistical data of CPU time (in hours) of the three algorithms independently executed 10 times are shown in Table 2.

In Table 2, Max, Min, Mean, and Std respectively represent the maximum, minimum, mean, and standard deviation of CPU time of each algorithm executing independently for 10 times. It can be seen from Table 2 that the proposed offspring generation strategy and the EP update algorithm adopted by the redesigned MOEA/D increase a certain amount of computation, making its computation time slightly higher than that of classical MOEA/D, but the computational efficiency of these two algorithms is significantly higher than that of the NSGA-II.

TABLE 3. Statistical data of S value by the three algorithms.

	Max	Min	Mean	Std
$S(R)$	0.0404	0.0390	0.0396	4.02×10^{-4}
$S(M)$	0.0490	0.0471	0.0481	6.13×10^{-4}
$S(N)$	0.0423	0.0412	0.0417	3.92×10^{-4}

TABLE 4. Statistical data of C value by the three algorithms.

	Max	Min	Mean	Std
$C(R, M)$	58.50	48.20	53.25	2.9255
$C(M, R)$	16.50	9.00	12.60	2.0618
$C(R, N)$	38.00	29.50	34.55	2.2427
$C(N, R)$	22.50	16.00	19.25	1.8962

To evaluate the distribution uniformity of the approximate Pareto optimal solutions, the performance indicators of Spacing (abbreviated as S) is used to measure the standard deviation of the minimum distance from each solution to the other solutions [34]. The smaller the value of S is, the more uniform the solution set is. The statistical data of S value obtained by independently executing the three algorithms for 10 times are shown in Table 3.

In Table 3, $S(R)$, $S(M)$, and $S(N)$ respectively represent the S value of the approximate Pareto solution set obtained by the three algorithms. As can be seen from Table 3, the value of $S(R)$ is slightly less than $S(N)$, and the value of $S(M)$ is relatively large. In other words, the approximate Pareto solution set obtained by the redesigned MOEA/D has the most uniform distribution, followed by the NSGA-II, and the uniformity of the results obtained by the MOEA/D is relatively the worst.

In this experiment, the performance indicators of Coverage (abbreviated as C) is used to evaluate the dominant relationship between the two sets of approximate Pareto optimal solutions [35]. Assume that A and B are two sets of approximate Pareto optimal solutions obtained by different algorithms, then $C(A, B)$ is defined as the ratio of the number of solutions in B dominated by at least one solution in A to the total number of solutions contained in B. Therefore, if $C(A, B) > C(B, A)$, that means solution set A is superior to B. The statistical data of C value obtained by independently executing the three algorithms for 10 times are shown in Table 4.

In Table 4, R, M, and N respectively represent the approximate Pareto solution set obtained by the three algorithms. As can be seen from Table 4, $C(R, M) > C(M, R)$ and $C(R, N) > C(N, R)$, besides, the difference value between $C(R, M)$ and $C(M, R)$ is more significant than that of $C(R, N)$ and $C(N, R)$. In other words, the approximate Pareto optimal solution set obtained by the redesigned MOEA/D is slightly better than that of the NSGA-II, and the result obtained by the MOEA/D is relatively the worst.

Based on the above results of this experiment, compared with the classical MOEA/D and NSGA-II algorithms, although the computational efficiency of the proposed algorithm is slightly lower than that of the MOEA/D, the approximate Pareto optimal solution set obtained by the redesigned MOEA/D is relatively optimal and the distribution is also relatively uniform when solving the problem of evasive maneuver strategy set in this paper. Furthermore, the effectiveness of the proposed algorithm is further validated.

V. CONCLUSION AND FUTURE WORK

Dogfight is often a continuous and multi-round process with missile attacks. The fighter needs to ensure the ability and superiority to complete the attack mission while ensuring a certain degree of survivability. This paper studies nondominated maneuver strategy set with tactical requirements for the fighter against the missile in a dogfight. The amalgamative tactical requirements of achieving multiple contradictory evasive objectives are taken into account, such as higher miss distance, less energy consumption and higher terminal superiority. Therefore, there is no absolute optimal solution for this problem, but a set of approximate Pareto optimal solutions (i.e., maneuver strategies).

The evasion problem is defined and reformulated into a multi-objective optimization problem in this paper. Firstly, dynamical models and constraints of the fighter and the missile in three-dimensional space are established, as well as simulation end conditions. Then the optimization model including multiple objective functions is designed on this basis. The redesigned MOEA/D is proposed to solve the model, which is improved from the aspects of encoding mechanism, normalized aggregate function, offspring generation strategy and EP update. Finally, the feasibility and effectiveness of the proposed method are verified through scenario-based simulations. A set of nondominated evasive maneuver strategies are obtained, which can satisfy different tactical requirements of the fighter while ensuring security. The pilot can choose the appropriate maneuver strategy according to the tactical requirements in the current situation of a dogfight.

The proposed method can be used for off-line calculation in a variety of scenarios. Then the results could be loaded into the airborne computer for selecting online. Furthermore, the results could also be applied for the fighter in the daily air combat evasive tactics training to improve combat efficiency. Future research directions will mainly include considering the uncertainty of information, building more accurate models, enhancing algorithm efficiency, and improving objective functions.

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